# Economic Hybrid Non-linear Model Predictive Control of a Dual Circuit Induced Draft Cooling Water System

C.J. Muller[a,b], I.K. Craig[b,*]

[a]*Sasol, Sasolburg, South Africa.*
[b]*Department of Electrical, Electronic, and Computer Engineering, University of Pretoria, Pretoria, South Africa.*

**Abstract**

Petrochemical plants require the addition and removal of energy to and from the process and the movement of material to, from, and within the process piping and vessels. These fundamental mass and energy transfer requirements are typically achieved through the use of process utilities, which include electricity, steam, fuel gas, cooling water and compressed air. Utilities are responsible for a significant portion of the operating cost of a plant. Therefore, reduction in the consumption of utilities is a common process optimisation area. The situation is different when it comes to the generation and transportation of these utilities, which are often overlooked with regard to optimisation. In this paper, the potential benefits of utility optimisation are illustrated with particular focus on the generation and transportation areas. The main objectives are reductions in electrical energy consumption and cost and are illustrated for a dual circuit cooling water system. This system is non-linear and also hybrid in the sense that it contains both continuous and discrete input variables, which significantly complicates the design and implementation of control and optimisation solutions. This paper illustrates how the cost and energy consumption of a hybrid system can be reduced through the implementation of Hybrid Non-linear Model Predictive Control (HNMPC) and Economic HNMPC (EHNMPC). The results are

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*Corresponding author. Tel.: +27 12 420 2172.
  *Email addresses:* nelis.muller@sasol.com, nelismuller@gmail.com (C.J. Muller),
icraig@postino.up.ac.za (I.K. Craig)

compared to that of a base case and an Advanced Regulatory Control (ARC) case, showing that significant additional benefit may be achieved through the implementation of these advanced control and optimisation techniques. The paper further illustrates that additional capital is not necessarily required for the implementation of these techniques.

## 1. Introduction

The movement of energy and mass associated with the operation of a petro-chemical plant is mainly achieved through the use of process utilities which include electricity, steam, fuel gas, cooling water and compressed air. The generation, preparation and transportation of these utilities also require energy (mostly in the form of electricity), though they are often overlooked as areas for improvement and optimisation.

The potential benefits of utility optimisation have been shown to be substantial in some cases and should therefore be explored. In [1], the losses and inefficiencies encountered in typical steam systems are analysed, revealing significant potential for improvement. In [2], [3], [4] and [5], the benefits of control improvements for cooling water systems are illustrated. In [6] and [7], the optimisation of an industrial fuel gas system is presented resulting in a significant reduction in operating cost. In [8], the losses encountered in compressed air systems are shown and in [9], [10] and [11], the benefits in optimising pumping systems are explored.

In this paper, a dual circuit cooling water system is used to illustrate the benefit of the application of Advanced Process Control (APC) techniques on hybrid utility systems. A description of the process is first given followed by a discussion on the development of two control and optimisation schemes. The first is a Hybrid Non-linear Model Predictive Control (HNMPC) configuration aimed at a reduction in energy consumption while honouring process constraints.

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The next is an extension of the first where Time-of-Use (TOU) electricity rates are used in an Economic HNMPC (or EHNMPC) solution for the dynamic

25 optimisation of electricity cost. The results are then discussed and the cases are compared to each other and to that of a base case and an Advanced Regulatory Control (ARC) case.

## 2. Process Description

The system considered in this study is a dual circuit, induced draft, counter

30 flow cooling water system as shown in Figure 1.

The system consists of two water circuits. The first is the tempered water (TW) circuit which is a closed system containing treated water. The second is the cooling water (CW) circuit which contains untreated water (apart from standard dosing). Each of the circuits is equipped with its own bank of five

35 parallel centrifugal pumps. The CW circuit also contains a bank of four cooling towers (CTs). The TW circuit runs through the plant heat exchanger network where it collects heat from the process. It then transfers the heat to the CW through an interconnecting bank of plate heat exchangers. The CW circuit then expels the heat through the cooling towers where the main mechanism for

40 cooling is the partial evaporation of a portion of the water. On the TW side of the common heat exchangers is a bypass line with a control valve, which is used to bypass a portion of the TW to reduce total cooling. There are also control valves on the discharges of the CW pumps initially intended to prevent pumps from running beyond capacity. A detailed account of the system model is given

45 in [4], [5] and [12].

The controlled variables for the system are the TW supply temperature, $T_{TWS}$, the TW differential temperature, $\Delta T_{TW}$, the total power consumption, $W_T$, and the total electricity cost, $C_T$.

The manipulated variables for the system are the number of running TW

50 pumps, $U_{TW}$, the number of running CW pumps, $U_{CW}$, the number of running CT fans, $U_{CT}$, the temperature control valve opening, $OP_{TV}$, and the open-
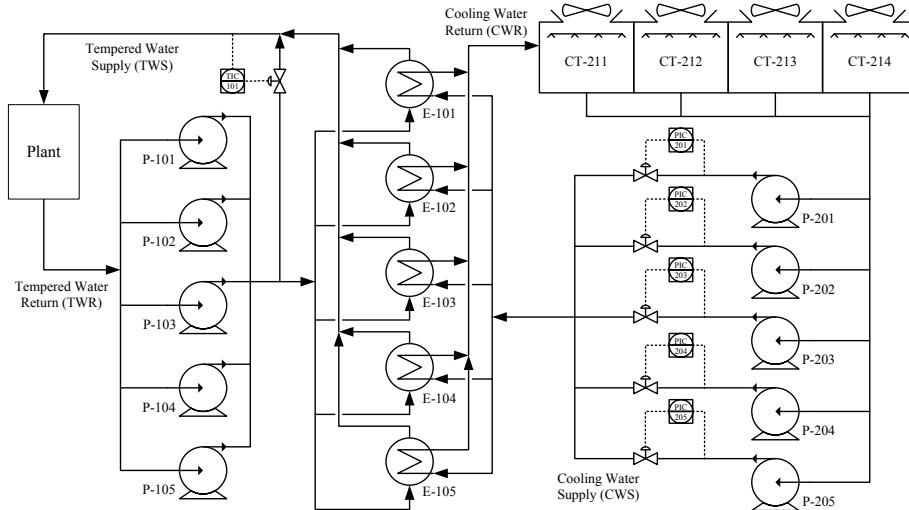
Figure 1: Dual circuit induced draft counter flow cooling water system [4, 5, 12].

ings of the pressure control valves, $OP_{PV}$ (the same valve opening is written to all the discharge valves of the running CW pumps). The running signals are discrete inputs whereas the valve openings are continuous handles. This combi-
nation of discrete and continuous handles classifies the cooling water system as a hybrid system, which complicates the formulation of control and optimisation solutions [13]. Furthermore, the system exhibits non-linear behaviour and is highly interactive.

The measured disturbance variables are the plant duty, $Q_P$, the ambient temperature, $T_a$, and the relative humidity, $RH$. The resultant model consists of 8 state equations, 14 algebraic equations, 5 inputs, and 3 measured disturbance variables together with 29 model parameters.

## 3. Methods

This section describes the application of various control and optimisation schemes to the cooling water system with the aim of reducing electricity con-sumption and/or cost while honouring process constraints.

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### 3.1. Simulation Set-up

Two operating scenarios are analysed. The first scenario covers a period of 7 days of artificial plant data during which step-like and ramp-like changes are made to the plant duty and sinusoidal changes are made to the ambient temperature and relative humidity (both influencing the wet-bulb temperature, $T_{wb}$). The second scenario uses 6 days of real plant data during a period where significant load disturbances occurred (the same data that was used for the model verification in [4], [5] and [12]). Figures 2 and 3 show the plant duty and wet-bulb temperature for the two scenarios. The wet-bulb temperature is calculated as proposed in [14].

