Opportunities for the development of understanding in Grade 8 mathematics classrooms

by

Gerdi de Jager

Submitted in partial fulfilment in accordance with the requirements for the degree

Master of Education

(Science, Mathematics and Technology Education)

at the

Faculty of Education

UNIVERSITY OF PRETORIA

Supervisor: Dr Hanlie Botha
Co-supervisor: Dr Sonja van Putten

NOVEMBER 2016

Acknowledgements

To have achieved this milestone in my life, I give thanks to God for giving me the ability and opportunity. I would like to thank Dr Hanlie Botha, Dr Sonja van Putten and Prof Gerrit Stols for their continued motivation, advice and support. Without you, I would not have been able to complete this dissertation. Thank you to the three teachers who participated in this study and who allowed me into their classrooms and hearts. Thank you to my many family and friends with whom I could discuss my research and who were always willing to give advice and encouraged me throughout the process. Finally, I would like to thank my husband for his support and encouragement.

Abstract

Learner performance in South Africa is poor in comparison with other countries as a result of poor teaching. At the core of the concern about learners' performance in mathematics in South Africa lies a controversy regarding how mathematics should be taught. The purpose of this study was to explore Grade 8 mathematics teachers' creation and utilisation of opportunities for learners to develop mathematical understanding in their classrooms. To accomplish this, an explorative case study was conducted to explore three mathematics teachers' instructional practices by using Schoenfeld et al.'s (2014) five dimensions of Teaching for the Robust Understanding of Mathematics (TRU Math) scheme, namely, the mathematics, cognitive demand, access to mathematical content, mathematical agency, authority and identity and uses of assessment. The three participants were conveniently selected from three private schools in Mpumalanga. The data collected consist of a document analysis, two lessons observations and a post-observation interview per teacher.

This study revealed that only one of the three teachers applied all Schoenfeld et al.'s (2014) TRU Math dimensions. The dimension identified which the teachers applied most in their classrooms was the mathematics. The dimensions identified where teachers still lack skills were cognitive demand, access to mathematical content, agency, authority and identity, and uses of assessment. This study revealed that the content of most tasks and lessons was focused and coherent, and built meaningful connections. However, the content did not engage learners in important mathematical content or provided opportunities for learners to apply the content to solve real-life problems. Due to the small sample used, the results from this study cannot be generalised. However, I hope that the findings will contribute to student-teacher training and in-service teacher training in both government and private schools. Future research could possibly build on this study by examining the learners and how they learn with understanding by using the TRU Math dimensions.

Key Terms:

Mathematics; Teachers; Grade 8; Learners; Instructional practice; TRU Math dimensions, Cognitive demand; Access to mathematical content; Agency, authority and mathematical identity; Uses of assessment.

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Ethical Clearance Certificate

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA
Faculty of Education

RESEARCH ETHICS COMMITTEE

CLEARANCE CERTIFICATE	CLEARANCE NUMBER: SM 15/09/01
DEGREE AND PROJECT	MEd Opportunities for the development of understanding in Grade 8 mathematics classrooms
INVESTIGATORS	Gerdi de Jager
DEPARTMENT	Science, Mathematics and Technology Education
APPROVAL TO COMMENCE STUDY	27 October 2015
DATE OF CLEARANCE CERTIFICATE	28 October 2016

Please note:

For Master's application, Ethics Clearance is valid for 2 years
For PhD application, Ethics Clearance is valid for 3 years

CHAIRPERSON OF ETHICS COMMITTEE: Prof Liesel Ebersöhn

CC Bronwynne Swarts
Hanlie Botha
Sonja van Putten

This Ethics Clearance Certificate if issued subject to the following conditions:

1. A signed personal declaration of responsibility
2. If the research question changes significantly so as to alter the nature of the study, a new application of ethical clearance must be submitted
3. It remains the student's responsibility to ensure that all the necessary forms for informed consent are kept for future queries

Please quote the clearance number in all enquiries

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Declaration

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA
Faculty of Education

Department: Department of Science, Mathematics and Technology Education

Supervisor: Dr. Hanlie Botha

Student name: Gerdi de Jager

Student number: 29042730

Assignment number: Dissertation

Date submitted: 18/11/2016

Declaration:

1. I understand what plagiarism is and I am aware of the University's policy in this regard.

2. I declare that this assignment is my own original work. Where other people's work has been used (either from a printed or electronic source), this has been properly acknowledged and referenced in accordance with departmental requirements.

3. I have not used work previously produced by another student or any other person to hand in as my own.

4. I have not allowed, and will not allow, anyone to copy my work with the intention of passing it off as his or her own work.

Signature (s): ____________________

Signature of supervisor: ____________________

Co-supervisor: ____________________

Table of Contents

List of abbreviations

CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
IEA	International Association for the Evaluation of Educational Achievement
IEB	Independent Examinations Board
NCS	National Curriculum Statement
NCTM	National Council of Teachers of Mathematics
TIMSS	Trends in International Mathematics and Science Study
TRU Math	Teaching for Robust Understanding in Mathematics

List of Figures

List of Tables

1. CHAPTER 1: Introduction and contextualisation

1.1 Introduction

Mathematical knowledge and skills are integrated into many facets of everyday life and many occupations require mathematical knowledge. Mathematics also plays a vital role in the way people approach their private, communal and civil lives (Kilpatrick, Swafford, & Findell, 2001). Mathematics develops some of the essential skills needed to be successful in the 21st century. Friedman (2007) argues that people who are mathematically proficient are more adaptable than others in a changing economy. The level of a person's mathematical proficiency may therefore open or close doors to a better future (Van De Walle, Karp, & Bay-Williams, 2013). According to Setati (2002), mathematics is also a key to higher education and higher paying jobs in South Africa. The improvement of mathematics education may therefore contribute to the growth of South Africa.

With the importance of mathematics in mind, it is a distressing fact that South African learners struggle to understand mathematics and are among the worst performers in international comparative studies (Spaull & Venkatakrishnan, 2014). A possible reason for learners' poor understanding of mathematics is ineffective and poor teaching (Stols, 2013). Effective teaching develops learners' ability to solve problems, think critically, transfer knowledge, and apply the knowledge in new settings (Darling-Hammond, 2008; McTighe & Seif, 2003).

1.2 Background

Traditionally, the central focus of teaching was on memorisation and application of rules instead of the development of conceptual understanding and problem solving (Anthony & Walshaw, 2009b). This often led to passive learning where learners follow mathematical procedures without understanding the relationships and connections between concepts (Armstrong, 2012). Much research (Hiebert et al., 2003; Hiebert, Gallimore, & Stigler, 2002; Hill & Ball, 2004; Loveless, 2003; Stigler & Hiebert, 1999) has been conducted to improve the teaching and learning of mathematics. Because of this research, most countries' education systems have moved away from a traditional teacher-centred approach towards a learner-centred approach. A learner-centred approach has also been promoted by the Department of Education (DoE) in South Africa (DoE, 2000).

Anthony and Walshaw (2009a) explain that the use of discovery and problem solving strategies provides productive learning opportunities for learners. Meaningful learning requires learners to be actively engaged by constructing their own knowledge through exploration and observation. To change from traditional teaching to teaching for understanding, the teacher's role during instruction should change from being the provider of information to creating productive learning environments and opportunities where meaningful learning can take place (Nelissen & Tomic, 1993). Instead of informing learners, teachers should create opportunities where learners can reason, investigate their own work, make mistakes, justify and improve their own approaches (Bishop, 1988).

1.3 Rationale

I had the privilege to be a member of staff at a new private school that was established in 2015. The Grade 8 learners of this newly established school come from a variety of schools from all over Mpumalanga. In my experience, learners struggle to understand and apply basic mathematical concepts. I believe a possible problem is that the main focus of teaching in South Africa remains on practising by repetition and not on teaching and learning for understanding. According to Grasha (2001), teachers have the power to influence learners' achievement by either supporting or obstructing their ability to understand new knowledge. Wayne and Youngs (2003) believe that teachers should be held accountable for their learners' achievement. Skemp (1989) also states that learners of any age will only be successful in learning mathematics if they are taught in ways which empower them, rather than by rote learning. Teachers therefore carry an enormous responsibility to develop learners' mathematical understanding since all learners have the ability to become mathematically proficient (Taylor, 1990). The question that remains is: How do mathematics teachers create and utilise opportunities to develop mathematical understanding in their classrooms?

1.4 Problem statement

National and international comparisons of learner achievement continue to make headlines, provoke public opinion and pressure governments to improve instruction (Arends, 2008; Erasmus & Mda, 2008; Van de Walle et al., 2013). The Trends in International Mathematics and Science Study (TIMSS) 1999 Video Study comparison of Grade 8 mathematics teaching in seven countries highlights the importance of developing learners' understanding and problem solving skills (McTighe & Seif, 2003). An example of a high-performing nation is Japan, as Japan is one of the top-performing countries in mathematics at Grade 7 and 8 levels (Smith, 1996). Japan's foremost goal is to develop learners' conceptual understanding (Takahashi, 2006). Teaching should focus on the key development of concepts rather than on artificial facts and processes (McTighe & Seif, 2003). Weiss, Pasley, Smith, Banilower and Heck (2003) found that the focus in effective classrooms is on the understanding of important mathematical ideas and the application of knowledge to unfamiliar problems.

International comparative studies such as the TIMSS gives South Africa an opportunity to benchmark the quality of education against other countries in the world (McTighe & Seif, 2003). The TIMSS indicates that learners' achievement in South Africa is almost the worst of all the participating countries. South Africa came last in 2003, did not participate in 2007 and came second last in 2011. However, in 2011, Grade 9 learners participated and not Grade 8 learners (Spaull & Venkatakrishan, 2014). There was a slight improvement in learners' mathematical achievement, but not enough to lift South Africa out of the bottom rankings (Maree & Van der Walt, 2007). Despite all the efforts, such as transformation in schools through desegregation and expanding access to make mathematics education more effective, there is no evidence of meaningful improvement in South Africa, as reflected in the TIMSS results (Weber, 2008).

In 2011, the TIMSS was conducted in 45 countries by the International Association for the Evaluation of Educational Achievement (IEA). The mathematics score of an average learner of the top seven countries surpassed a South African learner's performance at the 95th percentile (IEA, 2011). The implication is that the best learners in South Africa are less proficient than an average-performing learner in top-performing countries like Finland, Singapore, Chinese Taipei, Republic of Korea and Japan (Human Science Research Council (HSRC), 2011). According to the TIMSS (2011), South African learners were ranked highly with recalling facts or answering questions involving procedural knowledge. However, much discussion is focused on South African learners' poor results, particularly with regard to problem solving skills and higher-level cognitive abilities involving understanding (Spaull, 2013). The public perception is that South African learners do not measure up to their Japanese and Chinese counterparts. At the core of the concern about learners' performance in mathematics in South Africa lies a controversy regarding how mathematics should be taught.

There is considerable political pressure to improve the education system in South Africa and to raise standards. Poor teaching is one factor that can lead to a state of economic recession (Writer, 2015). According to a former Science and Technology Minister, Naledi Pandor, South Africa has not been capable of improving mathematics teaching in our schools, "and this has created a bottleneck in the expansion of our university system and unemployment for many young people" (Writer, 2015, p. 1). In order to break this cycle, it is important to understand how South African mathematics teachers create and utilise opportunities to develop mathematical understanding in their classrooms.

1.5 Purpose of the study

The purpose of this study is to explore Grade 8 mathematics teachers' creation and utilisation of opportunities for learners to develop mathematical understanding in their classrooms. For the purpose of this study, a teacher's instructional practice is defined as all approaches that a teacher uses to actively engage learners in the learning process (Saskatchewan Education, 1991). The data of this study were collected and analysed according to a specific framework for analysing instructional practices, namely the Teaching for the Robust Understanding of Mathematics (TRU Math) scheme, developed in recent years by Schoenfeld and colleagues. The TRU Math scheme is designed to capture teachers' instructional practices in a way that can enlighten teachers' professional development and can be used as a sufficient tool to analyse effective mathematics instruction. Observing various teachers' instructional practices can give an indication where the problems and challenges lie in teachers' attempts to develop learners' understanding (Schoenfeld et al., 2014). The findings may explain how teachers promote (or not) the development of mathematical understanding. The ultimate purpose of the study is to make a possible contribution towards improving the quality of teaching and learning in South African mathematics classes.

1.6 Research questions

The following primary and secondary research questions guided the study:

Primary research question:

How do Grade 8 mathematics teachers create and utilise opportunities to develop mathematical understanding in their classrooms?

Secondary research questions:

The following five sub-questions capture the five dimensions of TRU Math as explained by Schoenfeld et al. (2014). The TRU Math framework is discussed under the theoretical framework in Chapter 2. The first question focuses on the mathematical content, the second on cognitive demand, the third on opportunities for learners to access the mathematical content, the fourth on mathematical agency, authority and identity of the learner, and the last on the uses of assessment in the classroom:

1. How focused and coherent is the mathematics and how are the connections between procedures, concepts and contexts (where appropriate) addressed? *(The mathematics)*
2. What opportunities do learners have to make sense of mathematical ideas? *(Cognitive demand)*
3. How do teachers invite and support the active engagement of all of the learners in the classroom with the core mathematics being addressed in the lesson? *(Access to mathematical content)*
4. What opportunities do learners have to explain their own ideas and to respond to each other's mathematical ideas? *(Agency, authority and identity)*

5. How does instruction build on learners' ideas or address emerging misunderstandings? *(Uses of assessment)*

1.7 Methodological considerations

This qualitative case study explores teachers' instructional practices to obtain an in-depth understanding of the opportunities the teachers create to develop mathematical understanding in their classrooms. The research paradigm that underpinned this study is social-constructivism, which is based on the epistemological assumption that "social life is a distinctly human product and that human behaviour is affected by knowledge of the social world" (Nieuwenhuis, 2007, p. 59). This study is subjective in nature, with the ontological assumption that reality is understood through words and is the result of individual awareness (Cohen, Manion & Morrison, 2011).

The three participating teachers taught in private schools in Mpumalanga in South Africa. As part of the data collection process, two lessons presented by each of the participants were observed using an observation schedule, and one post-observation interview was conducted per participant. Deductive coding was done using categories identified from literature and set out in the theoretical framework in Chapter 2. Trustworthiness was addressed via member checking. This ensured that my interpretation of the document analyses, lesson observations as well as the post-observation interview was reliable.

1.8 Concept clarifications

Since there are various definitions of the concepts in literature, as discussed in the literature review, it is necessary to clarify the relevant concepts in the context of this research and how they are used in this study:

- Adaptive reasoning: The ability to reflect, think logically, reason and justify. It is one of the five strands of mathematical proficiency (Kilpatrick et al., 2001).
- Agency: The capacity and willingness to engage mathematically (Schoenfeld et al., 2014).
- Authority: Recognition for being mathematically solid (Schoenfeld et al., 2014).

- Classroom environment: Involves a wide-range of educational concepts, including the psychological environment created through social contexts and several instructional components associated with teacher behaviours and characteristics (Miller & Cunningham, 2011).
- Conceptual understanding: Refers to knowledge that is constructed by learners when understanding mathematical concepts, processes and relationships. It is one of the five strands of mathematical proficiency (Kilpatrick et al., 2001).
- Connections between concepts: New learning results either in increasing the amount of connections between concepts or causing radical changes in mental structures (Hiebert & Carpenter, 1992).
- Curriculum and Assessment Policy Statement (CAPS): A policy document in South Africa with the Learning Area Statements, Learning Programme Guidelines and Subject Assessment Guidelines for all the subjects listed in the National Curriculum Statement Grades R-12 (Department of Basic Education (DBE), 2011).
- Effective teaching: The teacher creates learning opportunities and a productive learning environment, contributing to effective learning (Schoenfeld et al., 2014).
- Instructional practices: All approaches that a teacher applies to actively engage learners in the learning process (Saskatchewan Education, 1991).
- Internal representation: Also called mental representation and is defined as an appearance to the mind in the form of a concept or idea (Hiebert & Carpenter, 1992).
- Mental representations: Cognitive processes that are established by information structures in the mind (Davis, 1992).
- Procedural fluency: The ability to carry out procedures with confidence. It is one of the five strands of mathematical proficiency (Kilpatrick et al., 2001).
- Productive disposition: The habit of making an effort with mathematics and making it practical, valuable and meaningful. It is one of the five strands of mathematical proficiency (Kilpatrick et al., 2001).
- Productive learning environment: A setting that will produce powerful mathematical thinkers (Schoenfeld et al., 2014).

- Representation: All the different ways to capture a general relationship that expresses similarities between objects (for example, written work, oral descriptions, models with manipulative resources and the mental processes one uses to do mathematics) (National Council of Teachers of Mathematics (NCTM), 2000a).

1.9 Possible contributions of the study

This study is an attempt to make a small contribution to both private and government education in South Africa by using knowledge that already exists in other countries (the TRU Math scheme) to make mathematics teachers in South Africa aware of how they can create opportunities for learners to develop conceptual understanding. Schoenfeld et al. (2014) provide an in-depth analytical framework for describing important dimensions of instructional practices which improve the quality of teaching and learning of mathematics in many other countries as well as in South Africa. The findings may explain how classrooms promote (or not) the development of mathematical understanding.

