ENTROPY GENERATION OF NANOFLUID FLOW OVER A CONVECTIVELY HEATED STRETCHING SHEET WITH STAGNATION POINT FLOW HAVING NIMONIC 80A NANOPARTICLES: BUONGIORNO MODEL

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ABSTRACT

This research investigates characteristics of entropy generation on stagnation point flow of Nimonic 80a nanoparticles over a convectively-heated stretching sheet in porous medium. The Buongiorno model is used for the Nimonic 80a metal nanoparticles of brick shape and includes both tBrownian motion and thermophoresis effect with thermal radiation. Similarity transformation variables are used to simplify the governing flow problem. Numerical solutions for temperature distribution, velocity of fluid, concentration of nanoparticles and entropy profile are established and examined. Moreover, the results obtained from the present methodology are validated when compared with research articles in the existing literature. Excellent agreement is obtained. Expressions for skin friction coefficient and Nusselt number are also taken into consideration and presented via tables.

INTRODUCTION

In recent years, nanotechnology has attracted the attention of distinctive from various authors because of its vast applications in modern technology Furthermore enhancement of heat transfer in mechanical and thermal systems was . observed. Different Newtonian and non-Newtonian base liquids such as oil, ethylene, glycols and water have minimum thermal conductivity. Such fluids have poor heat transfer. For this purpose an ingenious technique has been introduced to improve heat transfer of thermal systems by suspending a homogeneous mixture of ultrafine nanometre-sized particles in the base fluid which enhances conventional heat transfer. These fluids are known as nanofluids. The thermal conductivity of nanoparticles relies mainly on the particle size, shape, volume fraction, temperature and base fluid. Nanoparticles are metal oxide or metals that comprise of unique chemical and physical features. Due to these novel features, nanofluids are very helpful and applicable in active heat transfer. Solar collectors have received significant importance in chemical processing, thermal heating, and power generation. When a certain amount of nanoparticles is suspended in Sa base fluid server of solar collectors, the efficiency of the solar collector is enhanced. Nanofluids are also helpful in enhancing the thermal conductivity of nuclear reactor system, electronic devices, hybrid power engines, chillers, domestic refrigerators, cooling and lubrication of machine parts etc.

Ellahi et al. [1] investigated the influence of MHD and slip on nanofluid (non-Newtonian) flow towards a coaxial porous cylinder. Rosmila et al. [2] examined free convection flow of an MHD nanofluid through a porous stretching sheet using the Lie symmetry group transformation technique. They observed that the impact of thermal stratification and nanoparticle volume fraction is relevant in conventional heat transfer. Sheikholeslami et al. [3] explored analytically laminar flow of a nanofluid through a semi-porous channel in the presence of MHD by least squares and Galerkin methods. They observed that the magnetic field impact with nanoparticle volume fraction and velocity boundary layer thickness has minimal effect on velocity distribution. Javaherdeh and Ashorynejad [4] explored the impact of magnetic field on force convection fluids through a porous partially filled channel using the Lattice Boltzmann method. They determined that an increment in nanoparticle volume fraction enhances the average Nusselt number while it also increases slowly with an increment in the magnetic field. Noor et al. [5] studied flow of a nanofluid with heat transfer past a moving porous surface with coflowing fluid. They combined the two nanofluid models namely, Buongiorno [6] and Tiwari and Das [7] and obtained multiple solutions when free stream and the plate move in opposite directions. Recently, Bakar et al. [8] discussed thermal properties of a nanofluid past a stretching surface using the Buongiorno model.

Many systems dealing with heat transfer with the mechanism of irreversibility illustrate entropy generation and correspond to mass transfer, viscous dissipation, heat transfer and magnetic field. Various researchers/scientists applied the second law of thermodynamics [9-11]. To optimize such irreversibility for instance, Mahmud and Fraser [12] considered MHD free convection flow with entropy generation through a porous cavity. They determined that an increment in magnetic field leads to an increase in entropy generation. Further investigation of entropy generation under the influence of MHD and slip flow on a rotating disk in a porous medium having variable properties was presented by Rashidi et al. [13]. Komurgoz et al. [14] explored entropy generation with magnetic field towards an inclined porous planar channel. It was observed that maximum entropy generation can be obtained in the absence of magnetic field and porosity. A numerical study was conducted by Qing et al. [15] on entropy generation. They discussed Casson fluid flow over a stretching/shrinking porous sheet.

The prime interest of the current communication is to examine entropy generation on stagnation point flow through a connectively heated stretching sheet in a water based nanofluid. Such flows are very important in many engineering processes for example melt-spinning, wire drawing, paper production, aerodynamics suspension of plastic sheets and the production and rubber sheets. The present flow problem is simplified with appropriate use of a similarity transformation and solved by the shooting method. Impact of all physical parameters of interest is discussed numerically.

