# THE DEVELOPMENT OF MATHEMATICAL MODELING FOR NANOFLUID AS A POROUS MEDIA IN HEAT TRANSFER TECHNOLOGY

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# ABSTRACT

Nanofluids as combinations of nanoparticles and base fluids are introduced to be used as working mediums in heat transfer and thermal fluid technologies. The solid parts or nanoparticles have high thermal conductivity property and can enhance overall heat transfer properties when they are mixed with the base fluids which have low thermal conductivity. In the nanofluid-flow field, the nanoparticles could be assumed to be distributed uniformly throughout the base fluid, this flow could also be presumed as the nanofluid flow through the uniform porous media (the solid parts) with nanofluid properties. The current work presented the developed mathematical model of the nanofluid flow; Al<sub>2</sub>O<sub>3</sub> nanoparticles and water flow, as the steady fluid flow with the nanofluid properties through the porous medium with the Al<sub>2</sub>O<sub>3</sub> properties. The simulated nanofluid flow was under fully developed laminar flow conditions through a rectangular pipe. The governing equations written in terms of the 3-D dimensionless variables were solved through the developed in-house program by using the finite volume method with the SIMPLE algorithm. Effects of the porous media characteristics; porosity, thermal conductivity and permeability, on accuracies of simulated results were investigated when the porosity value of 0.98 was considered to be equal to the nanofluid volume fraction of 0.02; as a synopsis relationship between the porosity and the volume fraction. The mixing thermal conductivity model; Yu and Choi model coupled with Maxwell model, was applied to be the thermal conductivity model of the porous media part. From the comparisons between the simulated and experimental results, the assumed relationships between the porosity and the volume fraction could be proved to be gratified and implied that the nanoparticles were distributed uniformly throughout the fluid and the nanofluid flow could be taken as the fluid with the nanofluid properties flowing through the porous media as well. The current developed model using the mixing thermal conductivity model with the porous media assumption could improve the model performance and supported its excellent potential in the nanofluid simulation as the fluid flow through the porous media.

# NOMENCLATURE

$C_P$	[J/Kg.K]	Specific heat
D	[mm]	Diameter
$D_h$	[mm]	Hydraulic diameter
h	$[W/m^2K]$	Heat transfer coefficient
k	[W/m.K]	Thermal conductivity
K	$[1/m^2]$	Permeability
L	[m]	Length of pipe
Ρ	[Pa]	Pressure
Pe	[-]	Peclet number
Pr	[-]	Prandtl number
q	$[W/m^2]$	Heat flux
ће	[-]	Reynolds number
Т	[K]	Temperature
$T_{ave}$	[K]	Average temperature of the wall
$T_m$	[K]	Constant inlet temperature
U,V,W	[m]	Velocities in each coordinate
u,v, w	[m/s]	Velocities
X, Y, Z	[m]	Cartesian axis direction
Greek symb	ools	
ß	[-]	Ratio of the nanolayer thickness to the original particle
$\rho$		radius
З	[-]	Porosity of porous medium
$\rho$	[kg/m <sup>3</sup> ]	Density
$\phi$	[-]	Nanofluid volume fractions
μ	[kg/m.s]	Dynamic viscosity
Subscripts		
eff		Effective
f		Fluid
nf		Nanofluid
p		Nanoparticle
*		

#### INTRODUCTION

Nanofluid is a modern combined solid-fluid materials which is introduced as working fluids for thermal equipments and devices. These combinations consisted of nanoparticles such as  $Al_2O_3$ , CuO, Cu, SiO, and TiO and base fluids, for an example, coolants in electronic-device heat pipes. Significantly, the conductivities of the nanoparticles; 1–100 nm diameter metallic or nonmetallic particles, are higher than those of the base fluids which normally have low thermal conductivities and the nanofluids have shown their enhancements in the effective thermal conductivities and the convective heat transfer coefficients even at low concentrations [1, 2, 3].

