THE INTERACTION BETWEEN TWO SPHERES FALLING IN A VISCOELASTIC FLUID

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NOMENCLATURE

x	[m]	Cartesian horizontal axis direction
z	[m]	Cartesian vertical axis direction
t	[s]	Time
D	[mm]	Sphere diameter
S	[mm]	Separation between spheres centers
td	[s]	Elapsed time between spheres releasing
P1G60		Test fluid with 1% concentration in mass of polyacrylamide and 60% of glycerin
P1G90		Test fluid with 1% concentration in mass of polyacrylamide and 90% of glycerin
P2G60		Test fluid with 2% concentration in mass of polyacrylamide and 60% of glycerin

Subscripts

1	First sphere released
2	Second sphere released
i	Initial value
C	Critical value

ABSTRACT

The time evolution of the distance between two identical spheres falling in a viscoelastic fluid has been analyzed experimentally. The working fluid was a dilute solution of polyacrylamide in an aqueous solution of glycerin. Several concentrations of polyacrylamide and glycerin have been used in order to analyze fluids with different viscoelastic properties. Two experimental configuration allow analyzing the effect of the boundary conditions. A CCD camera was used to record the movement of the spheres allowing the computation of evolution of the space separation between them. It has been found that beyond certain critical initial elapsed time (td_c) the distance between the spheres grows in time, while the distance decrease for elapsed times smaller than the td_c. Reduction of the shear-thinning effect tends to eliminate the repulsion interaction.

INTRODUCTION

The interaction between particles falling in a non newtonian fluid has been a subject of great attention in the last years. The sedimentation of particles in viscoelastic fluids is of interest for fundamental reasons as well as for its relevance in practical applications. In many industrial processes, for example in the production of food, cosmetics, paints, etc, there are concerned materials largely dependent on the microstructure developed during the flow of the suspension. Moreover, in the area of biology, the problem of small clots interacting with the blood flow in which it is immersed [1; 2], has similar characteristics since blood is a viscoelastic fluid of the type shear-thinning. Experiments studying particle settling has been conducted for a variety of viscolastic fluids, with very different results in relation with the characteristics of the particle-particle interactions. Riddle et al. [3] presented the results of an experimental investigation studying the falling of two spheres along the same vertical line in shear-thinning viscoelastic fluids. These authors found that for all the fluids considered, the spheres attract if they are initially close and separate if the initial separation is larger than a critical distance [4]. Daugan et al. [5] conducted similar experiments also with shear-thinning fluids and did not observed repulsion between the spheres. They found that the spheres come together if the initial distance is less than a critical distance and move at the same velocity if the separation is larger than this critical distance [6]. The presence of large values of normal stress differences change the characteristics of the particle-particle interactions. Bot et al [7] investigated the motion of spheres falling in a fluid of constant viscosity (Boger fluid). They observed that spheres achieve a unique stable final distance, independent of the initial separation distance. This means that the spheres repel if the initial distance is less than the stable distance and attract otherwise. The behavior of the interaction is the opposite to the one reported by Riddle et al. [3]. Some authors concluded that the experimental observations suggest that shear-thinning effects favors the aggregation of spheres while fluid elasticity causes the spheres to separate [7]. However, this simple rule does not allow to explain all the mentioned experimental observations. These facts show that the mechanism of interaction between spheres it is still not well understood. In particular, how takes place the crossover from a shear-thinning fluid to a Boger fluid it is not known.

The present work study experimentally the dependence of the interaction between two spheres falling along a centre line of a tube with the polymer concentration and solvent viscosity.

EXPERIMENTAL SETUP

The releasing used spheres are made of steel with diameters of 1 mm and 1.6 mm. The studied fluids consist on shear-thinning solutions of Polycarylamide (92560-50G Sigma-Aldrich) in mixtures of water and glycerin (Tab.1).

Table 1. Polyacrylamide and glycerin percentage of mass concentration for each test fluid.

