A STUDY OF THE EFFECT OF TRIANGULAR AND RECTANGULAR SURFACE WAVES ON THE NATURAL CONVECTIVE HEAT TRANSFER FROM A CIRCULAR UPWARD FACING HEATED HORIZONTAL ISOTHERMAL SURFACE

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ABSTRACT

Natural convective heat transfer from a horizontal, upward facing circular isothermal heated surface imbedded in a large flat adiabatic surface has been numerically studied. The heated circular surface is covered with a series of waves which are arranged in an axially-symmetric (concentric) manner around the vertical axis of the horizontal circular surface. Surface waves with a triangular cross-sectional shape and with a rectangular cross-sectional shape have been considered. Conditions under which the flow is laminar, transitional, and turbulent have been considered. The flow has been assumed to be axially-symmetric and steady. Fluid properties have been assumed constant except for the density change with temperature that gives rise to the buoyancy forces which has been treated using the Boussinesq approach. The commercial CFD solver ANSYS FLUENT© was used to obtain the numerical solutions. The standard *k*-epsilon turbulence model was employed with full account being taken of buoyancy force effects. The heat transfer rate from the heated surface expressed in terms of the Nusselt number is dependent on the Rayleigh number, the number of surface waves, the shape of the surface waves, the dimensionless height of the surface waves, and the Prandtl number. Results have only been obtained for the case where there are five surface waves and for a Prandtl number of 0.74. A detailed study of the variation of the Nusselt number with Rayleigh number for various dimensionless surface wave heights has been undertaken. The heat transfer rates for the wavy surface have been compared with those which would exist in the case of a non-wavy (wave height of zero) surface.

NOMENCLATURE

d	[m]	outside diameter of circular surface
g	[m/s^2]	gravitational acceleration
H	[-]	dimensionless height of surface waves, h/d
h	[m]	height of surface waves
Nu	[-]	mean Nusselt number based on d and on the mean transfer rate from surface per unit base area
Nu_0	[-]	mean Nusselt number for the case where there are no surface waves
n	[-]	number of surface waves considered
Pr	[-]	Prandtl number
Ra	[-]	Rayleigh number based on d
T_f	[K]	undisturbed fluid temperature
T_w	[K]	heated surface temperature

Greek Symbols

α	[m^2/s]	thermal diffusivity
β	[1/K]	bulk coefficient of thermal expansion
ν	[m^2/s]	Kinematic viscosity

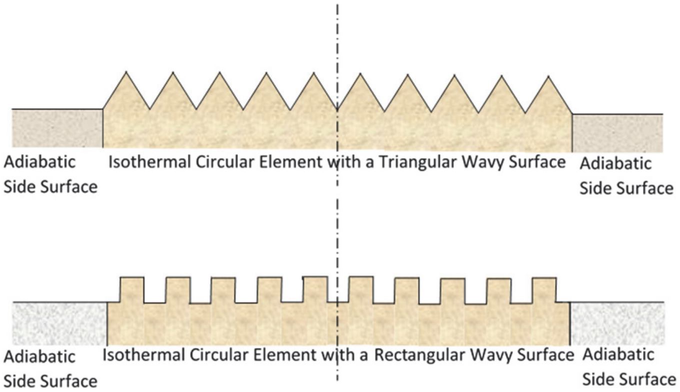

Figure 1 Flow situations considered

INTRODUCTION

The results of a numerical study of natural convective heat transfer from a horizontal upward facing circular isothermal heated surface that is imbedded in a large flat adiabatic surface are presented and discussed here. The heated circular surface is covered with a series of surface waves which are arranged in an axially-symmetric (concentric) manner around the vertical axis of the surface, i.e., the surface waves are circular. These surface waves are introduced in an attempt to increase the heat transfer rate above that which would exist with a non-wavy (plane) surface. For the case considered here the temperature of the circular heated wavy surface is higher than the temperature of the surrounding fluid. Surface waves with a triangular shape and with a rectangular shape (see Fig. 1) have been considered. A relatively wide range of the governing parameters has been considered and allowance has been made for the possibility that laminar, transitional and fully turbulent flows can occur. The results have mainly been used to study the effect of the wave cross-sectional shape and of the dimensionless height of the waves on the heat transfer rate from the heated wavy circular surface in these laminar, transitional and turbulent flow regions.