There are several approaches which can be used to simulate fluid flow with solid particles such as single-phase flow, two-

phase flow and fluid flow through porous media. In the singlephase approach, the combination is considered as one homogeneous fluid or no slip condition. Although several numerical and experimental studies concerning the heat convection in heated enclosures containing nanofluids were published, most of them concentrated on the nanofluids in cavities and only few of them considered the nanofluid as a porous medium [3].

Özerinç et al. [4] compared their results which were obtained from the single-phase heat transfer model by the finite different method with the experimental data of the heat transfer coefficients in the Al<sub>2</sub>O<sub>3</sub> fully-developed laminar flow at different Al<sub>2</sub>O<sub>3</sub> nanofluid volume fractions; 1%, 1.5%, 2% and 2.5%. Their simulated results at 2 % volume fraction were in very good agreement with the experimental data at the same nanofluid volume fraction. They presented that taking variable thermal conductivity and variable thermal dispersion into account in nanofluid analysis significantly improved the numerical accuracy. Since the single-phase model is simple, easy to understand, and less complicated, one can developed the single-phase model easily when the nanofluid is assumed to flow similar to other fluid, therefore, they should flow as the single-phase flow. Özerinç et al. [4] proved that the simple assumptions and flow model could be used to estimate the heat transfer coefficients of the new nanofluids efficiently [3].

Tongkratoke et al. [5] applied the single-phase model as the numerical method to analyze the heat transfer performance of the Al<sub>2</sub>O<sub>3</sub>/water nanofluid, based on the theoretical models. The nanofluid was considered as the flow under laminar fully developed flow conditions through a rectangular pipe as in the circuit application. This work also studied the effects of the Maxwell and Yu and Choi models as the thermal conductivity models. Different thermal conductivity models were used in the simulation domain alternately, named the mixing models, which were different from the works done before which used only one effective model in each simulation. From their numerical results, applying only the Yu and Choi model [6] gave similar results to the mixing thermal conductivity models which Yu and Choi model [6] coupled with Maxwell model [7] at the wall boundary in Peclet number (Pe) between 4000 and 6000. But, in Pe between 2500 and 4000, the mixing thermal conductivity models provided closer results to the experimental results than those of the single thermal conductivity models. They noticed that the thermal conductivity models at the wall boundary affected the accuracy of the calculation in the low Pe range. The lower errors of these mixing models in the low Pe range; from 2500 to 4000, may be caused by heat accumulation, which were calculated, and the errors affected the heat transfer coefficients directly [3].

Tongkratoke et al. [8] developed the single-phase model as the numerical method to analyze the heat transfer performance of the  $Al_2O_3$ /water nanofluid flow as the base flowing through a porous media. In this study, the effects of the porous media model, the permeability and the effective mixing thermal conductivity model were conducted for the  $Al_2O_3$ /water nanofluid flow and compared with the experimental results obtained from Lee and Mudawar [9]. Firstly, the digit numbers of the porosity values did not affect the heat transfer coefficients. The different permeability occurred in materials, which were manufactured differently and in different shapes, influenced the heat transfer coefficients differently, the more permeability the closer heat transfer coefficients to the experimental ones. Finally, the mixing thermal conductivity model which could improve the simulation performance in the previous work was examined and coupled with the porous model. The final results showed that the mixing thermal conductivity model could improve the numerical results to be closer to the experimental results trivially; by the forth digit. From this study, the developed mathematical model was found to demonstrate excellent potential in the nanofluid simulation as the porous media [3].