Four cases are considered: the first case illustrates the current system where no optimisation is performed in terms of equipment switching and only the temperature controller is active; the second case represents an Advanced Regulatory Control (ARC) scheme with conditional switching logic as described in [4] and [5]; the third is a Hybrid Non-linear Model Predictive Control (HNMPC) implementation whereas the fourth is an Economic HNMPC (EHNMPC) scheme where Time-of-Use (TOU) tariffs are included in the formulation. Each case was simulated for both the operating scenarios mentioned above.

The system constraints are:

- $26°\text{C} \leq T_{TWS} \leq 36°\text{C}$

- $\Delta T_{TW} \leq 6°\text{C}$

- $1 \leq U_{CT} \leq 4$

- $2 \leq U_{CW} \leq 5$

- $2 \leq U_{TW} \leq 5$

### 3.2. Base Case and Advanced Regulatory Control

The purpose of the base case is to illustrate the unoptimised operation of the plant. One pump on each bank and one cooling tower fan are used for
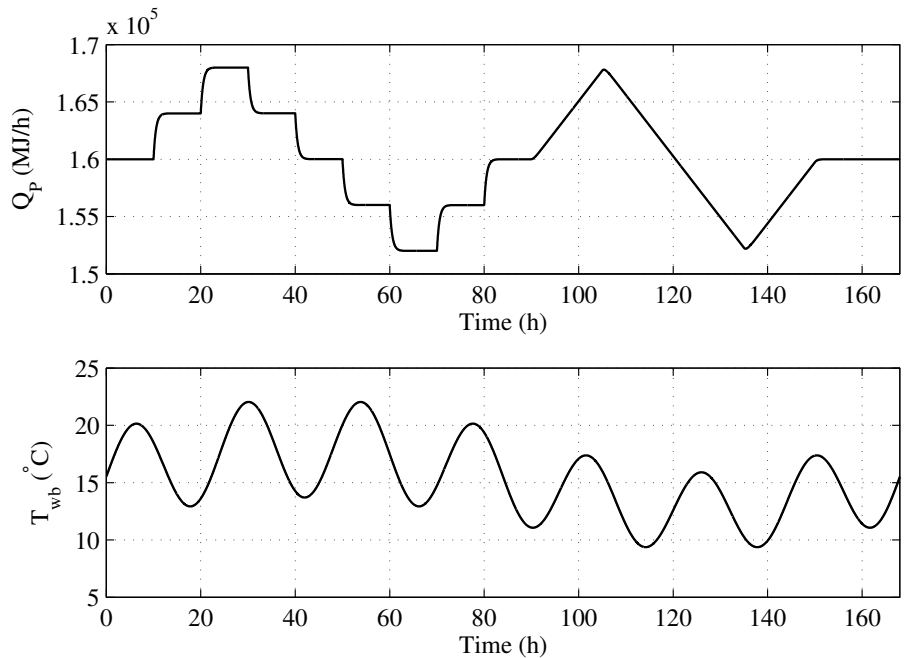
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Figure 2: Disturbance data for first simulation.

spares and therefore four pumps are running on each of the circuits with three

<sup>95</sup> cooling towers. The only feed-back controller that is activated in this case is the temperature controller on the TW side with a set-point of 26°C (TIC-101 in Figure 1). The pressure control valves on the discharges of the CW pumps are fully open (PIC-201 to PIC-205 in Figure 1). The results for both simulations are given in [4] and [5]. These results are compared to that of the solutions

<sup>100</sup> described in this paper in Section 4.3.

Advanced Regulatory Control (ARC) refers to control techniques that are implementable on most modern control systems without the need to purchase additional hardware or software. These techniques provide more functionality than what is typically achievable with only Proportional Integral Derivative

<sup>105</sup> (PID) control [15, 16]. For this study, the techniques that were deployed include override selector control, cascade control and conditional switching logic. This
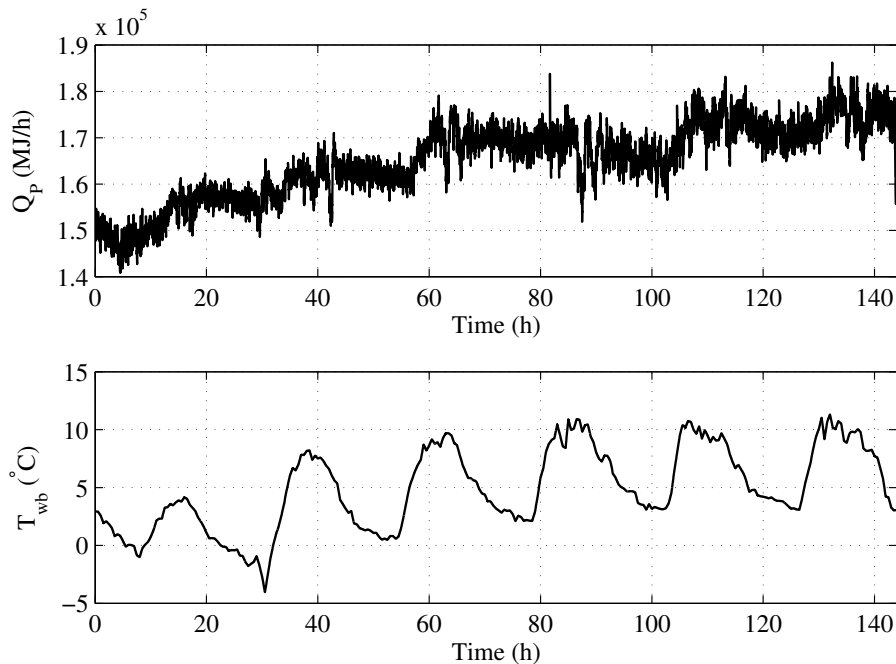
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Figure 3: Disturbance data for second simulation.

provides a degree of optimisation that is not present in the base case through switching of pumps and fans to meet cooling demand. The TW temperature controller is active with the same set-point as in the base case. The CW pump discharge valves are used for CW flow control as opposed to the original under-pressure control. Two additional temperature controllers measuring the TW supply temperature were implemented with set-points representing the high and low limits and are configured in a mid-of-three override selector control strategy to the CW flow with the desired nominal flow set-point, $f_{CW}^{SP}$, as the third input to the override selector. This enables the temperature controllers to manipulate the CW flow rate when constraint violations occur on the TW supply temperature. Figure 4 illustrates this scheme [4, 5].

The aim of the switching logic is to switch unnecessary equipment off when more cooling is provided than what is required. The effect is a reduction in
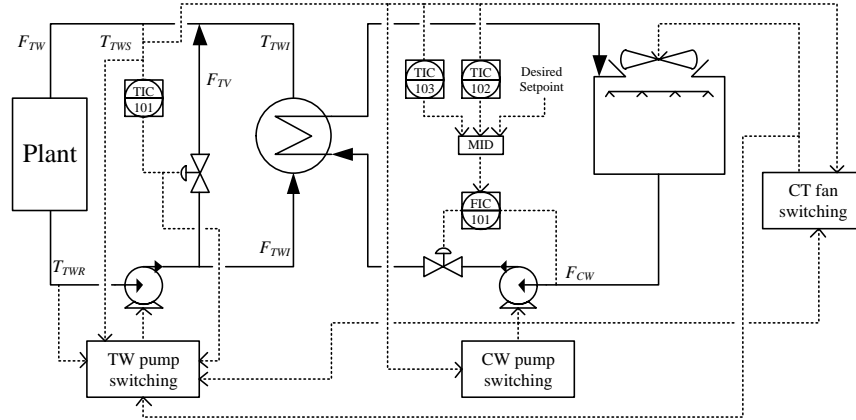
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Figure 4: ARC scheme illustration [4, 5].

power consumption. For more detail on the switching logic and the results for both simulations, refer to [4] and [5]. A comparison of the results from the ARC scheme to that of the solutions described in this paper is given in Section 4.3.

