The use of assessment outcomes to inform the teaching of mathematics

by

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ABSTRACT

The poor learner outcomes in the TIMSS assessment, the SACMEQ assessment, and the Grade 9 ANAs led to this study being conducted. The purpose of the study was to explore whether Grade 9 mathematics teachers' teaching can improve learner outcomes. This study therefore investigated the literature regarding mathematics teachers' classroom practices with an emphasis on teachers' Pedagogical Content Knowledge (PCK), Pedagogical Content Knowledge and Skills (PCK&S), and how these teachers used assessment outcomes to inform their teaching of mathematics.

In developing learners' conceptual understanding and higher-order thinking skills, mathematics teachers not only need subject matter knowledge, but also PCK and the skill to implement their planning efficiently during instruction. The conceptual framework for this study is based on Gess-Newsome's (in Berry et al., 2015) Model of Teachers' Professional Knowledge and Skills. Based on this, PCK was examined in the planning and executing of topic-specific instruction and as a skill when teaching this content to the learners for enhanced learner outcomes.

The research approach was qualitative and the research design was a case study. Two Grade 9 mathematics teachers from one school were selected through purposive sampling. The data were collected using a baseline test, three classroom observations, one semi-structured interview and a formative test. Both teachers used Direct Instruction in all their lessons, but proficiently used various representations when explaining the work, and integrated the topic into other mathematical topics and real-life scenarios. The teachers admitted that they did not usually make use of baseline tests to inform their teaching due to time constraints, but found it valuable during this endeavour. The findings from the two tests showed many learners still have the same misconceptions regarding the concepts of surface area, volume and capacity; continued to make the same typical mistakes in finding formulae for surface area and volume; and still found it difficult to convert between the SI units. There was, however, significant improvement in learner outcomes, but the positive outcomes regarding all typical mistakes and learner difficulties were still below 46%.

Key words: Classroom practice; integration, learner outcomes, mathematics teachers, pedagogical content knowledge, prior knowledge, teaching strategies, various representations.

EDITING CERTIFICATE

Exclamation Translations

To whom it may concern

The dissertation titled, "The use of assessment outcomes to inform the teaching of mathematics" has been edited and proofread as of 22 September 2017.

As a language practitioner, I have a Basic degree in Languages, an Honours degree in French and a Master's degree in Assessment and Quality Assurance. I have been translating, editing, proofreading and technically formatting documents for the past seven years.

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LIST OF ACRONYMS

ANA	Annual National Assessment
CAPS	Curriculum and Assessment Policy Statement
CK	Curriculum Knowledge
DBE	Department of Basic Education
FET	Further Education and Training
GET	General Education and Training
MCK	Mathematical Content Knowledge
PCK	Pedagogical Content Knowledge
Personal PCK	Personal Pedagogical Content Knowledge
Personal PCK&S	Personal Pedagogical Content Knowledge and Skill
SBA	School Based Assessment
SACMEQ	Southern and East African Consortium for Monitoring Educational Quality
SMK	Subject Matter Knowledge
TIMSS	Trends in International mathematics and Science Study
TPKB	Teacher Professional Knowledge Base
TPK&S	Teacher Professional Knowledge and Skill
TSPK	Topic Specific Professional Knowledge

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CHAPTER 1 AN OVERVIEW OF THE INQUIRY

1.1 INTRODUCTION

International comparative studies, such as the Trends in International mathematics and Science Study (TIMSS), provide reliable and timely data on the mathematics and science achievement of United States' learners compared to that of learners in other countries. TIMSS is conducted every four years on Grade 8 learners, but from 2003, South Africa together with Botswana and Honduras participated at Grade 9 level (Reddy et al., 2012). The TIMSS results (Long, 2007) indicated that South African learners perform poorly in mathematics due to being the lowest performing country of the 50 participating countries in 2003, not participating in 2007, and being ranked second last of the participating countries in 2011 and 2015. Although still scored second last, South Africa showed the highest improvement of all the countries. Another assessment, the Southern and East African Consortium for Monitoring Educational Quality (SACMEQ), assesses learners' outcomes on a regular basis to monitor the quality of teaching and learning. Of the 15 participating countries in 2011, South Africa was ranked 10^{th} (Ijeh & Nkopodi, 2013).

In 2012, a standardised summative assessment tool called the Annual National Assessment (ANA) was introduced in South Africa to measure the quality of languages and mathematics education in the General Education and Training (GET) band (Grades 0-9) in South Africa, with Grade 0 excluded from the ANA. The purpose of the ANAs is to help role players, subject facilitators, and officials in education to evaluate the performance of the education system and to improve it.

In 2013, Minister of Basic Education, Angie Motshekga made a statement that the diagnostic report, which was generated from the 2013 ANA results, had helped and guided the education sector on how to address challenges in the education sector (South African Government, 2014). These challenges included poor learning conditions, a lack of learning materials, ineffective principals, and unqualified and unmotivated teachers (AISA, 2014). In September 2014, the ANAs were written for the third time by Grade 1-6 learners and Grade 9 learners, with the results for Grade 9 mathematics being poor with a national average of 10.7% (DBE, 2014).

Based on the analyses of the results obtained from TIMSS, SACMEQ and the ANAs, two of the aspects that need to be investigated in the South African education system are teachers' classroom practices and their knowledge base. According to researchers (Berry, Friedrichsen & Loughran, 2015; Franke, Kazemi & Battey, 2007; Lampert, 2005; Stein, Engle, Smith & Hughes, 2008), the mathematics teachers' classroom practices are described in terms of: their knowledge of how to plan the mathematical content to be taught; how to teach the mathematical content effectively; how to use learners' prior knowledge in designing a lesson; their knowledge of how to teach difficult topics and how to rectify learners' misunderstandings. This specific knowledge that teachers need to make the mathematics comprehensible to the learners is known as Pedagogical Content Knowledge (PCK) (Botha, 2011). However, Brodie and Sanni (2014) conclude that teachers need both Mathematical Content Knowledge (MCK) and PCK to teach mathematics effectively. Thus developing content knowledge is pivotal as it is the core of PCK, a core that many teachers lack (Ball, Thames & Phelps, 2008; Brodie & Sanni, 2014).

Kriek and Grayson (2009) also report that in South Africa, in both mathematics and science education, there is cause for concern as "teachers have limited content knowledge, ineffective teaching approaches and unprofessional attitudes" (Kriek & Grayson, 2009, p. 185). Although there are many reports on the findings from international and national assessments, there is a lack of research with regard to Grade 9 mathematics teachers' PCK and the influence thereof on their learners' outcomes in mathematics. PCK forms the focus of this study, describing teachers' PCK in using learner outcomes from a baseline test to inform their teaching. The Department of Basic Education (DBE) (2014b) believes that by enhancing the quality of teaching and learning, learner performance may improve.

1.2 Rationale

I have been involved in the teaching of mathematics at a secondary school for 13 years and during that time, I was the Head of Department of mathematics for six years. I have thus experienced that Grade 9 mathematics can be a challenging subject as many learners have expressed the opinion that mathematics is a subject for clever learners. Therefore, Grade 9 mathematics teachers need special knowledge and skills to teach and make the subject comprehensible for learners. Teachers also play a significant role in getting learners motivated, involved and enthusiastic about mathematics, especially if these learners have had a negative experience with mathematics during their primary school years.

At the end of Grade 9, learners have to choose between mathematics and Mathematical Literacy for Grades 10-12, the latter being a less demanding subject. Learners who feel excluded from mathematics or perform poorly normally choose Mathematical Literacy, or are advised to rather choose Mathematical Literacy. Learners who already know in Grade 9 that they are going to choose Mathematical Literacy are not motivated and do not perform as they could. This may be one of the reasons why learners perform poorly in the ANAs. Personally, I value and acknowledge the ANA paper as a good assessment tool that could contribute to better learner outcomes. In studying learner performance in such assessments, it has been found that learners' performance may be influenced by prior knowledge and misconceptions, age, gender, race, language, motivation, and mental and physical abilities (Berry et al., 2015).

1.3 PROBLEM STATEMENT

The poor learner outcomes in the TIMSS assessment (Reddy et al., 2012), the SACMEQ assessment (AISA, 2014), and the Grade 9 ANAs (DBE, 2014a, 2014b, 2014c) led to this study being conducted. The SACMEQ assessment indicates that poverty is not a factor in poor learner outcomes in mathematics and science as South Africa was ranked 10th behind poor countries such as Kenya, Swaziland, Tanzania and Zimbabwe (Ijeh & Nkopodi, 2013). The findings from the SACMEQ and TIMSS 2011 assessments concluded that factors influencing the quality of mathematics and science education "are likely to be deeply rooted in the learner, national curriculum, subject matter and pedagogical flexibility of teachers" (Ijeh & Nkopodi, 2013, p. 463). In TIMSS 2015, South Africa was ranked second last out of 36 participating countries, but improved more than any other country. However, school teachers, principals, DBE subject specialists, learners and parents have shown concern over the poor outcomes of learners in mathematics and science. It has been posited that the problem lies in teachers having limited content knowledge and ineffective teaching approaches (Kriek & Grayson, 2009), which needs further investigation.

1.4 PURPOSE OF THE STUDY

This study therefore aimed to investigate the literature on mathematics teachers' classroom practices, exploring teachers' PCK and how they use assessment outcomes to inform their teaching of mathematics. Observing teachers' instructional strategies allowed an observation of their PCK with emphasis on their ability to address learners' conceptions and typical learner mistakes, as revealed in a baseline test. I furthermore wanted to see how they used this feedback to support learners in constructing new knowledge. This will have an impact on Grade 9 mathematics teachers' (and my own) PCK and PCK&S, particularly in terms of effectively using learners' outcomes from baseline tests in the planning and presenting of mathematics lessons.

1.5 CONCEPT CLARIFICATION

In the literature, there are various definitions or views of particular concepts. In this study, the following operational definitions (Table 1.1) will be used for the core concepts.

Table 1.1: Clarification of concepts used in this study

Concept	Explanation
ANA	Annual National Assessment (ANA). This was put in place for Grade 1-6 and Grade 9 by the DBE as a strategy to annually measure improvement in learner achievement (DBE, 2014).
Baseline assessment	Baseline assessment is used to determine the basic skills and knowledge levels of learners in a specific mathematical topic (DBE, 2012).
Classroom Practice	The interaction between Personal Pedagogical Content Knowledge (Personal PCK) and Personal Pedagogical Content Knowledge & Skill (Personal PCK&S); and how they influence each other (Berry et al., 2015).
Content knowledge	"Content knowledge, includes knowledge of the subject and its organizing structures" (Shulman, 1987, p. 2).

Concept	Explanation
Curriculum knowledge	Curriculum knowledge refers to knowledge of the goals, structure, scope, sequence and assessment of mathematics (Botha, 2011).
Diagnostic assessment	Diagnostic assessment informs the teacher about mathematical problem areas that learners may experience that can hinder their mathematical performance (DBE, 2012).
Formative assessment	Formative assessment informs the teaching and learning process, it is also known as assessment *for* learning (DBE, 2012).
Instructional strategies	"Teachers qualitative dimensions in the teaching and learning process" (Botha, 2011, p. 40). It involves specific teaching strategies, such as independent study, direct instruction, indirect instruction, interactive instruction and experimental learning that will guide the instruction that takes place in the classroom.
Integration	Three levels of integration with reference to mathematics: "Integration of the various components of mathematics; between mathematics and everyday real-world knowledge; and where appropriate, across learning areas" (Adler, Pournara & Graven, 2000, p. 3).
Learner outcomes	Demonstration of learners' knowledge and problem-solving skills.
Learners' prior knowledge	Prior knowledge refers to individual learners' correct conceptions and typical learner mistakes regarding particular mathematical concepts (Mavhunga, 2012).
Learning style	The way in which an individual learner will concentrate, process, evaluate, synthesise, absorb and retain new and difficult content.
Misconception	A mistaken idea or view resulting from a misunderstanding or faulty thinking on a concept.
PCK	The knowledge a teacher has that goes beyond mathematical content knowledge, which distinguishes a teacher from a subject specialist. Knowledge that a teacher needs to make the subject comprehensible for the learners. PCK is needed to understand

Concept	Explanation
	what learners do not understand and to rectify their misunderstandings (Botha, 2011).
Personal PCK	“Personal PCK is the knowledge of, reasoning behind and planning for teaching a particular topic in a particular way for a particular purpose to particular learners for enhanced learner outcomes (Reflection *on* Action, Explicit)” (Berry et al., 2015, p. 31).
Personal PCK&S	“Personal PCK&S is the act of teaching a particular topic in a particular way for a particular purpose to particular learners for enhanced learner outcomes (Reflection *in* Action, Tacit or Explicit)” (Berry et al., 2015, p. 31). Therefore, Personal PCK&S is to improvise in the classroom during instruction.
Pre-concepts	Pre-concepts are the mathematical concepts of particular topics prescribed in the curriculum that were taught in previous grades, which learners need to understand before moving on to new concepts in that particular topic (DBE, 2012).
Reflective practice	Reflective practice is a readiness to constantly evaluate and review your classroom practice in the light of new learning (Moon, 1999).
Representations	Various representations, for example, illustrations, analogies, explanations and demonstrations, enhance learners’ understanding of a concept. Representations can be used through written or oral language, diagrams, manipulatives, computers or calculators.
SACMEQ	Southern and East African Consortium for Monitoring Educational Quality is a Sub-Saharan regional systemic assessment that assesses learner outcomes in mathematics and science in order to monitor the quality of teaching (Ijeh & Nkopodi, 2013).
Summative assessment	Summative assessment is carried out after the completion of a mathematics topic, it is also known as assessment *of* learning (DBE, 2012).

Concept	Explanation
Systematic mistakes	Recurrent wrong responses methodologically constructed and produced across space and time (Luneta & Makonye, 2010).
Teaching strategies	Teaching strategies involve the principles and methods used by the mathematics teacher during instruction.
TIMSS	The Trends in International mathematics and Science Study is an international assessment that provides reliable and timely data on the mathematics and science achievement of United States' learners compared to that of learners in other countries (Reddy et al., 2012).
Typical learner mistakes	Mistakes are a reflection on the manner in which learners reason. They illuminate the processes through which learners attempt to construct their own knowledge (Olivier, 1989).
Unsystematic mistakes	Unintended, non-recurring wrong answers that learners can readily correct by themselves (Luneta & Makonye, 2010).

1.6 RESEARCH QUESTIONS

The following main research question guided this research project:

How are assessment outcomes used to inform the teaching of mathematics?

To answer the main question, the following sub-questions were asked:

1. What is the learners' prior knowledge, as revealed in a baseline test?
2. How do the outcomes of the baseline test inform the teachers' Personal Pedagogical Content Knowledge (Personal PCK)?
3. How do the outcomes of the baseline test inform the teachers' Personal Pedagogical Content Knowledge and Skills (Personal PCK&S), as demonstrated in their instruction?
4. What are the learners' outcomes after the teachers' teaching of the topic, as revealed in a formative assessment?

1.7 WORKING ASSUMPTIONS

The following are assumptions based on my own beliefs regarding the teaching of mathematics, the prior knowledge of learners, the instructional practises of teachers, and learner outcomes. These assumptions may have had an influence on how the study was conducted and the conclusions made.

Assumption 1: A teacher's ability to teach will positively impact the learners' outcomes.

Assumption 2: All learners have a prior knowledge base for each specific topic in mathematics.

Assumption 3: The better developed Personal PCK and Personal PCK&S that a teacher has, the more effective a teacher's classroom practice is, and the more likely it will positively impact on the improvement of learners' outcomes.

Assumption 4: An analysis of the outcomes of a baseline assessment will impact teachers' Personal PCK and Personal PCK&S, and will positively impact teachers' instructional strategies.

Assumption 5: The learners' outcomes will improve if teachers adapt their instruction to meet the learners' needs, as shown in the baseline assessment.

1.8 CONCEPTUAL FRAMEWORK

The conceptual framework for this study was adapted from Gess-Newsome's "Model of teacher professional knowledge and skill, including PCK and influences on classroom practice and student outcomes" (Berry et al., 2015, p. 31). The main concepts retained from this model were learner outcomes and a teachers' classroom practice. Classroom practice includes a teachers' Personal PCK and Personal PCK&S. Teachers' Personal PCK and Personal PCK&S are defined as follows (Berry et al., 2015, p. 31):

- "Personal PCK is the knowledge of, reasoning behind, and planning for teaching a particular topic in a particular way for a particular purpose to particular learners for enhanced learner outcomes (Reflection *on* Action, Explicit)."

- "Personal PCK&S is the act of teaching a particular topic in a particular way for a particular purpose to particular learners for enhanced learner outcomes (Reflection *in* Action, Tacit or Explicit)."

I used these concepts of teachers' Personal PCK and Personal PCK&S to guide the study in determining to what extent teachers use assessment outcomes to inform their teaching.

1.9 RESEARCH PARADIGM, ONTOLOGY, EPISTEMOLOGY AND METHODOLOGY

The research paradigm in this study was constructivist and interpretative; where reality is subjective and a product of the individuals involved. The reality in this case was the teachers' Personal PCK and Personal PCK&S, and how they could use assessment to inform their teaching to make the content more comprehensible for learners. The individuals involved were myself as the researcher, the Grade 9 mathematics teachers and the Grade 9 learners. One of the goals was to search for evidence that was valid and reliable in terms of the existing phenomenon.

The ontological assumption (what reality is) that underpinned this study was that of nominalism. The nominalist approach observes human behaviour and uses their words as data. The nominalist approach is often used where a large amount of qualitative data can be categorised (Athanasou & Maree, 2012). The qualitative paradigm concerns interpretation and finding meaning in the phenomenon. Merriam (2009) states that it concerns being "interested in understanding how people interpret their experiences, how they construct their worlds, and what meaning they attribute to their experiences" (p. 5). In this study, the teachers' instructions were observed to determine their Personal PCK&S, and through interviews their Personal PCK was explored. The emphasis was on how they dealt, among other things, with learners' prior knowledge and difficulties, different representations, learning strategies, and integration.

The epistemological assumption refers to the researcher's interaction with the participants. This study held an interpretive position where knowledge was used to interpret the data based on personal experience. I was thus subjectively involved in the study in order to become familiar with the phenomenon (Athanasou & Maree,

2012) as I observed how the teachers taught their mathematics lessons. I was also subjectively involved in conducting interviews with them to gain further clarification of their actions in the classroom. This also pertains to the methodology utilised to conduct the study (Ponterotto, 2005). Through these personal interactions with the teachers, I explored and interpreted the findings regarding teachers' ability to use learners' outcomes to inform their teaching.

In terms of the methodology, a qualitative research method was used. An exploratory case study design was used, which provided me with insight into and an understanding of the selected Grade 9 mathematics teachers' ability to use assessment to inform their teaching. Purposive sampling was used to select the participants for this research, who comprised two Grade 9 mathematics teachers and two classes for each (one high performing and one poor performing class each) at one public secondary school in the Tshwane North District (D3). This school was chosen based on performance in mathematics, the level of experience of the Grade 9 mathematics teachers, and the language of instruction. The data collection strategies that were used in this study were classroom observations, an individual semi-structured interview, and two tests, one baseline and one formative test on the topic of Surface area, volume and capacity of 3D objects. The tests, which were piloted at a similar school before being used in this study, were set by me and moderated by my supervisors.

1.10 VALIDITY AND RELIABILITY OF THE STUDY

Validity is "the development of sound evidence to demonstrate that the test interpretation matches its proposed use" (Creswell, 2012, p. 159). The validity of the study can be addressed through the data gathered, the participants, extent of triangulation and the objectivity of the research (Cohen, Manion & Morrison, 2011). Reliability is "a synonym for dependability, consistency and replicability over time, over instruments and over groups of respondents" (Cohen et al., 2011, p. 199). Reliability also refers to an instrument being precise, accurate and the ease with which other researchers can replicate it. The sample for this study was small and the observed lessons few, which influences the extent to which the sample was representative of the population. However, the multiple data collection strategies increased the validity and reliability of the study.

1.11 ETHICAL CONSIDERATIONS

Ethical clearance was requested from the Ethics Department at the University of Pretoria and the Gauteng Department of Education. The ethics application was submitted and approved by the Ethics Department at the University of Pretoria and the Gauteng Department of Education before the fieldwork was conducted. The application for ethical clearance addressed issues like the sensitivity level of the research activities, the research design and methodology, the participants involved, the data collection process, voluntary participation, informed consent, confidentially, anonymity and risk. Letters of consent were signed by the principals, teachers and parents and letters of assent were signed by the Grade 9 mathematics learners. In the letters of consent and assent the parties involved were informed about the purpose of the study, the strategies that were used during the research and information regarding confidentiality, anonymity and possible risks.

1.12 CHAPTER OUTLINE

Chapter 1: An overview of the inquiry

In Chapter 1, a broad overview was given of the study. The chapter started with an introduction, background, rationale and problem statement. This was followed by the purpose of the study, the clarification of the concepts that emerged from the literature review, the research questions and the working assumptions. A brief description was given of the conceptual framework, the research paradigm, ontology, epistemology and research methodology. The chapter concludes by addressing the validity and reliability of the study as well as the ethical considerations applicable to the study.

Chapter 2: Literature review

In Chapter 2, an in-depth literature review is given on international and national research that has been conducted with regard to assessment and classroom practice, especially mathematics teacher's knowledge, PCK, Personal PCK, Personal PCK&S and topic specific PCK. This chapter also outlines the conceptual framework that was used in the study, as well as an investigation on various aspects related to learner outcomes. The concepts used in the conceptual framework are fairly new and therefore this chapter also includes the gaps in the existing literature.

Chapter 3: Research design and methodology

In Chapter 3, the research design, selection of participants, the data collection strategies and the process of the data analysis and interpretation are discussed. The strengths and limitations of the study with regard to the research design and methodology are also highlighted.

Chapter 4: Presentation and discussion of the data

In Chapter 4, a detailed outline is given of the results of the study according to the conceptual framework and research questions. The different themes and sub-themes that emerged from the data collected, are also discussed. The findings are analysed and discussed according to the literature.

Chapter 5: Discussion of the findings

In this chapter, I discuss and then compare each teacher's classroom practice. This entails a general overview of the teachers' awareness and knowledge of pre-concepts; her use of the learner outcomes in general; and her instructional strategies used. The second part concerns the teachers' classroom practices with regard to three aspects: learner conceptions; typical learner mistakes; and learner difficulties. For each of these aspects, the learner outcomes from the baseline test; what the teacher did and did not do during instruction; what the teacher said in the interview and how the learner outcomes changed are discussed.

Chapter 6: Conclusions and recommendations

In Chapter 6, the study is summarised and the research questions are answered with reference made to the literature. The gaps in the existing literature are identified and the limitations and implications of the study are addressed. The chapter concludes with recommendations for further studies and a brief reflection on the study.

CHAPTER 2 LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

2.1 INTRODUCTION

In this chapter, a literature review is conducted on various researchers' findings to justify the research endeavour. The literature review focuses on the mathematics teachers' knowledge, the effective teacher and the reflective teacher, the mathematics teacher's classroom practices and different types of assessment. Finally, the literature review concludes with the conceptual framework used, which is based on theory obtained from the literature.

2.2 MATHEMATICS TEACHERS' KNOWLEDGE

In this section, I discuss some of the domains of mathematics teachers' knowledge, namely, Mathematical Content Knowledge (MCK); PCK, Personal PCK, and Personal PCK&S.

2.2.1 Domains of mathematics teachers' knowledge

Many studies (Hill, Ball & Schilling, 2008; Shulman, 1986) have been conducted to define teachers' knowledge. According to Shulman (1986), teachers' content knowledge is divided into three categories, namely, subject matter content knowledge (SMK), PCK; and Curricular Knowledge (CK). It was Shulman (1986) who introduced the concept of PCK, emphasising that it is specialised knowledge that a teacher needs that goes beyond mathematical content knowledge only.

Hill et al. (2008) based their categories of mathematical knowledge on Shulman's work. To conceptualise their ideas, they developed certain domains and sub-domains of teachers' knowledge. In the domain map below (Figure 2.1), it can be seen that they divided mathematical knowledge for teaching into two domains: Subject Matter Knowledge (SMK) and PCK. The domain of SMK consists of the sub-domains of common content knowledge, knowledge at the mathematical horizon, and specialised content knowledge. These three sub-domains lie outside of Shulman's conceptualisation of PCK. The right-hand side of the oval in Figure 2.1 is the domain of PCK, which is divided into the sub-domains of knowledge of content and learners, knowledge of content, and teaching and knowledge of the curriculum. These domains entail, among many other things, how to build on learners' thinking and learning and how to address learners' conceptions and typical learner mistakes.

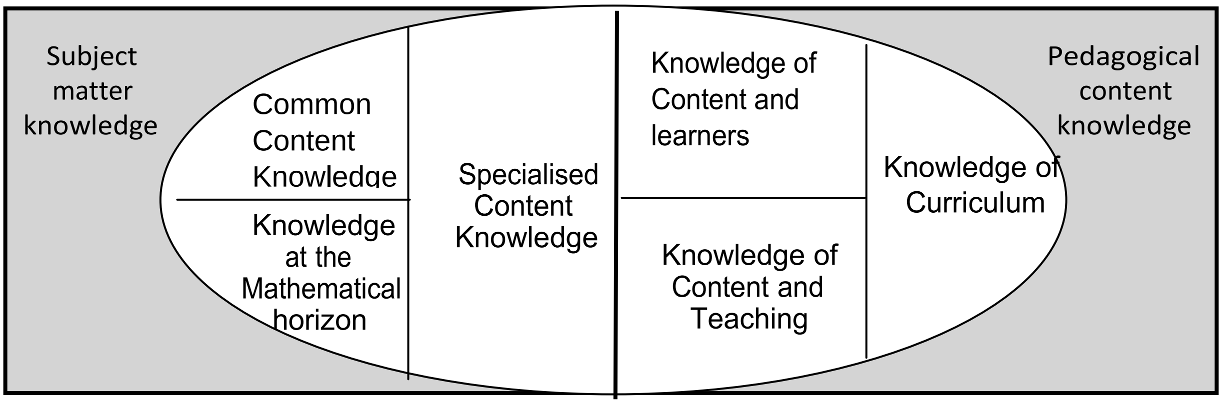


Figure 2.1: Domain map for mathematical knowledge for teaching (Hill et al., 2008)

Since this is a study of limited scope, I only focus on MCK, PCK, Personal PCK and Personal PCK&S in the rest of this study.

2.2.1.1 Mathematical Content Knowledge

Shulman (1986) stated that there are three categories of teacher knowledge: Subject Matter Knowledge (SMK), PCK and CK. In the last 30 years, various researchers have based their research on Shulman's notion of PCK (Ball, Thames & Phelps, 2005; Berry et al., 2015; Hill et al., 2008; Plotz, Froneman & Nieuwoudt, 2012). To be an effective mathematics teacher, a strong mathematical content knowledge base is a fundamental requirement (Fennema & Franke, 1992; Ijeh & Nkopodi, 2013).

MCK is required to identify and order mathematical concepts into teachable lessons in the classroom by using different representations (Shulman, 1986). A strong MCK will guide a teacher in preparing and presenting a lesson that will challenge the learners' cognitive thinking in order for them to become mathematically proficient. The teachers with a strong MCK will be able to use different instructional strategies, for example, illustrations, analogies, explanations and demonstrations in order to make the mathematical subject matter more comprehensible to the learners in their classroom (Hume & Berry, 2011). Teachers with a strong MCK will also be able to adapt or change their teaching based on the learners' knowledge of pre-concepts, the learners' prior knowledge, which includes the learners' conceptions and typical learner mistakes in mathematics. Effective teachers with a good MCK will therefore be able to present and align the learning of mathematical content and challenge learners' cognitive mathematical thinking (Ijeh & Nkopodi, 2013).

2.2.1.2 Pedagogical Content Knowledge

Shulman (1987) introduced PCK as one of three knowledge bases that teachers need. Shulman (1987) originally defined PCK as "the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learning, and presented for instruction" (p. 8), and "the particular form of content knowledge that embodies the aspects most germane to its teachability" (p. 9). Shulman (1987) described PCK as "the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful" (p. 15). Over 30 years, various researchers (Berry et al., 2015; Hill & Ball, 2004; Shulman, 1986) proposed models of PCK to guide their own thinking around PCK. One of the PCK models that was most often used for PCK was further developed by Magnusson, Krajcik and Borko (1999). Magnusson et al. (1999) believe that the teaching of science is influenced by numerous factors like teachers' curriculum knowledge, appropriate instructional strategies and the different types of assessments used by teachers. These factors will have an effect on the learners' understanding of the subject matter and, eventually, the outcomes achieved by the learners.

Hill et al. (2008) define PCK as teachers' content knowledge interwoven with knowledge of the curriculum, knowledge of teaching and knowledge of the learners. Therefore, PCK becomes evident when a teacher designs lesson plans, teaches learners in class, and reflects afterwards on their teaching. By using focused questioning during interviews, researchers can determine what the teacher planned for the learners and why they made certain decisions during the instruction of the learners (Berry et al., 2015). However, it is difficult to determine and measure a teacher's PCK. PCK as a theoretical construct is a hallmark in many disciplines of teaching, but its exact nature, the measurement thereof and how to improve teachers' PCK is still to be agreed upon (Mavhunga & Rollnick, 2013). Mavhunga and Rollnick (2013, p. 14) argue that "'a teacher is not just a teacher, but rather an 'English teacher' or a ‘Chemistry teacher.’” Therefore, effective teaching concerns the quality of teaching a specific topic.

During a 2014 summit on PCK, it became evident that too many ideas surrounded PCK. Gess-Newsome (Berry et al., 2015) refined the thinking around PCK and came up with the model of teacher professional knowledge and skills (TPK&S), including PCK. In this model, the Teacher Professional Knowledge Base is the first level and includes assessment knowledge, pedagogical knowledge, content knowledge, knowledge of learners, and curricular knowledge. Cochran-Smith and Lytle (1992) define these knowledge bases as knowledge for practice (Berry et al., 2015). The next level is Topic Specific Professional Knowledge, and includes knowledge of instructional strategies, various representations, the organising of content to build understanding, misconceptions, and how to integrate concepts of mathematics and science into real-life contexts (Berry et al., 2015). The first two levels focus on knowledge that is held and accessed by teachers. However, when teaching a subject, knowledge is personalised by the teacher. Thus, a teacher’s beliefs, the learners’ prior knowledge and the context of the classroom will influence a teachers’ teaching and these factors will act as filters in the teaching process.

