ECOLOGICAL AND ECOP PERFORMANCE OF AN IRREVERSIBLE TWO-STAGE ABSORPTION REFRIGERATION CYCLE

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ABSTRACT

An irreversible finite-time thermodynamics models of a two-stage AR (absorption refrigeration) system with variable-temperature heat reservoirs is established, the ecological function and the ECOP (ecological coefficient of performance) criteria are studied. In the model, heat transfer losses between external heat reservoir and internal cycle working medium in six heat exchangers, the internal irreversible loss during the cycling process of the working medium, and the heat leakage losses between surrounding and external heat reservoir are considered. The ecological function characteristic relations and the ECOP characteristic relations are derived. Using illustrative calculation, the heat exchange inventory of six heat exchangers are redistributed and the performances are optimized based on different optimization objective function, and the characteristic relationships among optimal ecological function, optimal ECOP, optimal cooling load, optimal cycle entropy generation rate and COP are investigated.

INTRODUCTION

Absorption systems (including AR, AHP (the first absorption heat pump) and AHT (absorption heat transformer)) must be a finite-size and finite-time devices, therefore, the performance bound obtained through classical thermodynamic analysis is an upper bound that cannot be realized forever [1-4]. Thus, finite-time thermodynamic theory [5-19] has been used to the bound analysis and performance optimization of absorption system considering finite-size and finite-time constraints [20-23].

In case of single-stage AR systems, optimal COP and cooling load performances were analyzed by using endoreversible [24,25] and irreversible [26] THR (three-heat-reservoir) models, endoreversible [27] and irreversible [28] FHR (four-heat-reservoir) models, irreversible FTL (four-temperature-level) models [29,30] with constant-temperature [29] and variable-temperature [30] heat reservoirs. Besides of cooling load and COP performance, some other objective functions

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have been analyzed for absorption systems. Ecological function and ECOP are two of these important objective functions. Ecological function had been introduced and developed by Angulo-Brown [31], Yan [32], and Chen et al. [33-35]. Tao et al. [36] established the ecological function of AR systems based on endoreversible FHR models and obtained its ecological characteristics. Qin et al. [37, 38] established ecological function of AHP systems [37] and AHT systems [38] based on irreversible FTL models, and obtained their ecological characteristics. ECOP had been introduced and developed by Ust et al. [39-41] and Ngouateu Wouagfack et al. [42-45]. Ngouateu Wouagfack et al. [42-45] established ECOP function of AR system based on THR [42-44] and FHR [45] irreversible cycle models, and obtained their ECOP characteristics.

Besides of single-stage AR systems, aimed at a parallel flow double-effect AR system, Medjo Noudjde et al. [46] established an irreversible cycle model and obtained its COP and ECOP characteristics. Aimed at a two-stage AR system, Huang and Sun [47] established a model considering the irreversibility of finite-rate heat transfer and the internal irreversibility of the working medium, and analyzed its cooling load and COP performances. Zhang [48] and Zhang et al. [49] established an irreversible cycle model considering the irreversibility of finite-rate heat transfer, the internal irreversibility of the working medium and heat leakages for two-stage AR systems with constant-temperature reservoirs, and analyzed its cooling load and COP performances.

In this paper, an irreversible model of a two-stage AR system with variable-temperature reservoirs will be established, and its cooling load, COP, ecological function and ECOP performances will be investigated.

SYSTEM DESCRIPTION AND IRREVERSIBLE CYCLE MODEL

The schematic illustration of a two-stage AR system is displayed in figure 1. It has six main assembly units: condenser, evaporator, HPA (high pressure absorber), LPA (low pressure absorber), HPG (high pressure generator) and LPG (low pressure generator). Six corresponding external heat reservoirs exchange heats with internal cycle working medium.