### 3.3. Hybrid Non-linear Model Predictive Control

Model Predictive Control (MPC) is an Advanced Process Control (APC) technique which has gained popularity especially in the process industry. A model of the process is used to predict the future behaviour of the system based on past and present data. The prediction is then used by an optimiser that determines the best set of input vectors in order to drive the process to an optimal operating point in an optimised fashion. The definition of optimality is determined by the formulation of a cost function and is usually subject to constraints. The first set of calculated input moves are implemented where-after the process is repeated based on the latest measurements. This is referred to as the receding horizon approach and provides feedback to the controller [6, 10, 15, 17, 18].

The cooling water system described in Section 2 is of a hybrid non-linear nature. This type of optimisation problem is generally referred to as mixed-integer

non-linear programming (MINLP). Conventional approaches for dealing hybrid systems involve segregation of the continuous and discrete optimisation into two different layers or transforming the system, whereas the non-linearity is typically cated for by linearising the system around certain operating points. Some examples of techniques capable of solving MINLP problems include generalised benders decomposition, branch and cut, outer approximation, and extended cutting plane. It should however be noted that these are limited to convex problems [19, 20]. Some algorithms capable of solving these problems directly include the genetic algorithm (GA) and particle swarm optimisation which are stochastic in nature.

In this case, the problem is treated as a single optimisation layer using a GA. The GA has several advantages including: Very little initial state data is required; it is capable of handling discrete and continuous optimisation variables in the same problem formulation; it does not require gradient information; constraints are easily incorporated into the problem definition; it is less likely to get stuck at local optima due to its stochastic nature; it can be very robust if configured correctly [21, 22, 23, 24]. Due to these advantages, the GA is especially attractive to problems that are difficult to formulate mathematically, exhibit strong non-linearities, have strong interaction between variables, contain discontinuities, are constrained, are of a hybrid nature, are non-convex, have ill-defined starting points, are time-variant and/or contain randomness or noise [5]. Therefore, the GA is a very attractive option for the cooling water problem illustrated in this paper. The main disadvantages of the GA are high computational intensity (which is less of a concern with modern computing capacity) and the fact that optimality cannot be guaranteed[1].

The resulting solution can be described as Hybrid Non-linear Model Predic-

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[1]The average convergence time of the algorithm is approximately 90 seconds on a 2011 model Apple MacBook Pro with dual core 2.7 GHz Intel i7 processor and 8 GB RAM. Comparing this to the controller iteration time of 30 minutes shows that the solution is realistically implementable even with modest computing power.
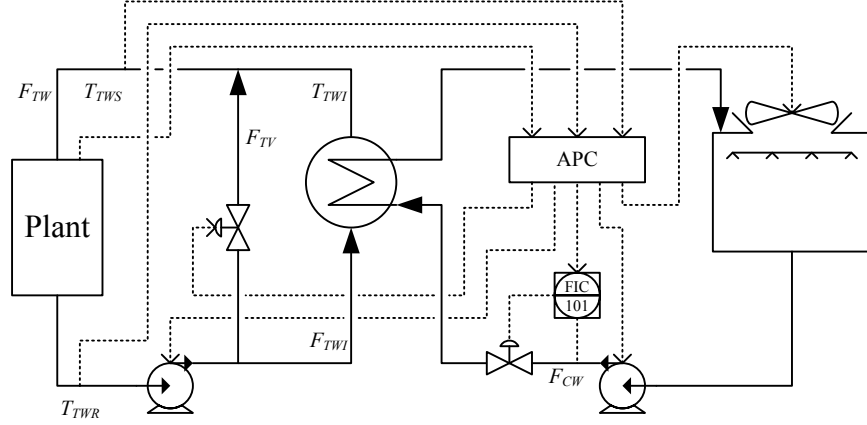
Figure 5: HNMPC control scheme illustration [5].

tive Control (HNMPC). Figure 5 illustrates this control scheme. A population size of 10 with a maximum number of generations of 15 were chosen for the GA based on the average convergence rate observed per iteration. To achieve the required constraint handling, a penalty function is minimized as opposed to the fitness function where the penalty function includes a term for infeasibility. It is then combined with binary tournament selection which scrutinises selection and pairing based on the feasibility of the individuals [25]. Specialised crossover and mutation functions are used together with a truncation procedure for integer restriction to accommodate the mixed-integer problem [26].

The fitness function is a weighted sum of the squares of the deviations from the allowable ranges on $T_{TWS}$ and $\Delta T_{TW}$ with a linear cost component proportional to the total power consumption, $W_T$. In addition, a linear cost component is added for the CW pump discharge valve openings which only becomes effective when the valves open fully, thereby providing wind-up detection for the CW flow control (the controller will know not to further increase the flow set-point if the valves are already saturated).

The fitness function is given by

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$$J = \frac{1}{j}\sum_{i=1}^{j}(Q_{T_{TWS}}E_{T_{TWS},i}^2 + Q_{\Delta T_{TW}}E_{\Delta T_{TW},i}^2 \tag{1}$$
$$+Q_W W_{T,i} + Q_{PV}E_{OP_{PV},i})$$

where $j$ represents the number of samples in the prediction frame (with a sampling time of 30 seconds (1/120 hours) resulting in $j = 720$ over the 6 hour prediction time), $E_{T_{TWS}}$, $E_{\Delta T_{TW}}$ and $E_{OP_{PV}}$ are the constraint violations for $T_{TWS}$, $\Delta T_{TW}$ and $OP_{PV}$ (which are zero when operating within the constraints), and $Q_{T_{TWS}} = 20$, $Q_{\Delta T_{TW}} = 15$, $Q_W = 10$ and $Q_{PV} = 100$ are the weighting variables.

Table 1 shows the specific controlled variable (CV) and manipulated variable (MV) limits for the HNMPC case. Safety margins are used for $T_{TWS}$ and $\Delta T_{TW}$ resulting in slightly more conservative limits than mentioned in Section 3.1.

Table 1: CV and MV limits for the HNMPC case.

| Variable | Low limit | High limit |
|---|---|---|
| $T_{TWS}$ | 28 | 34 |
| $\Delta T_{TW}$ | 0 | 5.5 |
| $U_{CT}$ | 1 | 4 |
| $U_{CW}$ | 2 | 5 |
| $U_{TW}$ | 2 | 5 |
| $f_{CW}^{SP}$ | 500 | 4500 |
| $OP_{TV}$ | 0 | 100 |

The MV limits are treated as hard constraints by the optimiser and will therefore not be violated under any circumstances. The CV limits are soft limits that are only used in the fitness function. Therefore, CV violations will merely cause weak performance and will not cause infeasible solutions when constraints cannot be met. This allows for a robust solution. The solution has not been tested on inherently unstable processes such as levels.

The results for the HNMPC case are discussed in Section 4.1.

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*3.4. Economic Hybrid Non-linear Model Predictive Control*

In the HNMPC case discussed in the previous section, the optimisation variable is the total power consumption. When a constant electricity rate is applied, this results in a minimised cost as well. However, when Time-of-Use (TOU) tariffs are enforced by the electricity supplier, a different approach is required if the goal is to minimise cost. This leads to a trade-off between energy usage and cost. The TOU rates used here are according to the rates applicable in South Africa at the time of writing and are defined by:

$$
p(t) = \begin{cases} p_o & \text{for} \quad t \in [0,6) \cup [22,24) \\ p_s & \text{for} \quad t \in [6,7) \cup [10,18) \cup [20,22) \\ p_p & \text{for} \quad t \in [7,10) \cup [18,20) \end{cases}
\tag{2}
$$

where $t$ is the time of day in hours, $p_s = 0.7423$ R/kWh is the standard rate, $p_o = 0.4031$ R/kWh is the off-peak rate, and $p_p = 2.4503$ R/kWh is the peak rate (for the high demand season from June to August). R is the symbol for the South African currency (Rand).

