

## Numerical results for selected optimisation problems

Table 1 show respectively: the name of the optimisation test problem (Problem), the name of the algorithm (Alg), number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations required by the algorithm to solve the problem (nfev), the number of minimisers generated as starting points by the algorithm (nlmin), the number of unique local minima mapped by the algorithm (nulmin), whether successful convergence to the global minima was achieved (Success), the CPU run time measured in seconds (Runtime) and finally the number of function evaluations per unique local minima (nfev/nulmin). For all these test problems the algorithm was terminated if the algorithm ran for longer than 10 minutes.

The optimisation runs were done on a computer with the following specifications:

- CPU: Intel Core i7-6700K CPU @ 4.2GHz
- Kernel: x86\_64 Linux 4.12.10-1-ARCH
- RAM: 15973MiB

Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
All	bh	0	1358408	0	0	NaN	NaN
	de	0	934804	0	0	NaN	NaN
	shgo-simplicial	0	72240	238	238	NaN	NaN
	shgo-sobol	0	29694	225	225	NaN	NaN
	tgo	0	63533	403	393	NaN	NaN
Average	bh	0	7546	0	0	NaN	0.108971
	de	0	5193	0	0	NaN	0.188172
	shgo-simplicial	0	401	1	1	NaN	1.115545
	shgo-sobol	0	164	1	1	NaN	0.004778
	tgo	0	352	2	2	NaN	0.008672
Ackley01	bh	2	16107	0	0	True	0.298839
	de	2	3423	0	0	True	0.190420
	shgo-simplicial	2	54	1	1	True	0.001750
	shgo-sobol	2	52	1	1	True	0.041898
	tgo	2	52	1	1	True	0.001998
Ackley02	bh	2	11844	0	0	True	0.090117
	de	2	456	0	0	True	0.010810
	shgo-simplicial	2	90	1	1	True	0.001905
	shgo-sobol	2	88	1	1	True	0.001738

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Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Ackley03	tgo	2	88	1	1	True	0.001615
	bh	2	2370	0	0	False	0.040504
	de	2	421	0	0	True	0.013166
	shgo-simplicial	2	59	1	1	True	0.001444
	shgo-sobol	2	57	1	1	True	0.001529
Adjiman	tgo	2	57	1	1	True	0.001432
	bh	2	2070	0	0	False	0.046875
	de	2	532	0	0	True	0.037358
	shgo-simplicial	2	26	1	1	True	0.003626
	shgo-sobol	2	36	1	1	True	0.004907
Alpine01	tgo	2	36	1	1	True	0.004410
	bh	2	32928	0	0	True	0.303166
	de	2	4423	0	0	True	0.138957
	shgo-simplicial	2	55	1	1	True	0.001360
	shgo-sobol	2	53	1	1	True	0.001466
Alpine02	tgo	2	53	1	1	True	0.001400
	bh	2	1617	0	0	True	0.024743
	de	2	492	0	0	True	0.014723
	shgo-simplicial	2	153	5	5	True	0.005421
	shgo-sobol	2	62	1	1	True	0.001830
BartelsConn	tgo	2	108	3	3	True	0.002290
	bh	2	19857	0	0	True	0.205387
	de	2	1282	0	0	True	0.036180
	shgo-simplicial	2	55	1	1	True	0.001298
	shgo-sobol	2	53	1	1	True	0.001491
Beale	tgo	2	53	1	1	True	0.001306
	bh	2	6306	0	0	False	0.045135
	de	2	4803	0	0	True	0.127161
	shgo-simplicial	2	63	1	1	True	0.001226
	shgo-sobol	2	61	1	1	True	0.001339
BiggsExp02	tgo	2	61	1	1	True	0.001239
	bh	2	3009	0	0	True	0.079360
	de	2	4003	0	0	True	0.177575
	shgo-simplicial	2	147	2	2	True	0.005318
	shgo-sobol	2	128	1	1	True	0.004324
	tgo	2	128	1	1	True	0.004133

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Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
BiggsExp03	bh	3	5812	0	0	True	0.134723
	de	3	10564	0	0	True	0.492391
	shgo-simplicial	3	145	1	1	True	0.007089
	shgo-sobol	3	151	1	1	True	0.005064
	tgo	3	151	1	1	True	0.004900
BiggsExp04	bh	4	13095	0	0	True	0.295152
	de	4	29765	0	0	True	1.368383
	shgo-simplicial	4	1091	1	1	True	0.167113
	shgo-sobol	4	384	1	1	True	0.011662
	tgo	4	384	1	1	True	0.011372
BiggsExp05	bh	5	14346	0	0	False	0.468451
	de	5	7632	0	0	False	0.461430
	shgo-simplicial	5	607	1	1	True	0.023766
	shgo-sobol	5	583	1	1	True	0.019824
	tgo	5	583	1	1	True	0.019717
Bird	bh	2	2421	0	0	False	0.038661
	de	2	695	0	0	True	0.021481
	shgo-simplicial	2	42	1	1	True	0.001750
	shgo-sobol	2	43	1	1	True	0.001341
	tgo	2	43	1	1	True	0.001209
Bohachevsky1	bh	2	3510	0	0	True	0.037407
	de	2	2763	0	0	True	0.077462
	shgo-simplicial	2	9	1	1	True	0.000461
	shgo-sobol	2	7	1	1	True	0.000564
	tgo	2	7	1	1	True	0.000511
Bohachevsky2	bh	2	3471	0	0	True	0.037333
	de	2	2923	0	0	True	0.080887
	shgo-simplicial	2	9	1	1	True	0.000520
	shgo-sobol	2	7	1	1	True	0.000620
	tgo	2	7	1	1	True	0.000496
Bohachevsky3	bh	2	3438	0	0	True	0.033962
	de	2	3043	0	0	True	0.081093
	shgo-simplicial	2	9	1	1	True	0.000478
	shgo-sobol	2	7	1	1	True	0.000603
	tgo	2	7	1	1	True	0.000481
BoxBetts	bh	3	8096	0	0	True	0.181494

