A Lock-In CCD Camera Based Method for Thermal Diffusivity Measurement by an Improved Photothermal Beam Deflection Slope Method

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ABSTRACT
Photothermal beam deflection is a well-established technique for measuring thermal diffusivity. A linear relationship that arises in this technique is that given by the phase lag of the thermal wave as a function of the distance to a punctual heat source when unidimensional heat diffusion can be guaranteed. This relationship is useful in the calculation of the sample’s thermal diffusivity, which can be obtained straightforwardly by the so-called slope method, if the pump beam modulation frequency is well-known. The measurement procedure requires the experimenter to displace the probe beam a given distance from the heat source, measure the phase lag at that offset and repeat this for as many points as desired. This process can be quite lengthy in dependence of the number points. Here we present a detection scheme that overcomes this limitation and simplifies the experimental setup using a Web-cam that substitutes all detection hardware utilizing motion detection techniques and software digital signal post-processing. The used Logitec C920 camera does not have on board lock-in capabilities, so specialized self-referenced lock-in software was developed for this application. Basically, a video is recorded for several pump beam excitation cycles and is post-processed so that a phase image is obtained. The usefulness of the method is demonstrated by measurements of test samples.

The pump beam is focused into a small spot on the sample surface and is modulated in intensity at a given frequency, f, in order to generate thermal waves.

The thermal waves will propagate through the sample and will heat the surrounding media. The refractive index is a physical property which is affected by temperature. Temperature variations in a medium will translate to refractive index variations. A refractive index inhomogeneity will produce a deflection on a light beam going through the medium[3]. The deflection of the light beam can then be sensed with a quadrant photodetector (QPD) or similar device.

One of the ways to determine thermal diffusivity, α, using thermal waves, is through the study of their spatial behavior. In order to sense this, the offset between the probe beam and the pump beam must be varied. The experimenter must adjust this offset and take one measurement for each of the desired number of points capturing the phase and amplitude of the deflection at each point. From the slope of the phase-lag versus offset curve the thermal diffusivity can be obtained straightforwardly if the pump beam modulation frequency is well-known. This procedure can be lengthy. It is our objective in this work to improve on the measuring times by using many probe beams and determining the thermal diffusivity of a sample with only one measurement.

INTRODUCTION
Photothermal beam deflection, or mirage effect, is a tried and tested technique where the behavior of a temperature field is inferred by the changes of the refractive index of its surrounding medium [1]. The modulated orthogonal surface skimming variant [2] utilizes a probe laser beam for sensing and an orthogonally positioned laser pump beam as the heating source.

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THEORY
Fig. 1. Shows schematically a typical experimental configuration. Suppose that the sample with a lower thermal conductivity than that of its surrounding medium ($k_s \ll k_f$) is optically opaque and being periodically heated by a light source, such as a laser beam, within an area as small as possible and that
the plane of study is normal to the object’s surface. Let us also affirm that the refraction index \((n)\) inhomogeneity is directly proportional to a small variation in temperature of an object in thermal contact with the transparent media. The object is also assumed to be semi-infinite in the \(x\) dimension. The spatial distribution of the thermal field generated in the surrounding medium, \(T_y\), is given by\([4]\):

\[
T_y(y) = \frac{P_0}{4\pi k_s} \int_0^\infty \delta l_0(\delta y) \exp\left(-\frac{(\delta\alpha)^2}{4}\right) \frac{1}{\beta_s} \times \Psi d\delta
\]

\[
\Psi = \frac{1 + \exp(-2\beta_s l)}{1 - \exp(-2\beta_s l)} \exp(-2\beta_s l)
\]

where \(P_0\) is the exciting beam power, \(a\) is the beam radius defined at \(1/e^2\) of the beam’s intensity, \(k_s\) is the sample’s thermal conductivity, \(J_0\) denotes the Bessel function of zero order, sub-indexes \(\gamma\) and \(s\) represent the medium and the sample respectively and \(\beta_i = \delta^2 + \frac{i\omega}{\alpha_i} (i = \gamma, s)\), with \(\delta\) as the integration variable.