![Figure 1 The cycle diagram of a two-stage AR system](image)

Figure 2 shows the irreversible models with finite heat capacity reservoirs. Six external heat reservoirs all are variable-temperature ones, the inlet temperatures are \( T_{Hg}^{in}, T_{Lg}^{in}, T_{e}^{in}, T_{Ha}^{in}, T_{La}^{in}, T_{c}^{in} \), and condenser temperature \( T_{c}^{in} \), the outlet temperatures are \( T_{Hg}^{out}, T_{Lg}^{out}, T_{e}^{out}, T_{Ha}^{out}, T_{La}^{out}, T_{c}^{out} \) respectively. The internal cycle working medium is working in HPG temperature \( T_{Hg}^{i} \), LPG temperature \( T_{Lg}^{i} \), HPA temperature \( T_{Ha}^{i} \), LPA temperature \( T_{La}^{i} \), evaporator temperature \( T_{e}^{i} \), and condenser temperature \( T_{c}^{i} \), add up to six different temperature levels.

The internal cycle working medium rejects heats to HPA, LPA and condenser heat reservoirs, the heat rejection rates are \( Q_{Hg}^{i}, Q_{Lg}^{i} \) and \( Q_{c}^{i} \). Defining the parameter \( a \) as the allocation ratio of absorber heat transfer rate in the total rejected heats to external heat reservoir, \( a = Q_{Hg}^{i} / (Q_{Hg}^{i} + Q_{Lg}^{i} + Q_{c}^{i}) \). Defining the parameter \( b \) as the allocation ratio of the internal cycle fluid heat rejection rate between HPA and LPA, \( b = Q_{Lg}^{i} / (Q_{Hg}^{i} + Q_{Lg}^{i}) \). The internal cycle working medium absorbs heats from HPG, LPG and evaporator heat reservoirs, the heat addition rates are \( Q_{Hg}^{i}, Q_{Lg}^{i} \) and \( Q_{e}^{i} \). Defining the parameter \( c \) as the allocation ratio of the internal cycle fluid heat addition rate between HPG and LPG, \( c = Q_{Lg}^{i} / (Q_{Hg}^{i} + Q_{Lg}^{i}) \).

The heat transfer rates \( Q_{Hg}^{i}, Q_{Lg}^{i}, Q_{e}^{i}, Q_{Ha}^{i}, Q_{La}^{i} \) and \( Q_{c}^{i} \) can be written as

\[
\begin{align*}
Q_{Hg}^{i} &= C_{Hg}E_{Hg}(T_{Hg}^{in} - T_{Hg}^{i}), \\
Q_{Lg}^{i} &= C_{Lg}E_{Lg}(T_{Lg}^{in} - T_{Lg}^{i}), \\
Q_{e}^{i} &= C_{e}E_{e}(T_{e}^{in} - T_{e}^{i}), \\
Q_{Ha}^{i} &= C_{Ha}E_{Ha}(T_{Ha}^{in} - T_{Ha}^{i}), \\
Q_{La}^{i} &= C_{La}E_{La}(T_{La}^{in} - T_{La}^{i}), \\
Q_{c}^{i} &= C_{c}E_{c}(T_{c}^{in} - T_{c}^{i})
\end{align*}
\]

where \( C_{i} \) (i=Hg, Lg, e, Ha, La, c) is heat capacity of \( i \)th heat reservoir, \( E_{i} \) is effectiveness of \( i \)th heat exchanger; \( E_{i} = 1 - \exp(-U_{i}A_{i}C_{i}) \), \( U_{i}A_{i} \) is heat exchanger inventory of \( i \)th heat exchanger.

According to the energy conservation law,
The surroundings temperature is \( T_s \), and six external heat reservoirs have heat leakages to surroundings. The heat leakage \( Q_{l_h}, \ Q_{l_g}, \ Q_{l_a}, \ Q_{i_h}, \ Q_{i_g}, \ Q_{i_a} \) and \( Q_{i_e} \) can be written as
\[
Q_{l_h} = k_l^i (T_{l_h} - T_s), \quad Q_{l_g} = k_l^i (T_{l_g} - T_s),
\]
\[
Q_{i_h} = k_i^i (T_{i_h} - T_s), \quad Q_{i_g} = k_i^i (T_{i_g} - T_s),
\]
\[
Q_{i_a} = k_i^i (T_{i_a} - T_s), \quad Q_{e} = k_i^i (T_{e} - T_s)
\]
where \( k_i^i \) (i=Hg, Lg, e, Ha, La, c) is heat leakage coefficient.