Fluid	Polyacrylamide (%)	Glycerin (%)
P1G60	1	60
P1G90	1	90
P2G60	2	60

The rheology of the different fluids was performed with a Anton Paar Physica MCR 301 rheometer, using a coaxial cylinder measuring system. The different flow curves were obtained working at 20°C and in the range of $\dot{\gamma}$ from 1 to 100 s $^{-1}$. The measured shear stress (τ) as a function of the shear rate ($\dot{\gamma}$) for the different fluids are presented in Fig. 1.

Power law index obtained from the data fitting to the Eq. (1) are show in Tab. 2.

$$\tau = b\dot{\gamma}^n \qquad (1)$$

We notice that the case $n = 1$ corresponds to a constant viscosity fluid. From Tab. 2, P1G90 presents the value of n more close to 1, therefore its properties are the most similar to those of a Boger fluid among the considered fluids.

The experimental setup comprises the fluid container and the release mechanism (Fig. 2). The recipients are prismatic and made of plexiglass, with square cross section in the horizontal plane and being its longer side on the z coordinate.

A scheme of the release mechanism is shown in Fig. 3. It consists of a horizontal cylindrical pipe and a piston with plain

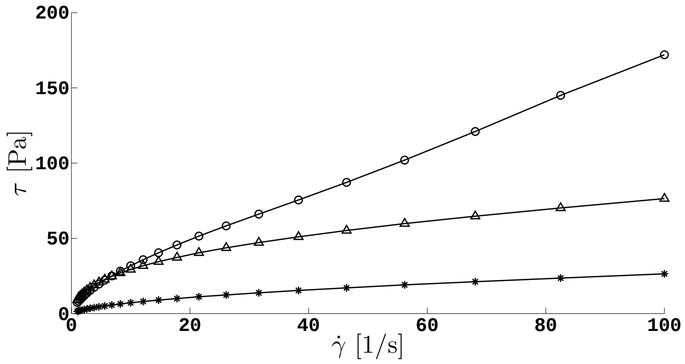

Figure 1. Rheological study of the dependence of τ with $\dot{\gamma}$ for the three fluids considered: P1G60 (asterisks), P2G60 (triangles) and P1G90 (circles).

Table 2. Values of the power law index from Eq. (1) obtained from the data fitting of the rheological properties of the fluids, and the corresponding values of the coefficient of determination R^2 for each fitting.

Fluid	b $[Pa \cdot s^n]$	n	R^2
P1G60	1.9	0.57	0.9994
P1G90	5.3	0.74	0.9945
P2G60	10.7	0.43	0.9989

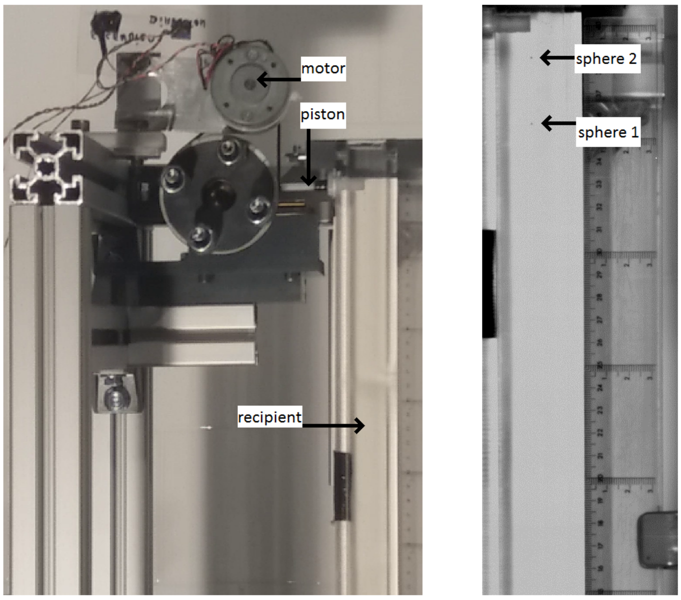

Figure 2. Experimental setup: on the left, a photography of the experimental apparatus; on the right, an image taken of the settling of the spheres during one experiment

Table 3. Dimensions of the two prismatic recipients.