There have been a number of studies of natural convective heat transfer from smooth upward facing horizontal surfaces, e.g. see [1-13]. However, relatively little attention has been given to horizontal surfaces having a wavy (or rough) surface, e.g., see [14–18]. In the majority of these available studies of heat transfer from wavy horizontal surfaces, consideration has been given only to situations in which the flow remains laminar. Most existing studies of natural convective heat transfer from wavy surfaces have considered cases where the waves are straight whereas in the present case circular waves have been considered. The study described here is part of a broader study of natural convective heat transfer from heated horizontal surfaces, e.g., see [19-25].

SOLUTION PROCEDURE

The mean flow has been assumed to be steady. The Boussinesq approximation has been adopted and fluid properties have thus been assumed to be constant except for the density change with temperature which gives rise to the buoyancy forces. Radiant heat transfer effects have not been considered. The flow has been assumed to be axisymmetric about the surface center-line. The possibility that turbulent flow may develop has been allowed for by using the standard k-epsilon turbulence model with standard wall functions and with full account being taken of buoyancy force effects. From previous studies it has been established that for flows of the type considered here, e.g., see [26-32] this approach yields relatively good predictions of the conditions for which turbulence develops and of the flow in the transitional region. The governing equations based on the use of the assumptions discussed above and subject to the boundary conditions have been solved numerically using the commercial CFD solver ANSYS FLUENT©. Quite extensive grid independence and convergence-criteria independence testing was conducted demonstrating that with the meshes selected for this study, the heat transfer results, i.e., the derived Nusselt number values, are grid- and convergence criteria independent to within approximately one per cent. The number of grid points used to achieve this depended on the flow parameters.

RESULTS

The solution depends on:

- The Rayleigh number, Ra, based on the outside diameter, d, of the circular heated surface and on the difference between the surface temperature and the temperature of the fluid far from the heated surface, i.e.:

$$Ra = \frac{\beta g d^3 (T_w - T_f)}{\nu\alpha}$$

- the number of surface waves, n
- the shape of the surface waves, i.e., triangular or rectangular,
- the dimensionless height of the surface waves, $H = h/d$, the surface height h being defined as shown in Fig. 2.
- the Prandtl number, Pr, of the fluid surrounding the surface.

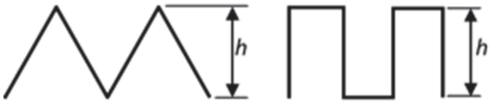

Figure 2 Definition of the surface wave height, *h*

Results have only been obtained here for a Prandtl number of 0.74. Attention has also been restricted to the case where there are five circular waves. Rayleigh numbers of between approximately 10^5 and 10^{14} have been considered. The heat transfer rate per unit base area (see Fig. 3) has been considered and has been expressed in terms of a mean Nusselt number, Nu, based on the outside diameter, d, of the circular surface, and on the difference between the surface temperature and the temperature of the fluid far from the heated surface. Nu, which depends on the wave shape considered, will be a function of Ra and H since this study considers fixed values of Pr and n.

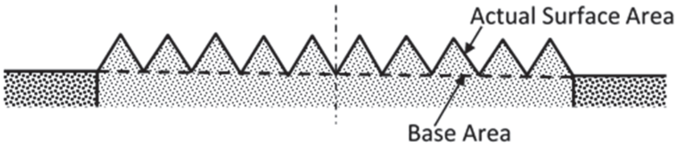

Figure 3 Base area used in defining heat transfer rate

Typical variations of the Nusselt number with Rayleigh number for various values of the dimensionless wave height are shown in Figs. 4 to 7 for the triangular surface wave case and in Figs. 8 to 11 for the rectangular surface wave case. For comparison purposes, the variation of the Nusselt number with Rayleigh number for the non-wavy (plane) surface case is also shown in these figures. It will be seen from these figures that the presence of the surface waves does produce an increase in the heat transfer rate at lower Rayleigh numbers, i.e., in the laminar flow region, and in some cases at higher Rayleigh numbers, i.e., in the turbulent flow region. However, at intermediate Rayleigh numbers, i.e. in the transitional flow region, the presence of the surface waves can produce a decrease in the heat transfer rate. This decrease is associated with changes in the conditions under which transition to turbulent flow starts to occur as a result of the use of surface waves.

