NUMERICAL STUDY OF NATURAL CONVECTIVE HEAT TRANSFER FROM HORIZONTAL HEATED ELEMENTS OF RELATIVELY COMPLEX SHAPE HAVING A UNIFORM SURFACE HEAT FLUX

Oosthuizen, P.H.[1]* and Kalendar, A.Y.[2]
*Author for correspondence
[1]Department of Mechanical and Aeronautical Engineering, Department of Mechanical and Materials Engineering
Queen's University, Kingston, Ontario, Canada, K7L 3N6
Email: oosthuiz@queensu.ca

[2]Department of Mechanical Power and Refrigeration Technology College of Technological Studies (CTS)
Public Authority for Applied Education and Training, Shuwaikh, Kuwait
Email: ay.kalendar1@paaet.edu.kw

ABSTRACT

With natural convective heat transfer from horizontal elements of relatively complex shape that have a uniform surface temperature it has been found that if a length scale based on the ratio of the element surface area to its perimeter is used in defining the Nusselt and Rayleigh numbers then, for a given Prandtl number, the variation of Nusselt number with Raleigh number is effectively the same for all element shapes. However, in a number of practical situations the element surface is not isothermal but there is effectively a uniform heat flux over the surface. Few studies have been undertaken to determine whether natural convective heat transfer rates from horizontal heated elements having a uniform surface heat flux and relatively complex shapes can be correlated using the same procedure that applies when there is an isothermal surface. A fuller investigation of whether the results for surfaces with a uniform heat flux can be correlated in this way has been undertaken. Attention has been given to an element having a circular shape with an inner circular adiabatic section, to an I-shaped element, and to a plus (+)-shaped element. Results for a square and a circular element are also presented for comparison purposes. The elements considered are imbedded in a larger surrounding flat adiabatic surface, attention being restricted to the case where the heated elements are facing upwards. The heat transfer from the element has been assumed to be to air. The results have been obtained numerically using the commercial CFD code ANSYS FLUENT©. The possibility that turbulent flow can occur in the system has been allowed for by using the basic k-epsilon turbulence model. The results have been used to investigate whether the heat transfer rates in the laminar, transitional and turbulent flow regions for the element shapes considered here can be correlated for each of the separate flow regions by using the same length scale that has been found to apply for elements with a uniform surface temperature.

INTRODUCTION

In the natural convective cooling of devices such as non-computer electronic and electrical components that are used in various industrial applications the heat transfer often effectively occurs from horizontal elements of relatively complex shape. In the case of such elements that effectively have a uniform surface temperature it has been found [1-3] that if a length scale based on the ratio of the element surface area to its perimeter is used in defining the Nusselt and Rayleigh numbers then, for a given Prandtl number, the variation of Nusselt number with Raleigh number is effectively the same for all element shapes. But in some practical situations the element surface is not isothermal and there is instead effectively a uniform heat flux over the element. A very limited number of studies exists to determine whether natural convective heat transfer rates from horizontal heated plane elements having a uniform surface heat flux and relatively complex shapes can be correlated using the same procedure that applies when there is an isothermal element. The present study therefore was undertaken to more fully investigate whether the results for elements with a uniform heat flux can be correlated in this way.

Attention has been given to a heated element having a circular shape with an inner circular adiabatic section, to a heated I-shaped element, and to a heated plus (+)-shaped element (see Fig. 1). For comparative purposes results have also been obtained for a heated circular element without an inner adiabatic section and for a square element (see Fig. 2). As shown in Fig. 3 the elements considered are imbedded in a larger surrounding flat adiabatic surface. The surface of the heated element is in the same plane as the surface of the surrounding adiabatic surface. Attention has been restricted to the case of upward facing heated elements and it has been assumed that the heat transfer from the elements is to air due to the applications considered here. The configurations considered are simplified models of those existing in most practical occurrences but the findings should be quite adequate to indicate whether the results for uniform surface heat flux elements can be correlated using the same length scale as that which has been found to apply with isothermal elements.