To the best of our knowledge not many researchers have combined both models in their research as highlighted in our literature review. Furthermore in our model we consider entropy generation with convective boundary condition which has not been considered previously in the combined models.

MATHEMATICAL FORMULATION

Consider stagnation point flow of a nanofluid with incompressible and irrotational features over a stretching porous surface (plate) at $y=0$. We choose a Cartesian coordinate i.e. the plate is considered along the horizontal direction $x-$axis and the $y-$axis is considered normal to the plate (see Fig. 1). It is assumed that at the lower surface, the sheet is heated convectively with temperature T_f with heat transfer coefficient h_f. We combine two mathematical models suggested by Buongiorno [6] and Tiwari and Das [7].

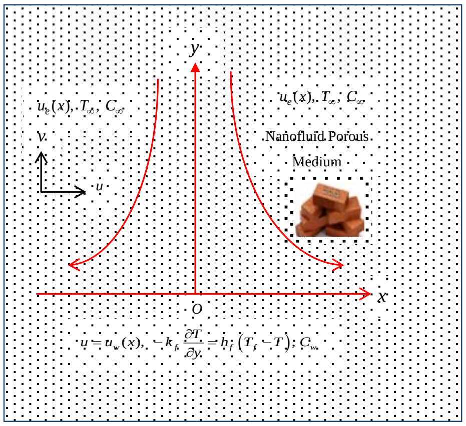

Figure 1 Physical diagram of the problem.

Then the physical equations that govern the flow are written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_{nf}\frac{\partial^2 u}{\partial y^2} + u_\infty \frac{du_\infty}{dx} - \frac{\nu_{nf}}{K_1}\left(u - u_\infty\right), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k^*(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B \frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T^2}{\partial y^2}\right), \tag{4}$$

The boundary conditions are

$$u = u_w(x) = cx, v = 0, -k_f \frac{\partial T}{\partial y} = h_f\left(T_f - T\right), C = C_w \text{ at } y = 0,$$
$$u = u_\infty(x) = ax,\; T \to T_\infty,\; C \to C_\infty \text{ as } y \to \infty. \tag{5}$$

where u and v are velocity components in the $x-$ and $y-$ directions respectively, μ_{nf} is the viscosity of the nanofluid, ρ_{nf} is the density of nanofluid, K_1 is the permeability of the porous medium, T is the temperature, T_∞ is the free stream temperature, C is the concentration of nanoparticles, C_∞ is the ambient concentration of nanoparticles, D_B and D_T are the coefficients of Brownian and thermophoresis diffusion respectively, k_{nf} is the thermal conductivity of nanofluid, $(\rho c_p)_{nf}$ is the specific heat capacitance of nanofluid defined as:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s,\; (\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s,$$
$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},\; \frac{k_{nf}}{k_f} = \frac{\left(k_s + (m+1)k_f\right) - (m+1)\left(k_f - k_s\right)\phi}{\left(k_s + (m+1)k_f\right) + \phi\left(k_f - k_s\right)}. \tag{6}$$

m is the shape factor which is taken to be 3.7 (Nanoparticles type: bricks).

Now, we introduce the similarity transformation:

$$\eta = y\sqrt{\frac{c}{\nu_f}},\; \psi = \sqrt{c\nu_f}\,x f(\eta),\; \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty},\; h(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \tag{7}$$

Here η is the similarity variable, ν_f is the kinematic viscosity of the base fluid, ψ is the stream function.

In view of relation (7), equations (2)-(6) are transformed into the following ordinary differential equations

$$\frac{1}{(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)} f''' + ff'' - f'^2 + B^2 - \frac{1}{(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)} K\left(f' - B\right) = 0\,, \tag{8}$$

$$\left(1+\frac{4}{3}N_r\right)\frac{k_{nf}/k_f}{\Pr\left(1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)}\theta'' + f\theta' + Nb\theta' h' + Nt\left(\theta'\right)^2 = 0\,, \tag{9}$$

$$h'' + Scfh' + \frac{Nt}{Nb}\theta'' = 0\,, \tag{10}$$

subject to the boundary conditions

$$f(0) = S,\; f'(0) = 1,\; \theta(0) = 1,\; h(0) = 1,$$
$$f'(\infty) \to B,\; \theta(\infty) \to 0,\; h(\infty) \to 0. \tag{11}$$

The prime denotes differentiation with respect to η, $\Pr = \mu_f c_p / k_f$ is the Prandtl number, $B = a/c$ is the ratio b/w free stream to stretching velocities, $Nb = \tau D_B \left(C_w - C_\infty\right)/ v_f$ is the Brownian motion parameter, $Nt = \tau D_T \left(T_f - T_\infty\right)/ T_\infty v_f$ is the thermophoresis parameter, $N_r = 4\sigma^* T_\infty^3 / k_{nf} k$ is the thermal radiation parameter, $K = v_f / cK_1$ is the porosity parameter and $Sc = v_f / D_B$ is the Schmidt number.