Recipient	Length (cm)	Cross section (cm×cm)
A	70	3×3
B	25	5×5

top and fitting the internal diameter of the pipe. They are fully immersed into the test fluid and positioned nearby the fluid free surface. The spheres are positioned into the pipe and the movement of the piston pushes them to fall. The motor moves the piston smoothly in two steps (pulses). The time between pulses is known and controlled by a National Instruments interface (NI-6216). For each of these steps one of the spheres is dropped through the center axis of the recipient.

The experiments are recorded on a CCD camera (a typical obtained image is shown on the right in Fig. 2). The horizontal and vertical coordinates (x,z) of the settling spheres as a function of time can be obtained from the digital recorded images.

It was taken into account the reproducibility of the experiments comparing the results from realizations with different resting time of the fluid and measuring the separation rate between the spheres as a function of time for the same value of the elapsed time between releasing the two spheres (td). We found that an interval of about 5 hours is needed to wait between experiments. The temperature of the fluid was kept around 21°C attempting to remain the fluid properties unchanged during the experiments.

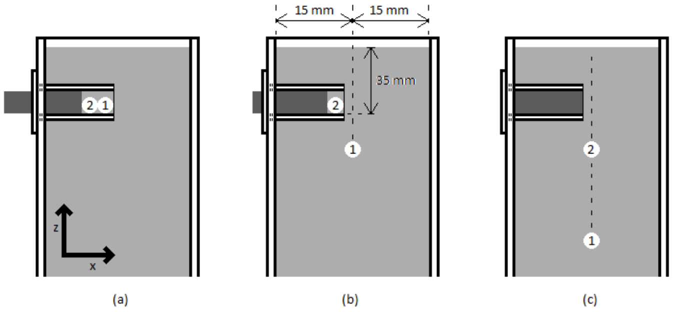

Figure 3. Scheme of the central vertical plane of the experimental apparatus as seen from the CCD camera. The three draws correspond to different stages along a typical measurement: (a) before the releasing of the first sphere; (b) after the first pulse of voltage hast been sent to the driven motor and the first sphere has fallen; (c) after the second pulse was sent and the two spheres are both falling

RESULTS

We have investigated experimentally the interaction between two settling spheres for different conditions: varying the sphere diameters, recipient lengths, and polymer and glycerin concentrations. The separation of the spheres is computed from the two-dimensional distance measured (x,z) of the spheres, *i.e.*: $S = \sqrt{(x_1 - x_2)^2 + (z_1 - z_2)^2}$. The initial separation between the spheres (S_i) is modified by changing td. The horizontal relative position $x_1 - x_2$ (subscripts 1 and 2 refer to the two spheres, as shown in Fig. 3) is also computed to check the presence of lateral movement. In this case, only fluctuations of less than $D/2$ are observed and then the separation between the spheres is mainly due to the difference between vertical coordinates $z_1 - z_2$.

The results are shown in terms of non-dimensional variables. For that, every length was divided by the corresponding sphere diameter (D), the vertical velocity (v_z) was divided by the terminal vertical velocity of a single sphere falling in the fluid (U^*), that was previously measured for each fluid, and the time was divided by $t^* = D/U^*$.