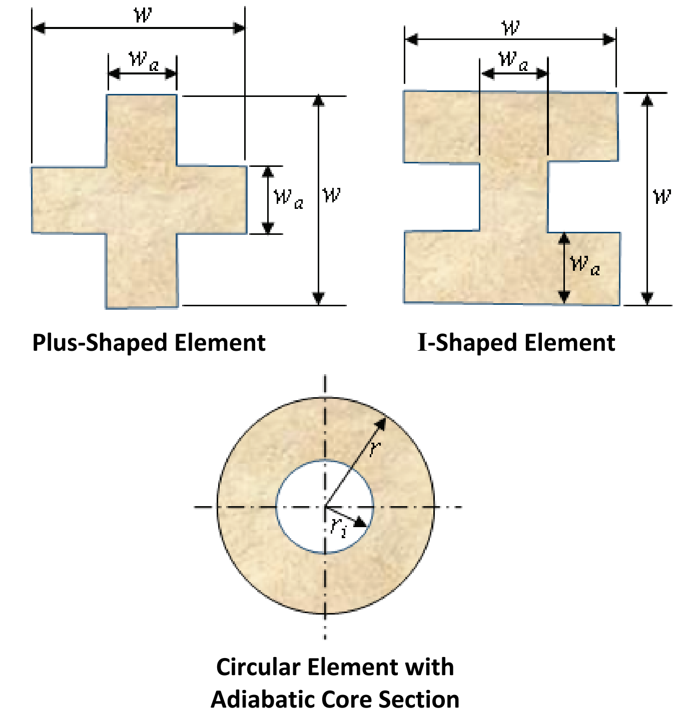

Figure 1 Complex element shapes considered

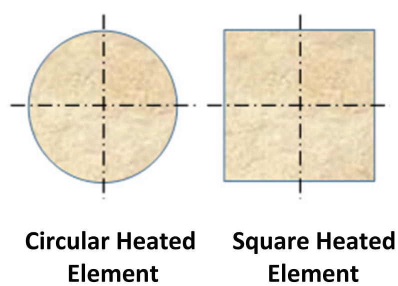

Figure 2 Circular and square elements

Results have been obtained numerically for a relatively wide range of the governing parameters and allowing for the possibility that laminar, transitional and fully turbulent flows can occur in the system. The main aim of the present study was to use the results obtained to determine whether, for the element shapes considered, the heat transfer rates in these three flow regions can be correlated for each of these separate flow regions by using the same length scale that has been found to apply for elements with a uniform surface temperature.

There have been many studies of natural convective heat transfer from heated horizontal elements. Most of these studies have dealt only with isothermal elements having a relatively simple shape and only with conditions under which laminar flow exists, e.g., see [4-15]. Some studies have considered elements having more complex shapes but again have mainly

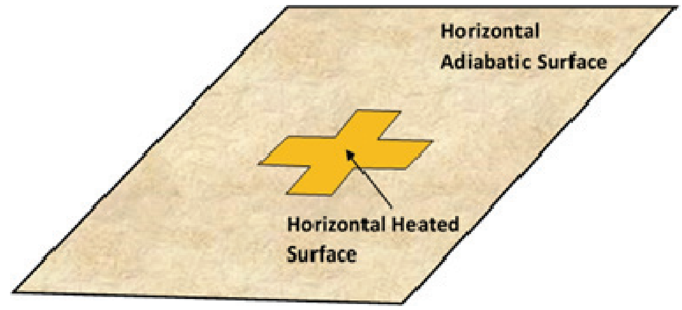

Figure 3 Flow situation considered

considered isothermal elements and conditions under which laminar flow exists, e.g. see [3, 16-18]. The present study therefore considers situations that are different from those considered in almost all past studies in that it deals with elements which have a relatively complex shape over whose surface there is uniform heat flux and with conditions under which laminar, transitional, and turbulent flow exists.

The present study is part of a larger overall investigation of natural convective heat transfer from horizontal and near horizontal heated elements for conditions in which laminar, transitional, and turbulent flow occurs. Representative other studies that are part of this larger investigation are described in [3, 19-26].

