EVIDENCE OF PERSISTENCE IN U.S. SHORT AND LONG-TERM INTEREST RATES USING LONG-SPAN MONTHLY AND ANNUAL DATA

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Abstract

This study examines the time series behavior of U.S. short- and long-run real expost interest rates within a long memory approach with non-linear trends using a long span of monthly and annual data. Recursive estimates of the integration order for the short- and long-run rates suggest long memory for the monthly data for the whole period of time, while we cannot reject the I(0) hypothesis for the annual short and long run interest rates.

JEL classification: C22, E43, G12.

Keywords: interest rates, long memory, non-linear trends

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1.   Introduction

The real interest rate is a key variable in many macroeconomic and financial models, such as consumption-based asset pricing models (Lucas, 1978; Hansen and Singleton, 1982; 1983; Rose, 1988), neoclassical growth models (Cass, 1965), Fisher equation (Fisher, 1896; 1930), investment decisions (Tobin, 1965) or the term structure of interest rates (Modigliani and Shiller, 1973), and this justifies the great interest in understanding the time series properties of this variable.

The time series properties of long and short-run real interest rates will shed some light on the degree of stationarity and persistence of these variables. For instance, if these variables are stationary, shocks have transitory effects and monetary policy intervention will only have transitory effects on these variables. Furthermore, the degree of persistence of this variable could help us to evaluate the theoretical implications of different macroeconomic and financial models. The relevant monetary policy implications of the time series properties of real interest rates explains the ample literature on the econometric modeling of this variable using different methodologies, such as unit root tests (Rose, 1988; King et al., 1991; Galí, 1992; Mishkin, 1992), cointegration tests (Bierens, 2000; Wallace and Warner, 1993), or fractional integration models (Lai, 1997; Tsay, 2000). However, despite the vast literature on the integration order or persistence of this variable, the results are not yet conclusive.[1]

The historical U.S. real interest rate data reveals that this variable has varied a lot since the beginning of the 19th century, and has been affected by several shocks, such as the Civil War (1861), World War I (1914), the Great Depression (1929), World War II (1939), oil price crises (1973 and 1978), or the most recent financial crisis (2007), several episodes that may have caused several disruptions, and thus, potential non-

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[1]   See Section 2 in this paper for the literature review on interest rate modeling.

linearities in the temporal evolution of this variable. The non-linear behavior of this variable has been mostly modeled in the literature by the inclusion of structural breaks (see, for example, Garcia and Perron, 1996; Caporale and Grier, 2000; Bai and Perron, 2003, among others), while only a few attempts have been made modelling non-linearities (see, for example, Kapetanios et al., 2003; Maki, 2005; Lanne, 2006).

The contribution of this paper is three-folded. First, we provide evidence of the long memory properties of the real short- and long-run ex-post interest rates for a long span of data allowing for non-linear deterministic trends in the form of Chebyshev polynomials in time. Second, including in the analysis both short- and long-run interest rates will allow us to compare time series properties of this variable at different maturities, allowing us to determine whether monetary policy intervention will be more or less effective when directed to interest rates at different maturities. Third, analyzing both monthly and yearly data will shed some light on the different degree of persistence of interest rates data at different time frequencies.

The remainder of the paper is structured as follows: Section 2 revises the literature on the econometric modeling on the interest rates. Section 3 describes the methodology and justifies its application in the context of interest rates. Section 4 presents the data and the main empirical results, while Section 5 contains some concluding comments and policy implications.

2.   Literature review

Modeling the dynamic behavior of interest rate series has become a relevant research area based on the relevance of this variable on many macroeconomic and financial models. Thus, a growing literature has examined the persistence of interest rate series using different time series techniques, different sample of countries and time periods.

For example, Rose (1988) examined the orders of integration of inflation rates and nominal interest rates for 18 OECD countries using annual, quarterly and monthly data, and concluded that any linear combination of stationary inflation rate and nonstationary nominal interest rates, and thus real interest rates, follow nonstationary processes. The same result of nonstationarity in real interest rates is found in Mishkin (1992), who applied Augmented Dickey-Fuller (ADF, Dickey and Fuller, 1979) tests to monthly data for US real interest rates for the period 1953-90. In contrast, King et al. (1991), Galí (1992) or Rapach and Weber (2004), among others, provide evidence that real interest rates are stationary using quarterly data of US interest rates. Wallace and Warner (1993) find unit root evidence for nominal interest and inflation rates and support the hypothesis of cointegration between the two variables, suggesting that US real interest rates are stationary. Bierens (2000) also finds evidence of cointegration between interest rates and inflation rates using monthly US data for 1954-94, concluding again that real interest rates are stationary.

