

Leader-following consensus for upper-triangular multi-agent systems via sampled and delayed output feedback

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Abstract

In this paper, the leader-following consensus problem is investigated for upper-triangular nonlinear MASs (multi-agent systems) via sampled and delayed output feedback. The sampled outputs transmitted through the communication channels are used to construct the output feedback controllers directly. Then, the output feedback control signals transmitted through the other communication channels are used to update the MASs. Two kinds of transmission delays, i.e., transmission delays from the MASs to the output feedback controllers and transmission delays from the output feedback controllers to the MASs, make the MASs and the output feedback controllers to be updated at different time instants. A method of interval decomposition is proposed such that the MASs and the output feedback controllers are updated at the same interval. Then, sufficient conditions are presented to ensure the MASs reach leader-following consensus. At last, an example is provided to verify efficiency of the proposed methods.

Index Terms

Upper-triangular, nonlinear systems, leader-following consensus, sampled-data, output feedback, interval decomposition

I. INTRODUCTION

With the development of artificial intelligence, study on MASs has received increasing attention in the field of communication and control. As a core of the distributed coordination problem for MASs, the consensus

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problem is currently a very hot research topic. Generally speaking, the consensus problem comprises leaderless consensus [1]–[12] and leader-following consensus [13]–[23]. Previous studies on consensus problems focus on MASs with simple dynamics [1], [8] or double integrators dynamics [3]–[5], [9], [16] or linear dynamics [2], [11]–[15]. Subsequently, researchers devoted to consensus problem for MASs with nonlinear dynamics [7], [14], [19]–[21], [24], which is hard to deal with because of their complex nonlinear dynamics. In practice, information are often sampled at discrete time instants and transmitted through a communication network among agents. Therefore, input delays or transmission delays are frequently encountered in a MAS. Thus it is necessary and important to study the sampled-data consensus problem of MASs with transmission delays.

More recently, some progresses have been made on sampled-data consensus of MASs [10], [19], [25]–[27]. For example, the consensus problem of directed nonlinear MAS was investigated in [25]. Based on the delayed-input approach, the authors transformed the sampled-data MAS into a nonlinear system with time-varying delay. Then, they obtained sufficient conditions to ensure that the MAS reach consensus [25]. The event-triggered sampled-data consensus problem for distributed MAS was also studied in [27]. Such a problem was converted to a stability problem of a time-varying system based on new sampled-data consensus protocols. New sufficient conditions were given such that the MAS reached consensus. The authors in [19], considered a more complex case, i.e., each agent had high-order nonlinear and heterogeneous dynamics. A consensus tracking protocol based on the asynchronous sampled data was proposed to solve the consensus problem of heterogeneous nonlinear MAS. Since many nonlinear systems can be converted to nonlinear systems with the structure of lower-triangular or upper-triangular, therefore, some developments have been made in the study of consensus problem for lower-triangular MASs [28]–[33]. In [28], the authors proposed full state and output consensus protocol to solve the leader-following consensus problem for nonlinear MASs. The leader-following consensus problem for high-order lower-triangular nonlinear MASs was studied in [32]. A recursive design approach was proposed to solve this problem.

However, there exist some shortcomings in the above literatures, for instance, transmission delays are not considered, or only transmission delays from the systems to the controller are considered. On the other hand, the upper-triangular systems can be applied to represent some physical systems, for example, the celebrated cart-pendulum system [34], [35] and the planar vertical takeoff and landing aircraft [36]. To the best of our knowledge, there are few research results on sampled-data consensus for upper-triangular nonlinear MASs with two kinds of transmission delays, transmission delays from the MASs to the output feedback controllers and transmission delays from the output feedback controllers to the MASs.

In this paper, we consider the leader-following consensus problem for upper-triangular nonlinear MASs via sampled and delayed output feedback. The main contributions include: a) Inspired by the ideas in the literatures [39]–[41], the sampled outputs and output feedback control signals transmitted through the communication channels are used to update the output feedback controllers and the MASs at different time instants, respectively. b) A method of interval decomposition is proposed such that the MASs and the output feedback controllers are updated at the same interval. c) Sufficient conditions are presented to ensure the MASs reach leader-following consensus.

This paper is organized as follows. Section II presents some definitions of graph theory, problem description and some assumptions. In Section III, we give the main results: leader-following consensus for upper-triangular MASs via sampled and delayed output feedback. A simulation example is provided to verify the theoretical results in Section IV. At last, Section V concludes this paper.

II. PRELIMINARIES

A. Graph theory

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denote a undirected graph of order N , where $\mathcal{V} = \{1, \dots, N\}$ is the set of nodes, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. The neighborhood of node i is denoted by $N_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}, j \neq i\}$. Graph \mathcal{G} is called connected if there is a path between any two nodes in \mathcal{G} , otherwise, we call that it is disconnected. $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{N \times N}$ denote the adjacency matrix of \mathcal{G} , where $a_{ij} = 1$, if $(i, j) \in E$, and $a_{ij} = 0$ otherwise. The degree matrix Δ is defined as $\Delta = \text{diag}(\Delta_1, \dots, \Delta_N)$, where $\Delta_i = \sum_{j \in N_i} a_{ij}$. The Laplacian matrix $\mathcal{L} = \Delta - \mathcal{A}$ is symmetric. Let graph $\bar{\mathcal{G}}$ contain graph \mathcal{G} and a leader 0 and edges between the leader and its neighbors. Let $\mathcal{H} = \mathcal{L} + \mathcal{D}$, where $\mathcal{D} = \text{diag}(d_1, \dots, d_N)$. If the leader and node i are neighbors, $d_i = 1$; otherwise, $d_i = 0$.

B. Problem description

Consider a upper-triangular nonlinear multi-agent system consisting of $N + 1$ agents, where the dynamic of i th agent ($i = 0, 1, \dots, N$) is expressed as

$$\begin{cases} \dot{x}^i(t) = A_0 x^i(t) + f(x^i(t)) + b u^i(t), \\ y^i(t) = C x^i(t), \quad i = 0, 1, \dots, N, \end{cases} \quad (1)$$

where

$$A_0 = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix}, b = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix},$$

$x^i(t) = (x_1^i(t), x_2^i(t), \dots, x_n^i(t))^T \in \mathcal{R}^n$, $u^i(t) \in \mathcal{R}$, and $y^i(t) \in \mathcal{R}$ are the agent i 's state, input and output, respectively, and $u^0(t) = 0$. The nonlinear function $f(x^i(t)) = (f_1(x^i(t)), f_2(x^i(t)), \dots, f_n(x^i(t)))^T$ has the form of $f_m(x^i(t)) = f_m(x_{m+2}^i(t), \dots, x_n^i(t))$, $m = 1, 2, \dots, n-2$, and $f_{n-1}^i(x^i(t)) = f_n^i(x^i(t)) = 0$.

Remark 1: The nonlinear system in the form of (1) is called the upper-triangular nonlinear system, which can be applied to represent some physical systems, for example, the celebrated cart-pendulum system [34], [35] and the planar vertical takeoff and landing aircraft [36]. The state feedback stabilization and output feedback stabilization of the upper-triangular system have been well-investigated in [37] and [38], respectively, and some interesting results have also been established.

We state the following useful results which can be found in [20], [42] for later use.