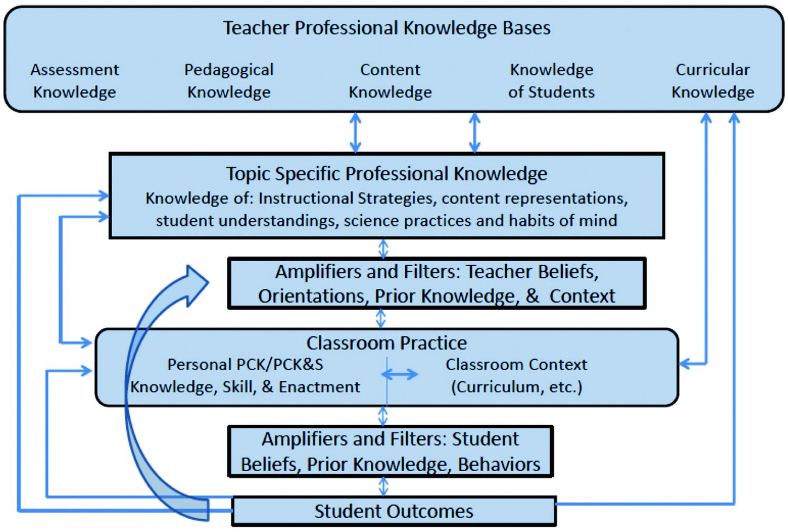


Figure 2.2: Model of Teacher Professional Knowledge and Skill, including PCK (Berry et al., 2015, p. 31)

The next levels of this model involve classroom practice with regard to the Personal PCK and Personal PCK&S, as well as learner outcomes. In this model, PCK is firstly defined as personal knowledge and not as a public knowledge base, as used earlier in the definitions of PCK. Therefore, the definition of Personal PCK was established in this model. Secondly, Personal PCK is context specific, Personal PCK is therefore the application of the personal knowledge that each teacher has to teach mathematics. Personal PCK can be seen in the lesson plans of the teacher and in their reflection before and after a lesson is presented. Therefore, Personal PCK is seen as Reflection on Action (Schon, 1983).

In this model Personal PCK&S was also introduced for the first time. Personal PCK&S can be observed in the classroom when a teacher attempts to carry out their lesson plans and while doing so pays attention to the learners' involvement and questions, and adjusts their classroom instruction based on the learners' reaction. Personal PCK can therefore be seen in the execution of a lesson plan in the classroom and on their reflection during the lesson. Therefore, Personal PCK&S is seen as Reflection in Action (Schon, 1983).

The model also takes into consideration amplifiers and filters such as student beliefs, prior knowledge and behaviours. The teachers and the learners can act as amplifiers and filters. It should always be borne in mind that teachers, on the one hand, interpret information, such as learners' outcomes, in different ways based on their teaching experience, their knowledge of content and the learners; which are referred to as teacher filters (Gess-Newsome, in Berry et al., 2015). Learners, on the other hand, also interpret instruction in different ways based on their prior knowledge and behaviour, among many other aspects, which is referred to as learner filters. The amplifiers and filters help us to understand that there is no direct relationship between the classroom instruction given by the teacher and the outcomes of the learners. Learner outcomes are the end result of the instruction that was given by the teachers, based on the teachers' Personal PCK and Personal PCK&S, and can therefore be seen as a learning opportunity for reflective teachers.

2.2.1.3 Personal PCK

PCK is defined by many researchers as the knowledge used to transform SMK into knowledge that is more comprehensible for learners (Shulman, 1986). Therefore PCK is used to adapt SMK for pedagogical purposes through a process that researchers call the following (Park & Oliver, 2008):

Shulman (1987)	“Transformation”
Ball (1987)	“Representation”
Veel and MaKinster (1999)	“Translation”
Bullough (2001)	“Professionalizing”
Dewey (1902/1983)	“Psychologizing”

PCK is the application of knowledge to teaching and is found in the instructional plans of teachers. Personal PCK, however, is the knowledge that teachers apply when reflecting on their instruction (Berry et al., 2015). Personal PCK is defined as ‘knowledge-on-action’, but also as knowledge through ‘reflection-on-action’ because it is a reflection after the teaching instruction has taken place. If teachers reflect on their actions, they will realise the need for elaboration or modification of their lesson planning on a particular topic. They may also adapt their repertoires used for teaching in their classrooms. The mathematics teachers will make additions, reorganise or modify their existing PCK on a specific topic. PCK that is developed and enacted on like this is stable and static (Park & Oliver, 2008).

As a result of the 2014 summit and contributions of the participants, consensus was reached around the definition of Personal PCK and the related constructed PCK&S. The following definition was given of Personal PCK (Berry et al., 2015, p. 36):

> Personal PCK is the knowledge of, reasoning behind and planning for teaching a particular topic in a particular way for a particular purpose to particular learners for enhanced learner outcomes (Reflection *on* Action, Explicit).

2.2.1.4 Personal PCK&S

Personal PCK&S is knowledge through ‘reflection-in-action’. From this definition, it can be seen that Personal PCK&S concerns the actions that take place *during* teaching, and Personal PCK is the action that takes place *after* instruction. Personal PCK&S becomes evident when teachers experience an unexpectedly challenging moment in class. Teachers need to transform this challenge into a teaching experience (Park & Oliver, 2008) using and applying all their PCK skills in their classrooms through appropriate instruction. Teachers need to observe the learners’ involvement and must be able to change their instruction based on what they observe from the learners (Berry et al., 2015). Most of the instruction that happens in the mathematics classroom is planned, but some action can happen unexpectedly and the teacher needs to improvise and with this reflection on action, the Personal PCK&S of the teacher will become evident. The development and enactment of Personal PCK&S is therefore active and dynamic.

2.3 THE EFFECTIVE MATHEMATICS TEACHER

There is a belief that what learners learn will depend on what teachers know and how effectively they teach. There is, however, a weak relationship between teacher content knowledge and the learners' outcomes (Berry et al., 2015). To understand mathematics, learners need to construct their own knowledge and ideas of the mathematical concepts (Ollerton & Watson, 2001). For learners to construct a coherent network of mathematical knowledge, the construction must be encouraged and guided by the teacher. Effective teaching is a skilled and purposeful endeavour that involves complex processes of pedagogical mathematical reasoning. This requires purposeful instruction in order to insure that learners learn (Shulman, 1987). For effective teaching, the teacher must therefore fulfil many roles in the classroom. The DBE (2003) states that there are seven roles of a teacher as outlined in the Norms and Standards for Educators: Learning mediator; Interpreter and designer of learning programmes and materials; Leader administrator and manager; Scholar, researcher and lifelong learner; Community, citizenship and pastoral role; Assessor; and Learning area/subject discipline/phase specialist.

In the past ten years, traditional teaching methods have undergone significant changes due to social, cultural and technological innovations. In order to develop an effective teaching style in the mathematics classroom, it is important for teachers to experiment with different styles and know how to engage with the learners in the learning process. The teacher should therefore use activities where the learners need to explore, interact and discuss their learning of new knowledge that they have constructed. The more cognitively demanding the task, the more learners will engage in more complex forms of thinking and reasoning and problem solving. McLaughlin and Talbert (1993) emphasise the importance of teaching learners higher-order thinking skills (Wenglinsky, 2001). In so doing, it is important to promote understanding rather than transfer information. Learners that are taught higher-order thinking skills will either learn new concepts to solve different problems, or will solve problems and learn the concepts afterwards (Blum, 2002). If learners understand the structure of mathematics, they will find it easier to recall, adapt and use mathematics. To further enhance learners' understanding, content can be presented in different ways such as the use of oral or written language, diagrams, tables, graphs, models, manipulatives, computers or calculators (Botha, 2011).

Kilpatrick (2001) developed the notion of mathematical proficiency, which involves five strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Conceptual understanding refers to the integrated and functional understanding of mathematical concepts and ideas. Procedural fluency refers to the skill learners have in order to carry out procedures in a flexible, accurate, efficient and appropriate way. Strategic competence refers to the ability of learners to formulate, represent and solve mathematical problems in the classroom as well as in real-life contexts. Adaptive reasoning refers to the capacity of learners to have logical thoughts about the mathematical concepts involved, to reflect, to explain and to justify their reasoning. Productive disposition is the inclination to see mathematics as sensible, useful and worthwhile, linked to the belief in diligence and one's own efficacy (NRC, 2001). These five strands are interwoven and interdependent.

2.4 THE REFLECTIVE MATHEMATICS TEACHER

Dewey (1933) introduced the concept of reflective thought, describing reflection as "an active and deliberative cognitive process which involves the sequence of interconnected ideas that take into account the underlying beliefs and knowledge" (Pedro, 2006, p.130). Teachers that have a reflective approach to their teaching will collect data about teaching and teaching practices and use the information to reflect on their own teaching and classroom practices before teaching, and/or after teaching (Schon, 1983).

Reflection *before* teaching (reflection on action), using Personal PCK, can be done when a teacher is planning a lesson to anticipate the possible outcomes of the lesson or how learners will act in the lesson. As a result of this reflection the teacher may adapt her teaching with regard to different modes of representation, strategies and integration.

Table 2.1: Reflection *before* teaching, and Personal PCK

	Reflection before teaching
1. Mathematics learners	Teacher's ability to reflect on what mathematics the learners will understand, but also understand why that mathematics is comprehensible to the learners; what mathematics the learners will not understand, but also understand why that mathematics is incomprehensible to the learners; how the learners will come to understand the mathematics; how learners will approach an activity; what pre-concepts learners will have; what prior knowledge learners will have.
2. Mathematics teaching	Teacher's ability to reflect on the use of various representations; the use of different instructional strategies; their own practices for the improvement of assessment outcomes; the integration with other mathematical topics, other subject areas and real-life contexts.

When the teacher teaches (reflection in action) for enhanced learner outcomes, the teacher's Personal PCK&S becomes evident. Personal PCK&S becomes evident when a teacher reflects *during* teaching, thus adapting their explanation during a lesson. This means the teacher has the ability to see what learners do, knows how to listen and hear what they think, and then is able to act appropriately as a mentor to facilitate the learning process.

Reflection can also take place *after* the lesson (reflection on action) by reflecting about the elements of and learners' reaction to the lesson. During the teaching practice of the 4th year student teachers at the University of Pretoria, the following reflection questions are asked of them after they present a lesson to reflect on their knowledge, reasoning and planning for teaching a specific topic.

1. What did I pay attention to during my planning that contributed to the success of my lesson?
2. What did I overlook or forget to pay attention to?
3. Did my introduction grab the learners' attention and link the new knowledge to their everyday lives?

4. Did my introduction progress according to my expectations; what could I have done differently?
5. Did my lesson progress according to my expectations?
6. What difficulties did I encounter during my lesson; what could I have done differently?
7. How did I establish whether, and to what extent, learners had achieved the lesson outcomes?
8. What did I do well and what could I improve on?

2.5 Classroom practice

The mathematics classroom is the primary venue where teachers and learners interact and where instruction is given on mathematical concepts and higher-order thinking skills. Researchers (Berry et al., 2015; Franke et al., 2007; Lampert, 2005; Stein et al., 2008) have different views on classroom practice. In this study, I adopt the idea of classroom practice that includes Personal PCK and Personal PCK&S, as described by Berry et al. (2015).

Lampert (2005) states that in every classroom, there should be key instructional activities or routines of practice. Teachers should learn these routines of practice in order to develop and expand the different domains of mathematical knowledge. Stein et al. (2008) propose five practices in a model to describe the actions of teachers: “anticipating likely student responses to rich mathematical tasks, monitoring learners’ responses to the tasks during the explore phase, selecting particular learners’ responses to present their mathematical responses during the discuss-and summarize phase, purposefully sequencing the learners’ responses that will be displayed, helping the class make mathematical connections between different learners’ responses” (Stein et al., 2008, p. 321). From these five practices, it can be seen that teachers need to make ample decisions and take action during and after teaching, which requires sufficient Personal PCK and Personal PCK&S.

Franke et al. (2007) focus on three key aspects of teachers’ classroom practice: discourse, norms and building relationships. Learners need to have opportunities in a specific classroom context to engage and interact around mathematical problems. Therefore, teachers must have good Personal PCK&S for the actions that take place *during* teaching or while solving mathematical problems in the classroom. Berry et al. (2015) describe the mathematics teacher's classroom practice in terms of the teachers' Personal PCK and Personal PCK&S (Berry et al., 2015). The learners' knowledge of pre-concepts, their prior knowledge, different modes of representations of mathematical concepts, different instructional strategies to teach topics, as well as the integration of mathematics into other subject areas influence teachers' instruction.

2.5.1 Learners' pre-concepts

Learners construct their own mathematical concepts, ideas and principles. When learning takes place, the existing knowledge on pre-concepts is reorganised, expanded or modified. To understand new mathematical concepts, learners need to structure existing knowledge on pre-concepts and ideas. When observing teachers' instruction, it is possible to see whether opportunities are given to learners to construct new ideas. For teachers to efficiently plan the instruction, they need to be acquainted with the mathematics curriculum. This knowledge is referred to as Curriculum Knowledge (CK), referring to knowledge of the syllabus designed for the teaching of different topics in mathematics for the preceding and following years. Teachers must also understand the sequencing and progression of topics in the curriculum (DBE, 2003, 2012).

2.5.2 Learners' prior knowledge

By asking questions and building on learners' responses, the teacher can determine the learners' thinking and prior knowledge, and build instruction on that. To become mathematically proficient, learners need to restructure their prior knowledge by engaging with the mathematical problem, examining, questioning, conjecturing, experimenting and justifying their answers. These processes must be adequately facilitated by the teacher using their Personal PCK&S (Kilpatrick, 2001; Stein et al., 2000). Some topics or concepts are difficult for learners to understand and the teacher must be able to identify those and do their planning accordingly. When these tasks are carefully designed and selected, the learners can connect new knowledge to prior knowledge through actively engaging with the problem (Botha, 2011). Due to the reasoning required, the outcomes of high-level tasks are sometimes less predictable and unintended outcomes can occur.

2.5.3 Various representations

When teaching new or difficult topics, it is crucial for the teacher to plan and use as many different representations to enable learners to understand the content. The teacher needs provide background materials such as written work, oral work, visuals (such as textbooks, newspaper articles), electronic media, posters, video material, pictures, traditional stories or models to aid learning. By observing, reading, listening and doing, learners restructure prior knowledge and gain the new knowledge (Väyrynen, 2004). The teachers must be able to adjust their representations and instructional strategies for better understanding and to eliminate typical learner mistakes.

2.5.4 Instructional strategies

Different instructional strategies can be used in the mathematics classroom to make the curriculum more accessible to learners and to provide every learner the opportunity to reconstruct their knowledge. In Figure 2.3 below, different instructional strategies are given that can be used in the mathematics classroom. The four key instructional strategies are: independent study, direct instruction, interactive instruction and experiential learning (Keesee, 2014).

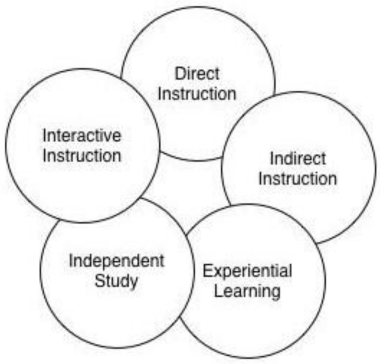


Figure 2.3: Different instructional strategies (Keesee, 2014)

Independent study refers to an instructional strategy where an individual can study on their own. The direct instruction strategy is teacher-centred and is most commonly used by teachers. The direct instruction strategy includes lecturing, didactic questioning, explicit teaching, practice, and drill and demonstrations. Interactive instruction refers to strategies that rely on discussions and sharing ideas. Experiential learning is learner-centred and the focus is on activities. Reflection about learning and applying the acquired knowledge to other contexts are the keys in experiential learning.

2.5.5 Integration

In the South African curriculum, the key driving principle is integration (Adler, Pournara, & Graven, 2000). Teachers need to implement integrated teaching into their lessons. However, teachers do not always have the required knowledge and some have to adopt a new pedagogical approach to teaching and learning (Mwakapenda & Dhlamini, 2010). Integration can be interpreted in various ways and is thus not well defined. However, Adler, Pournara and Graven (2000) have identified three levels of integration with reference to mathematics: "integration of the various components of mathematics; between mathematics and everyday real-world knowledge; and where appropriate, across learning areas" (p. 3). Adler et al. (2000) argue that integration is desirable, but the extent of the demands placed on mathematics teachers makes integration in the classroom extremely difficult.

2.6 Assessment

The foundation of effective teaching is built on an integrated structure of curriculum, instruction and assessment (Gareis & Grant, 2015). When teachers understand the integrated structure of curriculum, instruction and assessment, they will be effective teachers (Marzano, 2003). Without assessment, teaching will only focus on the teachers' input and classroom instruction. According to Gareis and Grant (2015), there are three fundamental roles of assessment, namely, pre-assessment (prior to learning); formative assessment (integrated into the act of teaching); and summative assessment (at the end of instruction).

The purpose of assessment is to identify the needs of learners, to plan learning, to decide where the learning programme should start, track learners' progress, identify learners' difficulties, help learners improve their knowledge, adjust the focus and pace of teaching and learning, provide feedback to all the role players, review the effectiveness of the curriculum, and to assess and reflect on own teaching (Departement of Education, 2002). Four steps can be identified in the assessment process: “generating and collecting evidence of achievement; evaluating this evidence; recording the findings and using this information to understand and thereby assist the learner’s development in order to improve the process of learning and teaching” (DBE, 2012, p. 154). The various methods of assessment used to measure the performance of learners will now be discussed.

2.6.1 Informal and formal assessment

Informal assessment is used for formative purposes, assisting mathematics teachers with their daily lesson planning, such as the choice of representations to use during classroom instruction, an allowing them to make professional judgements. Formal assessment provides learners the opportunity to demonstrate their proficiency in mathematics and is thus used collectively over the academic year to measure the mathematic learners’ competencies and progress. In Table 2.3, a summary is given of the different types of informal and formal assessments.

Table 2.2: Different methods of informal and formal assessments (DBE, 2012)

INFORMAL ASSESSMENT	FORMAL ASSESSMENT
Observations	Tests
Discussions	Examinations
Informal classroom interactions	Assignments
Practical demonstrations	Investigations
Learner-teacher conferences	Projects
Self-assessment and peer assessment	

Assessment should be appropriate to the cognitive level and age of the learner. The four cognitive levels that formal assessment should adhere to are: knowledge; routine procedures; complex procedures; and problem solving (DBE, 2012). In mathematics, baseline assessment, formative assessment, summative assessment, diagnostic assessment and systemic assessment are very useful (DBE, 2012). In this study, the preference was formal assessment in the form of a baseline test and a formative test.

2.6.1.1 Baseline assessment

Baseline assessment establishes the ‘starting point’ of the student's understanding. Baseline assessment, as described by the DBE (2012), is used by teachers to establish whether the learners have the necessary skills and prior knowledge required to learn a specific topic in mathematics. Knowing the learners' level of mathematical proficiency, the teacher can plan instruction accordingly to meet the learners' needs and to address the identified misconceptions and mistakes. Baseline assessment is therefore used at the beginning of a new learning experience, for example, a new grade, phase or new content to determine the learners' prior knowledge.

2.6.1.2 Formative assessment

Formative assessment is used to aid the teaching and learning process, and is thus assessment for learning. It is a continuous process of reflection to give constant feedback to the learners. Formative assessment can also be used by teachers to inform or adapt their instruction (DBE, 2012) and can be used in various forms at any given time during a mathematics lesson such as short class work during or at the end of a lesson or verbal questioning during a lesson.

2.6.1.3 Summative assessment

Summative assessment is also known as assessment of learning. It serves the purpose of gathering information on learner outcomes to improve their learning. Summative assessment is done after the completion of a topic or certain level in mathematics; normally at the end of a term or year and result in an overall assessment of the learners' outcomes at that moment in time. A judgement is made based on the learners' outcomes in tests, half year or final year examinations.

2.6.1.4 Diagnostic assessment

Diagnostic assessment is similar to formative assessment, but involves intervention or remedial processes in order to assist learners' understanding and to address typical learner mistakes.

2.6.1.5 Systemic assessment

In South Africa, systemic assessment is used to monitor the education system through an external body, for example, the Department of Education. Learners' outcomes are measured nationally or provincially with different measuring instruments, for example, the ANA in Grade 9 and the Senior Certificate in Grade 12.

2.7 CONCEPTUAL FRAMEWORK

The conceptual framework for this study was adapted from Gess-Newsome's (Berry et al., 2015) Model of Teacher Professional Knowledge and Skills (TPK&S), as discussed as discussed in Section 2.2.1.2 in this chapter and presented in Figure 2.2. The use of this conceptual framework was based on the fact that I wanted to explore teachers' classroom practices in order to see how they used learner outcomes to improve their teaching. The role of PCK is crucial in this, and thus I adapted the concepts of Personal PCK and Personal PCK&S (Berry et al., 2015, p. 36) in this study. Classroom practice in terms of the teacher's PCK and PCK&S, as well as the role of learner outcomes are now discussed, as presented in Figure 2.4 below.

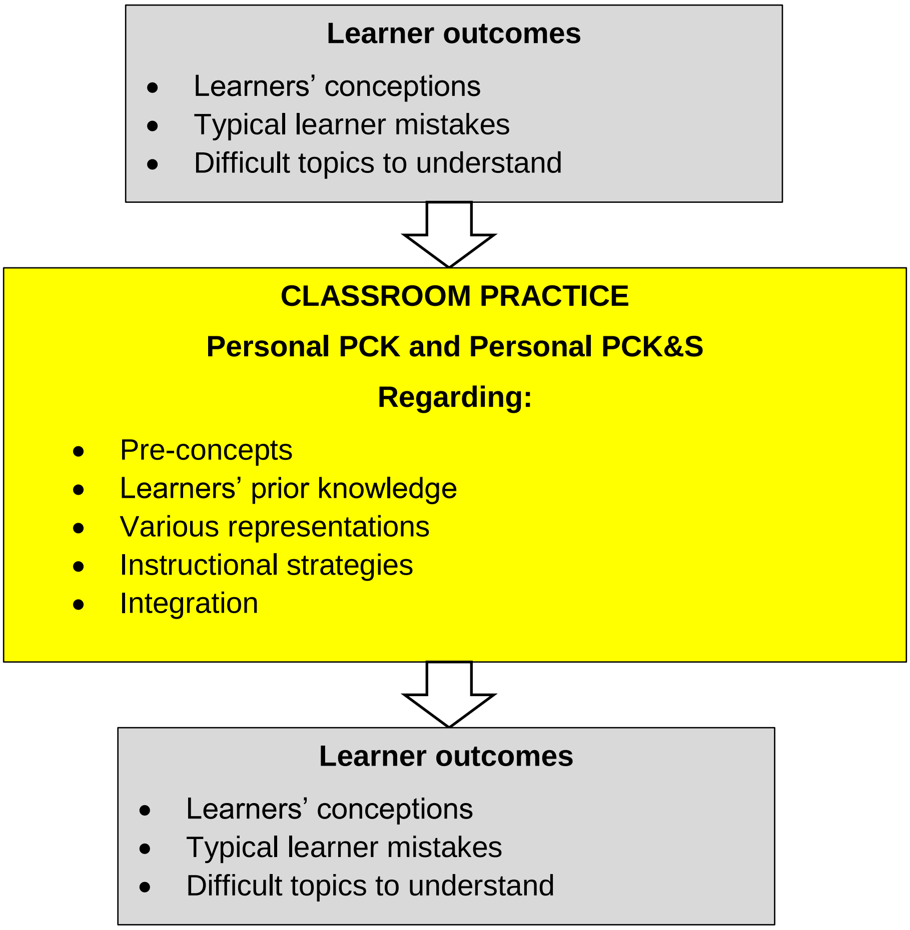


Figure 2.4: Conceptual framework: Classroom practice with Personal PCK and Personal PCK&S (Adapted from Gess-Newsome, in Berry et al., 2015).

2.7.1 Learner outcomes

In this study, learner outcomes entail learners' conceptions, typical learner mistakes, and difficult topics to understand. Learner conceptions entail the learners' prior knowledge and understanding of pre-concepts. Typical learner mistakes are a reflection of the manner in which learners reason while attempting to construct their own knowledge. To explain topics that are difficult to understand, teachers need to plan and use different representations and link abstract concepts with concrete examples in real-life. These three aspects of learner outcomes from a baseline test were initially used to determine the learners' prior knowledge and to further determine how the teachers used their learners' outcomes to inform their planning and teaching. When teachers plan their instruction based on the outcomes of the baseline assessment, their Personal PCK becomes evident, while during the actual teaching, their PCK&S becomes evident. After the informed instruction by the teacher, formative assessment will be used to determine whether instruction was effective and there is a change in learner outcomes.

2.7.2 Classroom practice

Informed by the learners' outcomes from the baseline test and filtered by teachers' teaching experience and knowledge of the content, learners, and the context, teachers' classroom practice was observed. Classroom practice is divided into two sections, namely, Personal PCK (data obtained by interviews) and Personal PCK&S (data obtained by observations). The teachers' Personal PCK and Personal PCK&S were investigated in terms of their knowledge regarding (Botha, 2011; Hill et al., 2008; Shulman, 1987):

- Pre-concepts – The teacher's knowledge with regard to the individual learners' pre-existing understanding of mathematical concepts, schema or models. Therefore, pre-concepts with regard to the topic of surface area, volume and capacity are the concepts that were taught in previous grades, as prescribed in CAPS (DBE, 2012). This is further discussed under Section 4.5.2.1.
- Learners' prior knowledge - The teacher must be able to assess learners' prior knowledge, which includes correct conceptions, typical learner mistakes and concepts that are difficult to understand.

- Various representations – The teachers should use various representations to enhance learners' understanding of a concept.
- Instructional strategies - The teacher must use different instructional strategies such as independent study, direct instruction, indirect instruction, interactive instruction and experiential learning.
- Integration – Teachers should integrate the content of the lesson with other mathematical topics, other subject areas and real-life contexts.

The next chapter presents the research methodology used to obtain data in this study.

CHAPTER 3 METHODOLOGY

3.1 INTRODUCTION

In Chapter 3, a description of the methodology used in this study is given. To understand the phenomenon being studied, I firstly discuss my research paradigm and assumptions, followed by the research approach and design used for studying this phenomenon. The research site, sample selection, and data collection strategies are described. Lastly the data analysis strategies used are discussed, as well as the trustworthiness, validity, reliability and ethical considerations applicable to the study.

3.2 RESEARCH PARADIGM AND PARADIGMATIC ASSUMPTIONS

The research paradigmatic assumptions, namely, ontology, epistemology and methodology are discussed below.

3.2.1 Research paradigm

Constructivism suggests that learners construct their own (mathematical) knowledge, i.e. concepts, ideas and principles, and are therefore active participants in their knowledge acquisition (Lerman, 1989). To understand new mathematical concepts, learners need to restructure existing knowledge, ideas and concepts. When observing teachers' instruction, it is possible to see whether opportunities are given to learners to construct new ideas. The discussion of new ideas to solve mathematical problems is a key principle in learners' learning and may positively impact learner outcomes.

The research paradigm in this study is constructivism, where reality is subjective and a product of the individuals involved. The reality, in this case, was what teachers' Personal PCK and Personal PCK&S were, and how these teachers could use assessment to inform their teaching to make the content more comprehensible for learners. The individuals involved were me as the researcher, two Grade 9 mathematics teachers, and their Grade 9 learners. One of the goals was to search for evidence that was valid and reliable in terms of the existing phenomenon. To obtain knowledge and to understand the phenomenon, as well as the obtained findings, it is important to inform the reader how I view and what understanding I have of the world, and what the purposes of the study were (Cohen et al., 2011).

3.2.2 Paradigmatic assumptions

In terms of the ontological understanding of reality assumed in this study, I held a nominalist position, relating to words and if they have existence or whether this reality is “the product of one’s mind” (Burrell & Morgan, 1979, p. 1). The epistemological assumption of this study, relating to the nature of the knowledge (Holden & Lynch, 2004), was interpretivist as it relates to knowledge that is gained and based on experience of and insight into the world. Lastly, the methodological assumption in this was idiographic methodological as the focus was on the subjective experience of the teachers who designed, presented, and reflected on and interpreted their lessons.

3.3 RESEARCH APPROACH AND DESIGN

Table 3.1 below provides a summary of all aspects of the research methodology of this study.

Table 3.1: Research methodology

Research approach	Qualitative			
Research design	Case Study: Exploratory (Yin, 2013). The purpose of a case study is to do an in-depth investigation and to understand the problem in its current situation. This case study focused on two Grade 9 mathematics teachers' classroom practices with emphasis on their personal PCK, personal PCK&S and how it impacts learner outcomes.			
Main question	**How are assessment outcomes used to inform the teaching of mathematics?**			
Research sub-questions	**Question 1** What is the learners' prior knowledge, as revealed in a baseline test?	**Question 2** How do the outcomes of the baseline test inform the teachers' Personal Pedagogical Content Knowledge (Personal PCK)?	**Question 3** How do the outcomes of the baseline test inform the teachers' Personal Pedagogical Content Knowledge and Skills (Personal PCK&S), as demonstrated in their instruction?	**Question 4** What are the learners' outcomes after the teachers' teaching of the topic, as revealed in a formative assessment?
Objectives of the sub-questions	To determine the learners' prior knowledge on a specific mathematical topic.	To gain insight into the teachers' Personal PCK regarding the topic they teach.	To gain insight into the teachers' Personal PCK&S regarding the topic they teach.	To determine the effect of teachers' informed teaching on the outcomes of the learners.

Participants	Two Grade 9 mathematics teachers from one public school, one having five more years of experience than the other.			
Data collection strategies	One baseline test for four Grade 9 classes with two different teachers in one school. Three observations per teacher for four different Grade 9 classes. One in-depth semi-structured interview per teacher. One formative test for four Grade 9 classes.			
Strategies for each sub-question	Baseline test	Interviews	Observations	Formative assessment test
Data analysis	According to Cohen et al. (2011): "Qualitative data analysis involves organizing, accounting for and explaining the data; in short, making sense of data in terms of the participants' definitions of the situation, noting patterns, themes, categories and regularities" (p. 537). A deductive-inductive data analysis approach was used. Data were analysed according to a set of predetermined themes and categories, as indicated in the conceptual framework, and emerging themes were identified.			

3.3.1 Research approach

The research approach for this study was qualitative and themes, or categories, were used, which resulted from the analysis of the data (inductive analysis), and from the literature before the data were analysed (deductive analysis). In terms of carrying out the gathering of said data, it was essential for me to interact positively with all the participants, keep current field notes, make backups of data and be able to reflect on experiences. I was always prepared and reflected on previous experiences obtained from observations, interviews and learners' tests. Communication with the participants was also very important in order to reduce misunderstandings.

The phases in this qualitative fieldwork were planning, execution and follow-up. After careful planning of the observations, the semi-structured interviews and the assessment of learners' mathematical knowledge, the research was executed. I had to prepare for certain challenges during data collection (for example, sports days and changes in time tables) and tried to always have a backup plan ready. After gathering the data, I followed up with the schools, teachers and learners.

3.3.2 Research design

The research design here was a case study comprising an exploration of the teachers' classroom practices in terms of their Personal PCK through interviews, as well as their Personal PCK&S through lesson observations between the time that the baseline and formative tests were written. Any challenges were eliminated or avoided by taking the necessary precautions. One of the challenges involved a sports day at the school on the day of observation so I had to reschedule it. Another challenge was that both teachers were sick during the periods in which I did the classroom observations, therefore I had to return in the fourth semester.

A further challenge is ensuring that the participants act naturally during observations and that there is a relationship of trust between the researcher and participants (Cohen et al., 2011). Before the fieldwork commenced I thus addressed these challenges and took precautions to ensure that the relationship between the participants and myself was relaxed in the interviews, and that the teachers and learners felt free to do what they normally did in their classrooms. The participants were then observed and interviewed and the learners' tests were analysed.

