Periodically intermittent control strategies for α-exponential stabilization of fractional-order complex-valued delayed neural networks

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Abstract

This paper is concerned with the global α-exponential stabilization for a class of fractional-order complex-valued neural networks with time delay. To end this, some new fractional-order differential inequalities are established, which improve and generalize previously known criteria. Then, a suitable periodically intermittent control scheme with time delay is proposed for the global α-exponential stabilization of the addressed networks, which include feedback control as a special case. By using the new fractional-order differential inequalities and coupling with the Lyapunov method and some other inequality techniques, some novel delay-independent criteria in terms of real-valued algebraic inequalities are obtained to ensure global α-exponential stabilization of the discussed networks, which are very simple to implement in practice and avoid complex computation on the matrix inequalities. Finally, an illustrative example with numerical simulations is given to demonstrate the effectiveness of the theoretical results.

Keywords: Fractional-order; Complex-valued delayed neural network; α-exponential stabilization; Periodically intermittent control; Inequality technique.

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1 Introduction

Fractional differential systems are the generalizations of the classical integer order differential systems. Fractional differential systems have gained an increasing attention in recent years due to their potential applications in many fields of science and engineering. Many significant contributions have been made in the theory of fractional differential systems [1–8]. Meanwhile, due to the finite speed of the signal transmission, time delay often exists in almost every system, and time delay and the order of system could affect the dynamic behavior of system [9]. At present, fractional calculus has been integrated into artificial neural networks, and fractional-order neural networks are a kind of potentially applicable networks. many interesting and important results about stability of fractional-order neural networks with or without delays have been presented [10–14]. However, there are many fractional-order neural networks that are unstable in nature. In this case, the controllers have to be added to the neural networks to guarantee the corresponding asymptotic behaviors.

To improve system performance, various control strategies such as track control [15], adaptive control [16], feedback control [17], impulsive control [18] and intermittent control [19] are adopted based on the actual control requirements. Now, various stabilization criteria are also established such as exponential stabilization [20], mean square stabilization [21], guaranteed cost stabilization [22], finite-time stabilization [23, 24], delay-independent stabilization [25], sampled-data stabilization [26]. As a whole, these stabilization strategies have been adopted in the light of different system structure analysis. On the other hand, an important subject in system analysis is to seek less control cost, simple and efficient methods for system control. The intermittent control method is often effective and robust compared with continuous control, since each period in this kind of control scheme is composed of work time(or control time) and rest time and the controller is activated in each work time and is off in the rest time. So, the system output is measured intermittently rather than continuously. Owing to those merits, intermittent control has been successfully applied to stabilize and synchronize neural networks [27–30].

As an extension of real-valued neural networks, complex valued neural networks (CVNNs) with complex-valued state, input, connection weight and activation function have been one of the most important research topic in many research areas. In fact, the major goal of studying CVNNs is not only to explore new dynamic performance but also to resolve some problem which cannot be solved in real-valued networks. For instance, the xor problem and the detection of symmetry [31] cannot be settled with a single real-valued neuron, but these can be settled by using a single complex-valued neuron with the orthogonal decision boundaries. At present, the authors [32, 33] investigated stability for integer-order CVNNs with delays. In [34–36], the authors presented some results on the existence, uniqueness and stability of the equilibrium point for fractional-order CVNNs with delays. In [37], the Lagrange α-exponential stability and α-exponential convergence of a class of fractional-order CVNNs are studied by utilizing fractional-order differential inequalities. The authors [38, 39] studied the stabilization of fractional-order neural networks by using linear state feedback controls. In [40], the authors proposed two types of intermittent schemes to control the chaos in fractional-order system. The synchronization for fractional-order CVNNs [41] is investigated by means of linear delay feedback control. In [42], the authors researched stabilization of fractional-order singular uncertain systems using the state feedback and output feedback control. To our knowledge, there are few literatures focusing on designing periodically intermittent controllers to stabilize fractional-order CVNNs.

Motivated by the above analysis, the aim of this paper is to study α-exponential stabilization of fractional-order CVNNs with delay via periodically intermittent control. We will first establish some new fractional-order differential inequalities and propose a kind of intermittent control scheme. Besides, some new sufficient conditions in terms of real-valued algebraic inequalities for α-exponential stabilization are obtained based on the new fractional-order differential inequalities. Different from traditional exponential stabilization, our results include feedback control as a special case and show that the α-exponential convergence rate depends on the ratio of control width to control period and the order of differentiation of the system and p-norm, but does not depends on control width or control period.

This paper is organized as follows. In Section 2, some preliminaries are given. Some new fractional-order differential inequalities are established in Section 3. The main results are stated in Section 4. Section 5 presents a numerical example with simulations to verify the main results and finally a summary is given in Section 6.

2 Preliminaries

Notations: Throughout this paper, R, $\mathbb{C}$, R^n, $\mathbb{C}^n$, $R^{m\times n}$ and $\mathbb{C}^{m\times n}$ denote the set of real numbers, complex numbers, n-dimensional real vector, n-dimensional complex vector, $m \times n$ real and complex matrices, respectively. i represents the imaginary unit, i.e., $i = \sqrt{-1}$. For $a \in \mathbb{C}$, $|a|$ denotes the module of a, a^* is the complex conjugate of a. M^* show the conjugate transpose of complex matrix M. $\mathrm{sgn}(\cdot)$ denote sign function. Let $\mathscr{F} = \{1, 2, \ldots, n\}$, $N_+ = \{0, 1, 2, \ldots\}$. For

$z = (z_1, z_2, \dots, z_n)^T$ with $z_j = x_j + iy_j$ and $x_j, y_j \in R$ for $j \in \mathscr{F}$, $\|z\|_p = \big(\sum\limits_{j=1}^{n} (|x_j|^p + |y_j|^p) \big)^{\frac{1}{p}}$, where p is a positive integer.

In this section, we will introduce some definitions and some useful lemmas. Throughout this paper, we choose the Caputo fractional-order derivative.

Definition 1 [2]. The fractional integral of order α of a function $f(t) : [t_0, +\infty) \to R$ is defined as

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} f(s) ds, \ t \geq t_0,$$

where $\alpha > 0$, $\Gamma(\cdot)$ is the Gamma function.

Definition 2 [2]. Caputo derivative of order α of a function $f \in C^n([t_0, +\infty), R)$ is defined by

$$D^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{f^{(n)}(s)}{(t-s)^{\alpha-n+1}} ds, \ t \geq t_0,$$

where n is a positive integer such that $n-1 < \alpha < n$. Particularly, when $0 < \alpha < 1$

$$D^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{f'(s)}{(t-s)^\alpha} ds.$$

Consider the following fractional-order CVNN with time delay

$$D^\alpha z(t) = -Cz(t) + Af(z(t)) + Bg(z(t-\tau)) + J, \tag{1}$$

where $0 < \alpha < 1$ denotes the order of fractional-order derivative, $z(t) = (z_1(t), z_2(t), \dots, z_n(t))^T \in \mathbb{C}^n$ corresponds to the state vector, $C = \text{diag}\{c_1, c_2, \dots, c_n\}$ represents the neuron charging time matrix with $c_j > 0$ for $j \in \mathscr{F}$, τ is the time delay, $f(z(t)) = (f_1(z_1(t)), f_2(z_2(t)), \dots, f_n(z_n(t)))^T : \mathbb{C}^n \to \mathbb{C}^n$ and $g(z(t)) = (g_1(z_1(t-\tau)), g_2(z_2(t-\tau)), \dots, g_n(z_n(t-\tau)))^T : \mathbb{C}^n \to \mathbb{C}^n$ are the vector-valued complex-valued activation functions without and with time delay. $A = (a_{jk})_{n\times n} \in \mathbb{C}^{n\times n}$ and $B = (b_{jk})_{n\times n} \in \mathbb{C}^{n\times n}$ are the connection weight matrix without and with time delays, respectively. $J = (J_1, J_2, \dots, J_n)^T$ denotes external input vector.

The initial condition of network (1) is of the form

$$z(s) = \varphi(s) + i\psi(s), s \in [-\tau, 0],$$

where $\varphi(s) = (\varphi_1(s), \varphi_2(s), \dots, \varphi_n(s))^T$ and $\psi(s) = (\psi_1(s), \psi_2(s), \dots, \psi_n(s))^T$ are continuous real-valued vector functions on $[-\tau, 0]$.

Definition 3. The point $\tilde{z} = (\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_n)^T \in \mathbb{C}^n$ is called an equilibrium point of (1) if and only if

$$-C\tilde{z} + Af(\tilde{z}) + Bg(\tilde{z}) + J = 0.$$

To stabilize the unstable equilibrium point $\tilde{z}$ of (1), the control model of (1) is described by

$$D^\alpha z(t) = -Cz(t) + Af(z(t)) + Bg(z(t-\tau)) + J + U(t), \tag{2}$$

where $U(t) = u(t) + iv(t)$ with $u(t), v(t) \in R^n$ is a periodically intermittent controller which need be designed. Note $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T, v(t) = (v_1(t), v_2(t), \ldots, v_n(t))^T$.

Definition 4. Network (2) with intermittent effects is said to be α-exponentially stable if there exist constants $M > 0$ and $\lambda > 0$ such that

$$\|z(t) - \tilde{z}\| \leq M \sup_{-\tau \leq t \leq 0} \|z(s) - \tilde{z}\| e^{-\lambda t^\alpha}$$

holds for $\forall t > 0$ and any initial values $z(s) = \varphi(s) + i\psi(s)$ with $s \in [-\tau, 0]$.