The knowledge of what the rates are at any given time of day allows the cost to be accurately calculated at any given moment. Therefore, it allows for the dynamic optimisation of the cost over the prediction horizon which is integrated into the MPC solution rather than in a separate Real-Time Optimisation (RTO) layer performing steady-state economic optimisation. This puts the current solution in the category of Economic MPC [27] and more specifically Economic Hybrid Non-linear MPC (EHNMPC) for this system. Figure 5 and Table 1 are also applicable to this case.

The fitness function for the EHNMPC case is similar to that of the HNMPC case with the power consumption component substituted with an electricity cost term as follows:

$$
\begin{aligned}
J &= \frac{1}{j} \sum_{i=1}^{j} (Q_{T_{TWS}} E_{T_{TWS},i}^2 + Q_{\Delta T_{TW}} E_{\Delta T_{TW},i}^2 \\
&\quad + Q_C C_{T,i} + Q_{PV} E_{OP_{PV},i})
\end{aligned}
\tag{3}
$$

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where $C_{T,i}$ is the total electricity cost at sampling instant $i$ (calculated using $W_{T,i}$ and $p(i)$) and $Q_C = 4$ is the weight for the cost component (which was adjusted iteratively to find a fair compromise between cost reduction and constraint violations).

The results for the EHNMPC case are given in Section 4.2.

## 4. Results and Discussion

The four cases discussed in the previous sections were evaluated through simulation studies to determine the level of optimisation achievable for each case in terms of power consumption/cost minimisation, while still honouring process constraints (see Section 3.1). The detailed results for the base case and the ARC case are given in [4] and [5] and those for the HNMPC and EHNMPC cases are presented in this section followed by a detailed comparison of all four cases [5].

### 4.1. Hybrid Non-linear Model Predictive Control Results

The CV values for the HNMPC case are shown in Figure 6 for the first simulation. The TW supply temperature is controlled within range more effectively than both the base and ARC cases and exhibits negligible constraint violations. The TW differential temperature is pushed toward the upper constraint and also has negligible constraint violations. The total power is lower than both previous cases. The CW pump discharge valve opening is included in the formulation of the fitness function to prevent wind-up on the CW flow controller. See [4], [5] and Tables 2 to 4 for a complete comparison.

The MV values for this simulation are given in Figure 7. Even though the control and optimisation results are superior to that of the base and ARC cases, these results are achieved with visibly fewer switching activities (see [4] and [5] for comparison). Although the optimisation algorithm actively tries to minimise the number of pumps that are running, it still allows the maximum number to run when required to prevent sustained constraint violations. It is also able
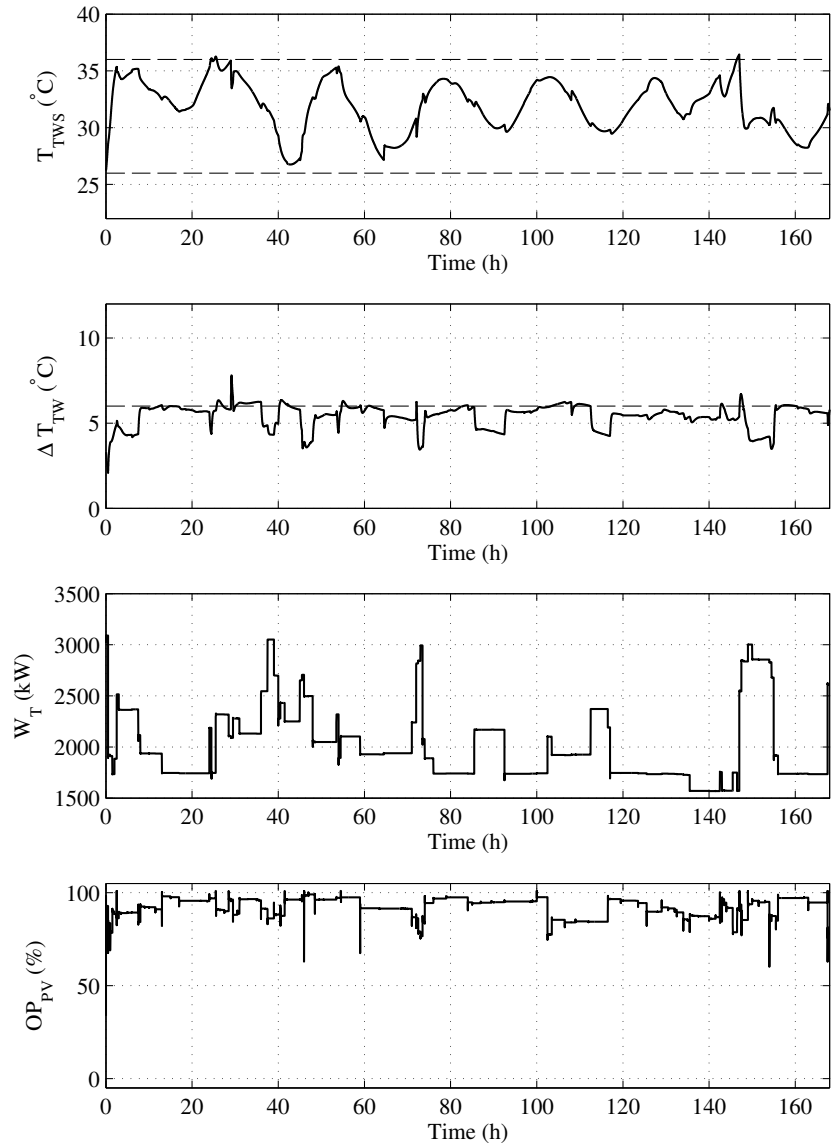
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Figure 6: Controlled variables (HNMPC case) for the first simulation.

to switch multiple pieces of equipment in the same execution cycle, which allows for superior disturbance rejection compared to the ARC (which follows a sequential switching pattern). The TW temperature control valve opening

and the CW flow controller set-point (the continuous control handles) are actively manipulated. The CW flow controller set-point can only be increased within the limitations of the valve openings as mentioned above. This serves to dynamically adjust the upper MV limit for this variable.

Figure 8 gives the CV values for the second simulation. Once again, the TW supply temperature is maintained within limits with negligible constraint violations (which was challenging for the base and ARC cases – see [4] and [5] for comparison). The TW differential temperature is also driven towards the upper constraint as in the first simulation allowing for the fewer TW pumps to be running. The total power consumption is lower that the previous cases. The CW pump discharge valve opening is again included in the formulation of the optimisation problem to prevent wind-up by telling the controller not to increase the flow controller set-point when the valves are 100% open.

Figure 9 gives the MV values for the second simulation. As with the first simulation, the pump and fan running signals (the discrete handles) are actively minimised to save energy as long as the CVs are within constraints. The continuous handles are also actively manipulated to honour CV limits.

The results for both simulations indicate clearly that the HNMPC solution allows for a further reduction in power consumption while simultaneously providing better constraint control than the base and ARC cases. See [4], [5] and Tables 2 to 4 for a complete comparison.