NOMENCLATURE

A	[m^2]	Surface area of heated element
d	[m]	Outer diameter of circular element
g	[m/s^2]	Gravitational acceleration
k	[W/mK]	Thermal conductivity
l	[m]	Reference element size, $4A/P$
L	[-]	Dimensionless mean size of element, l/d or l/w
Nu	[-]	Nusselt number based on w or d
Nu_L	[-]	Nusselt number based on the reference heated element size, l
P	[m]	Total perimeter of the heated element
Pr	[-]	Prandtl number
$\underline{q'}$	[W]	Mean heat transfer rate per unit area over element surface
r	[m]	Outer radius of circular elements
r_{in}	[m]	Radius of inner circular adiabatic section
R	[-]	Dimensionless outer radius of circular element, r/d
R_{in}	[-]	Dimensionless radius of inner circular adiabatic section, r/r_{in}
Ra^*	[-]	Heat Flux Rayleigh number based on w or d
Ra^*_L	[-]	Heat Flux Rayleigh number based on the reference heated element size, l
T_f	[K]	Undisturbed fluid temperature
$\overline{T_w}$	[K]	Mean element surface temperature
w	[m]	Side length of non-circular heated element
w_a	[m]	Width of "arms" of I-shaped and + -shaped elements
W_a	[m]	Dimensionless width of "arms" of I-shaped and + -shaped elements, w_a/w

Greek symbols

α	[m^2/s]	Thermal diffusivity
β	[1/K]	Bulk coefficient of thermal expansion
ν	[m^2/s]	Kinematic viscosity

SOLUTION PROCEDURE

In the present study it has been assumed that the flow is steady and that the fluid properties are constant except for the density change with temperature which gives rise to the buoyancy forces, the Boussinesq approximation being used in dealing with this. Attention has been restricted to the case where the heated element is facing upwards. Radiant heat transfer has been neglected. In dealing with the case of an element having a circular shape with an inner circular adiabatic section it has also been assumed that the flow is axisymmetric. In the case of the other element shapes considered, the flow has been assumed to be symmetrical about the centre-lines shown in Figs. 2 and 4. The possibility that turbulent flow will develop has been dealt with by using the basic k-epsilon turbulence model with standard wall functions and with account being taken of buoyancy force effects. This model is applied under all conditions considered and determines when transition to turbulent flow occurs. A number of previous studies, e.g., [27-33], have shown that the use of this approach gives relatively good predictions of the conditions under which turbulence develops in flows of the type being considered here. The governing equations based on the use of the assumptions discussed above and subject to the boundary conditions have been solved numerically using the commercial CFD solver ANSYS FLUENT©.

Grid independence and convergence-criteria independence testing showed that with the meshes used in obtaining the results presented here the heat transfer results, i.e., the derived Nusselt number values, are grid- and convergence criteria independent to within approximately one per cent.

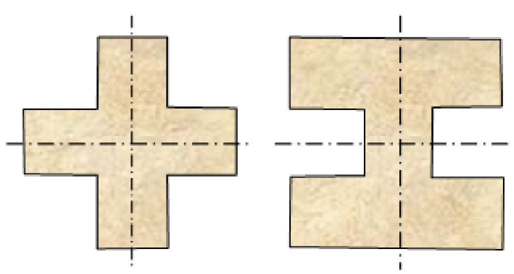

Figure 4 Centre-lines about which symmetry is assumed for the +-shaped and I-shaped elements.

RESULTS

The solution depends on the following parameters:

- the Heat Flux Rayleigh number, Ra^*, based on the outside size of the element, i.e., on the outer diameter, d, in the case of the circular element and on the outer size, w, in the case of the square , I-shaped, and +-shaped elements (see Fig. 2) and on the uniform surface heat flux, i.e.:

$$Ra^* = \frac{\beta g d^4 (\overline{T}_w - T_f)}{\nu\alpha} \quad \text{or} \quad Ra^* = \frac{\beta g w^4 (\overline{T}_w - T_f)}{\nu\alpha} \tag{1}$$

- the element shape being considered
- the Prandtl number, Pr.

Because of the applications that motivated this study, results have only been obtained for a Prandtl number of 0.74, i.e., effectively for the value for air. Heat Flux Rayleigh numbers of between approximately 10^5 and 10^{16} have been considered.

For the case being considered here where there is a uniform specified heat flux per unit area, q', over the element surface, the mean surface temperature relative to the temperature of the undisturbed fluid well away from the element, T_f, has been expressed in terms of a mean Nusselt number based on the outer diameter, d, of the circular element and on the outer size, w, of the elements of other shapes considered, i.e., in terms of:

$$Nu = \frac{\overline{q}' d}{k(\overline{T}_w - T_f)} \quad \text{or} \quad Nu = \frac{\overline{q}' w}{k(\overline{T}_w - T_f)} \tag{2}$$

Since a fixed value of Pr is here being considered, Nu will be a function of Ra^*, the relation between Nu and Ra^* depending on the shape of the element being considered.