Nield and Bejan [10] introduced a definition of porous media as a material consisting of a solid with an interconnected void; typical natural porous media are as the human lung tissue, sandstone, and honeycomb. Common properties of the porous media are area to volume ratios and thermal conductivity: when these properties are high, convective heat transfer coefficients are increased. Since structures of the porous media are heterogeneous, the effective thermal property and velocity distribution of the porous media vary. Researches on the porous media behaviour and properties have to be extended and continued by integrating various sciences such as heat transfer and fluid dynamics together. More porous media information can advocate many modern sciences and engineering [11]. There are several approaches which can be used to simulate fluid flow with solid particles such as single-phase flow, twophase flow, and fluid flow through porous media. In the singlephase approach, the combination is considered as one homogeneous fluid with no slip condition. Although several numerical and experimental studies; concerning the convective heat transfer in heated enclosures containing nanofluids, were published, most of them concentrated on the nanofluids in cavities and only a few of them considered the nanofluid as a porous medium. The fluid convection in the porous medium has been studied by many groups of researchers including Horton and Rogers [12], Lapwood [13], Nield [14], Rudraiah and Malashetty [15], Murray and Chen [16], Malashetty [17], Pop and Ingham [18], and Vadasz [19]. The dispersion of nanosized particles in the traditional fluid increased the thermal conductivity of the fluid and the presence of porous media enhanced the effective thermal conductivity of the base fluid. Thus, the use of nanofluids in porous media would be very much helpful in the heat transfer enhancement. There was a study presenting the steady-state free convective heat transport in a triangular enclosure which a heater was placed on its vertical wall, saturated with nanofluids and filled with a porous medium [11, 20]. If one considers nanparticles which distribute inside the base fluid, at one moment, the nanoparticles can be considered to be spread uniformly as a network structure. Therefore, the nanoparticles inside the base fluid were assumed to behave as the porous media. Since the base fluid was enhanced its properties and convective heat transfer coefficients by the nanoparticles, the properties of the enhanced fluid was considered as the nanofluid properties.

As seen from the literatures [1,5], most of the experimental studies on the thermal properties of the nanofluids proved that

the thermal conductivities of nanofluids depended upon the nanofluid properties such as volume fractions and Brownian motion effect. In our previous works [1, 8] which related to the nanofluid flow as the working fluids flowing through the porous media, the porous model was proved to be used in the Al<sub>2</sub>O<sub>3</sub>/water nanofluid flow simulation. In our current work, we also considered the nanofluid flow as the fluid flowing through a porous media and analyzed the heat transfer performance of the Al<sub>2</sub>O<sub>3</sub>/water nanofluid with the numerical method based on the theoretical models from the literature; Zeinali Heris et al. [21]. The mathematical model of the nanofluid flow has been developed as the steady flow of the fluid with nanofluid properties through the porous medium with the Al<sub>2</sub>O<sub>3</sub> properties. The simulated nanofluid flow was under fully developed laminar flow condition through a rectangular pipe as in electronic applications. The effects of the grid independence, the effects of the porous media characteristics; porosity, thermal conductivity and permeability, were investigated. The porosity values of 0.98 were considered to be equal to the nanofluid volume fraction of 0.02 as the synopsis relationship between the porosity and the volume fraction. This work also focused on studying the mixing thermal conductivity model; Yu and Choi model [6] coupled with Maxwell model [7]. The governing equations written in terms of the three-dimension dimensionless variables were solved through the developed inhouse program by using the finite volume method with the SIMPLE algorithm. The developed model using the mixing thermal conductivity model coupled with the porous media and the suitable permeability [22] assumption were concluded [3].

# MATHEMATICAL MODELING

#### Geometry

The rectangular pipe used in the simulations had a width and a height of 6 mm and a length of 1 m. The fluid was assumed to enter the pipe with a constant inlet temperature,  $T_m$ , of 300.15 K, Reynolds number (Re) ranging from 400 to 900 as shown in Figure 1 and thermophysical properties of water as the base fluid and the  $Al_2O_3$  nanoparticles taken at T equaled to 293.15 K were shown in Table 1 [3].



Figure 1 Domain geometry of the rectangular pipe [3] (not to scale pictures).