Heat addition (rejection) rates of six external heat reservoirs are \( Q_{i_h}, \ Q_{i_g}, Q_{i_a}, \ Q_{l_h}, \ Q_{l_g}, \ Q_{l_a}, \) and \( Q_{c} \), and they can be written as
\[
Q_{i_h} = Q_{i_h}' + Q_{l_h}' + Q_{l_g}' + Q_{l_a}' + Q_{c}' - Q_{i_h} - Q_{i_g} - Q_{i_a}, \quad Q_{c} = Q_{c}' - Q_{i_c}'.
\]

As shown in Figure 2, a factor \( I \) is imported to represent internal irreversibility [48, 49]
\[
I = \frac{(Q_{i_h} + Q_{l_h}' + Q_{l_g}' + Q_{l_a})}{(Q_{l_h} + Q_{l_g}' + Q_{l_a})} \geq 1
\]

Based on the irreversible model mentioned above, the cooling load \( R \) and the COP of \( \varepsilon \) of two-stage AR system are.
\[
R = Q_s
\]
\[
\varepsilon = Q_s / (Q_{i_h} + Q_{i_g})
\]

The relations among the heat rejection (addition) rates of external heat reservoir, cooling load and COP can be derived as
\[
Q_{i_h} = (1-a)M_s + Q_{l_h}'
\]
\[
Q_{l_h} = (1-a)(1-b)M_s - Q_{l_h}'
\]
\[
Q_{i_g} = M_s + Q_{l_g}'
\]
\[
Q_{l_g} = (1-c)M_s + Q_{l_g}'
\]
\[
Q_{i_a} = aM_s - Q_{l_a}'
\]
where \( M_s = R_{s} + R + Q_{c} - Q_{l_h}' - Q_{l_g}' - Q_{l_a}' \) and \( M_s = R_{s} - Q_{l_h}' - Q_{l_g}' - Q_{l_a}' \).

The general equation among the COP, the cooling load, heat leakages, irreversibility factor and heat transfer irreversibilities of a two-stage AR cycle can be derived as follows
\[
M_1 (M_e + M_a + M_s - M_s) = M_1 (M_e + M_s - M_s)
\]
where
\[
M_1 = [\frac{T_{l_h}^{o}}{E_{l_h}} + \frac{M_s}{C_{i_h} E_{l_h}}]^{-1}, \quad M_2 = [\frac{T_{l_h}^{o}}{E_{l_h}} + \frac{M_s}{(1-a)(1-b) C_{i_h} E_{i_a}}]^{-1},
\]
\[
M_3 = \frac{T_{l_h}^{o} + M_s}{a C_{i_h} E_{l_h}}, \quad M_4 = \frac{T_{l_h}^{o} - M_s}{C_{i_h} E_{l_h}}, \quad M_5 = \frac{T_{l_h}^{o} - M_s}{1-c C_{i_h} E_{l_h}}
\]
\[
M_6 = I(\frac{T_{l_h}^{o}}{c C_{l_h} E_{l_h}} - \frac{M_s}{C_{l_h} E_{l_h}})
\]

**ECOLOGICAL FUNCTION AND COP**

The exergy output rates \( EX_{i_h}, \ EX_{i_e}, \ EX_e \) and \( EX_e \) of the irreversible model mentioned above can be derived as follows
\[
EX_{i_h} = \int_{T_i}^{T_f} C_{i_h} (1 - \frac{Q_s}{C_{i_h}}) dT = Q_{i_h} - C_{i_h} T_s \ln(1 + \frac{Q_{i_h}}{C_{i_h} T_s})
\]
\[
EX_{i_e} = \int_{T_i}^{T_f} C_{i_e} (1 - \frac{Q_s}{C_{i_e}}) dT = Q_{i_e} - C_{i_e} T_s \ln(1 + \frac{Q_{i_e}}{C_{i_e} T_s})
\]
\[
EX_e = \int_{T_i}^{T_f} C_e (1 - \frac{Q_s}{C_e}) dT = Q_e - C_e T_s \ln(1 - \frac{Q_e}{C_e T_s})
\]