*4.2. Economic Hybrid Non-linear Model Predictive Control Results*

The CV results for the first simulation of the Economic HNMPC case are shown in Figure 10 and the MVs are shown in Figure 11. The results for the second simulation are given in Figures 12 and 13. The constraint handling and use of the MVs are similar to that of the HNMPC case with negligible constraint violations and effective minimisation of total power consumption. The success of this scheme in further reducing energy cost is illustrated in the case comparison results given in the next section which reveals superior cost control for both simulations. Furthermore, Figures 14 and 15 illustrate the power consumption
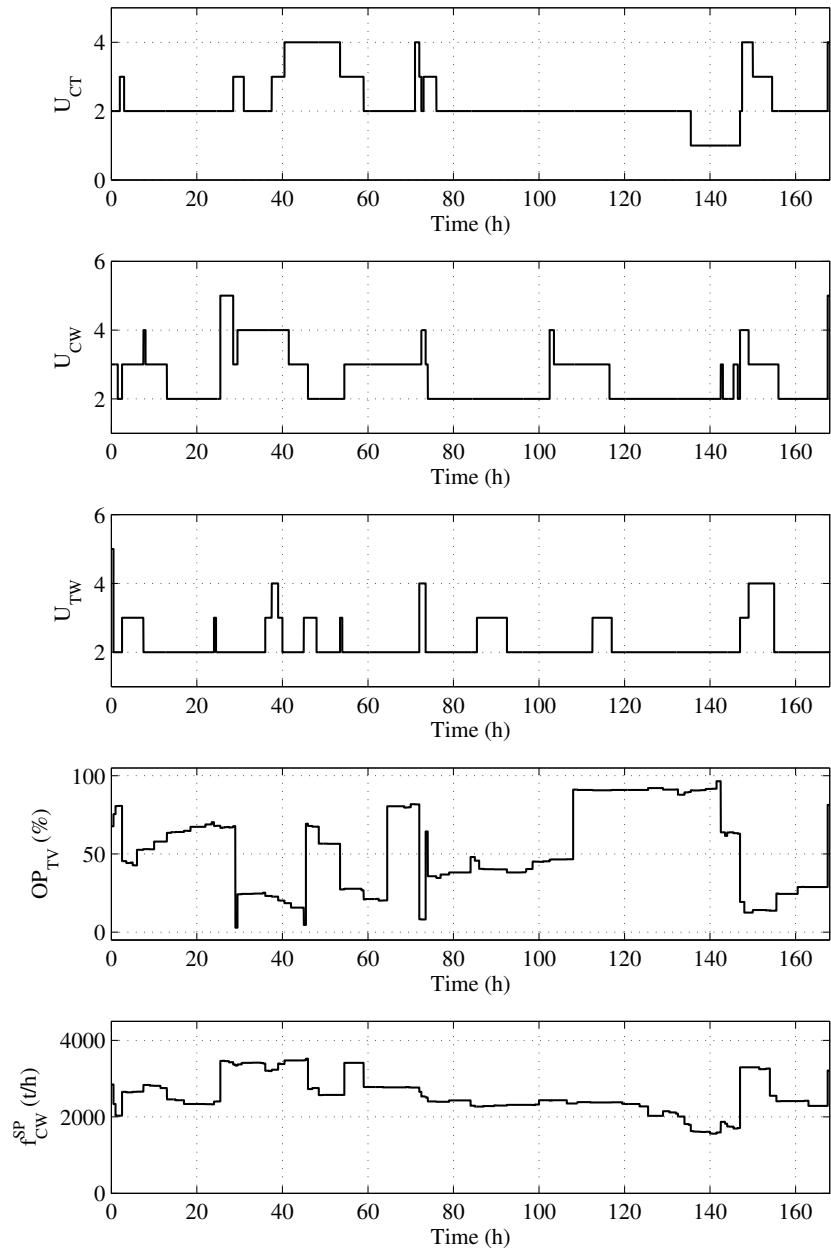
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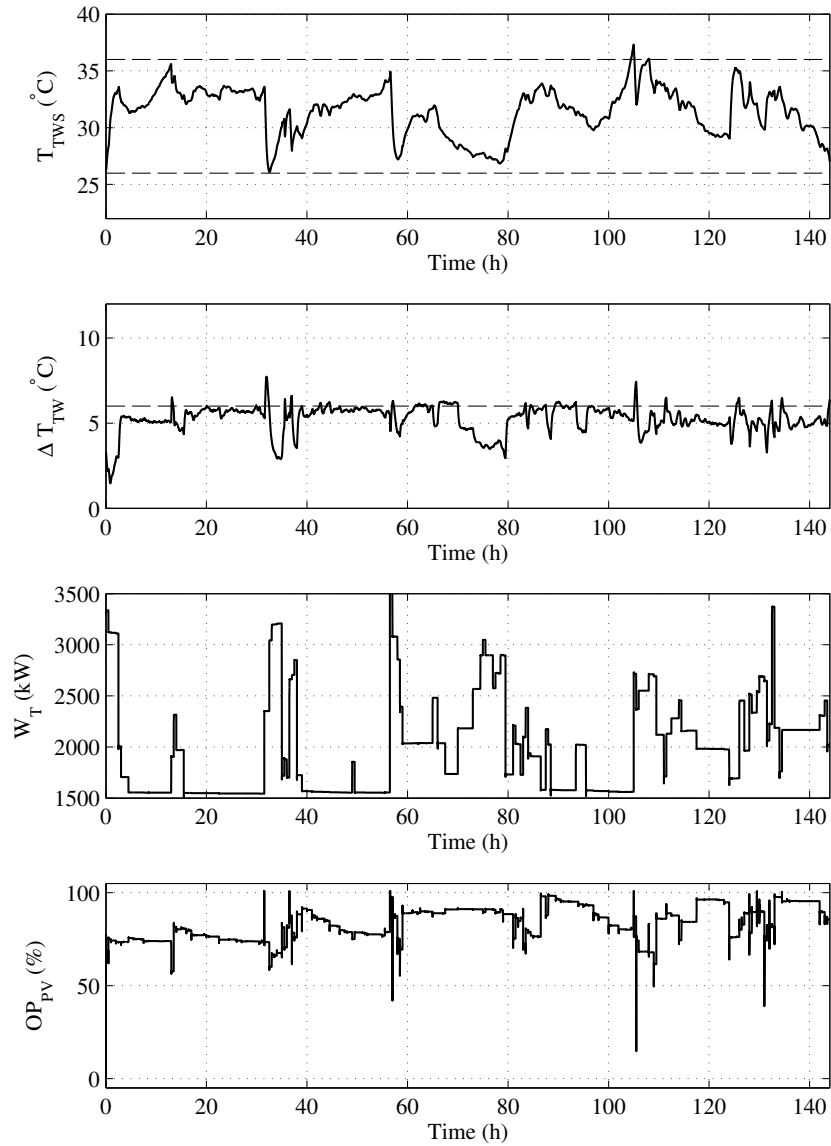Figure 7: Manipulated variables (HNMPC case) for the first simulation.

Figure 8: Controlled variables (HNMPC case) for the second simulation.