Attention will first be given to the case of an element having a circular shape with an inner adiabatic circular section. If the outer radius of the element is r and the radius of the inner adiabatic section is r_{in} then the element shape will be defined by:

$$R_{in} = r_{in} / r \tag{3}$$

Results have been obtained for R_{in} values between 0 (the case where there is no inner adiabatic circular section) and 0.425.

Now it has often been assumed (e.g., see [1-3]) that for natural convective heat transfer from horizontal heated elements, if a mean element size, l, defined by:

$$l = 4\frac{A}{P} \tag{4}$$

is introduced and if Nusselt and Rayleigh numbers based on this mean element size are used then the variations of Nusselt number with Rayleigh number will be the same for all element shapes. In the above equation, A is the surface area of the heated element and P is the total perimeter of the heated element.

For a circular element with an unheated adiabatic inner section:

$$l = 4\frac{\left[r^2 - r_{in}^2\right]}{2\left(r + r_{in}\right)} = 2\left(r - r_{in}\right) \tag{5}$$

from which it follows that the dimensionless mean element size, $L = l / d$, is given by:

$$L = 2\left(R - R_{in}\right) \tag{6}$$

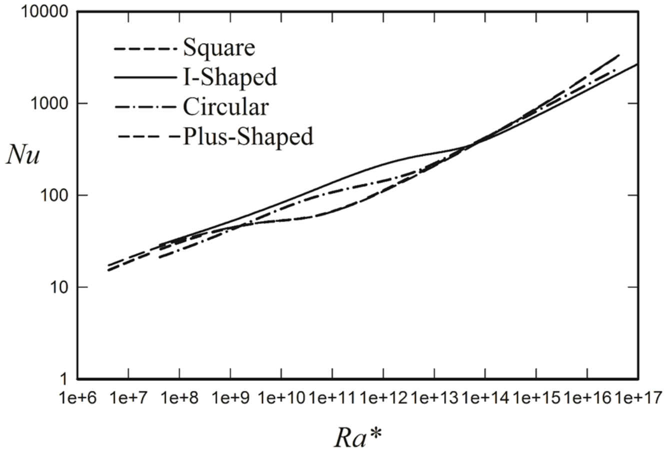

Figure 5 Variation of Nusselt number based on the outer diameter of element with Heat Flux Rayleigh number based on the outer diameter of element for a circular element having an inner circular adiabatic section.

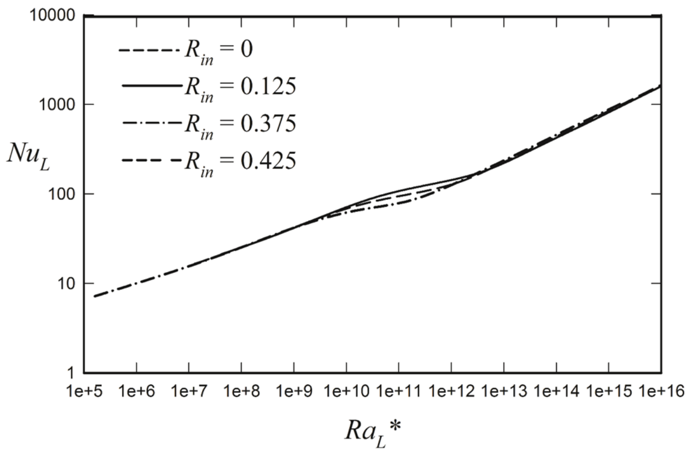

Figure 6 Variation of Nusselt number based on the mean element size, *l*, with Heat Flux Rayleigh number based on the mean element size, *l*, for a circular element having an inner circular adiabatic section.

The Nusselt and Rayleigh numbers based on *l* will be designated as Nu_L and Ra_L* respectively, i.e.:

$$Nu_L = \frac{\overline{q}' l}{k(\overline{T}_w - T_f)} \quad \text{and} \quad Ra_L^* = \frac{\beta g l^4 (T_w - T_f)}{\nu\alpha} \tag{7}$$

Typical variations of *Nu* with *Ra** for various values of R_{in} are shown in Fig. 5 while the corresponding variations of Nu_L with Ra_L* for various values of R_{in} are shown in Fig. 6. It will be seen from Fig. 6 that the variations of Nu_L with Ra_L* in the laminar flow region and in the fully turbulent flow region are essentially the same for all values of R_{in}. However, in the transitional flow region there is a difference between the variations for different values of R_{in}. This difference mainly results from the fact that the value of *Ra** at which transition occurs varies with R_{in}.