The entropy generation rate \( \sigma \) of the whole cycle can be written as
\[
\sigma = -\sigma_{i_h} - \sigma_{i_e} - \sigma_e + \sigma_{i_h} + \sigma_{i_e} + \sigma_e
\]
where \( \sigma_i \) (i=Hg, Lg, e, Ha, La, c) is entropy generation rate of \( i \)th heat reservoir.

The \( \sigma_{i_h}, \ \sigma_{i_g}, \ \sigma_{i_a}, \ \sigma_{i_h}, \ \sigma_{i_g}, \ \sigma_e \) and \( \sigma_e \) can be derived as follows
\[
\sigma_{i_h} = \int_{T_i}^{T_f} C_{i_h} dT = -C_{i_h} \ln(1 - \frac{Q_{i_h}}{C_{i_h} T_{i_h}})
\]
\[
\sigma_{i_g} = \int_{T_i}^{T_f} C_{i_g} dT = -C_{i_g} \ln(1 - \frac{Q_{i_g}}{C_{i_g} T_{i_g}})
\]
\[
\sigma_{i_e} = \int_{T_i}^{T_f} C_{i_e} dT = -C_{i_e} \ln(1 - \frac{Q_{i_e}}{C_{i_e} T_{i_e}})
\]
\[
\sigma_{i_h} = \int_{T_i}^{T_f} C_{i_h} dT = C_{i_h} \ln(1 - \frac{Q_{i_h}}{C_{i_h} T_{i_h}})
\]
\[
\sigma_{i_g} = \int_{T_i}^{T_f} C_{i_g} dT = C_{i_g} \ln(1 - \frac{Q_{i_g}}{C_{i_g} T_{i_g}})
\]
\[
\sigma_{i_e} = \int_{T_i}^{T_f} C_{i_e} dT = C_{i_e} \ln(1 - \frac{Q_{i_e}}{C_{i_e} T_{i_e}})
\]

Substituting equations (15)-(20) and (8) into equation (14) yields
\[
\sigma = C_{i_h} \ln(1 - \frac{c M_s + Q_{i_h}'}{C_{i_h} T_{i_h}}) + C_{i_g} \ln(1 - \frac{(1-c)M_s + Q_{i_g}'}{C_{i_g} T_{i_g}})
\]
\[
Q_{i_h} + Q_{i_g} + Q_{i_e} + Q_{i_a} + Q_{e} - Q_{i_c} - M_s
\]
\[
\sigma = C_{i_h} \ln(1 - \frac{(1-a)M_s + Q_{i_h}'}{C_{i_h} T_{i_h}}) + C_{i_g} \ln(1 - \frac{(1-a)(1-b)M_s + Q_{i_g}'}{C_{i_g} T_{i_g}})
\]
\[
Q_{i_h} + Q_{i_g} + Q_{i_e} + Q_{i_a} + Q_{e} - Q_{i_c} - M_s
\]
\[
C_{i_e} \ln(1 - \frac{R}{C_{i_e} T_{i_e}})
\]

**Ecological Function and the Relations among E, R, and \( \varepsilon \)**

Based on the unified ecological objective function [33], the Ec of a two-stage AR system is
\[
E = EX_e - T_s \sigma = EX_{i_h} + EX_{i_e} + EX_e - EX_e - T_s \sigma
\]
Substituting equations (10)-(13), (21) and (8) into equation (22) yields
\[ E = 2(M - TM_s - R - Q_{in} - Q_{out}) - C_{\mathrm{fr}} T_m \ln(1 - \frac{cM_s + Q_{\mathrm{fr}}}{C_{\mathrm{fr}} T_m}) - C_{\mathrm{fr}} T_i \ln(1 - \frac{(1-c)M_i + Q_{\mathrm{fr}}}{C_{\mathrm{fr}} T_i}) - \frac{R}{\varepsilon} \]  

(23)

Equations (23), (21) and (9) are the relations among \( E, \sigma, R \) and \( \varepsilon \) of the irreversible two-stage AR system. Using equations (23), (21) and (9), one can discuss the ecological function performance of two-stage AR cycles.