1.10 The structure of the dissertation

The dissertation consists of six chapters. Chapter 2, which consists of the literature review and theoretical framework, provides an in-depth analysis and synthesis of the relevant literature and explains the theoretical framework on which this study is based. In Chapter 3, the methodology used in this study is explained. The selection of the participants, data collection instruments, and data analysis procedures are discussed, as well as the trustworthiness of the study and ethical considerations. Chapter 4 details the analysis of the findings based on the data obtained by way of the document analyses, lesson observations and post-observation interviews. In Chapter 5 the findings are discussed in light of the literature review and theoretical framework and the research questions are answered, followed by a cross-case analysis and summary. Chapter 6 contains the discussion of conclusions and implications and comprises a discussion of the research questions, a chapter summary, concluding remarks concerning the study, recommendations and limitations of the study, and lastly, a final reflection on the study.

2. CHAPTER 2: Literature review and theoretical framework

In this chapter, current research on the teaching and learning of mathematics is critically discussed in the context of the research problem, and gaps and weaknesses are identified to justify this new investigation. The literature review provides an overview of mathematical understanding, what teaching for understanding entails and possible factors influencing effective classroom environments. The chapter concludes with a discussion of the theoretical framework for this study.

2.1 Understanding mathematics

New ideas and knowledge are developed through seeing, hearing or touching, or through our own thoughts and ideas (Van De Walle et al., 2013). One of the main findings of the United States (US) National Science Foundation project about how people learn (Bransford, Brown, & Cocking, 2000, p. 14) reveals that, if learners' "initial understanding is not engaged, they may fail to grasp the new concepts and information that are taught". Sherman, Richardson and Yard (2013) explain that learners build their understanding on previous knowledge and experiences. Learning is based on previous accomplished understandings; therefore, the quality of new knowledge depends highly on knowledge and experiences already obtained (Kilpatrick et al., 2001). Bransford et al. (2000) state that learners may struggle to comprehend new knowledge if the learners' existing understanding is not engaged in the learning process.

2.1.1 Mental representations

Understanding occurs when knowledge is represented mentally in the manner that the internal representations, also called mental representations, are structured (Sherman et al., 2013). Davis (1992) explains that understanding occurs when a new concept can fit into a larger structure of previously formed concepts. Understanding develops through representing, building on and connecting ideas (Bransford et al., 2000). The Common Core Standards Initiative (2010, p. 3) in the US explains that learners understand mathematics if they can express the mathematical relationship in their own words or else when they can explain the origin of a rule.

To be able to recognise and use connections among mathematical ideas requires systematic representation of knowledge (Van De Walle et al., 2013). However, Bransford et al. (2000) warn that this often requires more time and is harder to achieve than only memorising. Systematic representation of knowledge requires strong connections between concepts, and thorough building on previous knowledge and experiences.

2.1.2 A network of knowledge

Understanding can only take place when a mental representation is part of a bigger network. Representations should be connected to form a large network of knowledge, which is called a schema. It is therefore important for learners to build connections between concepts instead of memorising the concepts (Sherman et al., 2013). To connect new knowledge to existing knowledge, reflective thought is required. New learning results either in increasing the number of connections between concepts or causing radical changes in mental structures (Hiebert & Carpenter, 1992). It is more likely that it will be possible to retrieve information when the concept is connected to a bigger network. If what needs to be recalled does not come to mind, reflecting on related ideas and concepts may lead to recall of the desired concept (Van De Walle et al., 2013).

2.1.3 Quality of mental representations

The quality of a learner's mental knowledge of a concept depends on and is shaped, among other things, by external situations represented in the classroom (Bransford et al., 2000). Hiebert and Carpenter (1992) explain that the quantity and strength of connections to a network determine the degree of understanding. To thoroughly understand a mathematical concept, process or fact, it needs to be linked to existing networks with many and strong connections. To increase the chance that learners will form and integrate an emerging concept into a rich network, learners should be given opportunities to contemplate and investigate different methods (Van De Walle et al., 2013).

Hiebert and Carpenter (1992) argue that learners who interact with manipulatives, such as attribute blocks, fraction pieces, different colours and sizes of geometric shapes, base ten blocks and plastic counting cubes, will be on a higher level than learners who are only exposed to written symbols as they make mental representations in the learning process. Hence, mathematics classroom tasks must be carefully chosen so that they will assist learners to formulate strong connections between concepts. Understanding grows as internal networks increase and relationships become more organised (Hiebert & Carpenter, 1992). Understanding is a measure of the quality and quantity of connections that a new idea has with existing ideas (Sherman et al., 2013).

2.1.4 Conceptual understanding

Conceptual understanding is about well-structured knowledge; it is also seen as the primary knowledge of mathematics (Kilpatrick et al., 2001). Mental representations, a network of knowledge and the quality of mental representations are three important aspects of conceptual understanding. Conceptual understanding refers to knowledge that is constructed by learners when concepts are combined with existing theories, mathematical ideas, principles or elements. These internal representations involve the understanding of mathematical concepts, processes and relationships (Baroody, Feil, & Johnson, 2007). When learners have a conceptual understanding of a mathematical concept, they will be more likely to remember it and to apply that knowledge to other contexts. Learners who have conceptual understanding have less to learn and learn with less effort (Kilpatrick et al., 2001). One of the aims listed in the CAPS document is to develop deep conceptual understanding to make sense of mathematics (DBE, 2011).

2.1.5 Problem solving and the transferability of knowledge

Learning with understanding makes the learning of new concepts easier because strong connections are formed between previous knowledge and experiences and new concepts (Bransford et al., 2000). Flexible and adaptive learning can only take place when learners have the capacity to transfer knowledge that they have already learned to new settings. Effective learning means that learners have an in-depth, flexible and adaptive knowledge of mathematical concepts (Ma, 1999). Once concepts are a part of a strong network, transferability is enhanced and so are a learner's problem-solving skills (Lester & Cai, 2016). Solving real-life problems requires internal representations that are connected. Enhancing the capability to move among and between concepts expands learner understanding and retention (Van De Walle et al., 2013). Learners are likely to know when to use a specific method to solve a problem when they understand the association between a situation and a context (Lester & Cai, 2016). The ability to use mathematical knowledge outside the classroom is an indication of a good understanding of mathematics (Bransford et al., 2000). This will enable learners to think flexibly and solve problems in different contexts, from school to everyday-life situations. To be able to transfer knowledge between formal school settings and informal real-life settings is essential to cope with everyday life challenges (Lester & Cai, 2016). One of the main aims of the South African curriculum is to prepare learners to cope with real-life situations and to adapt to new situations using their previous knowledge and experiences. The South African curriculum for teaching and learning mathematics intends to promote the application of mathematics to familiar and unfamiliar situations (DBE, 2011). Fan (2008) identified three main challenges that encounter teachers to provide opportunities for learners to solve real-life problems. These three challenges are time constraints, lack of resources and a feeling of inadequacy.

2.1.6 Understanding versus procedural fluency

In South Africa, the CAPS highlights the ability to perform procedures for basic operations. The DBE (2012) describes computing as the ability to perform mathematical procedures flexibly, accurately, efficiently and appropriately. To be able to compute efficiently, it is essential that learners have a strong sense of numbers, are able to perform a range of operations fluently and easily and lastly, to do so quickly and correctly (DBE, 2012). Kilpatrick et al. (2001) define procedural fluency as the skill of carrying out procedures flexibly, accurately, efficiently and appropriately. Procedural fluency occurs when learners have the knowledge and ability to apply rules and carry out mathematical procedures with confidence as well as to represent mathematics using symbols (Van De Walle et al., 2013). Flexibility with procedures will support learners with computations in different situations (Kilpatrick et al., 2001). Flexibility, precision and efficiency can be improved by practising procedures. It is also important for learners to be able to estimate the result or answer of a procedure before doing the actual calculation. This could help them to compare their results and to make an informed judgment about the correctness and reasonableness of the results (Van De Walle et al., 2013).

Boaler (2015) cautions that procedural fluency should not to be interpreted as the opposite of understanding. It is not mere rote memorisation. In this regard, Kilpatrick et al. (2001) posit that procedural fluency and conceptual understanding are interwoven. Practising procedures in the absence of conceptual understanding may result in a lack of recall and increased mistakes (Van De Walle et al., 2013). The flexible use of procedures requires an in-depth understanding of associated mathematical concepts (Kilpatrick et al., 2001). Several researchers have reported that conceptual understanding is pivotal in developing procedural proficiency (Bransford et al., 2000; National Mathematics Advisory Panel, 2008; NCTM, 2000a). This is well illustrated by Parish (2014, p. 159), who explains procedural fluency in the context of number sense as “knowing how a number can be composed and decomposed and using that information to be flexible and efficient with solving problems”.

2.1.7 Summary

The literature suggests that understanding takes place when knowledge is represented mentally by means of structured internal representations. The effectiveness of the mathematical learning process is influenced by learners' previous knowledge and experiences. Understanding occurs when a mental representation is part of a bigger network (Hiebert & Carpenter, 1992). New learning happens when there are strong connections between concepts. Understanding is a measure of the quality and quantity of connections between a new idea and existing ideas. The learner's understanding plays an important role in whether the learner can transform new knowledge into usable – and understandable – concepts. Instructional practices which provide support to create meaningful representations, make connections, build on previous knowledge and transfer knowledge to new situations might develop a deeper mathematical understanding (Bransford, et al., 2000). In this section, what understanding mathematics is has been explained. Now that understanding mathematics has been discussed, it is essential to know how mathematics must be taught for understanding to occur.

2.2 Teaching for understanding

The nature of teaching affects the outcomes of learning (Lim & Morris, 2009; Sankaram, 2001). The understanding of what meaningful learning entails has shifted over time and generations. New aims and objectives for learning and teaching have been identified in the 21st century. Understanding also has become more important than memorising (Van De Walle et al., 2013). Traditionally, it has been argued that teaching revolves around the teacher, also known as behaviourism. In contrast, a more productive approach is to focus on learners' construction of knowledge. The following is a brief discussion about behaviouristic and constructivist approaches.

2.2.1 Behaviouristic approach

Behaviourism suggests a direct teaching style, where limited learner individuality is allowed and classroom tasks are performed under clear instructions of the teacher. Learners in more traditional classes often do mathematics by emulating what the teacher demonstrates to them. Such instructional practices often require learners to listen, duplicate, memorise, drill and calculate. When learners learn through constant repetition of a task, mathematics can easily seem overwhelming with never-ending lists of rules and symbols. It has been demonstrated that a behaviouristic teaching approach is not effective (Gamoran & Nylstrand, 1991), as this approach limits learner participation and understanding. In a traditional classroom, the teacher is seen as the authoritative source of knowledge and the only one who has to validate learners' work. The teacher's role in traditional teaching is to demonstrate, prescribe and check answers (Curro Centre for Educational Excellence, 2012).

2.2.2 Constructivist approach

A constructivist approach builds on learners' prior knowledge and teachers build on their learners' ideas (Cohen & Amidon, 2004; Gamoran & Nylstrand, 1991). Thus, a constructivist approach will support learners to take their understanding of mathematical concepts to the next level by connecting it to their previous knowledge and experiences (Fillingim & Barlow, 2010). The effectiveness of the mathematical learning process is influenced by learners' previous experiences (Brooks & Brooks, 1993). Classroom tasks must be selected in such a manner that they will support the learners to form connections between their current knowledge and the new concepts. The concepts therefore should be aligned with the developmental level of the learners (Reys, Suydam, Lindquist, & Smith, 1999). Learning is only effective when learners learn with an understanding and build new understandings on previously known concepts. Thus, the starting point of each topic in a grade should build on previous knowledge of the former grade level or previous knowledge and experiences. In a constructivist classroom, learners work cooperatively and take responsibility for their own learning. The emphasis is on empowering learners to make sense of mathematics and to understand it, rather than to imitate prescribed methods (Curro Centre for Educational Excellence, 2012).

Effective teaching entails an understanding of what learners already know and what learners are required to learn, and then challenging and supporting learners to acquire a robust understanding of those concepts (NCTM, 2000b). The teacher acts as a facilitator to ensure that learners have effective opportunities for learning. The teacher has the role of setting appropriate problems, organising interaction between learners, and negotiating the style of learning with learners (Curro Centre for Educational Excellence, 2012). According to Protheroe (2007), effective mathematics teaching includes the following characteristics: learners being actively engaged with mathematics, solving challenging problems, making various representations, sharing ideas to communicate mathematically, and lastly, proficiently using tools (for example, protractor and graph paper), technology and models with manipulative resources (for example, probability spinners and geoboards). Effective teaching is therefore not only about the actions of a teacher, but more about the learning environment that a teacher creates. According to Gifford and Lantham (2013, p. 33), "teachers hold the power to create, or remove, glass ceilings on children's mathematical attainment". What learners understand about mathematics is almost entirely dependent on the experiences that the teacher creates daily in the classroom. Effective teachers are those teachers that create an effective classroom environment for effective learning to occur. This leads to the question that is explored in the next section: What is an effective classroom environment?

2.3 Effective classroom environments

A considerable amount of literature (Cobb, Gravemeijer, Yackel, & Whitenack, 1997; Cohen & Lotan, 1997; Engle, 2011; Oakes, Joseph, & Moore, 2001; Schoenfeld et al., 2014) has been published on classrooms that will engage learners and will yield powerful mathematical thinkers. Teachers have the responsibility to create an environment where effective learning can take place (Bransford et al., 2000). There are many different opinions about the nature of effective classrooms. This section identifies different components of effective classroom environments from the literature. The following is a brief discussion on the beliefs of Bransford et al. (2000), the NCTM (2007) and Kilpatrick et al. (2001) on what an effective classroom environment should look like.

Bransford et al. (2000) identified four main principles of an effective classroom environment, namely, being learner-centred, being knowledge-centred, being assessment-centred and being situated within the community. Although the three principles of being learner-centred, knowledge-centred and assessment-centred are influenced by the bigger community, each of these principles should be valued as equally important. These principles are part of a bigger system where the elements are interconnected to each other and support one another.

Kilpatrick et al. (2001) identified five strands which learners should develop to learn and understand mathematics successfully. These are recognised as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Learners who are mathematically proficient will be able to cope with real-life challenges and continue with studies after secondary school. According to Kilpatrick et al. (2001), mastery of the five strands develops with time and should always be considered by teachers who teach for understanding. An effective classroom environment should create opportunities for learners to develop all five identified strands.

The NCTM (2007) identified six key requirements that will allow learners to develop mathematical understanding in a mathematics classroom, namely, creating an environment that offers equal opportunity of learning to all learners, focusing on a balance between conceptual understanding and procedural fluency, ensuring active learner engagement (through problem solving, connections, reasoning, communication and representation), integrating multiple assessments aligned with instructional goals and practices, helping learners to be aware of the power of comprehensive reasoning and accountability, and using technology to improve understanding (NCTM, 2007). To foster active learning in the classroom, learners need to be intellectually engaged in constructing new knowledge (NCTM, 2000a; NCTM, 2014; Sherman et al., 2013). Instructional practices that focus on active learning include problem-solving tasks, questioning, and inquiry (NCTM, 2000b). Problem solving requires learners to be actively engage in a task for which the explanation or method is not known beforehand (NCTM, 2000b). When a classroom environment allows learners to be comfortable in solving problems and sharing their thoughts on the problems with others, they see themselves as being capable of understanding (Kilpatrick et al., 2001).

The following aspects of an effective classroom environment were identified in the previous three paragraphs and are discussed in the following sections: learner-centeredness, active learning, resources, assessment, problem-centeredness, development of logical reasoning, and the development of a positive attitude.

2.3.1 Learner-centeredness

A number of studies have examined the term learner-centeredness (Hinton, Fischer, & Glennon, 2013; Moller, Deci, & Ryan, 2006; Trilling & Fadel, 2009; Wilson & Peterson, 2006; Zull, 2011). Walters et al. (2014) claim that a learner-centred classroom is a supportive learning environment that promotes meaningful engagement with mathematics. To create a classroom environment where learners are making sense of mathematics takes effort and does not happen by accident (Van De Walle et al., 2013). As discussed previously in the literature review, the key to mathematical understanding is to build on prior knowledge and experiences. It is therefore important for teachers to build on learners' prior knowledge and experiences. Each learner is unique and as such brings his or her existing understanding and experiences to the classroom (Bransford et al., 2000).

Teachers can view learners' prior knowledge either as an obstacle or as an asset. Learners come to class with a diversity of worldviews and social backgrounds. Worldviews and social backgrounds have different influences on learners' understanding of mathematics concepts. Effective learning occurs when learners are capable of constructing meaning based on their own understandings based on their worldviews (Fakudze, 2004). Considering learners' worldviews and social backgrounds can contribute to the teacher's choice of a teaching approach towards mathematics as well as promote equity in the classroom (Kazeni & Onwu, 2013). This is also valued by the South African National Curriculum Statement (NCS), where it identified as valuing indigenous knowledge systems as an important principle (DoE, 2002).