Assumption 1. Let $z = x + iy$ and $\hat{z} = \hat{x} + i\hat{y}$, $f_j(z)$, $g_j(z)$ for $j \in \mathscr{F}$ are separated into their real and imaginary part as follows

$$f_j(z) = f_j^R(x, y) + if_j^I(x, y), g_j(z) = g_j^R(x, y) + ig_j^I(x, y),$$

where $f_j^R(\cdot, \cdot), f_j^I(\cdot, \cdot), g_j^R(\cdot, \cdot), g_j^I(\cdot, \cdot) : R^2 \to R$ and satisfy

$$|f_j^R(\hat{x}, \hat{y}) - f_j^R(x, y)| \leq F_j^{RR}|\hat{x} - x| + F_j^{RI}|\hat{y} - y|,\ |f_j^I(\hat{x}, \hat{y}) - f_j^I(x, y)| \leq F_j^{IR}|\hat{x} - x| + F_j^{II}|\hat{y} - y|,$$

$$|g_j^R(\hat{x}, \hat{y}) - g_j^R(x, y)| \leq G_j^{RR}|\hat{x} - x| + G_j^{RI}|\hat{y} - y|,\ |g_j^I(\hat{x}, \hat{y}) - g_j^I(x, y)| \leq G_j^{IR}|\hat{x} - x| + G_j^{II}|\hat{y} - y|,$$

where F_j^{RR}, F_j^{RI}, F_j^{IR}, F_j^{II}, G_j^{RR}, G_j^{RI}, G_j^{IR}, G_j^{II} are known positive constants for $\forall x, \hat{x}, y, \hat{y} \in \mathbb{R}$.

Let $e_j(t) = z_j(t) - \tilde{z}_j = e_j^x(t) + ie_j^y(t)$, $z_j(t) = x_j(t) + iy_j(t)$, $\tilde{z}_j = \tilde{x}_j + i\tilde{y}_j$, where $e_j^x(t), e_j^y(t), x_j(t), y_j(t), \tilde{x}_j, \tilde{y}_j \in R$ for $j \in \mathscr{F}$, then network (2) can be transformed into

$$\begin{aligned} D^\alpha e_j^x(t) = &- c_j e_j^x(t) + \sum_{k=1}^n a_{jk}^R[f_k^R(x_k(t), y_k(t)) - f_k^R(\tilde{x}_k, \tilde{y}_k)] - \sum_{k=1}^n a_{jk}^I[f_k^I(x_k(t), y_k(t)) - f_k^I(\tilde{x}_k, \tilde{y}_k)] \\ &+ \sum_{k=1}^n b_{jk}^R[g_k^R(x_k(t-\tau), y_k(t-\tau)) - g_k^R(\tilde{x}_k, \tilde{y}_k)] \\ &- \sum_{k=1}^n b_{jk}^I[g_k^I(x_k(t-\tau), y_k(t-\tau)) - g_k^I(\tilde{x}_k, \tilde{y}_k)] + u_j(t), \end{aligned} \tag{3}$$

$$\begin{aligned} D^\alpha e_j^y(t) = &- c_j e_j^y(t) + \sum_{k=1}^n a_{jk}^R[f_k^I(x_k(t), y_k(t)) - f_k^I(\tilde{x}_k, \tilde{y}_k)] + \sum_{k=1}^n a_{jk}^I[f_k^R(x_k(t), y_k(t)) - f_k^R(\tilde{x}_k, \tilde{y}_k)] \\ &+ \sum_{k=1}^n b_{jk}^R[g_k^I(x_k(t-\tau), y_k(t-\tau)) - g_k^I(\tilde{x}_k, \tilde{y}_k)] \\ &+ \sum_{k=1}^n b_{jk}^I[g_k^R(x_k(t-\tau), y_k(t-\tau)) - g_k^R(\tilde{x}_k, \tilde{y}_k)] + v_j(t). \end{aligned} \tag{4}$$

Lemma 1 [10]. Let n be a positive integer such that $n-1<\alpha<n$. If $y(t)\in C^{n-1}[a,b]$, then

$$I^{\alpha}D^{\alpha}y(t)=y(t)-\sum_{i=0}^{n-1}\frac{y^{(k)}(a)}{k!}(t-a)^{k}.$$

In particular, if $0<\alpha\leq 1$ and $y(t)\in C[a,b]$, then

$$I^{\alpha}D^{\alpha}y(t)=y(t)-y(a).$$

Lemma 2 [16]. Suppose that $x(t)\in C[a,b]$ and meets

$$D^{\alpha}x(t)=f(t,x(t))\geq 0,\quad 0<\alpha<1 \tag{5}$$

for $\forall t\in[a,b]$, then $x(t)$ is monotonously non-decreasing for $0<\alpha<1$. If

$$D^{\alpha}x(t)=f(t,x(t))\leq 0,\quad 0<\alpha<1 \tag{6}$$

for $\forall t\in[a,b]$, then $x(t)$ is monotonously non-increasing for $0<\alpha<1$.

Lemma 3(Young inequality). Let $a>0$, $b>0$, $p>1$, $q>1$ and $\frac{1}{p}+\frac{1}{q}=1$, then the inequality $ab\leq\frac{1}{p}a^{p}+\frac{1}{q}b^{q}$ holds, and the equality holds if and only if $a^{p}=b^{q}$.

3 Some new differential inequalities related to the Caputo fractional derivative

Lemma 4. Assume that $x(t)$, $y(t)$ are continuous and differentiable on $[a,b)$ for $\forall a\in R$, if $D^{\alpha}x(t)\leq D^{\alpha}y(t)$ for $0<\alpha<1$ and $x(a)=y(a)$, then

$$x(t)\leq y(t),t\in[a,b).$$

Proof. Let $D^{\alpha}x(t)=f(t)$, $D^{\alpha}y(t)=g(t)$, then $f(t)\leq g(t)$. From Lemma 1, one has for $t\in[a,b)$

$$x(a)=x(t)-\frac{1}{\Gamma(\alpha)}\int_{a}^{t}(t-s)^{\alpha-1}f(s)ds,$$
$$y(a)=y(t)-\frac{1}{\Gamma(\alpha)}\int_{a}^{t}(t-s)^{\alpha-1}g(s)ds.$$

According to $x(a)=y(a)$, one has

$$x(t)-y(t)=\frac{1}{\Gamma(\alpha)}\int_{a}^{t}(t-s)^{\alpha-1}(f(s)-g(s))ds,$$

Since $a\leq s\leq t\Rightarrow t-s\geq 0$, $0<\alpha<1\Rightarrow\Gamma(\alpha)>0$, and $f(t)\leq g(t)\Rightarrow f(t)-g(t)\leq 0$. So, we can conclude on the basis of the above equation that $x(t)\leq y(t)$.

Remark 1. When $a = 0$, Lemma 4 here turns into Theorem 2.4 in [4]. So, Lemma 4 here is applicable to any continuous and differentiable interval for $x(t)$, $y(t)$ and more general.

Lemma 5. Let $G(t)$ be a real-valued continuous function on $[a, b)$ for $\forall a \in R$, and there exist a constant θ such that

$$D^\alpha G(t) \leq \theta G(t),$$

where $0 < \alpha \leq 1$, then

$$G(t) \leq G(a)e^{\frac{\theta}{\Gamma(\alpha+1)}(t-a)^\alpha}.$$

Proof. According to Definition 1 and Lemma 1, we can have

$$G(t) \leq G(a) + \frac{\theta}{\Gamma(\alpha)} \int_a^t (t-s)^{\alpha-1} G(s)ds, t \geq a.$$

From the famous Gronwall inequality, we have

$$G(t) \leq G(a)e^{\int_a^t \frac{\theta}{\Gamma(\alpha)}(t-\tau)^{\alpha-1}d\tau} = G(a)e^{\frac{\theta}{\Gamma(\alpha+1)}(t-a)^\alpha}.$$

The proof is completed.

Remark 2. When $a = 0, b = +\infty$, Lemma 5 here becomes into Lemma 4 in [10]. It is obvious that Lemma 5 here is an extension of Lemma 4 in [10] at any continuous interval.