The fractional integration methodology has also been applied to US interest rates, finding in most of the cases evidence of long-memory and mean-reverting behavior (see, for example, Lai, 1997, and Tsay, 2000), with the estimates of the order of fractional integration ranging from 0.7 to 0.8, significantly above 0 and below 1. In another paper, Shea (1991) investigates the consequences of long memory in interest rates for tests of the expectations hypothesis of the term structure. The author finds that allowing for the possibility of long memory significantly improves the performance of the model, even though the expectations hypothesis cannot be fully resurrected. In a related work, Backus and Zin (1993) observe that the volatility of bond yields does not decline exponentially when the maturity of the bond increases; in fact, they notice that the decline is hyperbolic, consistent with the fractionally integrated specification. Lai

(1997) and Phillips (1998) provided evidence based on semiparametric methods that ex-ante and ex-post US real interest rates are fractionally integrated. Tsay (2000) employs an Autoregressive Fractionally Integrated Moving Average (ARFIMA) model to provide evidence that the US real interest rate can be described as an $I(d)$ process. Further evidence can be found in Barkoulas and Baum (1997), Meade and Maier (2003) and Gil-Alana (2004a,b). Couchman, Gounder and Su (2006) estimate ARFIMA models to ex-post and ex-ante interest rates for sixteen countries. Their results suggest that, for the majority of countries, the fractional differencing parameter lies between 0 and 1, and it seems to be considerably smaller for the ex-post real rates than for the ex-ante rates. Evidence of persistence is also found in Neely and Rapach (2008), who survey the empirical literature on the time series properties of real interest rates and use different methodologies (unit root and cointegration tests, fractional integration, regime switching and structural breaks) to model the U.S. ex post real interest rate during the period 1953:Q1-2007:Q2 concluding that post-war real interest rates are very persistent. Haug (2014), while analysing the persistence property of long-term interest rates of ten industrialized countries (including the U.S.), using a long span of annual data (1880-2006), showed that persistence has varied over time, with periods when the real rate is covariance stationary and other periods when it follows a unit root process. Similar conclusions were also drawn by Apergis et al., (2015) based on a panel data approach of twenty OECD countries, which also included the U.S. In general, as far as persistence of real interest rate for the U.S. is concerned, one of the key stylized facts found in the literature is the considerable persistence found in real interest rates (Mishkin, 1992; King et al., 1991; Galí, 1992; Rapach and Weber, 2004; Neely and Rapach, 2008), especially in the most recent periods (Haug, 2014), but also that, persistence has been time-varying (Haug, 2014; Apergis et al., 2015).

Our paper examines the long memory properties of short and long run interest rates using a long span of annual (1800-2013) and monthly data (1871:M1-2015:M4), and allowing for non-linear deterministic trends in the form of Chebyshev polynomials in time, in order to account for the significant disruptions observed in the temporal evolution of real interest rates. Although different non-linear models have been used to model interest rates (see, for example, Kapetanios et al., 2003; Lanne, 2006), to our knowledge this is the first paper that allows for both long memory and non-linear deterministic trends to analyze the time series properties of ex-post US real interest rates. Furthermore, our results suggest that U.S. real interest rates are less persistent than suggested in the previous literature.

3.   Methodology

The methodology used in this paper is based on the concept of fractional integration. For this purpose we need to define first an I(0) process. We define a process $\{x_t, t = 0, \pm 1, ...\}$ as integrated of order 0 (and denoted as $x_t \approx I(0)$) if it is a covariance stationary process with a spectral density function that is positive and finite at the zero frequency. Having said this, a process is integrated of order d, (and denoted as $x_t \approx I(d)$) if it can be represented as

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, ..., \tag{1}$$

with $x_t = 0$ for t $\leq$ 0, and d > 0, where L is the lag-operator ($Lx_t = x_{t-1}$) and u_t is I(0). By allowing d to be fractional, we permit a much richer degree of flexibility in the dynamic specification of the series, not achieved when using the classical approaches based on integer differentiation, i.e., d = 0 and d = 1. Processes with $d > 0$ in (1) display

the property of "*long memory*", characterized in this way because the spectral density function of the process is unbounded at its origin. The methodology employed here to estimate the fractional differencing parameter is based on the Whittle function in the frequency domain (Dahlhaus, 1989).

