THERMODYNAMIC MODELING AND PERFORMANCE OPTIMIZATION FOR SIMPLE-CYCLE GAS TURBINE WITH AIR COOLING

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NOMENCLATURE

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Greek symbols

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Greek symbol

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INTRODUCTION

One of the most effective technological innovations to enhance specific power output and efficiency of gas turbine cycle is to enhance outlet temperature of the combustion chamber or the inlet temperature of the turbine. To prevent the turbine blades from hot corrosion, part of compressed air in the compressor must be bled to cooling the front blade stages of the turbine [1, 2]. In general, the cooling air should be so sufficient that it will cool the blades effectively, but bleeding too much compressed air will decrease the mass flow rate of main working fluid in the later flow path and then decrease the power output and efficiency of the gas turbine cycle [3]. How to determine the optimal cooling air percentages with respect to different cooling measures is very difficult. To solve this problem, many scholars have performed lots of research work. Ref.[2] presented a predicting model of cooling air percentage without involving the thermodynamic modelling of gas turbine cycle. Based on Ref.[2], Horlock et al [4, 5] and Jordal [6] pursued further studies with considering ideal air as working fluid [4, 5], and estimating the cooling air percentage with convection cooling and air film cooling [6]. Yong and Wilcock [7, 8] studied the thermodynamic performance of single-shaft ideal gas turbine cycle by considering the air cooling and the specific heat ratio of the air changes with temperature only, and the established thermodynamic model couldn't reflect the real operation process as the mass flow rate ratio of the fuel and the air was taken as perfect. Refs.[9-11] built the thermodynamic model of gas turbine cycle with the help of ASPEN soft, and...
reckoned the cooling air percentage for a single-shaft, simple-cycle gas turbine. Refs. [12-18] estimated or analyzed the characteristic performance based on the simple-cycle gas turbine plant considering air cooling but didn’t optimize the cycle key characteristic parameters such as the pressure ratio, etc. with respect to the cycles. Shi et al [19] proposes a new cooling method which integrates steam and air for gas turbine vane cooling with the aim to solve the problem of a very high thermal load at the trailing edge region of a steam-cooled gas turbine vane. Moskalenko et al [20] studied the cooling efficiency of the first stage turbine blade for different parameters of cooling mediums of air and water vapor, respectively. Sanjay [21] studied the thermoeconomics of gas turbine cycle with air film blade cooling.

In this paper, a predicting model of cooling air percentages for different turbine blades with respect to simple-cycle triple-shaft gas turbine plant considering the thermophysical properties of the air and the gas will be established. The thermodynamic performance of the cycle will be investigated. The calculation flow chart of the power output and the efficiency will be exhibited, and the verification computation will be performed based on the design performance data for DJNI80-II-type industrial gas turbine plant developed by Ukraine. The maximum output power, the maximum efficiency and their corresponding cooling air percentages will be obtained by optimizing the pressure ratio of the low-pressure compressor and the total pressure ratio, respectively.

**CYCLE MODELLING**

Fig.1 shows a simple-cycle triple-shaft gas turbine (GT) plant considering air cooling. In the figure, m_a denotes inlet air mass flow rate of the low pressure compressor (LC), m_0 denotes the cooling air mass flow rate that is bled from the outlet of the high-pressure compressor (HC). In the combustion chamber (CC), the fuel (m_f denotes fuel mass flow rate) is ignited and burned with the air and the product (gas, via. the mixture of the residual air and pure gas) mass flow rate at the outlet of the combustion chamber is m_a. LC and HC are driven by the low-pressure turbine (LT) and the high-pressure turbine (HT), respectively. The power turbine (PT) drives the load solely. Considering the thermophysical properties of the fluid (air in compressors and gas in GT hot section) is changeable with its temperature and components as it passes along the flow path.

\[
I = a_0 + a_1T + a_2T^2 + a_3T^3 + a_4T^4 + a_5T^5
\]

\[
lg\pi^0 = b_0 + b_1T^3 + b_2T^4 + b_3T^5 + b_4T^6
\]

where \( a_i \) and \( b_i \) (\( i = 0, 1, 2, \ldots, 5 \)) are the fitting coefficients respectively, and the relative pressure \( \pi^0 \) is the ratio of the local pressure \( p \) and the standard pressure \( p_0 \) under the reference condition.