Lemma 6. Suppose that a continuous function $V(t) : [t_0 - \tau, +\infty) \to [0, +\infty)$ meets the following differential inequality:

$$D^\alpha V(t) \leq aV(t) + bV(t-\tau), \forall t \geq t_0, \tag{7}$$

where $a > 0, b > 0$ are constants. Then

$$V(t) \leq \sup_{t_0-\tau\leq s\leq t_0} V(s)e^{\frac{a+b}{\Gamma(\alpha+1)}(t-t_0)^\alpha}.$$

Proof. From (7), it is clear that

$$D^\alpha V(t) \leq (a+b)\max\{V(t), V(t-\tau)\}.$$

After fractional integration on both sides of the above inequality from t_0 to t, one has

$$V(t) \leq V(t_0) + \frac{a+b}{\Gamma(\alpha)} \int_{t_0}^t (t-s)^{\alpha-1} \max\{V(s), V(s-\tau)\}ds. \tag{8}$$

Definite

$$W(t) = \frac{a+b}{\Gamma(\alpha)} \int_{t_0}^t (t-s)^{\alpha-1} \max\{V(s), V(s-\tau)\}ds.$$

Using Definition 2 to calculate the fractional-order derivatives of $W(t)$, then

$$D^{\alpha}W(t) = (a+b)\max\{V(t), V(t-\tau) \geq 0.$$

From Lemma 2, $W(t)$ is monotonously non-decreasing, namely, $W(t) \geq W(t-\tau)$, one has

$$\begin{aligned} V(t-\tau) \leq & V(t_0) + \frac{a+b}{\Gamma(\alpha)} \int_{t_0}^{t-\tau} (t-\tau-s)^{\alpha-1} \max\{V(s), V(s-\tau)\} ds \\ \leq & V(t_0) + \frac{a+b}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} \max\{V(s), V(s-\tau)\} ds. \end{aligned} \tag{9}$$

Combining (8) with (9), we get

$$\max\{V(t), V(t-\tau)\} \leq V(t_0) + \frac{a+b}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} \max\{V(s), V(s-\tau)\} ds$$

From the famous Gronwall inequality, one can obtain

$$V(t) \leq \max\{V(t), V(t-\tau)\} \leq V(t_0) e^{\frac{a+b}{\Gamma(\alpha+1)}(t-t_0)^{\alpha}} \leq \sup_{t_0-\tau \leq s \leq t_0} V(s) e^{\frac{a+b}{\Gamma(\alpha+1)}(t-t_0)^{\alpha}}.$$

Lemma 7. Let p be a positive integer, if $x(t) \in R$ is a continuous and differentiable function, then

$$D^{\alpha}|x(t)|^{p} \leq p|x(t)|^{p-1} D^{\alpha}|x(t)|, 0 < \alpha \leq 1, t \geq t_0. \tag{10}$$

Proof. When $p = 1$, the inequality (10) is clearly true.

When $p \geq 2$, it is equivalent to prove that

$$D^{\alpha}|x(t)|^{p} - p|x(t)|^{p-1} D^{\alpha}|x(t)| \leq 0. \tag{11}$$

If $x(t) > 0$, according to the Definition 2, one has

$$D^{\alpha}|x(t)| = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{\dot{x}(s)}{(t-s)^{\alpha}} ds = D^{\alpha}x(t),$$

and if $x_j(t) < 0$, then

$$D^{\alpha}|x(t)| = -\frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{\dot{x}(s)}{(t-s)^{\alpha}} ds = -D^{\alpha}x(t).$$

Therefore

$$D^{\alpha}|x(t)| = \operatorname{sgn}(x(t)) D^{\alpha}x(t). \tag{12}$$

So, we have

$$D^{\alpha}|x(t)|^{p} = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{p|x(s)|^{p-1}\operatorname{sgn}(x(s))\dot{x}(s)}{(t-s)^{\alpha}} ds.$$

Define $u(s) = |x(s)|^p - p|x(t)|^{p-1}|x(s)| + (p-1)|x(t)|^p$. From Lemma 3, we can obtain

$$u(s) \geq 0, u(t) = 0.$$

Combining (12) with Definition 2, we can obtain

$$\begin{aligned}
&D^{\alpha}|x(t)|^p - p|x(t)|^{p-1}D^{\alpha}|x(t)| \\
=&\frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{p\big[|x(s)|^{p-1} - |x(t)|^{p-1}\big]\mathrm{sgn}(x(s))\dot{x}(s)}{(t-s)^{\alpha}}ds \\
=&\frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{1}{(t-s)^{\alpha}}du(s) \\
=&\frac{1}{\Gamma(1-\alpha)}\lim_{s\to t_-}\frac{u(s)}{(t-s)^{\alpha}} - \frac{1}{\Gamma(1-\alpha)}\frac{u(t_0)}{(t-t_0)^{\alpha}} - \frac{\alpha}{\Gamma(1-\alpha)}\int_{t_0}^{t}u(s)(t-s)^{-\alpha-1}ds. \qquad (13)
\end{aligned}$$

From the first term of the expression (13), we have

$$\begin{aligned}
\frac{1}{\Gamma(1-\alpha)}\lim_{s\to t_-}\frac{u(s)}{(t-s)^{\alpha}} =& -\lim_{s\to t_-}\frac{p|x(s)|^{p-1}\mathrm{sgn}(x(s))\dot{x}(s) - p|x(t)|^{p-1}\mathrm{sg}(x(s))\dot{x}(s)}{\alpha\Gamma(1-\alpha)(t-s)^{\alpha-1}} \\
=& -\frac{1}{\alpha\Gamma(1-\alpha)}\lim_{s\to t_-}\big[p|x(s)|^{p-1} - p|x(t)|^{p-1}\big]\mathrm{sgn}(x(s))\dot{x}(s)(t-s)^{1-\alpha} \\
=&0.
\end{aligned}$$

Owing to $u(t) \geq 0$, one has

$$\begin{aligned}
&D^{\alpha+}|x(t)|^p - p|x(t)|^{p-1}D^{\alpha+}|x(t)| \\
=& -\frac{1}{\Gamma(1-\alpha)}\frac{u(t_0)}{(t-t_0)^{\alpha}} - \frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}u(s)\alpha(t-s)^{-\alpha-1}ds \\
\leq&0.
\end{aligned}$$

The above inequality indicates that the inequality (10) holds. This completes the proof.

Remark 3. When $p = 2$, Lemma 7 here becomes into Lemma 1 in [5]. Meanwhile, the case for $\alpha = 1$ corresponds to $\frac{d}{dt}|x(t)|^p \leq p|x(t)|^{p-1}\frac{d}{dt}|x(t)|$, it can be considered as a particular case of Lemma 7 here. So, our result is more general.

4 Main results

In this section, we design a class of periodically intermittent controllers to guarantee α-exponential stability of system (2).

The periodically intermittent controller $U(t) = u(t) + iv(t)$ is defined by

$$\begin{cases} \text{if } mT \leq t < mT + \delta, \\ \qquad u_j(t) = -d_j e_j^x(t) - \text{sgn}(e_j^x(t)) \sum\limits_{k=1}^{n} (s_{jk}|e_k^x(t-\tau)| + h_{jk}|e_k^y(t-\tau)|), \\ \qquad v_j(t) = -q_j e_j^y(t) - \text{sgn}(e_j^y(t)) \sum\limits_{k=1}^{n} (r_{jk}|e_k^x(t-\tau)| + l_{jk}|e_k^y(t-\tau)|), \\ \text{if } mT + \delta \leq t < (m+1)T, \quad u_j(t) = v_j(t) = 0. \end{cases} \tag{14}$$

Under special circumstances, the controller (14) turns into

$$\begin{cases} \text{if } mT \leq t < mT + \delta, \\ \qquad u_j(t) = -\varepsilon_j e_j^x(t) - \text{sgn}(e_j^x(t)) \sum\limits_{k=1}^{n} \rho_{jk}|e_k^x(t-\tau)|, \\ \qquad v_j(t) = -\epsilon_j e_j^y(t) - \text{sgn}(e_j^y(t)) \sum\limits_{k=1}^{n} \varrho_{jk}|e_k^y(t-\tau)|, \\ \text{if } mT + \delta \leq t < (m+1)T, \quad u_j(t) = v_j(t) = 0. \end{cases} \tag{15}$$

The parameters $d_j, q_j, s_{jk}, r_{jk}, h_{jk}, l_{jk}, \varepsilon_j, \epsilon_j, \rho_{jk}, \varrho_{jk}$ are all positive constants determined later. $T > 0$ denotes the control period and $\delta = \beta T (0 < \beta < 1)$ is called the control width and β is called the control width index.

Theorem 1. Under Assumption 1, network (2) with (14) is α-exponentially stable if there exist some positive scalars θ, μ, ν and λ such that the following conditions are satisfied for positive integer $p \geq 2$

$$\begin{cases} -p(c_j + d_j) + (p-1) \sum\limits_{k=1}^{n} (|a_{jk}^R|(F_k^{RR} + F_k^{RI}) + |a_{jk}^I|(F_k^{II} + F_k^{IR})) \\ \qquad\qquad + \sum\limits_{k=1}^{n} (|a_{kj}^R|(F_j^{RR} + F_j^{RI}) + |a_{kj}^I|(F_j^{IR} + |F_j^{II})) \leq -\theta p, \\ -p(c_j + q_j) + (p-1) \sum\limits_{k=1}^{n} (|a_{jk}^R|(F_k^{IR} + F_k^{II}) + |a_{jk}^I|(F_k^{RR} + F_k^{RI})) \\ \qquad\qquad + \sum\limits_{k=1}^{n} (|a_{kj}^R|(F_j^{IR} + |F_j^{II}) + |a_{kj}^I|(F_j^{RR} + |F_j^{RI})) \leq -\theta p. \end{cases} \tag{16}$$

$$\begin{cases} |b_{jk}^R|G_k^{RR} + |b_{jk}^I|G_k^{IR} \leq s_{jk}, \quad |b_{jk}^R|G_k^{RI} + |b_{jk}^I|G_k^{II} \leq h_{jk}, \\ |b_{jk}^R|G_k^{IR} + |b_{jk}^I|G_k^{RR} \leq r_{jk}, \quad |b_{jk}^R|G_k^{II} + |b_{jk}^I|G_k^{RI} \leq l_{jk}. \end{cases} \tag{17}$$