South Africa is a country that is rich in culture. There are many different cultures and beliefs that need to be considered when teaching. Each learner is different and therefore requires instructions that accommodate his or her needs to develop mathematical proficiencies (NCTM, 2000a). Instructional practices should therefore include a variety of methods. Teachers should build on the different cultural backgrounds brought into the classroom to support teaching (Bransford et al., 2000). Effective instructional practices embody and build on what the learners brings to the classroom.

2.3.2 Active learning

To teach for understanding requires effort, as learners' concepts and connections are built over time (Van De Walle et al., 2013). Teachers cannot make these connections for the learners. They can only create opportunities in a positive learning environment for learners to make these connections. Learners organise their own knowledge and connect ideas to what they already know. Teaching for understanding revolves around learners and therefore the responsibility of teachers should shift away from being the source of information towards the creation of an effective classroom environment which provides opportunities for learners to be actively involved in their own knowledge development (Bransford et al., 2000).

Active learning is identified as a principle in the South African NCS. A classroom environment should encourage an active approach to learning, rather than rote learning of given processes (DoE, 2002). It has been shown that allowing learners to participate in their own learning process and making maths more meaningful to them result in positive learner attitudes towards learning mathematics (Cohen & Amidon, 2004). Active learning as a teaching strategy implies experimental work, discussion, group work (pairs, mixed or cooperative) and discovery. Action verbs such as estimate, represent, construct, discover, explain, describe, develop, justify, formulate, investigate, solve, compare, predict and prove will most likely engage learners in doing mathematics. These actions lead to opportunities for making sense and figuring out problems as they involve higher-level thinking. Learners will actively think about the mathematical concepts that are involved when they are engaged in these kinds of actions. An effective classroom environment should make it possible for learners to reason, query, solve problems, and debate their ideas, solutions and strategies (Van De Walle et al., 2013). The environment must be such that learners are willing to take risks, share ideas and defend mathematical concepts.

2.3.3 Resources

Throughout this study, the term ‘resources’ refers to still and technology media used in a classroom environment. The choice of teaching resources should focus on developing the learners’ ability to their full potential by developing the necessary skills to solve problems effectively inside and outside of school (Bransford et al., 2000). The teaching resources should engage the learners so that they develop a deeper understanding and the ability to apply their knowledge inside and outside of school. These resources should allow learners to enter problems through multiple entry points and to apply strategies that make sense to them. It is through sense making that learners are able to develop and deepen their mathematical understanding (NCTM, 2000b). Mathematical development requires working on tasks which involve a productive intellectual challenge; tasks which promote critical thinking and reasoning, which can lead to higher achievements (Hiebert & Stigler, 2004). According to the NCTM (2000a), tasks with high-level cognitive demand provide opportunities for learners to communicate, reflect, and make connections between mathematical ideas or representations.

2.3.4 Assessment

Different types of assessments have different functions (Van De Walle et al., 2013). The South African DBE (2011) defines formative assessment as assessment for learning. The purpose of the assessment should not only focus on the traditional summative way of testing, but should rather be used to make learners' thinking visible to themselves, their peers and their teachers. This study mainly focuses on formative assessment, as formative assessment is part of the learning process. A key aspect of an effective learning environment is formative assessment. According to Bransford et al. (2000, p. 139), "[t]he key principles of assessment are that they should provide opportunities for feedback and revision and that what is assessed must be congruent with one's learning goals". It is vital that assessment focus on understanding rather than the skill to repeat facts and procedures to promote mathematical understanding (Bransford et al., 2000). Assessment should serve the purpose of providing opportunities for learners to improve their understanding (Leahy, Lyon, Thompson, & Wiliam, 2005). The purpose of assessment is to give both the learners and the teacher feedback. It should inform the teacher to make high-quality instruction choices (NCTM, 2000a). The type of assessment that is decided on reflects the teacher's objectives on what kind of mathematical proficiencies and skills are valued (NCTM, 2000a). Mathematics teachers who provide assessment that is fair, transparent and equitable support learners' ongoing learning (Ontario Ministry of Education, 2011). Assessment and instructions could be used interactively to promote meaningful development of mathematical proficiencies (Van De Walle et al., 2013). Formative assessment can be used in a variety of forms, for example discussions, observations, practice presentations and self-assessment (Bransford et al., 2000).

A main characteristic of formative assessment is constant feedback to the learners (DBE, 2011). Feedback goes hand-in-hand with the aims and learning outcomes. Feedback supports learning by helping learners to set up objectives and take accountability for own learning, and promoting independence (NCTM, 2000b). Immediate feedback provides learners with opportunities to revise and improve their thinking and helps teachers to identify problems in learners' thinking (Bransford et al., 2000). Immediate feedback takes place throughout learning and therefore supports learning (Reddy, 2004). Feedback on a daily basis assists learners to establish goals and develop self-regulation (Van De Walle et al., 2013), in terms of effectively monitoring and regulating performance and problem-solving methods (Andreassen & Braten, 2011). During a lesson, teachers need to constantly monitor the progress of learners and give appropriate corrective feedback (Mercer, Mercer, & Pullen, 2014). In order to monitor learners' progress, teachers need to verify and, if necessary, clarify learners' understanding by asking open-ended and probing questions (Allsopp, Kyger, & Lovin, 2007).

Formative assessment is used to inform the teacher about the teaching and learning process (Curro Centre for Educational Excellence, 2012). It can support the teacher to monitor progress (Organisation for Economic Co-operation and Development (OECD), 2005). The use of frequent feedback can indicate limited understanding and therefore possible modification and improvement to instruction (OECD, 2005). Teachers can use feedback to monitor learners' progress in order to determine how they should adjust their teaching approach or spend more or less time on a particular concept (Lerner & Johns, 2009; Mercer et al., 2014).

2.3.5 Problem-centeredness

An effective classroom environment inspires learners to be actively engaged in solving problems (Van De Walle et al., 2013). Kilpatrick et al. (2001) explain that effective mathematics classrooms consist of balanced teaching approaches and a variety of problem-solving engagements. Mathematically productive classrooms where critical thinking and problem-solving skills are developed, will develop mathematical understanding (Ontario Ministry of Education, 2011). Learners' current understanding and the way they construct knowledge influence the way they solve problems (Kilpatrick et al., 2001). The problem-centred approach does not impose formal methods on learners, but legitimises and builds on the intuitive and informal knowledge children already possess (Curro Centre for Educational Excellence, 2012). To be good at solving problems, it is necessary to make connections between concepts. Problem situations that make sense to learners are used as point of departure for new work and provide the contexts wherein learners can experience mathematical concepts and procedures as something that makes sense (Curro Centre for Educational Excellence, 2012).The teacher's role is thus to help learners build new concepts, solve problems that arise in the classroom and in everyday life, apply a diversity of strategies, and reflect on the process of mathematical problem solving (Loyens, Magda, & Rikers, 2008). In 2011, the Grade 9 learners in South Africa wrote the Grade 8 TIMSS tests. The TIMSS 2011 results indicated that many of the South African learners that participated in the study had basic knowledge and could do routine procedures effectively, but could not apply their knowledge to solve real-life problems; neither could they solve unfamiliar problems (Spaull, 2013). Even the best performing learners could only do items that required procedural proficiency, but struggled to solve unfamiliar problems and items that required adaptive reasoning (Stols, 2013).

Kilpatrick et al. (2001) use the term ‘strategic competence’ to refer to the skill to take a mathematical problem, represent it in mathematical notation, formulate the problem and solve it (Kilpatrick et al., 2001). It is the skill to express symbols and to solve mathematical problems in and out of school. This proficiency will help learners to solve the problems accurately as problems of mathematical nature arise in difficult and different situations in their everyday lives (Kilpatrick et al., 2001). Productive struggling and scaffolding, which are discussed in the next sections, are related to a problem-centered teaching approach.

2.3.5.1 Productive struggling

Research done by Bay-Williams (2010) and Hiebert and Grouws (2007) on instructional practices found that making connections between concepts can engage learners in productive struggling, which can result in understanding mathematics. According to Schoenfeld et al. (2014), success in mathematics can be determined by the persistence and the willingness to work hard for 22 minutes to gain sense of something that most people would give up on after 30 seconds. At some point, learners will experience difficulty in developing new concepts; therefore, it is important for learners to acknowledge that struggling is part of the learning process (Carter, 2008). This struggle should become part of their life and, in the process, they will learn to feel a sense of accomplishment once the problem has been solved. This newly acquired knowledge will help the learners to embrace the struggle and, as such, improve their mathematical problem solving skills (Carter, 2008). Learners must be able to cope with frustration by demonstrating persistence.

2.3.5.2 Scaffolding

Sawyer (2006) defines scaffolding as the support given during the learning process with the purpose of helping learners achieve their learning goals. Although scaffolding is a key instrument in creating opportunities to learn (Schoenfeld, 2013), it is important to withhold scaffolding once learners become more confident with the concept in order to provide them with the opportunity to develop independence (Van De Walle et al., 2013). Effective scaffolding requires knowledge of the gap between the learner's current understanding and what should be learnt. Vygotsky (1978) introduced the term 'zone of proximal development' (ZPD) to describe the distance between the learners' current level of understanding and what they can learn under the guidance of a teacher (Goos, 2004). Teachers need to scaffold in such a way that learners develop understanding, scaffolding learners' understanding instead of attempting to transfer knowledge through chalk and talk that result in rote memorisation. Silver (2011) and Vygotsky (1978) mention that one way to reduce the ZPD for struggling learners is to break up a task into smaller and more manageable parts.

In 2003, Verenikina published a paper in which she examined the ways that scaffolding has been understood, defined and applied. She presented an analysis of the concept 'scaffolding' in its connection to the ZPD. The data gathered in her study suggested that scaffolding can be an interruption rather than a support for learners' development depending on the context of its use. However, a deeper understanding of the concept of scaffolding will promote its inventive and informed use by teachers (Verenikina, 2003). These results differ from McMahon's (2000), which found that teachers who constantly use scaffolding are more likely to enhance learning. The findings further support the idea of Verenikina (2003) of further understanding the types of scaffolding that would enhance the teaching and learning process. In another study, Verenikina (2008) found that student teachers demonstrated understanding of some basic methods of scaffolding which are easy to grasp and implement, such as demonstration, modelling and breaking the tasks into smaller pieces. Moreover, the understanding of scaffolding principles will allow teachers to implement scaffolding methods provided by recent research.

2.3.6 Development of logical reasoning

Adaptive reasoning is when learners are capable of reflection, logical thinking, reasoning and justification (Kilpatrick et al., 2001). Any appropriate mathematical procedure and any appropriate computational method can be justified or rejected logically (Curro Centre for Educational Excellence, 2012). Logical thinking helps us to understand whether and why a solution makes sense (Van De Walle et al., 2013). Learners with this capability ask 'why' questions, they do not just accept a mathematical answer. Adaptive reasoning implies constructing sustainable arguments and evaluating one's own reasoning and the reasoning of others (Common Core State Standards Initiative, 2010). Adaptive reasoning will help learners to justify the conclusion they came up with or give alternative solutions for the problem presented (Kilpatrick et al., 2001). This strand is known as the "glue that holds everything together" (Kilpatrick et al., p. 129), steering towards the end goal. Deductive reasoning is directly linked to strategic competence and can therefore be used to acquire adaptive reasoning. Learners should form arguments to solve mathematical problems efficiently (Kilpatrick et al., 2001). Learners need to develop the habit of providing justification through logical arguments. They should be able to reason, explore mathematical conjectures, assess mathematical arguments and use different kinds of reasoning methods (Van De Walle et al., 2013).

2.3.7 The development of a positive attitude

Productive reasoning is a permanent disposition to see the value, sensibility and usefulness of mathematics as well as believing in attentiveness and in one's own worth (Van De Walle et al., 2013). A productive disposition is about having a positive attitude towards solving problems. Learners with a productive disposition are willing and committed to take on challenges, to try to solve problems and to make sense of them (Kilpatrick et al., 2001). A productive disposition is the habit of making an effort with mathematics and making it practical, valuable and meaningful. Much research (Aiken, 1970; Bursal & Paznokas, 2006; Ma, 1999; Swars, Daane, & Giesen, 2006; Weck, 2006) has been conducted on attitudes towards mathematics. Research has reported that the teachers' positive approach towards teaching the subject mathematics results in their learners being fonder of mathematics (Karp, 1991). When mathematical concepts are well understood, learners have a tendency to cultivate an affirmative self-concept and self-confidence in learning and understanding mathematics (Van De Walle et al., 2013).

In contrast, learner helplessness and anxiety occur when learners think they are unable to do mathematics and when they do not even try to solve mathematical problems (Kilpatrick et al., 2001). Learners can easily feel overwhelmed in a learning session if they get lost or do not understand basic concepts or principles (Van De Walle et al., 2013). Mike Ellicock, chief executive of the campaign group National Numeracy, warned that, when procedural skills are over-emphasised or when learners are shown how to do things, they may easily feel that mathematics is a subject they are unable to do (Adams, 2012). Learners may feel overwhelmed when they cannot relate the new content to existing knowledge. Teachers should not introduce the procedural rule first, but rather help learners to understand why the rule was invented and how it works. Teachers should rather focus on building confidence and competence in problem solving and mathematical reasoning by challenging learners to apply their own knowledge (Adams, 2012).

2.4 Theoretical framework

The purpose of this study is to explore Grade 8 mathematics teachers' creation and utilisation of opportunities for learners to develop mathematical understanding in their classrooms. The literature review revealed the importance of understanding mathematics and highlighted some important aspects of an effective classroom environment. Schoenfeld et al. (2014) developed an analytic framework to analyse the effectiveness or productiveness of a classroom environment. The framework is called the TRU Math scheme and was used as a theoretical framework for this study. This framework addresses the aspects discussed in the literature review. Schoenfeld (2013, p. 618) argues that the TRU Math scheme has 'the potential to be necessary and sufficient for the analysis of effective classroom instruction'. The TRU Math scheme consists of a general framework of five dimensions (see Table 2.1).

Table 2.1: The five dimensions of mathematically productive classrooms (Schoenfeld et al., 2014, p. 2)

The mathematics	The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Learners should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.
Cognitive demand	The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to learners' mathematical development. There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so large that learners are lost at sea.
Access to mathematical content	The extent to which classroom activity structures invite and support the active engagement of all of the learners in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in which a small number of learners get most of the "air time" (p. 2) is not equitable
Agency, authority and identity	The extent to which learners have opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically solid), resulting in positive identities as doers of mathematics.
Uses of assessment	The extent to which the teacher solicits learner thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction "meets learners where they are" (p. 2) and gives them opportunities to move forward.

These dimensions are: the mathematics, cognitive demand, access to mathematical content, mathematical agency, authority and identity, and uses of assessment. The TRU Math scheme is a tool for developing and reflecting on instructional practices that promote learner understanding. The five dimensions assist teachers in creating productive learning environments that will support learners to become powerful problem solvers as well as mathematical thinkers. In the following sub-sections, each dimension of the TRU Math scheme is discussed.

2.4.1 The mathematics

According to Schoenfeld et al. (2014), a productive classroom requires the mathematics discussed to be focused and coherent. Learners should have the opportunity to build meaningful connections between procedures, concepts, topics and contexts. The instruction must provide opportunities to learn important mathematical content and practices which develop productive mathematical habits of mind. The key aspects of the nature of the mathematics are discussed below.

2.4.1.1 Focused and coherent

The way mathematics is presented has an impact on understanding mathematics (Hiebert & Carpenter, 1992; NCTM, 2000a). Studies have revealed (Hiebert et al., 2003; Redeker, 2000; Stigler & Perry, 1998) that focused and coherent mathematics lessons may help learners to understand mathematics better and to learn mathematics conceptually. Schmidt (2008) argues that the mathematics discussed needs to flow in a logical sequence in order to be coherent. Wang and Murphy (2004) report that a focused lesson is content-focused and goal-oriented. The content and classroom activities should be purposefully designed to serve effective learning outcomes. According to Fernandez, Yoshida and Stigle (1992), lesson proceedings that are coherent relate to each other in ways that allow learners to draw inference relationships among proceedings.

2.4.1.2 Building meaningful connections

As discussed in section 2.1, it is critical to learn mathematics with understanding (NCTM, 2000b). Learners will learn mathematics with understanding when they actively build new knowledge on prior experience and knowledge (Van De Walle et al., 2013). Understanding can only take place when teachers ensure that learners' mental representations are part of bigger networks. It is therefore important for teachers to provide opportunities for learners to build connections between concepts instead of letting them memorise concepts (Bransford et al., 2000). Mathematical concepts are effectively learned when concepts are developed and linked to other concepts (see sections 2.1.2 and 2.1.3). To build understanding around big ideas will support learners to see that mathematics is integrated and not a collection of remote parts and sections (Van De Walle et al., 2013). Learning is enriched when learners need to apply their prior knowledge, make connections and comparisons, justify their own and other's ideas, make mathematical conjectures, test ideas, and develop sense-making skills (Van De Walle et al., 2013).

