Research Article

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What If Quantum Theory Violates All Mathematics?

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Abstract: It is shown by using a rather elementary argument in Mathematical Logic that *if* indeed, quantum theory does violate the famous Bell Inequalities, then quantum theory *must* inevitably also violate *all* valid mathematical statements, and in particular, such basic algebraic relations like $0 = 0,\ 1 = 1,\ 2 = 2,\ 3 = 3, \ldots$ and so on ...
An interest in that result is due to the following three alternatives which it *imposes* upon both Physics and Mathematics :
1) Quantum Theory is inconsistent.
2) Quantum Theory together with Mathematics are inconsistent.
3) Mathematics is inconsistent.
In this regard one should recall that, up until now, it is *not* known whether Mathematics is indeed consistent.

Keywords: Quanta, Bell Inequalities, Violation Conundrum, Inconsistency of Quantum Theory

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1 Bell Inequalities Conundrum

1.1 The Bell Inequalities belong to Pure Mathematics, and NOT at all to Physics ...

Recently it was shown, [1–3], that the Bell Inequalities, [4], are *not* violated by the quantum theory, *contrary* to the long ongoing customary and rather unanimous claim by many quantum physicists, a claim which simply turns out to be but the consequence of an elementary error in connecting mathematical abstractions with statistical data. Consequently, the *irrelevance* of the Bell Inequalities in Physics, in the sense that neither classical, nor quantum systems violate the Bell Inequalities.

Given the above, obviously a number of misconceptions still keep persisting today regarding both the Bell Inequalities as such, and their relation to Quantum Physics. Also, it is hardly at all known, let alone understood, among many physicists that the Bell Inequalities are *purely* mathematical statements, that is, they follow logically from elementary algebraic properties of the real numbers in $\mathbb{R}$, and thus do *not* need any sort of physical arguments in order to prove them, [1, 5]. In fact, the Bell Inequalities are mathematical consequences of inequalities proved in the Appendix of George Boole's 1854 classic book "The Laws of Thought", [5], thus long *before* the emergence of Quantum Theory.

Amusingly however, the triplet of seemingly independent recent papers [6–8], claims to have - at long last - completed the proof of the more than half a century old allegation that quantum physics violates the Bell Inequalities, and the many co-authors of these papers argue that they have closed all the loopholes in the previously presented various proofs, see also [9]. They use a variation of Bell's inequality that is called the Clauser-Horn-Shimony-Holt inequality, which is precisely the inequality discussed by Leggett, [10], and of which the Bell Inequality is a special case, namely, when $D = A$, in Leggett's notation.

Following soon after the mentioned triplet of papers, rather surprising statements regarding the above conundrum appeared in [11, 12].
The novelty in these two papers is that the mentioned conundrum is shown to reach to such *deep* levels that, in fact, it can be seen as a "clash between whole mathematical theories", which in this case are, on one hand, various possible mathematical theories of quantum theory, and on the other hand, theories of pure mathematics. Namely :

(A) As already pointed out in [1–3, 5], the Bell Inequalities do belong to Mathematics, and *not* to Physics, hence here in this paper they are once again treated accordingly, thus a considerable amount of misunderstandings in widely used Physics literature, discourse and thinking gets simply set aside.

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(B) Consequently, the Bell Inequalities *cannot* be violated by quantum theory, but *only and only* by one or another *mathematical theory* of quanta, and specifically, by some mathematical statement which happens to belong to one or another *mathematical theory* of quanta. After all, and all too obviously, quanta as such do *not* belong to Mathematics, while clearly, *only* the modern theories of quanta do so.

(C) In view of (A) and (B) above, it is shown - as a rather simple and elementary consequence in Mathematical Logic - that in case "quantum theory" would indeed violate the Bell Inequalities, then "quantum theory" would in fact violate the *whole* of Mathematics as well, thus in particular for instance, "quantum theory" would *have to* violate relations like $0 = 0,\ 1 = 1,\ 2 = 2,\ 3 = 3, \ldots$ and so on ...