Considering an adiabatic and irreversible thermodynamic process, the air isentropic relative pressure ratio \( \pi^0 _{2v} \) and enthalpy \( I_2 \) at LC outlet can be written as

\[
\pi^0 _{2v} = \pi^0 _{1v}\pi^0 _{HC} , I_2 = I_1 + (I_{2v} - I_1)/\eta_{HC}
\]

where \( \pi^0 _{1v} \), \( \eta_{HC} \) and \( \pi^0 _{HC} \) denote the air inlet relative pressure ratio, inlet enthalpy, efficiency, pressure ratio and isentropic outlet enthalpy of the LC.

Also, the air isentropic relative pressure \( \pi^0 _{3v} \) and enthalpy \( I_3 \) at HC outlet can be written as

\[
\pi^0 _{3v} = \pi^0 _{2v}\pi^0 _{HC} , I_3 = I_1 + (I_{3v} - I_2)/\eta_{HC}
\]

where \( \pi^0 _{2v} \), \( \eta_{HC} \) and \( \pi^0 _{HC} \) denote the air inlet relative pressure ratio, inlet enthalpy, efficiency, pressure ratio and isentropic outlet enthalpy of the HC.

According to mass and energy conservation, one has

\[
\begin{align*}
m_{LT} &= m_a + m_f + m_0, \\
m_{HC} &= m_{LT} - m_f - m_0\eta_{HC} = m_{LT} - (m_f + m_0)\eta_{HC} + m_f(1 - \eta_f)(I_{1v} - I_{2v})
\end{align*}
\]

where \( \eta_f \) is the efficiency of the air compressor; \( \eta_{HC} \) is the efficiency of the high-pressure compressor; \( \eta_{2v} \) is the efficiency of the low-pressure compressor; \( \eta_{3v} \) is the efficiency of the high-pressure turbine; \( \eta_{3v} \) is the efficiency of the low-pressure turbine; \( \eta_{HT} \) is the efficiency of the high-pressure turbine; \( \eta_{3v} \) is the efficiency of the low-pressure turbine.

\[
P = m_{PT}\eta_{PT}(I_{1v} - I_{3v})/m_a, \quad \eta = Pm_a / (m_{HC}\theta_{HT})
\]

where \( m_{PT} \), \( \eta_{PT} \) and \( I_{3v} \) denote the gas mass flow rate, efficiency and isentropic outlet enthalpy of the power turbine (PT). Obviously, \( m_{PT} = m_{LT} \).

**COOLING BLADE MODELLING**

There are two major blade cooling processes: convection cooling process and film cooling process.

For convection cooling process performed in the turbine blades (Fig.2), the cooling air flows into and out the hollow blades through inner passages, takes away quantity of heat from the turbine blades, decreases working temperature of the surface of turbine blade, and then blends into the main gas flow. In Fig.2, \( V_g \) and \( V_r \) are gas and cooling air velocities, respectively. \( T_g \) and \( T_r \) are gas inlet and outlet temperatures, respectively. \( T_v \) and \( T_o \) are cooling air inlet and outlet temperatures.
temperatures, respectively. \( A_g \) and \( T_{bl} \) are blade surface area and temperature, respectively. \( A_g \) is the effective area of the blade inner passage.

\[
\begin{align*}
\dot{m}_g V_g & \quad T_g \quad A_g \quad T_{bl} \\
\dot{m}_c V_c & \quad T_{wi} \quad T_{co} \\
\dot{m}_g + \dot{m}_c & \quad & \quad & \quad \text{blade root}
\end{align*}
\]

**Figure 2** Convection cooling thermal model for turbine blade

In order to analyse problem conveniently, the air bleeding from the compressor outlet to cool CC is ignored. According to the law of conservation of energy, one has

\[
\dot{Q}_{co} = \dot{m}_g c_{pc}(T_d - T_{wi}) = \dot{m}_g \int_{T_{wi}}^{T_d} c_{pc}(T_d - T_{wi}) dT = \alpha_g A_g (T_d - T_{wi}) \tag{9}
\]

where \( \alpha_g \) denotes the heat exchange coefficient between the blade surface and the gas \([4, 5]\).