Now, the changes in the Nusselt number produced by the use of the surface waves is the result of the increase in the surface area above that for a non-wavy surface and by changes in the flow pattern produced by the introduction of the surface waves. The change in the surface area produced by the use of the surface waves is indicated by the results presented in Fig. 12 which shows the variation the ratio of the actual surface area, A, to the base surface area A_0, which is equal to the surface area that would exist with no surface waves, with the dimensionless wave height for both triangular and rectangular surface waves.

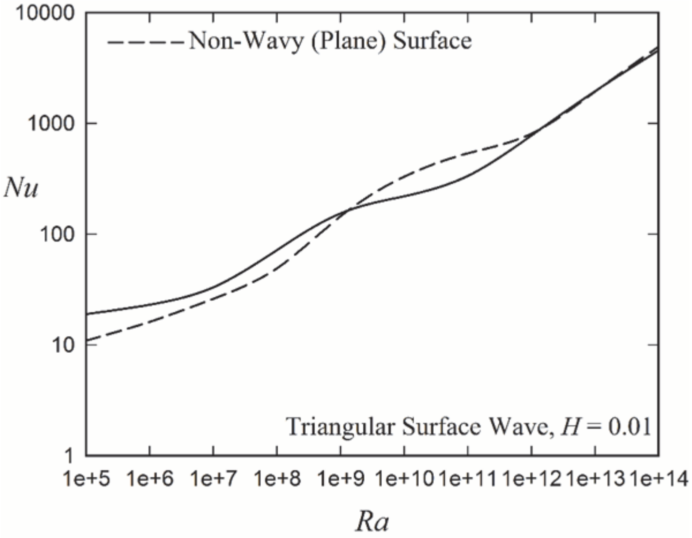

Figure 4 Variation of Nusselt number with Rayleigh number for a horizontal circular surface with triangular surface waves with a dimensionless surface wave height of 0.01 and for a non-wavy circular surface

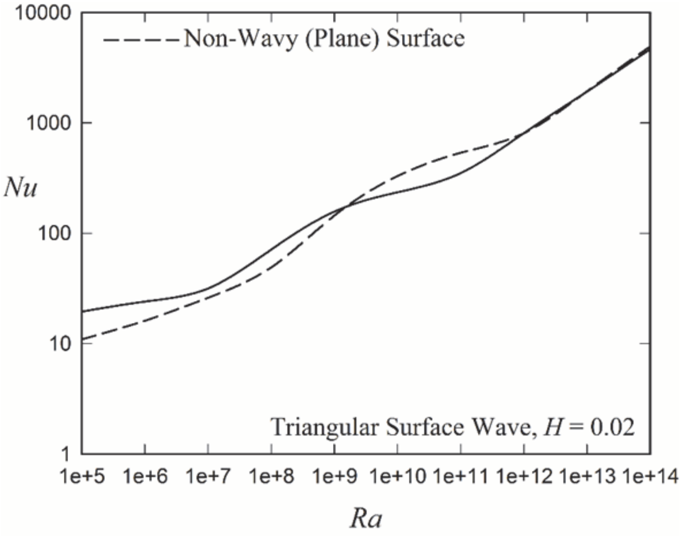

Figure 5 Variation of Nusselt number with Rayleigh number for a horizontal circular surface with triangular surface waves with a dimensionless surface wave height of 0.02 and for a non-wavy circular surface

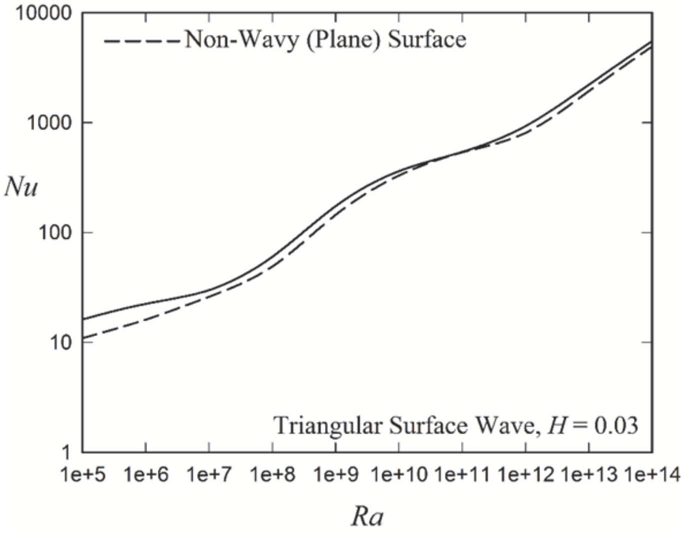

Figure 6 Variation of Nusselt number with Rayleigh number for a horizontal circular surface with triangular surface waves with a dimensionless surface wave height of 0.03 and for a non-wavy circular surface