**ECOP and the Relations among ECOP, R, and \( \varepsilon \)**

According to Refs. [42-46], the ECOP of AR system was defined as \( ECOP = R / (T_i \sigma) \). In the definition of ECOP, \( R \) reflects energy output (not exergy), \( T_i \sigma \) reflects exergy loss. Thus, the ratio of energy output and exergy loss as the definition of ECOP is not completely reasonable. The reasonable definition of ECOP based on exergy-analysis views should be \( ECOP = EX / (T_i \sigma) \), and the ECOP reflects the ratio of exergy output and the exergy loss.

Aimed at a two-stage AR system, the ECOP is

\[ ECOP = \frac{EX}{T_i \sigma} = \frac{EX_H + EX_L + EX_C - EX_s}{T_i \sigma} \]  

(24)

Substituting equations (21) and (8) into equation (24), the ECOP can be derived as

\[ ECOP = \frac{(M - TM_s - R - Q_{in} - Q_{out})}{T_i \sigma} \cdot \frac{T_m}{C_{\mathrm{fr}} T_m} \ln(1 - \frac{cM_s + Q_{\mathrm{fr}}}{C_{\mathrm{fr}} T_m}) + \frac{C_{\mathrm{fr}} T_i}{C_{\mathrm{fr}} T_i} \ln(1 - \frac{(1-c)M_i + Q_{\mathrm{fr}}}{C_{\mathrm{fr}} T_i}) + \frac{Q_{\mathrm{fr}}}{C_{\mathrm{fr}} T_i} + \frac{Q_{in}}{C_{\mathrm{fr}} T_i} + \frac{Q_{out}}{C_{\mathrm{fr}} T_i} + \frac{Q_{\mathrm{fr}}}{C_{\mathrm{fr}} T_i} + M_i \]  

(25)

Equation (25), (21) and (9) are the relations among ECOP, \( \sigma, R \) and \( \varepsilon \) of the irreversible two-stage AR systems. Using equations (25), (21) and (9), one can discuss the ECOP and the entropy generation rate performance of two-stage AR cycle.

**OPTIMAL PERFORMANCE**

The investment cost of a two-stage AR system is closely related to the total heat exchanger inventory \( UA \) ( \( UA = U_{\mathrm{lb}} A_{\mathrm{lb}} + U_{\mathrm{lg}} A_{\mathrm{lg}} + U_{\mathrm{lh}} A_{\mathrm{lh}} + U_{\mathrm{ha}} A_{\mathrm{ha}} + U_{\mathrm{ca}} A_{\mathrm{ca}} \) ) and the total heat transfer surface area \( A \) ( \( A = A_{\mathrm{lb}} + A_{\mathrm{lg}} + A_{\mathrm{lh}} + A_{\mathrm{ha}} + A_{\mathrm{ca}} \) ). If \( UA \) or \( A \) is a constant, selecting the optimal value of \( U_i A_i, \sigma \) or \( A, (i=Hg, Lg, c, Ha, La, c) \), the optimal characteristics can be obtained.