Lemma 1 ([20]): If H is positive definite and (A_0, b) is controllable, then there exists a row vector K such that the matrix $I_N \otimes A_0 - H \otimes bK$ is Hurwitz. Furthermore, there exists a positive definite matrix Q satisfying

$$(I_N \otimes Q)(I_N \otimes A_0 - H \otimes bK) + (I_N \otimes A_0 - H \otimes bK)^T(I_N \otimes Q) = -I.$$

Lemma 2 ([42]): For a given positive definite symmetric matrix $R^T = R > 0$, a real number $\gamma > 0$, and a vector function $\omega(s)$ defined on the interval $[0, \gamma]$, we have

$$\left[\int_0^\gamma \omega(s) ds \right]^T R \left[\int_0^\gamma \omega(s) ds \right] \leq \gamma \left[\int_0^\gamma \omega^T(s) R \omega(s) ds \right].$$

Figure 1 shows the sampling scheme for the MASs. Suppose the system output $y^i(t)$ is measured at sampling instants t_k , and t_k satisfies $0 = t_0 < t_1 < \dots < t_k < \dots$. The sampling period $T = t_{k+1} - t_k$. We also assume that transmission delay from the i th neighbor agents j to agent i is τ_k^{ji} , and d_k^i is the transmission delay from the controller to the ZOH (zero-order hold). Note that the sampled-data from i and its neighbor j may reach the output feedback stabilizer at different time instants. Therefore, we employ a common buffer to deposit the information until all the information is available. Let $\tau_k^i = \max\{\tau_k^{ji}\}$, $\tau_k = \max\{\tau_k^i\}$, and $d_k = \max\{d_k^i\}$.

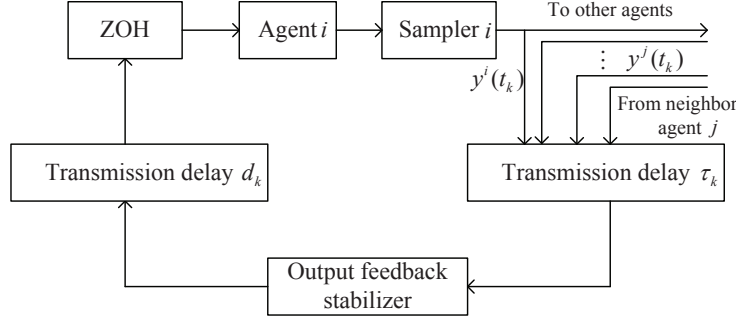


Figure 1. Sampling scheme for the MASs

The aim of this paper is to design output feedback controllers, such that the i th follower's state $x^i(t)$ asymptotically approaches the leader's state $x^0(t)$, as $t \rightarrow +\infty$. Based on the sampling scheme shown in Figure 1, we design the following output feedback controllers

$$\begin{cases} \dot{\hat{x}}^i(t) = A_0 \hat{x}^i(t) + Ma(x_1^i(t_k) - \hat{x}_1^i(t_k)) + f(\hat{x}^i(t)) + bu^i(t), \\ \dot{\hat{x}}^0(t) = A_0 \hat{x}^0(t) + Ma(x_1^0(t_k) - \hat{x}_1^0(t_k)) + f(\hat{x}^0(t)), \\ u^i(t) = -KW \left[\sum_{j \in N_i} a_{ij} (\hat{x}^i(t_k) - \hat{x}^j(t_k)) + d_i (\hat{x}^i(t_k) - \hat{x}^0(t_k)) \right], \\ \hat{x}^i(t_{k+1} + \tau_k) = \lim_{t \rightarrow (t_{k+1} + \tau_{k+1})^-} \hat{x}^i(t), t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}), k \geq 0, i = 1, 2, \dots, N, \end{cases} \quad (2)$$

where $\hat{x}^i(t) = (\hat{x}_1^i(t), \hat{x}_2^i(t), \dots, \hat{x}_n^i(t))^T$, $M = \text{diag}\{\frac{1}{L}, \frac{1}{L^2}, \dots, \frac{1}{L^n}\}$, $W = \text{diag}\{\frac{1}{L^n}, \frac{1}{L^{n-1}}, \dots, \frac{1}{L}\}$ with $L \geq 1$, $K = (k_1, k_2, \dots, k_n)$ is given in Lemma 1, $a = (a_1, a_2, \dots, a_n)^T$, where $a_m > 0$, $m = 1, 2, \dots, n$, are some design parameters.

From (1) and (2), we can obtain

$$\begin{cases} \dot{x}^i(t) = A_0 x^i(t) + f(x^i(t)) - bKW \left[\sum_{j \in N_i} a_{ij} (\hat{x}^i(t_k) - \hat{x}^j(t_k)) + d_i (\hat{x}^i(t_k) - \hat{x}^0(t_k)) \right], \\ t \in [t_k + \tau_k + d_k, t_{k+1} + \tau_{k+1} + d_{k+1}). \end{cases} \quad (3)$$

Note that the system (2) and the system (3) are updated at the time instants $t_k + \tau_k$ and $t_k + \tau_k + d_k$, respectively. In order to analysis the stability of the MASs, we assume that the time delays d_k and τ_k satisfy $d_k < \bar{m}_1 T$ and $\tau_k < \bar{m}_2 T$ for all $k > 0$, where \bar{m}_1 and \bar{m}_2 are positive integers. Then, $t_k + \tau_k + d_k < t_k + (\bar{m}_1 + \bar{m}_2)T$ and $t_{k+1} + \tau_{k+1} + d_{k+1} < t_k + (\bar{m}_1 + \bar{m}_2 + 1)T$. Therefore, there exist positive integers $m_{k_1} < \bar{m}_1 + \bar{m}_2$ and $m_{k_2} < \bar{m}_1 + \bar{m}_2 + 1$ such that

$$t_k + \tau_k + d_k \in [t_{k+m_{k_1}} + \tau_{k+m_{k_1}}, t_{k+m_{k_1}+1} + \tau_{k+m_{k_1}+1}),$$

and

$$t_{k+1} + \tau_{k+1} + d_{k+1} \in [t_{k+m_{k_2}} + \tau_{k+m_{k_2}}, t_{k+m_{k_2}+1} + \tau_{k+m_{k_2}+1}).$$

Without loss of generality, let $m_{k_1} < m_{k_2}$. Then

$$\begin{aligned} [t_k + \tau_k + d_k, t_{k+1} + \tau_{k+1} + d_{k+1}) &= [t_k + \tau_k + d_k, t_{k+m_{k_1}+1} + \tau_{k+m_{k_1}+1}) \\ \bigcup_{s=m_{k_1}+1}^{m_{k_2}-1} [t_{k+s} + \tau_{k+s}, t_{k+s+1} + \tau_{k+s+1}) &\bigcup [t_{k+m_{k_2}} + \tau_{k+m_{k_2}}, t_{k+1} + \tau_{k+1} + d_{k+1}). \end{aligned} \quad (4)$$

Now, we introduce the following notation \tilde{k} which is given by

$$\tilde{k} = \begin{cases} k + m_{k_1}, t \in [t_k + \tau_k + d_k, t_{k+m_{k_1}+1} + \tau_{k+m_{k_1}+1}), \\ k + s, t \in [t_{k+s} + \tau_{k+s}, t_{k+s+1} + \tau_{k+s+1}), s = m_{k_1} + 1, \dots, m_{k_2} - 1, \\ k + m_{k_2}, t \in [t_{k+m_{k_2}} + \tau_{k+m_{k_2}}, t_{k+1} + \tau_{k+1} + d_{k+1}). \end{cases} \quad (5)$$