3.4 RESEARCH SITE AND SAMPLING

An in-depth case study was conducted, requiring high quality, rich and saturated data. Therefore, only one public secondary school in the Tshwane North District (D3; in the Northern part of Pretoria) was purposively and conveniently chosen as a research site. The first criterion for selection was that the school should have different societal, economic and racial influences as these factors can impact teachers' classroom practices. The second criterion was that the school should have an average performance in the 2014 ANA results. I then used convenient sampling to select one school from the list of schools that adhered to these two criteria. The sampled school followed the school mathematics curriculum as outlined in the CAPS document (DBE, 2012), and used GDE approved textbook and curriculum materials.

Through convenient, non-probability sampling, the population for this study was the Grade 9 mathematics teachers and their learners. The two criteria for inclusion were the experience of these two teachers and the language of instruction. One teacher had five years more experience than the other in teaching mathematics at Grade 9 level. The gender and race of the teachers were not considered in the sampling.

Two different classes from the two teachers were selected purposively based on their performance – one poorly performing and one average/high performing class for each teacher. The one teacher had two English classes, while the other had one Afrikaans class and one English class. The inclusion and exclusion criteria for selection in this study are listed in Table 3.2 below:

Table 3.2: Inclusion and exclusion criteria

	Inclusion criteria	**Exclusion criteria**
School	Average performing Afrikaans or English medium	
Teachers	Experience of teaching Grade 9 mathematics Afrikaans or English speaking	Gender and race of the teachers
Classes	Different performing classes Afrikaans or English speaking	Gender and race of the learners

3.5 Data collection strategies

The data collection strategies that were used in this study were three classroom observations per teacher, one in-depth semi-structured interview per teacher, and two tests of which one was a baseline and the other a formative test. The learners first wrote a baseline test based on typical learner mistakes and concepts that are difficult to grasp and understand. After the baseline test the teachers were observed in their classrooms to gain insight in their Personal PCK&S and how they used learner outcomes to inform their teaching. Afterwards the teachers were interviewed individually to gain insight into their Personal PCK with regard to different aspects in their classroom practices. After the in-depth semi-structured interview with both teachers, the learners wrote a formative test to determine the influence of the teachers' instruction on learner outcomes. The data collection took place during the third quarter of 2016. In Table 3.2 a timeline is given of the data collection process.

Table 3.3: Timeline of the data collection process

Data gathering instrument	Participants	Date (2016)
Baseline test	All Grade 9 learners	1 September
Observation 1, Class 1	Alice	2 September
Observation 1, Class 2	Alice	2 September
Observation 2, Class 1	Alice	9 September
Observation 2, Class 2	Alice	9 September
Observation 1, Class 1	Mary	9 September
Observation 1, Class 2	Mary	9 September
Observation 3, Class 1	Alice	16 September
Observation 3, Class 2	Alice	16 September
Observation 2, Class 1	Mary	16 September
Observation 2, Class 2	Mary	16 September
Observation 4, Class 1	Alice	26 September
Observation 4, Class 2	Alice	26 September
Observation 3, Class 1	Mary	13 October
Observation 3, Class 2	Mary	13 October
Interview	Alice	14 October
Interview	Mary	17 October
Formative assessment	All Grade 9 learners	24 October

3.5.1 Baseline test

In this study, a baseline and a formative test were written. The CAPS document (DBE, 2012) was used as guideline to set up these tests. The learners' baseline test (see Addendum C) was set by my supervisors and myself. We ensured that typical learner mistakes and concepts that are difficult to grasp and understand were addressed in this test. The baseline test was administered by four invigilating teachers during the test period in which the learners wrote it. The tests were marked by me and I used the outcomes to assist me in observing how the teachers' Personal PCK and Personal PCK&S were informed by the outcomes of the baseline test. The marked scripts were handed back to the teachers prior to their teaching of the topic.

3.5.2 Observations

Cohen et al. (2011, p. 474) state that the use of observations is a "powerful tool for gaining insights into situations." However, one of the challenges when observing people is that they may change their behaviour if they know that they are being observed. To reduce the anxiety of the teachers with the observations, I sat at a table that was outside of their teaching space. Each teacher was observed three or four times with two different classes during their normal school timetable so that neither the school nor the teachers were disrupted. An observation schedule (See Addendum D) was used with every observation and all the observations were video-recorded.

3.5.3 Interviews

The purpose of interviews is to evaluate a person, to test or construct hypotheses, to collect data or to sample people's opinions (Cohen et al., 2011). In the interviews conducted with the Grade 9 mathematics teachers I also asked some questions on the influence of the classroom context on the their Personal PCK. The audio-recorded interviews took place in each teacher's classroom, lasted about half an hour each and were conducted at school time in their free periods.

3.5.4 Formative assessment

After the classroom observations and interview with each teacher, the learners wrote a formative test (See Addendum F) to determine the influence of the teachers' instruction on learner outcomes by comparing it with the written answers and results of the baseline test. The formative tests were administered by the mathematics teachers during class time. By comparing the answers qualitatively, I could determine if typical mistakes and difficult topics to teach and understand were addressed during instruction prior to the formative assessment.

3.6 DATA ANALYSIS STRATEGIES

Cohen et al. (2011, p. 537) state that "qualitative data analysis involves organizing, accounting for and explaining the data; in short, making sense of data in terms of the participants' definitions of the situation, noting patterns, themes, categories and regularities." Qualitative data analysis is also "an iterative, back-and-forth process" where you can have multiple interpretations (Cohen et al., 2011, p. 537). The data analysis strategy for this study was therefore explorative and both deductive and inductive.

To determine the influence of Grade 9 mathematics teachers' ability to teach for the improvement of learner outcomes, I needed to describe, portray, interpret, prove or demonstrate, explore, test and examine the same phenomenon but in different contexts. The data I obtained from the actions mentioned above were detailed and rich. These data were tabulated in a template according to the pre-determined themes or categories as obtained from the literature. Through a content analysis, the data will be presented teacher by teacher and by emerging themes or key issues amalgamated. Content analysis is a process where the data obtained from observations, interviews and tests can be reduced by classifying it into fewer categories; this involves "coding, categorizing, comparing and concluding" (Cohen et al., 2011, p. 564). Not only was the data from the observations of the teacher's two classes compared, but the data from the observations and interviews were also compared. Lastly the assessment outcomes from the two tests were also compared in order to draw conclusions. The process of content analysis, as used in this study, can be summarised in an eleven-step strategy (Cohen et al., 2011):

Step 1: Define the research questions to be addressed.

Step 2: Define the population from which texts are obtained.

Step 3: Define the included sample.

Step 4: Define the context of document generation (i.e. sources of data).

Step 5: Define the units of analysis (codes used to describe data).

Step 6: Decide on the codes.

Step 7: Decide on the categories.

Step 8: Conduct coding and categorising (i.e. using ATLAS.ti7 to present data).

Step 9: Conduct data analysis.

Step 10: Summarise (i.e. identify key factors, issues, concepts and areas).

Step 11: Make speculative inferences (i.e. draw conclusions and consider implications for further research).

3.7 QUALITY ASSURANCE CRITERIA

Aspects such as validity and reliability, and the possible Hawthorne and Halo effect were taken into consideration in this study and are discussed further below.

3.7.1 Validity

Validity is "the development of sound evidence to demonstrate that the test interpretation matches its proposed use" (Creswell, 2012, p. 159). Validity is a requirement for qualitative research and can be addressed through the data gathered, the participants, extent of triangulation and the objectivity of the research (Cohen, 2011). The data instruments were piloted first to make sure the pre-determined categories were appropriate and that the baseline test offered the data needed. I also remained objective during the interviews. The interview instrument was also piloted first to ensure that the questions were interpreted in the same way by different participants. The two participants also read through the transcriptions to ensure accuracy and validity.

3.7.2 Reliability

Reliability is "a synonym for dependability, consistency and replicability over time, over instruments and over groups of respondents" (Cohen et al., 2011, p. 199). A study is declared reliable if other researchers can replicate key findings with similar participants in a similar context. I acknowledge the fact that my presence during the observations in class might have influenced the teachers' instructional strategies, referred to as the Hawthorne effect (Cohen et al., 2011). The interview was scheduled after the last observation to determine the teachers' Personal PCK in relation to their observed Personal PCK&S. To ensure the reliability of what happened in the classroom, I took some notes and the lessons were video-recorded in order to have an accurate record of what happened in class. Regarding the interview, the same interview procedures were followed and the same questions were asked in the same order for the two teachers during the interviews to ensure reliability. The interviews were audio-taped to have a clear and accurate record of what had been discussed.

3.7.2.1 The Hawthorne effect

The Hawthorne effect is observed when there is a positive change in the performance of participants taking part in a study due to the fact that their perceptions are investigated (Cohen et al., 2011). My presence in the class might have influenced the teachers' behaviour during the classroom observations. To minimise the possible influence on the data, the interviews were conducted after the classroom observations. I assured the teachers that my purpose was not to report their performances to their head of department or principal. To enhance the trustworthiness of the study, all of the observed classroom lessons were video-taped, field notes were taken, and after each observation the teacher had to verify my field notes. To ensure trustworthiness of the interviews, it was important that the interviewees be honest. The same interview schedules were used for both teachers and the interview questions were short in order to avoid misunderstandings.

3.7.2.2 The Halo effect

The Halo effect (Cohen et al., 2011) is a cognitive bias that can influence the observer's overall impression, feelings and thoughts about the participants in the study. To enhance the trustworthiness of the baseline test (see Addendum C), I also piloted the test at a school in the same district. The interview was also piloted to refine the questions and to ensure the questions were interpreted the same way by different teachers and that it did not take too much time, as the teachers' work schedules had to be taken into consideration.

3.8 ETHICAL CONSIDERATIONS

Ethical clearance was requested from the Ethics Department at the University of Pretoria and the Gauteng Department of Education. The dignity of the participants in the study was maintained. I kept all information confidential by using pseudonyms to keep the participants' identities protected and assigned letters of the alphabet to the participating classrooms at the specific school. The level of sensitivity for this study was medium. The participants' lessons were video-taped and the interviews were audio-taped to have a record of all activities and conversations. The interviews were conducted in private during school hours.

Participation in the study was voluntary and participants could withdraw at any time without consequences. I initially made a telephonic appointment with the school principal to discuss this study. The principal then discussed it with the Head of Department of mathematics. Letters of consent requiring their participation were given to the principal, teachers and parents, and letters of assent were given to the learners to explain my presence in class. A letter of consent (see Addendum A) was signed by all the parties involved. However, for the parents that did not give consent, I positioned myself in the class in such a way that those learners were seated behind me. Those learners' participation did not contribute to the study. In the letters, the parties involved were informed about the purpose of the study, the instruments that were to be used during the research and the information on confidentiality, anonymity and possible risks. Most of the communication and arrangements were made directly with the two participants and sometimes I worked through the Head of Department of mathematics.

The data obtained was not sensitive. The participants were not harmed physically or psychologically during this study. However, the teachers might have experienced anxiety because the lessons were video-taped and during the interviews which were audio-taped they might have felt uncomfortable sharing and discussing their Personal PCK and Personal PCK&S with me.

This chapter reported on the research paradigm, assumptions, approach and design. The research site, sampling procedures, data collection and analysis strategies were discussed. The chapter concluded with a discussion of the quality assurance criteria and the ethical considerations. In Chapter 4, the data will be presented and discussed.

CHAPTER 4 PRESENTATION AND DISCUSSION OF THE DATA

4.1 INTRODUCTION

In Chapter 4, I report on the data collection process, the coding of the data based on the conceptual framework, the analysis of the data, and give information regarding the two Grade 9 teachers' lessons. I present and discuss my findings per participant, relate the findings to the literature review, and explain the different trends. The two themes in this study that are Personal PCK and Personal PCK&S.

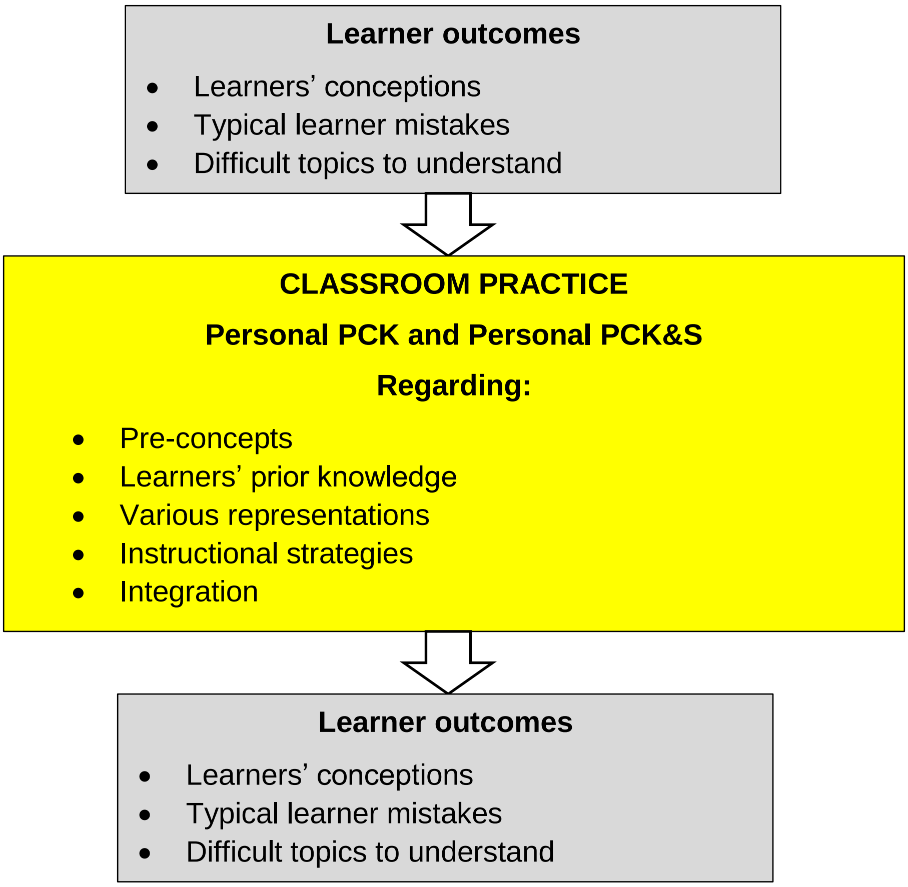


Figure 4.1: Conceptual framework: Classroom practice with Personal PCK and Personal PCK&S (Adapted from Gess-Newsome, in Berry et al., 2015)

The following research questions guided this research project.

Main question: How are assessment outcomes used to inform the teaching of mathematics?

Sub-questions:

1. What is the learners' prior knowledge, as revealed in a baseline test?
2. How do the outcomes of the baseline test inform the teachers' Personal Pedagogical Content Knowledge (Personal PCK)?
3. How do the outcomes of the baseline test inform the teachers' Personal Pedagogical Content Knowledge and Skills (Personal PCK&S), as demonstrated in their instruction?
4. What are the learners' outcomes, after the teachers' teaching of the topic, as revealed in a formative assessment?

4.2 THE DATA COLLECTION PROCESS

The data collection took place at a secondary school in the Tshwane North (D3) district in Pretoria in September and October of 2016 in two different classes for each of the two Grade 9 teachers. Pseudonyms were used for the two participants. The data collection process involved a baseline test; four observations for Alice's classes; three observations for Mary's classes; an individual, in-depth semi-structured interview after the observations with each teacher; and lastly a formative test. All video data, as well as the audio-taped data were transcribed verbatim. The teachers and I finally read all the transcripts together to ensure accuracy. I then analysed the data using an Excel spreadsheet. The data analysis strategy for the baseline test and formative test were both deductive and inductive.

4.3 CODING OF THE OBSERVATION AND INTERVIEW DATA

I obtained data from the video and audio recordings. Every theme and sub-theme were ascribed a code and according to these codes, the data were analysed using ATLAS.ti7. The data for the teachers' Personal PCK were collected from in-depth semi-structured interviews with the two teachers, while the Personal PCK&S was collected from the classroom observations. The five sub-themes that describe the teachers' Personal PCK and Personal PCK&S are: Pre-concepts; learners' prior knowledge; various representations; instructional strategies; and integration. In Table 4.1, a summary is given of the inclusion criteria used and the different codes assigned to the data.

Table 4.1: Summary of description of the indicators used to code teachers' Personal PCK and Personal PCK&S

THEME	SUBTHEME	CODES	DESCRIPTION OF THE TEACHERS' PERSONAL PCK AND PERSONAL PCK&S INDICATORS (CODES)
Personal PCK (Interview)	A. Pre-concepts	PPCKA1	• Teachers' mathematical content knowledge on how to assess the learners' pre-concepts using a baseline assessment.
		PPCKA2	• The teachers' knowledge of baseline assessment and how to assess the learners' pre-concepts.
		PPCKA3	• Reasoning behind and planning for teaching a particular lesson in a particular way based on the learners' pre-concepts.
	B. Learners prior knowledge	PPCKB1	• Reasoning behind and planning for teaching a particular lesson in a particular way based on the learners' prior knowledge
		PPCKB3	• Teachers' deal with topics that is difficult to grasp and understand.
	C. Various representations	PPCKC1	• Teachers use different modes of representations.
	D. Instructional strategies	PPCKD1	• Teachers used different instructional strategies.
		PPCKD2	• Mathematical mistakes made by teachers, the teachers' difficulties or misrepresentations.
		PPCKD3	• Teachers highlight mathematical mistakes made by learners, the learners' misconceptions or misrepresentations.
	E. Integration	PPCKE1	• Teachers integrate the content of the mathematics lessons with other mathematics topics, other subject areas and real-life contexts.

THEME	SUBTHEME	CODES	DESCRIPTION OF THE TEACHERS' PERSONAL PCK AND PERSONAL PCK&S INDICATORS (CODES)
Personal PCK&S (Observations)	A. Pre-concepts	PPCKSkillA1 PPCKSkillA2 PPCKSkillA3	• Teachers' ability to assess existing knowledge of learners prior to formal teaching; ideas, information, beliefs and attitudes that learners bring with them to the classroom by either questioning or listening. • Teachers' understanding of the aim of the baseline assessment and to identify learners' pre-concepts, starting points, goals, targets, interests and difficulties in learning. • Teachers' ability to provide detailed feedback after the baseline assessment that help learners recognise their strengths and weaknesses, and to link their knowledge of pre-concepts with new knowledge.
	B. Learners' prior knowledge	PPCKSkillB1 PPCKSkillB2	• Teachers' selection of tasks that are of high cognitive demand where learners need to engage with the mathematical activities, examine, question, conjecture, experiment and justify their answers in order to reconstruct their prior mathematical knowledge to become more mathematically proficient. • Teachers' ability to assess the nature and understand learners' misconceptions/misunderstandings, typical mistakes they make and to select tasks that will ensure that learners gain new knowledge, skills, abilities, beliefs in order to become mathematically proficient. Teachers' interpretation of the assessment outcomes, and planning instruction purposefully to rectify learners' prior knowledge. Teachers' understanding of learners' prior knowledge by assessing learners' work to

THEME	SUBTHEME	CODES	DESCRIPTION OF THE TEACHERS' PERSONAL PCK AND PERSONAL PCK&S INDICATORS (CODES)
		PPCKSkillB3	see what typical mistakes learners make and challenging concepts that are difficult to teach and understand by learners. • Teachers adapting and adjusting their instruction based on the learners' prior knowledge, including correct conceptions and typical learner mistakes. Teachers' ability to scaffold learners' thinking and reasoning to maintain high level of cognitive demand of activities.
	C. Various representations	PPCKSkillC1 PPCKSkillC2 PPCKSkillC3	• Teachers' knowledge of which concrete representations to use in order to enhance learners' understanding and to address learners' different learning styles. What teachers intend the learners to learn about the topic and to address misconceptions or misunderstanding by using different concrete representations. • Teachers' ability to use visual representation to enable learners to understand mathematical content that is difficult to grasp or understand. • Teachers' planning of the multiple representations and whether the teacher understands the sequencing and progression of topics in the curriculum and how different topics dealt with in one grade link with topics done in the next grade and adapt their instruction accordingly.
	D. Instructional strategies	PPCKSkillD1 PPCKSkillD2	• Teachers' use of different strategies of instruction; independent study, direct instruction, interactive instruction and experiential learning. Teachers' knowledge of how to include high cognitive demand activities that will contribute to better understanding of the concepts.

THEME	SUBTHEME	CODES	DESCRIPTION OF THE TEACHERS' PERSONAL PCK AND PERSONAL PCK&S INDICATORS (CODES)
			• Teachers' ability to adjust their instruction for better understanding and to eliminate misconceptions. Teacher-centered or learner-centered instructional approach or both. Teachers' use of an authority/delegator/facilitator/ traditional/demonstrator teaching style.
	E. Integration	PPCKSkillE1	• Teachers' ability to plan their instruction in such a way that the content of the lesson integrates with other mathematical topics and subject areas. Teachers' understanding of the structure of mathematics and good knowledge of the curriculum to integrate the mathematical topics and concepts into real-life problems.

4.4 INFORMATION REGARDING THE TWO GRADE 9 TEACHERS

Alice and Mary were teaching at a Section 21 (former Model C) school in the Tshwane North District (D3) with 1 016 learners (446 male and 570 female) where 44% of the learners were Black, 42% were White and 14% were a combination of Coloured, Indian and Asian learners.

4.4.1 Alice

At the time of this study, Alice was 33 years old and had completed a BEd FET Natural Science degree in 2006 at the University of Pretoria with mathematics, Botany and Zoology as her major subjects. She had nine years' teaching experience in various subjects, four years of which was in teaching Grades 9 and 10 mathematics. I observed two different classes where the medium of instruction was English in both classes. There were 37 learners in each class. Alice was a very enthusiastic teacher and started every class with a comment or anecdote projected on the white board. The learners were not afraid to ask questions and were actively involved in solving problems. However, Alice was teaching in her second language and sometimes struggled to explain the work to the learners.

4.4.2 Mary

At the time of this study, Mary was 31 years old and had completed a BEd FET General degree in 2007 at the University of Pretoria with mathematics, Zoology and Remediation as her major subjects. She had nine years' teaching experience in Grade 9 mathematics. I observed two different classes where the medium of instruction was English in the one class and Afrikaans in the other class. In the mathematics classes where English was the medium of instruction, there were 37 learners in the class, while in the mathematics class where Afrikaans was the medium of instruction there were only 24 learners in the class. Mary was a very dedicated teacher and worked very hard and purposefully with the learners in the class. The learners were not afraid to ask questions and were actively involved in solving problems.

4.5 PRESENTATION AND DISCUSSION OF DATA COLLECTED FROM ALICE

4.5.1 Baseline test

The data obtained from the baseline test (See Attachment A) written by 71 learners in Alice's classes are now presented and discussed in terms of learner conceptions, typical learner mistakes and concepts that are difficult to understand. After the analysis of this data, emerging issues that were not part of the deductive analysis process are discussed.

4.5.1.1 Learner conceptions

Question 1 assessed the learners' prior knowledge and understanding of the pre-concepts: surface area, volume, and capacity. The pre-concepts in teaching Grade 9 mathematics are derived from the prescribed curriculum (DBE, 2012) for Grade 7 and Grade 8 (see Table 4.10 under Section 4.5.2.1). This question was answered poorly. Learners were unable to write down the meaning of these three pre-concepts in their own words. Below are the definitions as stated in the memorandum, but similar meanings in the learners' own words were also considered and taken as correct. Surface area = sum of all the areas of all the surfaces of an object; volume = amount of space occupied by an object; and capacity = amount of substance contained by an object. Some of the answers obtained from the learners' scripts are given below in Table 4.2.

Table 4.2: Learner conceptions regarding surface area, volume, and capacity

Surface Area	**Volume**	**Capacity**
1.1 The flat ground of an area.	2.1 The height or filling of an object.	3.1 Capacity is the level of something.
1.2 A surface area is the flat part of a shape.	2.2 Volume is the inside of the surface.	3.2 Capacity is the surface area joined with volume.
1.3. The area around the surface of an object.	2.3 The mass and the weight of the object.	3.3 How heavy an object is whether it is in kg or g.
1.4. The amount of surface in an object or area.	2.4 The measurement of how long and tall an object is.	3.4 The final answer and the units that go with it.
1.5. The surrounding area of the object/the bottom part.	2.5 The height and width of the shape.	3.5 How much weight is inside the shape.

It was clear that most learners had misconceptions regarding the difference between these concepts and although a few learners had correct conceptions, there was an inability to clearly define the meaning of these pre-concepts in their own words. The learner that wrote the response in Number 1.3 had the correct idea, but lacked the vocabulary or terminology needed to clearly define surface area. Learners understood that the net of the shape forms the surface area of the object (see Number 1.2), but did not specify that you need to add all of those areas together. Furthermore, 19.7% of the learners thought that volume involves the amount of mass or depth. Another 19.7% of the learners thought capacity involves the amount of weight or pressure. In Number 2.2 it can be seen that the learner had a misconception as capacity, instead of volume, was described. A summary of the keywords used by learners for the definitions of these three pre-concepts is given in Table 4.3 below.

Table 4.3: Keywords used by learners for the definitions of surface area, volume and capacity

Surface area	'total length of the front part', 'outside perimeter', 'bottom area', 'base area', 'bottom of the shape', 'area of the net form'
Volume	'height', 'weight', 'total size', 'depth' and 'mass'
Capacity	'amount of empty space', 'amount of pressure' and 'number of types of shapes'.

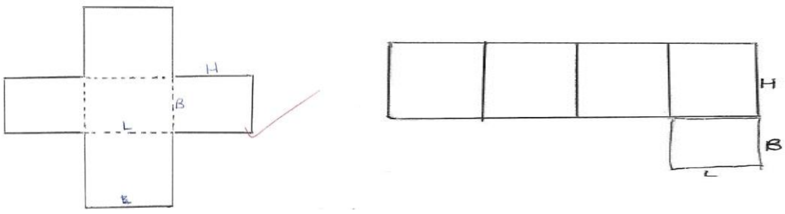
The learners' prior knowledge of drawing and investigating the nets of cubes and rectangular prisms were assessed in Question 2 of the baseline test. Table 4.5 shows that more than 50% of the learners had the correct concept of a net and were able to correctly draw the net of the rectangular prism.

Table 4.4: Learner conception of drawing the net of an open rectangular prism

Type of net drawn	Number of learners	Percentage of learners
Correct net with dimensions	25	33.3
Correct net with wrong dimensions	19	25.3
Closed rectangular prism with wrong dimensions	11	14.7
3D rectangular prism	8	10.7
Closed rectangular prism with right dimensions	4	5.3

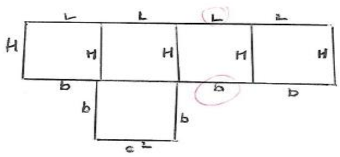
Type of net drawn	Number of learners	Percentage of learners
Correct net without dimensions	4	5.3
Closed rectangular prism without dimensions	2	2.7
Incorrect net	2	2.7

The net that occurred the most in the scripts was where the learners drew the net of the open rectangular prism with the base in the centre and the rest of the faces adjacent to the base, as can be seen on the left of Picture 4.1. There were also a few learners who provided an alternative representation of the net for the open rectangular prism, as can be seen on the right of Picture 4.1.



Picture 4.1: Two of the conceptions of the net of the open rectangular prism

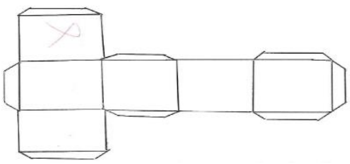
Some of the learners were able to draw the net, but struggled to identify the length, breadth and height of an object.



Picture 4.2: Correct net of an open rectangular prism with the incorrect dimensions

4.5.1.2 Typical learner mistakes

The purpose of the second question of the baseline test was to elicit conceptual understanding of using a net to derive a formula for surface area, which learners generally found difficult to do and typical learner mistakes emerged here. Some of the learners drew an incorrect net for the rectangular prism and did not indicate the dimensions L, B and H. This was due to the fact that they had drawn the net of a closed rectangular prism, as seen in Picture 4.3.



Picture 4.3: Incorrect net of an open rectangular prism without dimensions

Another mistake that 42.3% of the learners made was to write down the volume formula for a rectangular prism instead of the surface area of a rectangular prism. Most of the learners were able to draw the net correctly, but only a few managed to write in their own words how to calculate the surface area by using the net. The most common mistake learners made was in finding a formula for the surface area of the open vase. Some of the learners' mistakes are given in Table 4.5 below:

Table 4.5: Learner mistakes regarding formulae for surface area of an open vase

Learners' answers	**Number of learners**	**Percentage of learners**
LBH	30	42.3
Answers like: 4LH+LB, 2LB, BH, 5LBH, 8LB, 5LH, 0.5LBH, 2LBH, $L^2+B^2+H^2$, LH+LB+BH	17	23.9
L+B+H	12	16.9
LB	11	15.5
2LB+2LH+2BH	1	1.4
LB+2LH+2BH (Correct answer)	0	0

None of the learners were able to use the net to derive the surface area formula. Only one learner wrote down the correct formula. However, this learner drew the net incorrectly and gave the formula for a closed rectangular container. The most popular answers (74.7% of the learners) involved either the correct volume formula for a prism (42.3%) or an incorrect version where the length, breadth and height were added instead of multiplied (16.9% of the learners) or only length and breadth were multiplied (15.5% of the learners). The other 23.9% responses included a variety of answers.

4.5.1.3 Difficult concepts to understand

It seemed the learners could not understand and do conversions between units in the SI system. The learners were unable to manipulate the formula to calculate the height of the container and were also confused about the unit of measurement to use for the height. Some of the answers that were obtained are given below in Table 4.6.

Table 4.6: Learners' answers for conversions and height calculations

Conversions		Height of the water
1 m^3 to cm^3	1 m^3 to litre	
1 m^3 = 1000 cm^3	1 m^3= 100 litre	15 m/25 m
1 m^3 = 10 cm^3	1 m^3 = 1000 litre	50 ml
1 m^3 = 1 cm^3	1 m^3 = 30 litre	5 m^2
1 m^3 = 100 cm^3	1 m^3 = 1 litre	5 cm
1 m^3 = 300 cm^3	1 m^3 = 10 litre	80 (without unit of measurement)

Table 4.7 presents the answers in converting 1 m^3 to cm^3

Table 4.7: Analysis of different answers obtained when converting 1 m^3 to cm^3

Possible answers to convert m^3 to cm^3	Percentage of learners
100	32.9
1 000	23.3
10	17.8
1 000 000 (correct answer)	**6.8**
1	2.7
0.1	2.7
No answer	2.7
Incorrect answers that occurred only once in the learners' scripts (3, 30, 300, 60, 10 000, 100 000)	8.4

Most of the learners (79.4% of the learners) did not have the correct number of zeros in their answer, which included incorrect multiples of 10 (highlighted). Only 6.8% of the learners answered correctly. Table 4.8 presents the learners answers in converting 1 m^3 to litre.