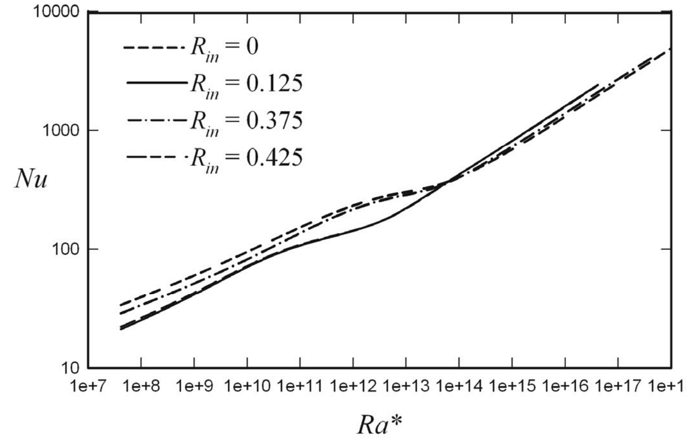

Figure 7 Variation of Nusselt number based on the outer size of element, *w*, with Heat Flux Rayleigh number based on the outer size of element, *w*, for a square element, a circular element with no inner circular adiabatic section, an I-shaped element, and a + -shaped element.

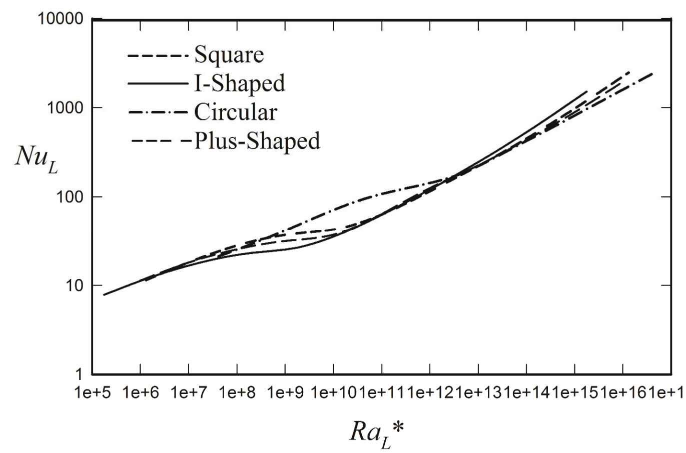

Figure 8 Variation of Nusselt number based on the mean element size, *l*, with Heat Flux Rayleigh number based on the mean element size, *l*, for a square element, a circular element with no inner circular adiabatic section, an I-shaped element, and a + -shaped element.

Attention will next be turned to heated elements having an I-shape or a + -shape. For these elements the dimensionless mean element size, $L = l / w$, is given for the I-shaped element by:

$$\text{I-shaped: } L = 4\left(\frac{3W_a - 2W_a^2}{6 - 2W_a}\right) \tag{8}$$

and for the +-shaped element it is given by:

$$\text{+ -shaped: } L = 2W_a - W_a^2 \tag{9}$$

Results have only been obtained for an I-shaped element with W_a = 0.25 and a + -shaped element with W_a = 0.5. Therefore, L = 0.4546 for the I-shaped element and L = 0.75 for the + -shaped element. These results will be compared to those obtained for a square element and a circular element having no inner adiabatic centre section, i.e., for the case where R_{in} = 0. Typical variations of Nu with $Ra*$ for these element shapes are shown in Fig. 7 while the corresponding variations of Nu_L with Ra_L* for these element shapes are shown in Fig. 8. It will be seen from Fig. 8 that the variations of Nu_L with Ra_L* for all element shapes considered are essentially the same in the laminar flow region. However, with these element shapes there are very significant differences between the variations of Nu_L with Ra_L* in the transitional flow and fully turbulent flow regions. The results therefore indicate that for complex shaped elements having a uniform surface heat flux the introduction of l as the characteristic length scale does not provide a means of correlating the results for all element shapes in the transitional and the fully turbulent flow regions.

CONCLUSIONS

The results of the present numerical study indicate that:

1. The heat transfer rates for a circular element having an inner circular adiabatic section and a uniform surface heat flux are well-correlated in the laminar and the fully turbulent flow regions by expressing the results in terms of the mean element size defined in eq. (4). However, this approach does not provide a means of correlating the results in the transitional flow regime.