2.4.1.3 Engagement in key practices

Mathematics is a discipline that helps humans to understand and discover rules and patterns surrounding us and their relationship to us and each other (DBE, 2011). Mathematics is also used to express arithmetical, geometrical and graphical relationships. It is vital to know the basics of mathematics to participate effectively in society. Mathematics requires and enhances creative thinking, decision making, and critical and logical thinking skills. Mathematics is not only a language which is expressed by symbols and notations, but also a human activity. For learners to be able to identify qualitative relationships and patterns in our physical, social and economic world, mathematics involves observation. Mathematical understanding allows us to understand the physical world and to solve problems connected to the world (DBE, 2011). The CAPS focuses on equipping learners with the required knowledge and skills to solve everyday-life problems through critical and reflective thinking (DBE, 2011). Everyday-life problems such as economic, social, cultural, political health, scientific, and environmental problems should be incorporated into all segments whenever suitable (DBE, 2011). Teachers should create classroom environments where learners have opportunities to learn important mathematical knowledge and skills, and to develop productive mathematical habits of mind (Schoenfeld et al., 2014).

2.4.2 Cognitive demand

Cognitive demand is about creating and maintaining an intellectual challenge that encourages learners to improve mathematically (Schoenfeld et al., 2014). In order to maintain an intellectual challenge, it is important to stress the connections between the learners' prior knowledge and experience of a task in everyday contexts with the new task or concept being learned (Vygotsky, 1978). Productive struggling and scaffolding (including the ZPD) are discussed in the following paragraphs.

2.4.2.1 Productive struggling

Hiebert and Grouws (2007) explain that productive struggling is fundamental to developing conceptual understanding. As discussed in section 2.1.4, conceptual understanding is more than knowing isolated facts and methods. Conceptual understanding is about understanding mathematical ideas and having the ability to transfer prior knowledge and experiences into new situations and apply it to new contexts (NCTM, 2014). Teachers should implement strategies such as self-assessment, time management and goal setting to encourage learners to make choices about what they learn, to develop their ability to become self-directed learners and to foster the skill of self-management (Meyer, 2013). As discussed in section 2.3.5.1, a productive classroom environment will generate and sustain productive struggling and sense making by building understandings and engaging in mathematical practices. Several studies (Hyland, 2003; McRae, 2007; Rypisi, Malcom, & Kim, 2009) have examined factors that have an influence on maintaining learner engagement. Teachers may use probing questions to encourage learners to make connections (Van De Walle et al., 2013). Learners must be able to explain their reasoning and justify their ideas (Mueller & Maher, 2009).

2.4.2.2 Scaffolding

Problems should be at a level on which learners will be challenged, but not demotivated, and problems should not be too easy for the learners as they will then not be engaging with mathematical ideas (Van De Walle et al., 2013). For productive struggling to happen, the gap between learners' current level of understanding and what they can learn is a crucial aspect. Vygotsky (1978) introduced the term ZPD to describe the distance between the learners' current level of understanding and what they can learn under the guidance of a teacher (Goos, 2004). A way to ensure that the challenges are not too large is to reduce the ZPD for struggling learners by using scaffolding. Scaffolding can take on different forms, for example collaborative work, breaking a problem up into manageable parts, or the provision of guidance and structure by a teacher (see section 2.3.5.2). Teachers should create classroom environments where there is a good balance between aiding and leaving the learners (Schoenfeld et al., 2014).

2.4.3 Access to mathematical content

In addition to rich mathematical discussions, an effective classroom environment should provide the opportunity for all learners to participate in discussions. Active participation and equal access are discussed in the following paragraphs.

2.4.3.1 Active participation

Promoting learner participation is a key aspect of an effective learning environment (Ontario Ministry of Education, 2011). It is essential to establish a classroom culture that enhances, supports, facilitates and maintains high level discussions (Curro Centre for Educational Excellence, 2012). Alpert (1987) identified three kinds of classroom discourse, namely: silent – the teacher rarely asks questions and talks all the time; controlled – the teacher acts as an expert above learners; and active – the learners participate in discussions while the teacher facilitates. There should be clear expectations for different ways to participate in and contribute to classroom discussions (Schoenfeld, 2013). Classroom discussions develop learners' ability to reason mathematically and their ability to communicate that reasoning (NCTM, 2014). Through discussion, learners become aware of other methods or of possibilities for refining their own methods. This happens when they explain their own methods and methods are compared to those of others. Discussing problems in class can help learners to understand and construct deeper meaning of concepts (Hoffman, Breyfogle, & Dressler, 2009). Discussion involves representing, thinking, talking, agreeing, disagreeing, exchanging ideas and what the ideas entail (NCTM, 2014). Five teaching strategies for improving class discussions are: talk interchanges that engage learners in discussion, the art of questioning, using learner thinking to propel discussions, setting up a supportive environment and orchestrating the discussions (Chapin, O'Connor, & Anderson, 2009).

2.4.3.2 Equal access

To guarantee equal access, all learners must be actively engaged and must be part of classroom discussions. All learners should have access to the content and at the same time be encouraged to engage actively (Schoenfeld et al., 2014). It is essential to create an environment that offers an equal opportunity for all learners to learn (Van De Walle et al., 2013). Effective teaching requires a classroom culture where there are high expectations and strong support for all learners (NCTM, 2000b). Excellence in mathematics requires high-quality instruction, where all learners are equally treated regardless of their race, dysfunctions, upbringings or personal characteristics (Van De Walle et al., 2013). Each learner's ideas should be valued equally and included in classroom discussions (Van De Walle et al., 2013). In furtherance of creating equity in a classroom environment, norms have to be established by teachers. It is important to establish norms in a classroom to create a conducive learning environment. Classroom norms will determine learners' attitudes, which will also determine whether they will help each other. An environment should be established where learners feel safe to make mistakes, to ask questions and to make connections to the outside world (Bransford et al., 2000). Classroom discourse may differ from teacher to teacher (Cazden, 2001).

2.4.4 Agency, authority, and identity

Learners should have opportunities to build positive identities as mathematical scholars. The main aspects of learners' identity are the capacity and willingness to engage mathematically and to believe that they are knowledgeable about certain aspects of mathematics (Schoenfeld et al., 2014). The fundamental aspects of agency, authority and mathematical identity are discussed in the following paragraphs.

2.4.4.1 Agency

This study uses the definition for the term agency as suggested by Schoenfeld et al. (2014), who view it as a learner's perception that he or she can make progress in solving challenging mathematical problems and also can trust in the mathematical conclusion that he or she draws. According to De Corte, Mason, Depaepe and Verschaffel (2011), authority involves three aspects, namely, whom are learners allowed to ask for help, who is allowed to answer learners' mathematics-related questions and who is allowed to evaluate the correctness or legitimacy of learners' responses to word problems. Learners should validate their solutions themselves and should not guess and then seek confirmation from the teacher (Curro Centre for Educational Excellence, 2012). Agency is a function of learners' engagement with a task and their perceived ability to act on their own (Meyer, 2013). Learners should be willing to tackle given problems independently, although the problem might seem strange or new to them. Learners should get satisfaction from solving the problem and not from finishing first or pleasing the teacher (Curro Centre for Educational Excellence, 2012).

2.4.4.2 Authority

Having authority over mathematics means believing that one is knowledgeable about certain aspects of mathematics and that one is also recognised by others as having knowledge about certain facets of mathematics (Schoenfeld et al., 2014). Learners should be able to evaluate the thinking and methods of others and use the language of mathematics to express concepts accurately (Van De Walle et al., 2013). Teachers should create opportunities for learners to conjecture, explain, make mathematical arguments and build on one another's ideas in ways that contribute to their development (Schoenfeld et al., 2014). Learners should show interest in one another's solutions and methods, and should listen to the others' explanations and try to make sense of them. The learners are expected to discuss their interpretation of the problems, suggested solutions and solution methods with friends or the members of their groups. Learners are expected to explain their methods, criticise those of others and justify their own if necessary (Curro Centre for Educational Excellence, 2012). According to Gresalfi and Cobb (2006), authority is about who is allowed to make mathematical contributions in the classroom.

Working effectively, both as individuals and with others as members of a team, is listed as an important principle in the NCS (DBE, 2012). Attempting to articulate an idea to others is an effective way to grapple with or strengthen the understanding of a concept or network. Learners should be capable of organising their own thinking through articulation and communicate it logically and clearly to others (see section 2.3.6). Good communication about mathematics and active group discussions can promote interaction and investigation of the concepts (Cazden, 2001).

2.4.4.3 Mathematical identity

Schoenfeld et al. (2014) refer to mathematical identity as individuals having a sense of who they are mathematically. Mathematical identity shapes the way learners go about doing mathematics. Mathematical identity is derived from experiences with mathematics. If learners experience mathematics in a way that empowers them to see themselves as mathematically productive, they can develop a sense of themselves as mathematical thinkers (Schoenfeld et al., 2014). Effective learning takes place when learners are motivated and capable of using their skills to take on challenges (NCTM, 2000a). Effective teaching shapes learners' understanding of mathematics, gives them self-confidence to do mathematics and develops their skills to solve various problems (Bransford et al., 2000). Expanding learners' knowledge of mathematical concepts and encouraging them to try new ways to solve problems will most likely teach them to enjoy mathematical tasks (Van De Walle et al., 2013).

Learners should not hesitate to use primitive methods, making sense of the problem in their own way. A key principle of problem-centred mathematics teaching is that learners must act mathematically independent, which means that they should consistently decide for themselves how to tackle problems and which methods, including computational methods, to use (Curro Centre for Educational Excellence, 2012). The teacher should allow learners to use the methods they prefer to solve problems. Teachers should in no way put pressure on the learners to use certain methods, because it will affect their independence and create a risk that they may simply copy a method without actually understanding (Curro Centre for Educational Excellence, 2012).

2.4.5 Uses of assessment

Formative assessment practices can help to determine the direction and shape of classroom practices. Assessment that becomes a fundamental and ongoing part of the learning process can guide the teacher to enhance learners' individual and collective reasoning (Schoenfeld et al., 2014). Soliciting learner thinking and building on learner ideas and misunderstandings are discussed in the following paragraphs.

2.4.5.1 Soliciting learner thinking

When formative assessment becomes a central focus of a teacher's instructional practice, learners are constantly asked to articulate their thinking (Schoenfeld et al., 2014). The ability to articulate thinking is a benchmark of understanding. Learners can articulate their reasoning and understanding through explaining and justifying reasons or summarising critical ideas in a task (Schoenfeld et al., 2014). This underlines the importance of continuous informal assessment of learners' work in class and the explanation of their thoughts and ideas (Curro Centre for Educational Excellence, 2012). Teachers can use activities and tasks, or expand and improve questioning and classroom discourse to provoke learner thinking (Van De Walle et al., 2013). Questions that encourage rich mathematical conversation not only give the learners an opportunity to share and expand on their ideas, but allow the teacher to check for understanding of the content (Manouchehri, 2007).

Learner thinking can be gathered while learners are engaged in activities, tasks and discussions (Wiliam, 2008). Teachers can use techniques such as think-pair-share, wait time, cold calling, sharing learner generated solutions, and all learner response systems such as mini whiteboards and exit cards, to check for understanding during or right after a lesson (Wiliam, 2008). Cognitively guided instruction is a teaching style that builds on learner's natural problem solving strategies (Carpenter, Fennema, Franke, Levi, & Empson, 2000). Cognitively guided instruction helps teachers to understand how learners think so that they can guide them towards mathematical understanding. This knowledge helps teachers to determine what each learner understands and then the teacher can decide how to help the learner extend his or her understanding (Carpenter et al., 2000). Mathematically productive classrooms require teachers to be attentive to their learners' mathematical development and thinking. Effective teachers will listen to their learners in order to learn about their learners' understanding and thought processes instead of listening only for the correct answer (Curro Centre for Educational Excellence, 2012). Teachers should not just listen to the right answer, but listen for evidence about learner thinking to enlighten the next instructional steps (Wiliam, 2008).

2.4.5.2 Builds on learner ideas and misunderstandings

Gathering learner ideas creates the opportunity to refine instruction in response to learners' understanding and reasoning (Schoenfeld et al., 2014). Teachers' explanations should be grounded in learner ideas in order to productively shape their understandings, meet the learners where they are, build on learners' ideas and address emerging misunderstandings (Schoenfeld et al., 2014). Teachers should use specific references to learner ideas, such as talking about a specific learner's idea and leading the class to build upon it (Schoenfeld et al., 2014). Learners build their understanding on previous knowledge and experiences (Bransford et al., 2000) (see section 2.1.1). If learners do not understand, they will try to memorise without understanding.

Learners sometimes develop misunderstandings. Teachers can use formative assessment as a tool that allows them to identify misunderstandings and learners' weaknesses (Schnepper & McCoy, 2013). Identifying learners' struggles is the starting point to clarifying misunderstandings (Bransford et al., 2000). The older a misunderstanding, the more difficult it is to cure. Teachers should try to identify misunderstandings as early as possible. Frequently, one learner's misunderstanding is a communal misunderstanding in the class. After identifying learners' misunderstandings, the question becomes how to deal with them. A learner-centred teaching approach minimises the development of misconceptions

(Cakir, 2008; Longfield, 2009). Research (Ginsburg, 1977; Labinowicz, 1985) shows that responses from learners can be used as a tool to address misunderstandings and provide the necessary support to reconcile new learning. According to Longfield (2009), presenting learners with a contradicting event with an outcome that will be different to the learners' initial misunderstanding will prompt learners to reason through their misunderstanding.

3. CHAPTER 3: Research design

3.1 Introduction

This chapter provides a description of the research design. This is discussed in terms of the layers of research design as explained by Saunders and Tosey (2013). This research design model is known as the research onion because of the different layers, namely, research philosophy, the methodical choice, research strategies, and techniques and procedures.

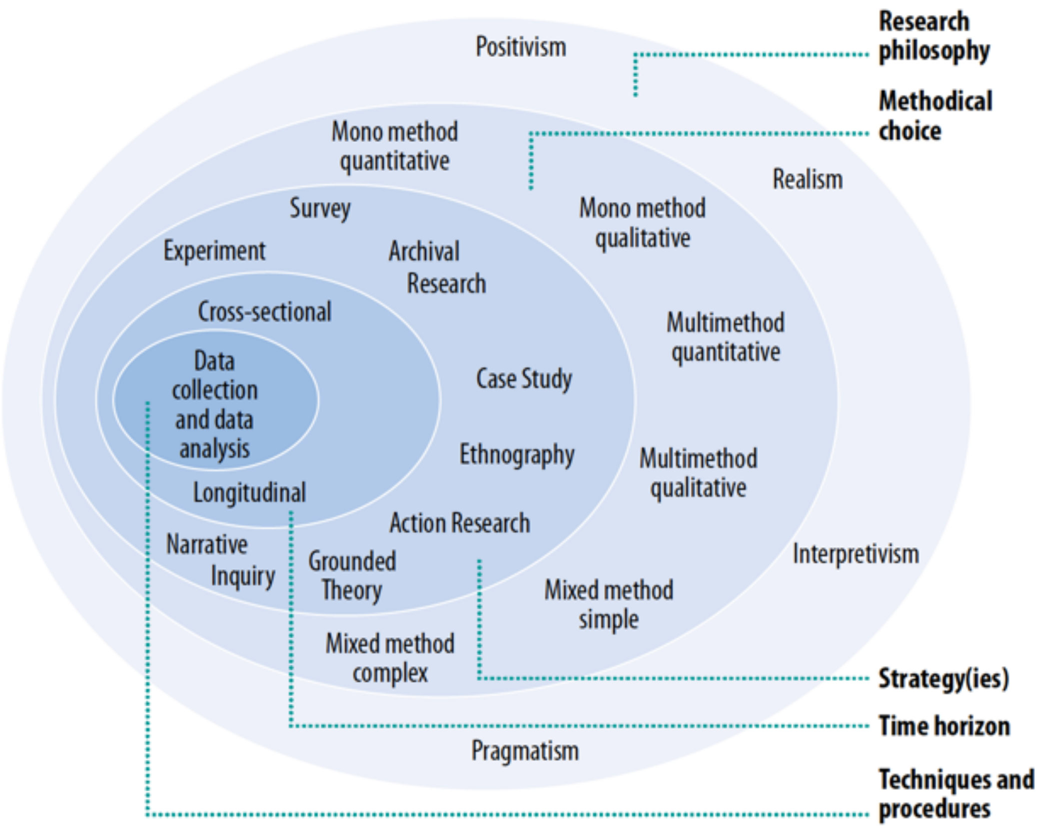

Figure 3.1: The research onion as adapted by Saunders and Tosey (2013)

This chapter is therefore divided into the different sections based on the different layers of the research onion as adapted by Saunders and Tosey (2013), but also includes a discussion about quality criteria and ethical considerations (see Figure 3.1). The discussion starts with a discussion of the research philosophy (outer layer) because that informs the methodical choice.

3.2 Research philosophy

The research philosophy that underpins this study is social-constructivism. Lincoln, Lynham and Guba (2011) found that experiences form our understanding of truth and will therefore be unique to each person. From an ontological viewpoint, constructivism holds that reality is insubstantial and created by the mind (cognitive constructivism). Furthermore, constructivism implies that multiple social realities can be constructed (social-constructivism). From this perspective, knowledge can either be constructed through cognitive processes or through the social interaction with an environment (Terre Blanche & Durrheim, 2006). The researcher constructs reality based on interpretations of data that are provided with the assistance of participants of the study. Even though reality is created by individuals, similarities may occur (Terre Blanche & Durrheim, 2006). Adopting a social-constructivist's approach enables the researcher to be flexible and open-minded to enable a thorough understanding of the participant's reality (Bisman & Highfield, 2012).