Table 4.8: Analysis of different answers obtained when converting m^3 to litres

Possible answers to convert m^3 to litres	Percentage of learners
100	27.4
1	17.8
1 000 (correct answer)	**15.1**

Possible answers to convert m^3 to litres	**Percentage of learners**
10	12.3
No answer	8.2
10 000	5.5
3	4.1
0.1	2.7
Incorrect answers that appeared only once in the learners' scripts (0.01, 1,1, 30, 150, 1 000 000)	7.0

Only 15.1% of the learners got the correct answer. Most of the learners (68.4% of the learners) did not have the correct number of zeros in their answers and the answers were incorrect multiples of 10 (highlighted). A few learners wrote that 1 m^3 is equal to 3 or 30 litres.

The last three questions in the baseline test were answered very poorly. Learners who could not do the conversions were unable to determine the volume and height of the water, and those who could struggled to find the height where the formula had to be manipulated.

4.5.1.4 Emerging issues

Some unforeseen issues emerged as many learners made the mistake of using the formula for volume instead of surface area, and used the wrong formula for the volume of the container. There were four variations that arose from this typical mistake: the volume formula (LBH) for surface area and volume; the incorrect formula (L+B+H) for surface area and volume; the incorrect formula (L+B+H) for surface area and the correct formula (LBH) for volume; and the volume formula (LBH) for surface area and the incorrect formula (L+B+H) for volume. Table 4.9 indicates the percentage of learners who made these typical mistakes.

Table 4.9: Variations of typical mistakes made in baseline test

Type of mistake	Percentage of learners in Question 2.3	Percentage of learners in Question 2.5
1. Using volume formula (LBH) for both questions.	42.3	23.9
2. Using an incorrect formula (L+B+H) for both questions.	16.9	5.6
3. Using an incorrect formula (L+B+H) for surface area question and LBH for volume question.	16.9	8.5
4. Using volume formula (LBH) for surface area question and an incorrect formula (L+B+H) for volume question.	42.3	8.5

These learner outcomes helped to show how the teachers' Personal PCK&S were informed by these outcomes. Among other things, I observed how the teachers addressed the learners' conceptions, the typical learner mistakes and concepts that were difficult to understand.

4.5.2 Classroom observations

I observed Alice to gain deeper insight into her Personal PCK&S. I have divided this theme in the conceptual framework into sub-themes, namely, pre-concepts, learners' prior knowledge, various representations, instructional strategies, and integration. Each of these will now be discussed.

4.5.2.1 Pre-concepts

In this section, I discuss the pre-concepts applicable to teaching surface area, volume, capacity and conversions to the Grade 9s (DBE, 2012). Thereafter, I discuss how Alice used these pre-concepts in her teaching.

- Pre-concepts according to the curriculum

The teacher should take cognisance of and revise these concepts before teaching the new concepts in Grade 9. Table 4.10 below provides the specifications of content for the topic *Surface area and volume of 3D objects* under the content area *Measurement* for Grades 7 to 9. The items that are highlighted in the table are new content that is introduced in that specific grade.

Table 4.10: Specifications of content for the topic: Surface area and volume for Grade 7 to Grade 9 (DBE, 2012, pp. 57, 108,146)

Grade 7	**Grade 8**	**Grade 9**
Use appropriate formulae to calculate the surface area, volume and capacity of • Cubes; • Rectangular prisms.	Use appropriate formulae to calculate the surface area, volume and capacity of • Cubes; • Rectangular prisms; • **Triangular prisms**.	Use appropriate formulae **and conversions between SI units to solve problems** and calculate the surface area, volume and capacity of • Cubes; • Rectangular prisms; • Triangular prisms; • **Cylinders**.
Describe the interrelationship between surface area and volume of the objects mentioned above.	Describe the interrelationship between surface area and volume of the objects mentioned above.	**Investigate how doubling any or all the dimensions of right prims and cylinders affects the volume.**
Solve problems involving surface area, volume and capacity.	Solve problems, with or without a calculator, involving surface area, volume and capacity.	Solve problems, with or without a calculator, involving surface area, volume and capacity.
Use and convert between appropriate SI units, including: • $mm^2 \leftrightarrow cm^2$ • $cm^2 \leftrightarrow m^2$ • $mm^3 \leftrightarrow cm^3$ • $cm^3 \leftrightarrow m^3$	Use and convert between appropriate SI units, including: • $mm^2 \leftrightarrow cm^2 \leftrightarrow m^2 \leftrightarrow km^2$ • $mm^3 \leftrightarrow cm^3 \leftrightarrow m^3$ • ml (cm^3) $\leftrightarrow$ l $\leftrightarrow$ kl • If 1 cm = 10 mm, then1 cm^3 = 1 000 mm^3 • If 1 m = 100 cm then 1 m^3 = 1000000 or 10^6 cm^3	Use and convert between appropriate SI units and note • If 1 cm = 10 mm, then 1 cm^3 = 1 000 mm^3 • If 1 m = 100 cm then 1 m^3 = 1000000 or 10^6 cm^3
Use equivalence between units when solving problems:	• **An object with a volume of 1 cm^3 will displace exactly 1 ml of water.**	• An object with a volume of 1 cm^3 will displace exactly 1 ml of water.

Grade 7	Grade 8	Grade 9
• **1 cm^3 ↔ 1 ml** • **1 m^3 ↔ 1 kl**	• **An object with a volume of 1 m^3 will displace exactly 1 kl of water.**	• An object with a volume of 1 m^3 will displace exactly 1 kl of water.
Emphasise that the amount of space inside a prism is called its capacity and the amount of space occupied by a prism is called the volume.	Emphasise that the amount of space inside a prism is called its capacity and the amount of space occupied by a prism is called the volume.	
Investigate the nets of cubes and rectangular prisms in order to deduce formulae for calculating their surface areas.	Investigate the nets of cubes and rectangular prisms in order to deduce formulae for calculating their surface areas.	
What is different to Grade 6? **In Grade 6 learners did not have to use formulae to calculate surface area and volume.**	**What is different to Grade 7?** **Surface area and volume of triangular prisms.**	**What is different to Grade 8?** **Surface area and volume of cylinders.**

In Grade 7 the learners learned the formulae for the surface area, volume and capacity of cubes and rectangular prisms, as well as conversions involving surface area, volume and capacity (DBE, 2012, p. 57). In Grade 8 the learners learned the formulae for the volume of a prism; surface area of a prism; volume of a cube; volume of a rectangular prism; the volume of a triangular prism; and the conversions between appropriate SI units, which include those for surface area, volume and capacity. Nets of cubes and rectangular prisms were investigated in order to deduce formulae for calculating their surface areas (DBE, 2012, p. 108). The Grade 9 curriculum prescribes the same content as in Grade 8, but introduces the formulae of the surface area and volume of cylinders (DBE, 2012, p. 146).

- Alice's use of pre-concepts

When Alice started the topic of surface area and volume, she spent some time on the pre-concept of a prism. According to Tapson (2006), a prism has two parallel faces that are equal in their dimensions and when a cross-section is present, it is congruent to those faces. Alice's explanation of the definition of a prism proceeded as follows:

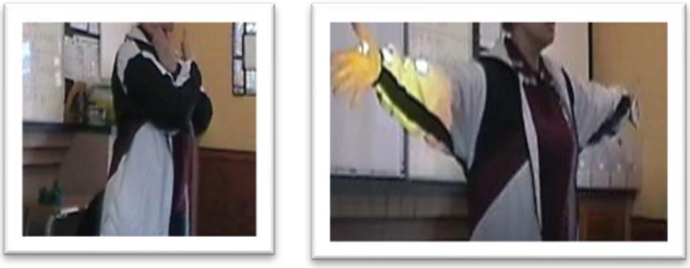
> *Teacher: A prism is an object with a base, the bottom and the top look the same. It's the same shape. Okay, so if I cut it in pieces, every little piece is going to be the same shape. Like for the triangle, a triangular prism. If I cut it like this, I'm going to have triangles. Even a cylinder, if I cut I'm going to have circles … A prism, it's the same shape right through.*

In Lesson 1, for the definition of surface area, Alice defined surface area as 'the area of all the faces added together'. In Lesson 3, she explained the pre-concept of volume as follows:

> *Teacher: Volume is the space that a block will take up. Okay? Capacity is what goes inside. It's basically the same thing, but the easiest way to remember it - capacity, you work with fluids. With your litre type of things, millilitre, kilolitre, litre. If you can pour water in here, that's capacity. Volume is the chocolate, capacity is melted chocolate.*

This explanation of volume and capacity was inadequate and will be further discussed under Section 5.5.2.5 'integration with real-life contexts'.

Alice assessed learners' knowledge of pre-concepts by asking questions, listening to their answers and explaining the concepts again. In all three lessons, she spent a lot of time on the pre-concepts of rounding of numbers when doing calculations, units of measurement and conversions. To explain the calculations with conversions, earlier in the year the learners needed to learn the rhyme "**k**ids **h**ate **d**oing **m**athematics **d**uring **c**loudy **M**ondays", which represents the pre-fixes of the powers of ten of the SI units: 'kilo, hecto, deca, meter, deci, centi, milli'. To explain conversions, she stood with her arms against her body (see Picture 4.4 below).



Picture 4.4: Alice demonstrating conversions with her arms

She then said that the larger units were on the left-hand side of the rhyme and the smaller units on the right-hand side. When doing a calculation with the conversions, she asked the learners to use the rhyme to determine if they went from a larger unit to a smaller unit. If they started with a larger unit and went to a smaller unit, she stretched out her arms to the sides for the larger unit and crossed her arms to move to the smaller unit, stating 'you therefore need to multiply' (see left side of Picture 4.4). If they started with a small unit and needed to go to a larger unit, with arms stretched out to the sides she said, 'you therefore need to divide' (see right side of Picture 4.4).

4.5.2.2 Learners' prior knowledge

Prior knowledge refers to the individual learners' conceptions; typical learner mistakes; and learner difficulties regarding the pre-concepts of surface area, volume, and capacity; the drawing of the nets of different prisms to derive the formulae; and conversions in the SI unit system.

- Learner conceptions

In this section, I discuss how Alice addressed and emphasised the learners' correct conceptions, their misconceptions, and how she concluded with the definitions of surface area, volume and capacity. I then discuss how, using models, she showed the learners how to draw the nets of various objects, and lastly how she derived the formulae of a cube, a rectangular prism and a triangular prism.

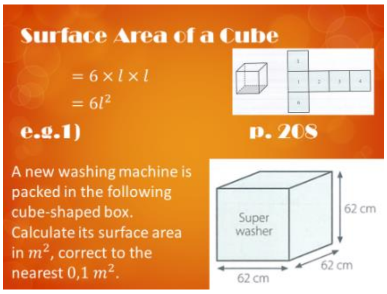
Learners' conceptions of surface area, volume and capacity

Learners' had several misconceptions regarding the concepts of surface area, volume and capacity. Alice started her explanation of the definition of surface area by showing a PowerPoint slide. For surface area, she took a 3D paper model and showed the learners that they needed a piece of carton board for every face of the model. Therefore, the area of every face of the object can be calculated with the formula for area of a square, or a rectangle or a triangle and added to give the surface area of the object. She told the learners that if you need to work out the surface area, you need to add all the areas of all the faces together. In the last lesson I observed, Alice used PowerPoint slides to explain the concept of volume and capacity. She told the learners that volume is 'the space that a block will take up' and capacity is 'what goes inside'. She concluded by saying it is 'basically the same thing'. Most of the learners had the correct conception of drawing the net of a rectangular prism in the baseline test. When Alice was explaining the drawing of the net of the different prisms in class, she demonstrated the nets with paper models that could unfold and corrected one of the learners who referred to the net as an 'open net'.

> *Teacher: If you draw the net of a rectangular container, it's the 3D thing, the/box that you must be able to put flat, right? That's a net…*
> *Learner: Must I draw an open net of a cylinder?*
> *Teacher: Yes, you don't have to say an open net, you can just say a net because the net is the open box. The sketch is the net.*

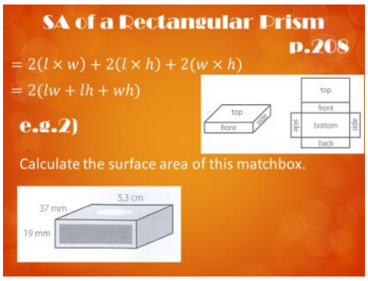
Surface area of the cube, rectangular- and triangular prism

In every lesson I observed, Alice started her lesson with a PowerPoint slide that included a thought or a joke. She explained the concept of surface area using a PowerPoint slide. The next slide included all the nets of the different prisms. While the slide was projected, she explained the nets while unfolding the models of the different prims. After the discussion of the nets, she explained the conversions and gave the learners a class activity on conversions. Thereafter, she explained the surface area of a cube using the example of a washing machine (see Picture 4.5).



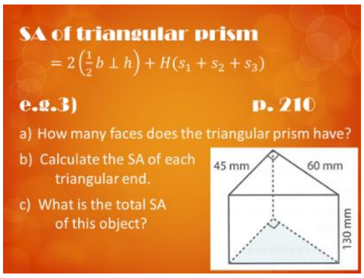
Picture 4.5: PowerPoint slide for explanation of surface area of a cube

For the rectangular prism, she explained the surface area by using an example of a match box (Picture 4.6).



Picture 4.6: PowerPoint slide for explanation of surface area of a rectangular prism

She explained the formula for surface area of a triangular prism with a net and did an example with the learners (see Picture 4.7).



Picture 4.7: PowerPoint slide for explanation of surface area of a triangular prism

<u>Applying a formula to a specific context</u>

After explaining the different nets and the surface area formulae for the different prisms, Alice gave the learners homework on calculating the surface area of different 3D objects. The homework was to calculate the surface area of a cube, a rectangular prism and a triangular prism correct to the nearest whole number. The learners were not asked to draw a net before attempting the homework questions. The learners struggled with the homework due to the fact that they were unable to derive the formulae for the different prisms and were unsure which formula to use for different prisms.

> *Teacher: All the sides are the same, then you know it's a cube. Then you use this formula, you substitute*, *type it into your calculator, round off… Okay, the surface area for rectangular prism… rectangular prism is a little bit different. Your surfaces are not the same size* [referring to the rectangular prism].

Alice explained the formula for the surface area of the rectangular prism as follows:

> *Teacher: Now you can't only work with one surface and then times it with 6* [referring to the surface area of the cube], *but you don't try to calculate all of them* [referring to the rectangular prism] *… you don't have to calculate this one* [showing to the base] *… you can calculate the top, times it with 2 because they are the same size. You can calculate the one side* [showing to the front face] *and times it with 2 because they are the same size. You can calculate the one side* [showing to the left side] *and they are the same size.*

Despite this, the learners were still confused about the dimensions of the rectangular prism and did not know which one was length, width and height.

> *Teacher: But listen, if it's a different length, width and height, in the end you are going to get the same answer. If you use this one as the length and the width and the height* [turned the sketch and changing the dimensions] *or if you turn it like this … it doesn't matter which way you turn it, you are going to get the same answer.*

This misconception was identified in the baseline test too, where learners could not indicate the dimensions correctly on their drawings.

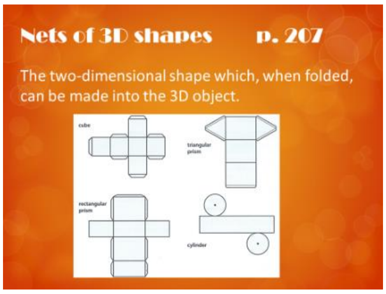
- Typical learner mistakes.

These comprise either unsystematic mistakes and systematic mistakes (Luneta & Makonye, 2010). In this study, I focused on systematic mistakes; recurrent wrong responses due to misunderstanding of the concept. In the baseline test, some of the learners struggled with the dimensions on the net of the rectangular prism. After one of the learners asked her a question on the dimensions, Alice then spent some time writing down the dimensions during her explanation of the triangular prism. The learners were unsure which side was the base, which side to use as height, and which one to use as perpendicular height. However, while she was explaining, one of the learners was very disruptive and some of the learners were not paying attention.

The learners in Alice's class were unable to use the net to derive a formula for the surface area of a rectangular prism. When the learners were asked in the baseline test to write down the formula for the surface area of a rectangular prism, 42.3% of the learners wrote the formula for volume.

> *Teacher: You must be able to recognise it … you must … if they give you a picture like this* [indicating the nets on the slide], *you must say … gives you a rectangular prism* [pointing to the net]*. Okay. This is a rectangular prism and the test was also a rectangular prism. It looks like a cube but they said it was a rectangular container.*

While teaching the formulae for the surface area of different prisms, she showed the learners a PowerPoint slide (see Picture 4.8) with the different nets, but did not derive the formulae for the surface area of the different prims for the learners. However, she did unfold the 3D paper models of every object while the slide was projected.



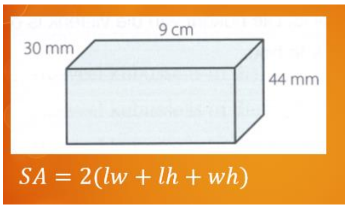
Picture 4.8: PowerPoint slide with the nets of different 3D objects

> *Teacher: Surface area is the area of all the faces added together. So I need a carton for the top and the bottom and two sides in the front and the back, it's all the sides, you add that together and … they are all the sides added together… is the area of all the faces added together.*

However, she forgot to mention that you must also add the left side and the right side as well. The learners also made systematic mistakes when they needed to adapt the formula of the surface area according to different scenarios, for example, when a container is open instead of closed. To address this difficulty, Alice explained the concept and formula for surface area of a rectangular prism. After Alice had explained the concept and formula, the learners wanted to use this exact formula for the surface area of an open container. She told the learners they had to know the basic formula for surface areas, but they should also be able to adjust the formula according to the context of the problem.

> *Teacher: They will tell you, this is the dog's cage and this side is open like the one of the test that you did, she* [referring to the researcher that compiled the baseline test] *said it is a rectangular container that's open at the top.*

Alice tried to pay attention to the nature of the learners' misunderstanding and typical mistakes that they made with regard to the surface area of different prisms. She selected tasks for homework from the textbook that ensured learners would gain new knowledge, skills and abilities, and would prevent learners from making the same mistakes. The homework she gave included a sum (see Picture 4.9 below) where the learners had to calculate the surface area of a rectangular prism.



Picture 4.9: Homework on surface area of rectangular prism

While she was marking the homework and explaining the formula for the surface area of the closed rectangular prism, she suddenly remarked to the learners that if one of the sides is not there, you have to 'adjust' the formula. The learners struggled to understand and she had to explain it again. She then explained the formula for the surface area of an open rectangular prism by using an example of a dog cage. The reason she did this was due to the fact that one of the learners was unsure about how to adapt the formulae of surface area for a closed rectangular prism to the formula for an open rectangular prism.

> *Teacher: You need to understand what the formulae is doing. If this is now a rectangular prism, if they say this is a little cage for a dog and that side is open, the dog must be able to get in there, the back that you don't see is closed okay? So, in other words, we calculate here width x height. I'm calculating the surface area but there isn't a block there, it's open but on that side is closed. So, I'm only calculating one rectangle wing* [indicating to the back of the rectangular prism] *…*

Learner: Mam, would you then write it outside the bracket because if you put it in the bracket it will be counted as two.

Teacher: Then you also multiply by two, so it depends on …

Learner: But I'm saying if … you say that the dog must be able to enter into the 44 mm side and then you have to put it outside the bracket otherwise it's going to be counted as both sides.

Teacher: Yes ... this is the dog cage, this side is open, the dog can go in there … the side is closed, so I don't have a carton there, I only have one here, so in other words this formulae says I calculate all the sides, but now a dog's cage you can go in there, there's no block here, which means this one which is the yellow, I mustn't calculate it two times, that's why I'm taking it out, so it's not there.

With this explanation, she attempted to address the problem of the open vase that was given in the baseline test, and tried to explain to the learners how to adapt the formula.

- Learner difficulties

The first difficulty that learners experienced in the baseline test was where they had to do conversions with the SI units. Alice attempted to rectify these learner mistakes by using her 'arm demonstration' as discussed under 'pre-concepts' (as discussed under Section 4.5.2.1). However, Alice concentrated on procedural fluency and the learners were rote learning the conversions by using the afore-mentioned rhyme to convert between units. By learning and applying this rhyme, the learners gained procedural fluency, but not conceptual understanding of how conversions work.

Teacher: You start with the large unit you end up with the small unit, where does your hands end … where it ends okay? So, if I go from metre to centimetres, square metres to square centimetres, metres is larger, I go from large too small. Where does my hands end?

Alice discussed it earlier in the year and still the learners struggled in the baseline test to convert between units of measurement.

Teacher: Okay, conversions, we have done it before … do you remember that?

Learners: No.

Teacher: Okay let me just recap on that again, if you go from a small unit to a large unit you divide, if you go from a large unit to a small unit, you multiply …

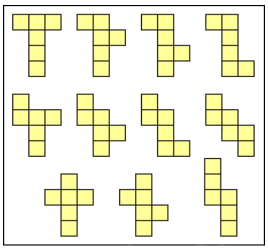
Secondly, the Grade 9 learners also found it difficult to change the subject of a formula yet Alice did not address this difficulty.

4.5.2.3 Various representations

Alice used a number of visual representations in the form of models of 3D shapes, including arm demonstrations as a visual representation to explain the conversion of units in the SI unit system as discussed under Section 4.5.2.1 'pre-concepts'. The models, like the one used in Picture 4.10 below, were made of paper and could easily unfold to show the net of the model. Alice used the paper model of the cube to show the learners that a cube can have different nets as shown in Picture 4.11.

Teacher: Okay, so if you see it's got squares and squares and squares, all of it is squares, you can have a cube … square and a square and a square … 1,2,3,4,5,6 [counting the squares] this is a net of a cube, this is also a net of a cube [unfolding a different net of another cube], these two can sit there, one square can sit there … one can sit there, that one can sit there, the net can look differently okay, so look out for the squares.

Picture 4.10: Alice using a model of a cube

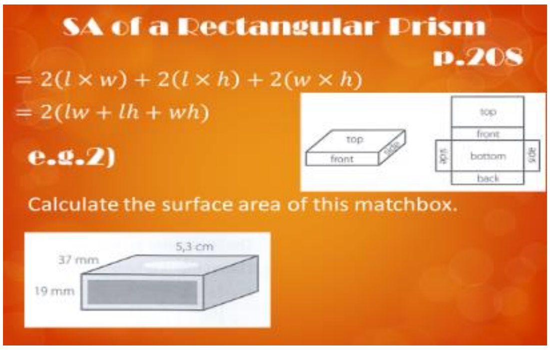


Picture 4.11: Different nets of the cube

Alice further used written and oral explanations and demonstrations in all her lessons to explain how to use the model to draw the net of objects; how to adapt the surface area formula if some of the faces are missing; and how to do conversions between different metric units.

4.5.2.4 Instructional strategies

Alice only used direct instruction, which was highly teacher-centred. The learners were, however, allowed to ask questions and engage with the problems. Alice firstly explained the concept of surface area of different prisms using the nets of those prisms. Secondly, she illustrated how the formulae for surface area for these different nets can be derived. Thirdly she demonstrated that volume is equal to the surface area of the base times the height. When they were busy with the surface area of a closed rectangular prism, she demonstrated, by using the model, how the formula for the surface area can be written in two different ways (see Picture 4.12 below).



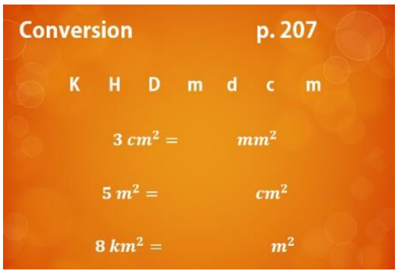
Picture 4.12: The two ways of writing the surface area formula for a rectangular prism

Her explanation of the derivation of the formulae above proceeded as follows (while she was pointing and referring to the diagrams in Picture 4.12):

> *Teacher: Okay, so if we just write it, length x width, if this is my base* [pointing to the base]*, this is my length, this is my width* [pointing to the sides of the container]*, so if I want to calculate the area of this bottom side it is the length x width* [pointing to the base]*, if I want to calculate this side* [pointing to the front part of the prism]*, it's the length x height … if I calculate the surface area it is length x height, width x height, length x width that's where we get the formula from, all that it means is, I work out the surface area of one of these blocks* [pointing to the base]*, let's say this block, and I times it with two, I don't need to calculate it twice, it's the same and this block* [pointing to the front face] *and I times it with two, and this block* [pointing to the right-hand side face]*, and I times it with two, on your right … the two letters next to each other means*

there's a x [multiplication sign] between them and two, two, two I take it out as common factor, so that's the simplified version of the equation.

When Alice was explaining conversions, she gave them a PowerPoint slide (see Picture 4.13) with the conversion rhyme, as well as the conversions they had to do as a class activity. The PowerPoint slide was projected on the white board and she filled in the answers as she was explaining.



Picture 4.13: The conversion of square units in the SI system

Her explanation of the conversions of the above class activity proceeded as follows (while she was pointing, demonstrating with her arms and writing on the white board):

> *Teacher: Okay, do these ones for me* [she gave them some time to complete the class activity]*. I say 3 square centimetres, how many square millimetres* [she wrote down the answer of 300 square millimetres]? *Five square metres, how many square centimetres* [she wrote down the answer of 50 000 square centimetres]*? Eight square kilometres is how many square metres … if I go from square centimetre to square millimetre, which one is larger? When you have sums like these in a test, I suggest that you write the rhyme and then with pencil you jump and then you can write the answer with your pen and you can jump again in pencil* [referring to a new sum].

4.5.2.5 Integration with real life contexts

Alice planned her instruction in such a way that the content of the lesson was integrated with other mathematical topics and real-life contexts, but not with other subject areas. Some examples will now be discussed with integration with real life contexts.

- Integration with other mathematical topics

When she explained the surface area of a rectangular prism, she showed the learners that they can simplify the equation by factorisation, as taught in the content area

Patterns, Functions and Algebra. She showed the learners the equations when two is taken out as common factor:

> Alice: *The two letters next to each other means there's a x (multiplication sign)* [pointing to the LB, LH and BH in surface area formula = 2LB+2LH+2BH] *between them and two, two, two* [pointing to the two in front of the LB, LH and BH terms] *I take it out as common factor, so that's the simplified version of the equation* [pointing to the formula of surface area = 2 (LB+LH+BH)].

When she explained the surface area of the triangular prism, she showed them how to use the Theorem of Pythagoras to work out the perpendicular height. This is dealt with in the content area of Measurement.

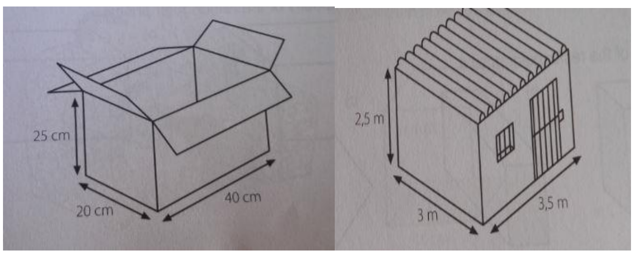
> Alice: *So we are looking for the height. So, I am going to say the perpendicular height is equal to … the square root of the hypotenuse minus the short side squared. Hypotenuse squared minus the shorter side squared, will give you the perpendicular height*.

Although the last part of the sentence above is incorrect, she indicated it correctly on the white board.

- Integration with real-life contexts

Alice included real-life contexts and cognitively demanding activities to enhance learners' understanding of doing conversions. One of the learners' difficulties was to determine in which unit of measurement to work. For homework, she gave them the following real-life problem about boxes that needed to fit in a store room.

> *Pumi has her own spaza shop. She buys sweets and chips in boxes* [see left-hand side of Picture 4.14]. *She has a small store room* [see right hand-side of Picture 4.14]. *How many boxes can Pumi store in her small store room?*



Picture 4.14: An example of one of the real-life context problems given for homework

She discussed the answers of the homework problems in the next lesson. The dimensions of the boxes were given in centimetres and the dimensions of the store room in meters, as seen in Picture 4.14 above. For learners to do the correct calculations, they needed to understand how to do the conversions with different units of measurements. Alice showed them two ways of solving this problem; either they had to convert the box's dimensions to metres or they could convert the store room's dimensions to centimetres.

Teacher: For the box, we can do it two ways. You can work in the centimetres then you get you answer in cube centimetres. Then you go from centimetre to metre. Two jumps. Centimetre to metre, divide. With six zeros. One, two, three, five, six, 0.02. Or you, you have to be in the same unit. So, either you take the box to cube metres, or you take the room to cube centimetres. But here you have to have the same unit. The volume of the room, 26,25 cube metres, divide by 0,02 cube metres. If you don't have the same unit, completely wrong answer, okay?

By using this homework activity, Alice integrated the conversions of SI units with a real-life context. Alice also explained the concept of volume with a real-life example. She derived the volume formula for a rectangular prism (LBH) by using an example of a box with slabs of chocolates.

Teacher: Good. If I have a rectangular prism ... And I packed the bottom layer with blocks of chocolate. I want a picture of blocks. Okay a slab. Okay, you want a slab. If this is my length and four blocks in the width. How many blocks can fit in the bottom layer?

Learner: Twenty.

Teacher: Okay, and how did you get that? You multiplied. And in the middle layer?

Learner: Twenty, is that not volume? [The learner was referring to the fact that she was explaining the concept of volume to the class].

Teacher: Ja, and then at the top layer also twenty… and when I add them, how many blocks of chocolate?

Learner: How many layers are there? Sixty.

Teacher: Sixty, okay? You have to understand what we're actually doing. But it's a bit of a long way. It's easier to remember area of the base times the height. So, it's one layer, how many blocks will fit into one layer? [Not waiting for response or discussion]. *How many layers are there?* [Not waiting for response or discussion]. *That is why I multiply with the height* [not stating that they need to multiply area with height].

The use of this real-life example of a slab of chocolates to explain the concept of volume grabbed the learners' attention and they were involved in the class discussion by giving the answers to Alice's questions. When Alice was explaining the last part of the volume and capacity concepts she did not give the learners a chance to respond, neither did she give the answers again. She concluded the discussion of volume as the area of the base times the height by comparing the layers of chocolate to the height of the object.

Alice then explained that the space a slab of chocolate occupies is volume, while the melted chocolate is capacity. However, this explanation can lead to a misconception among the learners because volume is the amount of space occupied by an object and is not necessary filled with substance.

> *Teacher: Okay, so volume is the space that a block will take up* [showing paper model of rectangular prism]*. Okay? Capacity is what goes inside* [open the paper model of the rectangular prism]*. It's basically the same thing, but the easiest way to remember it. Capacity, you work with fluids. With your litre type of things. Millilitre, litre, kilolitre. If you can pour water in here, that's capacity. Volume is if I put chocolate in. Capacity is melted chocolate.*
>
> *Learner: Volume is solids and capacity is fluids?*
>
> *Teacher: Yeah, it is almost something like that.*

4.5.3 Semi-structured interview

To understand Alice's Personal PCK, I divided this particular theme into the sub-themes: pre-concepts, learners' prior knowledge, various representations, instructional strategies, and integration. Each of these will now be discussed.

4.5.3.1 Pre-concepts

According to Alice, the only pre-concept that learners need is to understand the difference between surface area and volume: 'surface area is the outside and volume is the inside'. She also mentioned that they had done these concepts earlier in the year, as well as in Grade 8, and were thus supposed to know these concepts. When planning her lessons, she took cognisance of these pre-concepts, but due to time constrains, never used a formal or written baseline test to determine the learners' knowledge of these pre-concepts. Instead, she asked them oral questions before introducing the new topic.