1.2 Global Inconsistency ?

This paper attempts to makes more *explicit* the possibility shown in [11, 12] of the truly *deeper* implications which would inevitably result in case, when indeed, quantum theory would violate the Bell Inequalities.

Namely, as a rather trivial logical consequence of such a violation, the following *three* inevitable alternatives would sum up the situation in Physics and Mathematics :

1. Quantum Theory is inconsistent.
2. Quantum Theory together with Mathematics are inconsistent.
3. Mathematics is inconsistent.

In this regard one should recall that, up until now, it is *not* known whether Mathematics is consistent.

It follows that, perhaps, we should rather hope that, indeed, the Bell Inequalities are *irrelevant*, that is, quantum theory does *not* violate them, just as classical systems do not, since otherwise the ... Hell ... of the above three *inevitable* alternatives 1), 2) and 3) *would* break loose upon us ...

This is indeed the *stark* reality we may face, as argued in this paper ...

However, amusingly, due to latest developments in Mathematics and Mathematical Logic such a stark reality may as well be dealt with, as mentioned briefly in the following.

2 Modern Interaction between Physics and Mathematics

The presently predominant way of interaction between theories of Physics, and on the other hand, the various disciplines of Mathematics - interaction on the level of aiming at a more precise formulation of the former - can be traced back to Newton's master piece "Philosophiae Naturalis Principia Mathematica", published on 5 July, 1687, and in which the fundamentals of Classical Mechanics are presented in an *axiomatic* manner. In Mathematics itself, the axiomatic method goes back to Euclid's presentation of Geometry, in the 4th century BC. In Physics, however, the mentioned approach of Newton, coming about two millennia later, is considered to be the origin of axiomatization.

For convenience, a brief presentation of the essential aspects of the general mathematical axiomatic method can be found in the Appendix 1 of [11], and the notations there will be used also in the following.

And now, to the origin of the Bell inequalities.

As seemingly hardly known among physicists, the Bell Inequalities are in fact *purely mathematical* properties, that is, their formulation and proof does *not* need any other considerations, except for mathematical ones, [1–3, 5]. Furthermore, the Mathematics used in both stating and proving them is but a rather simple elementary Algebra of the real numbers in $\mathbb{R}$.

It was George Boole in his 1854 classic "The Laws of Thought" where, in the Appendix, he introduced and dealt with a class of inequalities which, when later extended mathematically, ended up by containing the Bell inequalities as well, and the respective extensions happened before and/or independently of the discovery of quanta. In this way, it is due to the typical limited familiarity with Mathematics on the part of many physicists which, in a way, resulted in John Bell's unintended and unconscious plagiarizing, when claiming originality with his inequalities.

And now to some brief and relevant considerations related to the axiomatic method.

Much of the theory of quanta - when formulated mathematically - is limited to the Mathematics determined by the axioms of the Zermelo - Fraenkel Set Theory, [13]. The meaning of that is in brief as follows.

Let us denote the set of all the Zermelo - Fraenkel axioms by $\mathcal{A}_{set}$, [11, 13]. Then as also follows from [11], we have $\mathcal{A}_{set} \subsetneqq \mathcal{F}_{set}$, where $\mathcal{F}_{set}$ is the set of all so called *well-formed formulas* - or briefly wff-s - in the alphabet A_{set} of Set Theory. Further, we have a set ρ_{set} of *logical deduction* *rules*, [11], which operate as follows

$$\mathcal{F}_{set} \supseteq P \xrightarrow{\rho_{set}} Q \subseteq \mathcal{F}_{set} \tag{1}$$

thus, to each subset P of wff-s - called *premises* - the logical deduction rules ρ_{set} associate a subset Q of wff-s - called *deductions* - and do so by a process which uses *exclusively* the classical binary valued Logic of Aristotle - Chrysippus.