A first set of experiments was made in the recipient A, using the test fluid P1G60 and spheres with $D = 1$ mm. After several experiments an approximate value of the transition elapsed time is found. It can be seen in Fig. 4 that when the initial separation between the spheres is near a critical value $S_c/D \approx 48$, the separation tends to remain constant in time. Below this value ($S_i < S_c$) spheres approach, denoting an attraction regime. The attraction is stronger for smaller initial separation. On the other hand, over the critical separation ($S_i > S_c$), the distance between the spheres tends to grow, denoting a repulsion regime. This qualitative behavior is in agreement with those obtained by other authors like Riddle et al. [3] and Joseph et al [4]. To obtain more information about how operates the interaction between the spheres, we considered the vertical velocities of the spheres for the case $S \approx S_c$, which are shown in Fig. 5. It can be seen that there is a mechanism that slows down the sphere 2 when approaching to sphere 1 while, at the same time, sphere 1 accelerates. After some time, the former sphere accelerates and the other decreases its speed, getting closer again. The result of this process is that the separations between the spheres remains approximately constant. In Fig. 6 vertical velocities of the sphere for the case $Si > Sc$ are shown, corresponding to curve a in Fig. 4. It can be observed that while initially the velocity of the sphere 2 approaches the velocity of the sphere 1, both velocities tend to two different constant values afterwards. Since the sphere 1 falls faster, they keep on separating. In Fig. 7 are shown the velocities for case $S_i < S_c$, curve e in Fig. 4. We can see how they approach exhibiting correlations trough oscillations that are in phase. The sphere 2 always moves faster, producing its approaching to the sphere 1.

The second set of measurements corresponds to the test fluid P2G60, recipient A and spheres with diameter $D = 1$ mm. In Fig. 8 it can be easily recognized the transition between repulsion and attraction behaviors about an initial separation of $S_i/D \approx 12$. The critical separation (S_c) observed for the fluid P1G60 is reduced a factor of 4 in the case of the fluid P2G60. This dependence of the critical separation with the polymer concentration is in agreement with the results obtained previously by Daugan et al [5]. In the case of our test fluids, when we increased the polymer concentration, the characteristic time is reduced [8] and then the critical separation is also reduced.

To check the influence of the wall effects on the behaviour of the falling spheres, experiments using the P2G60 fluid, the

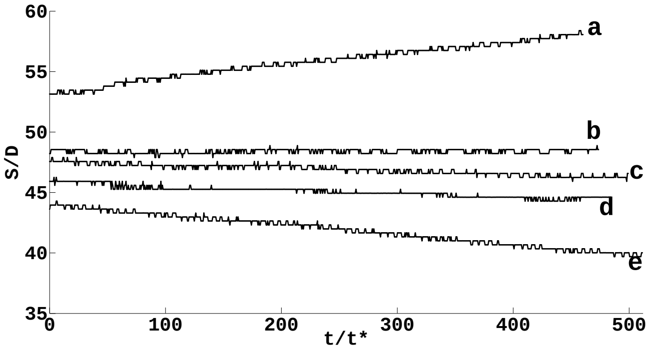

Figure 4. Non-dimensional separation S/D between the spheres as a function of the non-dimensional time t/t^* for the test fluid P1G60, $D = 1$ mm, recipient A. The value of the initial separation for each curve is: a) $S_i/D \approx 53.1$, b) $S_i/D \approx 48.2$, c) $S_i/D \approx 47.6$, d) $S_i/D \approx 45.9$ and e) $S_i/D \approx 43.6$.

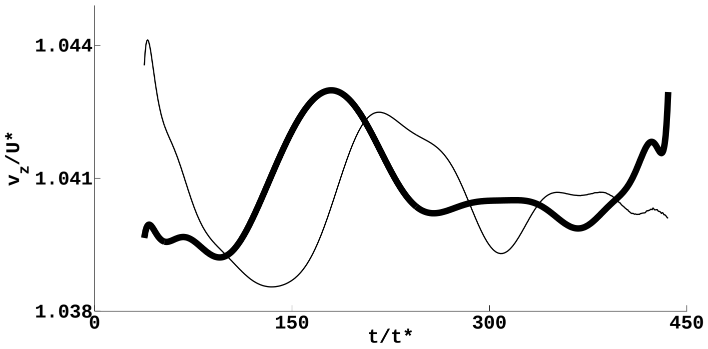

Figure 5. Non-dimensional vertical velocities of the spheres for the curve b ($S_i \approx S_c$) in Fig. 4. The thick line corresponds to the sphere 1 and the thin one corresponds to the sphere 2.