4.5.3.2 Learners' prior knowledge

Through oral questioning during her instruction over the past years, Alice came to realise that learners have limited or almost non-existent prior knowledge on surface area, volume and capacity. She normally created opportunities for the learners to engage with their prior knowledge, which would also inform her of the learners' conceptions, typical mistakes and difficulties. She believed that learners not only need to have conceptual understanding, but also need to develop procedural fluency; 'they (the learners) think that they are supposed to understand everything. Yes, they must understand the formula, but then they have to go home and study it'. It was important to her that the learners memorise the formulae of surface area, volume and capacity in order to know the type of calculation - addition or multiplication.

Alice stated that the learners' main misconception revealed in the baseline test was that they could not distinguish between surface area and volume, 'they got confused between the volume and the surface area'. She stated that even without conducting a baseline test, the teacher should know the learners' prior knowledge, and in particular, the difficulties they might experience, 'I know what they were supposed to know and then I kind of start to explain. I look at their faces to see, do they know this?' Some of the difficulties the learners experienced was in reading, understanding and interpreting the questions when the baseline test was conducted, 'they did not read to see that the vase that they used was open at the top, and then they do not know how to manipulate the formula to see, okay, which part of the formula must they take out'. When explaining difficult topics, she believed that the teacher needs to link the abstract problem to something more concrete by using models or real-life examples, and also that 'you have to try to explain it from a different angle'.

4.5.3.3 Various representations

Alice used various 3D paper models to explain surface area and volume. She believed that using these visual models, especially by making the faces with the same sizes the same colour, made the abstract concept more concrete for the learners. The use of colour on the models Alice used helped the learners to understand and visualise the different areas of each 3D object. She also used written and oral language, diagrams, computers and calculators in her classroom. She also explained that PowerPoint slides are neater than handwriting, quicker, and the pictures are also visible to all learners. However, the slides meant that her back was turned to the class when she was doing the calculations and writing on the white board. She mentioned that she did not include the answers of the sums, but rather did the calculations with the learners during class time.

4.5.3.4 Instructional strategies

Alice only used direct lecturing as instructional strategy for this topic due to time constraints and disciplinary problems. When asked if she might consider other instructional strategies, she replied that she would not consider it due to the above reasons.

4.5.3.5 Integration

Alice integrated various mathematical topics during her instruction of the topic, for example, the Pythagoras' Theorem, factorisation and simplifying equations. However, she only thought she was integrating this topic with the conversions of SI units of measurement. Alice also integrated the topic with real-life contexts by doing calculations from the textbook, but did not integrate this topic with other subject areas. When asked about integration of the content of the lesson with other mathematical topics and other subject areas, she mentioned that it was difficult for her, 'I do not really get to do the integration. Maybe the conversions'. Although Alice did real-life examples from the textbook, she did not emphasise the importance of this topic in learners' daily lives.

4.5.4 Formative test

After the four classroom observations for each of the two classes and the one interview with Alice, the learners wrote a formative test. The purpose was to determine the influence of Alice's instruction on the learner outcomes by comparing the outcomes of the formative test with the outcomes of the baseline test. By comparing the answers, I could determine if learners' conceptions, typical mistakes and difficult concepts to understand were addressed during instruction. The formative test was the same as the baseline test, only the numbers in the questions were changed.

4.5.4.1 Learner conceptions

Question 1 assessed the learners' acquired knowledge and understanding of the concepts: surface area, volume, and capacity. This question was still answered very poorly in the formative test as learners still did not know the meaning of these three concepts. Below are the definitions, as stated in the memorandum, but similar meanings in the learners' own words were also considered and taken as correct. Surface area = sum of all the areas of all the surfaces of an object; volume = amount of space occupied by an object; and capacity = amount of substance contained by an object. Some of the answers obtained from the learners' scripts are given below in Table 4.11.

Table 4.11: Learner conceptions regarding surface area, volume, and capacity

Surface Area	**Volume**	**Capacity**
1.1 The calculation of all of the faces of a shape.	2.1 The calculation of the size of a 3D object.	3.1 The calculation of what is inside the object.
1.2 Measurement of the shape.	2.2 Is how big the thing is or how large.	3.2 What is filled or what inside something.
1.3. Are the sides of the container.	2.3 How deep an object is and how many things can fit into that object.	3.3 The inside of the object.
1.4. Is the amount of space an objects takes.	2.4 Is the length of the object	3.4 The volume of a shape but carries liquids.
1.5. Space outside the container or 3D object.	2.5 Is the area of the container.	3.5 The total amount of all the sides of the shape.

It can be seen that the learners were more capable, as compared to the baseline test answers, of clearly defining the meaning of these concepts in their own words. Learners understood the net of the shape, the faces, forms the surface area of the object (see Number 1.1), but did not specify that you need to add all of those areas together. In Number 2.3, it can be seen that the learner had a misconception as capacity, instead of volume, was described. A summary of the keywords used by learners for the definitions of these three concepts is given in Table 4.12.

Table 4.12: Learners' keywords used in defining surface area, volume and capacity

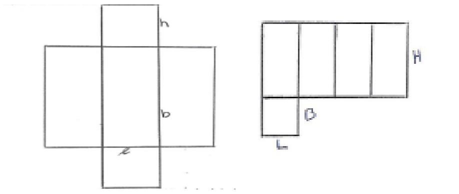
Surface area	'flat sides', 'space', 'outside', 'area around'
Volume	'amount hold', 'size', 'l x b x h'
Capacity	'weight', 'space'

It can be seen that there were fewer incorrect keywords used by learners in describing the concepts of surface area, volume and capacity. Most of the learners had the correct concept of a net and were able to correctly draw the net of the rectangular prism. In Table 4.13 it can be see that more learners in the formative test could draw the net correctly compared to the baseline test.

Table 4.13: Learner conception on the drawing of the net of an open rectangular prism

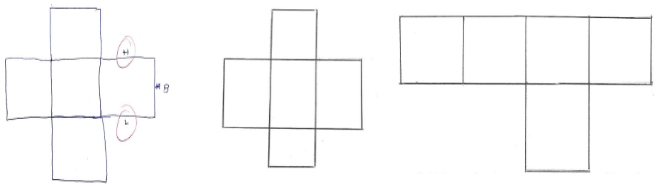
Type of net drawn	Number of learners	Percentage of learners
Correct net with dimensions	28	39.4
Correct net with wrong dimensions	19	26.8
Correct net without dimensions	14	19.7
Closed rectangular prims with wrong dimensions	3	4.2
Incorrect net	3	4.2
No response	2	2.8
Closed rectangular prism with dimensions	1	1.4
Closed rectangular prism without dimensions	1	1.4

The net that occurred the most in the scripts was when learners drew the net of the open rectangular prism with the base in the centre and the rest of the faces adjacent to the base, as can be seen on the left side of Picture 4.15. There were also a few learners who provided an alternative representation of the net for the open rectangular prism, as can be seen on the right side of Picture 4.15.



Picture 4.15: Two of the conceptions of the net of the open rectangular prism

In the formative test, most of the learners were able to draw the net, but some of them still struggled with the dimensions, some even leaving the dimensions out, as can be seen in Picture 4.16 below. The learners struggled to identify the length, breadth and height of an object, perhaps indicating a misconception.



Picture 4.16: Correct net of an open rectangular prism with incorrect or no dimensions

4.5.4.2 Typical learner mistakes

The most common error learners made in the baseline test was in finding the formula for the surface area of the open vase. In the formative test, most of the learners were again able to draw the net, but only a few managed to explain how to calculate the surface area using the net. After instruction, they were still unable to use the drawing of the net to find a formula for the surface area of the vase. Some of the learner mistakes in the formative test are given in Table 4.15 below.

Table 4.14: Learner mistakes regarding formulae for surface area of an open vase

Learners' answers	**Number of learners**	**Percentage of learners**
Answers like: 4L+4B, 2LBH, LB, LH, 4LB+2L^2, 2BL+2H, 0.5LBH, 2LB, 2$(LBH)^2$, 5LBH, 2L+2B+2H, LB-H, 4LBH, 2L^2+3LB, 2LB+2LBH, 3LB, LB+LBH	36	49.4
LBH	20	27.4
2LB+2LH+2BH	9	12.3
LB+2LH+2BH (Correct answer)	4	5.5

Learners' answers	Number of learners	Percentage of learners
LB	2	2.7
L+B+H	2	2.7

As seen above, nine learners wrote down the correct formula for a closed, rectangular container. However, they were unable to connect the idea of the net with the formula for surface area. Twenty of the 73 learners (27.4%) wrote down the volume formula for surface area, so their answers concerned the volume but not the total surface area of the vase. A greater variety of incorrect answers for the formulae of surface area appeared in the formative test compared to the baseline test. In Table 4.16, a comparison is drawn between the given formulae in the baseline and formative tests.

Table 4.15: Comparison between the formulae used for surface area in baseline test and formative test

Formulae used for Surface Area of open rectangular vase	Percentage of Alice's learners in Baseline Test	Percentage of Alice's learners in Formative Test
LBH	42.3	27.4
Incorrect answers	23.9	49.4
L+B+H	16.9	2.7
LB	15.5	2.7
2LB+2LH+2BH	1.4	12.3
LB+2LH+2BH (Correct answer)	0	5.5

In the baseline test, not one of the learners were able to write down the correct formulae for the surface area of an open rectangular container. In the formative test, more learners (12.3%) wrote the formula for a closed rectangular container compared to the 1.4% in the baseline test. There was a decrease from 42.3% to 27.4% in the number of learners who wrote the volume formula for a rectangular container down instead of the surface area formula for an open rectangular container. The percentage of learners who wrote down an incorrect answer increased significantly from 23.9% to 49.4%. Therefore, almost half of the learners in Alice's classes still had a totally incorrect answer for the formula of surface area after instruction.

The second problem that emerged from the baseline test was that if learners used the formula for volume instead of surface area, they made a mistake with the calculation for the volume of the container. After instruction, three of the four typical variations of typical mistakes occurred in the formative test as well. From Table 4.16 below it can be seen what percentage of learners made these typical mistakes in the baseline test compared to the formative test.

Table 4.16: Variations of typical mistakes in baseline test compared to formative test

Type of mistake	Percentage of learners in Question 2.3		Percentage of learners in Question 2.5	
	Baseline test	**Formative test**	**Baseline test**	**Formative test**
1. Using volume formulae (LBH) for both questions.	**42.3**	27.4	**23.9**	20.5
2. Using an incorrect formula (L+B+H) for both questions.	**16.9**	0	**5.6**	0
3. Using an incorrect formula (L+B+H) for surface area question and LBH for volume question.	**16.9**	2.7	**8.5**	16.4
4. Using volume formulae (LBH) for surface area question and an incorrect formula (L+B+H) for volume question.	**42.3**	27.4	**8.5**	2.7

4.5.4.3 Difficult concepts to understand

Learners seemingly struggled to understand and do conversions between units in the SI system. This was seen in the baseline as well as the formative test After Alice had spent a lot of time on conversions with arm demonstrations, a rhyme, and calculations, the learners were still unable to convert. In Table 4.17 a comparison between the answers obtained in the baseline and the formative test for Question 2.6 is given.

Table 4.17: Comparison of different answers obtained when converting m^3 to cm^3

Possible answers to convert m^3 to cm^3	**Percentage of learners in Baseline test**	**Percentage of learners in Formative test**
100	32.9	18.1
1 000	23.3	26.4
10	17.8	1.4
1 000 000 (correct answer)	6.8	26.4
1	2.7	0
0.1	2.7	2.8
No answer	2.7	8.3
1 000	1.4	5.6
Incorrect answers	8.4	11.2

Although there was an increase in the percentage of learners who did the conversion correctly, only 26.4% of the learners were able to convert m^3 to cm^3. There was also an increase in the percentage of learners who thought the answer is 1 000. In Table 4.18, the outcomes of the two tests are compared where learners had to convert from m^3 to litres.

Table 4.18: Comparison of different answers obtained when converting m^3 to litres

Possible answers to convert m^3 to litres	**Percentage of learners in Baseline test**	**Percentage of learners in Formative test**
100	27.4	9.7
1	17.8	4.2
1 000 (correct answer)	15.1	45.8
10	12.3	9.7
No answer	8.2	4.2
10 000	5.5	5.6
3	4.1	0
0.1	2.7	1.4
1 000 000	1.4	6.9
Incorrect answers	7.0	12.5

Although there was an increase in the percentage of learners who got the conversion right, it was still less than 50%. A great concern is that there was a bigger variety of other answers, including many decimal fractions. The learners may have been confused about whether they needed to multiply or divide.

4.6 PRESENTATION AND DISCUSSION OF DATA COLLECTED FROM MARY

4.6.1 Baseline test

The data obtained from the baseline test (see Attachment A) written by 58 learners in Mary's classes are now presented in terms of learner conceptions, typical learner mistakes, and concepts that are difficult to understand. After the analysis of this data, emerging issues that were not part of the deductive analysis process are discussed.

4.6.1.1 Learner conceptions

The first question focused on the learners' prior knowledge, conceptions, and typical mistakes in terms of the pre-concepts: surface area, volume, and capacity. The pre-concepts in teaching Grade 9 mathematics are derived from the prescribed curriculum (DBE, 2012) for Grade 7 and Grade 8 (see Table 4.10 under Section 4.5.2.1). This question was answered very poorly as learners were unable to write down the meaning of these three pre-concepts in their own words. Below are the definitions from the memorandum, but similar meanings in the learners' own words were also considered and taken as correct. Surface area = sum of all the areas of the surfaces of an object; volume = amount of space occupied by an object; capacity = amount of substance contained by an object. Some of the answers obtained from the learners' scripts are given below in Table 4.19.

Table 4.19: Learner conceptions regarding surface area, volume and capacity

Surface Area	Volume	Capacity
1.1 The area in which an object is calculated based on the top piece of the object.	2.1 Is the weight of shape or object that is being calculated with the mass as well.	3.1 The width and area determines the capacity.
1.2 Surface area is for example a box, now the bottom of the box is the surface area.	2.2 Volume is see the height of something for example the box we need to know what the height is.	3.2 Capacity is the weight of the box and we need to know the capacity of the box.

Surface Area	Volume	Capacity
1.3 Area around the surface	2.3 How much an object can be filled inside of it.	3.3. Capacity is how much an object weighs.
1.4 It is the surface of the 2D or 3D shape that you are working with.	2.4 It is the length, height and depth of a 3D shape that you are working with.	3.4 It is what is in the 3D shape. How full or empty is it?

As with Alice's classes, most learners had misconceptions regarding the difference between these concepts, but there were also a few learners who showed correct conceptions but an inability to clearly define the meaning of these pre-concepts in their own words. With regard to the surface area, 13.1% of the learners merely changed the word order and stated that 'surface area is the area of the surface'. Furthermore, 11.5% of the learners also said the surface area is the 'area around' (response 1.3). Another 8.2% of the learners thought surface area is only the area of the 'top piece' or the 'bottom of the box' (responses 1.1 and 1.2). Now with regard to volume, 4.9% of the learners thought volume involves the amount of mass or weight.   Another 21.3% of the learners thought capacity involves weight. Although the concept of mass and weight were not relevant to this study, learners did not understand these concepts and thought volume and mass are similar. Some of the learners had the right idea, but lacked the necessary vocabulary to define these pre-concepts (response 3.4). In Table 4.20, one of the learners defined capacity as the 'space in the object'. This learner also had the right idea, but lacked the vocabulary to define capacity. A summary of the keywords that learners used to define these three concepts is given in Table 4.20.

Table 4.20: Learners' keywords in defining surface area, volume and capacity

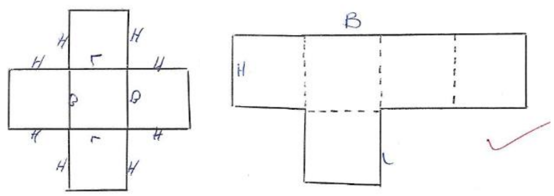
Surface Area	'outside of the object', 'size of the object', 'bottom part', 'area outside', 'LBH'
Volume	'l, b, h', 'weight or mass', 'height of the object', 'amount inside'
Capacity	'space in the object', 'size of the object', 'width and area', 'area of the object'

Question 2 of the baseline test assessed learners' prior knowledge of drawing and investigating the nets of cubes and rectangular prisms and they were able to answer this question. In Table 4.21, it can be seen that 72.4% of the learners were able to correctly draw the net of the rectangular prism.

Table 4.21: Learners’ conception of a net of an open rectangular prism

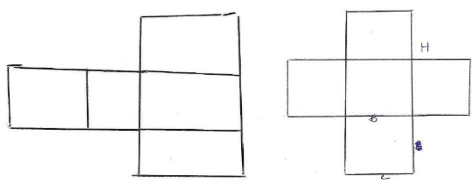
Type of net drawn	Number of learners	Percentage of learners
Correct net with dimensions	25	43.1
Correct net with wrong dimensions	14	24.1
3D rectangular prism	7	12.1
Closed rectangular prism with wrong dimensions	6	10.3
Correct net with no dimensions	3	5.2
Closed rectangular prism with dimensions	2	3.4
Incorrect net	1	1.7

The net that occurred the most in the scripts was where the learners drew the net of the open rectangular prism with the base in the centre and the rest of the faces adjacent to the base, as can be seen on the left in Picture 4.17. There were also a few learners who provided an alternative representation of the net for the open rectangular prism, as can be seen on the right in Picture 4.17.



Picture 4.17: Two of the conceptions of the net of the open rectangular prism

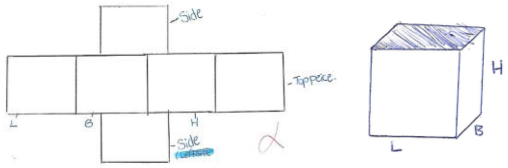
Some of the learners were able to draw the net, but struggled with the dimensions of the net as can be seen on the left in Picture 4.18. Some of the learners tried but struggled to identify the length, breadth and height of an object, as can be seen on the right of Picture 4.18. Also, the nets were not drawn to scale but the learners were not penalised for that.



Picture 4.18: Correct net of open rectangular prism: without and with incorrect dimensions

4.6.1.2 Typical learner mistakes

Learners generally found using a net to derive a formula for surface difficult to do and typical learner mistakes emerged here. Some of the learners drew an incorrect net for the rectangular prism and with the dimensions L, B and H on the wrong sides, as can be seen on the left of Picture 4.19. Some of the learners also struggled to understand the concept of drawing a net and redrew the picture of the rectangular vase as can be seen on the right of Picture 4.19.



Picture 4.19: Incorrect nets of the open rectangular prism

Another mistake that 36.2% of the learners made was to write down the volume formula for a rectangular prism instead of the surface area of a rectangular prism

Most of the learners were able to draw the net correctly, but only could write in their own words how to calculate the surface area by using the net. The most common mistake learners made was finding a formula for the surface area of the open vase. Some of the learner mistakes are given in Table 4.22 below.

Table 4.22: Learner mistakes regarding formulae for surface area of an open vase

Learners' answers	**Number of learners**	**Percentage of learners**
Answers like: 4LH+LB, 4HB+LB, 2LB, BH, 5LBH, 4L, LH+B, 0.5LBH, LB+L, 2L^2+LB, L^{2}4LB, 2LBH, 2(L+B)+2(B+L)+2(H+B), L^2+B^2+H^2.	22	37.9
LBH	21	36.2
L+B+H	12	20.7
LB	3	5.2
2LB+2LH+2BH	0	0
LB+2LH+2BH (Correct answer)	0	0

From the table, it is clear that not even one of the learners was able to use the net to derive the surface area formula. None of the learners wrote down the correct formula for a closed rectangular container. The most popular answers (62.1%) involved either the correct volume formula for a prism (36.2%) or an incorrect version where the length, breadth and height were added instead of multiplied (20.7%) or only length and breadth were multiplied (5.2%). The other 37.9% responses included a variety of answers.

4.6.1.3 Difficult concepts to understand

Learners seemingly struggled with understanding and doing conversions between units in the SI system. Here they were required to convert from m^3 to cm^3 and then to convert 1 m^3 to litre. The learners were also unable to manipulate the formula to calculate the height of the container, and were also confused about the unit of measurement to use for the height. Some of these answers are given in Table 4.23.

Table 4.23: Learner answers for conversions and height calculations

Conversions		**Calculating height of the water**
1 m^3 to cm^3	1 m^3 to litre	
1 m^3 = 3 cm^3	1 m^3 = 10 litre	20 cm
1 m^3 = 100 cm^3	1 m^3 = 1 000 litre	100 cm
1 m^3 = 10 cm^3	1 m^3 = 30 litre	10 cm^3
1 m^3 = 30 cm^3	1 m^3 = 1 litre	30 cm
1 m^3 = 0.1 cm^3	1 m^3 = 10 litre	500 (without unit of measurement)

Table 4.24 presents the responses in converting 1 m^3 to cm^3 and in Table 4.25, the learners had to convert 1 m^3 to litre.

Table 4.24: Analysis of different answers obtained when converting 1 m^3 to cm^3

Possible answers to convert m^3 to cm^3	**Percentage of learners**
1 000	33.9
100	16.9
3	6.8
1 000 000 (correct answer)	**5.1**
10	5.1
0.1	5.1
0.01	5.1
1	3.4

Possible answers to convert m^3 to cm^3	Percentage of learners
4.64	3.4
Incorrect answers that occurred only once in the learners' scripts (0.001, 1/5, 3.3, 9, 12, 30, 500, 3 000, 10 000)	15.2

Most of the learners (72.9%) did not have the correct number of zeros in their answers; and the answers were incorrect multiples of 10 (highlighted). Only 5.1% of the learners got the correct answer. Possible reasons for this and other misconceptions will be discussed in Section 5.2.2.3.

Table 4.25: Analysis of different answers obtained when converting 1 m^3 to litres

Possible answers to convert m^3 to litres	Percentage of learners
1 000 (correct answer)	**22.0**
100	22.0
1	13.6
10	10.2
10 000	5.1
3	5.1
No answer	5.1
0.01	3.4
0.001	3.4
3 000	3.4
Incorrect answers that appeared only once in the learners' scripts (0.0005, 5, 30, 1 000 000)	6.8

Although only 22% of the learners had the correct answer, this question had the highest percentage of correct answers of all the conversion questions. More than half of the learners (59.4%) did not have the correct number of zeros in their answers; and again, the answers were incorrect multiples of 10 (highlighted). A few learners wrote that 1 m^3 is equal to 3, 30 or 3 000 litres.

The last three questions in the baseline test were answered very poorly. The learners were guided to do a conversion first, then to determine the volume of the container that was not filled by water, and lastly to determine the height of the water in the container. Due to the fact that learners struggled to do the conversion, they were unable to determine the volume and the height of the water. Some learners were able to do the conversion right, then struggled with the last part where they had to manipulate the equation to make height the subject.

4.6.1.4 Emerging issues

The same issues emerged as from Alice's learner outcomes regarding calculating the surface area of an open vase (Question 2.3) and the volume of the container (Question 2.5). In most cases, the learners made the mistake of using the formula for volume instead of surface area. They also used the wrong formula for the volume of the container. There were four variations that arose from this typical mistake that occurred, namely, using: the volume formula (LBH) for surface area and volume; using the incorrect formula (L+B+H) for surface area and volume; using the incorrect formula (L+B+H) for surface area and the correct formula (LBH) for volume; and using the volume formula (LBH) for surface area and the incorrect formula (L+B+H) for volume. Table 4.26 indicates the percentage of learners who made these typical mistakes.

Table 4.26: Variations of typical mistakes made in the baseline test

Type of mistake	**Percentage of learners in Question 2.3**	**Percentage of learners in Question 2.5**
1. Using volume formulae (LBH) for both questions	36.2	15.5
2. Using an incorrect formula (L+B+H) for both questions	20.7	1.7
3. Using an incorrect formula (L+B+H) for surface area question and LBH for volume question	20.7	17.2
4. Using volume formulae (LBH) for surface area question and an incorrect formula (L+B+H) for volume question	36.2	8.6

These results assisted me in observing how the teachers' Personal PCK&S were informed by these outcomes. Among other things, I observed how the teachers addressed the learners' conceptions, typical learner mistakes, and concepts that are difficult to understand.

4.6.2 Classroom observations

As before, I divided this theme into the sub-themes: pre-concepts, learners' prior knowledge, various representations, instructional strategies, and integration. Each of these sub-themes will now be discussed.

4.6.2.1 Pre-concepts

In this section, I discuss how Mary integrated the pre-concepts applicable to teaching surface area, volume, capacity and conversions into her teaching of the curriculum content as prescribed in CAPS (DBE, 2012).

- Pre-concepts according to the curriculum

The pre-concepts in teaching Grade 9 mathematics are the same as for Alice's practice and are discussed under Section 4.7.2.1.

- Mary's use of pre-concepts

During all of Mary's lessons, she assessed the learners' existing knowledge on pre-concepts regarding conversions between different units of measurement; rounding off numbers; and various objects and their formulae prior to her formal teaching of the day's lesson. This was done in the form of discussing learners' ideas and testing their prior-knowledge through questioning. She continuously linked the learners' prior-knowledge with their new knowledge.

In a classroom discussion during the first lesson that I observed, Mary asked the learners why we need to do calculations involving surface area. The learners' answers involved real-life context examples like painting, tiling and gift wrapping. Before explaining the formulae of the surface area of different objects, Mary briefly explained the difference between 2D and 3D objects. She explained that a cube is 3D and asked the learners to look around the classroom and give examples of any 3D object they observed. The learners responded by giving examples like the cupboard, the paper and the posters on the wall. The discussion continued as follows:

> *Teacher: Wrapping a gift, how about covering your text book, that will mean surface area?* [She was summarising the classroom discussion]. *Right people, you know 3D, 3 dimensional objects, what is the difference between 3D and 2D?*

Learner: 2D is like a flat shape and 3D is a bigger picture …

Teacher: Good, so point at anything that you see in this class that is 3D.

Learner: The cupboard [indicating to the cupboard in the classroom].

Teacher: That's right, the cupboard, yes.

Learner: Paper, posters [pointing to the posters against the walls of the classroom].

Teacher: [Without correcting the learner, Mary continued as follows] *Alright people we are going to look first of all to the cube, right, the cube is 3D, what is then the 2D of a cube?*

Learner: Square.

Teacher: A square, right, so you need to know that a cube has six faces and they all look exactly the same, right? So, if we take the projector as a cube [indicating to the overhead projector that she was using in front of the classroom]*, explain to me what will the face be?*

Learner: A square.

Teacher: One of the squares remember, so these squares look exactly the same squares, all four sides are equal in length. So, can anybody tell me how do you calculate the area of a square?

Learner: Length times breadth.

Teacher: Okay, length times the breadth can work as well, but the breadth and width is the same. The length and the width is the same, because they have the same length, so remember L squared or S-squared… any of those will work.

From the above, it can be seen that Mary involved the learners in her explanations and probed with questions to determine their knowledge of the pre-concepts regarding properties of 2D and 3D objects and surface area of 3D objects. She did not give them the definition of the surface area of an object, but in a classroom discussion she asked the learners to provide real-life examples of where surface area is used. The real-life examples included single word answers like tiling, painting and gift wrapping. Her explanation of surface area was not always proficient due to the fact that she was not teaching in her mother tongue and did not elaborate or follow-up on learners' answers. One of the learners also mentioned that paper and posters were 3D and Mary did not correct the answer and continued with her explanation. However, Mary's learners cooperated and tried to answer all her questions.

In the third lesson, Mary explained the pre-concepts of volume and capacity. However, when she explained the pre-concepts of this topic, she did not use a real-life context and told the learners only to write down the definitions as follows:

Volume is the amount of space taken up by the object Capacity is the amount of space available inside the container or object

She continued with the following discussion:

> *Teacher: So today we are going to write down the definitions … Volume is the amount of space taken up by the object. So, it is the amount of space taken up by the object. So, it refers to the exterior part of the object, okay? Next word, capacity. Capacity is the amount of space available inside the container or the object. So, capacity is the amount of space available inside the container, or the object you are working with. Alright we are looking at formulae. Okay, there we have the first, what is the first diagram?*
>
> *Learner: A cube.*
>
> *Teacher: A cube, right and a cube's volume, is what? Okay, I refer to a side as L, so it will be L to the power of three, or L x L x L.*

From the above, it can be seen that she emphasised the point that volume refers to the *exterior part of the object* and capacity the amount of space available *inside the object*.

To explain the calculations involving conversions, she asked the learners about the relationship that exists between SI units, asking questions such as the following:

How many millimetres are there in 1 centimetre? One metre equals how many centimetres? How many metres are there in a kilometre?

Learners needed to memorise the basic relationships in the SI unit system and then had to convert to another SI unit of measurement. By doing this, learners had the opportunity to gain conceptual understanding of the relationship between the different units of measurement. The following is an example of such discourse in class:

Teacher: First of all let us quickly go and have a look at our conversions, so this is study work please. Conversions, so impress me today please [referring to the fact that they have done this earlier in the year]*. How many millimetres are there in 1 centimetre?*

Learner 1: 1 000 [incorrect answer].

Learner 2: 100 [incorrect answer].

Teacher: Okay, relax … 1 centimetre squared equal to 10 millimetres times 10 millimetres squared, remember times 10. So, $1cm^2$ will then be equal to 100 mm^2. Okay, right, the next one, 1 metre equals to how many centimetres? It is not the first time we are looking at this, please come on, so $1m^2$ will be equal to 100 centimetre x 100 centimetre, so that will be 10 000cm^2. Right, then the last one, how many metres are there in one kilometre?

Learner: 1 000.

Teacher: Great, now at last you can say 1 000 that is correct, so $1km^2$ is equal to 1 000 metres times 1 000 metres; 1 million metres squared equal to one kilometre squared right?

In all three lessons, Mary spent a lot of time on the pre-concepts, thus prior knowledge, on rounding off numbers when doing calculations; units of measurement; and conversions between different units of measurements.

4.6.2.2 Learners' prior knowledge

Prior knowledge refers to individual learners' conceptions; typical learner mistakes; and learner difficulties regarding the pre-concepts of surface area, volume, and capacity; the drawing of the nets of different prisms to derive the formulae; and conversions in the SI unit system.

- Learners' conceptions

In this section, I discuss how Mary dealt with learners' correct conceptions and misconceptions.

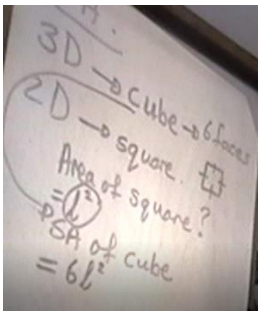
<u>Learner conceptions of surface area, volume and capacity</u>

Learners' explanations of the meaning of the concepts of surface area, volume and capacity in the baseline test showed several misconceptions. Mary started her explanation of the definition of surface area by asking learners questions. Although the learners' answers were inadequate during the classroom discussion, it can be seen that they had the correct conceptions about the concept of surface area. However, they were unable to describe the meaning of surface area in the baseline test in their own words. With regard to the concepts of volume and capacity, Mary only gave the learners the definitions of volume and capacity to write down in their scripts. She told the learners that 'Volume is the amount of space taken up by the object', therefore it refers to 'the exterior part of the object.' For capacity, she stated that 'Capacity is the amount of space available inside the container or the object'. Although Mary did not elaborate on the incorrect answers learners gave in the baseline test, she gave them the necessary vocabulary to define the concepts of surface area, volume and capacity in their own words.