Table 1 T	hermophysical	properties	of water a	nd Al <sub>2</sub> O <sub>3</sub>
n	anoparticles at	T = 293.15	5 K [3, 21]	]

Thermophysical	$C_p$	ρ	k
properties	(J/kg.K)	$(kg/m^3)$	(W/m.K)
water	4181.90	998.20	0.598
aluminium oxide	880.00	3920.00	42.340

### The dimensionless governing equations

The dimensionless governing equations [3] of steady, laminar flow, forced convection flow in the porous medium expressing; the continuity, momentum, and energy equations in three dimensional forms for the Al<sub>2</sub>O<sub>3</sub>/water nanofluid flow in the rectangular pipe were written as following

Continuity Equation;

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0 \tag{1}$$

Momentum Equation; X-component

$$\frac{1}{\varepsilon} \left( U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + W \frac{\partial U}{\partial Z} \right) = -\varepsilon \frac{\partial P}{\partial X} + \frac{1}{\operatorname{Re}} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} \right)$$

$$-\varepsilon \frac{\mu_{nf}}{\kappa} \cdot U \cdot \frac{D_h}{\rho_{nf} U_i}$$
(2a)

Y-component

$$\frac{1}{\varepsilon} \left( U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + W \frac{\partial V}{\partial Z} \right) = -\varepsilon \frac{\partial P}{\partial Y} + \frac{1}{\operatorname{Re}} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} \right)$$
$$-\varepsilon \frac{\mu_{nf}}{\kappa} \cdot U \cdot \frac{D_h}{\rho_{nf} U_i} \tag{2b}$$

Z-component

$$\frac{1}{\varepsilon} \left( U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} + W \frac{\partial W}{\partial Z} \right) = -\varepsilon \frac{\partial P}{\partial Z} + \frac{1}{\operatorname{Re}} \left( \frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2} \right)$$

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$$-\varepsilon \frac{\mu_{nf}}{\kappa} \cdot U \cdot \frac{D_h}{\rho_{nf} U_i}$$
(2c)

Energy Equation;

$$\left(U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} + W\frac{\partial T}{\partial Z}\right) = \frac{1}{\operatorname{Re.Pr}}\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} + \frac{\partial^2 T}{\partial Z^2}\right) \quad (3)$$

Where the dimensionless parameters are defined as

$$U = \frac{u}{U_i}, V = \frac{v}{U_i}, W = \frac{w}{U_i},$$
$$X = \frac{x}{D_h}, Y = \frac{y}{D_h}, Z = \frac{z}{D_h}$$
(4a, 4f)

$$P = \frac{p - p_i}{\rho_i U_i^2}, \ T = \frac{T_{nf} - T_{wall}}{T_{i,nf} - T_{wall}}$$
(4g, 4h)

$$\operatorname{Re} = \frac{\rho_{nf}UD}{\mu_{nf}}, \ \operatorname{Pr} = \frac{C_{p,nf}\mu_{nf}}{k_{nf}}$$
(4i, 4j)

The convective heat transfer coefficients were calculated according to the definition obtained from the literature (Zeinali Heris et al.) [21] as

$$h = \frac{q_w}{T_{ave} - T} \tag{5}$$

#### Thermophysical properties of nanofluids

Thermophysical properties [3] of nanofluids can be calculated by using the effective density, specific heat, dynamic viscosity, effective thermal conductivity and mixing effective thermal conductivity equations as following equations. The effective density of the nanofluid containing suspended particles can be evaluated through the following equation [3, 23];

$$\rho_{nf} = \phi \rho_p + (1 - \phi) \rho_f \tag{6}$$

Pak and Cho [23] also proposed an equation to calculate the specific heat of nanofluids based on the heat capacity concept [3];

$$C_{P,nf} = \phi C_{P,p} + (1 - \phi) C_{p,f}$$
(7)

The nanofluid viscosity is an important parameter for practical applications since it directly affects the pressure drop in the forced convection. Therefore, to enable the usage of nanofluids in practical applications, the increasing viscosity of nanofluids with respect to pure fluids should be thoroughly investigated. In this paper the effective dynamic viscosity of Al<sub>2</sub>O<sub>3</sub>/water nanofluids was calculated by using following model [3, 24];

$$\mu_{nf} = (1 + 2.5\phi)\mu_f \tag{8}$$

In this work, the effective thermal conductivity models of the Cu-water nanofluids were calculated by using following two models; Yu and Choi [6] correlation and Maxwell [7] correlation.