spheres of diameter 1 mm and the recipient B were performed. In this case the cross section is about 5x5 cm instead of 3x3 cm from the container A (increasing the area in about 3 times). In Fig. 9 are shown the results, having found again $S_c/D \approx 12$, and in Fig. 10 are shown all the results obtained for the P2G60 test fluid (for both vessels, A and B), showing an excellent agreement. Thus, according to our observations, the effect of the walls are unimportant because when the spheres-wall distance is modified, when is modified the size of the container, notorious changes in the critical distance are not detected.

The effect of the spheres size was also analyzed. A set of experiments with spheres of diameter $D = 1.6$ mm settling into the P1G60 test fluid in container A were performed. Some results are shown in Fig. 11. Again there exists an initial critical separation.

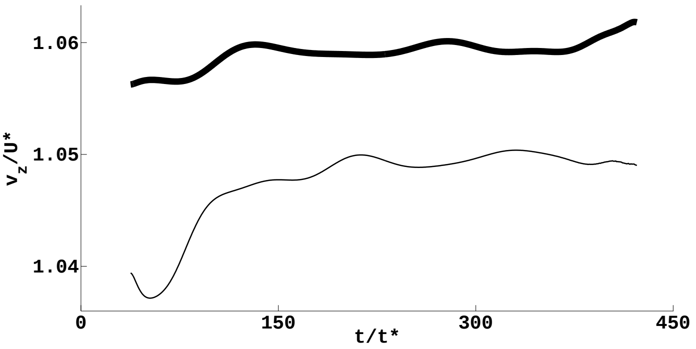

Figure 6. Vertical velocities of the spheres for the curve a ($S_i > S_c$) in Fig. 4. Same references than Fig.5

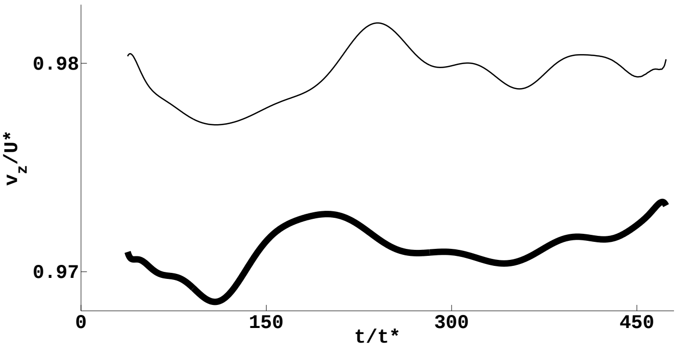

Figure 7. Vertical velocities of the spheres for the curve e ($S_i < S_c$) in Fig. 4. Same references than Fig.5

From the plot it is hardly possible to extract the exact transition value, but over 70 mm it is observed how the attraction effect is strongly diminished. In a different way than the $D = 1$ mm case where the critical separation is $S_c \approx 48$ mm, here the transition takes place after a quite larger initial separation after increasing the spheres diameter in about half a time.