Defining cooling efficiency \( \eta_{cool} = (T_{wi} - T_{co}) / (T_d - T_{wi}) \), combining with \( \dot{m}_g = \rho_g A_g V_g \) and Eq.(9) gives

\[
\xi = \frac{\dot{m}_i}{\dot{m}_g} = \frac{\Lambda S_g}{c_{pc}} \left( \frac{\tau_{pc}(T_g - T_{wi})}{\eta_{cool}(T_d - T_{wi})} \right) \tag{10}
\]

where \( \Lambda = A_g / A_s = 2Lc / (Ls \cos \alpha) \), \( \alpha \), \( c \), \( s \) and \( \tau \) denote the blades height, chord, pitch, flow outlet angle, respectively. Generally, \( s/c = 0.8 \) and \( \alpha = 75^\circ \) are designated in turbine blades cooling air percentage’s numerical calculation \([5]\); \( S_g = \alpha_s \left( \tau_{pc} \rho_g V_g \right) \); and \( \rho_g \) denotes the gas mass density.

Defining \( \epsilon_o = (T_g - T_{wi}) / (T_d - T_{wi}) \) and combining it with Eq.(10) gives

\[
\epsilon_o = K \epsilon_g / (1 - \epsilon_g) \tag{11}
\]

where \( K = C / \eta_{cool} \) and \( C = \Lambda S_g \tau_{pc} / c_{pc} \).

For film cooling process performed in the turbine blades (Fig.3), as the cooling air flows out the hollow blades through inner passages it forms a film in the high-pressure gas stream and covers the blades surface like fire wall. In Fig.3, \( T_{aw} \) denotes the air film temperature which is resulted from the mixing condition of hot gas and cooling air and different from \( T_{bl} \).

\[
\begin{align*}
\dot{m}_g V_g & \quad T_g \quad A_g \quad T_{aw} \\
\dot{m}_c V_c & \quad T_{wi} \quad T_{co} \\
\dot{m}_g + \dot{m}_c & \quad & \quad \text{blade root}
\end{align*}
\]

**Figure 3** Film cooling thermal model for turbine blade

According to the law of conservation of energy, one has

\[
\dot{Q}_{aw} = \alpha_g \epsilon_g (T_{aw} - T_{wi}) = \dot{m}_g \int_{T_{wi}}^{T_{aw}} c_{pc}(T_{aw} - T_{wi}) dT = \alpha_g A_g (T_{aw} - T_{wi}) \tag{12}
\]

where \( \alpha_{fg} \) denotes the heat exchange coefficient between the air film and blade surface.

Defining \( \epsilon_f = (T_g - T_{aw}) / (T_d - T_{aw}) \) and combining it with Eq.(12) gives

\[
\xi = \frac{\dot{m}_i}{\dot{m}_g} = C \left[ \epsilon_g - (1 - \eta_{cool}) \epsilon_f - \epsilon_g \epsilon_f / \eta_{cool} \right] / \left[ \eta_{cool}(1 - \epsilon_g) \right] \tag{13}
\]

where \( C = \Lambda S_g \tau_{pc} / c_{pc} \).

The total pressure loss resulted from the mixture of the cooling air and the gas stream (Fig.4) will decrease the efficiency of the simple-cycle gas turbine plant.

\[
\xi = \frac{\dot{m}_i}{\dot{m}_g} = \frac{\Delta p / p_i}{-0.5 \Delta p / (1 + T_d / T_g - 2 \chi \cos \phi)} \tag{14}
\]

**Fig. 4** The mixture model for cooling air and the gas stream

The total pressure loss coefficient is determined as \([5]\)

\[
\Delta p / p_i = -0.5 \epsilon_f \chi (1 + T_d / T_g - 2 \chi \cos \phi) \tag{14}
\]

**MODEL VERIFICATION**

Using the simple-cycle triple-shaft gas turbine thermodynamic model and turbine blade air cooling model mentioned above, one makes approximate calculation about the thermodynamic performance and cooling air information and compare the result with design performance public-data of DH80UJ-type industrial gas turbine developed by Ukraine \([23]\).