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Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Branin01	de	3	11944	0	0	True	0.525109
	shgo-simplicial	3	89	1	1	True	0.003011
	shgo-sobol	3	76	1	1	True	0.002723
	tgo	3	76	1	1	True	0.002527
	bh	2	2229	0	0	True	0.027123
	de	2	615	0	0	True	0.017054
	shgo-simplicial	2	39	1	1	True	0.000982
	shgo-sobol	2	37	1	1	True	0.001067
Branin02	tgo	2	37	1	1	True	0.000935
	bh	2	2094	0	0	True	0.031920
	de	2	735	0	0	True	0.022043
	shgo-simplicial	2	111	2	2	True	0.004283
	shgo-sobol	2	44	1	1	True	0.001358
	tgo	2	71	2	2	True	0.001664
	bh	2	915	0	0	True	0.016313
	de	2	6443	0	0	True	0.176477
Brent	shgo-simplicial	2	9	1	1	True	0.000487
	shgo-sobol	2	7	1	1	True	0.000596
	tgo	2	7	1	1	True	0.000502
	bh	2	1857	0	0	True	0.038897
	de	2	4083	0	0	True	0.146456
	shgo-simplicial	2	34	1	1	True	0.001163
	shgo-sobol	2	36	1	1	True	0.001331
	tgo	2	36	1	1	True	0.001246
Brown	bh	2	663	0	0	False	0.014341
	de	2	815	0	0	True	0.021064
	shgo-simplicial	2	20	1	1	True	0.000664
	shgo-sobol	2	18	1	1	True	0.000764
	tgo	2	18	1	1	True	0.000643
	bh	2	17166	0	0	True	0.098578
	de	2	4103	0	0	True	0.110858
	shgo-simplicial	2	26	1	1	True	0.000751
Bukin04	shgo-sobol	2	24	1	1	True	0.001064
	tgo	2	24	1	1	True	0.000798
	bh	2	22014	0	0	False	0.179138
	de	2	2623	0	0	False	0.075753

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Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
CarronTable	shgo-simplicial	2	1007	5	5	True	0.021791
	shgo-sobol	2	741	3	3	True	0.012376
	tgo	2	1169	5	5	True	0.017350
	bh	2	1899	0	0	False	0.035184
	de	2	972	0	0	True	0.034849
Cigar	shgo-simplicial	2	36	1	1	True	0.001454
	shgo-sobol	2	31	1	1	True	0.001087
	tgo	2	31	1	1	True	0.000984
	bh	2	8193	0	0	True	0.088217
	de	2	3743	0	0	True	0.124895
Colville	shgo-simplicial	2	20	1	1	True	0.000687
	shgo-sobol	2	18	1	1	True	0.000797
	tgo	2	18	1	1	True	0.000693
	bh	4	12965	0	0	True	0.103124
	de	4	29685	0	0	True	0.837259
Corana	shgo-simplicial	4	225	1	1	True	0.014057
	shgo-sobol	4	1029	2	2	True	0.060954
	tgo	4	3039	10	2	True	0.054228
	bh	4	2555	0	0	False	0.073994
	de	4	4085	0	0	True	0.192973
CosineMixture	shgo-simplicial	4	23	1	1	True	0.002139
	shgo-sobol	4	12	1	1	True	0.000986
	tgo	4	12	1	1	True	0.000847
	bh	2	348	0	0	False	0.014602
	de	2	1166	0	0	True	0.037936
CrossInTray	shgo-simplicial	2	17	1	1	True	0.001120
	shgo-sobol	2	7	1	1	True	0.000641
	tgo	2	7	1	1	True	0.000552
	bh	2	1578	0	0	False	0.028986
	de	2	489	0	0	True	0.019297
CrossLegTable	shgo-simplicial	2	69	1	1	True	0.003238
	shgo-sobol	2	33	1	1	True	0.001108
	tgo	2	33	1	1	True	0.001020
	bh	2	17355	0	0	True	0.209052
	de	2	4783	0	0	False	0.154005
	shgo-simplicial	2	9	1	1	True	0.000545

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Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
CrownedCross	shgo-sobol	2	7	1	1	True	0.000651
	tgo	2	7	1	1	True	0.000640
	bh	2	17130	0	0	False	0.210953
	de	2	4263	0	0	False	0.136233
	shgo-simplicial	2	9	1	1	True	0.000558
Cube	shgo-sobol	2	7	1	1	True	0.000644
	tgo	2	7	1	1	True	0.000520
	bh	2	8529	0	0	True	0.053030
	de	2	5243	0	0	True	0.125904
	shgo-simplicial	2	146	1	1	True	0.002060
Damavandi	shgo-sobol	2	144	1	1	True	0.002167
	tgo	2	144	1	1	True	0.002071
	bh	2	1566	0	0	False	0.026171
	de	2	535	0	0	False	0.015558
	shgo-simplicial	2	578	2	2	True	0.034168
DeVilliersGlasser01	shgo-sobol	2	60	2	2	True	0.001825
	tgo	2	97	2	2	True	0.002025
	bh	4	16805	0	0	True	0.554629
	de	4	24265	0	0	True	1.200274
	shgo-simplicial	4	439	2	2	True	0.024691
Deb01	shgo-sobol	4	446	1	1	True	0.044395
	tgo	4	667	9	9	True	0.023041
	bh	2	5565	0	0	True	0.065591
	de	2	1532	0	0	True	0.048103
	shgo-simplicial	2	28	1	1	True	0.001334
Deb03	shgo-sobol	2	18	1	1	True	0.000828
	tgo	2	18	1	1	True	0.000741
	bh	2	5310	0	0	False	0.076148
	de	2	40103	0	0	False	1.453880
	shgo-simplicial	2	4234	4	4	True	0.082374
Decanomial	shgo-sobol	2	83	1	1	True	0.002579
	tgo	2	1476	2	2	True	0.029756
	bh	2	9465	0	0	True	0.117149
	de	2	3383	0	0	True	0.107160
	shgo-simplicial	2	256	1	1	True	0.005111
	shgo-sobol	2	200	1	1	True	0.004302

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Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Deceptive	tgo	2	200	1	1	True	0.004083
	bh	2	441	0	0	False	0.017110
	de	2	913	0	0	True	0.025888
	shgo-simplicial	2	449	9	9	True	0.017866
	shgo-sobol	2	533	4	4	True	0.012727
DeckkersAarts	tgo	2	667	5	5	True	0.015315
	bh	2	2844	0	0	True	0.026594
	de	2	670	0	0	True	0.016102
	shgo-simplicial	2	86	1	1	True	0.003099
	shgo-sobol	2	99	2	2	True	0.002026
DeflectedCorrugatedSpring	tgo	2	167	2	2	True	0.002544
	bh	2	2055	0	0	True	0.043273
	de	2	892	0	0	True	0.032436
	shgo-simplicial	2	9	1	1	True	0.000695
	shgo-sobol	2	7	1	1	True	0.000676
DixonPrice	tgo	2	7	1	1	True	0.000560
	bh	2	3639	0	0	True	0.059777
	de	2	4563	0	0	True	0.160772
	shgo-simplicial	2	627	3	3	True	0.039069
	shgo-sobol	2	93	2	2	True	0.002869
Dolan	tgo	2	92	2	2	True	0.002412
	bh	5	51084	0	0	True	0.390904
	de	5	78692	0	0	True	2.479181
	shgo-simplicial	5	264	1	1	True	0.007559
	shgo-sobol	5	240	1	1	True	0.004052
DropWave	tgo	5	240	1	1	True	0.003832
	bh	2	2337	0	0	True	0.036634
	de	2	1012	0	0	True	0.032547
	shgo-simplicial	2	9	1	1	True	0.000526
	shgo-sobol	2	7	1	1	True	0.000634
Easom	tgo	2	7	1	1	True	0.000524
	bh	2	303	0	0	False	0.013139
	de	2	83	0	0	False	0.001927
	shgo-simplicial	2	2126	1	1	True	0.139017
	shgo-sobol	2	2210	1	1	True	0.049775
	tgo	2	2210	1	1	True	0.323876