With the interval decomposition (4) and the notation \tilde{k} introduced by (5), we can consider the system (2) and (3) on the same interval. When $t \in [t_k + \tau_k + d_k, t_{k+m_{k_1}+1} + \tau_{k+m_{k_1}+1})$, or $t \in [t_{k+s} + \tau_{k+s}, t_{k+s+1} + \tau_{k+s+1}), s = m_{k_1} + 1, \dots, m_{k_2} - 1$, or $t \in [t_{k+m_{k_2}} + \tau_{k+m_{k_2}}, t_{k+1} + \tau_{k+1} + d_{k+1})$, the output feedback control laws can be given as

$$\begin{cases} \hat{x}^i(t) = A_0 \hat{x}^i(t) + Ma(x_1^i(t_{\tilde{k}}) - \hat{x}_1^i(t_{\tilde{k}})) + f(\hat{x}^i(t)) + b\bar{u}^i(t), \\ \hat{x}^0(t) = A_0 \hat{x}^0(t) + Ma(x_1^0(t_{\tilde{k}}) - \hat{x}_1^0(t_{\tilde{k}})) + f(\hat{x}^0(t)), \\ \bar{u}^i(t) = -KW[\sum_{j \in N_i} a_{ij}(\hat{x}^i(t_{\tilde{k}}) - \hat{x}^j(t_{\tilde{k}})) + d_i(\hat{x}^i(t_{\tilde{k}}) - \hat{x}^0(t_{\tilde{k}}))], \\ \hat{x}(t_{\tilde{k}+1} + \tau_{\tilde{k}+1}) = \lim_{t \rightarrow (t_{\tilde{k}+1} + \tau_{\tilde{k}+1})^-} \hat{x}(t). \end{cases} \quad (6)$$

Let $\eta(t) = t - t_k$, $\gamma(t) = t - t_{\tilde{k}}$. Then t_k and $t_{\tilde{k}}$ can be converted into

$$t_k = t - \eta(t), \quad t_{\tilde{k}} = t - \gamma(t). \quad (7)$$

When $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$, we have $\eta(t) \leq t_{k+1} + \tau_{k+1} - t_k \leq (\bar{m}_2 + 1)T \leq (\bar{m}_1 + \bar{m}_2 + 1)T$. We can also obtain $\gamma(t) < t_{k+1} + \tau_{k+1} + d_{k+1} - t_k \leq (\bar{m}_1 + \bar{m}_2 + 1)T$, for $t \in [t_k + \tau_k + d_k, t_{k+1} + \tau_{k+1} + d_{k+1})$.

With the help of (7), the system (3) and the system (6) can be rewritten as

$$\begin{aligned} \dot{x}^i(t) &= A_0 x^i(t) + f(x^i(t)) - bKW[\sum_{j \in N_i} a_{ij}(\hat{x}^i(t - \eta(t)) - \hat{x}^j(t - \eta(t))) \\ &\quad + d_i(\hat{x}^i(t - \eta(t)) - \hat{x}^0(t - \eta(t)))], \end{aligned} \quad (8)$$

$$\begin{cases} \hat{x}^i(t) = A_0 \hat{x}^i(t) + Ma(x_1^i(t - \gamma(t)) - \hat{x}_1^i(t - \gamma(t))) + f(\hat{x}^i(t)) + b\bar{u}^i(t), \\ \hat{x}^0(t) = A_0 \hat{x}^0(t) + Ma(x_1^0(t - \gamma(t)) - \hat{x}_1^0(t - \gamma(t))) + f(\hat{x}^0(t)), \\ \bar{u}^i(t) = -KW[\sum_{j \in N_i} a_{ij}(\hat{x}^i(t - \gamma(t)) - \hat{x}^j(t - \gamma(t))) + d_i(\hat{x}^i(t - \gamma(t)) - \hat{x}^0(t - \gamma(t)))]. \end{cases} \quad (9)$$

Now, we give the following definition.

Definition 1 ([20]): The leader-following consensus problem of the nonlinear multi-agent system (1) can be solved by the sampled and delayed output feedback controllers (2), if under the controllers (2), for any initial condition $x^i(0)$ ($i = 1, 2, \dots, N$), the state $x^i(t)$ of the follower agent i asymptotically approaches the state $x^0(t)$ of the leader, as $t \rightarrow +\infty$, that is,

$$\lim_{t \rightarrow +\infty} \|x^i(t) - x^0(t)\| = 0, \quad i = 1, 2, \dots, N.$$

Remark 2: In [20], the authors proposed continuous state and output feedback controllers for a class of upper-triangular nonlinear MASs. In [25], the authors designed sampled-data state feedback controllers without considering the effect of transmission delays. In this paper, we propose sampled-data output feedback controllers by using sampled and delayed outputs for the upper-triangular nonlinear MAS. The agent systems are continuous while the controllers are discrete, which stands for a typical computer-based control scheme.

Remark 3: Notice that $\hat{x}^i(t)$ are continuous on the interval $[t_k + \tau_k, t_{k+1} + \tau_k)$, and the functions $f^i(\cdot)$ which satisfy Assumption 2 are also continuous. It is obvious that $\hat{x}^i(t)$ are continuous on the interval $[t_0, +\infty)$. For the same reason, $x^i(t)$ are continuous on the interval $[t_0, +\infty)$ as well.

We also make the following assumptions.

Assumption 1: Graph \mathcal{G} is connected.

Assumption 2: Assume there exists $l > 0$ such that the nonlinear functions f_m satisfy Lipschitz condition

$$\|f_m(y) - f_m(z)\| \leq l\|y - z\|, \quad \forall y, z \in \mathcal{R}^n.$$

III. MAIN RESULTS

In what follows, based on the sampled-data output feedback controllers (2), we try to construct a Lyapunov-Krasovskii functional and present sufficient conditions to ensure the upper-triangular multi-agent systems with delayed and sampled outputs can reach consensus. The error dynamics are given by:

$$\dot{e}^i(t) = A_0 e^i(t) - M a e_1^i(t - \gamma(t)) + f^i + b u^i(t) - b \bar{u}^i(t), \quad i = 0, 1, \dots, N, \quad (10)$$

$$\dot{\xi}^i(t) = A_0 \xi^i(t) + M a (e_1^i(t - \gamma(t)) - e_1^0(t - \gamma(t))) + \hat{f}^i + b \bar{u}^i(t), \quad i = 1, 2, \dots, N, \quad (11)$$

where $e^i(t) = x^i(t) - \hat{x}^i(t)$, $f(x^i(t)) - f(\hat{x}^i(t)) = f^i = (f_1^i, f_2^i, \dots, f_n^i)^T$ with $i = 0, 1, \dots, N$, and $\xi^i(t) = \hat{x}^i(t) - \hat{x}^0(t)$, $f(\hat{x}^i(t)) - f(\hat{x}^0(t)) = \hat{f}^i = (\hat{f}_1^i, \hat{f}_2^i, \dots, \hat{f}_n^i)^T$ with $i = 1, 2, \dots, N$.

Consider the following variable transforms

$$\varepsilon^i(t) = (\varepsilon_1^i(t), \varepsilon_2^i(t), \dots, \varepsilon_n^i(t))^T, \quad \varepsilon_m^i(t) = L^m e_m^i(t), \quad i = 0, 1, \dots, N, \quad m = 1, 2, \dots, n,$$

$$z^i(t) = (z_1^i(t), z_2^i(t), \dots, z_n^i(t))^T, \quad z_m^i(t) = L^m \xi_m^i(t), \quad i = 1, 2, \dots, N, \quad m = 1, 2, \dots, n.$$

Hence,

$$\dot{\varepsilon}^i(t) = \frac{1}{L} A_0 \varepsilon^i(t) - \frac{1}{L} a \varepsilon_1^i(t) + \frac{1}{L} a \int_{t-\gamma(t)}^t \dot{\varepsilon}_1^i(s) ds + \tilde{f}^i + L^n b u^i(t) - L^n b \bar{u}^i(t), \quad i = 0, 1, \dots, N, \quad (12)$$

$$\dot{z}^i(t) = \frac{1}{L} A_0 z^i(t) + \frac{1}{L} a (\varepsilon_1^i(t - \gamma(t)) - \varepsilon_1^0(t - \gamma(t))) + L^n b \bar{u}^i(t) + \bar{f}^i, \quad i = 1, 2, \dots, N, \quad (13)$$

where $\varepsilon_1^i(t - \gamma(t)) = L e_1^i(t - \gamma(t))$, $\tilde{f}^i = (\tilde{f}_1^i, \tilde{f}_2^i, \dots, \tilde{f}_n^i)^T$, $\tilde{f}_m^i = L^m f_m^i$, and $\bar{f}^i = (\bar{f}_1^i, \bar{f}_2^i, \dots, \bar{f}_n^i)^T$, $\bar{f}_m^i = L^m \bar{f}_m^i$.