The results are listed in Tab.1.

| Tab.1 the calculation values and design value of DH80UJ plant |
|-----------------|-----------------|-----------------|-----------------|
| Name, unit      | Design value    | Calculation value | Relative error  |
| Low-pressure turbine outlet temperature \( T_{in} \), K    | 1070            | 1083.5          | 1.26%           |
| Power turbine outlet temperature \( T_T \), K           | 773             | 754.8           | 2.35%           |
| Low-pressure compressor inlet air mass flow rate \( m_{in} \), kg/s | 85              | 83.4            | 1.88%           |
| Plant’s efficiency \( \eta \)                          | 34.25%          | 36.81%          | 7.47%           |
| Cooling air percentage of high-pressure ture \( \xi_{HT} \) | 15.12%          | 13.40%          | 11.38%          |
| Cooling air percentage of low-pressure ture \( \xi_{LT} \) | 3.61%           | 2.95%           | 18.28%          |

It shows the relative errors of the temperatures and the inlet air mass flow rate are less than 3%, but the calculation results of the cooling air flow rates and the plant’s efficiency seem to be relatively large as comparing those to the design values. The
reasons maybe: (1) The mathematical models are not so accurate that more realities are should be taken into account; (2) Actually, the designers and the manufactures intentionally increase the cooling air mass flow rate to well ensure the gas turbine’s safety as it operates in the most atrocious condition. The mathematical model built above can give helpful guidelines in cooling system design of turbine blade.

**CYCLE PERFORMANCE OPTIMIZATION FLOW CHART**

The thermodynamic performance calculation of the gas turbine plant considering blade air cooling includes three steps: (1) estimate the quantity of the cooling air that needed necessarily in each blade row; (2) estimate the total energy and total pressure loss resulted from the heat transfer between gas stream and cooling air and mixture, et al; (3) the plant power output and efficiency calculation based on steps (1) and (2).

Considering the simple-cycle triple-shaft gas turbine displayed in Fig.1, one can divide the turbine into three parts: the high-pressure turbine (single-stage), the low-pressure turbine (single-stage) and the power turbine (multi-stage). The blades of the front three row (the high-pressure turbine stationary blade row A and rotor blade row B, the low-pressure turbine stationary blade row C) need air cooling while the latter blade rows needn’t as where the gas stream temperature decreases markedly. It is assumed that the cooling air is all bled from the high-pressure compressor outlet.

In the calculation, some initial values are set: the ambient temperature is \( T_0 = 300.15K \), the standard pressure is \( p_0 = 1.013 \text{bar} \), each GT component’s isentropic efficiencies are \( \eta_{LC} = 0.88 \), \( \eta_{HC} = 0.88 \), and \( \eta_{HT} = \eta_{LT} = \eta_{PT} = 0.96 \), the low calorific value of fuel is \( H_L = 42700 \text{kJ/kg} \), the total pressure loss coefficient of combustion chamber is \( \sigma_p = 0.02 \), and outlet temperature of combustion chamber is \( T_3 = 1700K \).

First, assuming the surface temperature of the stationary blade row \( A \) \( T_{eb} = 1073K \), \( \eta_{out} = 0.7 \) and \( \epsilon = 0.4 \) [4, 5], one can obtain \( \xi_a \) and \( \xi_b \). Then, taking it for granted that the rotor blade row \( B \) inlet gas stream temperature \( T_{eb} = (T_3 + T_1) / 2 \), one can obtain \( \xi_b \). Here, as the air cooling processes of stationary and rotor blades of the HT are considered separately, Eq.(6) should rewritten as