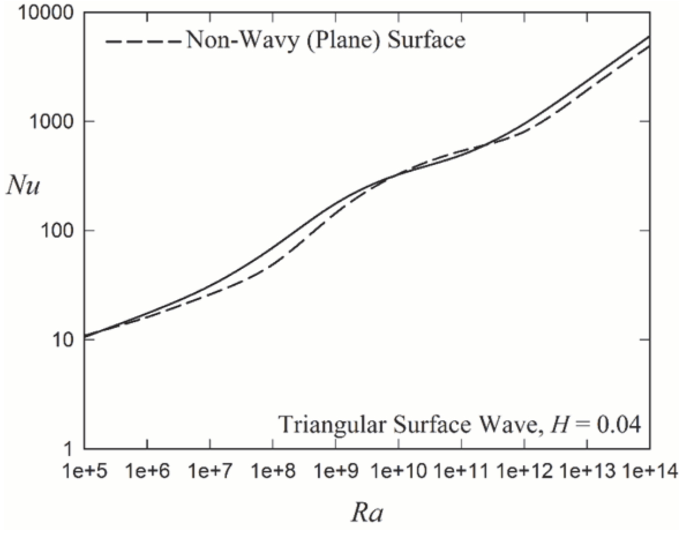

Figure 7 Variation of Nusselt number with Rayleigh number for a horizontal circular surface with triangular surface waves with a dimensionless surface wave height of 0.04 and for a non-wavy circular surface

It will be seen from Fig. 12 that the increase in surface area produced by the use of the surface waves is significantly greater at a given dimensionless wave height value with a rectangular surface wave than with a triangular surface wave.

Figures 13 and 14 show typical variations of Nu/Nu_0, Nu_0 being the Nusselt number that would exist at the same Rayleigh number with a non-wavy surface, with the dimensionless wave height for various Rayleigh number values for the triangular wave case and the rectangular wave case, respectively. These figures show how significantly the form of the variation of Nu/Nu_0 with H is dependent on the Rayleigh number. This dependence is in part associated with differences in the conditions under which the change from laminar flow to transitional flow and then the change from transitional flow to fully turbulent flow occurs.

By comparing the results given in Figs. 13 and 14 with the area ratios shown in Fig. 12 it will be seen that, except at the lower Rayleigh numbers considered, the fractional increase in the heat transfer rate produced by surface waves is less than the fractional increase in the surface area.

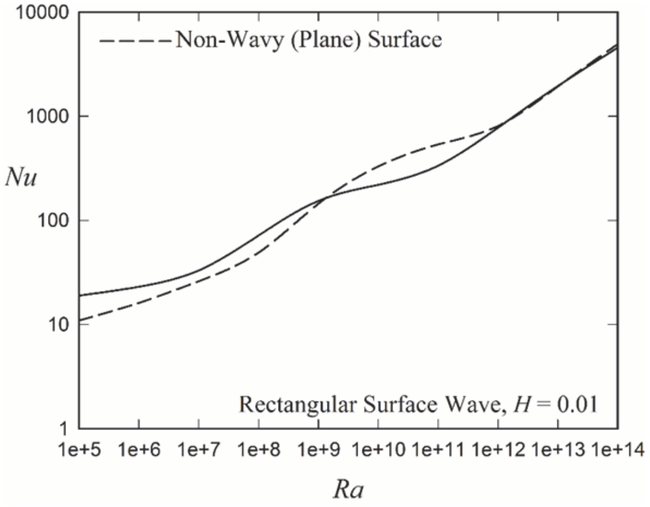

Figure 8 Variation of Nusselt number with Rayleigh number for a horizontal circular surface with rectangular surface waves with a dimensionless surface wave height of 0.01 and for a non-wavy circular surface