This study is an investigation into how Grade 8 mathematics teachers' instructional practices create and utilise opportunities to develop mathematical understanding in their classrooms. Within social-constructivism, the aim of research is to discover how people make sense of a situation at a specific point in time. Accordingly, a reality which is impartial can change at any time and is context-dependent. Hermeneutics is the study of explorative understanding or meaning within a certain context or situation (Bisman & Highfield, 2012). Findings emerge continuously throughout the research process (Bisman & Highfield, 2012). Dialectics was used as a method since the findings were created through exploring and interviewing concepts, viewpoints and opinions (Berniker & McNabb, 2006).

One of the tenets of social-constructivism is that reality is interpreted by using various mediums of communication. In this study, written communication (learner worksheets and assessments) and oral communication (lesson observations and individual post-observation interviews) were used to find meaning of the phenomenon. When analysing the data, the contexts of the teachers were taken into account to interpret and portray their experiences (Mertens, 2015). Data collection strategies such as lesson observations and post-observation interviews were used to understand the teachers' reality as they discussed and shared their experiences.

The methodological paradigm for this study is explorative as the study is grounded in a dialectical social-constructivist paradigm and through the teachers' actions and opinions. I wanted to interpret the data in order to come to an understanding of their instructional practices. Interpretivists believe that accepting the involved participant's behaviour, views, opinions and outlooks is key to interpreting and understanding a situation (Burr, 2003). For this reason, it is important that a situation should be understood and interpreted in context (Lincoln et al., 2011; Terre Blanche & Durrheim, 2006). From a social-constructivist's point of view, conceptual knowledge is not taught, but is built upon the interaction of the learner's existing knowledge, among other things. The ontological assumption was that of social-constructivism in which the phenomenon is understood through words and reality is regarded as a product of individual experiences (Andrews, 2012; Creswell, 2014, 2007; Maree, 2012). Finally, the epistemological assumption was that I came to understand through interacting and observing teachers. I subjectively described and interpreted the data from the document analysis, lesson observations and post-observation interviews of how the teachers created and utilised opportunities to develop mathematical understanding in their classrooms (Creswell, 2014, 2007; Maree, 2012).

3.3 Methodical choice

The research approach of this study is qualitative in nature as it seeks to explore how these teachers create and utilise opportunities in their classrooms to develop mathematical understanding. Thus, this study is interested in exploring the meaning teachers have constructed about the development of mathematics understanding and how they use their instructional practices to develop mathematical understanding. Real-life teaching situations were explored and used to collect data to make sense of how teachers teach for understanding in their classrooms. The intention was thus not to control or manipulate behaviour, but instead to let events unfold and to explore them as they occurred, with no interference from me.

Using a qualitative research approach enabled me to obtain information about human behaviour, opinions and experiences (Mack, Woodsong, MacQueen, Guest, & Namey, 2005). A qualitative research approach allowed me to observe and investigate a complex phenomenon within its context using a variety of data collection strategies. A variety of data collection strategies ensures that a situation can be explored through multiple lenses which allows the phenomenon to be understood through various facets (Baxter & Jack, 2008). Document analysis, lesson observations and post-observation interviews serve as data collection instruments to enable me to interpret the reality by becoming part of the lives of the teachers. Qualitative research strives to find meaning of a phenomenon from the participants' point of view. I wanted to gain a better understanding of the phenomenon to be able to eventually describe it (Hogan, Dolan, & Donnelly, 2009). Table 3.1 below provides an outline of the research methodology elements of this study.

Table 3.1: Outline of the research methodology elements

Research method	**Qualitative**				
Research strategy	Exploratory case study design Cross-sectional				
Main question	How do Grade 8 mathematics teachers create and utilise opportunities to develop mathematical understanding in their classrooms?				
Research sub-questions	Question 1 How focused and coherent is the mathematics and how are the connections between procedures, concepts and contexts (where appropriate) addressed?	Question 2 What opportunities do learners have to make their own sense of mathematical ideas?	Question 3 How do teachers invite and support the active engagement of all of the learners in the classroom with the core mathematics being addressed in the lesson?	Question 4 What opportunities do learners have to explain their own ideas and to respond to each other's mathematical ideas?	Question 5 How does instruction build on learners' ideas or address emerging misunderstandin gs?
TRU Math dimensions	The mathematics	Cognitive demand	Access to mathematical content	Agency, authority and identity	Uses of assessment
Participants	Three Grade 8 mathematics teachers from three different private schools in Mpumalanga.				
Data collection techniques	• Document analysis: mathematical tasks • Two lesson observations per participant (video recordings and field notes, in order to capture data during observations) • One semi-structured post-observation-interview, to get clarity about anything that was unclear during the observations				
Techniques per question	Document analysis Lesson observation Post-observation interview	Document analysis Lesson observation Post-observation interview	Lesson observation Post-observation interview	Lesson observation Post-observation interview	Lesson observation Post-observation interview

Data analysis	Deductive approach to data analysis • Establish units of analysis of the data • Use categories obtained from the relevant literature to interpret all the data • Create a 'domain analysis' • Establish relationships and links between the domains • Make speculative inferences • Summarise

3.4 Research strategy

A case study allowed me to explore how participants created and utilised opportunities to develop mathematical understanding in their classrooms. A case study generally attempts a profound exploration of a matter, with the emphasis on quality rather than on quantity (Rule & John, 2011; Yin, 2009). A case study aims to do an in-depth exploration of a phenomenon (Rule & John, 2011; Yin, 2009). A small sample size is applicable for observing and investigating existing phenomena within their real context, particularly when limitations among the phenomenon and context are unclear (Cohen et al., 2011; Lauckner, Paterson, & Krupa, 2012). A small sample size enabled me to use a variety of data collection strategies over a period of time to gather detailed information (Creswell, 2014). The units of analysis for this study were individuals (Baxter & Jack, 2008), while the cases were three Grade 8 mathematics teachers at different private secondary schools in South Africa.

Some advantages of a case study are that it exposes the participants to real-life situations and allows detail to be collected, which is otherwise difficult (Flyvbjerg, 2011). Despite the many advantages of using a case study approach, there are also some limitations to be considered. Findings cannot necessarily be generalised to the bigger population, because of the small sample sizes. I cannot assume that the reality of one unit of analysis reflects the reality of related entities. Supplementary research is necessary to determine whether findings can be generalised from one study to another (Flyvbjerg, 2011). A quantitative research approach aims to confirm hypotheses to a population, whereas a qualitative research approach seeks to explore phenomena (Mack et al., 2005). The aim of this research study is not to generalise to a bigger population, but to generalise to an existing theory. Therefore, this study is an exploratory study where the cases are explored in detail and in depth.

In this study, the context of each case differed. The participants in this study did not necessarily teach the same topic, so this study can be described as cross-sectional. The strategy of inquiry was a case study design which allowed the researcher to focus on all important dimensions of a productive mathematics classroom (Cohen et al., 2011). A case study design allows me to observe and investigate dissimilarities within and among cases (Baxter & Jack, 2008). The benefit of this is that there is space to investigate the matter to some extent and in depth. If common factors are found, the findings can be generalised to a limited degree (Rule & John, 2011).

3.5 Research techniques and procedures

The discussion of the research techniques and procedures includes an explanation of the selection of participants and sampling procedures, the data collection process, and the instruments that were used to collect and analyse the data.

3.5.1 Selection of participants and sampling procedures

Three teachers were selected using purposive and convenient sampling. Private schools (schools that are not sponsored or controlled by the government or local authorities) provide Independent School Education to learners from as early as three months up to Grade 12. The private school curriculum emphasises the development of skills and values, problem-solving and creative thinking. My preference for choosing private schools in Mpumalanga to conduct this research study was motivated by the fact that they followed a problem-solving approach regarding learning mathematics. I also chose schools in Mpumalanga because I lived in Mpumalanga at the time. Grade 12 learners also write the Independent Examinations Board (IEB) examination at the end of the year. The IEB strives to set well-constructed assessments that test the learners' understanding of what information applies in certain situations and how and why specific knowledge is applied. It also directs teachers in their instructional practices to improve logical thought in learners.

In this study, three Grade 8 mathematics teachers' instructional practices at different private secondary schools were observed and analysed separately. The instructional practices of these teachers were observed in their natural context to explore how these teachers create and utilise opportunities to develop mathematical understanding in their classrooms. The aim was to examine how these teachers create and utilise opportunities to develop mathematical understanding in their classrooms.

Table 3.2: Criteria for inclusion and exclusion in the sample

	Criteria
Inclusion	• Private schools • Grade 8 mathematics teachers • Male and female teachers • Teachers with at least four years' experience of teaching mathematics • Different races • Schools with different performance levels • Problem-solving approach regarding learning mathematics
Exclusion	• Government schools • Non-mathematics teachers

The selection of the participants was made using specific criteria; thus, purposive sampling was used. Purposive sampling involves pre-selected criteria relevant to a specific research question (Mack et al., 2005). The three participants were selected based on the following inclusion criteria: the Grade 8 mathematics teachers had to come from different private schools and had to have a minimum of four years teaching experience (see Table 3.2: Criteria for inclusion and exclusion in the sample). It was my assumption that experienced teachers have already developed content and pedagogical content knowledge and skills and should therefore have productive practices. Internships in tertiary education mainly offer support to professional development rather than practical experience (Ball & Cohen, 1999).

All private schools need to follow company policies and therefore need to obtain permission from head office before conducting any research. For this reason, I first approached the principals of different private schools, then the principals recommended participating teachers according to the pre-determined criteria and their willingness to participate in this research project. If permission was not granted, other schools were approached. Once permission was obtained from the head office and principals, teachers were contacted and received letters of informed consent. Only the three teachers who conformed to the inclusion criteria stated in Table 3.2 were part of the project. It was also more convenient to take the three schools nearest to me. The sample was therefore independent of sex or race. A letter of the alphabet was assigned to each participant to ensure anonymity and confidentiality.

3.5.2 Data collection process

Data collection was done at private inclusive mainstream schools in Mpumalanga. The principals of three schools were contacted to discuss the study and request their participation in the study. Two letters of invitation were sent to each school, one addressed to the principal and the other addressed to the Grade 8 mathematics teacher. During the data collection period, all communication and arrangements were made directly with the teachers. The data collection process involved one document analysis, two lesson observations on different days in one week on different topics, and an individual semi-structured post-observation interview just after the second lesson observation. Video recordings and field notes were used to capture data during observations, while an audio recorder was used to capture data during post-observation interviews and a document analysis schedule was used for the document analysis.

The data collection took place during the first quarter (February and March) of 2016. In Table 3.3, a timeline is given indicating the dates on which all three participants' documents were analysed, lessons were observed and interviews were conducted.

Table 3.3: Timeline of the data collection process

Participant	Data collection instrument	Date (2016)
Teacher A	Document analysis	1 February
	Lesson observation	1 February
	Document analysis	5 February
	Observation	5 February
	Post-observation interview	10 February
Teacher B	Document analysis	15 February
	Lesson observation	15 February
	Document analysis	18 February
	Observation	18 February
	Post-observation interview	25 February
Teacher C	Document analysis	29 February
	Lesson observation	29 February
	Document analysis	3 March
	Observation	3 March
	Post-observation interview	11 March

3.5.3 Instruments for data collection

The instruments that were used for data collection were document analysis, two lesson observations and one post-observation interview per teacher. The identification of the categories was based on the literature, as discussed in detail in section 2.4 and summarised in Table 2.1.

3.5.3.1 Document analysis

The first instrument was a document analysis to analyse mathematical tasks from learner textbooks. The document analysis consisted of structured questions to assist in answering sub-questions 1 and 2. Sub-question 1 is about the mathematics and sub-question 2 about cognitive demand (Addendum C). The aim of the analysis was to explore whether the mathematical tasks used in the classroom created or utilised opportunities to develop mathematical understanding in the teachers' classrooms. This was used to support findings made from the lesson observations and interviews. The teachers provided me with copies of the work given to the learners and I took photos of the mathematical tasks.

3.5.3.2 Lesson observations

The second instrument was a lesson observation schedule, which consisted of structured and open-ended questions that probed the instructional practices of the mathematics teachers concerning creating and utilising opportunities to develop mathematical understanding in their classrooms. A lesson observation schedule was used that was prepared in advance based on the theoretical framework and research questions. The lesson observation schedule included background information and specifics about the teaching and learning in the observed lessons (Addendum D). This enabled me to explore and observe all these aspects of a mathematically productive classroom as described in the theoretical framework. The data obtained from the lesson observations were used to answer all the sub-questions. The document analysis consisted of structured questions to assist in answering all sub-questions. Before each lesson observation, the teacher provided me with the relevant documentary sources that were prepared for the lesson. After each observation, the actions and words of the teacher were classified into the categories identified on the observation schedule.

I observed two lessons taught by each of the three participants. The second lesson of each participant was observed to obtain a clearer and more accurate view of the teacher's practice. Thus, two observations per week was done. Video recordings and field notes of all lesson observations describing responses and comments were used to capture the data during observations. The lessons were video recorded and transcribed afterwards to assist in the analysis of the data.

3.5.3.3 Post-observation interviews

The third instrument was a post-observation interview, which consisted of open-ended questions about the perceptions of the mathematics teachers concerning their instructional practices. The teachers were individually interviewed after the two observations. The purpose of the post-observation interview was for me to receive feedback and to have a clear and accurate record of all the communication that took place. During the post-observation interviews, the teachers were invited to make any other comments about their instructional practices. Through the interviews, the teachers had the opportunity to give meaning to their actions in class. The data obtained from the post-observation interviews were used to answer all the sub-questions. Each interview session was audio recorded and transcribed afterwards for analysis purposes. The aim of this post-observation interview was to receive feedback and to make sure a clear and accurate record of all the communication took place (Addendum E).

Table 3.4 provides an overview of the item analysis of the constructs, as well as the sources for measuring instruments. The table shows how each dimension of the TRU Math scheme, together with the indicators, was measured according to the instruments for data collection.

Table 3.4: Item analysis

		Five dimensions of the TRU Math scheme												
		1.The mathematics			2. Cognitive demand		3. Access to mathematical content			4. Agency, authority and identity			5. Uses of assessment	
Instruments / **Indicators**	A. Background information	1.1 Coherence	1.2 Connections	1.3 Engagement in key practices	2.1 Productive struggling	2.2 ZPD	3.1 Active participation	3.2 Equal access	3.3 Pattern of discourse in the classroom	4.1 Agency	4.2 Authority	4.3 Mathematical identity	5.1 Learner thinking	5.2 Feedback
Document analysis App. C	-	1.1.1	1.2.1	1.3.1	2.1.1	-	-	-	-	-	-	-	-	-
Lesson observation schedule App. D	A	1.1.1	1.2.1	1.3.1	2.1.1	2.2.1	3.1.1	3.2.1	3.3.1	4.1.1	4.2.1	4.3.1	5.1.1	5.2.1
Post-observation interview App. E	B-C	1.1.1	1.2.1	1.3.1	2.1.1	2.2.1	3.1.1	3.2.1	3.3.1	4.1.1	4.2.1	4.3.1	5.1.1	5.2.1

3.5.3.4 Data analysis procedures

Initially, a deductive approach was used to analyse the data according to a set of pre-determined aspects based on the theoretical framework (Table 2.1). For each of the instruments about how Grade 8 mathematics teachers create and utilise opportunities to develop mathematical understanding in their classrooms, answers were categorised in the following five categories: the mathematics, cognitive demand, access to mathematical content, agency, authority and identity, and use of assessment. The words and actions of the teacher during the observed lessons were categorised, either immediately or later, according to the same five categories included in the theoretical framework and other instruments. Each participant's document analysis, lesson observation and post-observation interview schedule were analysed and interpreted separately. A key principle of interpretive analysis is to stay close to the data and to interpret them from a position of empathic understanding. However, an inductive approach to data analysis was used where I created new coding labels that may have been discovered throughout the research process (Cohen et al., 2011). Open coding was used as well where new information came to light. Data analysis built from specific views to more general outlooks; consequently, an inductive approach was followed (Creswell, 2014). Each participant had his or her own context, background, teaching experience and the data had to be interpreted with that in mind (Cohen et al., 2011).

In the end, the three participants' data were compared in order to see if interesting similarities or differences emerged. The instruments were compared with each other to determine whether the participants created and utilised opportunities for developing mathematical understanding in their classrooms. From the data analysis, conclusions were drawn and recommendations were made for further research.

Each teacher's documentary sources, lesson observations and post-observation interview were coded and analysed separately. Each case is discussed separately so that a complete picture of each participant can be created. The data relating to each participant are presented and interpreted separately in terms of the document analysis, lesson observations and post-observation interview. An analysis of each teacher is presented, followed by a cross-case analysis. The results for each participant are organised, discussed and interpreted according to the research instruments. These research instruments are in turn linked to the secondary research questions (Table 3.1). The reason for this organisation of the results is my paradigmatic perspective of social-constructivism. Each participant's unique background and the context in which they taught are respected in the organisation of the results and the analysis of the data. For the cross-case analysis, three tables are used. Table 5.1 contains a summary of the categories addressed in the data analysis of each teacher. Table 5.2 contains a summary of the categories addressed in the observations by each teacher. Lastly, Table 5.3 contains a summary of the participants' beliefs regarding how Grade 8 mathematics should be taught.