On the other hand, it is well known that fractional integration, non-linearities and structural breaks are intimately related issues (Cheung, 1993; Diebold and Inoue, 2001; Giraitis et al., 2001; Kapetanios and Shin, 2003; Mikosch and Starica, 2004; Granger and Hyung, 2004; etc.). In particular, fractional integration can be an artifact generated by the presence of breaks that are not taken into account. Further, changes can occur smoothly rather than suddenly as implied by structural breaks; Ouliaris et al. (1989) therefore proposed regular polynomials to approximate deterministic components in the data generation process (DGP). However, as later pointed out by Bierens (1997), Chebyshev polynomials might be a better mathematical approximation of the time functions, since they are bounded and orthogonal; being cosine functions of time, they are a very flexible tool to approximate deterministic trends.

We then consider the following non-linear model:

$$y_t = \sum_{i=0}^{m} \theta_i P_{iT}(t) + x_t, \qquad t = 1, 2, \dots, \tag{2}$$

with *m* indicating the order of the Chebyshev polynomial, and x_t following an I(d) process of the form as in (1).

The Chebyshev polynomials $P_{i,T}(t)$ in equation (2) are defined as:

$$P_{0,T}(t) = 1,$$

$$P_{i,T}(t) = \sqrt{2}\cos(i\pi(t-0.5)/T), \qquad t = 1, 2, \dots, T; \quad i = 1, 2, \dots \tag{3}$$

(see Hamming (1973) and Smyth (1998) for a detailed description of these polynomials). Bierens (1997) uses them in the context of unit root testing. According to

Bierens (1997) and Tomasevic and Stanivuk (2009), it is possible to approximate highly non-linear trends with rather low degree polynomials. If *m = 0* the model contains an intercept, if *m = 1* it also includes a linear trend, and if *m > 1* it becomes non-linear - the higher *m* is the less linear the approximated deterministic component becomes.

4.   Data

In our paper, we work with ex post rather than ex ante real interest rate, where the latter is defined as the nominal interest rate minus the expected inflation rate, while the former is the nominal rate minus actual inflation. This is because the ex ante real rate is not directly observable because expected inflation is not directly observable. Of course, one could use some survey measure of inflation expectations, but there are problems to this approach, as economists are often reluctant to accept survey forecasts as measures of expectations (Neely and Rapach, 2008). Alternatively, one could use econometric forecasting methods to construct inflation forecasts, but then again, econometric forecasting models do not necessarily include all of the relevant information agents use to form expectations of inflation, and also such models could fail to change with the structure of the economy (Neely and Rapach, 2008). Given this, we use the actual inflation rate as a proxy for inflation expectations, as in Neely and Rapach (2008). Let us now see this more formally.

Suppose that the actual inflation rate at time $t(\pi_t)$ is the sum of the expected inflation rate $(E_{t-1}\pi_t)$ and a forecast error term (ε_t), i.e. $\pi_t = E_{t-1}\pi_t + \varepsilon_t$. If expectations are formed rationally, as argued by the literature on real interest rates, $E_{t-1}\pi_t$ should be an optimal forecast of inflation, and ε_t should therefore be a white noise process. The ex ante real rate $(r_{ea,t})$ can then be approximately expressed as: $r_{ea,t} = i_t - E_t\pi_{t+1}$, where i_t is the nominal interest rate. Given that $\pi_t = E_{t-1}\pi_t + \varepsilon_t$, and hence, $E_t\pi_{t+1} = \pi_{t+1} - \varepsilon_{t+1}$, $r_{ea,t} = i_t -$

(π_{t+1} -ε_{t+1}) or $r_{ea,t} = r_{ep,t} + \varepsilon_{t+1}$, where $r_{ep,t}$ is the ex post real rate ($i_t - \pi_{t+1}$). So under rational expectations, only a white noise component distinguishes the ex post from the ex ante real rate. More importantly, the ex post real rate and the ex ante real rate will have the same integration properties. Note that, this result holds if the expectation errors (ε_{t+1}) are stationary, and does not necessarily requires expectations to be formed rationally. Following the work of Rose (1988), by assuming that inflation expectation errors are stationary, the empirical literature, in general, tests the integration properties of the ex-post real rate as proxy for the ex-ante real rate, which is what we do as well.