Maxwell [7] proposed an equation to calculate the effective thermal conductivity of solid-liquid mixtures consisting of spherical particles and, in this equation, the effect of the size and shape of the particles and particle interactions were not included in Maxwell equation [3].

$$k_{nf} = \frac{k_p + 2k_f + 2(k_p - k_f)\phi}{k_p + 2k_f - (k_p - k_f)\phi}k_f$$
(9)

Yu and Choi [6] presented a model for the determination of the effective thermal conductivity of nanofluids by modifying the Maxwell model [7]. In the modification, the effect of the liquid nanolayers formed around nanoparticles was taken into account. The nanoparticle and the layer around it were considered as a single particle and the thermal conductivity of this particle was determined by using effective medium theory [3, 6].

$$k_{nf} = \frac{k_p + 2k_f + 2(k_p - k_f)(1 + \beta)^3 \phi}{k_p + 2k_f - (k_p - k_f)(1 - \beta)^3 \phi} k_f$$
(10)

In equation (9),  $\beta$  is the ratio of the nanolayer thickness to the original particle radius and  $\beta = 0.1$  was taken to calculate the nanofluid effective thermal conductivity [3, 6].

All thermophysical properties of the  $Al_2O_3$ /water nanofluid at the volume fractions 0.00 and 0.02 were calculated and shown in Table 2 [3].

**Table 2** Thermophysical properties of the Al<sub>2</sub>O<sub>3</sub>/water nanofluid at different volume fractions; 0 (base fluid) and 0.02 [3].

Thermophysical properties	$\phi = 0$ (base fluid)	<i>φ</i> = 0.02
$k_{nf}$ (W/m.K)	0.598	1056.6
$\rho_{nf}$ (kg/m <sup>3</sup> )	998.20	4115.862
$\mu_{nf}$ (kg/m.s)	$5.98 \times 10^{-4}$	0.000953
$C_{p,nf}$ (J/kg.K)	4181.90	0.637

### The Effective Mixing Thermal Conductivity Models;

Tongkratoke et al. [5] introduced the mixing effective thermal conductivity models, mixing models, or combinations of the two effective thermal conductivity models in only one considered domain. The combinations consisted of the following: (1) the Maxwell [7] model (at the channel nodes) coupled with the Brownian motion model [25] at the wall boundary; (2) the Maxwell [7] model (at the channel nodes) coupled with the Yu and Choi [6] model at the wall boundary; (3) the Bownian motion model [25] (at the channel nodes) coupled with the Maxwell [7] model at the wall boundary; (4) the Bownian motion model [25] (at the channel nodes) coupled with the Yu and Choi [6] model at the wall boundary; (5) the Yu and Choi [6] model (at the channel nodes) coupled with the Maxwell [7] model at the wall boundary; (6) the Yu and Choi [6] model (at the channel nodes) coupled with the Bownian motion model [25] at the wall boundary. From their results, the combinations which consisted of the Yu and Choi [6] model (at the channel nodes) coupled with the Maxwell [7] model at the wall boundary as shown in Figure 2 provided the best calculated results when this couple was compared with other (coupled) mixing effective thermal conductivity model. The results were caused by the Maxwell [7] model which was a static model and suitable for the flow part with low velocity as the channel wall nodes and the Yu and Choi [6] model which

was a dynamic model considering Brownian motion and suitable for the flow part with high velocity as the inside nodes of the channel [3].





# NUMERICAL METHOD

The governing equations were discretized by using the finite volume method. In this technique, the governing equations were integrated over the finite control volumes and their results in a set of algebraic equations can be solved numerically. Staggered grids have been used where the velocity components were calculated at the centre of the volume interfaces while the pressures including as well as other scalar quantities such as temperatures were computed at the centre of the control volumes. The algebraic discretization equations had been solved sequentially and iteratively throughout the physical domain by combining the lineby-line procedure and the well-known TDMA technique. Pressure and velocity were coupled using Semi Implicit Method for Pressure Linked Equations or SIMPLE [26]. Convergence of the iterative solutions was ensured when the residuals of all variables were less than  $10^{-6}$  and the grids used in the computations consisted of  $60 \times 60 \times 500$  and  $120 \times 120 \times 1000$  grid nodes in the x, y and z directions, respectively. Moreover, we have used the personal computer (PC), operating with Intel Core i7, 2.67 GHz and 2.79 GHz, 6 GB Memory and 64 bit operating system [2, 3, 5, 8].