3.6 Quality criteria

When doing qualitative research, there are a number of quality criteria that need to be taken into consideration. According to Cohen et al. (2011), a valid instrument measures what it is supposed to measure. Internal validity, also known as credibility, is important for this study. According to Spencer, Ritchie, Lewis and Dillon (2003), internal validity is when the data and findings are accurate. The data should be accurate, credible, neutral and consistent with the conclusions and interpretations (Cohen et al., 2011).

There are various types of validity that are unique, but not restricted to qualitative research (Spencer et al., 2003). In this study, internal validity was enhanced by spending at least two periods per participant in lesson observation, giving participants time to get used to the researcher’s presence in the classroom. Lesson observations were used in conjunction with post-observation interviews and document analysis in order to gather data. To assure that internal validity was achieved, every step of the research was peer reviewed. The purposive sampling that was used included participants with various types of qualifications and teaching experience, which helped to achieve internal validity. For this study, three Grade 8 mathematics teachers, each with at least four years’ experience of teaching mathematics and teaching at a private school, were chosen.

The second type of validity that is applicable to this research is construct validity. Construct validity is when the classifications of analysis for codes and instruments relate well with the literature (Cohen et al., 2011). In this study, I decided to use the theoretical framework as developed by Schoenfeld et al. (2014). The categories as described by the framework can be used for the coding of the data and for the development of themes. Member checking via post-observation interview after the completion of the document analysis and lesson observation created the opportunity to correct any misinterpretation and presentation of findings (Spencer et al., 2003).

Credibility was achieved through multiple data collection methods involving peer researchers and crystallisation. By triangulating the data of the instruments, I attempt to provide a convergence of evidence that strains credibility. Findings across datasets can be substantiated when information collected through different instruments is examined. The impact of potential biases that can occur in a single study is reduced through triangulation. According to Patton (1990), triangulation is a very effective method for helping the researcher to guard against the accusation that a study's findings can rely on a single method, single source or a single investigator's bias. Triangulation is defined as viewing a phenomenon from various angles with several data collection methods (Cohen et al., 2011). Concurrent validity is also achieved when triangulation is done. Concurrent validity takes place when a method's data reach an agreement with another method's data (Cohen et al., 2011; Yin, 2009). In this study, methodological triangulation was applied using three data collection methods: document analysis, lesson observations and a post-observation interview (see Figure 3.2). Theoretical triangulation is achieved by using various researchers' views on opportunities for the development of understanding in mathematics classrooms and incorporating them into the theoretical framework.

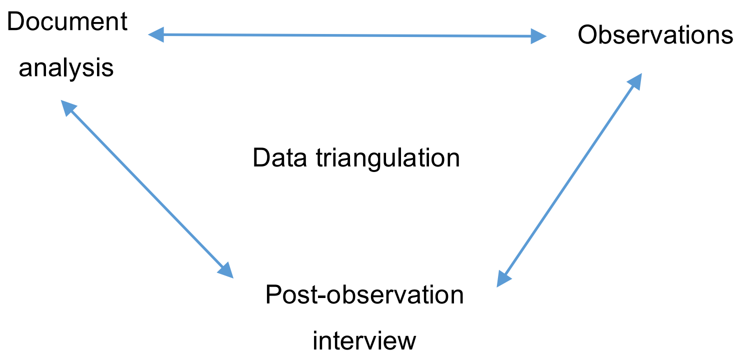

Figure 3.2: Data collection triangulation

There are numerous debates on whether reliability can be achieved in qualitative research (Spencer et al., 2003). In this study, documentary sources, lesson observations and post-observation interviews were selected to ensure accurate interpretations and analysis of the data. To ensure trustworthiness, member checking was used with regard to the interpretation of the lesson observation, as well as the post-observation interview interpretations. The informal feedback from the post-observation interview from the teachers improved the accuracy and credibility of the data.

3.7 Ethical considerations

The sources of data that were used in this study were lesson observations, a post-observation interview and document analysis. Confidentiality, anonymity and privacy were applied when collecting data. No names were mentioned of any school or participant during the dissemination phase of the study when the research report was written. To accomplish confidentiality, anonymity and privacy, a letter of the alphabet was assigned to each participant. The three participants are referred to as Teacher A, Teacher B and Teacher C. Anonymity and privacy were ensured by encrypting all digital video and audio recordings. Permission was obtained from the Ethics Committee at the University of Pretoria as well as the private schools' head offices. These applications were submitted after the proposal was successfully defended at faculty level and before fieldwork was conducted. Issues addressed in the application involve the sensitivity level of the research activities, the research approach, design and methodology, including full details regarding the participants, voluntary participation, informed consent, confidentiality, anonymity and risk. Informed consent was obtained from each private school's executive head and teacher participants and, for the video or audio recordings, from the learners' parents. Informed assent was obtained from the participants for the video and audio recordings during teacher interviews and lesson observations. Nobody was forced to participate in this study. The participants were allowed to withdraw at any time if they wished to do so. The data became institutional property of the University of Pretoria and was handed over to them for safekeeping. The data will be destroyed over time after completion of the process.

I informed the teachers verbally about the purpose of the study, their role in the study and that they could withdraw at any time. They also signed a letter of permission and consent (see Addendum A). The signed consent letters served as a further guarantee to the participants regarding the anonymity and confidentiality of the study. Consent needed to be obtained from parents or guardians and assent from learners because the researcher was present in the classroom during lesson observations. Teachers were video recorded where learners were only part of the background. If informed consent from a learner was not obtained, the learner sat in the back of the classroom where he or she was not video recorded. The interviews took place in a private environment.

3.8 Summary

In this chapter, I discussed social-constructivism as the paradigm for this study. Regarding the epistemology, the interpretation of the data collected was subjective and interpretive in nature. A qualitative research approach was used and an exploratory case study design was conducted with three teachers who taught Grade 8 mathematics at private secondary schools in Mpumalanga. Three instruments were used for data collection, namely, a document analysis, two lesson observations and one post-observation semi-structured interview per teacher. Categories obtained from the relevant literature were used to analyse and interpret all the data. Finally, quality criteria and the ethical considerations that were taken into account were discussed. In the next chapter, the data are presented and the findings are discussed.

4. CHAPTER 4: Results

4.1 Introduction

This chapter presents the findings from the data collection as they relate to the research questions composed for this study. The chapter commences with processes for data collection and data analyses, followed by information regarding the three participants and the coding of the data, and finally, the presentation of each participant's data. As this study is a case study, I discuss each case separately so that a complete picture of each participant can be created. The data relating to each participant are presented and interpreted separately in terms of the document analysis, lesson observations and post-observation interview. The categories that were used to interpret the data were based on the theoretical framework (see Table 3.4).

4.2 Processes for data collection and data analyses

Data collection started with a document analysis of the mathematical tasks the teacher planned to use during the lesson. This was provided by the teacher before each observation. The data obtained from the documents were recorded using a document analysis schedule (see Addendum C) to assist in answering sub-questions 1 and 2. Each mathematics teacher was observed twice over a period of one week on different topics with the purpose of exploring how Grade 8 mathematics teachers create and utilise opportunities to develop mathematical understanding in their classrooms. An observation schedule that contained five categories chosen from literature was used (see Addendum D) to assist in answering all sub-questions. Video recordings and field notes of all lesson observations describing responses and comments were used to capture the data during observations. I also took photos of the teaching material and the work explained on the whiteboard. Finally, a post-observation interview was conducted. The aim of this interview was to receive feedback and to clarify any aspects of the observations that were unclear (see Addendum E). During the interviews, the teachers were allowed to make comments about their instructional practices to assist in answering all the sub-questions. Through the interviews, the teachers had the opportunity to give meaning to their actions in class. Although the interview data are presented in this chapter, the data only become meaningful when discussed together with the findings from the document analysis and observations. This is done in Chapter 5. In the next section, background information regarding the three participants is discussed.

4.3 Information regarding the three participants

In this section biographical information regarding the three participants – Teacher A, Teacher B and Teacher C – is provided. A letter of the alphabet has been assigned to each participant to protect their identities.

4.3.1 Teacher A

Teacher A was a 29-year-old man who had a Baccalaureus Educationis (BEd) and a Baccalaureus Educationis Honours (BEd Hons) degree. Teacher A was a Phase Head at a private school in Mpumalanga where he taught Grade 8 mathematics, natural science, technology and bible study and Grade 5 mathematics. In total, he had eight years of teaching experience, including five years of teaching Grade 8 mathematics.

4.3.2 Teacher B

Teacher B was a 38-year-old woman who had a Baccalaureus Occupational Therapy degree and a Postgraduate Certificate in Education (PGCE). Teacher B was a mathematics teacher at a private school in Mpumalanga where she taught Grade 8 and 9 mathematics. In total, she had eight years of teaching experience, including four years of teaching Grade 8 mathematics.

4.3.3 Teacher C

Teacher C was a 34-year-old woman who had a BEd degree specialising in the FET phase with natural science as main subject. Teacher C was a mathematics teacher at a private school in Mpumalanga where she taught Grade 8 and 10 mathematics. In total, she had 11 years of teaching experience, including three years of teaching Grade 8 mathematics.

4.4 Coding of the data

In coding the data, I used a deductive approach based on my theoretical framework. According to the theoretical framework five dimensions were identified, namely, the mathematics, cognitive demand, access to mathematical content, mathematical agency, authority and identity, and uses of assessment. The transcripts were coded according to a set of pre-determined TRU Math dimensions and their indicators, which are given in Table 4.1.

A document analysis schedule (see Addendum C) was used to answer sub-questions 1 and 2 regarding the TRU Math dimensions. An observation schedule (see Addendum D) and post-observation interview schedule (see Addendum E) were used to answer all sub-questions regarding the five TRU Math dimensions: the mathematics, cognitive demand, access to mathematical content, agency, authority and identity and uses of assessment (Schoenfeld et al., 2014).

Table 4.1: TRU Math dimensions and dimension indicators as inclusion criteria for coding the data (Adapted from Schoenfeld et al., 2014, p. 2)

TRU Math dimension	Description of TRU Math dimension indicator
The mathematics	
Focused and coherent	Content is goal-oriented (focused) and logically sequenced.
Building meaningful connections	Building new knowledge on prior knowledge and experience and build understanding around big ideas (other topics).
Engagement in key practices	Opportunities to learn important mathematical content and applying it on solving real-life problems.
Cognitive demand	
Productive struggling	Intellectual activity and an opportunity for learners to develop conceptual understanding through productive struggling.
Scaffolding	Extent to which teacher provides scaffolding to reduce the ZPD.
Access to mathematical content	
Active participation	Active participation in class discussions.
Equal access	Providing mathematical access to a wide range of learners through equity.
Agency, authority and identity	
Agency	Learners' capacity and willingness to engage mathematically.
Authority	Opportunities to demonstrate how knowledgeable learners are; and being recognised for that by the teacher.
Mathematical identity	Positive doers of mathematics.
Uses of assessment	
Soliciting learner thinking	Gathering information about learner thinking.
Building on learner ideas and misunderstandings	Building on learners' ideas and addresses emerging misunderstandings.

The first column in Table 4.1 above indicates the different categories of these TRU Math dimensions and the second column presents the descriptions of the TRU Math dimension indicators. The next section presents the findings from each data collection instrument per teacher. The teachers' words during their two lessons and one interview are given in italics.

4.5 Teacher A

In this section, I discuss the results from the document analysis, the lesson observations, and the post-observation interview of Teacher A.

4.5.1 Document analysis

The document analysis was based on mathematical tasks from learner textbooks which the teacher planned to use during the lessons. The teacher provided the tasks he planned to use just before each observation. The mathematical tasks of each lesson were analysed. The tasks used in both lessons were from the textbook PracMaths Grade 8 (CAPS).

Task 1 (see Figure 4.1: Task 1) was about solving linear algebraic equations. Seven equations were given in this task requiring the learners to solve for x. In Lesson 1, the teacher provided the learners with a memo for Task 1.

7. Solve for x:

a) $-2x-1=11$	b) $x-5=-x+1$	c) $\frac{4x}{5}=-8$	d) $\frac{3x+1}{2}=5$
e) $7x-3x=13-17$	f) $\frac{x}{3}-4=-1$	g) $2x+5-(3x+2)=4x$	

Figure 4.1: Task 1

Task 2 (see Figure 4.2) involved solving angles of 2-D geometric shapes. The first question in Task 2 asked the learners to correct eight incorrect statements. The second question of Task 2 involved solving unknown sides and angles in triangles and quadrilaterals. The geometric problems involved solving unknown sides and angles of triangles and quadrilaterals by using previous knowledge about angles as well as angle relationships on straight lines.

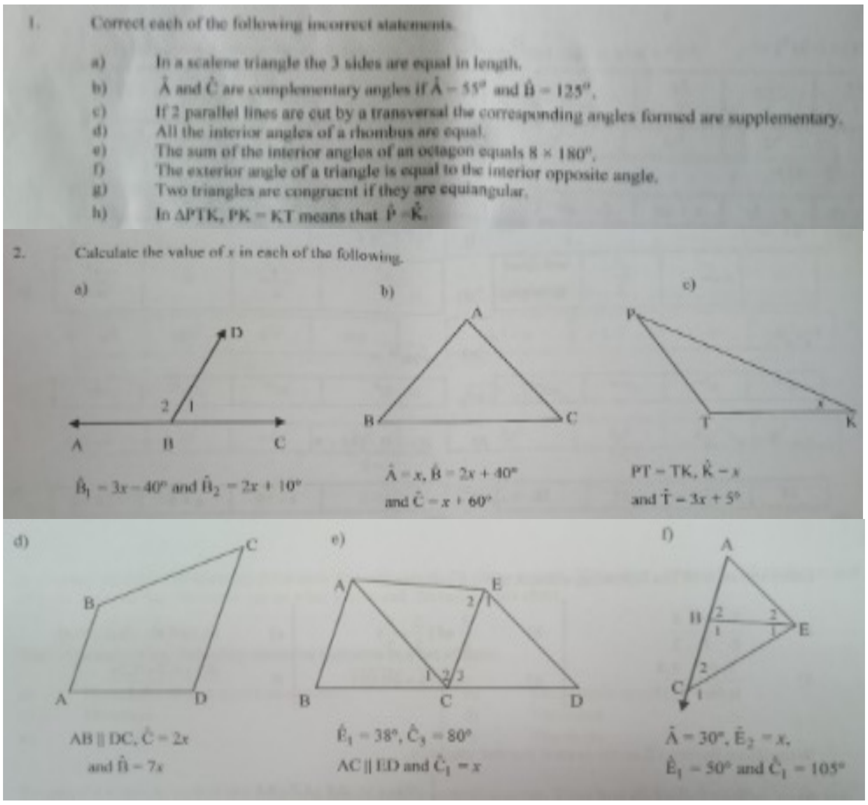

Figure 4.2: Task 2

Table 4.2: Teacher A (Document analyses of Tasks 1 and 2 below presents the analysis of Tasks 1 and 2 of Teacher A.

Table 4.2: Teacher A (Document analyses of Tasks 1 and 2)

TRU Math dimensions	Task 1	Task 2
The mathematics		
Focused and coherent	The knowledge was presented in a focused manner as it was goal-oriented. The goal in Task 1 was to teach learners how to solve linear algebraic equations. The goal was kept through all of Task 1 as all of the content focused on developing skills to solve linear algebraic equations (see Figure 4.1). The knowledge was presented in a coherent structure as it was offered in a logical way. The task showed a progression of learning as the questions escalated from easy to more difficult. For example, the first and last question of Task 1 were: $-2x - 1 = 11$ and $2x + 5 - (3x + 2) = 4x$ respectively.	The knowledge was presented in a focused manner as it was goal-oriented. The goal in Task 2 was to teach learners how to solve angles of 2-D geometric shapes. The goal was kept through all of Task 2 as all of the content focused on developing skills to solve angles of 2-D geometrical shapes (see Figure 4.2). The knowledge was presented in a coherent structure as it was offered in a logical way. Task 2 started by asking learners to recall theoretical rules of geometry angles and progressed to solving geometric problems involving unknown sides and angles in triangles and quadrilaterals using known properties and definitions.