As shown in the baseline test, most of the learners had the correct conception of drawing the net of a rectangular prism. To further strengthen their understanding of nets, Mary demonstrated the nets with paper models that could unfold. She further used the overhead projector to show the learners a sketch of the net where she indicated the dimensions on the sketch or next to the sketch. After she showed the different nets of the cube, the rectangular prism and the triangular prism, she directly derived the formulae for surface area of that particular prism. Mary emphasised the use of the net for deriving the formulae. When it comes to doing calculations, Mary wanted the learners to write down the dimensions on the net before substituting the values into the formula.

<u>Surface area of the cube, rectangular- and triangular prism</u>

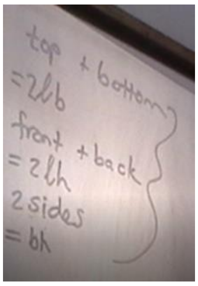
When she was explaining the concept of surface area of a cube, she showed the learners the net of the cube. She derived the surface area formula of a cube from the net by using the area of a square, stating that they had to calculate the sum of the areas of all six faces of the squares (see Picture 4.20). She explained this by using the same real-life context example as Alice, the example of a washing machine (see Picture 4.5 under Alice's 'Learner conceptions').



Picture 4.20: Explanation of surface area of a cube

She then moved over to the surface area formula of a rectangular prism. Mary started with the net and afterwards applied the new knowledge to a real-life context using the same matchbox example as Alice (see Picture 4.6 under Alice's 'Learner conceptions'). She explained the formula of the surface area by indicating to the different faces of the rectangular prism and numbering the faces on the net. From Numbers 1 and 3 on the net she derived the area formulae of the top and bottom part, and from Numbers 2 and 4 she derived the area formulae of the front and back. From Numbers 5 and 6 she derived the area formulae of the sides. She added all the area formulae to obtain the surface area formulae and proceeded as follows:

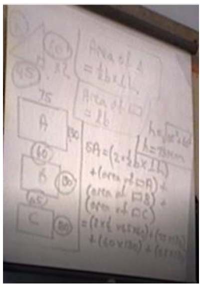
> *Teacher: Okay, so again, the net of a rectangular prism looks like this, the net is two dimensional. Okay so, right quickly, check Number 1 and Number 3 gives the top and the bottom, so it means that this rectangle Number 1 and rectangle Number 3 are the same size, okay? Then look at the front and the back, that would be Number 2 and Number 4, they will be exactly the same and then you have the sides, Number 5 and 6, you see that, the sides are the same, right?*



Picture 4.21: Explanation of surface area of a rectangular prism

From Picture 4.21, it can be seen that she wrote the area formulae for the two sides incorrectly, but corrected it as soon as she derived the surface area formula for the rectangular prism by adding the two in front of the term.

To explain the surface area of a triangular prism, she again started with a net and stated that the five faces consist of two triangles and three rectangles that are not the same size. She mentioned that the rectangles would be the same size if the triangular prism consisted of two equilateral triangles. She involved the learners by doing an example of a triangular prism with the learners (see Picture 4.22 below).



Picture 4.22: Explanation of the surface area of a triangular prism

From Picture 4.22, it can be seen that she first wrote down the area formulae for a triangle and a rectangle. After that, she explained to the learners how to calculate the perpendicular height using Pythagoras' theorem. She continued by labelling the three rectangles and wrote down their dimensions. Afterwards, she wrote down the formula for the surface area of a triangular prism, substituted the values, and obtained the answer with the unit of measurement.

Applying a formula to a specific context

The learners did not understand that you sometimes need to adapt the formula according to the specific scenarios. Mary addressed this by asking learners questions about finding the surface area in a real-life context. She gave them a homework activity where they had to work out the surface area of a rectangular house with a triangular roof. She guided their thinking on how to solve the calculation as follows:

> *Teacher: You have to calculate the surface area of that house, so quickly tell me if you paint a house on the outside, the exterior of the house, we will not paint between the roof and the base of the house, would we? No, okay, and we will not paint the bottom part of the house, right?*

- Typical learner mistakes

For the purpose of this study, I focused on the systematic mistakes made. Firstly, the outcomes of the baseline test showed that the learners struggled with the dimensions (L, B and H) or completely left out the dimensions of the net of the rectangular prism.

Mary spent a lot of time sketching the objects and writing down the dimensions on the drawing of the object. Before attempting an example in class, she always wrote down the dimensions and did the conversion (if necessary) before substituting it into the formula. Secondly, the learners in Mary's class were unable to use the net to correctly derive a formula for the surface area of a rectangular prism, as 36.2% of the learners gave the formula for volume instead of the surface area of a rectangular prism. When Mary was teaching the formulae for surface area of different prisms, she derived the formulae for the surface area of the different prisms from the net for the learners.

> *Teacher: So right, a rectangular prism would be something like the cupboard, so once again this is 3D okay, what will then be 2D?*
>
> *Learner: The faces are rectangles.*
> *Teacher: Rectangle, good, okay. But they are different sizes do you see that? Okay so again the net of a rectangular prism looks like this, the net is 2D… let us start with the top and the bottom … okay let us go to the front and the back … then the last part is the two sides …*

With this explanation, Mary attempted to help the learners to write down the formulae of different prisms. If the learners followed these procedures, they would have been able to adapt the surface area formula for different prisms, if necessary. Therefore, indirectly she addressed the problem of the open vase that was given in the baseline test and tried to explain to the learners how to derive or adapt a surface area formula.

Thirdly, Mary also tried to pay attention to the nature of the learners' misunderstanding and typical mistakes they made regarding the volume of different prisms. She selected specific tasks for homework from the textbook that had the potential to develop learners' knowledge, skills and abilities, and prevent them from making the same mistakes again. While she was explaining the formula for the volume of a cube and a rectangular prism, she emphasised that there is a multiplication sign between the letters in the volume formula of a rectangular prism. She thus addressed the problem of the volume of the open vase in the baseline test.

> *Teacher: Alright, we're looking at volume. Okay, there we have the first, what's the first diagram?*
>
> *Learner: A cube.*

Teacher: A cube, right, and a cube's volume is what? Okay, but did I refer to a side is L, so it will be L to the power of three, or L x L x L. Okay, are you writing down? You write down. The cube [referring to the heading], *you don't have to sketch the cube. A cube's volume or volume of a cube. So, if you want to write side cube, you may do that or side by side by side, or A x A x A. It doesn't matter, okay? The next one is rectangular prism, rectangular prism* [referring to the heading]. *Volume of a rectangular prism, okay, is the length times the breadth or the width times by the height. So, we are going to say it is L x B x H, LBH* [saying this while writing down the formula for volume of a rectangular prism on the overhead projector]. *So, there's nothing, no sign in between the letters, so what was it again?*

Learner: Multiplication.

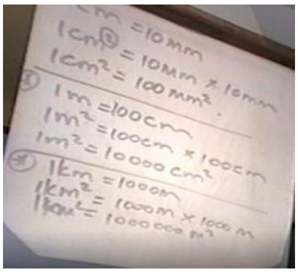
Teacher: Multiplication, ne? So, make sure you know that.

- Learner difficulties

The first difficulty learners experienced in the baseline test was in doing conversions with the SI units of measurements. Mary rectified this by using her three questions as discussed under Section 4.6.2.1, 'Mary's use of pre-concepts'. The learners had to rote learn or memorise how the conversions should be done. However, they could gain conceptual understanding if they visualised the conversion where Mary explained that one millilitre will fit into one cubic centimetre cube.

Teacher: Right, we have done the conversions. Millimetres cube to centimetres cube, we have done that. Right, we have written that down, but the following is the conversions between unit of volume and capacity. So, the volume will be in millimetres cubed, centimetres cubed, metres cubed, kilometres cubed. Right, you have that? But the capacity will then be in millilitres, litres, kilolitres, right? Okay and the first one that you should know is one centimetre cube is equal to a thousand millimetres cubed… and one cubic centimetre is equal to one millilitre. So just quickly think for yourself … look at your ruler. One centimetre, if you check the size of one centimetre, you see it is very small. You draw a cube with one centimetre by one centimetre by one centimetre, it is like tiny, tiny right? One millilitre can fit into that.

The learners struggled in the baseline test to convert between units of measurement despite doing it earlier that year. They also struggled with conversions of length, area and volume. Mary not only assessed and understood the learners' prior knowledge, but had a different way of explaining conversions and rectifying common mistakes the learners made. The learners needed to learn the basic conversions to change millimetres to centimetres and then to derive the rest of the squared and cubed units from the basic conversion (see Picture 4.23 in Mary's own words).



Picture 4.23: Explanation of conversion of SI units of measurement

From the explanation, it can be seen that the learners gave two wrong answers; they said that there are either 1 000 millimetres or 100 millimetres in a centimetre. After two learners gave the incorrect answers, Mary gave the right answer of 10. When she asked the class how many metres there are in a kilometre, one of the learners did give her the correct answer of 1 000. Moreover, the Grade 9 learners also found it difficult to change the subject of a formula, for example, when they were required to determine the height when volume is given. Mary did not address this in her class.

4.6.2.3 Various representations

Mary used a number of visual representations in the form of models of 3D shapes to show learners the net of the different objects to enhance the learners' understanding of surface area. During her explanations and demonstrations, Mary used written and oral language in all her lessons, making sure that the learners copied the work in their scripts. By using written as well as oral language at the same time, she enhanced the learners' conceptual understanding.

> *Teacher: Quickly. Number five, there's a box. You write down length, the breadth and the height of that box. On the next page, they give you a store room and you do the same. You write down the length, the breadth and the height of that store room. When you're done with that, you should see that they have*

different units. You see that? So, I want you to convert the centimetres to metres.

Another example of using various representations is when she was explained the problems based on a triangular prism. She instructed the learners to take the object and to label the different rectangles in order to avoid any possible mistakes and misunderstandings as the rectangles were not congruent and had different dimensions. She asked the learners to write down the dimensions of the different faces and to make sure they had the same units.

> *Teacher: So we have two of those* [referring to the triangles of the triangular prism]*, then we have three rectangles, if you check the one here and the front and then there's one that side, and one this side … then you will agree that they are not the same size … okay they are not the same size, so they are not congruent. So, let's start with the first rectangle, the first one that we have there, it starts with the one in front. We are going to name it rectangle A, okay, and the next one B and the last one C. Okay, so we are going to write in the measurements here.*

4.6.2.4 Instructional strategies

The instructional strategy Mary used in all her lessons was direct instruction, however, the learners were allowed to ask questions and engage with the problems. Mary followed a specific sequence when explaining concepts. Firstly, she explained the concept of surface area with real-life contexts. Secondly, she explained the concept of the surface area of different objects using the nets of those prisms. Thirdly, she explained that volume is equal to the surface area of the base times the height of the object. Moreover, when explaining the surface area of the triangular prism, Mary demonstrated two ways of solving the problem: either solving the different parts separately or doing it as one calculation.

> *Teacher: Okay, let's go to calculating the surface area of the triangular prism, so you going to calculate the area of two triangles, so it's going to be two times half of the base or base x perpendicular height okay? Then the next one we add the area of rectangle A plus the area of rectangle B and area of rectangle C, right and from here on it is substitution okay. So, let's look at the first bracket there, we have 2 x half x the base, 45 x 60 - can you see we have done all our dimensions that we need to use. The next one with rectangle A is going to be*

75 x 130, B will be 60 x 130 and lastly, C will be 45 x 130. So, all of that, you can either go and get the answers now separately for each bracket and then add them together or from start, type it into your calculator and get to the answer.

Mary gave the learners three basic conversions to rote learn based on the three questions discussed under ‘pre-concepts’ (Section 4.6.2.1 ‘pre-concepts’), and then asked them some questions on how to convert between the units of measurement. While the learners were giving the answers, she wrote them down on the overhead projector as can be seen in Picture 4.23 (above under ‘Learner difficulties’).

4.6.2.5 Integration

Mary planned her instruction in such a way that the content of the lesson was integrated with other mathematical topics and real-life contexts, but not with other subject areas. Some examples will now be discussed.

- Integration with other mathematical topics

When Mary explained the surface area of a rectangular prism, she also showed the learners that they can simplify the equation by factorisation, which is taught in the content area Patterns, Functions and Algebra. When she wrote down the formula for surface area, she swapped the length and the breadth around. One of the learners asked her if there is a difference between these two parts of the formula, namely LB and BL.

Learner: What is the difference? [Referring to the formula on the board]
Teacher: No, remember it is multiplication, so if you swap the breadth and the length and the height whatever, it stays exactly the same.

In the above discussion Mary referred to the commutative properties of numbers in algebra. Then, when she was explaining the surface area of the triangular prism, she also showed them how to use the Theorem of Pythagoras to work out the perpendicular height. This is dealt with in the content area of Measurement.

Teacher: Okay, let us start with Pythagoras. To calculate the hypotenuse here you will take the square roots, square the shorter side plus the other side and type it into your calculator and you will get to 75 millimetres.

Although she did not specify that they needed to square ‘the other side’ in her explanation, she indicated it correctly on the board.

- Integration with real-life contexts

Mary explained the concept of surface area with real-life contexts by using the overhead projector in front of the class.

> *Teacher: You need to know that a cube has six faces and they all look exactly the same, right? So, if we take the projector as a cube* [indicating to the overhead projector that she was using in front of the classroom]*, explain to me what will the face be?*
>
> *Learner: A square.*

Mary included real-life contexts and cognitively demanding activities to enhance learners' understanding of doing conversions. One of the learners' difficulties was in determining in which unit of measurement to work. For class work, she gave the same contextual problem as Alice about boxes that needed to fit in a store room (see Section 4.7.2.5). She discussed the learners' answers after she gave them some time in class to do the calculation on their own. The dimensions of the boxes were given in centimetres and the dimensions of the store room in metres. Here, learners needed to understand how to do the required conversions. Mary used her approach of first writing down the dimensions of the boxes and the store room and doing the conversions first before attempting the problem.

> *Teacher: Quickly. Number five there is a box. You write down the length, the breath and the height of the box. On the next page, they give you a storage room and you do the same. You write down the length, the breadth and the height of that store room. When you are done with that, you should see that they have different units. You see that? So, I want you to convert the centimetres to metres. So, the question here is how many of these boxes can fit into the store room? Okay, that's question five. How many boxes can fit into the store room? So, this is the calculation that you will do. You are going to take the length, or determine how many boxes can fit into the length. How many can fit into the breadth, and how many can be stacked on each other. In other words, the height. Okay, so for the length we are going to take the store room's length. So we do the same with the breadth.*

Mary guided the learners' thinking, simplifying this real-life context problem for the learners. The learners thus understood why they had to convert the dimensions of the boxes, however, she did most of the work and did not build her instruction on the learners' contributions.

4.6.3 Semi-structured interview

To understand Mary's Personal PCK, I divided this into the same sub-themes as the observations, namely: pre-concepts, learners' prior knowledge, various representations, instructional strategies, and integration. Each of these will now be discussed.

4.6.3.1 Pre-concepts

Mary had a good idea of the pre-concepts the learners were supposed to have about surface area, volume and capacity. According to Mary, learners must be able to define surface area, volume and capacity; differentiate between surface area and volume; and apply these concepts in their daily lives as they had already covered these topics in the Grade 8 syllabus. In Grade 8 the learners also learned to substitute the values in all the different formulae, but did not learn to manipulate a formula or to get the perpendicular height with regard to the triangular prism. When planning her lessons, Mary took cognisance of these pre-concepts, but due to time constrains, never used a formal or written baseline test to determine the learners' actual prior knowledge. Instead, she asked them oral questions before introducing the new topic. Mary also mentioned that the learners could obtain any mathematical formula or definition from Google and by doing so, revise their pre-concepts. However, the knowledge learners lacked the most was how to use or apply these formulae obtained from the teacher, the textbook or Google, especially in daily life.

4.6.3.2 Learners' prior knowledge

Mary graded the Grade 9 learners' prior knowledge on surface area, volume and capacity as limited or almost non-existent. She believed that the prior knowledge the learners lacked the most was in describing where they would apply surface area and volume in their daily lives. She also maintained that the more experience teachers gain, the more they will be able to plan their lessons based on the prior knowledge of the learners and anticipate any mistakes learners might make. To help learners to engage with their prior knowledge, she tried to connect the mathematical topic with a real-life context.

When learners continued to misunderstand a new concept, she would use peer-teaching, and sometimes the learners who understand the concepts would stand in front of the class or they would explain to each other in pairs or groups,

> *Even after I have explained something to the class and asked them who understand it … only about four or five raised their hands. So, I asked one of them [referring to one of the Grade 9 learners in the class] that is certain that they understand to explain to the rest of the class. There is one on their level [referring to a learner] and they, perhaps tell them something that I assumed they know how I got to it […] the boy went to the front, he explained just one thing that I did not explain where I got it from and suddenly everybody understands*

According to Mary, the misconceptions that were revealed in the baseline test were that learners were unable to differentiate between surface area and volume; manipulate the formula to determine the height; and did not know how to use the net to derive the formula. However, Mary always took the learners' prior knowledge into consideration when planning her lessons. She mentioned that most of the time she needed to start at the beginning because learners would react to the work as if they had never seen it before in their lives. To create and utilise opportunities for the learners to engage with their misconceptions and to reconstruct or adapt their knowledge, she always asked them questions, then provided them with definitions and connected the concept with something in their daily lives. When dealing with difficult topics, Mary would work slowly through an example and then give the learners one to try on their own. She would also let them write down the steps in words and connect the difficult concepts with something in their daily lives.

4.6.3.3 Various representations

Mary used 3D visual models of the different prisms to explain surface area and volume. Due to time constraints, she did not use other representations, but believed that different measuring cups and jugs would help learners with the concept of different measuring units. Mary used the overhead projector when explaining the concepts; the different formulae for surface area and volume of various prisms; and to do the step-by-step calculations with the learners. She did, however, remark on other resources she could have used to teach this topic more effectively, such as various actual objects and illustrations.

4.6.3.4 Instructional strategies

Mary only used direct lecturing as instructional strategy due to a lack of time. She thought she could have used discussions and case studies to teach surface area and volume, but because of the time constraint decided not to do so. She mentioned that group work would not have worked in these classes.

4.6.3.5 Integration

Mary integrated a lot of mathematical topics into her classroom. She also connected her lesson with another subject area, namely, technical drawing, when the net was used to create a three-dimensional object. Mary stated that by connecting the concepts of surface area and volume with real-life examples, you create and utilise opportunities for learners to engage with the misconceptions and to reconstruct or adapt their prior knowledge, as well as dealing with topics that are difficult to teach and understand.

4.6.4 Formative test

After the three classroom observations for each of the two classes and the one interview with Mary, the learners wrote a formative test. The purpose was to determine the influence of Mary's instruction on the learner outcomes by comparing the outcomes of the formative test with the outcomes of the baseline test. I could thus determine if learners' conceptions, typical mistakes and difficult concepts to understand were addressed during instruction. The formative test was the same as the baseline test, only numbers in the questions were changed.

4.6.4.1 Learner conceptions

The first question focused on the learners' acquired knowledge and typical learner mistakes. Question 1 assessed the learners' acquired knowledge and understanding of the concepts: surface area, volume, and capacity. This question was still answered very poorly in the formative test. Below are the definitions as stated in the memorandum, but similar meanings in the learners' own words were also considered and taken as correct. Surface area = sum of all the areas of all the surfaces of an object; volume = amount of space occupied by an object; and capacity = amount of substance contained by an object. Some of the answers obtained from the learners' scripts are given below in Table 4.27.

Table 4.27: Learner conceptions regarding surface area, volume, and capacity

Surface Area	**Volume**	**Capacity**
1.1 Is all the faces of an object added together.	2.1 Is when calculating the whole object.	3.1 The objects weight.
1.2 Is the base of the object.	2.2 The mass taken out of space.	3.2 Is the space available around the object.
1.3. Calculations that contain the length, height and breadth of any 3D object.	2.3 The amount inside of the container.	3.3 The amount of space taken out of a container.
1.4 The amount of the nets faces added together.	2.4 The amount that can go into a container.	3.4 The amount of space it can hold.
1.5. The shape all around added together.	2.5 The amount of the whole shape inside.	3.5 Multiply length, breadth and height with the amount of faces.

From the table above, it is clear that, compared to the baseline test answers, the learners were more capable of clearly defining the meaning of these concepts in their own words. Learners understood that the net of the shape forms the surface area of the object (see Number 1.1), but did not specify that you need to add all of those areas together. In Numbers 2.3, 2.4 and 2.5 it can be seen that the learners lacked the necessary vocabulary to describe volume as all the definitions included the word 'amount'. A summary learners' keywords in defining these three concepts is given in Table 4.28 below. It can be seen that learners used fewer incorrect keywords.

Table 4.28: Learners' keywords used in defining surface area, volume and capacity

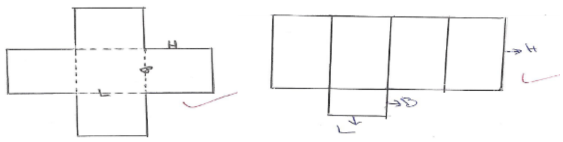
Surface area	'faces added', 'amount of faces', 'all around added', 'base of the object'
Volume	'size of inside', 'amount it can hold', 'size'
Capacity	'space', 'capacity to hold', 'weight', 'interior volume'

The learners' acquired knowledge of drawing and investigating the net of a rectangular prisms were assessed in Question 2. From Table 4.29, it can be seen that 70.2% had the correct concept of a net of an open rectangular prism with and were able to correctly draw the net of the rectangular prism.

Table 4.29: Learners' conception of the net of an open rectangular prism

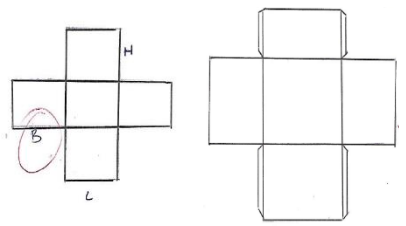
Type of net drawn	**Number of learners**	**Percentage of learners**
Correct net with dimensions	25	43.9
Correct net without dimensions	10	17.5
Correct net with wrong dimensions	9	15.8
Closed rectangular prism with wrong dimensions	7	12.3
Incorrect net	3	5.3
Closed rectangular prism with dimensions	2	3.5
Closed rectangular prism without dimensions	1	1.8

The net that occurred the most in the scripts was where the learners drew the net of the open rectangular prism with the base in the centre and the rest of the faces adjacent to the base (see the left side of Picture 4.24). There were also a few learners who provided an alternative representation of the net for the open rectangular prism as seen on the right in Picture 4.24 below.



Picture 4.24: Two of the conceptions of the net of the open rectangular prism

In the formative test, most of the learners were able to draw the net, but some of them still struggled with the dimensions of the net or leaving the dimensions out (see Picture 4.25). The learners struggled to identify the length, breadth and height of an object, this misconception will be discussed in more detail under Section 5.3.2.1.



Picture 4.25: Correct net of an open rectangular prism with incorrect or no dimensions

4.6.4.2 Typical learner mistakes

The most common error that learners made in the baseline test was in finding a formula for the surface area of the open vase (discussed in more detail in Section 5.3.2.1). In the formative test, most of the learners were again able to draw the net, but only a few managed to write in their own words how to calculate the surface area by using the net. After instruction, they were still unable to use the drawing of the net to find a formula for the surface area of the vase. I will compare the mistakes in the formative test, see Table 4.30, with those in the baseline test.

Table 4.30: Learners' mistakes regarding formula for surface area of an open vase

Learners' answers	**Number of learners**	**Percentage of learners**
Answers like: 4L+4B, $6L^2$, 2LBH,LB, $5L^3$, 2LB/H, BH, LH, $4LB+2L^2$, 2BL+2H, 0.5LBH, 2LB, $2(LBH)^2$, 5LBH, 2L+2B+2H, LB-H, 4LBH, $2L^2+3LB$, 2LB+2LBH, 3LB, LB+LBH	17	30.9
LBH	17	30.9
2LB+2LH+2BH	16	29.1
L+B+H	4	7.3
LB+2LH+2BH (Correct answer)	1	1.8
LB	1	1.8

Only one learner (1.8%) got the correct answer for the surface area of the open rectangular vase after instruction. Sixteen learners (29.1%) wrote down the surface area formula for a closed rectangular vase. A greater variety of incorrect answers appeared in the formative test. However, there was a decrease in the percentage of other answers for the formula of surface area in the formative test compared to the baseline test. In Table 4.31 below, a comparison is drawn between the formulae used in the baseline test compared to those in the formative test.

Table 4.31: Comparison between baseline test and formative test: surface area

Answers for surface area of an open rectangular vase	Percentage of learners in baseline test	Percentage of learners in formative test
Incorrect answers	37.9	30.4
LBH	36.2	30.3
L+B+H	20.7	7.1
LB	5.2	1.8
2LB+2LH+2BH	0	28.6
LB+2LH+2BH (Correct answer)	0	1.8

In the baseline test, none of the learners were able to write down the correct formula for the surface area of an open rectangular container compared to the one learner in the formative test. None of the learners wrote the surface area formula down for a closed rectangular container in the baseline test, however, 28.6% of the learners in the formative test almost got the formula correct but thought the container was closed. After instruction, there were fewer learners who used the wrong formulae, however the percentage remained high. The second problem that emerged from the baseline test was that learners were unsure about which formula to use for the volume of the container. After instruction, three of the four typical mistakes occurred in the formative test as well. From Table 4.32 below, it can be seen what percentage of learners made these typical mistakes.

Table 4.32: Comparison between baseline and formative test: typical mistakes

Type of mistake	Percentage of learners in Question 2.3		Percentage of learners in Question 2.5	
	Baseline test	Formative test	Baseline test	Formative test
1. Using volume formulae (LBH) for both questions.	**36.2**	30.3	**15.5**	10.7
2. Using an incorrect formula (L+B+H) for both questions.	**20.7**	7.1	**1.7**	5.4

Type of mistake	Percentage of learners in Question 2.3		Percentage of learners in Question 2.5	
	Baseline test	Formative test	Baseline test	Formative test
3. Using an incorrect formula (L+B+H) for surface area question and LBH for volume question.	**20.7**	0	**17.2**	0
4. Using volume formulae (LBH) for surface area question and an incorrect formula (L+B+H) for volume question.	**36.2**	30.3	**8.6**	8.9

4.6.4.3 Difficult concepts to understand

The learners tended to struggle with conversions between units in the SI system. This emerged in the baseline test, as well as the formative test as learners were unable to convert between units in the SI unit system. After Mary had spent a lot of time on conversions with analogies and calculations, learners were still unable to convert. In Table 4.33 an analysis of the answers obtained from Question 2.6 in the baseline test and the formative test are given.

Table 4.33: Comparison of different answers obtained when converting m^3 to cm^3

Possible answers to convert m^3 to cm^3	Percentage of learners in baseline test	Percentage of learners in formative test
1 000	33.9	17.9
100	16.9	41.1
3	6.8	0
1 000 000 (correct answer)	5.1	12.5
10	5.1	7.1
0.1	5.1	3.6
0.01	5.1	5.4
1	3.4	0
4.64	3.4	0
Incorrect answers	15.2	12.5

Although there was a small increase in the percentage of learners who did the conversion correctly, only 12.5% of the learners were able to convert m^3 to cm^3. There was an even bigger increase in the percentage of learners who thought the answer is 100. In Table 4.34, the outcomes of the two tests are compared where learners had to convert from m^3 to litres.

Table 4.34: Comparison of different answers obtained when converting m^3 to litres

Possible answers to convert m^3 to litres	**Percentage of learners in baseline test**	**Percentage of learners in formative test**
1 000 (correct answer)	22.0	42.9
100	22.0	12.5
1	13.6	12.5
10	10.2	16.1
10 000	5.1	3.6
3	5.1	0
No answer	5.1	0
0.01	3.4	0
0.001	3.4	7.1
3 000	3.4	0
Other answers	6.8	5.4

Although there was an increase in the percentage of learners who got the conversion right, it was still less than 50%. The 'other answers' that appeared in the formative test included either a decimal fraction or a number with the incorrect number of zeros.

4.7 Conclusion

In this chapter, I discussed the data collection process, which involved a baseline test; classroom observations; an in-depth semi-structured interview after the classroom observations; and lastly a formative test. The data were presented and discussed based on the conceptual framework and using a deductive approach to interpret the data. In the next chapter, the findings from the analysed data of both teachers are discussed.

CHAPTER 5 DISCUSSION OF THE FINDINGS

5.1 INTRODUCTION

In Chapter 4, the data were analysed and presented. In this chapter, the findings from the analysed data are discussed. The purpose of the study was to explore the ability of Grade 9 mathematics teachers to use learner outcomes from a baseline test to inform their teaching for the improvement of learner outcomes, as measured in a formative test. I investigated the existing literature regarding mathematics teachers' classroom practices, with an emphasis on teachers' PCK and how they use learner outcomes to inform their teaching of mathematics. By observing the teachers' instructional practices and conducting interviews, I explored their Personal PCK&S and Personal PCK respectively.

The discussion of each teacher entails a general overview of the teacher's awareness and knowledge of pre-concepts; her use of the learner outcomes in general; and the instructional strategies she used. The second part concerns the teacher's classroom practice with regard to: learner conceptions; typical learner mistakes; and learner difficulties. For each of these aspects, the learner outcomes from the baseline test, what the teacher did and did not do during instruction, what the teacher said in the interview, and how the learner outcomes changed are discussed.

5.2 DISCUSSION OF ALICE'S FINDINGS

5.2.1 General overview of classroom practice

In terms of the importance of pre-concepts, Alice firstly believed that the pre-concepts the learners needed to have about surface area, volume and capacity is to know the difference between these concepts and have the knowledge that 'Surface area is the outside and volume is the inside'. Secondly, she mentioned that they had done these concepts earlier in the year, and in Grade 8, and were supposed to know these concepts. Thirdly, when planning her lessons, she took into account these pre-concepts, but due to time constrains never used a formal or written baseline test but instead asked them oral questions based on the knowledge of the pre-concepts before introducing the new topic.

The baseline test in this study was written by all her Grade 9 learners to determine their prior knowledge, typical mistakes, and difficulties. Alice recognised that learners struggled with the concepts of surface area, volume, capacity, the definition of a prism, and the conversions of SI units of measurement. The outcomes of the baseline test made her realise that she needed to spend more time on the concepts of surface area, volume and capacity, which she did through demonstrations, explanations and calculations on the white board. Alice's Personal PCK&S became evident when she addressed the learner outcomes from the baseline test.