\[
\eta_{HT}(m_a + m_{air})(I_4 - I_{SP}) + \eta_{HT}(m_{air}I_{SP} - I_a) = m_a(I_1 - I_a)
\]

where \( \eta_{HT} \) and \( \eta_{HT} \) are isentropic efficiencies of the stationary blade row \( A \) and rotor blade row \( B \) of the HP, respectively, \( m_{air} \) is working fluid mass flow rate in row \( A \), \( I_{SP} \) and \( I_a \) are row \( A \) outlet gas stream isentropic and real enthalpy, respectively, and \( m_a \) is the air mass flow rate cooling the HT. Like \( \xi_a \) and \( \xi_b \), \( \xi_a \) can be obtained. Finally, according to Eq.(13) one can obtain the total pressure loss in the mixture process and efficiency loss of each turbine, in further, power output and efficiency, even exergy loss in each component, of the simple-cycle triple-shaft gas turbine plant are derived. The corresponding computer calculation flow chart based on Matlab is displayed in Fig.5.

**OPTIMIZATION RESULTS AND ANALYSES**

Fig.6 illustrates the characteristics of \( P \), \( \eta \) and \( \xi \) versus \( \pi_{LC} \). It shows that \( P \) and \( \eta \) decrease while \( \xi \) increases with increase in \( \pi_{LC} \) as the outlet temperature of combustion chamber \( T_3 \) is given and the turbine blades are appointed air cooling measure.

![Figure 6](image)

**Figure 6** The characteristics of \( P \), \( \eta \) and \( \xi \) versus \( \pi_{LC} \)

Fig.7 illustrates the characteristics of \( P \), \( \eta \) and \( \xi \) versus the total pressure rations \( \pi \) with selected \( \pi_{LC} = \sqrt{\pi} \). It shows that \( P \) and \( \eta \) increase first and then decrease while \( \xi \) increases with increase in \( \pi \). There exists an optimum \( \pi_{\max} \) which leads to \( P_{\max} \) with the corresponding \( \eta_{\max} \) and \( \xi_{\max} \). There also exists an optimum \( \pi_{\max} \) leads to \( \eta_{\max} \) with the corresponding \( P_{\max} \) and \( \xi_{\max} \). It is evident that \( \pi_{\max} < \pi_{\max} \).

Consideration both plant power and its efficiency, the suitable total pressure ratio should range from \( \pi_{\min} \) to \( \pi_{\max} \).
CONCLUSION

This paper establishes a predictive model of the air cooling for different turbine blades with respect to simple-cycle triple-shaft gas turbine plant considering the thermophysical properties of the air and the gas, and investigates the power and efficiency performance of the plant. Some characteristic parameters are compared with the design performance data for ДИ80Л-Type industrial gas turbine developed by Ukraine. The results indicate the model is reasonable and can predict the design performance of gas turbine cycle effectively. Further optimization is performed by taking the power output and efficiency of gas turbine plant as the objectives based on numerical example. The maximum power output, the maximum efficiency and their corresponding cooling air percentages are obtained by searching the optimal total pressure ratio and the optimal pressure ratio of the LC using the models established, and the effect of the outlet temperature of the combustion chamber on the thermodynamic performance of plant is analysed.

The results and the analyses indicate:

1) giving the outlet temperature of the combustion chamber and the total pressure ratio and considering the turbine blades are cooled by air, the power output and the efficiency decreases while the cooling air percentage increases with increase in the pressure ratio of the LC;

2) giving the outlet temperature of combustion chamber, there exist different total pressure ratios lead to the maximum power output and the maximum efficiency, respectively. The cooling air percentage increases with increase in the total pressure ratio;

3) the maximum power and the corresponding total pressure ratio and cooling air percentage increase while the corresponding efficiency decreases as the outlet temperature of the combustion chamber increases. The maximum efficiency and its corresponding total pressure ratio decrease while the corresponding power and cooling air percentage increase as the outlet temperature of the combustion chamber increases.

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REFERENCES


[23] Бондию Ю., and Михайлов А., Основные результаты опытно-промышленной эксплуатации ГТД ДН80Л №2 на КС