The data employed in this paper corresponds to both annual and monthly frequencies. Annual data on U.S. consumer price index (CPI, covering 1774-2014) is obtained online from the database maintained by Professor Robert Sahr (http://oregonstate.edu/cla/polisci/sahr/sahr), while the monthly data (covering 1871:1-2015:5) on the same is downloaded from the data segment of the website of Professor Robert J. Shiller (http://www.econ.yale.edu/~shiller/data.htm). Inflation rate is computed as the first-difference of the natural logarithms of the CPI multiplied by 100 and 1200 respectively to obtain annualized values in percentages of the inflation rate under annual and monthly data. Monthly data on the long-term interest rate (ten-year government bond yield) is obtained from the Global Financial database. This data is available at monthly frequency from January of 1800 (covering 1871:1-2015:5). However, since monthly CPI only starts in 1871, we cannot compute monthly real long-term interest rates from 1800, but only from 1871:1 to 2015:4. But since annual CPI data is available from 1774, we can compute annual real long-term interest rate from 1800, where the annual nominal long-term rate is derived by averaging over the 12 months. Our long-term real interest rate at annual frequency covers the period of 1800-2013, given the last entry of annual inflation being that of 2014. The monthly short-term

interest rate till 2013:12 comes from the website of Professor Amit Goyal (http://www.hec.unil.ch/agoyal/), and then updated till 2015:5 from the FRED database of the Federal Reserve Bank of St. Louis. Note that, the short-term rate is measured by the three-month Treasury bill rate from 1920 onwards, and prior to that is based on an estimation, as in Goyal and Welch (2008), using the Commercial paper rates for New York City, which in turn, are obtained from the National Bureau of Economic Research (NBER) Macrohistory data base. The monthly short-term interest rate covers the period of 1871:1-2015:4. The anual data on the nominal short-term interest rate is obtained from Homer and Sylla (2005) over the period of 1831-1870, and thereafter from the online data segment on Professor Shiller's website, ending in 2012. To be consistent with how Homer and Sylla (2005) measures their short-term interest rate, Professor Shiller indicates in his notes to the data that from 1953 onwards, the short-term rate is measured by the six-month commercial paper rate till 1997 (after which the series was discounted by the Federal Reserve Board) and then the six-month certificate of secondary market deposit rate till 2012, after which this series was discontinued as well. For the sake of consistency in measurement, we did not use the twelve-month average of the monthly three-months Treasury bill rate beyond 1871 or even just for the year 2013, so our annual short-term real interest rate series ends in 2012. So to summarize, our monthly real rates, both short and long covers the period of 1871:1-2015:4, while the anual data covers 1831-2012 for the real short-term interest rate and 1800-2013 for the real long-term interest rate.

Figure 1 displays the time series plots of the monthly and yearly real short- and long-term interest rates. The real rates, irrespective of their types (i.e., short or long) and across frequencies, seems to follow a similar pattern. In general, the rates are observed