The important physical quantities of interest are the local skin friction coefficient, the local Nusselt number and the local Sherwood Number and are defined as

$$C_{fx} = \frac{\tau_w}{\rho_{nf} u_w^2}, \quad Nu_x = -\frac{x q_w}{k_f (T_f - T_w)}, \; Sh_x = \frac{x m_w}{D_B (C_w - C_\infty)}, \tag{12}$$

where τ_w is the shear stress in the $x-$direction, q_w is the heat flux and m_w is the mass flux given by

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y}\right)_{y=0}, \; q_w = -k_{nf} \left(\left(1 + \frac{16\sigma T_\infty^3}{3 k k_{nf}}\right)\frac{\partial T}{\partial y}\right)_{y=0}, \quad m_w = -D_B \left(\frac{\partial C}{\partial y}\right)_{y=0}, \tag{13}$$

Using equation (9), we obtain

$$C_f \operatorname{Re}_x^{1/2} = \frac{1}{(1-\phi)^{2.5}} f''(0), \quad Sh_x \operatorname{Re}_x^{-1/2} = -h'(0), \quad Nu_x \operatorname{Re}_x^{-1/2} = -\frac{k_{nf}}{k_f}\left(1 + \frac{4}{3} N_r\right)\theta'(0). \tag{14}$$

where $\operatorname{Re}_x = x u_w(x) / v_f$ is the Reynolds number.

3. Entropy Generation Analysis

The entropy equation of a viscous fluid is written as

$$S_{gen}''' = \frac{k_{nf}}{T_\infty^2}\left[\left(\frac{\partial T}{\partial y}\right)^2 + \frac{16\sigma T^3}{3k}\left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu_{nf}}{T_\infty}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\mu_{nf}}{T_\infty K_1} u^2 + \frac{RD}{C_\infty}\left(\frac{\partial C}{\partial y}\right)^2 + \frac{RD}{T_\infty}\left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial C}{\partial y}\right) \tag{15}$$

Volumetric entropy generation has three factors, (i) Heat Transfer Irreversibility (HTI), (ii) Fluid Friction Irreversibility (FFI) and (iii) Diffusive Irreversibility. It is characterized as

$$S_0''' = \frac{k_f \left(\Delta T\right)^2}{l^2 T_\infty^2} \tag{16}$$

In dimensionless form

$$N_G = \frac{S_{gen}'''}{S_0'''} = \frac{\operatorname{Re} B_r}{\Omega}\left(\frac{\mu_{nf}}{\mu_f}\right) f''^2 + \operatorname{Re}\frac{k_{nf}}{k_f}\left(1 + \frac{4}{3} N_r\right)\theta'^2 + \frac{\operatorname{Re} B_r}{\Omega}\left(\frac{\mu_{nf}}{\mu_f}\right) K f'^2 + \operatorname{Re}\lambda\left(\frac{\varsigma}{\Omega}\right)^2 h'^2 + \operatorname{Re}\lambda\left(\frac{\varsigma}{\Omega}\right)\theta' h' \tag{17}$$

where $\Omega = \Delta T / T_\infty$ is the dimensionless temperature difference, $Br = \mu_f u_w^2 / k_f \Delta T$ is the Brinkman number, $\operatorname{Re}_L = cl^2 / v_f$ is the Reynolds number based on the characteristic length, $\varsigma = \Delta C / C_\infty$ is the dimensionless concentration difference and $\lambda = RDC_\infty / k_f$ is the diffusive constant parameter.

RESULTS AND DISCUSSION

This section explores theoretical and numerical results for all parameters included in the governing equations (8-11). The shooting method in Matlab is used to discuss novelties of the porous medium, nanoparticle volume fraction, radiation parameter, Brownian motion parameter, thermophoresis parameter, Brinkman and Reynolds numbers, respectively. Particularly, impact of these parameters on velocity, temperature distribution, concentration of nanoparticle and entropy profile is investigated. Table 1 represents thermo-physical properties of the base fluid and Nimonic 80a nanoparticles. Furthermore, comparison is also made with previously published work [16-19] by taking $K = 0,\ \phi = 0$ and $N_d = Nt = Nb = 0,\ \gamma \to \infty$ see Tables 2 and 3.