As already mentioned, an interesting and barely investigated phenomena is the transition from viscoelastic to Boger fluid. In the present work we reduced the viscosity variations by adding more glycerin to the mixture. Fig. 12 shows the results for the P1G90 test fluid in the vessel B. It can be observed how attraction effect has been notoriously reduced with respect to the P1G60 test fluid (Fig. 4). In that case, above $S_i/D \approx 10$ it is hardly seen any attraction effect. Moreover, for an initial separation of $S_i/D \approx 7$ the separation between spheres is reduced in an amount of 1 to 2 diameters along the whole fall. When the initial separation is above the critical separation, no attraction is observed, in the same way as was observed by Daugan et al, *i.e.*, attrac-

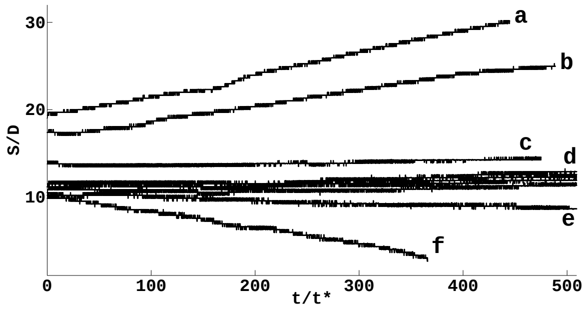

Figure 8. Separation between the spheres as a function of time for the test fluid P2G60, $D = 1$ mm, recipient A. The value of the initial separation for each curve is: a) $S_i/D \approx 19.0$, b) $S_i/D \approx 17.4$, c) $S_i/D \approx 13.8$, d) $S_i/D \approx 10.5, 11.2, 11.4$ (group of three curves) and e) $S_i/D \approx 9.8$

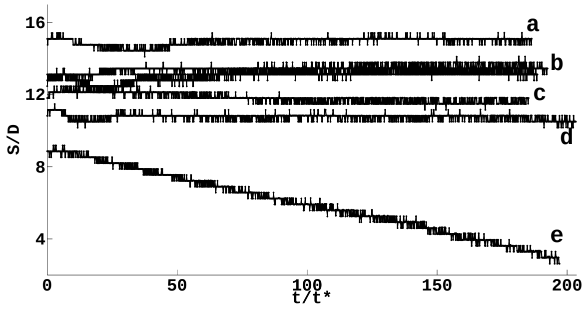

Figure 9. Separation between the spheres as a function of time for the test fluid P2G60, $D = 1$ mm, recipient B. The value of the initial separation for each curve is: a) $S_i/D \approx 15.1$, b) $S_i/D \approx 13.1, 12.5$ (group of two curves), c) $S_i/D \approx 11.8$, d) $S_i/D \approx 10.8$ and e) $S_i/D \approx 8.9$

tion only for small initial separations without interaction when $S_i > S_c$. So that, we obtained a change in the behavior of the fluids from that observed by Riddle et al [3] and the observed by Daugan et al [5]. The attraction reduction effect caused by the stabilization of the viscosity is in agreement with observations using Bogger fluids, in which case for small initial separation repulsion effects are observed instead of attraction [7]. However 90% of glycerin concentration seems to not be enough for observing this transition.

CONCLUSION

In this work we study the settling of two spheres in shear-thinning viscoelastic fluids along the center line of prismatic containers. We considered the dependence of the interaction between the spheres with the polymer concentration, solvent viscosity and sphere diameter. Two containers of different cross section were used to estimate the wall effects. For the low solvent viscosity (60% glycerin) we found a behavior that is in agreement with those obtained previously for some shear-thinning fluids. The spheres attract if their distance S_i is less than a critical value S_c and repel if S_i is larger than S_c. This behavior is similar for all sphere diameters and containers considered. This critical distance S_c has found to be dependent on the fluid properties and the sphere diameter D. S_c decreases with increasing the polymer concentration while increases with sphere diameter D. When used the same fluid and diameter in different containers, the critical distance S_c is almost the same, showing that in our experiments the wall effects are negligible. We considered the influence of reducing the shear-thinning effect increasing the glycerin concentration (fluid P1G90), in order to approach the fluid properties to those of Boger fluids. A qualitative change behavior was observed with this variation, since the repulsion in-