against the dynamic electricity price backdrop to illustrate how the controller attempts to minimise consumption during peak hours (within the limitations imposed by the CV constraints). Less apparent is the expected increase in
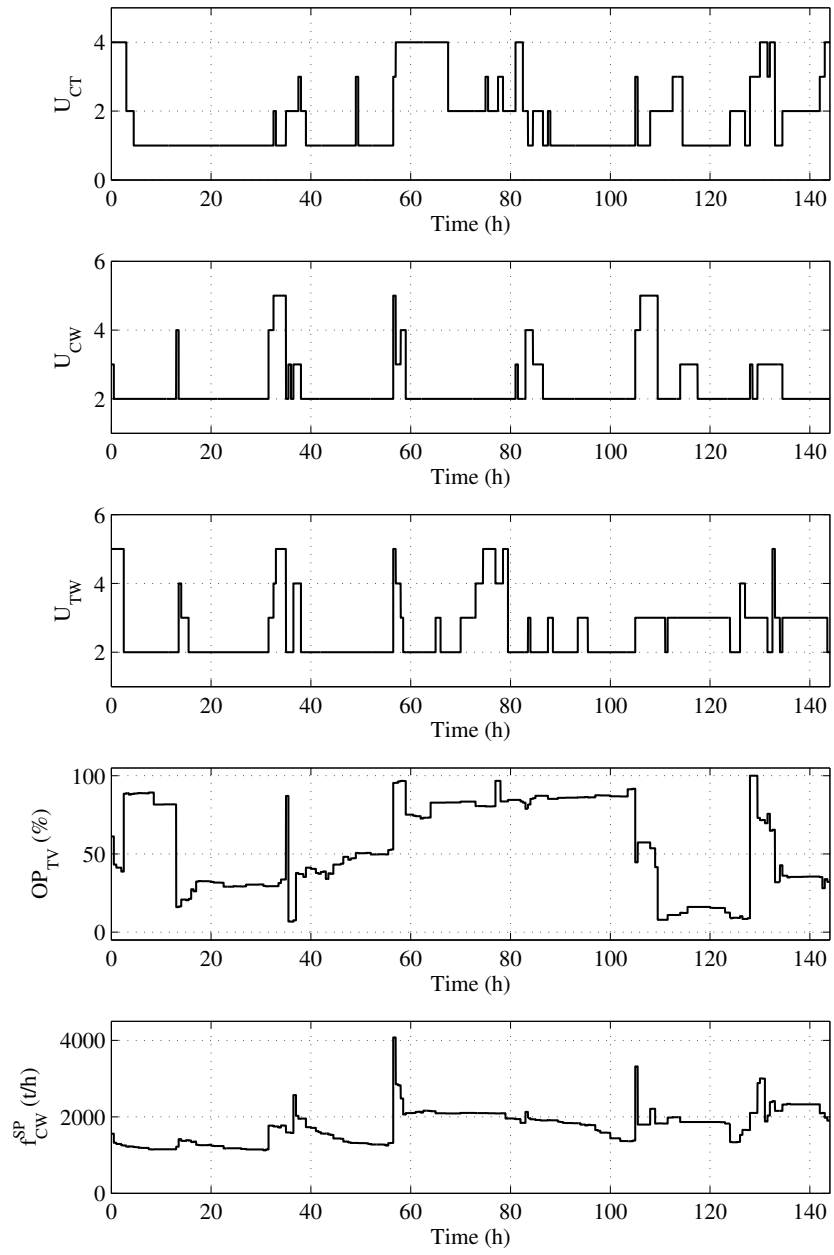
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Figure 9: Manipulated variables (HNMPC case) for the second simulation.

consumption during off-peak periods. This can be explained considering that the off-peak periods fall predominantly during night time when the ambient temperature drops, which results in an increased cooling efficiency. Therefore, the cooling demand is lower and there is no incentive for the controller to increase flow-rates even though the price is lower.

### 4.3. Case Comparison

In addition to the visual illustration of the controller performances for the different cases, the power and cost for each case are calculated and compared. The power consumption, total energy consumed, and energy cost for each case for the two simulations are shown in Tables 2 and 3. The incremental differences between cases are also shown.

The results for the first simulation indicate that the ARC can potentially provide a reduction in energy consumption of around 30%. The HNMPC and EHNMPC achieve a further 4% and 6% respectively. The ARC case also provides a cost saving in the order of 30% with the HNMPC and EHNMPC adding an additional 4% and 7% saving.

The results for the second simulation show a similar improvement for the ARC case of around 30% on both the power consumption cost, and there is still the significant additional benefit of being able to control the plant within constraints, which is not achieved in the base case. The results for the HNMPC and EHNMPC cases are also similar to that of the first simulation (37% and 40.6% reductions in energy with 35.2% and 41.6% reductions in cost). Therefore, the results obtained with these techniques as well as that of the ARC seem to be repeatable at different operating points.

In order to compare the constraint handling ability of the different control schemes, violation indices were calculated for the TW supply temperature ($e_{T_{TWS}}$) and TW differential temperature ($e_{\Delta T_{TW}}$) for the two simulations. The indices were calculated as the sum of violations over the simulation period divided by the number of samples in the period. The results are shown in Table 4. The constraint handling for the TW supply temperature improves as more
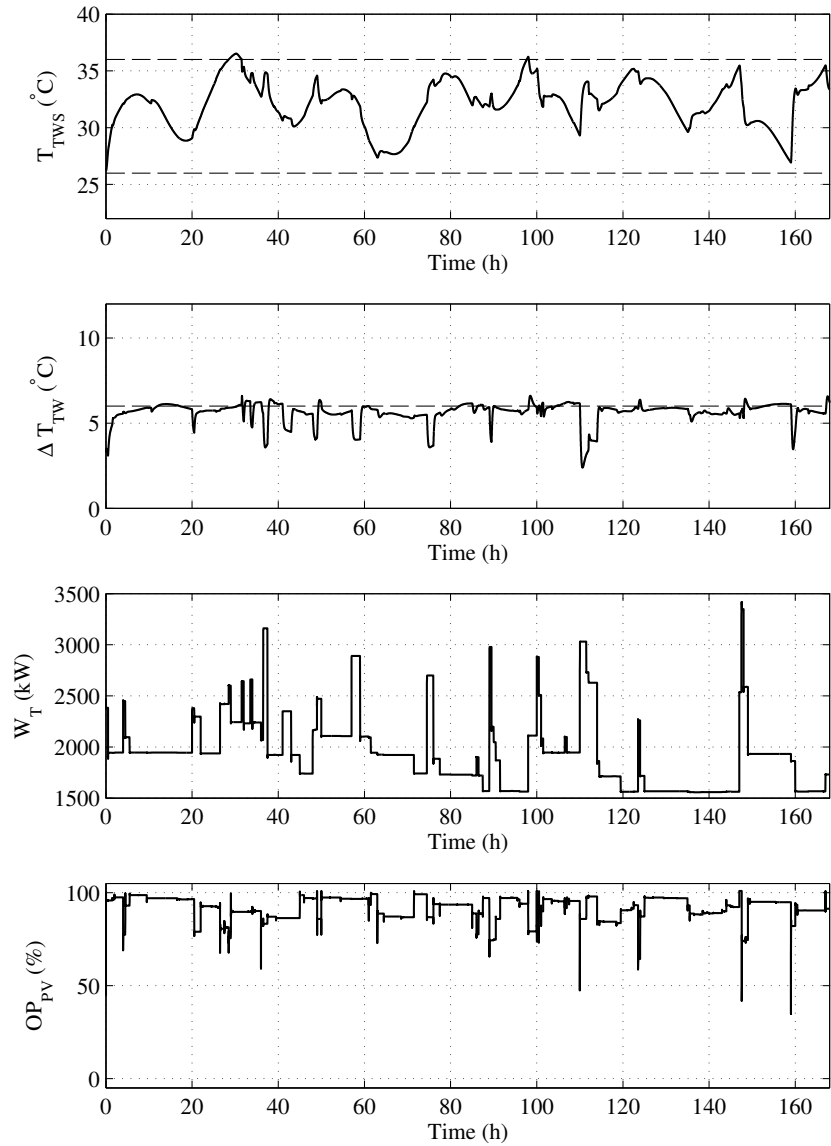
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Figure 10: Controlled variables (Economic HNMPC case) for the first simulation.