2. In the case of square heated elements, circular heated elements with no inner adiabatic section, I-shaped heated elements, and + -shaped heated elements with a uniform surface heat flux the results are well-correlated in the laminar flow region by expressing the results in terms of the mean element size defined in eq. (4). However this approach does not provide a means of correlating the results in the transitional flow region and in the fully-turbulent flow regime.

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REFERENCES

[1] Burmeister, L.C., 1993, *Convective Heat Transfer*, John Wiley & Sons, Inc., New York, 2nd Edition, pp. 636-637.
[2] Sucec, J., 1985, *Heat Transfer*, Wm. C. Brown Publisher, Dubuque, pp. 636-637.
[3] Oosthuizen, P.H., A numerical study of natural convective heat transfer from horizontal isothermal heated elements of complex shape, *Proceedings of the 1st Thermal and Fluids Engineering Summer Conference TFESC 2015*, 2015, Paper TFESC-12863.
[4] Clifton, J.V., and Chapman, A.J., Natural-convection on a finite-size horizontal plate, *International Journal of Heat and Mass Transfer*, Vol. 12, No. 12, 1969, pp. 1573-1584.
[5] Hassan, K.-E., and Mohamed, S.A., Natural convection from isothermal flat surfaces, *International Journal of Heat and Mass Transfer*, Vol. 13, No. 12, 1970, pp. 1873-1886.
[6] Kitamura, K., and Kimura, F., Heat transfer and fluid flow of natural convection adjacent to upward-facing horizontal plates, *International Journal of Heat and Mass Transfer*, Vol. 38, No. 17, 1995, pp. 3149-3159.
[7] Lewandowski, W. M., Radziemska, E., Buzuk, M., Bieszk, H., Free convection heat transfer and fluid flow above horizontal rectangular plates, *Applied Energy*, Vol. 66, No. 2, 2000, pp. 177-197.
[8] Martorell, I., Herrero, J., and Grau, F. X., Natural convection from narrow horizontal plates at moderate Rayleigh numbers, *International Journal of Heat and Mass Transfer*, Vol. 46, No. 13, 2003, pp. 2389-2402.
[9] Pretot, S., Zeghmati, B., and Le Palec, G., Theoretical and experimental study of natural convection on a horizontal plate, *Applied Thermal Engineering*, Vol. 20, No. 10, 2000, pp. 873-891.
[10] Radziemska, E., and Lewandowski, W.M., The effect of plate size on the natural convective heat transfer intensity of horizontal surfaces, *Heat Transfer Engineering*, Vol. 26, No. 2, 2005, pp. 50-53.
[11] Restrepo, F., and Glicksman, L.R., The effect of edge conditions on natural convection from a horizontal plate, *International Journal of Heat and Mass Transfer*, Vol. 17, No. 1, 1974, pp. 135-142.
[12] Rotem, Z., and Claassen, L., Natural convection above unconfined horizontal surfaces, Journal of Fluid Mechanics, Vol. 38, No. 1, 1969, pp. 173-192.
[13] Yousef, W.W., Tarasuk, J.P., and McKeen, W.J. Free convection heat transfer from upward-facing isothermal horizontal surfaces, J. Heat Transf., vol. 104, pp. 493-500, 1982.
[14] Kobus, C.J., and Wedekind, G.L., An experimental investigation into natural convection heat transfer from horizontal isothermal circular disks, *International Journal of Heat and Mass Transfer*, Vol.44, 2001, pp. 3381-3384.
[15] Kitamura, K., and Kimura, F., Fluid flow and heat transfer of natural convection over upward-facing, horizontal heated circular disks, *Heat Transfer-Asian Research*, Vol. 37, No. 6, 2008, pp. 339-351.
[16] Al-Arabi, M., and El-Riedy, M. K., Natural convection heat transfer from isothermal horizontal plates of different shapes, *International Journal of Heat and Mass Transfer*, Vol. 19, No. 12, 1976, pp. 1399-1404.
[17] Kozanoglu, B., and Lopez, J., Thermal boundary layer and the characteristic length on natural convection over a horizontal plate, *Journal of Heat and Mass Transfer*, Vol. 43, No. 4, 2007, pp. 333-339.
[18] Lloyd, J.R., and Moran, W.R., Natural convection adjacent to horizontal surface of various planforms, *Journal of Heat Transfer*, Vol. 96, No. 4, 1974, pp. 443-447.
[19] Oosthuizen, P.H., A numerical study of natural convective heat transfer from a horizontal isothermal square element imbedded in an adiabatic surface with a parallel adiabatic covering surface, *Proceedings of the 10th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics HEFAT 2014*, 2014, pp. 192-199.
[20] Oosthuizen, P.H., Natural convective heat transfer from a horizontal rectangular isothermal element imbedded in a plane adiabatic surface with a parallel adiabatic covering surface, *Proceedings of the ASME 2014 International Mechanical Engineering Congress and Exposition 2014*, 2014, Paper IMECE2014-36780.