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Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
EggCrate	bh	2	1935	0	0	False	0.025269
	de	2	3963	0	0	True	0.110584
	shgo-simplicial	2	9	1	1	True	0.000492
	shgo-sobol	2	7	1	1	True	0.000619
	tgo	2	7	1	1	True	0.000511
EggHolder	bh	2	1983	0	0	False	0.046486
	de	2	941	0	0	False	0.036749
	shgo-simplicial	2	616	2	2	True	0.042003
	shgo-sobol	2	233	4	4	True	0.007344
	tgo	2	293	5	5	True	0.008198
ElAttarVidyasagarDutta	bh	2	3219	0	0	True	0.031350
	de	2	1301	0	0	True	0.034577
	shgo-simplicial	2	33094	2	2	True	2.799852
	shgo-sobol	2	277	3	3	True	0.005578
	tgo	2	351	4	4	True	0.005266
Exp2	bh	2	2892	0	0	True	0.078883
	de	2	4003	0	0	True	0.184855
	shgo-simplicial	2	56	1	1	True	0.002597
	shgo-sobol	2	137	1	1	True	0.004752
	tgo	2	137	1	1	True	0.004762
Exponential	bh	2	1515	0	0	True	0.026072
	de	2	286	0	0	True	0.008401
	shgo-simplicial	2	9	1	1	True	0.000546
	shgo-sobol	2	7	1	1	True	0.000624
	tgo	2	7	1	1	True	0.000538
FreudensteinRoth	bh	2	5262	0	0	True	0.041467
	de	2	4103	0	0	True	0.100865
	shgo-simplicial	2	356	7	7	True	0.012095
	shgo-sobol	2	49	1	1	True	0.001195
	tgo	2	49	1	1	True	0.001008
Gear	bh	4	505	0	0	False	0.013874
	de	4	11445	0	0	True	0.336399
	shgo-simplicial	4	23	1	1	True	0.001465
	shgo-sobol	4	31	1	1	True	0.001925
	tgo	4	37	2	2	True	0.001130
Giunta	bh	2	2301	0	0	True	0.046305

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Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
GoldsteinPrice	de	2	449	0	0	True	0.015731
	shgo-simplicial	2	34	2	2	True	0.001792
	shgo-sobol	2	31	1	1	True	0.001276
	tgo	2	31	1	1	True	0.001143
	bh	2	4587	0	0	True	0.050597
	de	2	781	0	0	True	0.021106
Griewank	shgo-simplicial	2	85	1	1	True	0.001664
	shgo-sobol	2	83	1	1	True	0.001773
	tgo	2	83	1	1	True	0.001681
	bh	2	1872	0	0	False	0.039220
	de	2	3283	0	0	True	0.124090
	shgo-simplicial	2	9	1	1	True	0.000633
Gulf	shgo-sobol	2	7	1	1	True	0.000710
	tgo	2	7	1	1	True	0.000604
	bh	3	404	0	0	False	0.025843
	de	3	15244	0	0	True	0.924735
	shgo-simplicial	3	650	3	3	True	0.050251
	shgo-sobol	3	234	1	1	True	0.010897
Hansen	tgo	3	234	1	1	True	0.010778
	bh	2	3432	0	0	True	0.104063
	de	2	1341	0	0	True	0.071333
	shgo-simplicial	2	130	3	3	True	0.004946
	shgo-sobol	2	114	1	1	True	0.004238
	tgo	2	379	7	7	True	0.014041
Hartmann3	bh	3	7464	0	0	True	0.132592
	de	3	720	0	0	True	0.026820
	shgo-simplicial	3	70	1	1	True	0.002384
	shgo-sobol	3	54	1	1	True	0.001875
	tgo	3	53	1	1	True	0.001753
	Hartmann6	bh	6	16625	0	0	True
de		6	2230	0	0	True	0.091498
shgo-simplicial		6	181	1	1	True	0.026658
shgo-sobol		6	153	1	1	True	0.006292
tgo		6	443	3	2	True	0.010686
HelicalValley		bh	3	7688	0	0	True
	de	3	12124	0	0	True	0.361064

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
HimmelBlau	shgo-simplicial	3	1456	4	4	True	0.129628
	shgo-sobol	3	136	2	2	True	0.003249
	tgo	3	137	2	2	True	0.002614
	bh	2	2529	0	0	True	0.023512
	de	2	4683	0	0	True	0.114841
	shgo-simplicial	2	66	1	1	True	0.001178
HolderTable	shgo-sobol	2	45	1	1	True	0.001055
	tgo	2	45	1	1	True	0.000950
	bh	2	1857	0	0	False	0.031476
	de	2	415	0	0	True	0.012619
	shgo-simplicial	2	179	2	2	True	0.010071
	shgo-sobol	2	97	2	2	True	0.002625
Hosaki	tgo	2	117	3	3	True	0.002640
	bh	2	2526	0	0	False	0.029150
	de	2	335	0	0	True	0.008496
	shgo-simplicial	2	47	1	1	True	0.001053
	shgo-sobol	2	29	1	1	True	0.000968
	tgo	2	29	1	1	True	0.000827
Infinity	bh	2	2583	0	0	True	0.040846
	de	2	3803	0	0	True	0.126713
	shgo-simplicial	2	13	1	1	True	0.001057
	shgo-sobol	2	150	1	1	True	0.004148
	tgo	2	121	1	1	True	0.002820
	bh	2	10632	0	0	True	0.209818
JennrichSampson	de	2	904	0	0	True	0.033887
	shgo-simplicial	2	52	1	1	True	0.001704
	shgo-sobol	2	50	1	1	True	0.001765
	tgo	2	50	1	1	True	0.001640
	bh	2	3207	0	0	True	0.072541
	de	2	741	0	0	True	0.031098
Judge	shgo-simplicial	2	53	1	1	True	0.001520
	shgo-sobol	2	51	1	1	True	0.001924
	tgo	2	51	1	1	True	0.001817
	bh	2	474	0	0	False	0.025357
	de	2	2006	0	0	True	0.110219
	shgo-simplicial	2	9	1	1	True	0.000852