advanced control is applied. The TW differential temperature constraint vi-
olations deteriorate slightly as a shift is made from conservative operation in
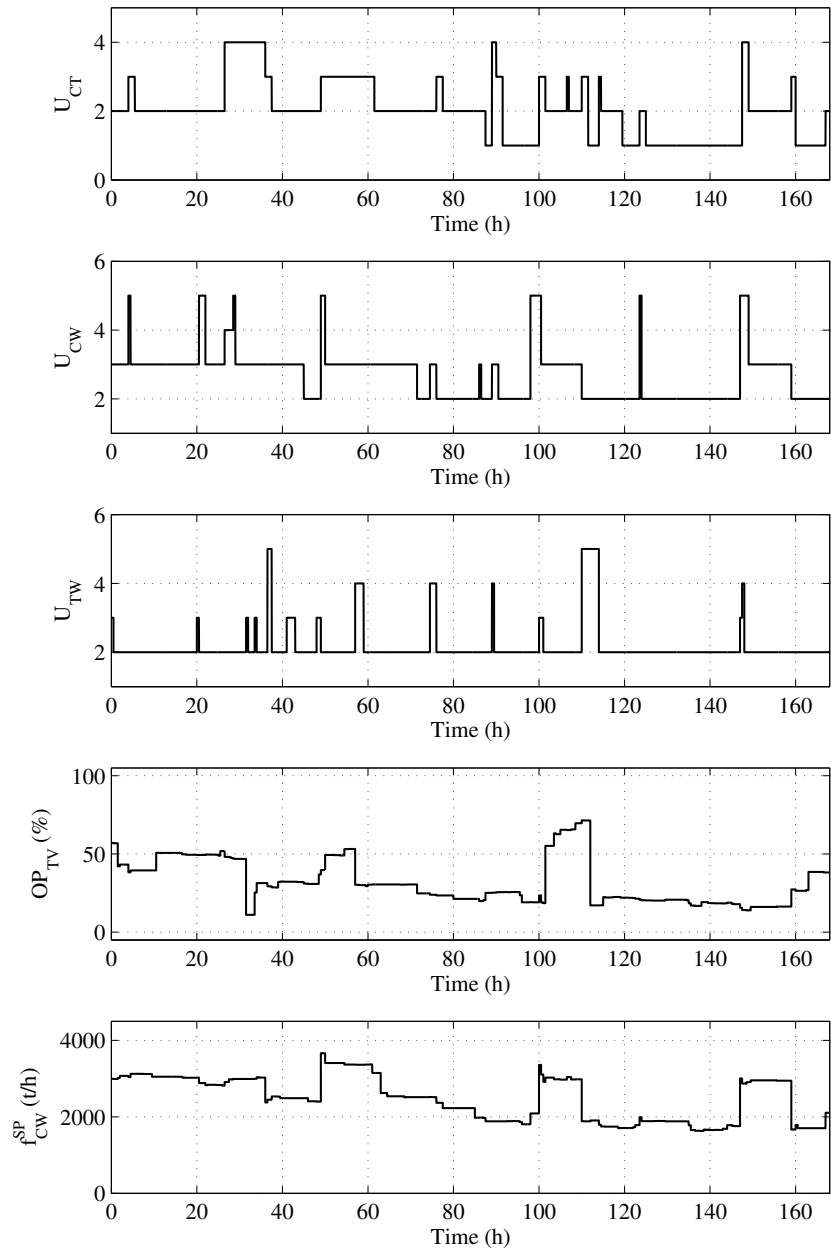315 exchange for energy/cost savings.

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Figure 11: Manipulated variables (Economic HNMPC case) for the first simulation.
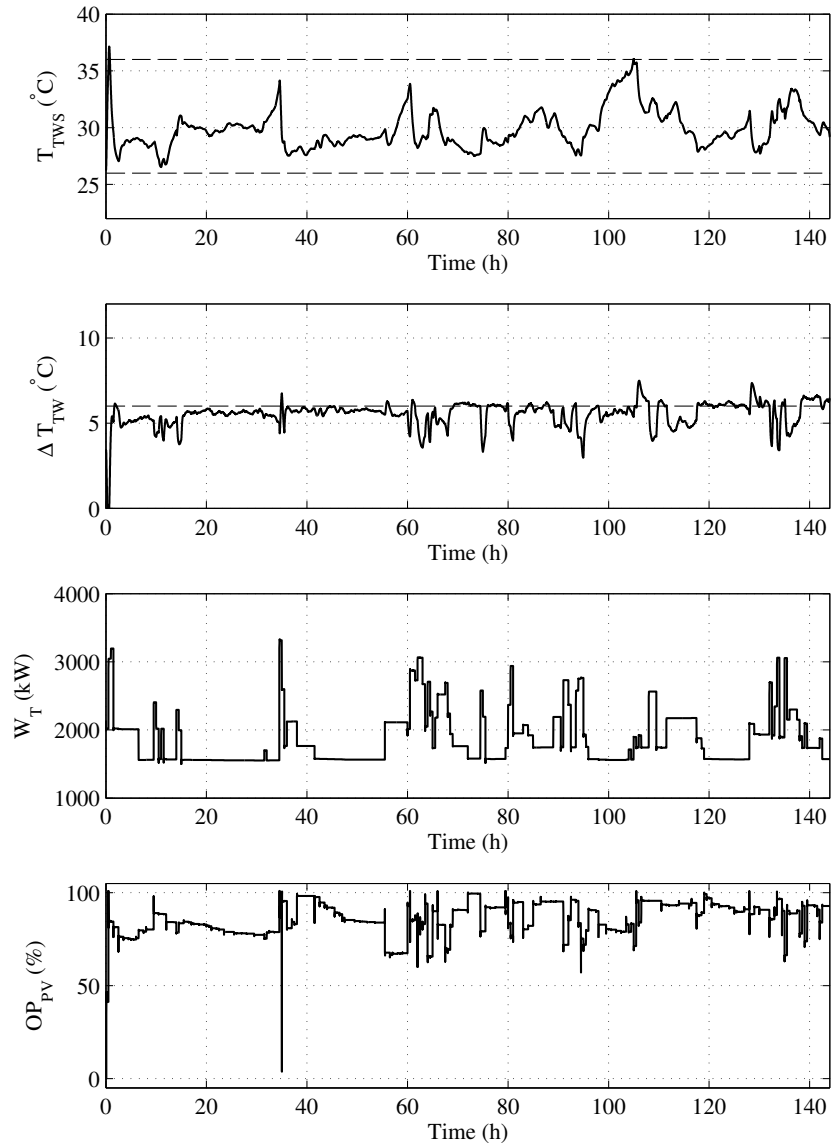
Figure 12: Controlled variables (Economic HNMPC case) for the second simulation.