Let $\varepsilon(t) = ((\varepsilon^1(t))^T, (\varepsilon^2(t))^T, \dots, (\varepsilon^N(t))^T)^T$, $\varepsilon_1(t) = (\varepsilon_1^1(t), \varepsilon_1^2(t), \dots, \varepsilon_1^N(t))^T$, $\varepsilon_1(t - \gamma(t)) = (\varepsilon_1^1(t - \gamma(t)), \varepsilon_1^2(t - \gamma(t)), \dots, \varepsilon_1^N(t - \gamma(t)))^T$, and $z(t) = ((z^1(t))^T, (z^2(t))^T, \dots, (z^N(t))^T)^T$.

Then, the systems (12) and (13) can be expressed in matrix form

$$\dot{\varepsilon}(t) = \frac{1}{L} (I_N \otimes A_0) \varepsilon(t) - \frac{1}{L} \varepsilon_1(t) \otimes a + \frac{1}{L} \int_{t-\gamma(t)}^t \dot{\varepsilon}_1(s) ds \otimes a + \tilde{f} + L^n u \otimes b - L^n \bar{u} \otimes b,$$

$$\dot{z}(t) = \frac{1}{L} (I_N \otimes A_0) z(t) + \frac{1}{L} (\varepsilon_1(t - \gamma(t)) - 1_N \otimes \varepsilon_1^0(t - \gamma(t))) \otimes a + L^n \bar{u} \otimes b + \bar{f},$$

where $\tilde{f} = ((\tilde{f}^0})^T, (\tilde{f}^1})^T, \dots, (\tilde{f}^N})^T)^T$, $\bar{f} = ((\bar{f}^1})^T, (\bar{f}^2})^T, \dots, (\bar{f}^N})^T)^T$, $u = (u^1(t), u^2(t), \dots, u^N(t))^T$, $\bar{u} = (\bar{u}^1(t), \bar{u}^2(t), \dots, \bar{u}^N(t))^T$.

Note that

$$L^n u \otimes b = -\frac{1}{L} (H \otimes bK) z(t_k), \quad L^n \bar{u} \otimes b = -\frac{1}{L} (H \otimes bK) z(t_{\bar{k}}).$$

Thus

$$\dot{\varepsilon}(t) = \frac{1}{L} (I_N \otimes A_0) \varepsilon(t) - \frac{1}{L} \varepsilon_1(t) \otimes a + \frac{1}{L} \int_{t-\gamma(t)}^t \dot{\varepsilon}_1(s) ds \otimes a + \tilde{f} + \frac{1}{L} (H \otimes bK) \int_{t-\gamma(t)}^t \dot{z}(s) ds - \frac{1}{L} (H \otimes bK) \int_{t-\gamma(t)}^t \dot{z}(s) ds, \quad (14)$$

$$\dot{z}(t) = \frac{1}{L} (I_N \otimes A_0 - H \otimes bK) z(t) + \frac{1}{L} (\varepsilon_1(t - \gamma(t)) - 1_N \otimes \varepsilon_1^0(t - \gamma(t))) \otimes a + \frac{1}{L} (H \otimes bK) \int_{t-\gamma(t)}^t \dot{z}(s) ds + \bar{f}. \quad (15)$$

Now, we present the following main results.

Theorem 1: Under the Assumptions 1 and 2, there exist sampled-data output feedback controllers in the form of (2) to make the multi-agent systems (1) to reach consensus, if there exists a matrix $P = P^T > 0$ such that

$$A_1^T P + P A_1 \leq -I,$$

and the sampling period T and the high gain L satisfy

$$T < \min \left\{ \frac{\lambda_q^2 L - 8n\sqrt{N}\lambda_p\lambda_q^2 l}{24\bar{m}\lambda_q^2(1+n\bar{a}_1+n^2N)+\bar{m}}, \frac{L-4n\sqrt{N}\lambda_q l}{8\bar{m}\lambda_{ah}^2+8\bar{m}n^2N}, \frac{2\lambda_q^2 L - 8n\lambda_p\lambda_q^2 l}{16\bar{m}\lambda_q^2(1+n\bar{a}_1+n^2)+\bar{m}}, \frac{-\gamma_2 + \sqrt{\gamma_2^2 + 4\gamma_1 L^2}}{2\bar{m}\gamma_1}, \frac{-\gamma_4 + \sqrt{\gamma_4^2 + 64\lambda_h^2 L^2}}{2\bar{m}\lambda_h^2}, \frac{-\gamma_2 + \sqrt{\gamma_2^2 + 4\gamma_3 L^2}}{2\bar{m}\gamma_1} \right\}, \quad (16)$$

$$L > \max \{1, l, 8n\sqrt{N}\lambda_p l, 4n\sqrt{N}\lambda_q l, 4n\lambda_p l\}, \quad (17)$$

where $\bar{a}_1 = \max\{a_i^2\}$, $\bar{m} = \bar{m}_1 + \bar{m}_2 + 1$, $\gamma_1 = 6n\bar{a}_1 + \frac{1}{4\lambda_q^2}$, $\gamma_2 = 4n\bar{a}_1\lambda_p^2 L + \frac{L}{4}$, $\gamma_3 = 4n\bar{a}_1 + \frac{1}{4\lambda_q^2}$, $\gamma_4 = 20\lambda_q^2\lambda_h^2 L$, and $\lambda_p = \lambda_{\max}(P)$, $\lambda_q = \lambda_{\max}(Q)$, $\lambda_h = \lambda_{\max}(H \otimes bK)$, $\lambda_{ah} = \lambda_{\max}(I_N \otimes A_0 - H \otimes bK)$,

$$r = 64n\bar{a}_1\lambda_q^2, r_0 = 64nN\bar{a}_1\lambda_q^2, A_1 = \begin{pmatrix} -a_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & \cdots & 1 \\ -a_n & 0 & \cdots & 0 \end{pmatrix}.$$

Proof: Consider the following Lyapunov-Krasovskii functional

$$V(t) = r(V_1(t) + V_4(t)) + V_2(t) + V_5(t) + r_0(V_3(t) + V_6(t)). \quad (18)$$

where $V_1(t) = \varepsilon(t)^T(I_N \otimes P)\varepsilon(t)$, $V_2(t) = z(t)^T(I_N \otimes Q)z(t)$, $V_3(t) = \varepsilon^0(t)^T P \varepsilon^0(t)$, $V_4(t) = \int_{t-\bar{m}T}^t \int_{\beta}^t \dot{\varepsilon}(s)^T \dot{\varepsilon}(s) ds d\beta$, $V_5(t) = \int_{t-\bar{m}T}^t \int_{\beta}^t \dot{z}(s)^T \dot{z}(s) ds d\beta$, $V_6(t) = \int_{t-\bar{m}T}^t \int_{\beta}^t \dot{\varepsilon}^0(s)^T \dot{\varepsilon}^0(s) ds d\beta$.