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Keane	shgo-sobol	2	7	1	1	True	0.000788
	tgo	2	7	1	1	True	0.000699
	bh	2	1566	0	0	True	0.023998
	de	2	7523	0	0	True	0.215796
	shgo-simplicial	2	1502	2	2	True	0.028949
Kowalik	shgo-sobol	2	14	1	1	True	0.000787
	tgo	2	7	1	1	True	0.000507
	bh	4	14660	0	0	True	0.248955
	de	4	6755	0	0	True	0.267475
	shgo-simplicial	4	240	1	1	True	0.006430
Langermann	shgo-sobol	4	229	1	1	True	0.005653
	tgo	4	228	1	1	True	0.005533
	bh	2	2877	0	0	True	0.098686
	de	2	692	0	0	True	0.035666
	shgo-simplicial	2	397	5	5	True	0.023237
LennardJones	shgo-sobol	2	49	1	1	True	0.002482
	tgo	2	49	1	1	True	0.002274
	bh	6	10374	0	0	True	0.058650
	de	6	15748	0	0	True	0.451860
	shgo-simplicial	6	124	1	1	True	0.001939
Leon	shgo-sobol	6	81	1	1	True	0.002031
	tgo	6	173	1	1	True	0.002409
	bh	2	6207	0	0	True	0.042320
	de	2	5363	0	0	True	0.129094
	shgo-simplicial	2	99	1	1	True	0.001523
Levy03	shgo-sobol	2	97	1	1	True	0.001644
	tgo	2	97	1	1	True	0.001533
	bh	2	2670	0	0	False	0.067339
	de	2	3803	0	0	True	0.163955
	shgo-simplicial	2	45	1	1	True	0.001729
Levy13	shgo-sobol	2	43	1	1	True	0.001749
	tgo	2	43	1	1	True	0.001647
	bh	2	4491	0	0	True	0.053878
	de	2	3803	0	0	True	0.114689
	shgo-simplicial	2	20	1	1	True	0.000724
	shgo-sobol	2	18	1	1	True	0.000825

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Matyas	tgo	2	18	1	1	True	0.000704
	bh	2	1803	0	0	True	0.018082
	de	2	4323	0	0	True	0.103091
	shgo-simplicial	2	9	1	1	True	0.000461
	shgo-sobol	2	7	1	1	True	0.000587
McCormick	tgo	2	7	1	1	True	0.000470
	bh	2	2073	0	0	False	0.023725
	de	2	495	0	0	True	0.012853
	shgo-simplicial	2	42	1	1	True	0.000948
	shgo-sobol	2	40	1	1	True	0.001059
Michalewicz	tgo	2	40	1	1	True	0.000956
	bh	2	4320	0	0	True	0.070726
	de	2	498	0	0	True	0.017048
	shgo-simplicial	2	50	1	1	True	0.001517
	shgo-sobol	2	48	1	1	True	0.001593
MieleCantrell	tgo	2	48	1	1	True	0.001480
	bh	4	9270	0	0	True	0.084157
	de	4	42965	0	0	True	1.297974
	shgo-simplicial	4	455	1	1	True	0.007732
	shgo-sobol	4	444	1	1	True	0.006351
Mishra01	tgo	4	443	1	1	True	0.006228
	bh	2	1830	0	0	False	0.025162
	de	2	406	0	0	True	0.011320
	shgo-simplicial	2	0	0	0	False	0.000000
	shgo-sobol	2	11	1	1	True	0.000743
Mishra02	tgo	2	11	1	1	True	0.000581
	bh	2	1752	0	0	False	0.028230
	de	2	566	0	0	True	0.017230
	shgo-simplicial	2	9	1	1	True	0.000513
	shgo-sobol	2	0	0	0	False	0.000000
Mishra03	tgo	2	0	0	0	False	0.000000
	bh	2	21105	0	0	False	0.191135
	de	2	2028	0	0	False	0.057982
	shgo-simplicial	2	70	1	1	True	0.001465
	shgo-sobol	2	68	1	1	True	0.001578
	tgo	2	68	1	1	True	0.001521

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Mishra04	bh	2	23679	0	0	False	0.212897
	de	2	1663	0	0	False	0.048371
	shgo-simplicial	2	599	3	3	True	0.016789
	shgo-sobol	2	4357	8	8	True	0.072626
	tgo	2	9363	18	18	True	0.149810
Mishra05	bh	2	7512	0	0	False	0.100622
	de	2	852	0	0	False	0.026962
	shgo-simplicial	2	50	1	1	True	0.001769
	shgo-sobol	2	142	2	2	True	0.003643
	tgo	2	263	3	3	True	0.005386
Mishra06	bh	2	2346	0	0	False	0.042102
	de	2	695	0	0	True	0.022795
	shgo-simplicial	2	62	1	1	True	0.002086
	shgo-sobol	2	121	2	2	True	0.003196
	tgo	2	170	2	2	True	0.003860
Mishra07	bh	2	1230	0	0	True	0.028053
	de	2	7043	0	0	True	0.250454
	shgo-simplicial	2	170	1	1	True	0.024871
	shgo-sobol	2	47	2	2	True	0.001797
	tgo	2	84	3	3	True	0.002268
Mishra08	bh	2	9831	0	0	True	0.122770
	de	2	2903	0	0	True	0.092307
	shgo-simplicial	2	243	1	1	True	0.004905
	shgo-sobol	2	235	1	1	True	0.005008
	tgo	2	235	1	1	True	0.004814
Mishra10	bh	2	303	0	0	False	0.010973
	de	2	923	0	0	True	0.021494
	shgo-simplicial	2	9	1	1	True	0.000476
	shgo-sobol	2	7	1	1	True	0.000583
	tgo	2	7	1	1	True	0.000472
Mishra11	bh	2	1140	0	0	True	0.022872
	de	2	6443	0	0	True	0.207037
	shgo-simplicial	2	9	1	1	True	0.000561
	shgo-sobol	2	7	1	1	True	0.000644
	tgo	2	7	1	1	True	0.000520
MultiModal	bh	2	21084	0	0	True	0.192647