$$\begin{cases} -pc_j + (p-1) \sum\limits_{k=1}^{n} (|a_{jk}^R|(F_k^{RR} + F_k^{RI}) + |a_{jk}^I|(F_k^{II} + F_k^{IR})) + \sum\limits_{k=1}^{n} (|a_{kj}^R|(F_j^{RR} + F_j^{RI}) + |a_{kj}^I|(F_j^{IR} + |F_j^{II})) \\ \qquad\qquad + (p-1) \sum\limits_{k=1}^{n} (|b_{jk}^R|(G_k^{RR} + G_k^{RI}) + |b_{jk}^I|(G_k^{IR} + G_k^{II})) \leq \mu p, \\ -pc_j + (p-1) \sum\limits_{k=1}^{n} (|a_{jk}^R|(F_k^{IR} + F_k^{II}) + |a_{jk}^I|(F_k^{RR} + F_k^{RI})) + \sum\limits_{k=1}^{n} (|a_{kj}^R|(F_j^{IR} + |F_j^{II}) + |a_{kj}^I|(F_j^{RR} + |F_j^{RI})) \\ \qquad\qquad + (p-1) \sum\limits_{k=1}^{n} (|b_{jk}^R|(G_k^{IR} + G_k^{II}) + |b_{jk}^I|(G_k^{RR} + G_k^{RI})) \leq \mu p. \end{cases} \tag{18}$$

$$\sum_{k=1}^{n}(|b_{kj}^{R}|+|b_{kj}^{I}|)(G_j^{RR}+G_j^{IR})\le \nu,\ \ \sum_{k=1}^{n}(|b_{kj}^{R}|+|b_{kj}^{I}|)(G_j^{RI}+G_j^{II})\le \nu. \tag{19}$$

$$-\theta\beta^{\alpha}+(\mu+\nu)(1-\beta)^{\alpha}+\lambda\le 0. \tag{20}$$

Proof. It is easy to get

$$D^{\alpha}|e_j^x(t)|=\mathrm{sgn}(e_j^x(t))D^{\alpha}e_j^x(t),\ \ D^{\alpha}|e_j^y(t)|=\mathrm{sgn}(e_j^y(t))D^{\alpha}e_j^y(t). \tag{21}$$

Let the Lyapunov function be in the form of

$$V(t)=\sum_{j=1}^{n}\frac{|e_j^x(t)|^p}{p}+\sum_{j=1}^{n}\frac{|e_j^y(t)|^p}{p}. \tag{22}$$

On the basis of Lemma 7, combining with (21) and calculating the fractional-order derivatives of $V(t)$ along the solutions of networks (3)-(4), we obtain

$$\begin{aligned}
D^{\alpha}V(t)\le&\sum_{j=1}^{n}|e_j^x(t)|^{p-1}D_{t_0}^{\alpha}|e_j^x(t)|+\sum_{j=1}^{n}|e_j^y(t)|^{p-1}D_{t_0}^{\alpha}|e_j^y(t)|\\
\le&-\sum_{j=1}^{n}c_j|e_j^x(t)|^p+\sum_{j=1}^{n}\sum_{k=1}^{n}|a_{jk}^R||e_j^x(t)|^{p-1}[F_k^{RR}|e_k^x(t)|+F_k^{RI}|e_k^y(t)|]\\
&+\sum_{j=1}^{n}\sum_{k=1}^{n}|a_{jk}^I||e_j^x(t)|^{p-1}[F_k^{IR}|e_k^x(t)|+F_k^{II}|e_k^y(t)|]\\
&+\sum_{j=1}^{n}\sum_{k=1}^{n}|b_{jk}^R||e_j^x(t)|^{p-1}[G_k^{RR}|e_k^x(t-\tau)|+G_k^{RI}|e_k^y(t-\tau)|]\\
&+\sum_{j=1}^{n}\sum_{k=1}^{n}|b_{jk}^I||e_j^x(t)|^{p-1}[G_k^{IR}|e_k^x(t-\tau)|+G_k^{II}|e_k^y(t-\tau)|]\\
&-\sum_{j=1}^{n}c_j|e_j^y(t)|^p+\sum_{j=1}^{n}\sum_{k=1}^{n}|a_{jk}^R||e_j^y(t)|^{p-1}[F_k^{IR}|e_k^x(t)|+F_k^{II}|e_k^y(t)|]\\
&+\sum_{j=1}^{n}\sum_{k=1}^{n}|a_{jk}^I||e_j^y(t)|^{p-1}[F_k^{RR}|e_k^x(t)|+F_k^{RI}|e_k^y(t)|]\\
&+\sum_{j=1}^{n}\sum_{k=1}^{n}|b_{jk}^R||e_j^y(t)|^{p-1}[G_k^{IR}|e_k^x(t-\tau)|+G_k^{II}|e_k^y(t-\tau)|]\\
&+\sum_{j=1}^{n}\sum_{k=1}^{n}|b_{jk}^I||e_j^y(t)|^{p-1}[G_k^{RR}|e_k^x(t-\tau)|+G_k^{RI}|e_k^y(t-\tau)|]\\
&+\sum_{j=1}^{n}|e_j^x(t)|^{p-1}\mathrm{sgn}(e_j^x(t))u_j(t)+\sum_{j=1}^{n}|e_j^y(t)|^{p-1}\mathrm{sgn}(e_j^y(t))v_j(t).
\end{aligned} \tag{23}$$

According to Lemma 3, we can obtain

$$|e_j^x(t)|^{p-1}|e_k^x(t)|\le\frac{p-1}{p}|e_j^x(t)|^p+\frac{1}{p}|e_k^x(t)|^p,\ \ |e_j^x(t)|^{p-1}|e_k^y(t)|\le\frac{p-1}{p}|e_j^x(t)|^p+\frac{1}{p}|e_k^y(t)|^p, \tag{24}$$

$$|e_j^y(t)|^{p-1}|e_k^x(t)| \le \frac{p-1}{p}|e_j^y(t)|^p + \frac{1}{p}|e_k^x(t)|^p,\ |e_j^y(t)|^{p-1}|e_k^y(t)| \le \frac{p-1}{p}|e_j^y(t)|^p + \frac{1}{p}|e_k^y(t)|^p. \quad (25)$$

When $mT \le t < mT + \delta$ for $\forall m \in N_+$, from (23)-(25), one has

$$\begin{aligned}
D^\alpha V(t) \le & \frac{1}{p}\sum_{j=1}^{n}\Big\{ -pc_j + (p-1)\sum_{k=1}^{n}(|a_{jk}^R|(F_k^{RR}+F_k^{RI}) + |a_{jk}^I|(F_k^{IR}+F_k^{II})) \\
& + \sum_{k=1}^{n}(|a_{kj}^R|(F_j^{RR}+F_j^{RI}) + |a_{kj}^I|(F_j^{IR}+|F_j^{II}))\Big\}|e_j^x(t)|^p \\
& + \sum_{j=1}^{n}\sum_{k=1}^{n}\Big[|b_{jk}^R|G_k^{RR} + |b_{jk}^I|G_k^{IR}\Big]|e_j^x(t)|^{p-1}|e_k^x(t-\tau)| \\
& + \sum_{j=1}^{n}\sum_{k=1}^{n}\Big[|b_{jk}^R|G_k^{RI} + |b_{jk}^I|G_k^{II}\Big]|e_j^x(t)|^{p-1}|e_k^y(t-\tau)| \\
& + \frac{1}{p}\sum_{j=1}^{n}\Big\{ -pc_j + (p-1)\sum_{k=1}^{n}(|a_{jk}^R|(F_k^{IR}+F_k^{II}) + |a_{jk}^I|(F_k^{RR}+F_k^{RI})) \\
& + \sum_{k=1}^{n}(|a_{kj}^R|(F_j^{IR}+|F_j^{II}) + |a_{kj}^I|(F_j^{RR}+|F_j^{RI}))\Big\}|e_j^y(t)|^p \\
& + \sum_{j=1}^{n}\sum_{k=1}^{n}\Big[|b_{jk}^R|G_k^{IR} + |b_{jk}^I|G_k^{RR}\Big]|e_j^y(t)|^{p-1}|e_k^x(t-\tau)| \\
& + \sum_{j=1}^{n}\sum_{k=1}^{n}\Big[|b_{jk}^R|G_k^{II} + |b_{jk}^I|G_k^{RI}\Big]|e_j^y(t)|^{p-1}|e_k^y(t-\tau)| \\
& + \sum_{j=1}^{n}|e_j^x(t)|^{p-1}\mathrm{sgn}(e_j^x(t))u_j(t) + \sum_{j=1}^{n}|e_j^y(t)|^{p-1}\mathrm{sgn}(e_j^y(t))v_j(t) \\
\le & \frac{1}{p}\sum_{j=1}^{n}\Big\{ -p(c_j+d_j) + (p-1)\sum_{k=1}^{n}(|a_{jk}^R|(F_k^{RR}+F_k^{RI}) + |a_{jk}^I|(F_k^{IR}+F_k^{II})) \\
& + \sum_{k=1}^{n}(|a_{kj}^R|(F_j^{RR}+F_j^{RI}) + |a_{kj}^I|(F_j^{IR}+|F_j^{II}))\Big\}|e_j^x(t)|^p \\
& + \sum_{j=1}^{n}\sum_{k=1}^{n}\Big[|b_{jk}^R|G_k^{RR} + |b_{jk}^I|G_k^{IR} - s_{jk}\Big]|e_j^x(t)|^{p-1}|e_k^x(t-\tau)| \\
& + \sum_{j=1}^{n}\sum_{k=1}^{n}\Big[|b_{jk}^R|G_k^{RI} + |b_{jk}^I|G_k^{II} - h_{jk}\Big]|e_j^x(t)|^{p-1}|e_k^y(t-\tau)| \\
& + \frac{1}{p}\sum_{j=1}^{n}\Big\{ -p(c_j+q_j) + (p-1)\sum_{k=1}^{n}(|a_{jk}^R|(F_k^{IR}+F_k^{II}) + |a_{jk}^I|(F_k^{RR}+F_k^{RI})) \\
& + \sum_{k=1}^{n}(|a_{kj}^R|(F_j^{IR}+|F_j^{II}) + |a_{kj}^I|(F_j^{RR}+|F_j^{RI}))\Big\}|e_j^y(t)|^p \\
& + \sum_{j=1}^{n}\sum_{k=1}^{n}\Big[|b_{jk}^R|G_k^{IR} + |b_{jk}^I|G_k^{RR} - r_{jk}\Big]|e_j^y(t)|^{p-1}|e_k^x(t-\tau)|
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}\sum_{k=1}^{n}\Big[|b_{jk}^R|G_k^{II}+|b_{jk}^I|G_k^{RI}-l_{jk}\Big]|e_j^y(t)|^{p-1}|e_k^y(t-\tau)| \\
&\leq -\theta V(t). \qquad (26)
\end{aligned}
$$