The derivatives of $V_1(t)$, $V_2(t)$ and $V_3(t)$ along the system (12) and the system (15) are given as follows

$$\begin{aligned} \dot{V}_1(t) &= \frac{1}{L}\varepsilon(t)^T [I_N \otimes (A_1^T P + P A_1)] \varepsilon(t) + \frac{2}{L}\varepsilon(t)^T (I_N \otimes P) \left(\int_{t-\gamma(t)}^t \dot{\varepsilon}_1(s) ds \otimes a \right) + 2\varepsilon(t)^T (I_N \otimes P) \tilde{f} \\ &\quad + \frac{2}{L}\varepsilon(t)^T (I_N \otimes P) (H \otimes bK) \int_{t-\eta(t)}^t \dot{z}(s) ds - \frac{2}{L}\varepsilon(t)^T (I_N \otimes P) (H \otimes bK) \int_{t-\gamma(t)}^t \dot{z}(s) ds \\ &\leq -\frac{3}{4L}\varepsilon(t)^T \varepsilon(t) + \frac{2}{L}\varepsilon(t)^T (I_N \otimes P) \left(\int_{t-\gamma(t)}^t \dot{\varepsilon}_1(s) ds \otimes a \right) + 2\varepsilon(t)^T (I_N \otimes P) \tilde{f} \\ &\quad + \frac{8}{L}\lambda_q^2\lambda_h^2 \left(\int_{t-\eta(t)}^t \dot{z}(s)^T ds \int_{t-\eta(t)}^t \dot{z}(s) ds + \int_{t-\gamma(t)}^t \dot{z}(s)^T ds \int_{t-\gamma(t)}^t \dot{z}(s) ds \right), \end{aligned} \quad (19)$$

$$\begin{aligned} \dot{V}_2(t) &= \frac{1}{L}z(t)^T [(I_N \otimes Q)(H \otimes bK + I_N \otimes A_0) + (H \otimes bK + I_N \otimes A_0)^T (I_N \otimes Q)] z(t) \\ &\quad + \frac{2}{L}z(t)^T (I_N \otimes Q) (H \otimes bK) \int_{t-\gamma(t)}^t \dot{z}(s) ds + \frac{2}{L}z(t)^T (I_N \otimes Q) [(\varepsilon_1(t - \gamma(t)) - 1_N \otimes \varepsilon_1^0(t - \gamma(t))) \\ &\quad \otimes a] + 2z(t)^T (I_N \otimes Q) \tilde{f} \\ &\leq -\frac{3}{4L}z(t)^T z(t) + \frac{4}{L}\lambda_q^2\lambda_h^2 \int_{t-\gamma(t)}^t \dot{z}(s)^T ds \int_{t-\gamma(t)}^t \dot{z}(s) ds + \frac{2}{L}z(t)^T (I_N \otimes Q) [(\varepsilon_1(t - \gamma(t)) \\ &\quad - 1_N \otimes \varepsilon_1^0(t - \gamma(t))) \otimes a] + 2z(t)^T (I_N \otimes Q) \tilde{f}, \end{aligned} \quad (20)$$

$$\begin{aligned} \dot{V}_3(t) &= \frac{1}{L}\varepsilon^0(t)^T (A_1^T P + P A_1) \varepsilon^0(t) + \frac{2}{L}\varepsilon^0(t)^T P a \int_{t-\gamma(t)}^t \dot{\varepsilon}_1^0(s) ds + 2\varepsilon^0(t)^T P \tilde{f}^0 \\ &\leq -\frac{3}{4L}\varepsilon^0(t)^T \varepsilon^0(t) + \frac{4}{L}\bar{m}n\bar{a}_1\lambda_p^2 T \int_{t-\gamma(t)}^t \dot{\varepsilon}^0(s)^T \dot{\varepsilon}^0(s) ds + 2\varepsilon^0(t)^T P \tilde{f}^0. \end{aligned} \quad (21)$$

Note that

$$(\tilde{f})^T \tilde{f} = \sum_{i=1}^N \sum_{m=1}^n (\tilde{f}_m^i)^2 = \sum_{i=1}^N \sum_{m=1}^n L^{2m} (f_m^i)^2 \leq \sum_{i=1}^N \sum_{m=1}^n \frac{nl^2}{L^4} \varepsilon^i(t)^T \varepsilon^i(t) \leq \frac{n^2 N l^2}{L^4} \varepsilon(t)^T \varepsilon(t), \quad (22)$$

$$\tilde{f}^T \tilde{f} = \sum_{i=1}^N \sum_{m=1}^n (\tilde{f}_m^i)^2 = \sum_{i=1}^N \sum_{m=1}^n L^{2m} (\hat{f}_m^i)^2 \leq \sum_{i=1}^N \sum_{m=1}^n \frac{nl^2}{L^4} z^i(t)^T z^i(t) \leq \frac{n^2 N l^2}{L^4} z(t)^T z(t), \quad (23)$$

$$\frac{2}{L} \varepsilon(t)^T (I_N \otimes P) \left(\int_{t-\gamma(t)}^t \dot{\varepsilon}_1(s) ds \otimes a \right) \leq \frac{1}{4L} \varepsilon(t)^T \varepsilon(t) + \frac{4}{L} n \bar{a}_1 \lambda_p^2 \int_{t-\gamma(t)}^t \dot{\varepsilon}_1(s)^T ds \int_{t-\gamma(t)}^t \dot{\varepsilon}_1(s) ds, \quad (24)$$

$$2\varepsilon^T(t) (I_N \otimes P) \tilde{f} \leq 2[\varepsilon(t)^T (I_{N+1} \otimes P) \varepsilon(t)]^{\frac{1}{2}} [\tilde{f}^T (I_{N+1} \otimes P) \tilde{f}]^{\frac{1}{2}} \leq \frac{2n\sqrt{N}\lambda_p l}{L^2} \varepsilon(t)^T \varepsilon(t), \quad (25)$$

$$2z(t)^T (I_N \otimes Q) \bar{f} \leq 2\lambda_q ((\bar{f})^T \bar{f})^{\frac{1}{2}} (z(t)^T z(t))^{\frac{1}{2}} \leq \frac{2n\sqrt{N}\lambda_q l}{L^2} z(t)^T z(t), \quad (26)$$

$$2\varepsilon^0(t)^T P \tilde{f}^0 \leq 2\lambda_p ((\tilde{f}^0)^T \tilde{f}^0)^{\frac{1}{2}} (\varepsilon^0(t)^T \varepsilon^0(t))^{\frac{1}{2}} \leq \frac{2n\lambda_p l}{L^2} \varepsilon^0(t)^T \varepsilon^0(t), \quad (27)$$

$$\begin{aligned} & \frac{2}{L} z^T(t) (I_N \otimes Q) [(\varepsilon_1(t - \gamma(t)) - 1_N \otimes \varepsilon_1^0(t - \gamma(t))) \otimes a] \\ & \leq \frac{1}{4L} z^T(t) z(t) + \frac{4}{L} n \bar{a}_1 \lambda_q^2 (\varepsilon_1(t - \gamma(t)) - 1_N \otimes \varepsilon_1^0(t - \gamma(t)))^2. \end{aligned} \quad (28)$$