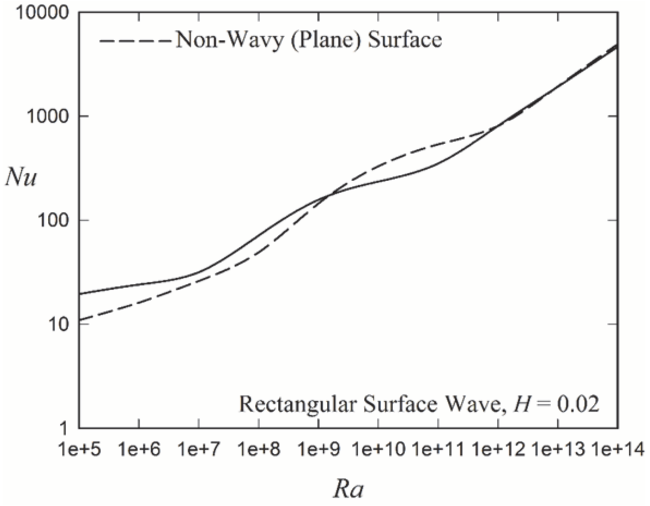

Figure 9 Variation of Nusselt number with Rayleigh number for a horizontal circular surface with rectangular surface waves with a dimensionless surface wave height of 0.02 and for a non-wavy circular surface

The results given in Figs. 13 and 14 also show that under almost all conditions the use of the rectangular surface waves does produce a larger increase in the heat transfer than that produced by the use of the triangular surface waves.

CONCLUSIONS

The results of the present study indicate that:

1. the presence of the surface waves does produce an increase in the heat transfer rate at lower Rayleigh numbers and, in some cases, at higher Rayleigh numbers. At intermediate Rayleigh numbers, however, the presence of the surface waves can produce a decrease in the heat transfer rate.
2. except at the lower Rayleigh numbers considered the fractional increase in the heat transfer rate produced by the surface waves is less than the fractional increase in the surface area produced by introducing the surface waves.
3. under almost all of the conditions considered here the use of rectangular surface waves results in a larger increase in the heat transfer than that produced by the use of triangular surface waves.

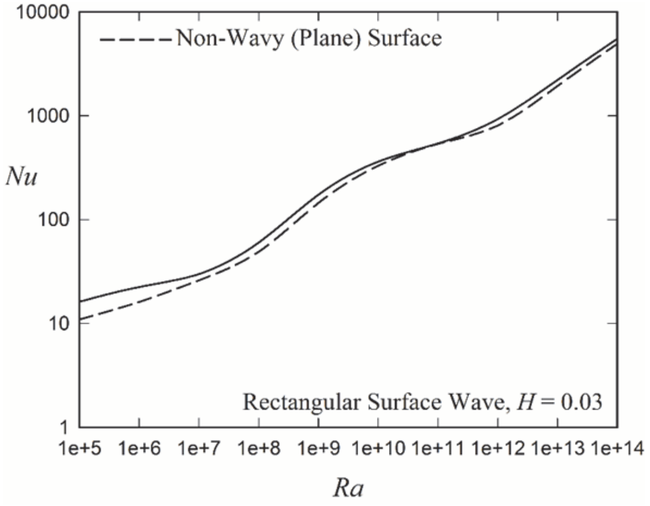

Figure 10 Variation of Nusselt number with Rayleigh number for a horizontal circular surface with rectangular surface waves with a dimensionless surface wave height of 0.03 and for a non-wavy circular surface

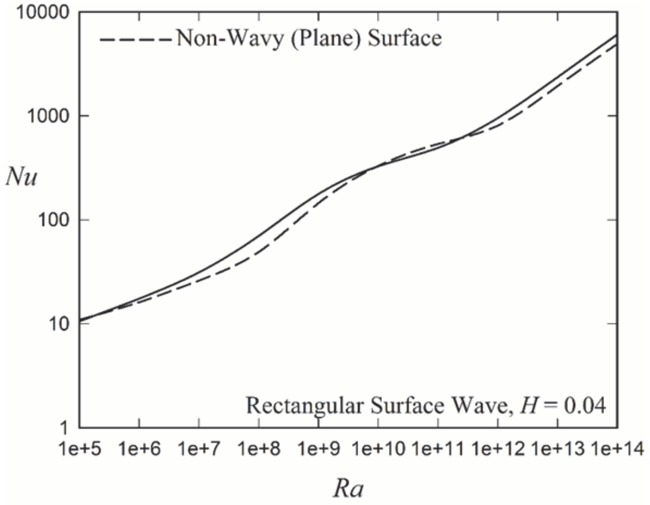

Figure 11 Variation of Nusselt number with Rayleigh number for a horizontal circular surface with rectangular surface waves with a dimensionless surface wave height of 0.04 and for a non-wavy circular surface