By Lemma 5, we get for $mT \leq t < mT+\delta$

$$
V(t) \leq V(mT)e^{\frac{-\theta}{\Gamma(\alpha+1)}(t-mT)^{\alpha}}. \qquad (27)
$$

From Lemma 3, we can obtain

$$
\begin{aligned}
|e_j^x(t)|^{p-1}|e_k^x(t-\tau)| &\leq \frac{p-1}{p}|e_j^x(t)|^p+\frac{1}{p}|e_k^x(t-\tau)|^p, \qquad (28)\\
|e_j^x(t)|^{p-1}|e_k^y(t-\tau)| &\leq \frac{p-1}{p}|e_j^x(t)|^p+\frac{1}{p}|e_k^y(t-\tau)|^p, \qquad (29)\\
|e_j^y(t)|^{p-1}|e_k^x(t-\tau)| &\leq \frac{p-1}{p}|e_j^y(t)|^p+\frac{1}{p}|e_k^x(t-\tau)|^p, \qquad (30)\\
|e_j^y(t)|^{p-1}|e_k^y(t-\tau)| &\leq \frac{p-1}{p}|e_j^y(t)|^p+\frac{1}{p}|e_k^y(t-\tau)|^p. \qquad (31)
\end{aligned}
$$

When $mT+\delta \leq t < (m+1)T$ for $\forall m \in N_+$, substituting (28)-(31) into (23), one has

$$
\begin{aligned}
D^{\alpha}V(t) \leq &\frac{1}{p}\sum_{j=1}^{n}\Big\{-pc_j+(p-1)\sum_{k=1}^{n}(|a_{jk}^R|(F_k^{RR}+F_k^{RI})+|a_{jk}^I|(F_k^{IR}+F_k^{II}))\\
&+\sum_{k=1}^{n}(|a_{kj}^R|(F_j^{RR}+F_j^{RI})+|a_{kj}^I|(F_j^{IR}+|F_j^{II}))\\
&+(p-1)\sum_{k=1}^{n}|b_{jk}^R|(G_k^{RR}+G_k^{RI})+(p-1)\sum_{k=1}^{n}|b_{jk}^I|(G_k^{IR}+G_k^{II})\Big\}|e_j^x(t)|^p\\
&+\frac{1}{p}\sum_{j=1}^{n}\sum_{k=1}^{n}(|b_{kj}^R|+|b_{kj}^I|)(G_j^{RR}+G_j^{IR})|e_j^x(t-\tau)|^p\\
&+\frac{1}{p}\sum_{j=1}^{n}\Big\{-pc_j+(p-1)\sum_{k=1}^{n}(|a_{jk}^R|(F_k^{IR}+F_k^{II})+|a_{jk}^I|(F_k^{RR}+F_k^{RI}))\\
&+\sum_{k=1}^{n}(|a_{kj}^R|(F_j^{IR}+|F_j^{II})+|a_{kj}^I|(F_j^{RR}+|F_j^{RI}))\\
&+(p-1)\sum_{k=1}^{n}|b_{jk}^R|(G_k^{IR}+G_k^{II})+(p-1)\sum_{k=1}^{n}|b_{jk}^I|(G_k^{RR}+G_k^{RI})\Big\}|e_j^y(t)|^p\\
&+\frac{1}{p}\sum_{j=1}^{n}\sum_{k=1}^{n}(|b_{kj}^R|+|b_{kj}^I|)(G_j^{RI}+G_j^{II})|e_j^y(t-\tau)|^p. \qquad (32)
\end{aligned}
$$

Combining with the inequalities (18)-(19) and (32), we get

$$
D^{\alpha}V(t) \leq \mu V(t)+\nu V(t-\tau). \qquad (33)
$$

From Lemma 6, based on the inequality (33), one has for $mT+\delta \le t < (m+1)T$

$$V(t) \le V(mT+\delta)e^{\frac{\mu+\nu}{\Gamma(\alpha+1)}(t-(mT+\delta))^{\alpha}}. \tag{34}$$

Based on the inequalities (27) and (34), then we have for $mT+\delta \le t < (m+1)T$

$$\begin{aligned} V(t) \le & V(mT)e^{\frac{-\theta}{\Gamma(\alpha+1)}\delta^{\alpha}+\frac{\mu+\nu}{\Gamma(\alpha+1)}(t-(mT+\delta))^{\alpha}} \\ \le & V(mT)e^{\frac{-\theta\beta^{\alpha}+(\mu+\nu)(1-\beta)^{\alpha}}{\Gamma(\alpha+1)}T^{\alpha}}. \end{aligned}$$

And

$$V((m+1)T) \le V(mT)e^{\frac{-\theta\beta^{\alpha}+(\mu+\nu)(1-\beta)^{\alpha}}{\Gamma(\alpha+1)}T^{\alpha}}.$$

From the above inequality, we can obtain

$$\begin{aligned} V(mT) \le & V((m-1)T)e^{\frac{-\theta\beta^{\alpha}+(\mu+\nu)(1-\beta)^{\alpha}}{\Gamma(\alpha+1)}T^{\alpha}} \\ \le & V((m-2)T)e^{2\cdot\frac{-\theta\beta^{\alpha}+(\mu+\nu)(1-\beta)^{\alpha}}{\Gamma(\alpha+1)}T^{\alpha}} \\ \le & \cdots \\ \le & V(T)e^{(m-1)(\frac{-\theta\beta^{\alpha}+(\mu+\nu)(1-\beta)^{\alpha}}{\Gamma(\alpha+1)}T^{\alpha}} \\ \le & V(0)e^{m(\frac{-\theta\beta^{\alpha}+(\mu+\nu)(1-\beta)^{\alpha}}{\Gamma(\alpha+1)}T^{\alpha}}. \end{aligned}$$

Therefore, when $mT \le t < (m+1)T$, we have

$$\begin{aligned} V(t) \le & V(mT)e^{\frac{-\theta\beta^{\alpha}+(\mu+\nu)(1-\beta)^{\alpha}}{\Gamma(\alpha+1)}T^{\alpha}} \\ \le & V(0)e^{(m+1)(\frac{-\theta\beta^{\alpha}+(\mu+\nu)(1-\beta)^{\alpha}}{\Gamma(\alpha+1)}T^{\alpha}} \\ \le & V(0)e^{\frac{-\lambda}{\Gamma(\alpha+1)}t^{\alpha}}. \end{aligned}$$

That is to say

$$\|e(t)\|_p \le \|e(0)\|e^{\frac{-\lambda}{p\Gamma(\alpha+1)}t^{\alpha}} \le \sup_{-\tau\le s\le 0}\|e(s)\|_p e^{\frac{-\lambda}{p\Gamma(\alpha+1)}t^{\alpha}}.$$

From Definition 4, network (2) with the controller (14) is α-exponentially stable.

Remark 4. In Theorem 1, we used delayed feedback periodically intermittent controller (14) to realize the stabilization of network (1). However, the real part and the imaginary part of the controller (14) depend on not only the past of the real part of states but also the past of the imaginary part of states. This will give some restrictions on the practical applications. To cope this, we design the controller (15), where the real part and the imaginary part of the controller (15) are related to the past of the real part and the imaginary part of states, respectively.

Remark 5. Because the limit of the Young inequality is $p \neq 1$, we solely discuss the case of $p = 1$ in the following Theorem 2.