From (19)-(28), we have

$$\begin{aligned} \dot{V}_1(t) & \leq -\frac{1}{2L} \varepsilon(t)^T \varepsilon(t) + \frac{2n\sqrt{N}\lambda_p l}{L^2} \varepsilon(t)^T \varepsilon(t) + \frac{4}{L} \bar{m} n \bar{a}_1 \lambda_p^2 T \int_{t-\gamma(t)}^t \dot{\varepsilon}(s)^T \dot{\varepsilon}(s) ds \\ & \quad + \frac{8}{L} \bar{m} \lambda_q^2 \lambda_h^2 T \left(\int_{t_k}^t \dot{z}(s)^T \dot{z}(s) ds + \int_{t-\gamma(t)}^t \dot{z}(s)^T \dot{z}(s) ds \right), \end{aligned} \quad (29)$$

$$\begin{aligned} \dot{V}_2(t) & \leq -\frac{1}{2L} z(t)^T z(t) + \frac{2}{L^2} n \sqrt{N} \lambda_q l z(t)^T z(t) + \frac{4}{L} \bar{m} \lambda_q^2 \lambda_h^2 T \int_{t-\gamma(t)}^t \dot{z}(s)^T \dot{z}(s) ds \\ & \quad + \frac{4}{L} n \bar{a}_1 \lambda_q^2 (\varepsilon_1(t - \gamma(t)) - 1_N \otimes \varepsilon_1^0(t - \gamma(t)))^2, \end{aligned} \quad (30)$$

$$\dot{V}_3(t) \leq -\frac{3}{4L} \varepsilon^0(t)^T \varepsilon^0(t) + \frac{2n\lambda_p l}{L^2} \varepsilon^0(t)^T \varepsilon^0(t) + \frac{4}{L} \bar{m} n \bar{a}_1 \lambda_p^2 T \int_{t-\gamma(t)}^t \dot{\varepsilon}^0(s)^T \dot{\varepsilon}^0(s) ds. \quad (31)$$

The derivatives of $V_4(t)$, $V_5(t)$ and $V_6(t)$ are given by

$$\begin{aligned} \dot{V}_4(t) & = \bar{m} T \dot{\varepsilon}(t)^T \dot{\varepsilon}(t) - \int_{t-\bar{m}T}^t \dot{\varepsilon}(s)^T \dot{\varepsilon}(s) ds \\ & \leq \frac{6}{L^2} \bar{m} T \{ [(I_N \otimes A_0) \varepsilon(t)]^2 + (\varepsilon_1(t) \otimes a)^2 + (\int_{t-\gamma(t)}^t \dot{\varepsilon}_1(s) ds \otimes a)^2 + L^2 \tilde{f}^T \tilde{f} \\ & \quad + [(H \otimes bK) \int_{t_k}^t \dot{z}(s) ds]^2 + [(H \otimes bK) \int_{t-\gamma(t)}^t \dot{z}(s) ds]^2 \} - \int_{t-\bar{m}T}^t \dot{\varepsilon}(s)^T \dot{\varepsilon}(s) ds \\ & \leq \frac{6}{L^2} \bar{m} T \{ \varepsilon(t)^T \varepsilon(t) + n \bar{a}_1 \varepsilon_1(t)^T \varepsilon_1(t) + \bar{m} n \bar{a}_1 T \int_{t-\gamma(t)}^t \dot{\varepsilon}_1(s)^T \dot{\varepsilon}_1(s) ds + L^2 \tilde{f}^T \tilde{f} \\ & \quad + \bar{m} \lambda_h^2 T (\int_{t_k}^t \dot{z}(s)^T \dot{z}(s) ds + \int_{t-\gamma(t)}^t \dot{z}(s)^T \dot{z}(s) ds) \} - \int_{t-\bar{m}T}^t \dot{\varepsilon}(s)^T \dot{\varepsilon}(s) ds, \end{aligned} \quad (32)$$

$$\begin{aligned} \dot{V}_5(t) & = \bar{m} T \dot{z}(t)^T \dot{z}(t) - \int_{t-\bar{m}T}^t \dot{z}(s)^T \dot{z}(s) ds \\ & \leq \frac{4}{L^2} \bar{m} T \{ [(I_N \otimes A_0 - H \otimes bK) z(t)]^2 + [(H \otimes bK) \int_{t-\gamma(t)}^t \dot{z}(s) ds]^2 + L^2 \tilde{f}^T \tilde{f} \\ & \quad + [(\varepsilon_1(t - \gamma(t)) - 1_N \otimes \varepsilon_1^0(t - \gamma(t))) \otimes a]^2 \} - \int_{t-\bar{m}T}^t \dot{z}(s)^T \dot{z}(s) ds \\ & \leq \frac{4}{L^2} \bar{m} T \{ \lambda_{ah}^2 z(t)^T z(t) + \bar{m} \lambda_h^2 T \int_{t-\gamma(t)}^t \dot{z}(s)^T \dot{z}(s) ds + L^2 \tilde{f}^T \tilde{f} \\ & \quad + n \bar{a}_1 (\varepsilon_1(t - \gamma(t)) - 1_N \otimes \varepsilon_1^0(t - \gamma(t)))^2 \} - \int_{t-\bar{m}T}^t \dot{z}(s)^T \dot{z}(s) ds, \end{aligned} \quad (33)$$

$$\begin{aligned}\dot{V}_6(t) &= \bar{m}T\dot{\varepsilon}^0(t)^T\varepsilon^0(t) - \int_{t-\bar{m}T}^t \dot{\varepsilon}^0(s)^T\varepsilon^0(s)ds \\ &\leq \frac{4}{L^2}\bar{m}T(1+n\bar{a}_1+n^2)\varepsilon^0(t)^T\varepsilon^0(t) + \left(\frac{4}{L^2}\bar{m}^2n\bar{a}_1T^2-1\right)\int_{t-\gamma(t)}^t \dot{\varepsilon}^0(s)^T\varepsilon^0(s)ds.\end{aligned}\quad (34)$$

From (22), (23) and the condition $L > l$, we have

$$\tilde{f}^T\tilde{f} \leq \frac{n^2N}{L^2}\varepsilon(t)^T\varepsilon(t), \quad \bar{f}^T\bar{f} \leq \frac{n^2N}{L^2}z(t)^Tz(t),$$

and

$$\begin{aligned}(\varepsilon_1(t-\gamma(t)) - 1_N \otimes \varepsilon_1^0(t-\gamma(t)))^2 &= \sum_{i=1}^N (\varepsilon_1^i(t-\gamma(t)) - \varepsilon_1^0(t-\gamma(t)))^2 \\ &= \sum_{i=1}^N [\varepsilon_1^i(t) - \varepsilon_1^0(t) - \int_{t-\gamma(t)}^t \dot{\varepsilon}_1^i(s)ds + \int_{t-\gamma(t)}^t \dot{\varepsilon}_1^0(s)ds]^2 \\ &\leq 4\varepsilon(t)^T\varepsilon(t) + 4\bar{m}T \int_{t-\gamma(t)}^t \dot{\varepsilon}(s)^T\dot{\varepsilon}(s)ds + 4\bar{m}NT \int_{t-\gamma(t)}^t \dot{\varepsilon}^0(s)^T\dot{\varepsilon}^0(s)ds + 4N\varepsilon^0(t)^T\varepsilon^0(t).\end{aligned}$$