Figure 1: Time series plots

Monthly real long interest rate

Monthly real short interest rate

Yearly real long interest rate

Yearly real short interest rate
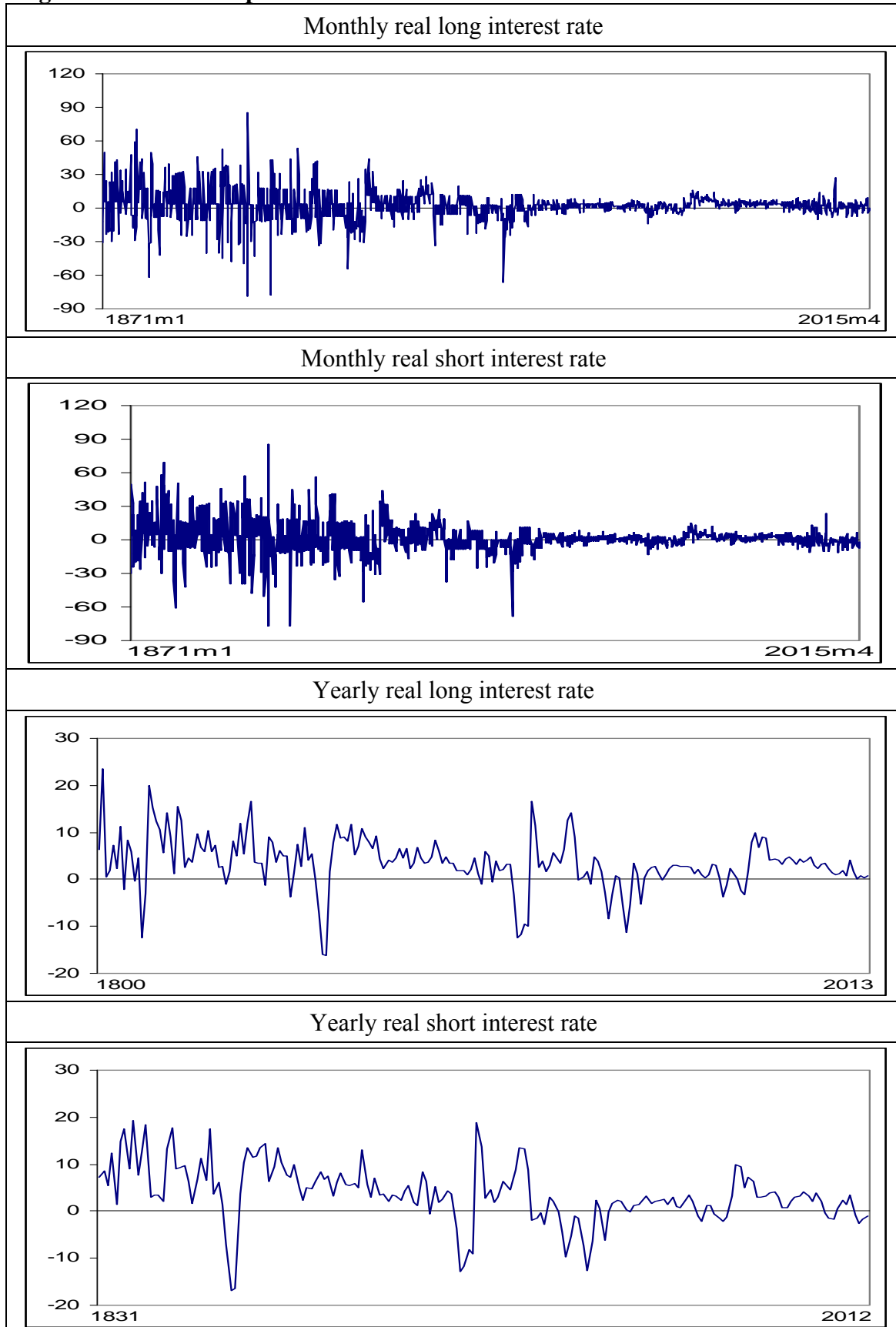
to be more volatile in the earlier part of the sample, and then becoming stable around zero percent. Casual observation tend to suggests that the interest rates rates to be mean-reverting.

5.   Empirical results

As is standard practice in the literature discussed above, we too conducted an wide-array of unit root tests, which besides the standard linear unit root tests, also included tests with structural breaks and non-linearities. All the tests indicated that the short- and long-term real interest rates are stationary, irrespective of the data frequency considered.[1] However, since unit root tests only reveal whether the data is mean-reverting or not, and does not speak on the precise degree of persistence (i.e., short or long-memory), we decided to concentrate on the long-memory approach. In cases of mean-reversion, the long-range dependence methods applied below helps us detect short- or long-memory, i.e., fast or slow-mean reversion or persistence. This is of paramount importance in case of a policy variable as crucial as the real interest rate, since a result of mean-reversion with strong persistence in the real interest rate, might also require intervention from the Federal Reserve Bank, depending on how persistent the rates are. Besides, from a statistical point of view, it is quite well-known that unit root tests have low power when a series is characterized by a fractional integration process (for a detailed discussion in this regard and more, see Gil-Alana et al., 2015).

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[1]The unit root tests that we conducted includes:the Augmented Dickey Fuller (ADF, 1979), the GLS-detrended Dickey-Fuller (Elliot, Rothenberg, and Stock, 1996), the Phillips-Perron (Phillips and Perron, 1988), the Kwiatkowksi, Phillips, Schmidt, and Shin (KPSS, 1992),the Ng and Perron (NP, 2001).Then we conducted four unit root tests with one (Zivot and Andrews , 1992), two (Lumsdaine and Papell, 1997 and Lee and Strazicich, 2003), and unknown (Enders and Lee, 2012) number of structural breaks. Further, we also carried out two non-linear unit root tests proposed by Kapetanios, Shin and Shell (KSS, 2003), and Sollis (2009). Finally, we also combined the KSS test with the Enders and Lee (2012) tests to accommodate for non-linearity due to regime switching and structural breaks. Complete details of these results are available upon request from the authors.

Starting with the fractional approaches, we first consider the following linear model,

$$y_t = \beta_0 + \beta_1 t + x_t, \quad t = 1, 2, \ldots \tag{4}$$

with x_t given by equation (1). We display in Table 1 the estimates of d along with their corresponding 95% confidence intervals, presenting the results for the three cases of no regressors ($\beta_0 = \beta_1 = 0$ a priori in (4)), an intercept (β_0 unknown and $\beta_1 = 0$ a priori) and an intercept with a linear trend (β_0 and β_1 unknown). The results show that the model with an intercept seems to be sufficient to describe the deterministic part of the process.[2] Table 1(i) focuses on the case of uncorrelated errors. We observe that d is about 0.23 for the two monthly rates; it is about 0.40 for the yearly long rate, and 0.54 for the yearly short run interest rate.