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
NeedleEye	de	2	3643	0	0	True	0.111857
	shgo-simplicial	2	9	1	1	True	0.000544
	shgo-sobol	2	7	1	1	True	0.000635
	tgo	2	7	1	1	True	0.000513
	bh	2	10005	0	0	True	0.088015
NewFunction01	de	2	247	0	0	True	0.004802
	shgo-simplicial	2	9	1	1	True	0.000515
	shgo-sobol	2	7	1	1	True	0.000609
	tgo	2	7	1	1	True	0.000502
	bh	2	20874	0	0	False	0.170837
NewFunction02	de	2	1683	0	0	False	0.045641
	shgo-simplicial	2	3813	6	6	True	0.064411
	shgo-sobol	2	1569	6	6	True	0.026975
	tgo	2	10079	23	23	True	0.155735
	bh	2	22662	0	0	False	0.186096
OddSquare	de	2	1721	0	0	False	0.045242
	shgo-simplicial	2	159	1	1	True	0.004481
	shgo-sobol	2	341	2	2	True	0.005782
	tgo	2	361	2	2	True	0.005829
	bh	2	303	0	0	False	0.015612
Parsopoulos	de	2	1238	0	0	True	0.042061
	shgo-simplicial	2	0	0	0	False	0.000000
	shgo-sobol	2	204	1	1	True	0.006156
	tgo	2	487	8	8	True	0.012439
	bh	2	1608	0	0	True	0.020883
Pathological	de	2	4163	0	0	True	0.110717
	shgo-simplicial	2	30	1	1	True	0.001201
	shgo-sobol	2	39	1	1	True	0.001080
	tgo	2	39	1	1	True	0.000929
	bh	2	8106	0	0	True	0.192009
Paviani	de	2	2498	0	0	True	0.109880
	shgo-simplicial	2	20	1	1	True	0.000988
	shgo-sobol	2	18	1	1	True	0.001041
	tgo	2	18	1	1	True	0.001183
	bh	10	13970	0	0	False	0.231402
	de	10	6088	0	0	True	0.261439

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
PenHolder	shgo-simplicial	10	1257	1	1	True	180.467203
	shgo-sobol	10	364	1	1	True	0.013044
	tgo	10	358	1	1	True	0.007403
	bh	2	1512	0	0	False	0.029009
	de	2	532	0	0	True	0.017184
	shgo-simplicial	2	117	2	2	True	0.004500
Penalty01	shgo-sobol	2	86	2	2	True	0.002352
	tgo	2	74	2	2	True	0.001852
	bh	2	3300	0	0	True	0.103837
	de	2	3803	0	0	True	0.192800
	shgo-simplicial	2	45	1	1	True	0.002049
	shgo-sobol	2	43	1	1	True	0.002108
PermFunction01	tgo	2	43	1	1	True	0.001994
	bh	2	4599	0	0	True	0.132834
	de	2	4963	0	0	True	0.250583
	shgo-simplicial	2	83	1	1	True	0.003318
	shgo-sobol	2	70	1	1	True	0.002988
	tgo	2	70	1	1	True	0.002867
PermFunction02	bh	2	4665	0	0	True	0.130644
	de	2	4563	0	0	True	0.228565
	shgo-simplicial	2	73	1	1	True	0.002946
	shgo-sobol	2	71	1	1	True	0.002968
	tgo	2	71	1	1	True	0.002847
	bh	2	3075	0	0	False	0.121511
Pinter	de	2	4043	0	0	True	0.230878
	shgo-simplicial	2	9	1	1	True	0.000793
	shgo-sobol	2	7	1	1	True	0.000814
	tgo	2	7	1	1	True	0.000695
	bh	2	303	0	0	False	0.012533
	de	2	283	0	0	True	0.007515
Plateau	shgo-simplicial	2	9	1	1	True	0.000512
	shgo-sobol	2	7	1	1	True	0.000612
	tgo	2	7	1	1	True	0.000497
	bh	4	14285	0	0	True	0.090999
	de	4	35285	0	0	True	0.939842
	shgo-simplicial	4	220	1	1	True	0.003550

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
PowerSum	shgo-sobol	4	209	1	1	True	0.002965
	tgo	4	208	1	1	True	0.002803
	bh	4	53785	0	0	True	1.136949
	de	4	80125	0	0	True	3.736967
	shgo-simplicial	4	386	1	1	True	0.011866
Price01	shgo-sobol	4	641	1	1	True	0.018445
	tgo	4	640	1	1	True	0.018284
	bh	2	921	0	0	True	0.016215
	de	2	4003	0	0	True	0.106303
	shgo-simplicial	2	14	1	1	True	0.000571
Price02	shgo-sobol	2	12	1	1	True	0.000679
	tgo	2	12	1	1	True	0.000559
	bh	2	1614	0	0	False	0.028523
	de	2	732	0	0	False	0.023422
	shgo-simplicial	2	9	1	1	True	0.000519
Price03	shgo-sobol	2	7	1	1	True	0.000657
	tgo	2	7	1	1	True	0.000557
	bh	2	5136	0	0	True	0.040620
	de	2	4283	0	0	True	0.107037
	shgo-simplicial	2	58	1	1	True	0.001118
Price04	shgo-sobol	2	56	1	1	True	0.001240
	tgo	2	56	1	1	True	0.001102
	bh	2	6984	0	0	True	0.050386
	de	2	40043	0	0	True	1.001060
	shgo-simplicial	2	9	1	1	True	0.000480
Quadratic	shgo-sobol	2	7	1	1	True	0.000593
	tgo	2	7	1	1	True	0.000484
	bh	2	1917	0	0	True	0.019071
	de	2	378	0	0	True	0.008401
	shgo-simplicial	2	35	1	1	True	0.000803
Quintic	shgo-sobol	2	33	1	1	True	0.000902
	tgo	2	33	1	1	True	0.000795
	bh	2	35934	0	0	True	0.653358
	de	2	3823	0	0	True	0.155548
	shgo-simplicial	2	114	1	1	True	0.003436
	shgo-sobol	2	112	1	1	True	0.003479

