DYNAMIC ANALYSIS OF HELICAL FIN WITH FIXED FIN TIP TEMPERATURE BOUNDARY CONDITION

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ABSTRACT

A dynamic analysis is performed on a traditional helical fin; but, with fixed fin tip temperature, rather than adiabatic fin tip boundary condition. A time dependent solution is provided. The final version of the solution is presented in an analytical and closed form equation. The problem is solved using Laplace transforms for partial differential equations. The complete governing equation is extensively in the form of the Bessel function. It is shown that the dynamic equation, when time reaches infinity, resolves to the steady state solution for the same problem.

INTRODUCTION

Traditionally, the helical fin is modelled using the adiabatic fin tip condition. This work investigates the same problem but with fixed tip temperature condition. The solution is derived from a microscopic and unsteady state energy balance. Differential equations, Laplace transforms, and dimensionless variables are all utilized in finding the solution.

The fin is modelled and shown in Figures 1 and 2, along with all the required and defined physical parameters.

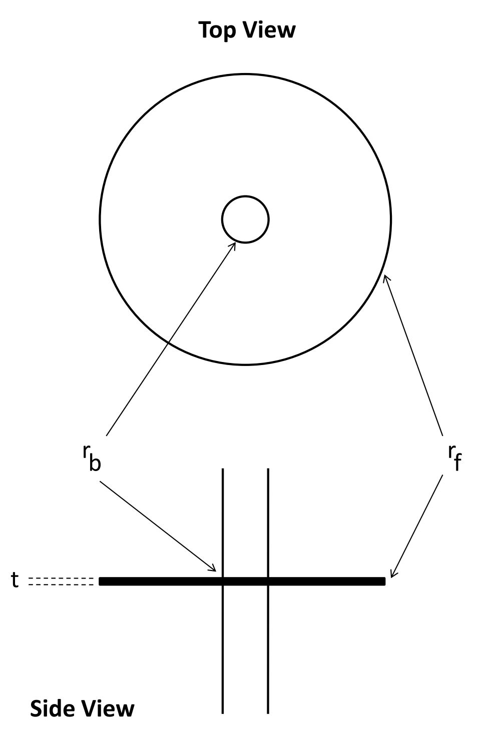

Figure 1 Top and side view of helical fin.

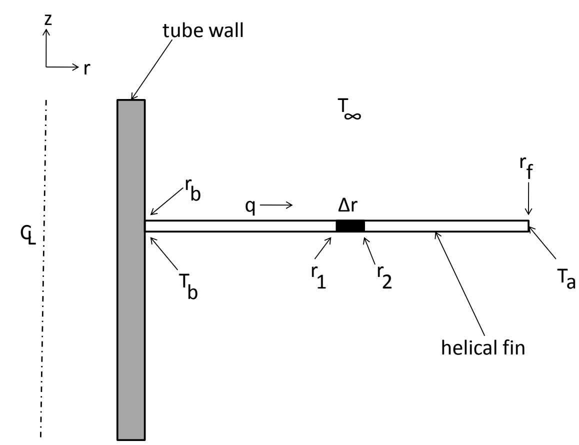

Figure 2 Physical dimensions of helical fin.

NOMENCLATURE

A	area
c_p	heat capacity
e	Euler's number, 2.7182 . . .
h	convection heat transfer coefficient
i	$(-1)^{0.5}$
J	Bessel function of the 1st kind
I	modified Bessel function of the 1st kind
K	modified Bessel function of the 2nd kind
k	thermal conductivity
l	fin length, $r_f - r_b$
q	heat transfer rate
R	dimensionless radius, $(r - r_b)/(r_f - r_b)$
r	radial coordinate
S	Laplace variable
s	time
T	temperature
t	fin thickness
y	$R+\lambda$
z	axial coordinate
α	thermal diffusivity, $k/\rho\text{-}c_p$
α_n	n^{th} pole of θ
Δ	change
η	fin efficiency
Θ	dimensionless temperature, $(T - T_\infty)/(T_b - T_\infty)$
θ	defined in equation 5
Λ^2	$2hl^2/kt$
λ	$r_b/(r_f - r_b)$
π	pi, 3.1415 . . .