Allowing for autocorrelated disturbances (throughout an AR(1) process and also by means of the model of Bloomfield, 1973)[3] very similar results are obtained for the monthly series (with estimates around 0.2), but now the estimates of d in the yearly series are much smaller and the I(0) null hypothesis cannot statistically be rejected with AR(1) disturbances and with the Bloomfield model in the case of the long real rate. Performing some specification tests, the results indicate that the two monthly series can be well described in terms of the model with white noise disturbances, while the yearly series seem to be more adequately described throughout the autoregressive model. Thus, the monthly rates display long memory I(d) behavior with a significantly positive value of d. However, the yearly rates seem to be I(0).

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[2]The deterministic terms were chosen according to the t-values of the coefficients in the d-differenced processes, which are supposed to be I(0).
[3] This is a nonparametric approach that produces autocorrelations decaying exponentially as in the AR case.

Table 1: Estimates of d and 95% confidence band

i) Uncorrelated (white noise) errors			
	No deterministic terms	An intercept	A linear time trend
Monthly real long interest rate	0.22 (0.19, 0.26)	**0.22 (0.19, 0.25)**	0.22 (0.19, 0.25)
Monthly real short interest rate	0.23 (0.20, 0.26)	**0.23 (0.19, 0.26)**	0.22 (0.19, 0.26)
Yearly real long interest rate	0.44 (0.32, 0.59)	**0.40 (0.29, 0.56)**	0.40 (0.27, 0.56)
Yearly real short interest rate	0.57 (0.45, 0.75)	**0.54 (0.40, 0.73)**	0.53 (0.38, 0.73)
ii) AR (1) errors			
	No deterministic terms	An intercept	A linear time trend
Monthly real long interest rate	0.20 (0.15, 0.26)	**0.20 (0.15, 0.25)**	0.19 (0.15, 0.25)
Monthly real short interest rate	0.22 (0.17, 0.27)	**0.21 (0.16, 0.27)**	0.20 (0.15, 0.25)
Yearly real long interest rate	0.07 (-0.16, 0.28)	**0.05 (-0.19, 0.22)**	-0.18 (-0.47, 0.14)
Yearly real short interest rate	0.09 (-0.37, 0.38)	**0.07 (-0.18, 0.29)**	-0.07 (-0.37, 0.16)
iii) Autocorrelated (Bloomfield-type) errors			
	No deterministic terms	An intercept	A linear time trend
Monthly real long interest rate	0.21 (0.16, 0.25)	**0.20 (0.16, 0.26)**	0.20 (0.16, 0.26)
Monthly real short interest rate	0.21 (0.17, 0.26)	**0.22 (0.16, 0.26)**	0.22 (0.16, 0.26)
Yearly real long interest rate	0.14 (-0.01, 0.32)	**0.10 (-0.01, 0.26)**	0.03 (-0.13, 0.24)
Yearly real short interest rate	0.26 (0.11, 0.47)	**0.22 (0.08, 0.38)**	0.11 (-0.07, 0.36)

Table 2: Parameter estimates in non-linear I(d) models

	d	θ_0	θ_1	θ_2	θ_3
i) Uncorrelated (white noise) errors					
Monthly real long interest rate	0.21 (0.18, 0.25)	2.526 (1.695)	1.123 (0.973)	1.399 (1.322)	0.277 (0.280)
Monthly real short interest rate	0.21 (0.18, 0.25)	1.850 (1.241)	1.839 (1.592)	1.690 (1.596)	0.108 (0.110)
Yearly real long interest rate	0.39 (0.25, 0.57)	3.657 (1.673)	1.812 (1.277)	0.836 (0.694)	-0.300 (-0.285)
Yearly real short interest rate	0.52 (0.36, 0.73)	3.375 (0.856)	2.667 (1.140)	1.131 (0.611)	-0.271 (-0.177)
i) Autocorrelated AR(1) errors					
Monthly real long interest rate	0.21 (0.18, 0.23)	2.526 (1.695)	1.123 (0.973)	1.399 (1.322)	0.277 (0.281)
Monthly real short interest rate	0.21 (0.19, 0.23)	1.850 (1.241)	1.839 (1.592)	1.690 (1.596)	0.108 (0.110)
Yearly real long interest rate	0.39 (0.31, 0.48)	3.657 (1.673)	1.812 (1.277)	0.836 (0.694)	-0.300 (-0.285)
Yearly real short interest rate	0.52 (0.43, 0.62)	3.375 (0.856)	2.667 (1.140)	1.131 (0.611)	-0.271 (-0.177)