### **RESULTS AND DISCUSSION**

This research centered on the nanofluid flow as the nanofluid flowing through the  $Al_2O_3$  porous media. The developed in-house computer code has been applied continuously and its numerical results were validated with the experimental results from Zeinali Heris et al. [21] who performed an experimental study on the fully developed laminar flow of an  $Al_2O_3$ /water nanofluid in a rectangular pipe. In the in-house code, the density correlation from Pak and Cho [23], the specific heat correlation from Pak and Cho [23], the viscosity correlation from Einstein [24], the mixing thermal conductivity model; Yu and Choi model [6] coupled with Maxwell model [7] as shown in Figure 2 and nanofluid volume fractions at 0.02, were engaged in this computer program.

#### **Numerical Model Validation**

Zeinali Heris et al. [21] carried out the experimental study on the fully developed laminar flow of  $Al_2O_3$ /water nanofluid. The thermophysical properties of  $Al_2O_3$ /water nanofluid can be obtained with the same procedures as written in the previous section. The calculated heat transfer coefficients were obtained from the governing equations and compared with the experimental heat transfer coefficients from Zeinali Heris et al. [21] in Reynolds number ranged from 400 to 900, shown in Figure 3, at the  $Al_2O_3$ /water nanofluid volume fraction of 0.02. In the high Reynolds number range from 600 to 900, the numerical results approached to the experiment results, the calculated heat transfer coefficients were different from the experimental heat transfer coefficients in an average error of 1.55%. From Figure 3, the average error percentage between the calculated and experimental results was 1.15% while the most error percentage was 4.87% at Reynolds number equalled to 400. However, in the low Reynolds number range from 400 to 600, the numerical values situated away from the experimental values noticeably, and this error may be caused by the effect of the boundary layer thickness for nanofluid flow.



Figure 3 Comparison between the simulated results and the experimental data obtained from Zeinali Heris et al. [21] to validate the numerical model.

#### The effect of the grid independence

To investigate the effect of the grid independence, comparisons between the simulated results with three different grid nodes  $(6 \times 6 \times 100, 60 \times 60 \times 500 \text{ and } 120 \times 120 \times 1000)$  to predict the heat transfer coefficients for the Al<sub>2</sub>O<sub>3</sub>/water nanofluid at 0.02 volume fractions and experimental data [10] were shown in Figure 4. The results from the  $60 \times 60 \times 500$  and  $120 \times 120 \times 1000$  grid sets provided the similar result curves and both of them were closer to the experimental results, but their time consumptions were different, the  $(60 \times 60 \times 500)$  set consumed less time. Since the  $(120 \times 120 \times 1000)$  was the gridindependent and the time consumption was one of important parameter to consider in the simulation, so the  $(60 \times 60 \times 500)$ was chosen to be carried on in the next examinations. About the CPU times for each grid set,  $(120 \times 120 \times 1000)$ ,  $(60 \times 60 \times 500)$ and  $(6 \times 6 \times 100)$  took 5.45, 4.20 and 2.30 hrs of operating times, respectively.

# The effects of the porous media characteristics

We would like to investigate the relationship between the volume fraction and the porosity, in Figure 5, the convective