TRU Math dimensions	Task 1	Task 2
Building meaningful connections	The task required learners to apply their prior knowledge of adding, subtracting, multiplying and dividing integers. The mathematics motivated learners to memorise concepts and use known procedures instead of building connections between concepts. Task 1 did not address connections between concepts of algebraic equations. The mathematics did not entirely support learners to develop mental representations as part of a bigger network of algebraic equations as Task 1 had only one main question, which was to solve x (see Figure 4.1). Task 1 did not require learners to make connections and comparisons, justify their own and other's ideas, make mathematical conjectures, test ideas or develop sense-making skills. Task 1 built understanding around big ideas as concepts were developed and linked to other concepts. For example, solving equations in algebra indicated to learners how algebra integrates with addition, subtraction, multiplication and division of integers.	There was evidence of connecting to prior experience and knowledge in Question 1 as learners had to correct incorrect statements by using previous knowledge. For example, in a scalene triangle, the three sides are equal in length (see Figure 4.2). Meaningful connections between procedures, concepts and contexts were promoted in Task 2, Question 2 as learners had to make comparisons, justify their own ideas, make mathematical conjectures and test ideas. For example, Question 2, number d, asked to calculate the value of x if $AB//CD, \hat{C} = 2x$ and $\hat{B} = 7x$. Task 2 built understanding around big ideas. Concepts were developed and linked to other concepts. For example, solving the geometric problems indicated to learners how geometry integrates with solving equations in algebra.
Engagement in key practices	The tasks included basic solving of algebraic equation concepts, but did not enhance creative thinking, decision making, or critical and logical thinking skills. There was no question that asked learners to analyse complex, real-life scenarios or to construct and use mathematical models to interpret and solve problems (see Figure 4.1).	The tasks included basic geometric concepts, but did not enhance creative thinking, decision making, or critical and logical thinking skills. No problems involving real-life scenarios were given in any of the questions of Task 2 (see Figure 4.2).

TRU Math dimensions	Task 1	Task 2
Cognitive demand		
Productive struggling	This task encouraged rote and superficial learning. Task 1 gave learners minimal opportunity to engage in conceptual understanding as the task involved constant repetition of information and rote learning. Although the activity escalated from easy to difficult questions, the procedures that would be applied to solve x remained the same. The questions focused on finding the correct answer by using procedures and required little mathematical understanding or engagement in mathematical practices. The task required following known procedures and solving routine problems without making new connections, for example, $-2x - 1 = 11$ (see Figure 4.1). Task 1 provided limited opportunities for learners to develop conceptual understanding as the task required learners to apply mathematical concepts and carry out mathematical procedures with precision and fluency. All the questions were similar; therefore, Task 1 did not engage learners in productive struggling through relevant, thought-provoking questions and problems that stimulate interest and elicit mathematical thinking. The only way learners would experience some productive struggling was by moving from easier questions to more difficult ones, for example, from 1c) $\frac{4x}{5} = -8$ to 1d) $\frac{3x+1}{2} = 5$.	Question 1 required recalling and reasoning of the angles of 2-D geometrical shapes. For example, Question 1 asked to correct the following statement: All the interior angles of a rhombus are equal (see Figure 4.2). Task 2, Question 2 gave learners the opportunity to engage in key practices such as problem solving and reasoning. Learners would need to construct and use mathematical models to interpret and solve problems in Question 2. Task 2, Question 2 required learners to build understanding and engage in mathematical practices by making connections, analysing information and drawing conclusions. For example, in Question 2f, learners had to calculate numerous angles before calculating x. Learners had to think conceptually to be able to calculate the value of x of the given figures. Task 2 allowed learners to make sense of some mathematical ideas as it involved conceptual understanding and promoted critical thinking and reasoning. Task 2, Question 2 helped learners to develop the meaning of mathematical concepts by the use of visual diagrams and symbols. This higher-level question required some degree of thinking as learners could not solve x mindlessly.

Table 4.3 below presents the summary of Tasks 1 and 2 of Teacher A.

Table 4.3: Teacher A (Summary of Tasks 1 and 2)

TRU Math dimensions	Task 1	Task 2
1. The mathematics		
1.1 Focused and coherent	Yes	Yes
1.2 Building meaningful connections	No	Yes
1.3 Engagement in key practices	No	No
2. Cognitive demand		
2.1 Productive struggling	No	Yes

4.5.2 Lesson observations

Teacher A was observed while teaching the topics of solving linear algebraic equations and solving angles of 2-D geometric shapes. There were 10 learners present in class during the first observation and 15 learners during the second observation. Learners sat in groups of four. The duration of each lesson was 46 minutes. The teacher used the whiteboard to teach. An introduction to the lesson was given in the form of the teacher writing examples on the whiteboard. After that, learners had the opportunity to complete their tasks. Both lessons ended by marking the lesson's task. In Lesson 1, the learners received a memo to mark their own work. In Lesson 2, the teacher read the answers to the learners while the learners marked their own tasks.

4.5.2.1 The mathematics

Focused and coherent

The content of Lessons 1 and 2 was focused and goal-oriented. The teacher's goal in Lesson 1 was to teach learners how to solve linear algebraic equations. He kept to his goal by doing two examples on the whiteboard, namely, $2x + 6 - (x + 2) = 4x$ and $\frac{x}{3} - 4 = -2$, and then the learners solved six linear equations (see Figure 4.1) which had different levels of difficulty, such as $x - 5 = -x + 1$ and $\frac{4x}{5} = -8$. The teacher's goal in Lesson 2 was to teach learners how to solve angles of 2-D geometrical shapes. He kept to his goal by doing one example on the whiteboard, namely, $\widehat{B_1} + \widehat{B_2} = 180°$, and then the learners had to correct eight incorrect statements about solving angles of 2-D geometrical shapes (see Figure 4.2) and solve the unknown sides and angles of six figures consisting of triangles and quadrilaterals (see Figure 4.2). Both these tasks had different levels of difficulty, for example: Correct the following incorrect statement: two triangles are congruent if they are equiangular; and: Solve ΔABC if $\hat{A} = x, \hat{B} = 2x + 40°$ and $\hat{C} = x + 60°$.

The content of the lessons was presented in a logical sequence. The content showed a progression of learning as the content in both lessons escalated from easy to more difficult. For example, in Lesson 1, the content of the lesson progressed from solving basic linear algebraic equations, such as $-2x - 1 = 11$, to solving more advanced linear algebraic equations, such as $\frac{3x+1}{2} = 5$. The content of

Lesson 2 was presented in a logical sequence as identifying errors required learners to think about angles of 2-D geometric shapes and then learners had to apply these properties by solving angles on a straight line and later solving angles of a quadrilateral geometric shape.

Building meaningful connections

There was referral to previous knowledge and concepts to first introduce linear algebraic equations and angles of 2-D geometric shapes in both lessons. Teacher A built new knowledge on prior knowledge in Lessons 1 and 2 when he referred to concepts which were discussed in the previous lessons. Expressions of fractions were used as prior knowledge in order to be able to complete the following equation, which the teacher explained on the whiteboard: $\frac{x}{3} - 4 = -2 \rightarrow \frac{x}{3} = -2 + 4 \rightarrow \frac{x}{3} = 2 \rightarrow x = 2 \times 3 \rightarrow x = 6$. The teacher explained that learners could get rid of fractions by getting rid of the denominator by multiplying with the denominator's inverse; thus, $\frac{1}{3}$ multiplied by 3 equals 1. He then emphasised the principle of comparison, which states that what is done on the one side of the equation must be done on the other side. In Lesson 2, he reminded learners of the angles associated with parallel lines and explained that one of the first steps was to identify the parallel lines. He then mentioned that corresponding angles are equal, alternate angles are equal and co-interior angles add up to 180°. He also used a visual representation in the form of the word FUN (see Figure 4.3) to clarify the different pairs of angles formed when working with parallel lines.

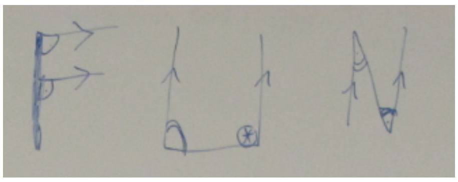

Figure 4.3: Explaining parallel lines and pairs of angles

Meaningful connections were built between procedures and concepts in Lessons 1 and 2 as the teacher introduced the topic of each lesson by doing examples on the whiteboard of problems similar to what the learners could expect in the task. Lessons 1 and 2 built understanding around big ideas as topics integrated with other topics. This is illustrated in the following examples: In Lesson 1, the teacher indicated to learners how solving equations in algebra integrates with addition, subtraction, multiplication and division of integers and order of operations. For example, the teacher said: *Two negatives make a positive*. In Lesson 2, the teacher indicated to learners how solving the geometric problems integrates with solving equations in algebra. For example, the teacher explained that learners should substitute the information from the sketch and form an algebraic equation in order to solve for x. The teacher showed the following on the whiteboard:

$\widehat{B_1} + \widehat{B_2} = 180° \quad (Supplementary\ angles)$

$(3x - 40°) + (2x + 10°) = 180°$

Engagement in key practices

The teacher taught important content of solving linear algebraic equations in Lesson 1. The first key practice was to procedurally move a number or variable from the one side to the other. For example, in the equation $-2x - 1 = 11$, the negative one was moved from the left side to the right side while changing the sign of the number from a negative to a positive. However, the teacher did not aim to make learners know the very reason behind every mathematical action, manipulation or understanding of why this rule governs this mathematical action. He did not emphasise properties of equation operation, such as that adding the same number to both sides of an equation keeps equality. The second key practice was to add like terms, for example: $2x - 3x + 5 - 2 \rightarrow -x + 3$. The last key practice was to simplify brackets first by expanding the brackets, for example: $2x + 5 - (3x + 2) \rightarrow 2x + 5 - 3x - 2$.

The teacher taught important content of solving angles of 2-D geometrical shapes in Lesson 2. The first key practice was to reason about the properties of angles of geometrical 2-D shapes. The second key practice was to solve angles by identifying geometrical proofs and applying them, such as angles on a straight line, interior angles of a triangle, exterior angles of a triangle, angles opposite to equal sides of a triangle and angles associated with parallel lines.

There were no opportunities for learners to apply solving linear algebraic equations or solving angles of 2-D geometrical shapes in real-life situations. The teacher also did not give examples where equations or geometry are applicable in real-life situations. Teacher A did not make the mathematics relevant to learners as he did not emphasise the association between situation and context.

4.5.2.2 Cognitive demand

Productive struggling

The teacher spoon-fed the learners in Lesson 1 as he told learners exactly what to do and when to do it. In Lesson 1, the teacher encouraged learners to learn procedural rules as he gave the learners steps to follow for solving algebraic equations. Learners had to memorise these steps and apply them in the right way. For example, in Lesson 1, Teacher A mentioned: *The first step to solving algebraic equations is to get* x *on the left-hand side and all the numbers on the right-hand side*. Learners were required to be intellectually active during Lesson 2 as they were asked to explain their reasoning for solving the problem and asked to justify their mathematical thinking. The geometric properties and theories were used to solve problems. For example, Teacher A mentioned the following: *Adjacent angles on a straight line add to 180°, because it is supplementary angles*.

Teacher A used very few probing questions to promote productive struggling during both lessons. The only examples of a probing questions are: *What do you have to do first?* (Lessons 1 and 2) and *What is the importance here?* (Lesson 2). Instead of using more probing, the teacher most of the times told learners exactly what to do. For example, in Lesson 1 the teacher said: *Remember at number g, take the five over to the left-hand side and make it a negative number. You have to times the minus into the brackets*. The teacher demonstrated in Lesson 2 how learners should calculate the sizes of angles in an equilateral and said: *You have to do it like this.*

Scaffolding

In Lesson 2, the teacher broke a problem up into manageable parts. For example, he said the following: *Look what they ask you. Draw what they ask and figure it out. What do you see?* The teacher did not have a good balance between providing assistance and leaving the learners in either of the lessons as he offered too much support. The teacher was eager to help learners and did not really leave learners to discover on their own. He told the learners exactly what to do, talked them through the tasks and sometimes even gave the answers. This is illustrated in the following examples from Lesson 1: The teacher told the learners to get all the similar variables on the same side instead of using scaffolding to guide learners to discover it for themselves. Teacher A sometimes helped a learner by taking the learner's pen in his own hand and showing the learner how to solve the problem in the task. In Lesson 1, the teacher said: *You have to put this in brackets. Use the steps given. In the second step you should times the brackets out.* In Lesson 2, the teacher gave instructions step-by-step: *Draw two parallel lines like train tracks. Draw the tracks at equal distance. Put in arrows like this. Then you should draw a transversal line like this. Use the following information to solve the problem. We indicate parallel lines with arrows and equal sizes with a little straight line. This is corresponding angles; therefore, it is an F of FUN. If this is an F, those angles are equal to each other.*

4.5.2.3 Access to mathematical content

Active participation

Learners could interact with the teacher, but not with each other. Discussions were initiated by the teacher; however, the discussions were kept short. The teacher remained the main source of information. Minimum opportunity was given to learners to ask questions about their struggles or to discuss their point of view. There was no opportunity for learners to explain their own methods or compare their methods to those of others. Class discussions did not involve thinking, representing, agreeing, disagreeing, or exchanging ideas. Teacher A used questioning as a technique to initiate class discussions. However, these questions did not invite learners to participate in class discussions as they were closed-ended questions. For example, in Lesson 1 the teacher asked: W*hat is on the left side? How many terms do you see here? How do I get the negative away? If 5x is equal to three, what will one be? What similar terms can you add or subtract?* In Lesson 2 the teacher asked: *What will the interior angles be of this triangle? What do we see is equal to each other?* Learners responded to the questions asked by the teacher. The answers that learners gave were short and did not lead to any deeper discussions into their understanding of the concept.

Equal access

The teacher provided individual assistance to any learner who struggled with a concept in both lessons. When learners asked for help by putting up their hand, the teacher immediately helped them without any hesitation. This is illustrated in the following examples from Lesson 1: *Yes, how can I help you? Are you still okay? Do you need help with that?* The following are examples from Lesson 2: *Okay, so let's look at the question. What question number are you at? Yes, I can look at your question.*

All learners were invited to participate in classroom discussions. For example, in Lesson 1, the teacher said: *I'd like someone to explain how the next step, and I will write it on the whiteboard. Who has an answer?* An example from Lesson 2 is: *Does anyone notice anything about the angles?* All learners were equally treated regardless of their race, dysfunctions, upbringings or personal characteristics. For example, the teacher said in Lesson 1: *Does anyone need help?* The following is an example from Lesson 2: *I want everyone to try.* Each learner's ideas were valued equally. For example, the teacher said the following in Lesson 1: *So, Euan, is that what you were saying?* Teacher A also gave recognition to learners' ideas. The following is an example from Lesson 2: *Gugu said six minus two is equal to four. Yes Quamo, that's it.*

4.5.2.4 Agency, authority and identity

Agency

The teacher talked the learners through the tasks and did not leave them to act on their own. Examples of this from Lesson 1 are: *Remember to add the similar variables. You have to expand the brackets first.* $7x$ *minus* $3x$ *equals* $4x$. *You cannot add* $3x$ *plus two. Minus eight times positive five equals negative 40 and not positive 40.* Examples of this from Lesson 2 are: *The interior angles will be 180 degrees together. If this is equal, that will be equal. The opposite angles will be equal to each other.* In the process of assisting the learners through the tasks, Teacher A asked questions and then answered the questions himself. For example, in Lesson 1 the teacher asked: *Can this be simplified?* Then he answered: *No, it is already in its simplest from.* An example of this from Lesson 2 is: *If the angles are 180 degrees together, we call it? Complementary angles. If the angles are 180 degrees together, we call it?* The teacher answered: *Supplementary angles.* Self-assessment was done during both lessons as learners marked their own tasks by the use of a memo.

Authority

Learners did not get an opportunity to demonstrate how knowledgeable they were as there were no opportunities to debate their ideas, solutions and strategies and they were not allowed to interact with each other. The teacher oversaw all communications that took place during both lessons. The teacher initiated all conversations and asked most of the questions. Learners' speech turns were short in both lessons. Little opportunity was given to learners to ask questions about their struggles or to discuss their point of view with each other or the teacher. There was no opportunity given in either of the lessons for learners to explain their ideas and reasons to each other. There were no examples of the teacher giving recognition for learners' ideas during either of the lessons.

Mathematical identity

The learners did not appear self-confident as they asked continuously for assistance. If learners got stuck, they immediately put their hands up and asked the teacher for help. The teacher motivated and encouraged learners by saying the following in Lesson 1: *Yes, well done. Alright. Okay, that's it. It doesn't look too difficult. Come on. You can do it.* In Lesson 2, the teacher said the following: *You did well. There you go. That's it. Good. Alright. Beautiful.* Learners were not encouraged to decide for themselves how to tackle problems and which methods to use. This is illustrated in the following examples from Lesson 1: *Underline the variable. All the x's must go to the one side and the numbers to the other side. First you are going to take the five over and then the x.* It is also illustrated in the following examples from Lesson 2: *Why did you do the question like that? Remember to solve the triangle first. Use the steps. Write them down. You used the correct steps.*

4.5.2.5 Uses of assessment

Soliciting learner thinking

Teacher A used questioning to solicit learner thinking. The only examples of this from Lesson 1 are: *What will you do? What will you do next?* The example from Lesson 2 is: *What is your understanding of this?* The teacher used the tasks to provoke learner thinking. Learner monitoring was done during each lesson as the teacher moved around in the classroom to check up on learners' work and to monitor the individual progress of each learner. However, the teacher did not ask learners why they had done problems in a certain way. Teacher A listened for the correct answer, instead of listening to the learners' thoughts and ideas. This is evident in the following examples from Lesson 1: *Number e is correct. No, the answer is not that. Can you see that you've done it wrong?* The examples from Lesson 2 are: *Number two is incorrect. You did the whole question wrong. Yes, that's the correct answer.*

Builds on learner ideas and misunderstandings

After marking the tasks, the teacher went through the class and gave feedback to each learner. Some misconceptions were addressed in the lessons. This is illustrated in the following examples from Lesson 1: *You should have taken minus five from the right hand-side to the left-hand side. You cannot subtract five from one* x. *You can only add like terms.* It is also illustrated in the following examples from Lesson 2: *Do you see that this is an isosceles triangle? That means that p is equal to the angle t. Therefore, change that part. Rather use interior angles of a triangle next time.* However, the teacher did not build on learners' ideas during instruction as his explanations were not based on learners' ideas.