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Rana	tgo	2	112	1	1	True	0.003377
	bh	2	1851	0	0	False	0.044188
	de	2	1055	0	0	False	0.041408
	shgo-simplicial	2	292	5	5	True	0.010485
	shgo-sobol	2	651	10	10	True	0.019660
Rastrigin	tgo	2	1157	20	20	True	0.032245
	bh	2	3198	0	0	True	0.045106
	de	2	2323	0	0	True	0.075224
	shgo-simplicial	2	20	1	1	True	0.000758
	shgo-sobol	2	18	1	1	True	0.000832
Ratkowsky01	tgo	2	18	1	1	True	0.000722
	bh	4	5355	0	0	False	0.108619
	de	4	3595	0	0	False	0.147932
	shgo-simplicial	4	286	1	1	True	0.008561
	shgo-sobol	4	187	1	1	True	0.005141
Ratkowsky02	tgo	4	186	1	1	True	0.005016
	bh	3	18628	0	0	True	0.325850
	de	3	2308	0	0	True	0.089465
	shgo-simplicial	3	124	1	1	True	0.005340
	shgo-sobol	3	111	1	1	True	0.002979
Ripple01	tgo	3	110	1	1	True	0.002856
	bh	2	5961	0	0	False	0.124923
	de	2	824	0	0	False	0.033875
	shgo-simplicial	2	153	3	3	True	0.006820
	shgo-sobol	2	476	1	1	True	0.016043
Ripple25	tgo	2	1879	29	29	True	0.061017
	bh	2	3426	0	0	False	0.068602
	de	2	815	0	0	True	0.030853
	shgo-simplicial	2	106	3	3	True	0.005244
	shgo-sobol	2	99	1	1	True	0.003343
Rosenbrock	tgo	2	372	9	9	True	0.009879
	bh	2	6324	0	0	True	0.100650
	de	2	4923	0	0	True	0.173339
	shgo-simplicial	2	118	1	1	True	0.002990
	shgo-sobol	2	116	1	1	True	0.003024
	tgo	2	116	1	1	True	0.002962

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
RosenbrockModified	bh	2	5790	0	0	False	0.058106
	de	2	898	0	0	True	0.024119
	shgo-simplicial	2	128	2	2	True	0.004098
	shgo-sobol	2	66	1	1	True	0.001674
	tgo	2	66	1	1	True	0.001431
RotatedEllipse01	bh	2	1566	0	0	True	0.019852
	de	2	3923	0	0	True	0.101885
	shgo-simplicial	2	9	1	1	True	0.000527
	shgo-sobol	2	7	1	1	True	0.000608
	tgo	2	7	1	1	True	0.000489
RotatedEllipse02	bh	2	1542	0	0	True	0.016783
	de	2	3763	0	0	True	0.091090
	shgo-simplicial	2	9	1	1	True	0.000463
	shgo-sobol	2	7	1	1	True	0.000584
	tgo	2	7	1	1	True	0.000468
Salomon	bh	2	7743	0	0	True	0.090562
	de	2	1489	0	0	False	0.048057
	shgo-simplicial	2	40	1	1	True	0.001145
	shgo-sobol	2	38	1	1	True	0.001219
	tgo	2	38	1	1	True	0.001105
Sargan	bh	2	1536	0	0	True	0.062461
	de	2	4003	0	0	True	0.234294
	shgo-simplicial	2	9	1	1	True	0.000796
	shgo-sobol	2	7	1	1	True	0.000806
	tgo	2	7	1	1	True	0.000697
Schaffer01	bh	2	3273	0	0	False	0.030547
	de	2	2883	0	0	True	0.076986
	shgo-simplicial	2	9	1	1	True	0.000513
	shgo-sobol	2	7	1	1	True	0.000602
	tgo	2	7	1	1	True	0.000492
Schaffer02	bh	2	4806	0	0	False	0.043977
	de	2	2803	0	0	True	0.077345
	shgo-simplicial	2	9	1	1	True	0.000485
	shgo-sobol	2	7	1	1	True	0.000590
	tgo	2	7	1	1	True	0.000489
Schaffer03	bh	2	6183	0	0	False	0.065953

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Schaffer04	de	2	3289	0	0	True	0.094391
	shgo-simplicial	2	0	0	0	False	0.000000
	shgo-sobol	2	870	2	2	True	0.014601
	tgo	2	6986	15	15	True	0.112689
	bh	2	4956	0	0	False	0.054692
	de	2	1769	0	0	True	0.051488
Schwefel01	shgo-simplicial	2	0	0	0	False	0.000000
	shgo-sobol	2	956	2	2	True	0.016079
	tgo	2	7424	16	16	True	0.120559
	bh	2	2592	0	0	True	0.034659
	de	2	4003	0	0	True	0.119275
	shgo-simplicial	2	9	1	1	True	0.002122
Schwefel02	shgo-sobol	2	7	1	1	True	0.000628
	tgo	2	7	1	1	True	0.000508
	bh	2	1530	0	0	True	0.051924
	de	2	4563	0	0	True	0.231409
	shgo-simplicial	2	9	1	1	True	0.000774
	shgo-sobol	2	7	1	1	True	0.000781
Schwefel04	tgo	2	7	1	1	True	0.000653
	bh	2	2856	0	0	True	0.047694
	de	2	4403	0	0	True	0.148907
	shgo-simplicial	2	47	1	1	True	0.001346
	shgo-sobol	2	45	1	1	True	0.001410
	tgo	2	45	1	1	True	0.001343
Schwefel06	bh	2	32622	0	0	True	0.224756
	de	2	4263	0	0	True	0.114865
	shgo-simplicial	2	150	1	1	True	0.002512
	shgo-sobol	2	148	1	1	True	0.002586
	tgo	2	148	1	1	True	0.002469
	bh	2	37062	0	0	True	0.251657
Schwefel20	de	2	3663	0	0	True	0.102275
	shgo-simplicial	2	72	1	1	True	0.001425
	shgo-sobol	2	70	1	1	True	0.001518
	tgo	2	70	1	1	True	0.001410
	bh	2	30786	0	0	True	0.151423
	de	2	4263	0	0	True	0.103540

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Schwefel22	shgo-simplicial	2	23	1	1	True	0.000652
	shgo-sobol	2	21	1	1	True	0.000780
	tgo	2	21	1	1	True	0.000653
	bh	2	34536	0	0	True	0.315124
	de	2	3903	0	0	True	0.119683
Schwefel26	shgo-simplicial	2	75	1	1	True	0.001700
	shgo-sobol	2	73	1	1	True	0.001798
	tgo	2	73	1	1	True	0.001667
	bh	2	1377	0	0	False	0.023562
	de	2	1763	0	0	True	0.059069
Schwefel36	shgo-simplicial	2	85	2	2	True	0.060432
	shgo-sobol	2	46	1	1	True	0.001913
	tgo	2	46	1	1	True	0.001716
	bh	2	531	0	0	False	0.012828
	de	2	741	0	0	True	0.017205
Shekel05	shgo-simplicial	2	593	1	1	True	0.029588
	shgo-sobol	2	514	1	1	True	0.010140
	tgo	2	80	2	2	True	0.001428
	bh	4	4765	0	0	False	0.077725
	de	4	2505	0	0	True	0.095390
Shekel07	shgo-simplicial	4	118	1	1	True	0.003752
	shgo-sobol	4	107	1	1	True	0.002920
	tgo	4	106	1	1	True	0.002766
	bh	4	5720	0	0	True	0.091560
	de	4	2590	0	0	True	0.099127
Shekel10	shgo-simplicial	4	134	1	1	True	0.004098
	shgo-sobol	4	123	1	1	True	0.003263
	tgo	4	122	1	1	True	0.003110
	bh	4	4365	0	0	False	0.073112
	de	4	2500	0	0	False	0.097044
Shubert01	shgo-simplicial	4	142	1	1	True	0.004259
	shgo-sobol	4	131	1	1	True	0.003466
	tgo	4	130	1	1	True	0.003383
	bh	2	3111	0	0	True	0.067104
	de	2	1467	0	0	True	0.061174
	shgo-simplicial	2	0	0	0	False	0.000000