heat transfer coefficients of the nanofluid at 0.02 volume fraction performed by Zeinali Heris et al. [21] and were compared with the calculated coefficients of the porous media model with porosity values at 0.975, 0.980 and 0.985, respectively. The comparison results showed that the calculated coefficients of the porous media model with the 0.980 porosity provided the best results compared with others. The calculated heat transfer coefficients were compared with the experimental heat transfer coefficients from Zeinali Heris et al. [21] shown in Figure 5 in Reynolds number ranged from 400 to 900 and at 0.02 Al<sub>2</sub>O<sub>3</sub>/water nanofluid volume fraction. The simulated results were close to the experimental result obtained from Zeinali Heris et al [21] when the porosity was at 0.98 and the Al<sub>2</sub>O<sub>3</sub>/water nanofluid volume fraction was at 0.02. Therefore, the assumed relationships between the porosity and the volume fraction could be proved to be in the good agreement; this agreement also implied that the nanoparticles were distributed uniformly throughout the fluid and the nanofluid flow could be taken as the nanofluid with the nanofluid properties flowing through the porous media with the nanoparticle properties.



Figure 4 Comparison between the simulated results with different grid nodes and the experimental data obtained from Zeinali Heris et al. [21] to check the grid independence effect.



**Figure 5** Comparison between the simulated results with the porosity, the volume fraction and experimental data obtained from Zeinali Heris et al. [21] in prediction of the heat transfer coefficient of the Al<sub>2</sub>O<sub>3</sub>/water nanofluid

# The effects of the nanoparticles permeability

The permeability  $(\kappa)$  is one of solid properties, nanoparticles as solid particles in the nanofluid, which indicates liquid penetration through the solid particles. The permeability is one of the flow field parameter as shown in the steadylaminar-forced convection flow in the porous media; Equations (2a), (2b) and (2c). The permeability ( $\kappa$ ) of the Al<sub>2</sub>O<sub>3</sub> nanoparticles was brought into the current work to investigate the heat transfer enhancement as in Figure 6; the relationships between the heat transfer coefficients and Reynolds number for the volume fractions such as 2% nanoparticles. Figure 6 also displayed the comparison between the simulated results obtained from the effective mixing thermal conductivity model and the experimental results from the literature; Zeinali Heris et al [21], by applying the grid independence  $(60 \times 60 \times 500)$  and varying the permeability of the  $Al_2O_3$  nanoparticles; 1, 0.1, 0.0001 and  $1 \times 10^{-14}$  m<sup>-2</sup>, to investigate the permeability effects. The permeability values from  $1 \times 10^{-14} \text{ m}^2$  which was the permeability of the Al<sub>2</sub>O<sub>3</sub> obtained from Narva and Vin [27] and the results from this value revealed the closest results to the experimental results from Zeinali Heris et al [21].



Figure 6 Comparison between the simulated results, with the different permeability values and the effective mixing thermal conductivity model, and the experimental data obtained from Zeinali Heris et al. [21] to investigate the nanoparticles permeability effects.

#### CONCLUSION

The developed in-house program has been applied to simulate the nanofluid flow through the porous media to improve the simulation accuracy. The fully developed laminar flow of the  $Al_2O_3$ /water nanofluid in a rectangular pipe was chosen in this current work. The numerical and experimental heat transfer coefficients were compared and discussed in Reynolds number range from 400 to 900. The effects of the grid independence, the effects of the porous media characteristics; porosity, thermal conductivity and permeability, were found. Firstly, the numerical code was validated with the experimental results from the literature. Next, the porosity values of 0.98 was assumed to be equal to the nanofluid volume fraction of 0.02 as the relationship between the porosity and the volume fraction and, then, they were proved to be in the good

agreement. This assumed relationship also implied that the nanoparticles were distributed uniformly throughout the nanofluid flow and could be considered as the nanofluid with the nanofluid properties flowing through the porous media with the nanoparticle properties. The grid dependent was investigated and it showed that the  $60 \times 60 \times 500$  grid set was suitable to be used in this nanofluid flow configuration with less time consumption. The simulated results obtained from the effective mixing thermal conductivity model; the Yu and Choi model (at the channel nodes) coupled with the Maxwell model (at the wall nodes) still showed its potential to improve the simulation performance from the previous work [5] when it was applied and coupled with the porous media model. The developed porous media model with the nanofluid assumption as the porous media utilizing the mixing thermal conductivity model could improve the performances of the porous media and mixing thermal conductivity models and supported thier excellent potentials in the nanofluid simulation as the porous media.

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