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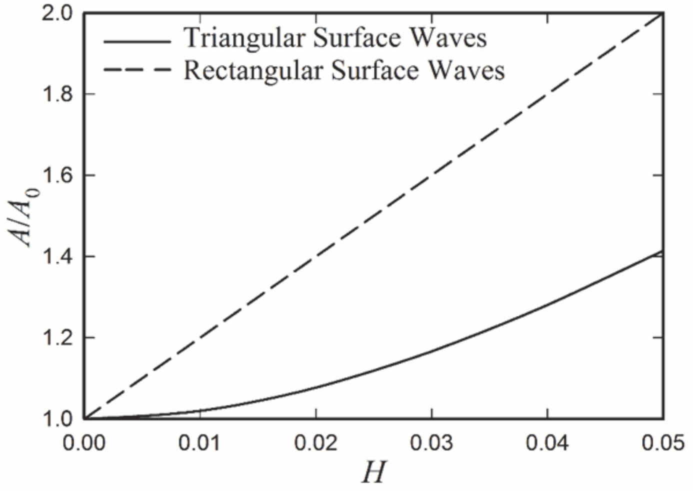

Figure 12 Variation of the ratio of the actual area of the wavy surface to the base area with dimensionless surface wave height for the triangular surface wave case and for the rectangular surface wave case

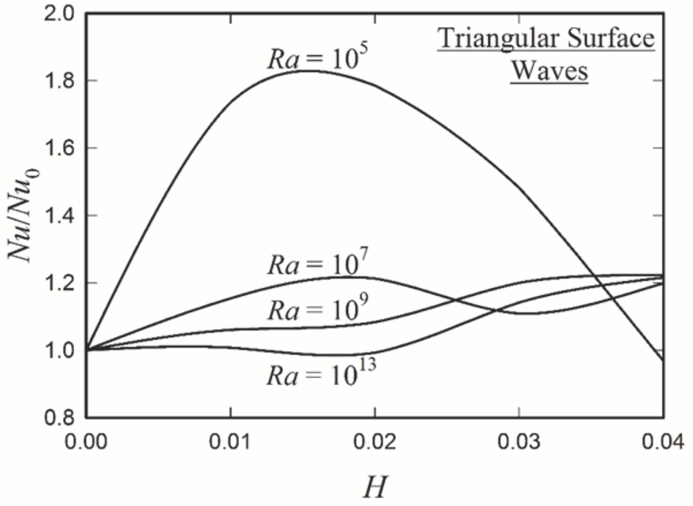

Figure 13 Variation of the ratio of the Nusselt number for a horizontal circular surface with triangular surface waves to the Nusselt number for a non-wavy circular surface under the same conditions with a dimensionless surface wave height for various Rayleigh numbers

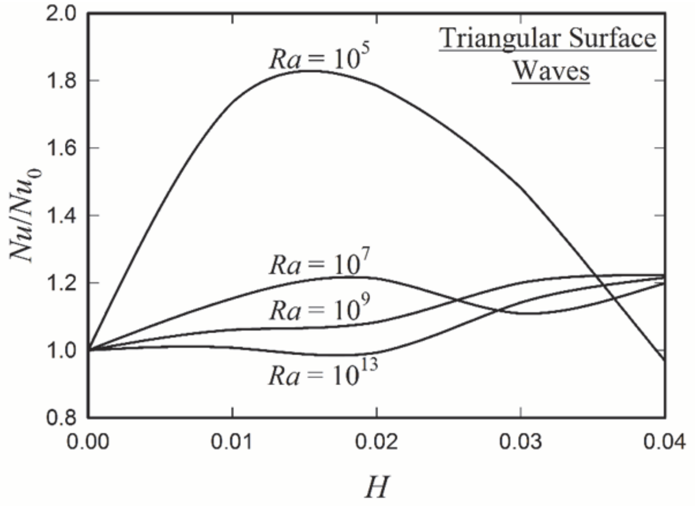

Figure 14 Variation of the ratio of the Nusselt number for a horizontal circular surface with rectangular surface waves to the Nusselt number for a non-wavy circular surface under the same conditions with a dimensionless surface wave height for various Rayleigh numbers

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