ρ density
τ dimensionless time, $\alpha s/(r_f - r_b)^2$

Subscripts

a arbitrary
b base
c cross sectional
cond conduction
conv convection
f fin tip
max maximum
s surface
∞ bulk air conditions

ENERGY BALANCE

An energy balance is performed on the fin shown in Figures 1 and 2 in conjunction with equations 1 and 2.

$$q_{cond} = -kA_c \frac{dT}{dr} \quad (1)$$

$$q_{conv} = hA_s(T(r) - T_\infty) \quad (2)$$

The energy balance yields equation 3.

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{2h}{kt}(T - T_\infty) = \frac{1}{\alpha}\frac{\partial T}{\partial s} \quad (3)$$

Equation 3 is then put into dimensionless terms:

$$\frac{\partial^2 \Theta}{\partial y^2} + \frac{1}{y}\frac{\partial \Theta}{\partial y} - \Lambda^2 \Theta = \frac{\partial \Theta}{\partial \tau} \quad (4)$$

$\Theta(y,0) = 1$
$\Theta(\lambda,\tau) = 1$
$\Theta(1+\lambda,\tau) = \Theta_a$

where $y = R + \lambda$, and R ranges from 0 to 1.

Equation 4 is then solved using Laplace transforms where the transform is defined in equation 5:

$$\theta = \int_0^\infty \Theta e^{-S\tau} d\tau \quad (5)$$

After applying the boundary conditions, the general solution for θ is obtained using the Bromwich integral, or otherwise known as Cauchy's residue theorem:

$$\Theta = \frac{1}{2i\pi}\int \theta e^{S\tau} dS = \sum_n \operatorname{Res}\left[\theta e^{(\tau\alpha_n)}\right] \quad (6)$$

RESULTS

The solution to equation 4 is given in equations 7 and 8. The steady state solution to equation 4 is given by equation 7, Carranza and Ospina [1]. Figure 3 is a plot of equation 7 utilizing an example problem (Example 1.3, physical dimensions and transport properties are borrowed) given by Kraus *et al*. [2] For all cases in this work, $\Theta_a = 0$, for simplicity. Note that Λ^2 is the Biot number.

$$\Theta = \frac{[K_0(\Lambda(1+\lambda)) - \Theta_a K_0(\Lambda\lambda)] I_0(\Lambda(R+\lambda))}{K_0(\Lambda(1+\lambda)) I_0(\Lambda\lambda) - I_0(\Lambda(1+\lambda)) K_0(\Lambda\lambda)} + \frac{[\Theta_a I_0(\Lambda\lambda) - I_0(\Lambda(1+\lambda))] K_0(\Lambda(R+\lambda))}{K_0(\Lambda(1+\lambda)) I_0(\Lambda\lambda) - I_0(\Lambda(1+\lambda)) K_0(\Lambda\lambda)} \quad (7)$$

Thus, the dynamic problem is simply equation 7 plus equation 8. Equations 7 and 8 are determined from equation 6. Equations 7 and 8 are plotted in Figure 4.

$$\sum_{n=1}^{\infty} -2i\lambda e^{-(\alpha_n^2+\Lambda^2\lambda^2)\,\tau/\lambda^2} \begin{bmatrix} \Theta_a \alpha_n^2 (f1 - f2) + (-f1 + f2 + f3)(\alpha_n^2 + \Lambda^2\lambda^2) \\ + (f5 - f6)(\Lambda^2\lambda^2) + f4(\alpha_n^2 - \Lambda^2\lambda^2) \end{bmatrix} / \begin{bmatrix} -2i\lambda^3\Lambda^2 f3 + 2i\lambda^3\Lambda^2 f4 - 4i\alpha_n^2\lambda f3 + 4i\alpha_n^2\lambda f4 - \\ \alpha_n^3\lambda f10 + \alpha_n^3 f7 + \alpha_n^3\lambda f7 + \alpha_n^3 f8 + \alpha_n^3\lambda f8 - \\ \alpha_n^3\lambda f9 - \alpha_n\Lambda^2\lambda^3 f10 + \alpha_n\Lambda^2\lambda^2 f7 + \alpha_n\Lambda^2\lambda^3 f7 + \\ \alpha_n\Lambda^2\lambda^2 f8 + \alpha_n\Lambda^2\lambda^3 f8 - \alpha_n\Lambda^2\lambda^3 f9 \end{bmatrix} \quad (8)$$