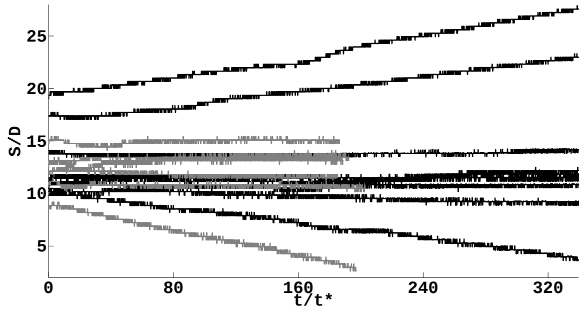

Figure 10. Separation between the spheres as a function of time for P2G60, $D = 1$ mm, for different values of S_i/D and for both recipients A (black lines) and B (grey lines)

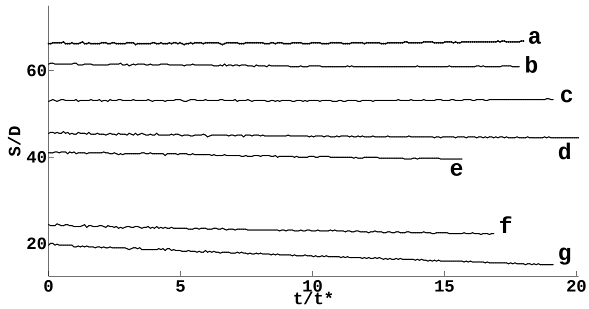

Figure 11. Separation between the spheres as a function of time for the test fluid P1G60, $D = 1.6$ mm, recipient A. The value of the initial separation for each curve is: a) $S_i/D \approx 66.3$, b) $S_i/D \approx 61.5$, c) $S_i/D \approx 52.9$, d) $S_i/D \approx 45.5$, e) $S_i/D \approx 41.0$, f) $S_i/D \approx 24.4$ and g) $S_i/D \approx 19.7$

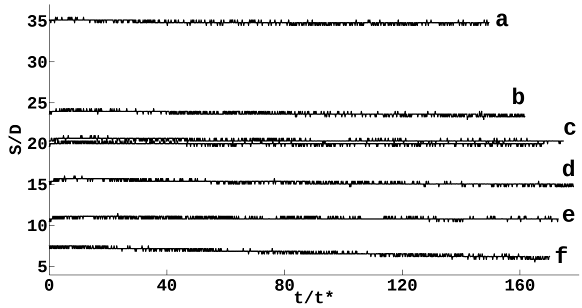

Figure 12. Separation between the spheres as a function of time for the test fluid P1G90, $D = 1$ mm, recipient B. The value of the initial separation for each curve is: a) $S_i/D \approx 35.1$, b) $S_i/D \approx 23.6$, c) $S_i/D \approx 20.3$, 19.7 (group of two curves) d) $S_i/D \approx 15.4$, e) $S_i/D \approx 10.5$ and f) $S_i/D \approx 7.6$

teraction was suppressed. In P1G90, the spheres attract if $S_i > S_c$ and move at a constant separation otherwise. We then obtained a change between different regimes that were observed in the past in two different fluids: A) attraction at $S < S_c$, repulsion $S > S_c$ [3] to B) attraction for short S and weak attraction at large S [5]. To the best of the authors acknowledge, a transition between two of these cases were not reported in previous experimental studies. The attraction that takes place for $S_i < S_c$ in P1G90 is less than those in P1G60. This effect due to viscosity stabilization is in agreement with observations using Boger fluids, in which case repulsion interactions are observed instead of attraction when spheres are close. However, it seems that 90% of glycerin concentration seems to not be enough for observing total suppression of attractive interaction when the spheres are at small separation. This will be a subject of future work.

ACKNOWLEDGMENT

The work has been supported by the Spanish Ministerio de Economía y Competitividad under project CTQ201346799-C21-P, and by Uruguayan institutions Comisión Sectorial de Investigación Científia (CSIC) and Programa de Desarrollo de las Ciencias Básicas (PEDECIBA).

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