Next we consider the possibility of non-linear trends. We consider the following model,

$$y_t = \sum_{i=0}^{m} \theta_i P_{iT}(t) + x_t, \qquad (1 - L)^d x_t = u_t, \qquad (5)$$

assuming that u_t is a white noise process (in Table 2(i)), and autocorrelated throught the AR(1) model (in Table 2(ii)).
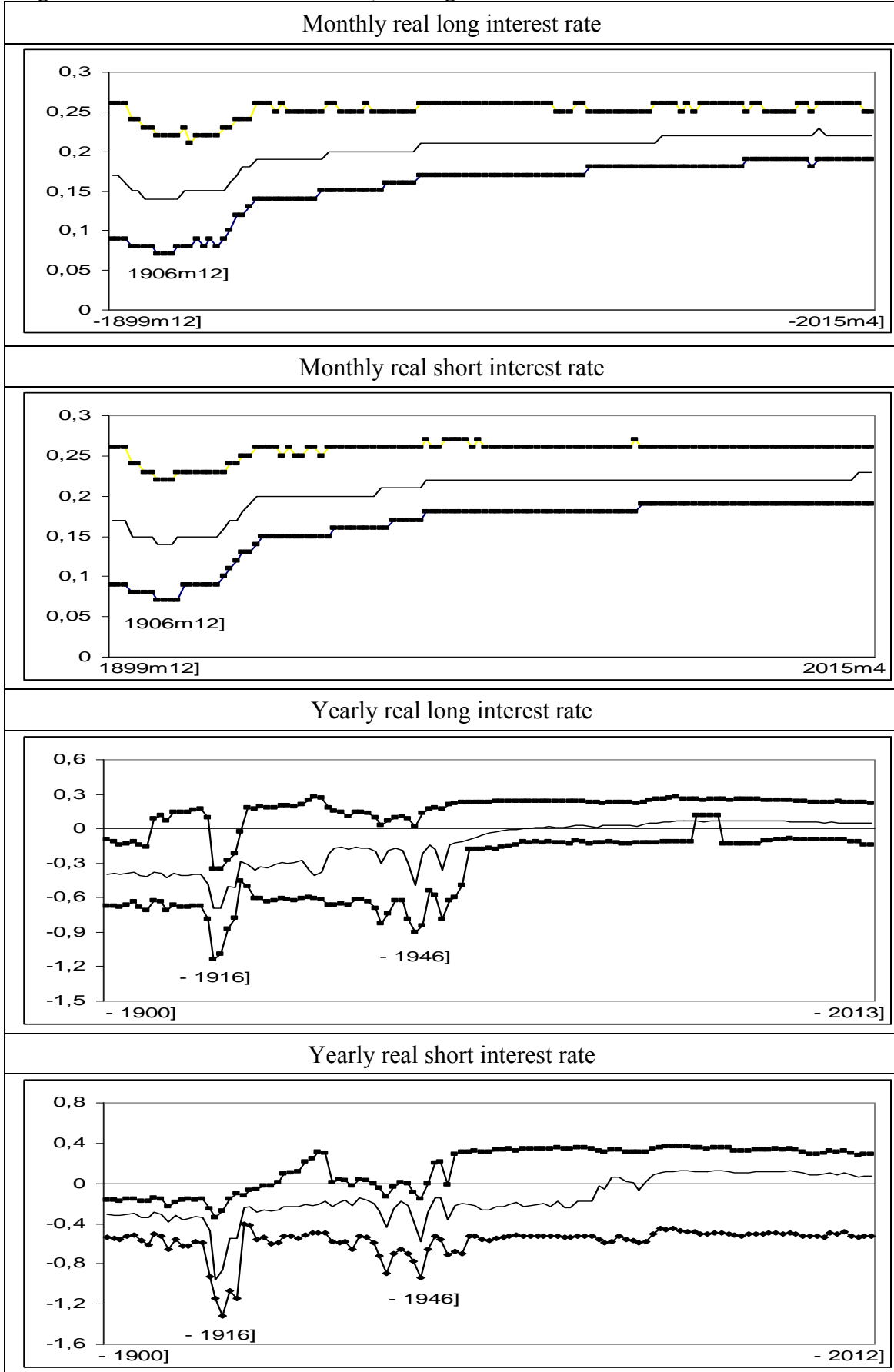
The results indicate that for the monthly rates, the orders of integration are slightly above 0.2 for the two rates, which is consistent with the results for the linear model presented in Table 1. For the yearly series, the estimates are above 0.4 and 0.52 respectively for the long and short real interest rates. However, the estimated coefficients for θ_2 and θ_3 are all found to be statistically insignificant, providing no evidence of non-linearities in the data.