We may divide the deflection \(\phi\) into two components, a transversal \((\phi_T)\) and a normal one \((\phi_N)\), which are related to the temperature by the following expressions\([4]\):

\[
\phi_T = -\frac{1}{n} \frac{dn}{dT} \frac{P_0}{2\pi K_s} e^{i\omega t} \int_0^\infty \delta \sin(\delta y) e^{-\frac{(\delta\alpha)^2}{4}\beta_s^{-1}} \Psi d\delta \hat{k}
\]

\[
\phi_N = -\frac{1}{n} \frac{dn}{dT} \frac{P_0}{2\pi K_s} e^{i\omega t} \int_0^\infty \delta \cos(\delta y) e^{-\frac{(\delta\alpha)^2}{4}\beta_s^{-1}} \Psi d\delta \hat{j}
\]

where \(n\) is the diffraction index and \(l\) is the sample’s length in the \(z\) direction.

Salazar \textit{et al} \([5]\) have shown that in the case of having a pump and a probe beam with the same diameter and with a minimal height from the sample surface, the phase of the transverse component of the perpendicular deflection and the pump to probe transverse offset, \(y_0\), will have a linear relationship. The straight line’s slope will be given by:

\[
m = -\frac{\pi f}{\alpha_s}
\]

where \(f\) is the pump beam modulation frequency and \(\alpha_s\) the sample’s thermal diffusivity.

**EXPERIMENTAL DETAILS**

When multiple beams are used a more sophisticated sensing technique must be implemented in order to process the information for many beams simultaneously. The multiple probe beams are generated using a diffraction grating, the grating is carefully selected and positioned for each experiment in order to obtain as many beams as possible. In order to intensify the deflection effect the sample is submerged in a liquid with high thermal conductivity, and the straight line’s slope will be given by:

The position tracking of the beams is achieved by segmenting the video into as many sections as beams there are with each segment containing only one beam for individual analysis. This process can be automated by analyzing the periodic variations in intensity through a Fourier transform of the image, peak localization among other methods. Some filtering and
thresholding is needed in order to identify the beams properly. Figure 3 shows an example of such sectioning. When the beam is deflected the lighting distribution in the section changes. If a vertical section midline is established and a comparison is made between the sum or weight of the values of the pixels to the right with those to the left of each frame in the video, then the beam transverse displacement can be easily obtained. This is pretty much the software implementation of a QPD, except that for sake of simplicity we are not processing the normal deflection, which could be obtained with the same process but by drawing a horizontal midline (Fig. 4).

**Figure 3** Sectioning of a multiple beam deflection video, in this case the sample is a CdTe bulk crystal. Not all sections are shown. This image has been digitally enhanced for visualization purposes.

**Figure 4** Deflection of the beam showed in Figure 3, segment 7. The red line shows the middle of the section, this is the boundary from which left and right pixels will be weighed against each other. The plot shows the evolution of said weight with each frame, modulation frequency for this measurement was 1 Hz. Dots represent experimental data while the solid line is merely an interpolation between them.

Once the deflection data has been obtained, lock-in detection can be implemented. Lock-in detection requires a sharp reference signal for it to be effective. In order to perfectly sync this signal with our video, a small number of pixels in each image receive a minor fraction of the light from the pump beam obtained by means of a beam splitter. The sinusoidal reference signal is obtained by applying a Fourier transform to the mean temporal evolution of these pixels and extracting the first harmonic. While it would be in theory possible to simply obtain the Fourier transform of the deflection data and calculate the phase, we have found that using lock-in amplification provides cleaner results.