The ARC has superior performance to the base case overall. The HNMPC and EHNMPC similarly outperform the ARC case. The HNMPC and EHNMPC cases are more similar in performance though the goal of achieving a further cost
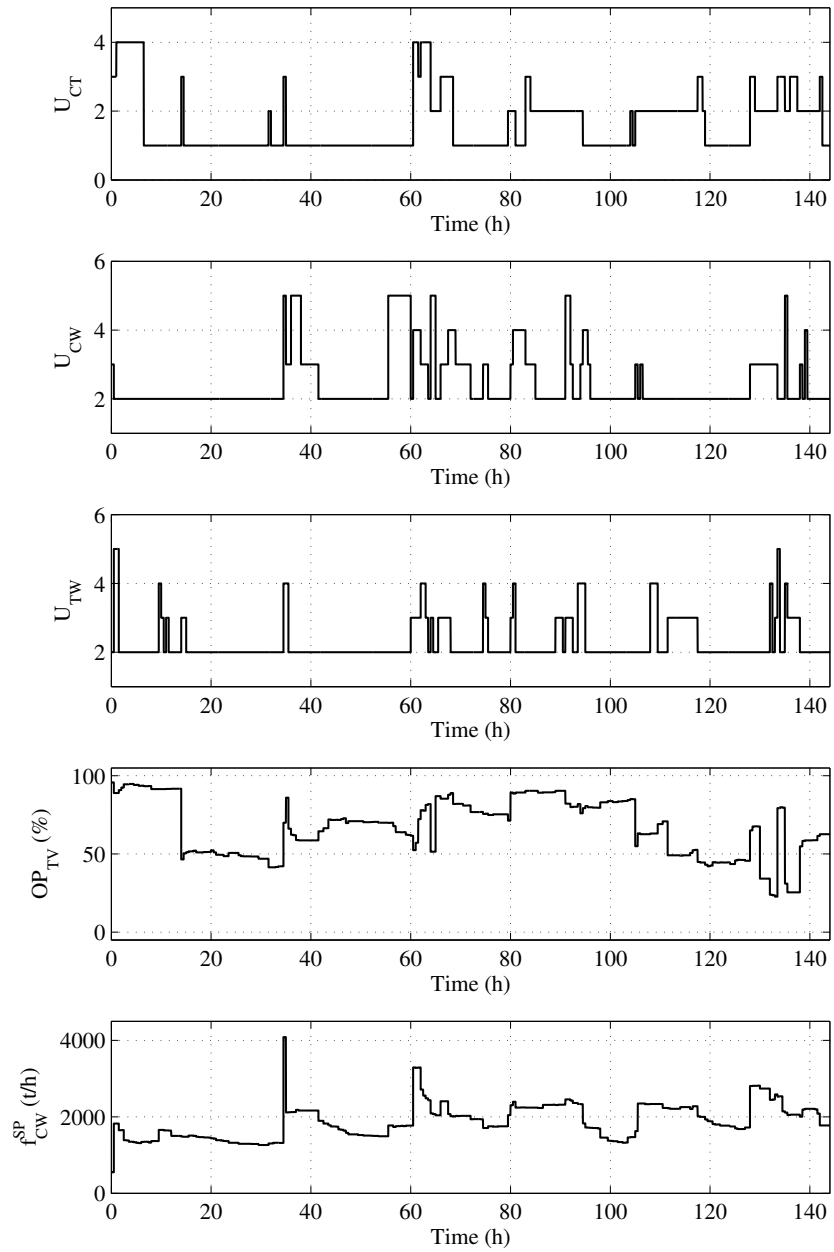
Figure 13: Manipulated variables (Economic HNMPC case) for the second simulation.
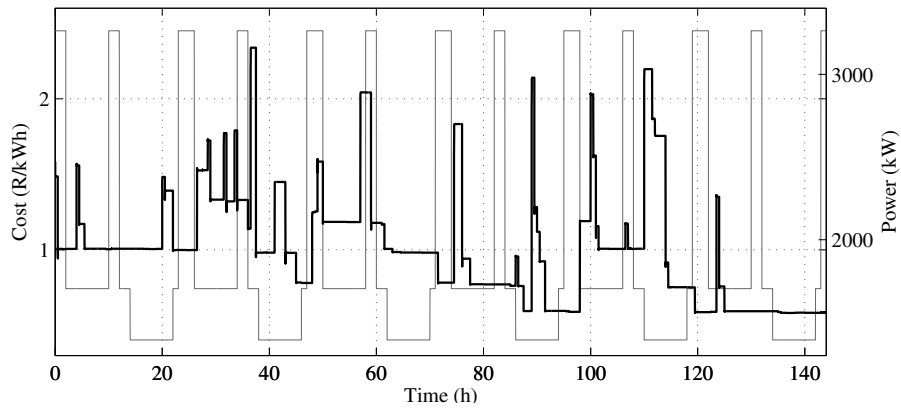
Figure 14: Power consumption (black line) and electricity cost (grey line) for the first simulation of the EHNMPC case.
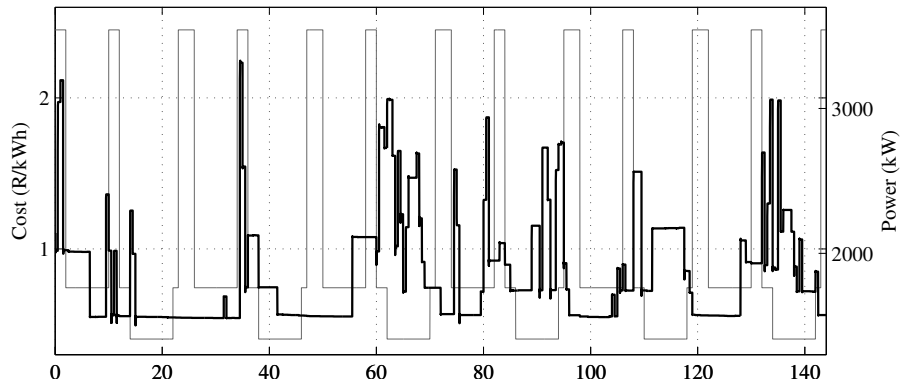


Figure 15: Power consumption (black line) and electricity cost (grey line) for the second simulation of the EHNMPC case.

saving is achieved by the EHNMPC and it is interesting that it achieves this while also achieving a further reduction in consumption. It would however be possible to achieve lower cost at higher consumption in certain circumstances though this would not likely be the case with the cooling water system, where the low-cost time periods coincide with lower cooling demand.

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Table 2: Average power and energy consumption comparison.

| Case | Average Power (kW) | Total energy (kWh) | Reduction from base | Incremental reduction |
|------|--------------------|--------------------|---------------------|------------------------|
| Simulation 1 | | | | |
| Base | 3,052 | 512,736 | - | - |
| ARC | 2,130 | 357,840 | 30.2% | 30.2% |
| HNMPC | 2,001 | 336,168 | 34.4% | 6.1% |
| EHNMPC | 1,935 | 325,080 | 36.6% | 3.3% |
| Simulation 2 | | | | |
| Base | 3,142 | 452,448 | - | - |
| ARC | 2,213 | 318,672 | 29.6% | 29.6% |
| HNMPC | 1,981 | 285,264 | 37.0% | 10.5% |
| EHNMPC | 1,865 | 268,560 | 40.6% | 5.9% |

## 5. Conclusion

The potential for energy/cost reduction in the generation and transmission of utilities can be substantial, yet these areas are often overlooked in terms of optimisation. A prime example of such a utility is the cooling water system discussed in this paper.

The main benefit in optimising this system is the reduction of running pumps and fans during times of over-cooling. Advanced control and optimisation techniques can provide the means for achieving the desired optimisation of the system and a wealth of information is available on these topics. When a system comprises of both continuous and discrete inputs, it is referred to as a hybrid system and there are several approaches that may be followed in optimising these systems. The conventional approach is to treat the continuous and discrete components in separate layers and also separates the dynamic and economic optimisation.

Table 3: Electricity cost comparison.

| Case | Total Cost (R) | Reduction from base | Incremental reduction |
|---|---|---|---|
| **Simulation 1** | | | |
| Base | 503,280 | - | - |
| ARC | 354,160 | 29.6% | 29.6% |
| HNMPC | 330,790 | 34.3% | 6.6% |
| EHNMPC | 316,860 | 37.0% | 4.2% |
| **Simulation 2** | | | |
| Base | 445,910 | - | - |
| ARC | 317,260 | 28.9% | 28.9% |
| HNMPC | 288,780 | 35.2% | 9.3% |
| EHNMPC | 260,610 | 41.6% | 9.8% |

This approach is followed in the development of the ARC solution where the continuous control and the discrete switching logic operate in two separate layers
340 and execute at different frequencies. The HNMPC scheme unifies multiple layers by combining the continuous and discrete elements into a single control and optimisation solution. The EHNMPC goes one step further in also combining the dynamic and economic optimisation into the same solution by including the TOU data in the control formulation.

345 The results indicate that the HNMPC and EHNMPC achieve superior energy and cost reductions compare to the base and ARC cases (as seen in Tables 2 and 3.). These results show that significant savings may be achieved through the use of modern control and optimisation techniques when applied to utility systems. Furthermore, these techniques do not necessarily require additional
350 capital investment and are able to accommodate hybrid interactive non-linear processes without having to transform or linearise the system.

Table 4: Constraint violation comparison.

| Case | $e_{T_{TWS}}$ | $e_{\Delta T_{TW}}$ |
|---|---|---|
| Simulation 1 | | |
| Base | 0.0691 | 0.0000 |
| ARC | 0.0207 | 0.0208 |
| HNMPC | 0.0014 | 0.0256 |
| EHNMPC | 0.0058 | 0.0312 |
| Simulation 2 | | |
| Base | 3.5254 | 0.0047 |
| ARC | 0.0190 | 0.0244 |
| HNMPC | 0.0032 | 0.0242 |
| EHNMPC | 0.0013 | 0.0576 |

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