The axiomatic method in the above particular case of the Zermelo - Fraenkel Set Theory, means that one *starts* with the specific axioms in the set $\mathcal{A}_{set}$, and applies to them the logical deduction rules ρ_{set}, obtaining thus a certain set of deductions. Then iteratively, one keeps applying the logical deduction rules ρ_{set} to the latest set of deductions obtained which now are considered as premises, and does so - in principle - unlimitedly many times.

The result of that axiomatic method is a corresponding set $\mathcal{T}_{set}$ of *theorems* which constitute all Mathematics expressible by Set Theory. Clearly, we have, [11]

$$\mathcal{A}_{set} \subsetneqq \mathcal{T}_{set} \subsetneqq \mathcal{F}_{set} \tag{2}$$

So much for Mathematics and Mathematical Logic, and let us now turn to Physics.

Let us denote by $\mathcal{T}_q$ the set of theorems of a large enough part of a given theory of quanta, when they are formulated in mathematical terms. Here "large enough" means that it contains the mathematical formulation of the quantum arguments regarding the alleged violation of the Bell Inequalities by quantum theory. Then obviously, it is assumed - even if only rather tacitly on the part of physicists - that

$$\mathcal{T}_q \subsetneqq \mathcal{T}_{set} \tag{3}$$

or in other words, *no* theorem $T \in \mathcal{T}_q$, that is, *no* theorem T of the mentioned part of the theory of quanta - when considered in its mathematical formulation - is supposed to be mathematically *incorrect*.

On the other hand, in view of the previous remarks regarding the Bell Inequalities, we have as well

$$T_{Bell} \in \mathcal{T}_{set} \tag{4}$$

where T_{Bell} denotes the *pure* mathematical theorem stating the Bell Inequalities.

3 Does Quantum Theory Violate the Bell Inequalities ?

Now, the mentioned widely accepted claim, supported by books of famous physicists, [10], that "the quanta violate the Bell Inequalities"- a claim to which recently the further claim was added in [6–8] that the last loopholes in its experimental verification were at long last closed - amounts in the above formulation to the existence of a mathematical theorem, let us denote it by S, such that

$$S \in \mathcal{T}_q \tag{5}$$

In other words, there exists a theorem S among the set $\mathcal{T}_q$ of theorems of the above mentioned relevant part of the theory of quanta, such that

$$S \Longrightarrow (non\ T_{Bell}) \tag{6}$$

Remark 3.1. Regarding the statement S in (5) let us note the following. In this case, the statement S is in fact the theoretical expression of the results of certain suitable *quantum experiments*, thus typically those results are expressed in *numbers* which describe the outcome of the experiments. For instance, according to the Copenhagen Interpretation, those numbers are computable from eigenvalues of Hermitian operators defining the observables which participated in the experiments.

Consequently, the expression of the particular claim that "quantum theory violates the Bell Inequalities" can only take the *pure mathematical* form in (6). And such an expression is possible, at least in principle, if *experimental facts* leading to the mentioned numbers get formulated as a *mathematical statement* S.

Indeed, in present day Physics, there is simply *no* other way to, so to say, "violate" Bell-type Inequalities, except to express the relevant experimental facts mathematically, see [10], including those which are claimed in [6–8] to have no longer loopholes ...

In other words, the claimed "violation" can only happen in the realms of Pure Mathematics, since the Bell Inequalities do belong to those realms. □

Let us now recall that it is a typical and general feature of all sets of theorems $\mathcal{T}$ in every axiomatic mathematical theory to have the following implication hold

$$(S' \in \mathcal{T},\ S'' \in \mathcal{F},\ S' \Longrightarrow S'') \Longrightarrow S'' \in \mathcal{T} \tag{7}$$

where, with the above notation, $\mathcal{F}$ is the set of all wff-s of the respective axiomatic theory, [11]. Similarly, we have as a typical and general feature of all sets of wff-s $\mathcal{F}$ in every axiomatic mathematical theory the implication

$$S' \in \mathcal{F} \Longrightarrow non\, S' \in \mathcal{F} \tag{8}$$

And then (5) - (7) give

$$non\ T_{Bell} \in \mathcal{T}_q \subsetneqq \mathcal{T}_{set} \tag{9}$$

which together with (3), (8) implies

$$T_{Bell},\ non\ T_{Bell} \in \mathcal{T}_{set} \tag{10}$$

that is, $\mathcal{T}_{set}$, and thus Set Theory, is contradictory, since it contains *both* the theorem T_{Bell} *and* its negation $non\ T_{Bell}$.