[21] Oosthuizen, P.H., Natural convective heat transfer from a horizontal isothermal circular element imbedded in a flat adiabatic surface with a parallel adiabatic covering surface, *Proceedings of the AIAA/ASME Joint Thermophysics and Heat Transfer Conference*, 2014, Paper AIAA-2014-3357.

[22] Oosthuizen, P.H., and Paul, J.T., Numerical study of the effect of vent positioning on the natural convective heat transfer rate from a horizontal isothermal heated surface mounted in a flat adiabatic base and surrounded by a protective surface, *Proceedings of the 13th UK Heat Transfer Conference UKHTC2013*, 2013, Paper UKHTC2013/66.

[23] Oosthuizen P.H., Natural convective heat transfer from an inclined isothermal square flat element mounted in a flat adiabatic surrounding surface, *Proceedings of the 15th International Heat Transfer Conference IHTC-15,* 2014, Paper IHTC15-8499.

[24] Oosthuizen, P.H., 2015, Laminar, transitional and turbulent natural convective heat transfer from a horizontal rectangular isothermal element imbedded in a flat adiabatic surrounding surface, *Proceedings of the 6th ICHMT International Symposium on Advances in Computational Heat Transfer CHT-15, 2015*, Paper CHT-15-145.

[25] Oosthuizen, P.H., A numerical study of natural convective heat transfer from a horizontal isothermal square element with an unheated square adiabatic inner section, *Proceedings of the 10th Int. Conf. on Heat Transfer, Fluid Mechanics and Thermodynamics HEFAT 2015*, 2015, Paper 570075655.

[26] Oosthuizen, P.H., and Kalendar, A.Y., A numerical study of natural convective heat transfer from a pair of adjacent horizontal isothermal square elements embedded in an adiabatic surface – effect of element spacing on heat transfer rate, *Proceedings of the 11th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics (HEFAT 2015)*, 2015, Paper 1570075659.

[27] Savill, A.M., Evaluating turbulence model predictions of transition. An ERCOFTAC special interest group project, *Applied Scientific Research*, Vol. 51, 1993, pp. 555-562.

[28] Schmidt, R.C., and Patankar, S.V., Simulating boundary layer transition with low-Reynolds-number k-ε turbulence models: Part 1- An evaluation of prediction characteristics, *Journal of Turbomachinery*, Vol. 113, 1991, pp. 10-17.

[29] Plumb, O.A., and Kennedy, L.A., Application of a k-e turbulence model to natural convection from a vertical isothermal surface, *Journal of Heat Transfer*, Vol. 99, 1977, pp. 79-85.

[30] Zheng, X., Liu, C., Liu, F., and Yang, C.-I., Turbulent transition simulation using the k-ω model, *International Journal of Numerical Methods in Engineering*, Vol. 42, 1998, pp. 907-926.

[31] Albets-Chico, X., Oliva, A., and Perez-Segarra, C.D., Numerical experiments in turbulent natural convection using two-equation eddy-viscosity models, *Journal of Heat Transfer*, Vol. 130, No. 7, 2008, pp. 072501-1–072401-11.

[32] Oosthuizen, P.H., and Naylor, D., A numerical study of laminar-to-turbulent transition in the flow over a simple recessed window-plane blind system, *Proceedings of the 4th Canadian Solar Buildings Conference*, M. Stylianou, ed., The Solar Buildings Research Network, Montreal, 2009.

[33] Xamán, J., Álvarez, G., Lira, L., and Estrada, C., Numerical study of heat transfer by laminar and turbulent natural convection in tall cavities of façade elements, *Energy and Buildings*, Vol. 37, 2005, pp. 787-794.