Theorem 2. Under Assumption 1, network (2) with the controller (15) is α-exponentially stable if there exist some positive scalars θ, μ, ν and λ such that the following conditions are satisfied

$$\begin{cases} -c_j - \varepsilon_j + \sum\limits_{k=1}^{n} (|a^R_{kj}| + |a^I_{kj}|)(|F_j^{RR} + F_j^{IR}) \leq -\theta, \ |b^R_{kj}| + |b^I_{kj}|)(G_j^{RR} + G_j^{IR} \leq \rho_{kj}, \\ -c_j - \epsilon_j + \sum\limits_{k=1}^{n} (|a^R_{kj}| + |a^I_{kj}|)(F_j^{II} + F_j^{RI}) \leq -\theta, \ |b^R_{kj}| + |b^I_{kj}|)(G_j^{RI} + G_j^{II} \leq \varrho_{kj}, \end{cases} \tag{35}$$

$$\begin{cases} -c_j + \sum\limits_{k=1}^{n} (|a^R_{kj}| + |a^I_{kj})|(F_j^{IR} + F_j^{RR}) \leq \mu, \quad \sum\limits_{k=1}^{n} (|b^R_{kj}| + |b^I_{kj}|)(G_j^{RR} + G_j^{IR}) \leq \nu, \\ -c_j + \sum\limits_{k=1}^{n} (|a^R_{kj}| + |a^I_{kj}|)(F_j^{II} + F_j^{RI}) \leq \mu, \quad \sum\limits_{k=1}^{n} (|b^R_{kj}| + |b^I_{kj}|)(G_j^{RI} + G_j^{II}) \leq \nu, \end{cases} \tag{36}$$

$$-\theta\beta^\alpha + (\mu + \nu)(1 - \beta)^\alpha + \lambda \leq 0. \tag{37}$$

Proof. Consider the following Lyapunov function:

$$V(t) = \sum_{j=1}^{n} |e_j^x(t)| + \sum_{j=1}^{n} |e_j^y(t)|.$$

Combining with (21), calculating the fractional-order derivatives of $V(t)$ along the solutions of system (3)-(4), we can obtain

$$\begin{aligned} D^\alpha V(t) \leq & -\sum_{j=1}^{n} c_j |e_j^x(t)| + \sum_{j=1}^{n}\sum_{k=1}^{n} |a^R_{jk}| [F_k^{RR} |e_k^x(t)| + F_k^{RI} |e_k^y(t)|] \\ & + \sum_{j=1}^{n}\sum_{k=1}^{n} |a^I_{jk}| [F_k^{IR} |e_k^x(t)| + F_k^{II} |e_k^y(t)|] \\ & + \sum_{j=1}^{n}\sum_{k=1}^{n} |b^R_{jk}| [G_k^{RR} |e_k^x(t-\tau)| + G_k^{RI} |e_k^y(t-\tau)|] \\ & + \sum_{j=1}^{n}\sum_{k=1}^{n} |b^I_{jk}| [G_k^{IR} |e_k^x(t-\tau)| + G_k^{II} |e_k^y(t-\tau)|] + \sum_{j=1}^{n} \mathrm{sgn}(e_j^x(t)) u_j(t) \\ & - \sum_{j=1}^{n} c_j |e_j^y(t)| + \sum_{j=1}^{n}\sum_{k=1}^{n} |a^R_{jk}| [F_k^{IR} |e_k^x(t)| + F_k^{II} |e_k^y(t)|] \\ & + \sum_{j=1}^{n}\sum_{k=1}^{n} |a^I_{jk}| [F_k^{RR} |e_k^x(t)| + F_k^{RI} |e_k^y(t)|] \\ & + \sum_{j=1}^{n}\sum_{k=1}^{n} |b^R_{jk}| [G_k^{IR} |e_k^x(t-\tau)| + G_k^{II} |e_k^y(t-\tau)|] \\ & + \sum_{j=1}^{n}\sum_{k=1}^{n} |b^I_{jk}| [G_k^{RR} |e_k^x(t-\tau)| + G_k^{RI} |e_k^y(t-\tau)|] + \sum_{j=1}^{n} \mathrm{sgn}(e_j^y(t)) v_j(t). \end{aligned} \tag{38}$$

When $mT \le t < mT + \delta$ for $\forall m \in N_+$, we have

$$\begin{aligned}
D^\alpha V(t) \le & \sum_{j=1}^{n} \Big\{ -c_j - \varepsilon_j + \sum_{k=1}^{n} (|a_{kj}^R| + |a_{kj}^I|)(F_j^{RR} + F_j^{IR}) \Big\} |e_j^x(t)| \\
& + \sum_{j=1}^{n}\sum_{k=1}^{n} \Big[(|b_{kj}^R| + |b_{kj}^I|)(G_j^{RR} + G_j^{IR}) - \rho_{kj} \Big] |e_j^x(t-\tau)| \\
& + \sum_{j=1}^{n} \Big\{ -c_j - \epsilon_j + \sum_{k=1}^{n} (|a_{kj}^R| + |a_{kj}^I|)(F_j^{II} + F_j^{RI}) \Big\} |e_j^x(t)| \\
& + \sum_{j=1}^{n}\sum_{k=1}^{n} \Big[(|b_{kj}^R| + |b_{kj}^I|)(G_j^{RI} + G_j^{II}) - \varrho_{kj} \Big] |e_j^x(t-\tau)| \\
\le & -\theta \sum_{j=1}^{n} |e_j^x(t)| - \theta \sum_{j=1}^{n} |e_j^y(t)| \\
= & -\theta V(t). \qquad (39)
\end{aligned}$$

Using Lemma 5, we can obtain

$$V(t) \le V(mT) e^{\frac{-\theta}{\Gamma(\alpha+1)}(t-mT)^\alpha}. \qquad (40)$$

When $mT + \delta \le t < (m+1)T$ for $\forall m \in N_+$, one has

$$\begin{aligned}
D^\alpha V(t) \le & \sum_{j=1}^{n} \Big\{ -c_j + \sum_{k=1}^{n} (|a_{kj}^R| + |a_{kj}^I|)(F_j^{IR} + F_j^{RR}) \Big\} |e_j^x(t)| \\
& + \sum_{j=1}^{n} \Big\{ \sum_{k=1}^{n} (|b_{kj}^R| + |b_{kj}^I|)(G_j^{RR} + G_j^{IR}) \Big\} |e_j^x(t-\tau)| \\
& + \sum_{j=1}^{n} \Big\{ -c_j + \sum_{k=1}^{n} (|a_{kj}^R| + |a_{kj}^I|)(F_j^{II} + F_j^{RI}) \Big\} |e_j^x(t)| \\
& + \sum_{j=1}^{n} \Big\{ \sum_{k=1}^{n} (|b_{kj}^R| + |b_{kj}^I|)(G_j^{RI} + G_j^{II}) \Big\} |e_j^x(t-\tau)| \\
= & \mu V(t) + \nu V(t-\tau). \qquad (41)
\end{aligned}$$

By the similar proof of Theorem 1, we can obtain

$$\|e(t)\|_1 \le \|e(0)\|_1 e^{\frac{-\lambda}{\Gamma(\alpha+1)} t^\alpha} \le \sup_{-\tau \le s \le 0} \|e(s)\|_1 e^{\frac{-\lambda}{\Gamma(\alpha+1)} t^\alpha}.$$

From Definition 4, network (2) with the controller (15) is α-exponentially stable.

Remark 6.

Remark 7. When $\beta = 1$, the periodically intermittent controllers (14) and (15) turn into the following linear delay feedback controllers, respectively:

$$\begin{cases} u_j(t) = -d_j e_j^x(t) - \mathrm{sgn}(e_j^x(t)) \sum\limits_{k=1}^{n} (s_{jk}|e_k^x(t-\tau)| + h_{jk}|e_k^y(t-\tau)|), \\ v_j(t) = -q_j e_j^y(t) - \mathrm{sgn}(e_j^y(t)) \sum\limits_{k=1}^{n} (r_{jk}|e_k^x(t-\tau)| + l_{jk}|e_k^y(t-\tau)|). \end{cases} \qquad (42)$$

$$\begin{cases} u_j(t) = -\varepsilon_j e_j^x(t) - \mathrm{sgn}(e_j^x(t)) \sum\limits_{k=1}^{n} \rho_{jk}|e_k^x(t-\tau)|, \\ v_j(t) = -\epsilon_j e_j^y(t) - \mathrm{sgn}(e_j^y(t)) \sum\limits_{k=1}^{n} \varrho_{jk}|e_k^y(t-\tau)|. \end{cases} \tag{43}$$

From the process of derivation of Theorem 1-2, we have the following corollaries:

Corollary 1. Under Assumption 1, network (2) with the controller (42) is α-exponentially stable if there exists a scalar $\theta > 0$ such that the following conditions are satisfied

$$\begin{cases} -p(c_j + d_j) + (p-1)\sum\limits_{k=1}^{n}(|a_{jk}^R|(F_k^{RR} + F_k^{RI}) + |a_{jk}^I|(F_k^{IR} + F_k^{II})) \\ \qquad\qquad + \sum\limits_{k=1}^{n}(|a_{kj}^R| + |a_{kj}^I|)(F_j^{RR} + F_j^{IR}) \le -\theta, \\ -p(c_j + q_j) + (p-1)\sum\limits_{k=1}^{n}(|a_{jk}^R|(F_k^{IR} + F_k^{II}) + |a_{jk}^I|(F_k^{RR} + F_k^{RI})) \\ \qquad\qquad + \sum\limits_{k=1}^{n}(|a_{kj}^R| + |a_{kj}^I|)(F_j^{RI} + F_j^{II}) \le -\theta, \end{cases}$$

$$\begin{cases} |b_{jk}^R|G_k^{RR} + |b_{jk}^I|G_k^{IR} - s_{jk} \le 0, \quad |b_{jk}^R|G_k^{RI} + |b_{jk}^I|G_k^{II} - h_{jk} \le 0, \\ |b_{jk}^R|G_k^{IR} + |b_{jk}^I|G_k^{RR} - r_{jk} \le 0, \quad |b_{jk}^R|G_k^{II} + |b_{jk}^I|G_k^{RI} - l_{jk} \le 0. \end{cases}$$