In Table 4 values of the skin friction, the Nusselt number and the Sherwood number versus ϕ for different values of B when $K = 0.5,\ Nb = 0.5,\ Nt = 0.5,\ N_d = 0.5,\ Sc = 1$ are presented.

Comparison with existing literature is made for multiple values of the stretching parameter and Prandtl number. Our results match closely, which assures us of the validity of the current methodology. According to recent studies [20-21], the Prandtl number for our present study is chosen as 6.2 (water) and nanoparticle volume fraction is chosen from 0 to 0.2 $(0 \le \phi \le 0.2)$, whereas $(\phi = 0)$ corresponds to a regular fluid.

Table 1. Thermo-physical features of base fluid and nanoparticles.

Physical properties	Nimonic 80a	Water
$k(W/mK)$	112	0.613
$c_p(J/kgK)$	448	4,179
$\rho(kg/m^3)$	8190	997.1

Table 2: Comparison of $f''(0)$ when $K = 0,\ \phi = 0$ for several values of B.

B	Mahapatra and Gupta [16]	Ishak et al. [17]	Khan et al. [18]	Present
0.01	-	-0.9980	-0.998028	-0.9980
0.1	-0.9694	-0.9694	-0.969387	-0.9694
0.2	-0.9181	-0.9181	-0.918107	-0.9181
0.5	-0.6673	-0.6673	-0.667262	-0.6673
2.0	2.0175	2.0175	2.017487	2.0175
3.0	4.7293	4.7294	4.729260	4.7293

Table 3: Comparison of $-\theta'(0)$ when $N_d = Nt = Nb = 0$, $\gamma \to \infty$ for several values of Pr and B.

Pr	α	Mahapatra and Gupta [16]	Hayat et al [19]	Khan et al. [18]	Present
1	0.1	0.603	0.602156	0.602157	0.6022
	0.2	0.625	0.624467	0.624471	0.6245
	0.5	0.692	0.692460	0.692451	0.6925
1.5	0.1	0.777	0.776802	0.776807	0.7768
	0.2	0.797	0.797122	0.797129	0.7971
	0.5	0.863	0.864771	0.864806	0.8648

Table 4: Values of the skin friction, the Nusselt number and the Sherwood number versus ϕ for different values of B when $K = 0.5,\ Nb = 0.5,\ Nt = 0.5,\ N_d = 0.5,\ Sc = 1$ are fixed.

B	ϕ	$C_f \mathrm{Re}_x^{1/2}$	$Nu_x \mathrm{Re}_x^{-1/2}$	$Sh_x \mathrm{Re}_x^{-1/2}$
0.5	0	-0.7540	0.3419	0.5918
	0.1	-1.0982	0.5396	0.5836
	0.2	-1.5086	0.7691	0.5868
0.8	0	-0.3307	0.3456	0.6751
	0.1	-0.4839	0.5520	0.6683
	0.2	-0.6653	0.7935	0.6703
1.2	0	0.3658	0.3513	0.7699
	0.1	0.5375	0.5666	0.7644
	0.2	0.7395	0.8199	0.7653
1.5	0	0.9750	0.3556	0.8329
	0.1	1.4359	0.5762	0.8283
	0.2	1.9764	0.8363	0.8288

CONCLUSION

In this manuscript, optimization of entropy generation with thermal radiation on stagnation point flow over a convectively-heated stretching sheet in a porous medium has been investigated. Nimonic 80a nanoparticles of bricks shape in a water based nanofluid were suspended using Buongiorno model. Similarity transformations have been applied to model the governing flow problem. The numerical results of the governing flow problem are obtained by using a shooting method. The important outcomes for the current analysis are:

- A higher value of the porosity parameter enhances the velocity profile and nanoparticle volume fraction enhances the velocity of the fluid when $B > 1$ and reduces it when $B < 1$. The temperature distribution and concentration of nanoparticles enhance for both cases $B > 1$ and $B < 1$.
- Temperature distribution and concentration of nanoparticles increase as the thermophoresis parameter increases.
- Increasing values of the Brownian motion parameter lead to increase temperature of the fluid and decrease the concentration of nanoparticles.
- Effect of the radiation parameter markedly boosts the temperature profile as well as the concentration of nanoparticles.
- The thermal and concentration boundary layers thicknesses enhance due to the convective parameter.
- Entropy profile is increased due to greater impact of Reynolds number and Brinkman number.
- The values of the skin friction, Nusselt number and Sherwood number are larger when $B > 1$ compared to $B < 1$.

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