From (29)-(34), one can obtain that

$$\begin{aligned}\dot{V}(t) &\leq -\frac{r}{L^2}\left[\frac{L}{2} - 2n\sqrt{N}\lambda_p l - 6\bar{m}T(1+n\bar{a}_1+n^2N) - \frac{16}{r}\bar{m}n\bar{a}_1T - \frac{16}{r}n\bar{a}_1\lambda_q^2L\right]\varepsilon^i(t)^T\varepsilon^i(t) \\ &\quad -r\left[1 - \frac{4}{L}\bar{m}n\bar{a}_1\lambda_p^2T - \frac{6}{L^2}n\bar{a}_1\bar{m}^2T^2 - \frac{16}{rL}n\bar{a}_1\lambda_q^2\bar{m}T - \frac{16}{rL^2}n\bar{a}_1\bar{m}^2T^2\right]\int_{t-\bar{m}T}^t \dot{\varepsilon}^i(s)^T\dot{\varepsilon}^i(s)ds \\ &\quad -\frac{r_0}{L^2}\left[\frac{3L}{4} - 2n\lambda_p l - 4\bar{m}T(1+n\bar{a}_1+n^2) - \frac{16N}{r_0}n\bar{a}_1\bar{m}T - \frac{16N}{r_0}n\bar{a}_1\lambda_q^2L\right]\varepsilon^0(t)^T\varepsilon^0(t) \\ &\quad -r_0\left[1 - \frac{4}{L}n\bar{a}_1\lambda_p^2\bar{m}T - \frac{4}{L^2}n\bar{a}_1\bar{m}^2T^2 - \frac{16N}{r_0L}n\bar{a}_1\lambda_q^2\bar{m}T - \frac{16N}{r_0L^2}n\bar{a}_1\bar{m}^2T^2\right]\int_{t-\bar{m}T}^t \dot{\varepsilon}^0(s)^T\dot{\varepsilon}^0(s)ds \\ &\quad -\frac{1}{L^2}\left(\frac{L}{2} - 2n\sqrt{N}\lambda_q l - 4\lambda_{ah}^2\bar{m}T - 4n^2N\bar{m}T\right)z(t)^Tz(t) \\ &\quad -\left(1 - \frac{20}{L}\lambda_q^2\lambda_h^2\bar{m}T - \frac{16}{L^2}\lambda_h^2\bar{m}^2T^2\right)\int_{t-\bar{m}T}^t \dot{z}(s)^T\dot{z}(s)ds \\ &\leq -\frac{r}{L^2\lambda_p}\left[\frac{L}{4} - 2n\sqrt{N}\lambda_p l - 6\bar{m}T(1+n\bar{a}_1+n^2N) - \frac{\bar{m}T}{4\lambda_q^2}\right]V_1(t) - \frac{1}{L^2\lambda_q}\left(\frac{L}{2} - 2n\sqrt{N}\lambda_q l\right. \\ &\quad \left.- 4\lambda_{ah}^2\bar{m}T - 4n^2N\bar{m}T\right)V_2(t) - \frac{r_0}{L^2\lambda_p}\left[\frac{L}{2} - 2n\lambda_p l - 4\bar{m}T(1+n\bar{a}_1+n^2) - \frac{\bar{m}T}{4\lambda_q^2}\right]V_3(t) \\ &\quad -\frac{r}{\bar{m}T}\left[1 - \frac{4}{L}n\bar{a}_1\lambda_p^2\bar{m}T - \frac{6}{L^2}n\bar{a}_1\bar{m}^2T^2 - \frac{\bar{m}T}{4L} - \frac{\bar{m}^2T^2}{4\lambda_q^2L^2}\right]V_4(t) - \frac{1}{\bar{m}T}\left(1 - \frac{20}{L}\lambda_q^2\lambda_h^2\bar{m}T\right. \\ &\quad \left.- \frac{16}{L^2}\lambda_h^2\bar{m}^2T^2\right)V_5(t) - \frac{r_0}{\bar{m}T}\left[1 - \frac{4}{L}n\bar{a}_1\lambda_p^2\bar{m}T - \frac{4}{L^2}n\bar{a}_1\bar{m}^2T^2 - \frac{\bar{m}T}{4L} - \frac{\bar{m}^2T^2}{4\lambda_q^2L^2}\right]V_6(t).\end{aligned}\quad (35)$$

The conditions (16) and (17) imply that

$$\dot{V}(t) \leq -\frac{\rho}{L^2}V(t), \quad t \in [t_0, \infty).$$

where $\rho = \min\left\{\frac{1}{\lambda_p}\left[\frac{L}{4} - 2n\sqrt{N}\lambda_p l - 6\bar{m}T(1+n\bar{a}_1+n^2N) - \frac{\bar{m}T}{4\lambda_q^2}\right], \frac{1}{\lambda_q}\left(\frac{L}{2} - 2n\sqrt{N}\lambda_q l - 4\lambda_{ah}^2\bar{m}T - 4n^2N\bar{m}T\right), \frac{1}{\lambda_p}\left[\frac{L}{2} - 2n\lambda_p l - 4\bar{m}T(1+n\bar{a}_1+n^2) - \frac{\bar{m}T}{4\lambda_q^2}\right], \frac{1}{\bar{m}T}\left[L^2 - 4Ln\bar{a}_1\lambda_p^2\bar{m}T - 6n\bar{a}_1\bar{m}^2T^2 - \frac{\bar{m}TL}{4} - \frac{\bar{m}^2T^2}{4\lambda_q^2}\right], \frac{1}{\bar{m}T}\left(L^2 - 20L\lambda_q^2\lambda_h^2\bar{m}T - 16\lambda_h^2\bar{m}^2T^2\right), \frac{1}{\bar{m}T}\left[L^2 - 4Ln\bar{a}_1\lambda_p^2\bar{m}T - 4n\bar{a}_1\bar{m}^2T^2 - \frac{\bar{m}TL}{4} - \frac{\bar{m}^2T^2}{4\lambda_q^2}\right]\right\}$.

Then, the error systems with the output feedback controllers (2) are asymptotically stable. In other words, the system (1) reaches leader-following consensus.

Remark 4: The results can be extended to the leader-following consensus problem of multi-agent systems with lower-triangular nonlinear terms:

$$\begin{cases} \dot{x}_1^i(t) = x_2^i(t) + f(x_1^i(t)), \\ \dot{x}_2^i(t) = x_3^i(t) + f(x_1^i(t), x_2^i(t)), \\ \vdots \\ \dot{x}_{n-1}^i(t) = x_n^i(t) + f(x_1^i(t), \dots, x_{n-1}^i(t)), \\ \dot{x}_n^i(t) = f(x_1^i(t), \dots, x_n^i(t)) + u^i(t), \\ y^i(t) = x_1^i(t), \quad i = 0, 1, \dots, N, \end{cases} \quad (36)$$

where $f(t)$ are continuous functions and satisfy global Lipschitz conditions. Then, the following similar output feedback controllers can be proposed:

$$\begin{cases} \dot{\hat{x}}^i(t) = A_0 \hat{x}^i(t) + Ma(x_1^i(t_k) - \hat{x}_1^i(t_k) + f(\hat{x}^i(t)) + bu^i(t), \\ \dot{\hat{x}}^0(t) = A_0 \hat{x}^0(t) + Ma(x_1^0(t_k) - \hat{x}_1^0(t_k)) + f(\hat{x}^0(t)), \\ u^i(t) = -KW \left[\sum_{j \in N_i} a_{ij} (\hat{x}^i(t_k) - \hat{x}^j(t_k)) + d_i (\hat{x}^i(t_k) - \hat{x}^0(t_k)) \right], \\ \lim_{t \rightarrow (t_{k+1} + \tau_{k+1})^-} \hat{x}^i(t) = \hat{x}^i(t_{k+1} + \tau_{k+1}), \\ t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}), \quad k \geq 0, \quad i = 1, 2, \dots, N, \end{cases} \quad (37)$$

where $b = (0, \dots, 0, 1)^T$, $M = \text{diag}(L, L^2, \dots, L^n)$, $W = \text{diag}(L^n, L^{n-1}, \dots, L)$. Then, there exists a proper gain L , a sampling period T and positive gains a_i such that the system (36) under the controllers (37) can reach leader-following consensus.

IV. SIMULATION

In order to validate the theoretical results, the following numerical example is presented.