Corollary 2. Under Assumption 1, network (2) with the controller (43) is α-exponentially stable if there exists a scalar $\theta > 0$ such that the following conditions are satisfied

$$\begin{cases} -c_j - \varepsilon_j + \sum\limits_{k=1}^{n}(|a_{kj}^R + |a_{kj}^I|)(F_j^{IR} + F_j^{RR}) \le -\theta, \\ \sum\limits_{k=1}^{n}(|b_{kj}^R| + |b_{kj}^I|)(G_j^{IR} + G_j^{RR}) - \sum\limits_{k=1}^{n}\rho_{kj} \le 0, \\ -c_j - \epsilon_j + \sum\limits_{k=1}^{n}(|a_{kj}^R| + |a_{kj}^I|)(F_j^{II} + F_j^{RI}) \le -\theta, \\ \sum\limits_{k=1}^{n}(|b_{kj}^R| + |b_{kj}^I|)(G_j^{RI} + G_j^{II}) - \sum\limits_{k=1}^{n}\varrho_{kj} \le 0. \end{cases}$$

Remark 7. In n-dimensional real space, network (1) turns into the following fractional-order real-valued neural networks

$$D^\alpha x_j(t) = -c_j x_j(t) + \sum_{k=1}^{n} a_{jk} f_k(x_k(t)) + \sum_{k=1}^{n} b_{jk} g_k(x_k(t-\tau)) + J_j(t), \tag{44}$$

where $c_j > 0, x_i(t), f_k(\cdot), g_k(\cdot), a_{jk}, b_{jk}, J_j(t) \in R$. And Assumption 1 become the following assumptions, respectively:

Assumption $\tilde{1}$. Let $x, y \in R$, functions $f_j(\cdot) \in R$ is Lipschitz-continuous on R with Lipschitz constants $F_j, G_j > 0$, .i.e. $|f_j(x) - f_j(y)| \le F_j|x-y|, |g_j(x) - g_j(y)| \le G_j|x-y|, j \in \mathscr{F}$.

The two periodically intermittent controller (14)-(15) turns into the controller as follow:

$$\begin{cases} \text{if } mT \le t < mT + \delta, \\ \qquad u_j(t) = -d_j e_j(t) - \mathrm{sgn}(e_j(t)) \sum_{k=1}^{n} s_{jk} |e_k^x(t-\tau)|, \\ \text{else} \\ \qquad u_j(t) = 0. \end{cases} \tag{45}$$

From the process of derivation of Theorem 1-2, we have the following result.

Corollary 3. Under Assumption $\tilde{1}$, network (44) is α-exponentially stabilized via the controller (45) if there exist a positive integer $p \ge 2$, some positive scalars θ, μ, ν and λ such that the following conditions are satisfied

$$\begin{cases} -pc_j - pd_j + (p-1) \sum_{k=1}^{n} (|a_{jk}|F_k + |a_{kj}|F_j) \le -\theta p, \\ |b_{jk}|G_k - s_{jk} \le 0, \\ -pc_j - pd_j + (p-1) \sum_{k=1}^{n} (|a_{jk}|F_k + |a_{kj}|F_j) + |b_{jk}|G_k \le -\theta p, \\ \sum_{k=1}^{n} |b_{kj}|G_j \le vp. \end{cases}$$

or

$$\begin{cases} -c_j - d_j + \sum_{k=1}^{n} |a_{kj}|F_j \le -\theta, \\ |b_{jk}|G_k - s_{jk} \le 0, \\ -c_j + \sum_{k=1}^{n} |a_{kj}|F_j \le -\theta, \\ \sum_{k=1} |b_{kj}|G_j \le v. \end{cases}$$

Remark 8: **Remark 8**. So far, there are some results concerning the exponential stabilization or synchronization of fractional-order neural networks with or without time delays [38, 39]. However, to the best of our knowledge, there are no results on the α-exponential stabilization of fractional-order CVNNs with delay or synchronization between two fractional-order CVNNs with delay via periodically intermittent control. Obviously, our results have optimality in the control cost for the stabilization, which include feedback control as a special case.

5 An Illustrative example

In this section, using the predictor-corrector scheme for solving nonlinear delay differential equations of fractional order [43], which have been used for fractional-order chaotic systems with delay [9, 44], we present a numerical example to show the effectiveness of the obtained results.

Example 1. Consider the following two-neuron fractional-order CVNN

$$D^{\alpha}z(t) = -Cz(t) + Af(z(t)) + Bg(z(t-\tau)), \tag{46}$$

where $\alpha = 0.95$, $\tau = 5$, let $z_j(t) = x_j(t) + iy_j(t)$, $x_j(t)$, $y_j(t) \in R$ for $j = 1, 2$. And

$$f_j(z_j(t)) = \tanh(x_j(t)) + y_j(t) + i(x_j(t) + \tanh(y_j(t))),$$

$$g_j(z_j(t)) = x_j(t) + \tanh(y_j(t)) + i(\tanh(x_j(t)) + y_j(t)),$$

$$C = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, A = \begin{bmatrix} 1+1i & 1 \\ -1 & 1+1i \end{bmatrix}, B = \begin{bmatrix} -1-1i & -1i \\ 1i & -1-1i \end{bmatrix}.$$

Let the initial value $z_1(s) = \sin(0.4\pi t) + i\cos(0.4\pi t) - 1$, $z_2(s) = \cos(0.4\pi t) + i\sin(0.4\pi t) - i$. It is easy to verify that Assumption 1 is satisfied with $l_j^{RR} = l_j^{II} = l_j^{RI} = l_j^{IR} = 1$. Fig.1 and Fig.2 depict the state curves z_1 and z_2 of (46) with $u(t) = v(t) = 0$ in 3-dimension space and 2-dimensional space, respectively. Fig.3 and Fig.4 depict the time responses of real part and imaginary parts of z_1 and z_2 of (46) with $u(t) = v(t) = 0$, respectively.

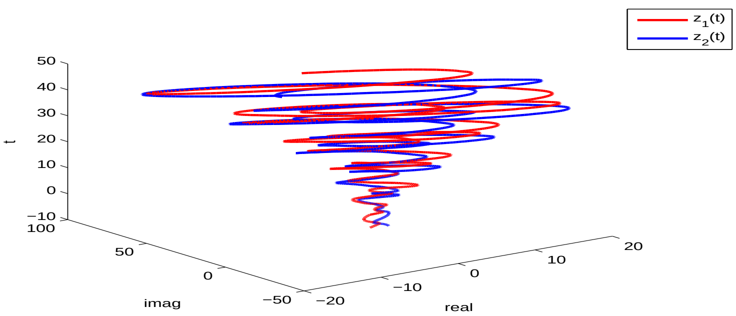

Fig.1. Curves z_1 and z_2 in 3-dimension space without control.

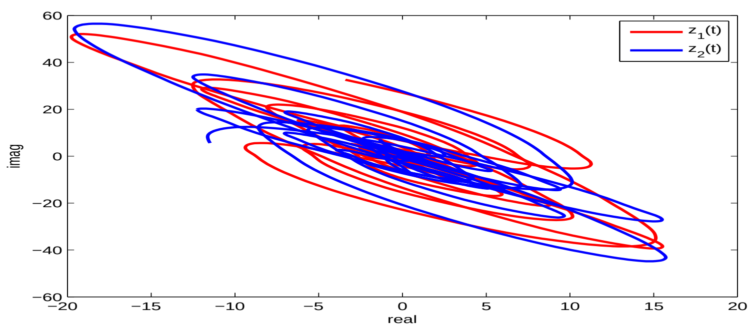

Fig.2. Curves z_1 and z_2 in 2-dimension space without control.

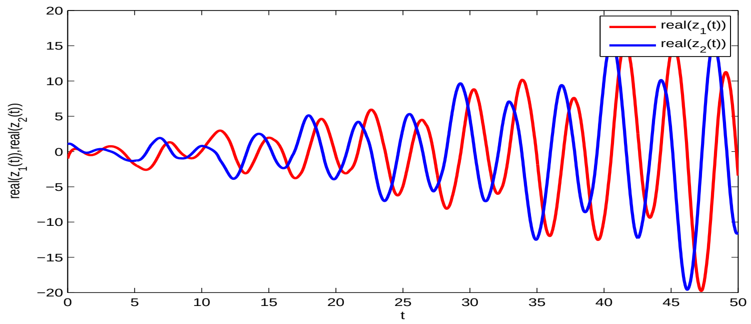

Fig.3. Real parts of z_1 and z_2 of network (46) without control.

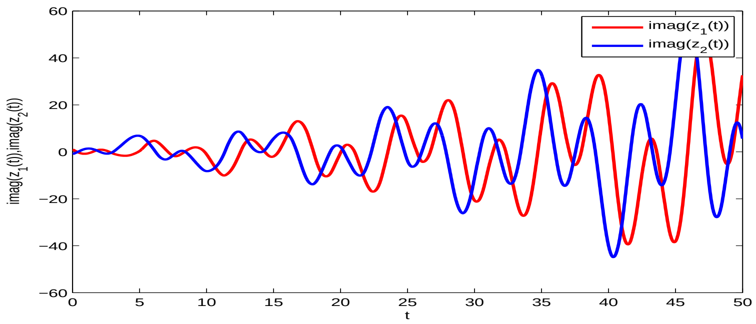

Fig.4. Imaginary parts of z_1 and z_2 of network (46) without control.