Defining \( R_{\text{UA}} \) and \( R_{\text{UA, max}} \), \( E_{\text{UA}} \) and \( E_{\text{UA, max}} \), \( E_{\text{OP}} \) and \( E_{\text{OP, max}} \), and \( \sigma_{\text{UA}} \) and \( \sigma_{\text{UA, max}} \) as the optimal cooling load and its maximum value, the optimal ecological function and its maximum value, the optimal ECOP and its maximum value, and the optimal entropy generation rate and its maximum value after optimization distribution of \( UA \). Using nonlinear programming algorithm, one can analyze \( E_{\text{UA}} / E_{\text{UA, max}} \) versus \( \psi \), \( E_{\text{OP}} / E_{\text{OP, max}} \) versus \( \psi \), \( \sigma_{\text{UA}} / \sigma_{\text{UA, max}} \) versus \( \psi \), and \( R_{\text{UA}} / R_{\text{UA, max}} \) versus \( \psi \) characteristics of irreversible two-stage AR cycle. In the numerical calculations, \( T_{\text{in}}^H = 355K, \  T_{\text{in}}^L = 345K, \  T_{\text{in}}^C = 284K, \  T_{\text{in}}^C = 305K, \  T_{\text{in}}^C = 303K, \  T_{\text{in}}^C = 307K, \  C_{\text{lg}} = C_{\text{lg}} = C_{\text{lg}} = C_{\text{lg}} = 200kW/K, \  C_{\text{c}} = C_{\text{c}} = 250kW/K, \  K_{\text{lg}} = K_{\text{lg}} = K_{\text{lg}} = K_{\text{lg}} = K_{\text{lg}} = 0.5kW/K, \  a = 0.3, \  b = 0.45, \  c = 0.5, \  \text{and} \  UA = 942kW/K \) are set. The “fmincon” function and “interior-point” algorithm in Matlab toolbox are selected as optimization tool.

As shown in figure 3, the \( E_{\text{UA}} \), \( E_{\text{OP}} \) and \( R_{\text{UA}} \) versus \( \psi \) characteristic all are loop-shaped curves, the \( \sigma_{\text{UA}} \) versus \( \psi \) characteristic is a parabolic-like curve. The COP values are not the same at the maximum \( E_{\text{UA}} \), \( E_{\text{OP}} \) and \( R_{\text{UA}} \) and the minimum \( \sigma_{\text{UA}} \) points. Aimed at different goals, one should select different objective function.

![Figure 3](image)

Table 1 lists the optimal performance parameters with different objective function. Compared with max\((R_{\text{UA}})\), max\((E_{\text{UA}})\) makes \( R \) decrease about 37%, but \( \varepsilon \) increase about 95% and \( \sigma \) decrease about 75%; max\((E_{\text{OP}})\) makes \( R \) decrease about 60%, but \( \varepsilon \) increase about 124% and \( \sigma \) decrease about 87%. Compared with max\((E_{\text{UA}})\), max\((E_{\text{OP}})\) makes \( \varepsilon \) decrease about 15% and \( \sigma \) increase about 158%, but \( R \) increase about 143%; max\((E_{\text{OP}})\) makes \( \varepsilon \) decrease about 2% and \( \sigma \) increase about 35%, but \( R \) increase about 36%. It can be seen that the max\((E_{\text{UA}})\) and the max\((E_{\text{OP}})\) points make the heating load (compared to the max\((E_{\text{UA}})\) point) increase with a little increase of \( \sigma \) and the COP (compared to the max\((R_{\text{UA}})\) point) increase with a little
CONCLUSION

An irreversible models with variable-temperature heat reservoirs into a two-stage AR cycle is established, the cooling load, the COP, the ecological and the ECOP objective function and the relations among them are derived.

The optimal performances through optimizing distribution of the total heat inventory are analyzed. Numerical illustration shows that the characteristic relations between the optimal ecological function, the optimal COP, the optimal cooling load and the COP are loop-shaped curves for an irreversible two-stage AR cycle. The operating point of max($E_{UA}$) and max(ECOP$_{UA}$) are all between the operating point of max($E_{UA}$) and max($R_{UA}$). The ecological function and the ECOP are valuable functions, which can attain a compromise in inter restricted relations of $\sigma$, $R$ and $\sigma$. The results can provide some new suggestions about parameter selection for practical two-stage absorption refrigerators.

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