Example 1: Consider a class of nonlinear upper-triangular multi-agent systems composed by 3+1 agents, the dynamic of agent i is described by

$$\begin{cases} \dot{x}_1^i(t) = x_2^i(t) + \log[1 + x_3^i(t)^2], \\ \dot{x}_2^i(t) = x_3^i(t), \\ \dot{x}_3^i(t) = u^i(t), \quad i = 0, 1, 2, 3. \end{cases}$$

the output feedback controllers are proposed as follows:

$$\begin{cases} \dot{\hat{x}}_1^i(t) = \hat{x}_2^i(t) + \frac{1}{L} a_1 (x_1^i(t_k) - \hat{x}_1^i(t_k)) + \log[1 + \hat{x}_3^i(t)^2], \\ \dot{\hat{x}}_2^i(t) = \hat{x}_3^i(t) + \frac{1}{L^2} a_2 (x_1^i(t_k) - \hat{x}_1^i(t_k)), \\ \dot{\hat{x}}_3^i(t) = u^i(t) + \frac{1}{L^3} a_3 (x_1^i(t_k) - \hat{x}_1^i(t_k)), \quad i = 0, 1, 2, 3, \end{cases}$$

where $u^0(t) = 0$, $u^i(t) = -KW[\sum_{j \in N_i} a_{ij}(\hat{x}^i(t_k) - \hat{x}^j(t_k)) + d_i(\hat{x}^i(t_k) - \hat{x}^0(t_k))]$ with $W = \text{diag}(\frac{1}{L^3}, \frac{1}{L^2}, \frac{1}{L})$.

The communication topology graph is shown in Figure 2. Choose the appropriate parameters $K = (3, 4, 6)$, $(a_1, a_2, a_3) = (2, 3, 4)$ and $L = 8$. The initial conditions $x_0^0 = [-1, -1, -1]^T$, $x_0^1 = [-2, -2, -2]^T$, $x_0^2 = [-3, -3, -3]^T$, $x_0^3 = [-4, -4, -4]^T$, and $\hat{x}_0^0 = [1, 2, 3]^T$, $\hat{x}_0^1 = [4, 5, 6]^T$, $\hat{x}_0^2 = [7, 8, 9]^T$, $\hat{x}_0^3 = [10, 11, 12]^T$. The sampling period is set as $T = 0.05s$. The transmission delays τ_k^{ji} and d_k^i are simulated by random numbers in the intervals $[0, 1.6T]$ and $[0, 1.8T]$.

The simulation results are shown in Figure 3 and Figure 4. Figure 3 shows that the trajectories of the state estimation errors $e_n^i(t) = x_n^i(t) - \hat{x}_n^i(t)$, ($n = 1, 2, 3, i = 0, 1, 2, 3$). Figure 4 shows that the trajectories of the state estimation errors between the follower agents and the leader agents $\varsigma_n^i(t) = x_n^i(t) - x_n^0(t)$, ($n = 1, 2, 3, i = 1, 2, 3$). Figure 5 shows that the trajectories of the state x_k^i .

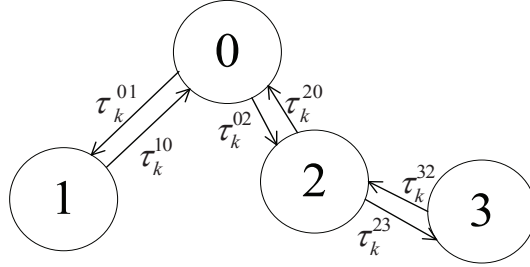


Figure 2. Connected graph

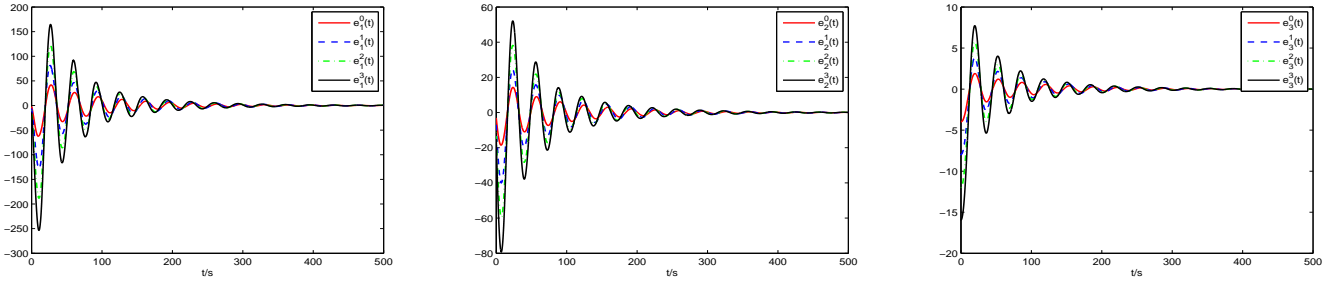


Figure 3. Trajectories of the state estimation errors $e_n^i(t) = x_n^i(t) - \hat{x}_n^i(t)$.

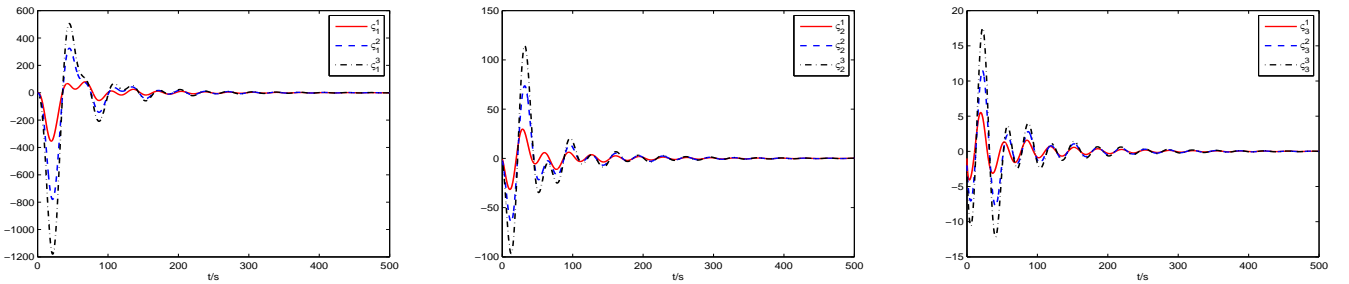


Figure 4. Trajectories of the estimation errors $\varsigma_n^i(t) = x_n^i(t) - x_n^0(t)$.

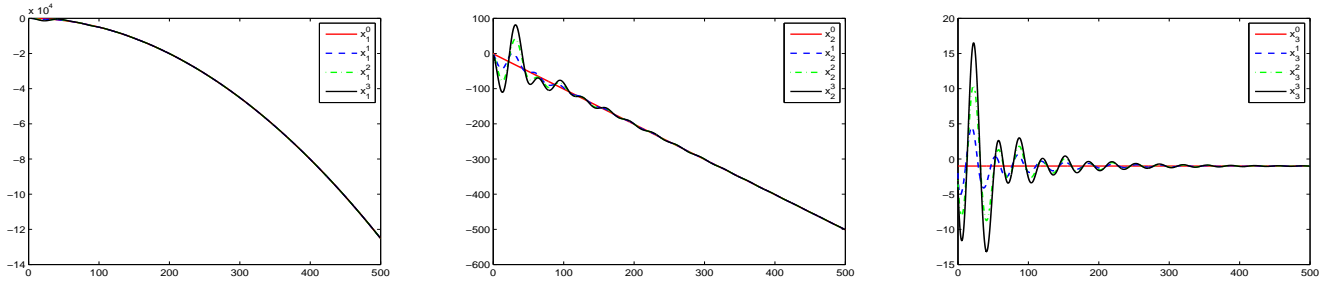


Figure 5. Trajectories of the state x_k^i .

V. CONCLUSION

In this paper, we investigated the leader-following consensus problem for upper-triangular nonlinear MASs via sampled and delayed output feedback. The sampled outputs and the sampled output feedback control signals were used to update the output feedback controllers and the MASs at different time instants, respectively. We proposed a method of interval decomposition such that the MASs and the output feedback controllers were updated at the same interval. Then, sufficient conditions were presented to ensure the MASs could reach leader-following consensus. In the future, we will research leader-following consensus problem for MASs with more general form via sampled and delayed output feedback.

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