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Shubert03	shgo-sobol	2	76	1	1	True	0.003538
	tgo	2	157	3	3	True	0.005496
	bh	2	3000	0	0	True	0.069237
	de	2	1055	0	0	True	0.045163
	shgo-simplicial	2	0	0	0	False	0.000000
Shubert04	shgo-sobol	2	51	1	1	True	0.002095
	tgo	2	51	1	1	True	0.001756
	bh	2	3024	0	0	True	0.069495
	de	2	1175	0	0	True	0.049131
	shgo-simplicial	2	0	0	0	False	0.000000
SineEnvelope	shgo-sobol	2	142	3	3	True	0.004916
	tgo	2	178	4	4	True	0.005449
	bh	2	1416	0	0	False	0.034150
	de	2	1449	0	0	False	0.053516
	shgo-simplicial	2	9	1	1	True	0.000597
SixHumpCamel	shgo-sobol	2	7	1	1	True	0.000680
	tgo	2	7	1	1	True	0.000574
	bh	2	3030	0	0	True	0.028893
	de	2	615	0	0	True	0.014982
	shgo-simplicial	2	175	1	1	True	0.009341
Sodp	shgo-sobol	2	42	1	1	True	0.001182
	tgo	2	42	1	1	True	0.000968
	bh	2	3648	0	0	True	0.051512
	de	2	4043	0	0	True	0.130966
	shgo-simplicial	2	9	1	1	True	0.000509
Sphere	shgo-sobol	2	7	1	1	True	0.000609
	tgo	2	7	1	1	True	0.000515
	bh	2	909	0	0	True	0.017210
	de	2	3603	0	0	True	0.101877
	shgo-simplicial	2	9	1	1	True	0.000493
Step	shgo-sobol	2	7	1	1	True	0.000607
	tgo	2	7	1	1	True	0.000493
	bh	2	303	0	0	False	0.012524
	de	2	1083	0	0	True	0.030122
	shgo-simplicial	2	9	1	1	True	0.000481
	shgo-sobol	2	7	1	1	True	0.000605

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Step2	tgo	2	7	1	1	True	0.000497
	bh	2	303	0	0	False	0.013313
	de	2	843	0	0	True	0.025589
	shgo-simplicial	2	9	1	1	True	0.000516
	shgo-sobol	2	7	1	1	True	0.000630
StretchedV	tgo	2	7	1	1	True	0.000509
	bh	2	1866	0	0	True	0.039634
	de	2	1529	0	0	True	0.055269
	shgo-simplicial	2	43	1	1	True	0.001558
	shgo-sobol	2	41	1	1	True	0.001483
StyblinskiTang	tgo	2	41	1	1	True	0.001373
	bh	2	2031	0	0	False	0.036926
	de	2	492	0	0	True	0.015949
	shgo-simplicial	2	41	1	1	True	0.001222
	shgo-sobol	2	48	1	1	True	0.001570
TestTubeHolder	tgo	2	48	1	1	True	0.001405
	bh	2	1563	0	0	False	0.027345
	de	2	852	0	0	True	0.025876
	shgo-simplicial	2	0	0	0	False	0.000000
	shgo-sobol	2	895	1	1	True	0.024863
ThreeHumpCamel	tgo	2	1101	31	30	True	0.023859
	bh	2	2247	0	0	True	0.022462
	de	2	4163	0	0	True	0.103089
	shgo-simplicial	2	9	1	1	True	0.000438
	shgo-sobol	2	7	1	1	True	0.000586
Treccani	tgo	2	7	1	1	True	0.000479
	bh	2	2658	0	0	True	0.023854
	de	2	2403	0	0	True	0.062638
	shgo-simplicial	2	9	1	1	True	0.000466
	shgo-sobol	2	7	1	1	True	0.000592
Trid	tgo	2	7	1	1	True	0.000476
	bh	6	9387	0	0	True	0.115005
	de	6	4178	0	0	True	0.146371
	shgo-simplicial	6	152	1	1	True	0.025636
	shgo-sobol	6	98	1	1	True	0.002638
	tgo	6	94	1	1	True	0.002139

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Trigonometric01	bh	2	6288	0	0	True	0.149112
	de	2	6843	0	0	True	0.316741
	shgo-simplicial	2	9	1	1	True	0.000654
	shgo-sobol	2	7	1	1	True	0.000729
	tgo	2	7	1	1	True	0.000627
Tripod	bh	2	27252	0	0	False	0.200039
	de	2	3863	0	0	True	0.111605
	shgo-simplicial	2	163	2	2	True	0.004474
	shgo-sobol	2	254	2	2	True	0.004403
	tgo	2	254	2	2	True	0.004049
Ursem01	bh	2	1911	0	0	False	0.023793
	de	2	332	0	0	True	0.008287
	shgo-simplicial	2	22	1	1	True	0.000700
	shgo-sobol	2	29	1	1	True	0.000926
	tgo	2	29	1	1	True	0.000811
Ursem03	bh	2	5286	0	0	False	0.072546
	de	2	782	0	0	True	0.019909
	shgo-simplicial	2	55	1	1	True	0.001412
	shgo-sobol	2	53	1	1	True	0.001487
	tgo	2	53	1	1	True	0.001377
Ursem04	bh	2	16347	0	0	True	0.138740
	de	2	591	0	0	True	0.013742
	shgo-simplicial	2	97	1	1	True	0.001742
	shgo-sobol	2	95	1	1	True	0.001855
	tgo	2	95	1	1	True	0.001740
UrsemWaves	bh	2	420	0	0	False	0.014496
	de	2	498	0	0	False	0.013866
	shgo-simplicial	2	13	2	2	True	0.000739
	shgo-sobol	2	19	1	1	True	0.000854
	tgo	2	19	1	1	True	0.000707
VenterSobieczczanskiSobieski	bh	2	2448	0	0	False	0.034364
	de	2	655	0	0	True	0.019093
	shgo-simplicial	2	9	1	1	True	0.000497
	shgo-sobol	2	7	1	1	True	0.000617
	tgo	2	7	1	1	True	0.000507
Vincent	bh	2	2805	0	0	True	0.056584