Given the lack of non-linearities, our next step was to investigate the potential presence of structural breaks in the data, as well as, time-varying persistence, depicted in Haug (2014) and Apergis (2015). For this purpose we start by conducting recursive estimates of the (fractional) differencing parameters. Thus, we produced estimates first with an initial subsample ending at 1899. Then, we added 12 observations (or 1 in case of the yearly data) each time and recomputed the estimates of d in the four series.

Starting again with the monthly data, we first notice that the estimated values of d are all statistically significantly higher than 0, implying long memory. Apparently we observe a slightly reduction in the value of d, with a minimum in the subsample ending at 1906m12. Then, the values start increasing being relatively stable around 0.2 for the rests of the subsamples. For the yearly data, apparently there are two breaks in relation with the estimation of the differencing parameter: one at around 1916 and the other one

Figure 2: Recursive estimates of d, adding one observation each time

Monthly real long interest rate

Monthly real short interest rate

Yearly real long interest rate

Yearly real short interest rate

at the time of the World War II (1946). The estimated value of d is negative for the first subsample and significantly negative till 1916. Then, for the period between 1916 and 1946, the estimate of d is also negative though the I(0) hypothesis cannot be rejected. After 1946, the estimated value of d becomes positive though still we cannot reject the I(0) hypothesis.

6. Concluding comments

This paper examines the time series behaviour of U.S. short and long-run real ex-post interest rates within a long memory approach, including the possibility of a non-linear trend and using a long span of monthly (1871:M1-2015:M4) and annual data (1800-2013). In a first step, we estimate the fractional order of integration (d) of our four interest rates (short-run monthly real interest rate, long-run monthly real interest rate, short-run yearly real interest rate and long-run yearly real interest rate) for the whole period. Based on this analysis, we conclude that monthly real interest rates display long memory I(d) behaviour with a significantly positive value of d, but significantly below 0.5, suggesting mean-reversion. In contrast, yearly real interest rates seem to be I(0) variables, that is, they do not display long memory. The policy implications of these results are very important. First, according to these results, ex-post real U.S. interest rates are not as persistent as shown in most of the papers in the literature (Neely and Rapach, 2008), which could be due to the longer span of data used in this paper. That is, our results substantially differ from one of the main findings in the empirical literature on U.S. interest rates persistence (Mishkin, 1992; King et al., 1991; Galí, 1992; Rapach and Weber, 2004; Neely and Rapach, 2008), suggesting lower persistence in real interest rates. Second, as yearly real interest rates do not display long memory, shocks to these interest rates (including interventions from the Federal Reserve Bank)

will only have temporary effects, while shocks on monthly interest rates will have longer effects, suggesting that policy interventions will only be effective in the short-run. Third, and as far as theoretical macroeconomic models are concerned, our empirical result of low persistence is in accordance to some of these models, such as the consumption-based asset pricing model (Lucas, 1978). According to this model, consumption growth and real interest rates and changes in consumption growth (which is a stationary process) should have the same integration properties, which according to most of the empirical literature on interest rates do not occur. In contrast, this conclusion holds according to our results.

In a second step, we consider the possibility of non-linear trends in the interest rates behaviour, providing no evidence of non-linearities in the data for any of the four interest rate variables. Finally, we conduct recursive estimates of the differencing parameter for each of the series, obtaining the following results. First, for the monthly interest rates, the estimated values of d are all statistically significantly higher than 0, implying again long memory. Second, for the yearly interest rates, we find evidence of structural breaks (such as in 1916 and 1946), a consistent finding with the literature. Third, we again find evidence that yearly interest rates do not display long memory for the post-war period. That is, even after accounting for structural breaks and time-varying persistence, our findings suggest again that real U.S. interest rates are less persistence than suggested by prior literature, challenging one of the key stylized facts on real interest rates persistence (Neely and Rapach, 2008). As mentioned, the policy implications of these results are important for both the interest rates policy interventions and the analysis of the theoretical conclusions of different financial and macroeconomic models.

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