Teachers should reflect on their own teaching and classroom practice, which can happen any time, during or after instruction (Schon, 1983). Alice mainly used direct instruction as an instructional strategy in all her lessons, a strategy which is highly teacher-centred (Keesee, 2014). Learners were only involved in discussing and elaborating on their answers obtained in the homework activities. By involving the learners in this way, misunderstandings and misconceptions were addressed and their conceptual understanding was enhanced (Franke et al., 2007). The learners in Alice's classes were very disruptive and not motivated, and a lack of discipline had a negative effect on the learners' learning process (Berry et al., 2015). Alice responded to the learners' disruptive behaviour and lack of involvement by trying to change her instructional strategy to interactive instruction. Changing to interactive instruction actually led to learners being confused and misunderstanding her further. This showed a lack of Personal PCK&S with regard to using various instructional strategies to address learner needs.

5.2.2 Classroom practice with regard to specific learner outcomes

5.2.2.1 Learner conceptions

To develop an understanding of mathematical concepts, learners need to construct their own knowledge and ideas of the concepts verbally (Ollerton & Watson, 2001). Two main misconceptions were identified from the baseline test: learners' concepts of surface area, volume and capacity; and drawing the net of an open rectangular vase with the correct dimensions. It was clear that the learners lacked mathematical vocabulary and were not mathematically proficient (Kilpatrick, 2001).

Surface area, volume and capacity

The majority of the learners struggled to define the concepts of surface area, volume and capacity due to a lack of conceptual understanding, while a few learners showed correct conceptions but a lack of mathematical vocabulary to clearly define the concepts. None (0%) of the learners correctly explained surface area; 21.4% explained volume; and 35.7% capacity. In response to the learner outcomes, Alice used various representations in the form of 3D models to explain surface area and for volume and capacity she used integration with a real-life context. When she improvised in class (Personal PCK&S) to explain the concepts of volume and capacity, her explanation of volume as chocolate and capacity as melted chocolate could have led to misunderstanding among the learners. In her interview, Alice mentioned that learners need to understand that 'surface area is the outside and volume is the inside' (Personal PCK). From this, it can be seen that Alice did not express herself correctly and her Personal PCK and Personal PCK&S about the three concepts were not always intact. The formative test showed that 42.5%, 26% and 52.1% of the learners could then correctly explain surface area, volume and capacity respectively. The learners had a better understanding of surface area and capacity, but the problem with the definition of volume persisted. It can therefore be concluded that Alice's instruction based on the learner outcomes from the baseline test improved understanding and learner outcomes, as revealed in the formative test.

Drawing of the net of an open vase

The baseline test presented the learners with a real-life context question of an open vase to determine if they could draw the net thereof and indicate the dimensions on the drawing. Most (63.9%) of the learners were able to draw the net of an open rectangular prism, but some struggled to write the dimensions on the net. As such, only 33.3% of the learners got it completely right. The learners who got the net incorrect drew the net of a closed rectangular prism, a 3D rectangular prism or a net with the incorrect number of faces. Learners had a misconception regarding the dimensions of an object, especially when the object is rotated and the base of the object changes position. During the observed lessons Alice explained the surface area of various prisms by using PowerPoint slides where the net was drawn next to the appropriate formula. None of the examples she explained included a calculation where the learners had to manipulate the basic formula. It was only when she marked the homework in the next lesson that she mentioned that if one of the sides is not there, you have to 'adjust' the formula (Personal PCK&S). She used the example of an open dog cage in an attempt to integrate these concepts into a real-life context. The learners struggled to understand and she explained again. In her interview, she mentioned that 'that you have to try to explain it from a different angle' (Personal PCK). The formative test showed that 85.9% of the learners could then draw the net correctly, but still struggled to write down the correct dimensions on the net. This is an increase of 34.4% of the learners who were able to draw the net, however, only 39.4% of the learners got it completely right and were able to write down the dimensions on the net as well. This implies an increase of 18.3% of the learners who got it completely right.

5.2.2.2 Typical learner mistakes

The first typical mistake that occurred in the baseline test was that 17 learners drew the net of a closed rectangular prism (22.7%) instead of an open rectangular prism. In the textbook used by the participating school, only one possibility of the net of a rectangular prism is given. Alice only indicated one net on her PowerPoint slide and did not place emphasise how to draw alternative representations and how to complete the dimensions on the net by using diagrams (Personal PCK&S). In the formative test only five (7.0%) of the learners drew the net of a closed rectangular prism. Therefore, there was a decrease of 70.6% in learners who drew the incorrect net of a closed rectangular prism.

The second typical mistake that occurred in the baseline test was where leaners had to find a formula for the surface area of the vase. None (0%) of the learners in the baseline test were able to find a formula for the surface area of the open rectangular prism. When Alice was explaining the surface area of different prisms, she projected the 3D object with the net and the surface area formula for the particular object on a PowerPoint slide. She also did an example to demonstrate to the learners how to use the formulae of the different prisms. In the formative test, there was only a small increase in correct answers, from none in the baseline test to 5.5% of the learners in the formative test.

Furthermore, an interesting issue emerged where the learners had to calculate the surface area and the volume of an open vase, as discussed in Section 4.7.1.4 under 'Emerging issues'. Four variations of these typical mistakes were made in the baseline test and three variations in the formative test. In the baseline test, almost half of the learners (42.3%) favoured the formula of volume to calculate surface area. The formative test showed that there was a marked improvement and there was a decrease in all the above variations, except for the second variation. For the first variation, there was a 35.2% decrease of learners who used the volume formula to calculate surface area, and a 14.2% decrease of learners who used the volume formula for volume. The second variation did not appear in the formative test; therefore, showing a 100% decrease in this variation. For the third variation, there was an 84% decrease of learners who used an incorrect formula (L+B+H) to calculate surface area and a 92.9% increase of learners who used LBH to calculate volume. For the fourth variation, there was a 35.2% decrease of learners who used the volume formula for surface area and a 68.2% decrease of learners who used the incorrect formula (L+B+H) to calculate volume. Therefore, more learners knew which formula to use for calculating surface area and volume.

5.2.2.3 Learner difficulties

In the baseline test, the learners experienced difficulties with regard to the conversions of SI units of measurement. Conversion between cubic units with different pre-fixes and from cubic units to litres proved to be challenging. Only 6.8% of the learners in the baseline test were able to convert 1 m^3 correctly to 1 million cm^3, where 1 000 cm^3 was the more popular answer. This may be due to the fact that they thought the exponent of three represents 1000. For the conversion of m^3 to litre, only 15.1% of the learners in the baseline test got the conversion right. Alice instructed the learners to rote-learn a rhyme, which represent the pre-fixes of the powers of ten of the SI units. The rhyme was learned earlier in the year and at the time of the study the learners had already forgotten how to use it. In the interview, Alice mentioned that when explaining difficult topics, she believed the teacher needs to link the abstract problem to something more concrete by using models or real-life examples (Personal PCK). However, she did not use real-life examples to demonstrate the relationship between units in the SI system of measurement. Although only 26.4% of the learners in the formative test were able to convert 1 m^3 correctly, representing an increase of 288.2%, the majority of learners still did not grasp this concept. Two interesting issues that arose in the formative test was that, firstly, the same percentage of learners (26.4%) got the answer of 1000 compared to the correct answer of 1 000 000. Secondly, more learners got the answer right than in the baseline test. The fact that the learners thought the answer should be 1 000 instead of 1 000 000 may be due to the fact that they still thought the exponent of three represents 1000. Although there was an increase in correct answers, the percentage of learners who got the answer right remained low at 26.4% and 45.8% for the two conversions respectively.

5.3 DISCUSSION OF MARY'S FINDINGS

5.3.1 General overview of classroom practice

Mary understood that learners struggle with the pre-concepts of 2D and 3D objects, defining surface area, volume and capacity, as well as conversions in the SI unit system. According to Mary, learners must be able to define surface area, volume and capacity; differentiate between surface area and volume; and apply these concepts to their daily lives. Normally, when planning her lessons, Mary took cognisance of the pre-concepts they had learned in Grade 8, but due to time constraints never used a formal or written baseline test to determine prior knowledge. Instead, she asked them oral questions before introducing the new topic. She also mentioned that the learners could obtain any mathematical formula or definition from Google. According to Mary, the knowledge learners lacked the most was how to use or apply these formulae and that learners were unable to make the connection between these formulae, concepts or definitions and application in their daily lives (Adler et al., 2000).

Mary spent time on the pre-concepts of 2D and 3D objects, surface area, volume and capacity by using paper models which could unfold in order to explain the concept of drawing the nets, calculating the surface area and volume. She also did surface area and volume examples on the overhead projector and integrated the calculations with a real-life context as well. For the conversions of SI units of measurement, she incorporated three basic problems as a starting point to explain the conversions and derived the rules. Mary's Personal PCK&S became evident when she elaborated on the learner outcomes in the baseline test with regard to the pre-concepts.

Mary used mainly direct instruction as an instructional strategy in all her lessons and learners were only involved in discussing and elaborating on their answers obtained in the homework activities (Keesee, 2014). However, from the classroom discussions, which involved surface area, it can be seen that Mary was not comfortable with English and this could influence learners' conceptions. Mary's poor competence in expressing herself in English could have led to misunderstandings and prevented even better learner outcomes. With regard to volume and capacity, Mary merely gave the learners the definition to write down in their scripts. She did not orally assess to see whether learners could explain in their own words the meaning or difference between these pre-concepts of volume and capacity. Mary reflected and improvised during her instruction and one could conclude that she had a developed Personal PCK&S in terms of knowledge about pre-concepts (Berry et al., 2015). However, she did not always respond to learners' answers to her questions and this could deny the learners the opportunity to further develop understanding. In this regard, her Personal PCK&S is considered limited.

5.3.2 Classroom practice with regard to specific learner outcomes

As with the outcomes of the baseline test in Alice's classes, two main misconceptions were identified: explaining the concepts of surface area, volume and capacity; and to draw the net of an open rectangular vase with the correct dimensions. From the answers obtained in the baseline test, it is clear that the learners lacked mathematical vocabulary and were not mathematically proficient (Kilpatrick, 2001).

5.3.2.1 Learner conceptions

Surface area, volume and capacity

Similar to the learners in Alice's classes, the majority of learners struggled to clearly define the pre-concepts of surface area, volume and capacity due to poor conceptual understanding, while others lacked the mathematical vocabulary to clearly define the concepts. Only 13.1% of the learners correctly explained surface area; 16.4% volume; and 36.1% capacity. In response to these learner outcomes, Mary used various representations in the form of 3D models to explain surface area and for volume and capacity, and used integration with a real-life context. When she started the topic of surface area, she asked the learners why they needed to do calculations involving surface area. After the learners gave examples of a real-life context, Mary also asked about the difference between 2D and 3D. One of the learners replied by saying '2D is like a flat shape and 3D is a bigger picture'. Mary continued with her explanation and did not asked him what he meant by that, which could have led to misunderstanding among the learners. This is an indication of limited Personal PCK&S. From the classroom discussion, which involved surface area, it can be seen that Mary was not comfortable with English and this could influence learners' conceptions. In her interview, Mary mentioned that the prior knowledge learners generally lacked was the application of surface area and volume in their daily lives. She further believed that if learners have knowledge regarding the difference between surface area and volume, they will find it easy to apply the different formulae. To help learners to engage with their prior knowledge, she tried to connect the mathematical topic with a real-life context (Personal PCK). The formative test showed that 16.4%, 41.8% and 41.8% of the learners could then explain surface area, volume and capacity respectively. From the percentages, learners had a better understanding of volume area and capacity, but the problem with the definition of surface area persisted. It can therefore be concluded that Mary's instruction based on the learner outcomes form the baseline test improved understanding and learner outcomes, as revealed in the formative test.

Drawing of the net of an open vase

The baseline test presented the learners with a real-life context question to determine if they could draw a net and indicate the dimensions on the drawing. Most of the learners (72.4%) were able to draw the net of an open rectangular prism, but some struggled to write down the dimensions on the net. Therefore, only 43.1% of the learners got it completely right. Learners had misconceptions with regard to the dimensions of an object, especially when the object is rotated and the base of the object changes position. During the observed lessons, Mary explained the surface area of various prisms with 3D paper models, which could unfold; nets were also drawn on the overhead projector next to the appropriate formula (Personal PCK&S). In her interview, Mary mentioned that if concrete models of prisms are used, learners would be able to identify and see the different parts of an object. She assumed that the learners would then be able to construct the net and connect that to the actual object (Personal PCK). The formative test showed that 77.2% of the learners could draw the correct net, but still struggled to write down the correct dimensions on the net. This shows an increase of 6.6% of learners who were able to draw the net. However, only 43.9% of the learners got it completely right and were able to write down the dimensions on the net, showing an increase of 1.9% of the learners who got it completely right.

5.3.2.2 Typical learner mistakes

The first typical mistake that occurred in the baseline test was that eight learners drew the net of a closed rectangular prism (13.8%) instead of an open rectangular prism. In the textbook, only one possibility of the net of a rectangular prism is given. Mary only indicated one net on the overhead projector and did not place emphasis on how to draw alternative representations or how to complete the dimensions on the net by using diagrams (Personal PCK&S). Mary relied on peer-teaching (Personal PCK) when repeated explanation did not work. However, in the observed lessons, Mary relied only on direct instruction. In the formative test 10 (17.5%) of the learners drew the net of a closed rectangular prism, which is an increase of 26.8% in drawing the incorrect net of a closed rectangular prism.

The second typical mistake that occurred in the baseline test was where leaners had to find a formula for the surface area of the vase. None (0%) of the learners in the baseline test were able to find a formula for the surface area of the open rectangular prism. When Mary was explaining the surface area of different prisms, she drew the 3D object with the net and the surface area formula for the particular object on the overhead projector. She also used examples to demonstrate to the learners how to use the formulae of the different prisms. In the formative test, there was only a small increase in correct answers from none in the baseline test to 1.8% of the learners in the formative test. Furthermore, an interesting issue emerged where the learners had to calculate the surface area and the volume of an open vase, as discussed in Section 4.8.1.4 under ‘Emerging issues’. Four variations of these typical mistakes were made in the baseline test and three variations in the formative test. The learners favoured the volume formula the most when calculating the surface area (36.2%) when calculating surface area. From the learner outcomes in the formative test, it can be seen that there was either a decrease or increase in all of the above variations. For the first variation, there was a 16.3% decrease of learners who used the volume formula to calculate surface area and a 31% decrease of learners who used the volume formula for volume. The second variation did not appear in the formative test, showing a 100% decrease in this variation. For the third variation, there was a 65.7% decrease of learners who used an incorrect formula (L+B+H) to calculate surface area and a 68.6% decrease of learners who used LBH to calculate volume. For the fourth variation, there was a 16.3% decrease of learners who used the volume formula for surface area and a 3.5% increase of learners who used the incorrect formula (L+B+H) to calculate volume. The learners were thus still unsure which formula to use for calculating surface area and volume.

5.3.2.3 Learner difficulties

In the baseline test, the learners struggled to convert between cubic units and to convert cubic units to litres. Only 5.1% of the learners in the baseline test were able to convert 1m^3 correct to 1 million cm^3, and 22% of the learners were able to convert 1m^3 correctly to 1000 litres. In the observed lessons, Mary spent a lot of time on the conversions between different SI units of measurements. She gave learners three basic relationships to learn in the SI unit system (Personal PCK&S). When she was explaining the calculations, she always wrote down the basic relationships and asked the learners questions based on the conversions. In the interview, Mary mentioned that she would work slowly through an example of a difficult concept and then give the learners one to do on their own. She would also let them write down the steps in words and connect the difficult concepts with something in their daily lives (Personal PCK) (Adler et al., 2000). Only 12.5% of the learners in the formative test were able to convert 1 m^3 correct to 1 million cm^3, an increase of 14.5%. Although there was an increase, the percentage of correct answers still remained low. An interesting issue that arose in the formative test was that 17.9% got the answer of 1000 compared to the correct answer of 1 million. This may be due to the fact that they thought the exponent of three represents 1000 (three zeros). For the conversion of m^3 to litres, 42.9% of the learners in the formative test did the conversion right, which is an increase of 95%. Although there was a significant increase in correct answers, the percentage of learners who got the answer right remains low at 12.5% and 42.9% for the two conversions respectively. Therefore, the assessment outcomes in terms of learner difficulties with conversions is below average.

5.4 COMPARISON BETWEEN ALICE AND MARY

While Mary only had five more years' teaching experience than Alice, I still believe it is worth comparing their practices as they planned their lessons together. Table 5.1 below lists some similarities and differences between their practices.

Table 5.1 Similarities and differences between Alice and Mary

	Alice (4 years of experience)	Mary (9 years of experience)
Similarities	English not her mother tongue. Direct lecturing style. Learner conceptions similar. Learners make the same typical mistakes. Learner difficulties the same.	English not her mother tongue Direct lecturing style Learner conceptions similar. Learners make the same typical mistakes. Learner difficulties the same.
Differences	Use oral more than written language Explain conversions with a rhyme and arm demonstration. Learners were not co-operating and ill-disciplined.	Use oral and written language to the same extent. Explain conversions with three basic conversions. Learners co-operated and were more disciplined than in Alice's classes.

In Table 5.2 the raw scores of the learner misconceptions, typical learner mistakes and learner difficulties from the baseline test and formative test are given as percentages. It can be seen that there was an improvement in the learner outcomes in the formative test compared to the baseline test.

Table 5.2: Raw scores of baseline test compared to formative test

	Alice's classes		Mary's classes	
	Baseline	Formative	Baseline	Formative
Correct net with dimensions	33.3%	39.4%	43.1%	43.9%
Using correct surface area formula	0%	5.5%	0%	1.8%
Conversion from m^3 to cm^3	6.8%	26.4%	5.1%	12.5%
Conversion from m^3 to litres	15.1%	45.8%	22.0%	42.9%

A better indication of the improvement in learner outcomes is the normalised gain (Hake, 1998). Normalised gain is the actual gain (Formative Test – Baseline Test) divided by the maximum possible gain (100 – Baseline Test), and expressed as a percentage, the formula is: $\frac{\text{Formative Test-Baseline Test}}{100-\text{Baseline Test}} \times 100$. In Table 5.4, the normalised gain of the two teachers' learner outcomes is given.

Table 5.3: Normalised gain of baseline test and formative test

	Alice's classes		Mary's classes	
	Actual gain	Normalised gain	Actual gain	Normalised gain
Correct net with dimensions	6.1%	0.09 (9%)	0.8%	0.01 (1%)
Using correct surface area formula	5.5%	0.06 (6%)	1.8%	0.02 (2%)
Conversion from m^3 to cm^3	19.6%	0.21 (21%)	7.4%	0.08 (8%)
Conversion from m^3 to litres	30.7%	0.36 (36%)	20.9%	0.27 (27%)

From Table 5.3, it can be seen that the learner outcomes of the formative test of Alice's classes are slightly better than Mary's classes, although Mary had more teaching experience and depended less on rote learning. Developing procedural knowledge is valuable, but it will be interesting to see what the long-term effect is if a similar test could be written again after a certain period. The question is, will they still remember the rhythms? However, the overall learner outcomes in the formative test for both teachers are quite similar.

5.5 Conclusion

I discussed the findings in this chapter in terms of 1) A general overview of each teacher's classroom practice in terms of her knowledge of pre-concepts, her use of learner outcomes; and instructional strategies used; and 2) Her practice with regard to learner conceptions, typical mistakes and difficulties. In the next chapter, the conclusions and implications gained from the findings and literature are discussed.

CHAPTER 6 CONCLUSIONS AND IMPLICATIONS

6.1 INTRODUCTION

In Chapter 5 the findings from the research endeavour were discussed. This chapter provides the research questions are answered, I reflect on the research and draw conclusions, make recommendations for further research, and list the limitations of the study.

6.2 DISCUSSION OF THE RESEARCH QUESTIONS

6.2.1 Q1: What is the learners' prior knowledge, as revealed in a baseline test?

The learner outcomes showed that the learners lacked the required prior-knowledge and had several misconceptions regarding surface area, volume and capacity; they also lacked vocabulary in writing down the meaning of these concepts. It was shown that 66% of all learners drew the correct net of the rectangular prism, but only 56% of them indicated the correct dimensions in their drawings. A typical difficulty became apparent when learners had to use the net to find a formula for the surface area of a vase as no learner could find the correct formula. Learners also made systematic mistakes when doing calculations regarding surface area and volume and in many cases used the wrong formula. Only one of the learners could manipulate the formula to make height the subject of the formula when volume was given. The most difficult concept for all learners was the conversions of SI units of measurement. Only 6% of all learners could correctly convert 1 m^3 to cm^3 and only 19% could correctly convert m^3 to litres. The learner outcomes overall, when taking into account that this topic had been done in both Grades 7 and 8 already, were poor.

6.2.2 Q2: How do the outcomes of the baseline test inform the teachers' Personal PCK?

These two teachers usually did not use a baseline test to determine the prior knowledge of the learners to inform and guide their teaching due to time constraints. Since the baseline test constituted an important aspect of the study, the teachers were obliged to use the outcomes, which they regarded as valuable. Alice believed that the pre-concept learners need to have about surface area, volume and capacity is to know the difference between these concepts. Alternatively, Mary believed that learners must be able to differentiate between surface area and volume and apply these concepts to their daily lives. Both teachers acknowledged that they took cognisance of these pre-concepts when planning their lessons.

Both teachers graded the Grade 9 learners' prior knowledge of surface area, volume and capacity as limited or almost non-existent. Alice believed that the learners' prior knowledge should not only be based on conceptual understanding, but they also need to develop procedural fluency, therefore, she taught the learners to memorise formulae and rhymes. Mary believed that the more experience teachers gain, the more they will be able to plan their lessons based on the prior knowledge of the learners and they will be able to anticipate mistakes learners might make. Both teachers believed that the main misconception learners revealed in the baseline test was that they could not distinguish between surface area and volume. The few learners who had better conceptions were unable to express their thoughts in words. The teachers believed this led to learners deriving incorrect surface area formulae when they had to use the net. Only Mary mentioned the difficulty that the learners experienced in manipulating the volume formula to determine height. According to Alice, the main difficulties the learners experienced in the test were to read, understand and interpret the questions.

Both teachers said that, due to discipline problems in the classes and time constraints, they preferred to use a direct instructional strategy. During the interviews, both teachers said that when they explained difficult topics, they believed that the teacher needs to explain the abstract problem with something more concrete such as models or use real-life contexts.

6.2.3 Q3: How do the outcomes of the baseline test inform the teachers' Personal PCK&S, as demonstrated in their instruction?

From the lessons I observed, it was clear that both the teachers did their lesson planning based on the learners' prior knowledge. They addressed the misconceptions regarding the difference between surface area and volume, and drawing the net of an open box without six faces. However, they did not spend sufficient time on addressing the typical mistakes of writing down the surface area formula if the container consists of five instead of six faces, or on adapting the formula to find height when volume was given. Both teachers had a specific skill in explaining the conversions of SI units. Alice used ‘arm demonstrations’ to illustrate the rule of conversions and required the learners to rote-learn a rhyme, while Mary gave the learners three basic relationships and asked relevant questions to elicit reasoning and to develop conceptual understanding.

Both teachers used a direct instructional strategy and followed a specific sequence when explaining the topic of surface area, volume and capacity. Alice used PowerPoint slides during her instruction, while Mary used the overhead projector. The strategies that the two teachers used included lecturing, questioning, drills and demonstrations. The learners in the four classes were allowed to ask questions and engage with the problems. There were many similarities between the two teachers in their use of instructional strategies, but they used different approaches for the conversion of units.

Both teachers, as well as the learners used calculators to do the calculations involving surface area, volume and capacity, as well as calculations to convert within the SI units of measurement. Alice and Mary planned their instruction in such a way that the content of their lessons to explain surface area, volume and capacity was integrated with other mathematical topics and real-life contexts. However, neither of the teachers connected this topic with other subjects like engineering, graphic design or physics.

6.2.4 Q4: What are the learners’ outcomes after the teachers’ teaching of the topic, as revealed in a formative assessment?

The purpose of the formative test was to determine the influence of the teachers’ instruction on the learner outcomes by comparing the outcomes of the formative test with the baseline test. These showed an increase in correct conceptions of the difference between surface area, volume and capacity, but many learners still lacked the vocabulary to describe the meaning of these concepts. Moreover, 82% (66% in the baseline test) of all learners were able to correctly draw the net of the rectangular prism, but only 42% (56% in baseline test) indicated the correct dimensions on their drawing. A typical mistake was using the net to find the formula for surface area of the vase, and only 4% (0% in the baseline test) could find the correct formula. The learners also made systematic mistakes when doing calculations regarding surface area and volume, and used the wrong formulae. As in the baseline test, only one of the learners could manipulate the formula to make height the subject of the formula when volume is given. The most difficult concept for all the learners was the conversions of SI units of measurement. Only 20% (6% in the baseline test) of the learners could correctly convert 1 m^3 to cm^3 and only 44% (19% in the baseline test) could correctly convert m^3 to litres. Overall, the learner outcomes after the instruction of this topic improved, but were still poor.

To answer the main question: Due to time constraints, the teachers usually did not use a baseline test to determine the prior knowledge of the learners. Not only did they acknowledge the value of the assessment outcomes in planning their lessons during the interviews, but this was observed during their instruction and seen in the learner outcomes of the formative test. They especially took cognisance of the learners' misconceptions, typical mistakes, and difficulties, and attempted to use various representations, and integrating the topic with real-life contexts. However, they did not plan various and more appropriate strategies that could have been used to develop learners' conceptual understanding. Being part of this process was a new experience for the teachers, and by continuing to use assessment outcomes, their Personal PCK and Personal PCK&S could develop, implying better informed and guided teaching.

6.3 PROVIDING FOR ERRORS IN THE CONCLUSION

Unknowingly and unintentionally, I might have been wrong in some of my conclusions in conducting this study. However, I attempted to enhance the credibility and trustworthiness of my research endeavour through triangulation by using a baseline test, at least three observations, an in-depth individual interview, and a formative test.

To reduce the Hawthorne effect, the observations were done prior to the interview as the interview questions could have influenced the teachers' instruction (Cohen et al., 2011). I emphasised the fact that I was interested in the uniqueness of each teacher's classroom practice and that my purpose was not to report their performance to the school. Furthermore, I used the same interview schedules, including the same questions in the same sequence for both interviewees. With all these considerations, I realise that my presence still had an influence on their practices.

6.4 Conclusions

Some conclusions regarding the relationship between learner outcomes and the teachers' classroom practice, which include the teachers' Personal PCK and Personal PCK&S, appear below.

Learner outcomes from the comparison between the data obtained from the baseline test and the formative test:

- Learners experienced difficulty with regard to finding a formula for the surface area of the open vase. There might be different reasons for this difficulty; firstly, the learners had just memorised the formulae of different prisms. Secondly, the learners could not visualise the object, or thirdly, there might be a lack of spatial thinking among the learners.
- Learners used the formulae for volume instead of surface area or variations thereof. A possible reason for this typical mistake is that the learners did not know the difference between the concepts of surface area and volume.
- Learners were unable to convert between SI units of measurement. The learners experienced the conversions between SI units as difficult to understand. This may be due to the fact that the learners lacked conceptual understanding, adaptive reasoning and strategic competence (Kilpatrick, 2001). The learners might also have lacked the building blocks of instruction with regard to mathematical operations. From the answers obtained, it is clear that the learners had an incomplete understanding of how to convert between SI units of measurement.

Teachers' classroom practice:

- Learners lacked knowledge of the pre-concepts of prisms, surface area, volume and capacity, as well as the conversion of SI unit of measurement. To address this problem, teachers can ask learners to find shapes in real life in order for them to understand the concepts of different prisms better.
- Learners' prior knowledge about the basic ideas in this topic was limited or non-existent and many typical mistakes were made. In order to eliminate these mistakes, teachers could emphasise the properties of polygons, specifically

quadrilaterals, more and could indicate the similarities and differences between the different prisms.

- Both teachers used a variety of representations in effective ways.
- Both teachers only used direct lecturing as the instructional strategy. To enhance understanding, teachers could use more mathematical terms in their classroom when explaining the concepts of surface area, volume and capacity.
- Both teachers integrated the mathematical topic of surface area, volume and capacity with other mathematical topics and real-life contexts, but not with other subject areas.

The relationship between the teacher's classroom practice and the learners' outcomes

- Neither teacher used a formal written baseline test prior to teaching a new topic due to time constraints.
- Both teachers acknowledged the value of the baseline test, especially with regard to typical learner mistakes (PCK&S). However, they had never used a baseline test before. Firstly, a baseline test could inform teachers on where to start their instruction with regard to a certain topic. Secondly, they could also use it to analyse the nature of misconceptions and typical mistakes and start form the origin of the misconception and/or mistake.
- Both teachers used direct lecturing, which did not contribute optimally to the improvement of the learner outcomes (PCK&S).
- Both teachers know the pre-concepts applicable to the topic (PCK), however after teaching this topic, the learners were still unsure about how to define these concepts.

6.5 RECOMMENDATIONS FOR FURTHER RESEARCH

Policy makers in education can use the recommendations based on the findings of this study to beneficially enhance learner outcomes by supporting teachers in their teaching practice. Several aspects, as examined in this study, require further research. These include investigation into:

- Developing a 'reflection before or after teaching' sheet for teachers (using their Personal PCK) to determine if they planned for enhanced learner outcomes by considering the pre-concepts, typical learner mistakes and topics that are difficult to understand.

- Developing a 'reflection during teaching' sheet for teachers (using their Personal PCK&S) to determine if they act appropriately or adapt their explanation, if necessary, to facilitate the learning process for enhanced learner outcomes.
- Developing baseline tests to determine learners' conceptions, typical learner mistakes, difficult topics to understand and prior knowledge of mathematical topics.
- Teaching and learning mathematical vocabulary to learners and training teachers how to use more mathematical terms in the classroom**.**
- The development of content knowledge on the teaching of conversions of SI units of measurement.
- The training of teachers in the use of various instructional strategies, including effective questioning techniques and different assessment types to enhance learner outcomes.
- The training of teachers in analysing the nature of learners' mistakes and misconceptions with regard to surface area, volume and capacity.
- The training of teachers on how to start from the origin of the mistake or misconception to enhance learner understanding and to adapt their instructional strategies for enhanced learner outcomes.
- Learners need to spend more time on construction procedures. Due to time constraints, teachers only indicate the nets and prisms and learners do not construct the objects themselves. By teaching construction procedures, learners will understand the different components of a prism.

These recommendations could further contribute to the findings of this research.

6.6 LIMITATIONS OF THE STUDY

The data were gathered from only two Grade 9 mathematics teachers and four mathematics classes and therefore the generalisation of the results is impossible. I realised that I had missed valuable communication between the teacher and the learners during classroom observations as I sat in front of the class so as not to intrude by moving around in the classroom with the camera. I also would also have used a better camera to tape the dialogue between the teacher and learners since many of the video clips were of poor quality and could not be used. This study was also bound by time and personal experience. Although my conclusions were scrutinised by my supervisors, the possibility that subjectivity may have influenced my findings cannot be ruled out.

6.7 LAST REFLECTION

It is my wish that this study will assist teachers to not only use baseline tests to inform their teaching, but constructively build on learners' prior knowledge, use various representations and instructional strategies, and integrate their lessons with other mathematical topics, content areas and other disciplines. This study had a major impact on my own practice in terms of my Personal PCK and Personal PCK&S.