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Watson	de	2	753	0	0	True	0.021509
	shgo-simplicial	2	42	1	1	True	0.001052
	shgo-sobol	2	31	1	1	True	0.001015
	tgo	2	31	1	1	True	0.000920
	bh	6	33320	0	0	True	1.415519
	de	6	23095	0	0	True	1.642898
	shgo-simplicial	6	337	1	1	True	0.042924
Wavy	shgo-sobol	6	283	1	1	True	0.016899
	tgo	6	279	1	1	True	0.015085
	bh	2	3465	0	0	False	0.054129
	de	2	2603	0	0	True	0.089450
	shgo-simplicial	2	9	1	1	True	0.000559
	shgo-sobol	2	7	1	1	True	0.000659
	tgo	2	7	1	1	True	0.000548
WayburnSeader01	bh	2	6933	0	0	True	0.052945
	de	2	4823	0	0	True	0.127303
	shgo-simplicial	2	111	1	1	True	0.001713
	shgo-sobol	2	109	1	1	True	0.001851
	tgo	2	109	1	1	True	0.001741
WayburnSeader02	bh	2	6732	0	0	True	0.055006
	de	2	5043	0	0	True	0.126818
	shgo-simplicial	2	150	1	1	True	0.002188
	shgo-sobol	2	148	1	1	True	0.002326
	tgo	2	148	1	1	True	0.002210
Weierstrass	bh	2	30213	0	0	False	1.013885
	de	2	3623	0	0	True	0.210147
	shgo-simplicial	2	2225	1	1	True	0.218606
	shgo-sobol	2	0	0	0	False	0.000000
	tgo	2	0	0	0	False	0.000000
Whitley	bh	2	5244	0	0	False	0.123444
	de	2	1618	0	0	False	0.071220
	shgo-simplicial	2	34	1	1	True	0.001424
	shgo-sobol	2	32	1	1	True	0.001498
	tgo	2	32	1	1	True	0.001402
Wolfe	bh	3	1156	0	0	False	0.015604
	de	3	16444	0	0	True	0.422252

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
XinSheYang01	shgo-simplicial	3	14	1	1	True	0.001167
	shgo-sobol	3	10	1	1	True	0.000682
	tgo	3	9	1	1	True	0.000555
	bh	2	7632	0	0	False	0.097735
	de	2	6663	0	0	True	0.220974
XinSheYang02	shgo-simplicial	2	153	1	1	True	0.003723
	shgo-sobol	2	77	1	1	True	0.002016
	tgo	2	149	1	1	True	0.003290
	bh	2	2691	0	0	False	0.046717
	de	2	4223	0	0	True	0.141718
XinSheYang03	shgo-simplicial	2	65	1	1	True	0.001683
	shgo-sobol	2	63	1	1	True	0.001770
	tgo	2	63	1	1	True	0.001675
	bh	2	312	0	0	False	0.017095
	de	2	1409	0	0	True	0.058742
XinSheYang04	shgo-simplicial	2	9	1	1	True	0.000640
	shgo-sobol	2	7	1	1	True	0.000714
	tgo	2	7	1	1	True	0.000594
	bh	2	1791	0	0	False	0.044684
	de	2	1795	0	0	True	0.065124
Xor	shgo-simplicial	2	77	1	1	True	0.002366
	shgo-sobol	2	75	1	1	True	0.002478
	tgo	2	75	1	1	True	0.002320
	bh	9	7940	0	0	False	0.207127
	de	9	1520	0	0	False	0.061928
YaoLiu04	shgo-simplicial	9	645	1	1	True	15.690898
	shgo-sobol	9	197	1	1	True	0.017347
	tgo	9	208	2	2	True	0.006470
	bh	2	29316	0	0	True	0.173835
	de	2	3903	0	0	True	0.100743
YaoLiu09	shgo-simplicial	2	23	1	1	True	0.000677
	shgo-sobol	2	21	1	1	True	0.000803
	tgo	2	21	1	1	True	0.000677
	bh	2	3300	0	0	True	0.049375
	de	2	2843	0	0	True	0.093783
	shgo-simplicial	2	20	1	1	True	0.000767

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Table 1: Comparison of the performance of SHGO, TGO, BH and DE over a wide selection of optimisation test problems. The columns are the name of the optimisation test problem, the algorithm (Alg) used, the number of dimensions ( $n$ ) of the optimisation problem, the number of function evaluations (nfev), the number of minimisers generated (nlmin), the number of unique local minima (nulmin), successful convergence to the global minima and the total CPU run time (runtime).

Problem	Alg	ndim	nfev	nlmin	nulmin	success	runtime
Zacharov	shgo-sobol	2	18	1	1	True	0.000867
	tgo	2	18	1	1	True	0.000758
	bh	2	2046	0	0	True	0.039036
	de	2	4043	0	0	True	0.143251
	shgo-simplicial	2	46	1	1	True	0.001841
ZeroSum	shgo-sobol	2	45	1	1	True	0.001503
	tgo	2	45	1	1	True	0.001399
	bh	2	20538	0	0	False	0.245236
	de	2	1743	0	0	False	0.056568
	shgo-simplicial	2	0	0	0	False	0.000000
Zettl	shgo-sobol	2	23	1	1	True	0.001420
	tgo	2	0	0	0	False	0.000000
	bh	2	4167	0	0	True	0.033280
	de	2	861	0	0	True	0.020092
	shgo-simplicial	2	116	1	1	True	0.001764
Zimmerman	shgo-sobol	2	114	1	1	True	0.001909
	tgo	2	114	1	1	True	0.001793
	bh	2	24543	0	0	False	0.277145
	de	2	6503	0	0	True	0.207111
	shgo-simplicial	2	3032	1	1	True	0.173071
Zirilli	shgo-sobol	2	1585	1	1	True	0.033359
	tgo	2	1585	1	1	True	0.041850
	bh	2	2562	0	0	False	0.023889
	de	2	575	0	0	True	0.013579
	shgo-simplicial	2	34	1	1	True	0.000779
	shgo-sobol	2	32	1	1	True	0.000948
	tgo	2	32	1	1	True	0.000811

## References