Case 1. Choosing $p = 2, d_j = q_j = 16$, $s_{jk} = h_{jk} = r_{jk} = l_{jk} = 2, \beta = 0.5$. We easily know that the conditions of Theorem 1 are all satisfied and network (46) with the controller (14) is α-exponentially stable, which is demonstrated in Figs.5-8. Fig.5 and Fig.6 depict the curves z_1 and z_2 of (46) with (14) in 3-dimension space and 2-dimensional space, respectively. Figs.7-8 depict the time responses of real part and imaginary parts of z_1 and z_2 of (46) with (14), respectively.

Case 2. Choosing $\varepsilon_j = \epsilon_j = 8$, $\rho_{jk} = \varrho_{jk} = 3, \beta = 0.75$. It is easy to know that the conditions of Theorem 2 are all satisfied and network (46) with the controller (15) is α-exponentially stable, which is demonstrated in Figs.9-12. Fig. 9 and Fig. 10 depict the curves z_1 and z_2 of (46) with the controller (15) in 3-dimension space and 2-dimensional space, respectively. Figs.11-12 depict the time responses of real part and imaginary parts of z_1 and z_2 of (46) with (15), respectively.

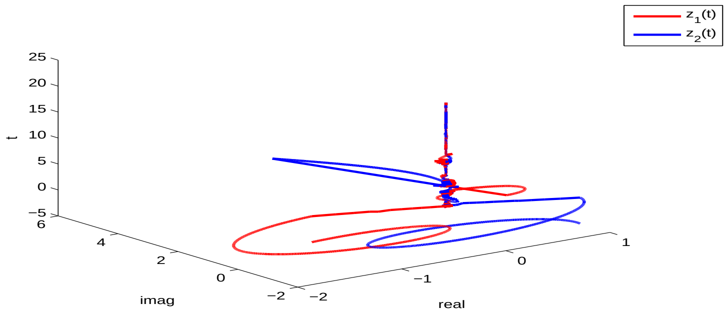

Fig.5. Curves z_1 and z_2 of (46) with (14) in 3-dimension space.

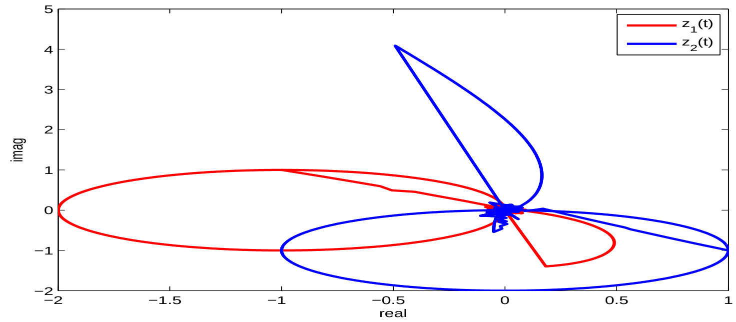

Fig.6. Curves z_1 and z_2 of (46) with (14) in 2-dimension space.

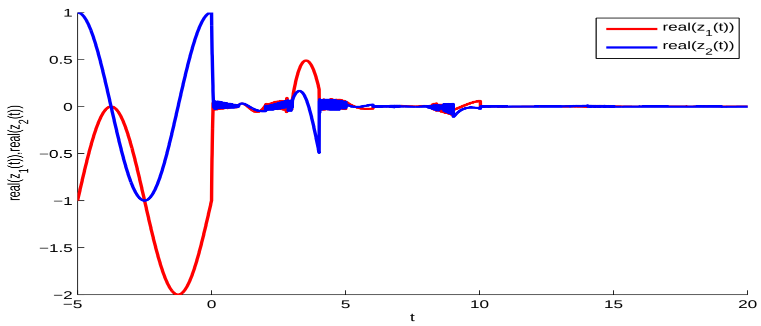

Fig.7. Real parts of z_1 and z_2 of (46) with (14).

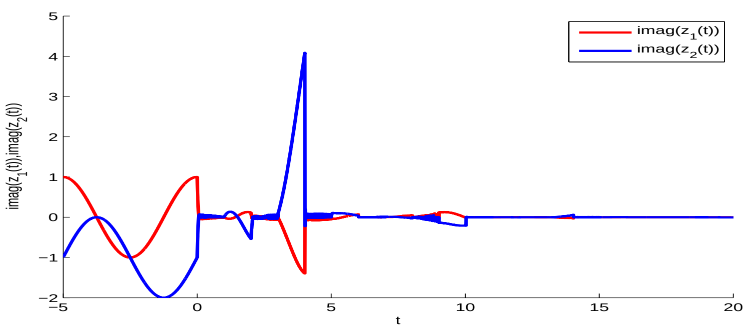

Fig.8. Imaginary parts of z_1 and z_2 of (46) with (14).

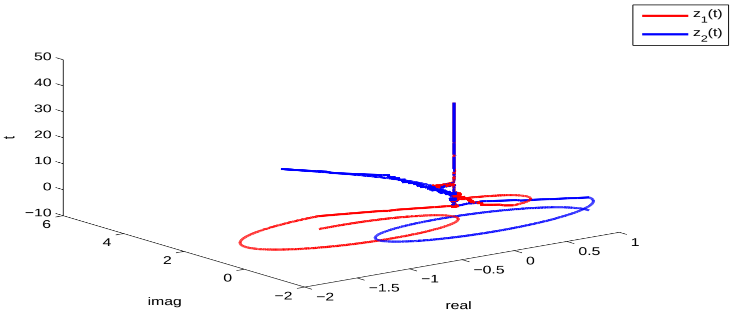

Fig.9. Curves z_1 and z_2 of (46) with (15) in 3-dimension space.

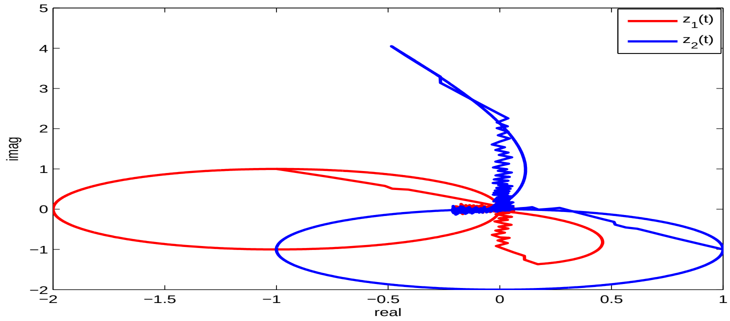

Fig.10. Curves z_1 and z_2 of (46) with (15) in 2-dimension space.

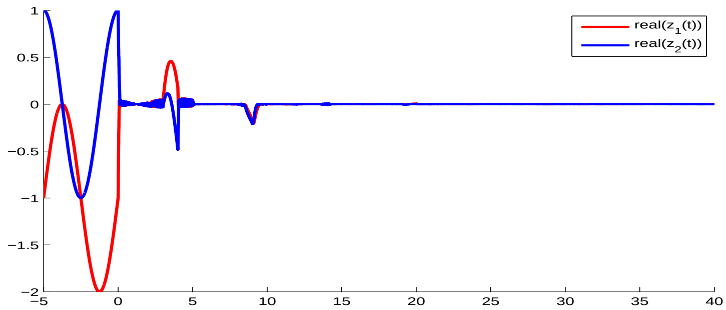

Fig.11. Real parts of z_1 and z_2 of (46) with (15).

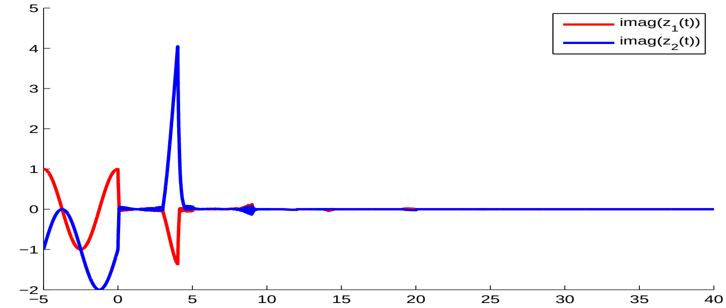

Fig.12. Imaginary parts of z_1 and z_2 of (46) with (15).

6 Conclusions

In this paper, we investigate the α-exponential stabilization for a class of fractional-order CVNNs with delay via periodically intermittent control. By using some fractional-order inequalities and coupling with the Lyapunov method, the periodically intermittent controllers are designed to achieve the α-exponential stabilization for the discussed fractional-order CVNNs. What's more, our results include feedback control as a special case and state that the α-exponential convergence rate relies on the ratio of control width to control period and the order of differentiation of the system and p-norm, but does not depends on control width or control period. So far, the periodically intermittent control method is successfully used to stabilize the addressed fractional-order neural networks with delay. For actual problems, we also can randomly choose the control period for achieving fractional-order CVNN stabilization. An illustrative example with simulations based periodically intermittent control method is presented to illustrate the effectiveness of the obtained results.

Acknowledgment

The authors are grateful for the support of the National Natural Science Foundation of China (11601268).

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