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Lock-in amplification involves mixing, i.e., multiplying, the experimental data with the reference signal and with an additional signal which has a 90° phase offset from this reference (Fig. 5). The mixed signals are then low-passed filtered and two values are obtained. The in-phase value, I, results from the mixing with the original reference signal while the quadrature value, Q, results from the mixing with the 90° offset signal [6]. The in-phase and quadrature values can then be used to obtain the amplitude of the input signal using the following simple equations:

\[ \varphi = \tan^{-1} \left( \frac{Q}{I} \right) \]  \hspace{1cm} (5)

and

\[ A = \frac{1}{2} \sqrt{I^2 + Q^2} \]  \hspace{1cm} (6)

An important consideration is that the camera’s speed limits the modulation frequency. The Nyquist sampling theorem tells us that the sampling rate of a band-limited signal must be at least twice as high in order to digitally reconstruct the signal properly, in our case, when working at full frame the maximum sampling frequency is 20 Hz, so modulation frequencies must be below 10 Hz [7]. A faster (but much more expensive) camera would allow for higher sampling frequencies, but the use of low frequencies can give appropriate results in this kind of experiments, as will be seen in the next section. Furthermore, at low frequencies 1/f noise becomes problematic, and a 4-30 Hz frequency range has been suggested by Bertolotti et al. for the conventional beam deflection technique [8].

**RESULTS AND DISCUSSION**

Three samples with known thermal diffusivities were chosen for measurement. Figure 6 shows a typical experimental result, in this case a CdTe crystal. For each measurement an optimal measuring frequency is experimentally obtained. Although higher frequencies are desired, these also mean a reduction in the signal’s amplitude, so that a compromise must be made.

<table>
<thead>
<tr>
<th>Material</th>
<th>Measured Diffusivity</th>
<th>Reported Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>AISI 1018 steel</td>
<td>0.001 cm²/s°C</td>
<td>0.001 cm²/s°C</td>
</tr>
<tr>
<td>Cadmium telluride</td>
<td>0.004 cm²/s°C</td>
<td>0.004 cm²/s°C</td>
</tr>
<tr>
<td>AISI D2 steel</td>
<td>0.003 cm²/s°C</td>
<td>0.003 cm²/s°C</td>
</tr>
</tbody>
</table>

Table 1 shows the measured diffusivity values. The values obtained for the AISI 1018 steel and the cadmium telluride film are in good agreement with the reported literature value. A deviation exists from the D2 steel measured value and the reported value, this is likely due to normal variation between manufacturers since the AISI D2 standard. As for the high apparent error in the cadmium telluride measurement, this is due to a limited number of fitting points due to a relatively small sample (<1.5 mm).
Figure 6  Experimental result for a CdTe sample, modulation frequency is 4 Hz. The dots show experimental data, the solid line the best linear fit and the dash lines show the 95% confidence interval for said fit.

Table 1  Results

<table>
<thead>
<tr>
<th>Sample</th>
<th>Measured $\alpha$-value</th>
<th>$\alpha$-literature value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AISI 1018 low carbon steel</td>
<td>$(1.3 \pm 0.3) \times 10^{-5}$ m$^2$/s</td>
<td>$1.33 \times 10^{-5}$ m$^2$/s [8]</td>
</tr>
<tr>
<td>D2 high carbon steel</td>
<td>$(3.9 \pm 0.6) \times 10^{-6}$ m$^2$/s</td>
<td>$4.0 \times 10^{-6}$ m$^2$/s [9]</td>
</tr>
<tr>
<td>CdTe</td>
<td>$(3.4 \pm 1.1) \times 10^{-6}$ m$^2$/s</td>
<td>$3.35 \times 10^{-6}$ m$^2$/s [10]</td>
</tr>
</tbody>
</table>

CONCLUSIONS

We have improved the photothermal beam deflection technique in its surface skimming variant by utilizing multiple laser beams and simultaneously sensing the refraction index changes at multiple points. The improvement utilizes digital signal and video processing techniques, replacing commonly utilized hardware, such as a QPD, with a digital video camera. Transversal deflection tracking was successfully implemented and amplified using a lock-in procedure. The method was validated by measuring three samples with known thermal diffusivities. The difference percentage between measured and literature values are less than 2%.

Acknowledgements

This work was partially financially supported by research grants SIP-IPN 1638 (projects 20140006 and 20150390) and CONACyT. The support from COFAA-IPN through the BEIFI and SIBE Programs is also acknowledged. EM and HC would like to thank the Consejo Nacional de Ciencia y Tecnología CONACYT for funding this work through the research project 264093.

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