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ADDENDUM A LETTER OF PERMISSION AND CONSENT

UNIVERSITEIT VAN PRETORIA
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Faculty of Education

Mrs. L. Zweers
Natural Sciences building
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University of Pretoria
liezell@7zweerse.co.za
Cell no: 079 602 1922

5 May 2016

Dear Sir/Madam

Letter of consent to the Gauteng Department of Education (GDE) Tshwane North (D3)

I hereby request permission to use public schools in the Tshwane North district (D3) for my research. I would like to invite Grade 9 mathematics teachers to participate in this research aimed at investigating how Grade 9 mathematics teachers use assessment outcomes to inform their teaching. This research will be reported upon in my Master's dissertation at the University of Pretoria.

Before the start of the research I need to select my sample, four teachers from two different schools. For this I need Tshwane North district to select participating schools according to the pre-determined criteria and willingness to participate in this study. The following criteria will be used to select the four participating teachers: The Grade 9 mathematics teacher with the most teaching experience and one with the least teaching experience. The teachers must be selected from two schools; two teachers from a top performing school and two teachers from a low performing school. The performances of the schools should be based on the 2015 yearend examination marks. The performance of the learners will be used to select two classes per teacher, one class must be top performing and the one class low performing.

If consent can be obtained from GDE, data will be collected by means of a baseline test, a classroom observation, an individual semi-structured interview, a formative test and another individual semi-structured interview.

- The teachers and I will set a baseline test based on the topic or content to be taught and the test will be written by the learners during class time. After the learners have written the test I will make copies to analyse by myself and the teachers will mark the tests as they normally do.
- After the tests have been marked I will observe one lesson in two different classes to be taught by each of the two Grade 9 mathematics teachers at a time convenient to the teachers as it should not disrupt the teachers' timetable and programme. The observations will be video recorded. This will allow for a clear and accurate record of the teacher's classroom practice. In cases where parents do not give permission for the video recording of the lessons, I will

position myself in such a way that the learners will not be part of the observation.

- After the observations I would like to interview each teacher individually at two different occasions, one after the observations and the second one at the end of the data collection process. The interviews should be conducted at a time and place convenient to the teacher and should not take longer than one hour. The interviews will be audio-taped by me in order to have a clear and accurate record of all the communication that took place. The first interview will be based on the prior knowledge of the learners and the classroom practice of the teacher.
- After the first interview the learners will write a formative test that will be set and marked by me. The formative test will be on the newly developed concepts that were taught by the teachers.
- The second interview will be conducted after the formative test and will be based on how the learners' outcomes were influenced after the teachers' instruction on the newly developed concepts on the topic, as revealed in the formative assessment.

Although I will analyse the learners' work in the baseline test and mark their formative tests, I will not be in direct contact with the learners. The learners will also be present in the class during the observations together with the researcher. The learners will receive a letter of assent to inform them about the research that will be conducted in the classroom and that we will use copies of their baseline tests and formative tests in this study. However the identity of the learners will be kept confidential and anonymous. The parents/guardians will receive a letter of informed consent for my presence in the class and the video recording of the lessons, as well as the use of the learners' baseline- and formative tests.

All participation is voluntary and once committed the schools, the teachers and learners may withdraw at any time. Pseudonyms will be used for all the parties involved to guarantee confidentiality and anonymity. The parties will not be identifiable in the findings of my research and only my supervisor and I will have access to the video/audio recordings which will be password protected. The data collected will only be used for academic purposes.

After the successful completion of my Master's degree, I will give feedback to the GDE in the form of a written report and if the GDE is willing, I would like to do a PowerPoint presentation of my findings to the mathematics subject facilitators.

For any questions before or during the research, please feel free to contact me. If you are willing to allow members of your staff to participate in this study, please sign this letter as a declaration of your consent.

Yours sincerely

05/05/2016

Researcher: Mrs L Zweers Date

05/05/2016

Supervisor: Dr JJ Botha

Date

Natural Science Building 4-1, Groenkloof Campus, UP, E-mail: hanlie.botha@up.ac.za

I hereby grant consent to Mrs L Zweers to conduct her research in Tshwane North District (D3) schools for her Master's research. I hereby also grand consent to Mrs L Zweers to video record the lessons, audio-tape the interviews and make copies of the learners' tests to be analysed by me.

District official for Tshwane North's name: ____________________

District official of Tshwane North's signature: ___________________

Date: __________________________

UNIVERSITEIT VAN PRETORIA
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Cell no: 079 602 1922

5 May 2016

Dear Sir/Madam

Letter of consent to the Principal for conducting a pilot study

I am currently enrolled for a Master's degree in mathematics Education at the University of Pretoria. My research is aimed at investigating how Grade 9 mathematics teachers use assessment outcomes to inform their teaching. I hereby request permission to use your school for a pilot study for my research. The pilot study is aimed at verifying whether my data collection instruments are valid. For this purpose I need to work with one of your Grade 9 mathematics teachers.

The data collection process will be as follows: The teacher will be required to set a baseline test together with me on the new topic that the teacher is about to introduce at that time. I am going to make copies of the learners' work to analyse and the teacher will mark the test as she normally does. After the teacher has marked the tests, I want to observe her lesson. After the observation I would like to interview the teacher; it will be based on the prior knowledge of the learners and the classroom practice of the teacher. After the first interview the learners will write a formative test. The test will be set and marked by myself. I would like to conduct a second interview after the learners have written the formative test. The second interview will be based on how the learners' outcomes were influenced after the teachers' instruction on the topic, as revealed in the formative assessment. The interviews will be conducted outside school hours at a time and place convenient for the teacher.

To have an accurate record of the data collected, I would like to video record the observed lesson and audio-tape the two interviews. All participation is voluntary and once committed the schools, the teachers and learners may withdraw at any time. Confidentiality and anonymity will be guaranteed at all times.

If you are willing to allow one member of your staff to participate in this research study, please sign this letter as a declaration of your consent.

Yours sincerely

________________________	05/05/2016 ________________________
Researcher: Mrs L Zweers	Date

____________________ 05/05/2016
____

Supervisor: Dr JJ Botha Date

Natural Science Building 4-1, Groenkloof Campus, UP
E-mail: hanlie.botha@up.ac.za

I hereby grant consent to Mrs L Zweers to conduct her pilot study in this school for her Master’s research.

School principal’s name: ____________________

School principal’s signature: ____________________

Date: __________________________

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Cell no: 079 602 1922

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

5 May 2016

Dear Sir/Madam

Letter of consent to the mathematics teacher for conducting a pilot study

You are invited to participate in a pilot study for my research project aimed at investigating how Grade 9 mathematics teachers use assessment outcomes to inform their teaching. The pilot study is aimed at verifying whether my data collection instruments: a baseline test, an observation, two semi-structured interviews and a formative test, are valid. It is proposed that you form part of the pilot study by setting a base line test, being observed teaching a Grade 9 mathematics class for one lesson, interviewed twice and administrating a formative test.

The process of data collection will be as follows:

- The Grade 9 mathematics learners will write a baseline test that will be set by you and me on the topic or content to be taught at that time. The test will be written by the learners during class time. After the learners have written the test I will make copies to analyse by myself and you will mark the tests as you normally do.
- After the tests have been marked I will observe you teaching a Grade 9 mathematics class for one lesson only (30-40 minutes). Note that you are not required to do anything extra than what you normally do during teaching mathematics lesson; no extra preparation is needed. The observation will be video recorded. This will allow for a clear and accurate record of your classroom practice. You however have an option to choose if you want to be video recorded or not. In cases where parents do not give permission for the video recording of the lesson, I will position myself in such a way that the learners will not be part of the observation.
- After the observation I would like to interview you in order discuss the lesson you presented (not longer than an hour). The interview will be conducted at a time and place convenient to you, outside of school hours. The interview will be audio-taped by me in order to have a clear and accurate record of all the communication that took place.
- After the first interview the learners will write a formative test set by me. After the formative test is written, a second interview will take place (not longer than 1 hour). The interview will be based on how the learners' outcomes, as revealed in the formative test, were influenced by the instruction.

Your participation is voluntary and confidentiality and anonymity will be guaranteed at all times. Once committed you may withdraw at any time, if you wish to do so. The data collected from the pilot study will not be part of the final thesis, but will be used to test the validity of my data collection instruments.
If you are willing to participate in this research study, please sign this letter as a declaration of your consent, i.e. that you participate willingly.

Yours sincerely

______________________	05/05/2016 ______________________
Researcher: Mrs L Zweers	Date
______________________	05/05/2016 ______________________
Supervisor: Dr JJ Botha	Date

Natural Science Building 4-1, Groenkloof Campus, UP
E-mail: hanlie.botha@up.ac.za

I hereby grant consent to Mrs L Zweers to observe my Grade 9 class, have access and make copies of the written tests and to conduct interviews with me for her Master's degree research. I also grant consent to Mrs L Zweers to analyse the written tests and audio-tape the interviews and video record the lessons.

Teacher's name: ____________________

Teacher's signature: ___________________

Date: __________________________

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UNIVERSITEIT VAN PRETORIA
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Faculty of Education

26 May 2016

Dear Sir/Madam

Letter of consent to the Principal

I hereby request permission to use your school for my research. I would like to invite two Grade 9 mathematics teachers to participate in this research aimed at investigating how Grade 9 mathematics teachers use assessment outcomes to inform their teaching. This research will be reported upon in my Master's dissertation at the University of Pretoria.

If consent can be obtained from you, the teachers and the parents, data will be collected by means of a baseline test, a classroom observation, an individual semi-structured interview, a formative test and another individual semi-structured interview.

- The teachers and I will set a baseline test based on the topic or content to be taught and the test will be written by the learners during class time. After the learners have written the test I will make copies to analyse by myself and the teachers will mark the tests as they normally do.
- After the tests have been marked I will observe one lesson in two different classes to be taught by each of the two Grade 9 mathematics teachers at a time convenient to the teachers as it should not disrupt the teachers' timetable and programme. The observations will be video recorded. This will allow for a clear and accurate record of the teacher's classroom practice. In cases where parents do not give permission for the video recording of the lessons, I will position myself in such a way that the learners will not be part of the observation.
- After the observations I would like to interview each teacher individually at two different occasions, one after the observations and the second one at the end of the data collection process. Although the interviews will be conducted at a time and place convenient to the teacher, it will be outside of school hours and should not take longer than one hour. The interviews will be audio-taped by me in order to have a clear and accurate record of all the communication that took place. The first interview will be based on the prior knowledge of the learners and the classroom practice of the teacher.
- After the first interview the learners will write a formative test that will be set and marked by me. The formative test will be on the newly developed concepts that were taught by the teachers.
- The second interview will be conducted after the formative test and will be based on how the learners' outcomes were influenced after the teachers'

instruction on the newly developed concepts on the topic, as revealed in the formative assessment.

Although I will analyse the learners' work in the baseline test and mark their formative tests, I will not be in direct contact with the learners. The learners will also be present in the class during the observations together with the researcher. The learners will receive a letter of assent to inform them about the research that will be conducted in the classroom and that we will use copies of their baseline tests and formative tests in this study. However the identity of the learners will be kept confidential and anonymous. The parents/guardians will receive a letter of informed consent for the video recording of the lessons, the use of the learners' baseline- and formative tests and my presence in the class.

All participation is voluntary and once committed to the research, the school, teachers and learners may still withdraw at any time. Confidentiality and anonymity will be guaranteed at all times by using pseudonyms for the school and the teachers. The school, the teachers and the learners will therefore not be identifiable in the findings of my research and only my supervisor and I will have access to the video- and audio recordings which will be password protected. The data collected will only be used for academic purposes. All data collected with public funding may be made available in an open repository for public and scientific use.

After the successful completion of my Master's degree, I will give feedback to the school in the form of a written report and if the school is willing, I would like to do a presentation of my findings to all mathematics teachers at that school.

For any questions before or during the research, please feel free to contact me. If you are willing to allow your staff to participate in this research study, please sign this letter as a declaration of your consent.

Yours sincerely

[signature]	26/05/2016
Researcher: Mrs L Zweers	Date
[signature]	26/05/2016
Supervisor: Dr JJ Botha	Date

Natural Science Building 4-1, Groenkloof Campus, UP
E-mail: hanlie.botha@up.ac.za

I hereby grant consent to Mrs L Zweers to conduct her research in this school for her Master's degree research. I also grant consent to Mrs L Zweers to analyse the tests, video record the lessons and audio record the interviews.

School principal's name: ___________________

School principal's signature: __________________

Date: _________________________

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UNIVERSITEIT VAN PRETORIA
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Faculty of Education

26 May 2016

Dear Sir/Madam

Letter of consent to the mathematics teacher

You are invited to participate in a study for my research project aimed at investigating how Grade 9 mathematics teachers use assessment outcomes to inform their teaching. The research will be reported upon in my Master's dissertation at the University of Pretoria. It is proposed that you form part of this study's data collection phase by setting a baseline test, being observed while teaching two Grade 9 mathematics classes for one lesson, interviewed twice and administrating a formative test.

The data collection process will be as follows:

- We must select two of your Grade 9 mathematics classes to participate in this study. One class must be top performing and the other class low performing. The Grade 9 mathematics learners of the two classes will write a baseline test that will be set by you and me on the topic or content to be taught at that time. The test will be written by the learners during class time. After the learners have written the test I will make copies to analyse by myself and you will mark the tests as you normally do.
- After the tests have been marked I will observe one lesson in the two different classes to be taught by you at a time convenient to you as it should not disrupt your timetable and programme. Note that you are not required to do anything extra than what you normally do during teaching; no extra preparation is needed. During my observations I will make notes on an observation sheet that has been prepared in advance, based on my research questions. If the principal, you and the parents give consent I would like to video record the lessons for an accurate record of you classroom practice. You have an option to choose if you want to be video recorded or not. In cases where parents do not give permission for the video recording of the lesson, I will position myself in such a way that the learners will not be part of the observation.
- After the observation I would like to interview you in order to discuss the lesson you presented (not longer than an hour). The interviews will be audio-taped by me in order to have a clear and accurate record of all the communication that took place. Although the interview will be conducted at your convenience it will be outside of school hours.
- After the first interview the learners will write a formative test set and marked by me. After the formative test is marked, a second interview will take place

(not longer than an hour). The interview will be based on how the learners' outcomes, as revealed in the formative test, were influenced by the instruction.

Should you declare yourself willing to participate in this research, you will be one of four teachers that form part of my research project. Your participation is voluntary and confidentiality and anonymity will be guaranteed at all times. This will be done by allocating pseudonyms to you and the school. You may decide to withdraw at any time without giving any reasons for doing so. You and your school will not be identifiable in the findings of my research and only my supervisor and I will have access to the video/audio recordings which will be password protected. You will have access to the interview transcriptions should you want to do so. The data collected will only be used for academic purposes. All data collected with public funding may be made available in an open repository for public and scientific use.

After the successful completion of my Master's degree, I will give feedback of my findings to the school in the form of a written report and if the school is willing, I would like to do a presentation of my findings to all mathematics teachers at your school.

If you are willing to participate in this research study, please sign this letter as a declaration of your consent, i.e. that you participate willingly and that you understand that you may withdraw at any time.

Yours sincerely

______________________	26/05/2016
Researcher: Mrs L Zweers	Date
______________________	26/05/2016
Supervisor: Dr JJ Botha	Date

Natural Science Building 4-1, Groenkloof Campus, UP
E-mail: hanlie.botha@up.ac.za

I hereby grant consent to Mrs L Zweers to observe my Grade 9 class, have access and make copies of the written tests and to conduct interviews with me for her Master's degree research. I also grant consent to Mrs L Zweers to analyse the written tests, video record the lesson and audio-tape the interviews.

Teacher's name: ____________________

Teacher's signature: ____________________

Date: ____________________________

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Cell no: 079 602 1922

5 May 2016

Dear Sir/Madam

Letter of consent to the parents/guardians

I am currently enrolled for a Master's degree at the University of Pretoria. My research is aimed at investigating how Grade 9 mathematics teachers use assessment outcomes to inform their teaching. The research will be reported upon in my Master's dissertation at the University of Pretoria. In order to do the research, I will observe your child's mathematics teacher in one of their lessons. I would like to video record this lesson as it will help me to have an accurate record of the teacher's classroom practice. I also need permission to analyse two of the tests your child will write; one before and one after the specific lesson that will be observed.

When video-recording the lesson, I will focus on the teacher and not on the learners in the class. The video recordings will be taken from the back of the class and I will, as far as possible, only film the teacher. All video recordings will be password protected and will only be used for my Master's degree. Your child's assessment outcomes form part of my research. All children's confidentiality and anonymity will be protected at all times and only my supervisor and I will have access to the recordings and tests. The data collected will only be used for academic purposes. All data collected with public funding may be made available in an open repository for public and scientific use.

All participation is voluntary and once committed your child may withdraw at any time. If you have any questions or concerns, please do not hesitate to contact me. If you are willing for your child to be present during the video recorded lessons and give me permission to analyse the tests, please sign this letter as a declaration of your consent.

Yours sincerely

26/05/2016

Researcher: Mrs L Zweers

Date

26/05/2016

Supervisor: Dr JJ Botha   Date

Natural Science Building 4-1, Groenkloof Campus, UP
E-mail: hanlie.botha@up.ac.za

I the undersigned, hereby grant consent to Mrs L Zweers to video record the lessons where my child will be present and to use my child's two written tests, for her Master's degree research. I am aware that my child will remain anonymous and that the findings of this research will be used to promote teaching and learning in the mathematics classroom.

Parent's/guardian's name: ____________________

Parent's/guardian's signature: ___________________

Date: __________________________

Child's name: _____________________________

Grade (e.g. 9C): ___________________________

ADDENDUM B LETTER OF ASSENT TO LEARNERS

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
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Faculty of Education

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5 May 2016

Dear learner

Letter of assent to the learners

I am enrolled for a Master's degree at the University of Pretoria and want to determine how mathematics teachers teach mathematics. This implies that I will not be teaching you. I want to film your teacher with a video camera while he/she is teaching mathematics. I will be standing at the back of the classroom and the video camera will be focused on your teacher and not you. The video will be used for my studies and no one will see the video recording but my supervisor and me.

I will also make some copies of the two tests you will be writing; one before and one after the lesson. That is the only way you will be involved in the research and you do not have to do anything except what your teacher expects you to do. Your participation is voluntary and once committed, you may withdraw at any time. If you have any questions you may contact me at any time.

Yours sincerely

Researcher: Mrs L Zweers	26/05/2016 Date
Supervisor: Dr JJ Botha	26/05/2016 Date

Natural Science Building 4-1, Groenkloof Campus, UP
E-mail: hanlie.botha@up.ac.za

I hereby grant assent to be present in the mathematics class when the lesson will be video recorded by Mrs L Zweers. Mrs Zweers may also make copies of the two tests I will be writing.

Learner's name: ___________________

Learner's signature: __________________

Date: _______________________

Grade (example 9C): __________________

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Mrs. L. Zweers
Natural Sciences building
Groenkloof campus
University of Pretoria
liezell@7zweerse.co.za
Cell no: 079 602 1922

5 Mei 2016

Beste leerder

Brief van instemming aan die leerders

Ek is 'n Meesters student van die Universiteit van Pretoria. My navorsing is gerig op hoe die Wiskunde onderwyser klas gee. Dit impliseer dat ek nie vir julle sal klas gee nie. Ek wil graag julle onderwyser afneem met 'n video kamera terwyl hy/sy vir julle wiskunde verduidelik. Ek gaan agter in die klas staan en die kamera gaan net op julle onderwyser(es) gefokus wees en glad nie op julle nie. Die video gaan slegs gebruik word vir my navorsing en niemand anders behalwe ek en my studieleier gaan na die video kyk nie.

Ek gaan afskrifte maak van die twee toetse wat jy gaan skryf; een voor en een na die les. Verder gaan jy geensins betrokke wees nie en hoef jy niks anders te doen, behalwe wat deur die onderwyser van jou verwag word nie. Jou deelname aan die studie is vrywillig en jy mag ter enige tyd onttrek van die studie. As jy enige navrae het, kan jy my enige tyd kontak.

Beste wense

_______________________	26/05/2016
Navorser: Mrs L Zweers	Datum
_______________________	26/05/2016
Studieleier: Dr JJ Botha	Datum

Natuurwetenskappegebou 4-1, Groenkloofkampus, UP
E-pos: hanlie.botha@up.ac.za

Ek stem hiermee in om teenwoordig te wees by die wiskunde les wat met ‘n video kamera opgeneem gaan word deur mev L Zweers. Ek gee ook toestemming aan mev L Zweers om afskrifte te maak van my twee toetse wat ek gaan skryf.

Leerder se naam: ___________________

Leerder se handtekening: ___________________

Datum: __________________________

Graad (bv. 9C): ___________________________

ADDENDUM C BASELINE TEST

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

GRADE 9 MATHEMATICS

BASELINE ASSESSMENT: VOLUME, SURFACE AREA AND CAPACITY

Name: ______________________________ Grade 9: _______

Instructions:

1. All questions must be answered on the question paper.
2. Show all calculations clearly.
3. You may use an approved scientific calculator.
4. Indicate units of measurement where applicable.
5. Round all the final answers off to two decimal places unless stated otherwise.

Question 1: [6 marks]

Explain in your own words the meaning of:

1.1 Surface area (2)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

1.2 Volume (2)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

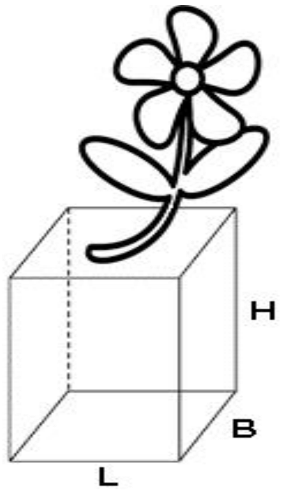
1.3 Capacity (2)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

Question 2: [29 marks]

Mrs. Mdiza has a rectangular container on her desk in her classroom. The container is open and does not have a lid.



2.1 Draw the net of the open container. Label all the sides with L, B and H. (4)

2.2 Explain in your own words how you will use the net to calculate the surface area of the container. (3)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

2.3 Use your answer in Question 2.2 and write down the formula for the surface area of the container. (3)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

2.4 Determine the surface area of the container with L = 15 cm, B = 10 cm and H = 20 cm. (5)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

2.5 Calculate the volume of the container with L = 15 cm, B = 10 cm and H = 20 cm. (4)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

2.6 Do the following conversions:

2.6.1 Complete: 1 m^3 = ______________________ cm^3 (1)

2.6.2 Complete: 1 m^3 = ______________________ liter (1)

2.7 Mrs. Mdiza pours 500 ml water in the container. Use the information to answer the following questions:

2.7.1 Convert 500 ml to cm^3 (2)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

2.7.2 Determine the volume of the container not occupied with water. (3)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

2.7.3 Determine the height of the water in the container. (3)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

Total: 35 marks

OBSERVATION SHEET

Assessing teachers' classroom practice (Personal PCK&S and classroom context) through observations (videotape lesson and make field notes during observations)

Name of school	
Name of researcher	Mrs. L. Zweers
Subject observed	Grade 9 mathematics
Number of learners present in class	
Number of learners absent from class	
Topic of the lesson	
Name of teacher	
Date of observation	

Table A: Personal Pedagogical Content Knowledge & Skill (Personal PCK&S)

Research Question 3:

How do the outcomes of the baseline test inform the teachers' Personal Pedagogical Content Knowledge and Skill (Personal PCK&S) as demonstrated in their instruction?

Aspects of Personal PCK&S	Comments (Support with examples)
A. Learners' prior knowledge	
1. Assessment of learners' prior knowledge (Questioning, listening)	
2. Nature of learners' prior knowledge	
3. Linking learners' prior knowledge with new knowledge	
B. Understanding learners' misconceptions	
1. Assessment of learners' misconceptions	
2. Nature of misconceptions	

3. Rectifying learners' misconceptions	
C. Modes of representation	
1. Appropriate and effective use of concrete representations	
2. Appropriate and effective use of visual representations	
3. Ability to use multiple representations	
D. Instructional strategies	
1. Various instructional strategies used	
2. Applicability of the strategies used (encourage and support learner involvement)	
E. Integration	
1. Integration with other mathematical topics	

2. Integration with other subject areas	
3. Integration with real-life contexts	

Table B: Classroom context

Research Question 4:

How can the classroom context be described?

A. Aspects of the classroom context	Comments (Support with examples)
1. Resources available and used	
2. Discipline	
3. Time management	
4. Learners (Motivation and level of proficiency)	

ADDENDUM E INTERVIEW SCHEDULE

SEMI-STRUCTURED INTERVIEW SCHEDULE FOR GRADE 9 MATHEMATICS TEACHERS

INTERVIEW 1
TO BE CONDUCTED AFTER THE CLASSROOM OBSERVATION

This interview schedule details the list of predetermined questions that will be asked to the Grade 9 mathematics teachers. The interview is semi-structured and therefore the researcher can ask additional questions in the interview to clarify or better understand the interviewees' responses. The additional questions are not included in the predetermined list of questions below. The interview will be conducted with the teachers on an individual basis.
The aim of this study is to examine the teacher's use of learner assessment outcomes to inform the teaching of mathematics. This interview consists of three sections, namely: A) General Information; B) Prior knowledge of the learners; and the C) Classroom context.

Date and time of interview: ______________________

Teacher: 1 or 2
Class: 1 or 2

Thank you for agreeing to take part in this interview. I want to audio-tape the interview so that I may have a clear and accurate record of our conversation. Remember that all information and opinions you reveal in this interview will be treated as confidential and a pseudonym will be used in the presentation of the data to protect your identity. Do you agree to be interviewed?

Please answer the questions truthfully.

SECTION A:

GENERAL INFORMATION

Please introduce yourself and give some background on your teaching career.

1. Age: 21-30____ 31-40____ 41-50____ 51-60____ 60+____

2. Gender: Male or Female____

3. Which other phase do you teach? __________

4. Years of teaching in the Senior phase (interrupted/uninterrupted):
   0-3____ 4-6____ 7-9____ 10+____

5. Years of teaching experience in (interrupted/uninterrupted):
   Public school(s) _____ Private school(s) ____

6. What are your qualifications?

____

____

7. What do you regard as important when teaching a mathematics lesson?

____

____

8. How do you think learners develop conceptual understanding in mathematics?

____

____

SECTION B:
PRIOR KNOWLEDGE OF THE LEARNERS

Research Question 1:

What is the learners' prior knowledge, as revealed in a baseline test?

1. What is your main goal of conducting a baseline assessment?

____

____

2. How will your grade the learners' prior knowledge on the topic of volume and surface area?

On par Sufficient Limited Non-existent

3. How do you analyse the learners' answers in the baseline test to determine their prior knowledge?

____

____

4. What were the misconceptions the learners revealed in the baseline test?

____

____

5. Do you use a baseline assessment before each new topic is introduced?
Yes or No _____
If yes, what type of baseline assessment do you use? (for example diagnostic test, reflective journal, concept maps, brainstorming, semantic map)

____

____

SECTION C:

CLASSROOM PRACTICE (Personal PCK and classroom context)

Subsection C1:

Personal PCK

Research Question 2:

How do the outcomes of the baseline test inform the teachers' Personal Pedagogical Content Knowledge (Personal PCK)?

1.a) What prior knowledge was learners supposed to have about volume and surface area?

____

____

b) What is the prior knowledge learners' lack the most?

____

____

2. Do you take the learners' prior knowledge into consideration when planning for the day's lesson? Yes or No _____
   If yes, how?

____

____

3. How do you create and utilise opportunities for learners to engage with the misconceptions and to reconstruct or adapt their knowledge?

____

____

4. How do you treat learners who continue to misunderstand a new concept?

____

____

5. How do you deal with topics that are difficult to teach?

____

____

6.a) Why did you use models as representation(s) when you explained surface area?

____

____

b) Are there any other representations you could have used to further explained surface area?

____

____

7.a) Why did you use direct lecturing as instructional strategy to explain and explore the new concepts?

____

____

b) Are there any other instructional strategies you could have used to further explained and explored the new concepts?

____

____

8. How did you integrate the content of the lesson with other mathematical topics and other subject areas?

____

____

Subsection C2:

Classroom context

Research Question 4:

How can the classroom context be described?

1. Why did you choose to use an overhead projector/projector as resource in presenting your lesson?

____

____

2. Which other resources that were not available did you need to effectively teach your lesson?

____

____

3.a) What are the discipline problems you experience in this class?

____

____

b) How did you plan to deal with those discipline problems in your class?

____

____

4. How do you plan to make effectively use of the available time for your lesson?

____

____

5. How motivated do you think the Grade 9 learners are to perform in mathematics?

____

____

6.a) How do the levels of mathematical proficiency differ in this class?

b) How do you deal or adapt your instruction to address these different levels of proficiency in one class?

7. Two different mathematics classes can also have different levels of mathematical proficiency. How do you deal or adapt your instruction with different levels of proficiency among classes?

ADDENDUM F FORMATIVE TEST

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

GRADE 9 MATHEMATICS

FORMATIVE ASSESSMENT: VOLUME, SURFACE AREA AND CAPACITY

Name: ______________________________ Grade 9: _______

Instructions:

1. All questions must be answered on the question paper.
2. Show all calculations clearly.
3. You may use an approved scientific calculator.
4. Indicate units of measurement where applicable.
5. Round all the final answers off to two decimal places unless stated otherwise.

Question 1: [6 marks]

Explain in your own words the meaning of:

1.1 Surface area (2)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

1.2 Volume (2)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

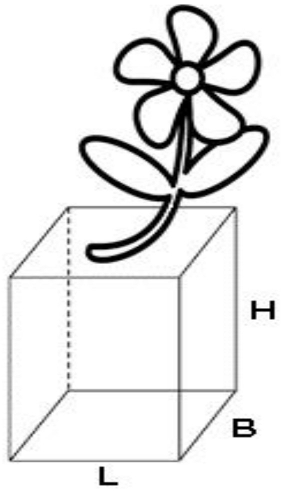
1.3 Capacity (2)

……………………………………………………………………………………………

……………………………………………………………………………………………

Question 2: [29 marks]

Mrs. Mdiza has a rectangular container on her desk in her classroom. The container is open and does not have a lid.



2.1 Draw the net of the open container. Label all the sides with L, B and H. (4)

2.2 Explain in your own words how you will use the net to calculate the surface area of the container. (3)

……………………………………………………………………………………………

……………………………………………………………………………………………

2.3 Use your answer in Question 2.2 and write down the formula for the surface area of the container. (3)

……………………………………………………………………………………………

……………………………………………………………………………………………

2.4 Determine the surface area of the container with L = 25 cm, B = 15 cm and H = 30 cm. (5)

……………………………………………………………………………………………

……………………………………………………………………………………………

2.5 Calculate the volume of the container with L = 25 cm, B = 15 cm and H = 30 cm. (4)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

2.6 Mrs. Mdiza pours 500 ml water in the container. Use the information to answer the following questions:

2.6.1 Complete: $1\ m^3$ = ______________________ cm^3 (1)

2.6.2 Complete: $1\ m^3$ = ______________________ liter (1)

2.6.3 Convert 750 ml to cm^3 (2)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

2.6.4 Determine the volume of the container not occupied with water. (3)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

2.6.5 Determine the height of the water in the container. (3)

…………………………………………………………………………………………………

…………………………………………………………………………………………………

Total: 35 marks