However, the set $\mathcal{T}_{set}$ of all mathematical theorems of Set Theory is considered to be *contradiction free*, although so far that property - or for that matter, the negation of that property - has *not* yet been proved.

Lastly, let us show that given any S in (5), then we have the following logical consequence, and at the same time, also a much *stronger* version of (6), namely

$$[S \Longrightarrow (non\ T_{Bell})] \Longrightarrow [\forall T \in \mathcal{T}_{set} : (S \Longrightarrow (non\ T)] \tag{11}$$

or in other words, the relation 6 holds not only in the *particular* case of the Bell inequalities given by the theorem T_{Bell}, but also for *all* valid mathematical statements in Set Theory, that is, for *all* theorems $T \in \mathcal{T}_{set}$.

In particular, as mentioned in [11, 12], it is true that :

> If a valid quantum statement - be it of theoretical or experimental nature, yet expressed mathematically by $S \in \mathcal{T}_q$ - violates the Bell Inequalities, then that statement S must also violate *all* other valid mathematical statements $T \in \mathcal{T}_{set}$, and thus in particular, relations such as $0 = 0,\ 1 = 1,\ 2 = 2,\ 3 = 3, \ldots$.

Indeed, the proof of (11) is as follows. Let us assume that (11) does not hold, and let us reformulate it conveniently in a logically equivalent form. For that purpose, we recall that, according to basic and elementary rules of the binary valued Logic of Aristotel - Chrysippus, we have the following *logical equivalences*, denoted by $\equiv$, for arbitrary wff-s $Q, Q', Q'' \in \mathcal{F}$

$$\begin{aligned} non\,(non\,Q) &\equiv Q \\ (Q \Longrightarrow Q') &\equiv (non\,Q) \vee Q' \\ non\,(Q \Longrightarrow Q') &\equiv Q \wedge (non\,Q') \end{aligned}$$

hence regarding (11), we have

$$\begin{aligned} (S \Longrightarrow (non\,T_{Bell})) &\equiv (non\,S) \vee (non\,T_{Bell}) \\ (non\,(S \Longrightarrow (non\,T_{Bell}))) &\equiv (S \wedge T_{Bell}) \end{aligned}$$

thus the expression in (11) is logically equivalent with

$$(S \wedge T_{Bell}) \vee (\forall T \in \mathcal{T}_{set} : (S \Longrightarrow (non\,T))$$

And then assuming that (11) does not hold, we obtain

$$((non\,S) \vee (non\,T_{Bell})) \wedge (\exists T_0 \in \mathcal{T}_{set} : S \wedge T_0) \tag{12}$$

In this way, both statements $((non\,S) \vee (non\,T_{Bell}))$ and $(\exists T_0 \in \mathcal{T}_{set} : S \wedge T_0)$ must be valid. And since in view of (5), S was supposed to be valid, it follows that $non\ T_{Bell}$ must also be valid. And then, we are again back to the contradiction in (10).

Thus (11) does indeed hold.

Remark 3.2.

1. Both (10) and (11) require - as we have seen above - the *basic assumption* that the set $\mathcal{T}_{set}$ of all mathematical theorems of Set Theory is contradiction free. Yet, so far, that remains an *unproven* assumption, just as is the case with its negation.
2. In case it may turn out, however, that nevertheless, the above basic assumption is not correct, and Set Theory is in fact contradictory, then one may turn to [14–19] and the references cited there, where relevant and major latest extensions of Mathematical Logic and Mathematics are pointed out in the literature.

Dedication: Dedicated to Marie-Louise Nykamp

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