where

$a = \alpha_n(R+\lambda)/\lambda$
$b = \alpha_n(1+\lambda)/\lambda$
$f1 = I_0(ai)K_0(\alpha_n i)J_0(\alpha_n)$
$f2 = I_0(\alpha_n i)K_0(ai)J_0(\alpha_n)$
$f3 = I_0(bi)K_0(\alpha_n i)J_0(\alpha_n)$
$f4 = I_0(\alpha_n i)K_0(\alpha_n i)J_0(b)$
$f5 = I_0(ai)K_0(\alpha_n i)J_0(b)$
$f6 = I_0(bi)K_0(ai)J_0(\alpha_n)$
$f7 = I_1(bi)K_0(\alpha_n i)J_0(\alpha_n)$
$f8 = I_0(ai)K_1(bi)J_0(\alpha_n)$
$f9 = I_1(\alpha_n i)K_0(\alpha_n i)J_0(b)$
$f10 = I_0(bi)K_1(\alpha_n i)J_0(\alpha_n)$

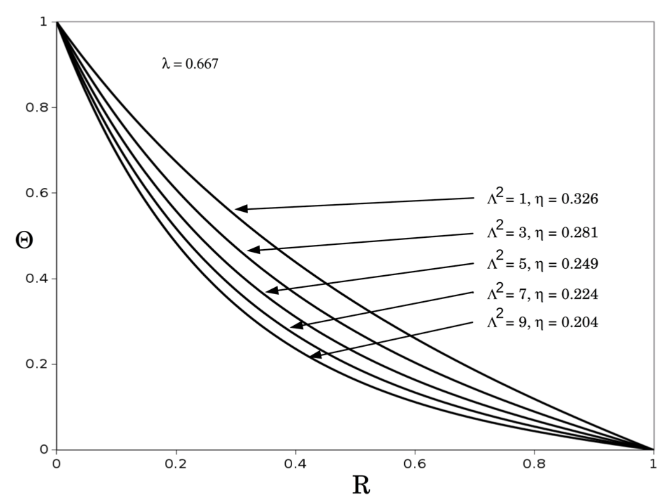

Figure 3 Steady state solution of equation 4.

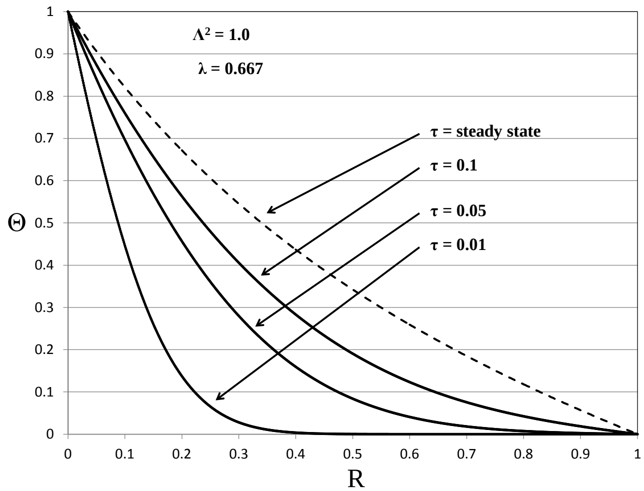

Figure 4 Dynamic solution of equations 7 and 8

REFERENCES

[1] Carranza R.G. and Ospina J., Fin Efficiency of Helical Fin with Fixed Fin Tip Temperature Boundary Condition, *Proceedings of the 17th International Conference on Fluids and Thermal Engineering*, Rome, Italy, September 2015.

[2] Kraus A.D., Aziz A., Welty J., *Extended Surface Heat Transfer*, John Wiley and Sons, 2001.