Table 4.4 below presents the summary of Lessons 1 and 2 of Teacher A.

Table 4.4: Teacher A (Summary of Lessons 1 and 2)

TRU Math dimensions	Lesson 1	Lesson 2
1. The mathematics		
1.1 Focused and coherent	Yes	Yes
1.2 Building meaningful connections	Yes	Yes
1.3 Engagement in key practices	No	No
2. Cognitive demand		
2.1 Productive struggling	No	No
2.2 Scaffolding	No	No
3 Access to mathematical content		
3.1 Active participation	No	No
3.2 Equal access	Yes	Yes
4. Agency, authority and identity		
4.1 Agency	No	No
4.2 Authority	No	No
4.3 Mathematical identity	No	No
5. Uses of assessment		
5.1 Soliciting learner thinking	No	No
5.2 Building on learner ideas and misunderstandings	No	No

4.5.3 Post-observation interview

Through the interview, the teacher had the opportunity to make comments about his instructional practice. The teacher's words during the post-observation interview are given in italics. The data from the post-observation interview are now presented.

Table 4.5: Teacher A (Presentation of the data from the post-observation interview)

TRU Math dimensions	Teacher A's comments about his instructional practice
The mathematics	
Focused and coherent	Teacher A was of the opinion that *teaching with the main goal is for your learners to truly understand the work completely. Not to get through the curriculum, not to pass the time but to understand.* He did not mention anything about the content being focused or logically sequenced.
Building meaningful connections	Teacher A believed that he should *use something cool that you know your learners are familiar with and find interesting to connect to their prior knowledge to give them a safe base to attack the new problem.* He did not mention anything about building understanding around big ideas.
Engagement in key practices	Teacher A mentioned that he believed the essence of learning mathematics successfully was to *learn for understanding and not just to pass the grade. In reality all mathematics is connected in some interesting way.* Therefore, *I usually provide lessons and activities that are connected to real-life applications.* Moreover, *learners can learn so much from investigations.*
Cognitive demand	
Productive struggling	Teacher A mentioned that, when learners struggled with a problem, he would give them mental representations, suggestions and connections to prior knowledge and sometimes he stepped back with a simpler problem or sum. He did not mention anything about learners being intellectually active.
Scaffolding	For scaffolding, Teacher A mentioned that *if you are truly turned in to your learner's level, it will happen automatically.* The most effective way to teach mathematics is to *teach at the level of the learners in your class.* The teacher recommended that *when learners ask for answers or methods only give them clues or suggestions. Better yet answer them with a well thought out question. I use prompting in all of my lessons.* He did not mention anything about reducing the ZPD.
Access to mathematical content	
Active participation	Teacher A's view of active participation was that *class discussions is a good way to get the ball rolling or to start a new lesson with.* He said that he believed in *giving definite boundaries and control participation by every member of a class or group. I do not see myself as the source of all mathematical knowledge.*
Equal access	Teacher A's view of equal access was that *all learners are allowed to discuss their progress with me. I strongly feel that learners need to get one-to-one time with the teacher.* He also mentioned *a teacher must be flexible and accommodate all the learners in their class.*

TRU Math dimensions	Teacher A's comments about his instructional practice
Agency, authority and identity	
Agency	Teacher A mentioned the essence of learning mathematics successfully was *the willingness to work hard. I think it is important for learners to make sense of their own mathematical ideas. If your goal of your teaching is for learners to understand the work, then they have to make sense of the problem.*
Authority	Teacher A mentioned that he *encourages learners to explore mathematics on their own by asking questions to spark their curiosity and then letting them research it and give a presentation about it.* With regard to creating opportunities to demonstrate how knowledgeable learners are, the teacher said *if your learners can stand in front of a class or group and explain their thinking in solving a problem, then you as teacher achieved your goal of teaching for understanding.*
Mathematical identity	Teacher A explained *the importance of giving learners self-confidence to do mathematics as it is one of his fundamental priorities as a mathematics teacher.* Therefore, *I always try to motivate learners to keep at it in a problem*. He said that *he gives feedback after every assessment to motivate, encourage and challenge learners.* He failed to mention the importance of mathematics being enjoyable and learners acting independently.
Uses of assessment	
Soliciting learner thinking	Teacher A mentioned the importance of knowing one's learners. He mentioned the importance to *know their level, ability and interests.* He explained that this knowledge of one's learners *makes it easy to help a child.* He also explained that, when he wanted to gather information about learner thinking, he would ask learners questions to ensure understanding is achieved.
Building on learner ideas and misunderstandings	For addressing misunderstandings, he mentioned that he would *make corrections on incorrect answers [as it] helps find the fault or mistake in understanding.* Teacher A believed *when you spot a common mistake, stop the lesson and re-explain the specific sum on the board for the class in order to clear out any misunderstandings.* He did not mention anything about building on learners' ideas.

Table 4.5 below presents Teacher A's comments about his instructional practice. The interview data are, as with the observations, presented based on the TRU Math dimensions and dimension indicators as inclusion criteria for coding the data (see Table 4.1). Table 4.6 below presents the summary of the post-observation interview of Teacher A.

Table 4.6: Teacher A (Summary of the post-observation interview)

TRU Math dimensions	Post-observation interview
1. The mathematics	
1.1 Focused and coherent	No
1.2 Building meaningful connections	No
1.3 Engagement in key practices	Yes
2. Cognitive demand	
2.1 Productive struggling	No
2.2 Scaffolding	No

TRU Math dimensions	Post-observation interview
3 Access to mathematical content	
3.1 Active participation	Yes
3.2 Equal access	Yes
4. Agency, authority and identity	
4.1 Agency	Yes
4.2 Authority	Yes
4.3 Mathematical identity	No
5. Uses of assessment	
5.1 Soliciting learner thinking	Yes
5.2 Building on learner ideas and misunderstandings	No

4.6 Teacher B

In this section, I discuss the results from the document analysis, lesson observations, and post-observation interview of Teacher B.

4.6.1 Document analysis

The document analysis was based on mathematical tasks from learner textbooks which the teacher planned to use during the lessons. The teacher provided the tasks she planned to use just before each observation. The mathematical tasks of each lesson were analysed. The tasks used in both lessons were from the textbook Curro Mathematics 8 (Brombacher & Associates, 2015). The textbook was in the form of an e-book on each learner's tablet.

Task 1 was about adding and subtracting integers (see Figure 4.4). Question 1 involved addition and subtraction with integers. Ten questions were given in this question, requiring the learners to calculate expressions involving integers. Question 2 involved recognising and using the commutative, associative and distributive properties of addition and multiplication of integers. Seventeen equations were given in this question, requiring the learners to complete number equations involving integers. The last part of Task 1 was an investigation about consecutive negative integers.

Adding and subtracting integers

Revision

Exercise 2

1. Calculate.
   a) $3 + 7$
   b) $3 - 7$
   c) $-3 + 7$
   d) $-3 - 7$
   e) $-7 + 7$
   f) $3 + (-7)$
   g) $-3 + (-7)$
   h) $3 - (-7)$
   i) $-3 - (-7)$
   j) $-3 - (-7 + 5)$

2. Copy and complete.
   a) $25 - 40 = \square$
   b) $-32 - 82 = \square$
   c) $\square - 55 = -100$
   d) $-10 = -90 + \square$
   e) $34 - \square = -87$
   f) $42 = \square - 18$
   g) $-63 + \square = -1$
   h) $7 - (-3) = \square$
   i) $22 = \square - (-7)$
   j) $36 - \square = 18$
   k) $36 - \square = 80$
   l) $10 + (-2) - (-9) + (-7) = \square$
   m) $28 - \square + (-7) = 30$
   n) $5\frac{1}{4} + \left(-\frac{1}{2}\right) = \square$
   o) $-3{,}5 - (1{,}7) = \square$
   p) $-1{,}8 = \square - (-1{,}7)$
   q) $-12\frac{1}{5} + \square = -13$

Investigation: Consecutive negative integers

What do you notice about the answers when you add or subtract four consecutive negative integers?

1. Take for example: -10 ; -9 ; -8 ; -7.
   Now, systematically start to add and subtract these numbers in all the different possible ways. For example:
   - $(-10) + (-9) + (-8) + (-7)$
   - $(-10) - (-9) + (-8) + (-7)$
   - $(-10) - (-9) - (-8) + (-7)$etc.

   There are many more possibilities. Try them all.

2. Calculate the value of each expression.

3. Choose a different set of four consecutive negative integers and repeat this process.

Figure 4.4: Task 1

Task 2 involved multiplying and dividing integers as well as some adding and subtracting of integers (see Figure 4.5). Concepts learned in Task 1 were integrated in Task 2. Question 1 of Task 2 only required the learners to multiply and divide with integers. Question 2 of Task 2 consisted of calculations with integers involving all four operations, namely, addition, subtraction, multiplication and division, but also required learners to recognise and use the additive and multiplicative inverses for integers. Questions 3 and 4 involved substitutions where learners had to determine the value of integer expressions when the variable was given. Question 5 was a word problem about positive and negative numbers.

Exercise 6

1. Calculate.

a) 3×-7
b) -3×-7
c) $-5(-6)$
d) 8×-3
e) $(-4)(-6)$
f) $2(-3) \times 5$
g) $-4(2)(-2)$
h) $(-2)(-5)(-3)$
i) $3 \times (-5) \times (-2)$
j) $-2 \times (-7) \times 3$
k) $-5 \times 4 \times (-2)$
l) $-7 \times 0 \times 2$
m) $4 \times (-5) \times 1$
n) $(-6+2) \times (-4+9)$
o) $7 \times -3 + (-3)$
p) $-5 + (-7) \times 5$

2. Copy and complete.

a) $-1 \times \square = 2$
b) $\square \times 5 = -10$
c) $-1 \times -2 \times \square = 48$
d) $-1 + \square = 3$
e) $-7 - 2 = \square \times -1$
f) $-5 - \square = 3(-4)$
g) $-2(\square) = 14 - (-6)$
h) $-5 - (-5) = 102 \times \square$
i) $-4(5-7) = 10 + \square$
j) $\square = -2 - 4 \times 3$
k) $-6(\square + 7) = 12$
l) $(-1)(-2)(-5) = 5 \times \square$

3. Copy and complete the following to determine the value of $12 + 3n$ when $n = -5$

$$12 + 3n$$
$$= 12 + 3 \times -5$$
$$= 12 + \cdots$$

4. Determine the value of $3(n-5)$ when $n = -7$.

5. If a is a positive number and b is a negative number which of the following is the greatest: $a-b$, $b-a$, $a+b$, $-a-b$ or you cannot tell?

Figure 4.5: Task 2

Table 4.7 below presents the analysis of Tasks 1 and 2 of Teacher B.

Table 4.7: Teacher B (Document analyses of Tasks 1 and 2)

TRU Math dimensions	Task 1	Task 2
The mathematics		
Focused and coherent	The goal in Task 1 was to teach learners how to add and subtract integers. The goal was kept through all of Task 1 as all of the content focused on developing skills to teach learners how to add and subtract integers (see Figure 4.4). The knowledge was presented in a coherent structure and was offered in a logical way. The task showed a progression of learning as the questions in both tasks escalated from easy to more difficult. This is illustrated in the following examples: Question 1: $-3+7$; Question 2: $-63+\square=-1$; and the next question from the investigation: What do you notice about the answers when you add or subtract four consecutive negative integers? The questions themselves showed also a progression of learning. For example, in Task 1, Questions 2a and 2f respectively asked learners to copy and complete the following: $25-40=\ \square$ and $-12\frac{1}{5}+\square=-13$.	The goal in Task 2 was to teach learners how to multiply and divide integers. The goal was kept through all of Task 2 as all of the content focused on developing skills to multiply and divide integers (see Figure 4.5). The knowledge was presented in a coherent structure and was offered in a logical way. The task showed a progression of learning as the questions in both tasks escalated from easy to more difficult. An example of Question 1 is: 8×-3. An example of Question 2 is: $-7-2=\square\times-1$. Questions 3 and 4 involved substitutions with integer numbers and an example of the last question is: If a is a positive number and b is a negative number, which of the following is the greatest?
Building meaningful connections	Task 1 promoted meaningful connections between procedures, concepts and contexts of adding and subtracting integers as learners had to reason why procedures were done. For example, in the investigation, learners had to investigate what happens to four negative integers by testing different negative integers (see Figure 4.4). The mathematics supported learners to develop mental representations as part of a bigger network of integers. The investigation motivated learners to make connections and comparisons, justify their own ideas, make mathematical conjectures, test ideas and develop sense-making skills.	Meaningful connections between procedures, concepts and contexts were promoted in Task 2 as learners had to connect to prior experience and knowledge to solve the problems. For example, Question 2f asked to complete the following equation: $-5-\square=3(-4)$. Learners had to apply previous knowledge, such as solving equations and subtracting negative integers. Question 5 motivated learners to make connections and comparisons, justify their own ideas, make mathematical conjectures, test ideas and develop sense-making skills (see Figure 4.5). Concepts were developed and linked to other concepts. For example, involved multiplying and dividing integers as well as some adding and subtracting of integers.

TRU Math dimensions	Task 1	Task 2
Engagement in key practices	The tasks included basic adding and subtracting concepts, but did not enhance creative thinking, decision making, or critical and logical thinking skills. There were no opportunities to solve problems involving real-life situations (see Figure 4.4).	The tasks included basic adding, subtracting, multiplying and dividing concepts, but did not enhance creative thinking, decision making, or critical and logical thinking skills. No problems involving real-life scenarios were given in any of the questions of Task 2 (see Figure 4.5).
Cognitive demand		
Productive struggling	Task 1 gave learners the opportunity to engage in key practices such as problem solving and reasoning. Learners needed to construct and use mathematical models to interpret and solve problems in the investigation of Task 1. The investigation required the solving of problems in contexts involving multiple operations with integers. For example, Question 3 of the investigation asked learners to choose a different set of four consecutive negative integers and repeat the process used in Question 2. Task 1 allowed learners to make sense of mathematical ideas as it involved conceptual understanding and promoted critical thinking and reasoning. For example, Question 2n asked learners to complete the following: $5\frac{1}{4}+\left(-\frac{1}{2}\right)=\square$. The investigation in Task 1 helped learners to develop the meaning of consecutive negative integers. The investigation provided opportunities for calculation in a context of experimenting, conjecturing and testing conjectures. Task 1 engaged learners in productive struggling through relevant, thought-provoking questions and problems that stimulated their interest and elicited mathematical thinking as the investigation required learners to explain and justify their thinking thoroughly and clearly (see Figure 4.4).	Task 2 gave learners the opportunity to engage in key practices such as problem solving and reasoning. Learners needed to construct and use mathematical models to interpret and solve problems in Questions 3 to 5. For example, Question 3 asked learners to choose determine the value of $3(n-5)$ when $n=-7$. Task 2 allowed learners to make sense of some mathematical ideas as it involved conceptual understanding and promoted critical thinking and reasoning. For example, Question 2l asked learners to complete the following: $(-1)(-2)(-5)=5\times\square$. Task 2, Question 5 required learners to build understanding and engage in mathematical practices by making connections, analysing information and drawing conclusions. For example: If a is a positive number and b is a negative number, which of the following is the greatest? This higher-level task required some degree of thinking as learners could not solve the questions mindlessly (see Figure 4.5).

Table 4.8 below presents the summary of Tasks 1 and 2 of Teacher B.

Table 4.8: Teacher B (Summary of Tasks 1 and 2)

TRU Math dimensions	Task 1	Task 2
1. The mathematics		
1.1 Focused and coherent	Yes	Yes
1.2 Building meaningful connections	Yes	Yes
1.3 Engagement in key practices	No	No
2. Cognitive demand		
2.1 Productive struggling	Yes	Yes

4.6.2 Lesson observations

Teacher B was observed while teaching the topics of adding and subtracting integers and multiplying and dividing integers in expressions and equations. There were 11 learners present in class during the first observation and 23 learners during the second observation. Learners sat in rows of three. The duration of each lesson was 45 minutes. The teacher used the overhead projector and whiteboard to teach. Both lessons started by marking the previous day's task. In Lessons 1 and 2, the teacher read the answers to the learners while the learners marked their own tasks. She also asked learners to give their answers and explain their workings and mathematical thinking. In both lessons, the teacher walked through the class and checked every learner's homework. After marking, the teacher started with an introduction. An introduction to the lesson was given in the form of a PowerPoint presentation. She showed a PowerPoint presentation of an investigation before the learners had to complete the tasks in both lessons. If the learners did not finish the task, it was homework.

4.6.2.1 The mathematics

Focused and coherent

The content of Lessons 1 and 2 was focused and goal-oriented. The teacher's goal in Lesson 1 was to teach learners how to add and subtract integers. She kept to her goal by starting with previous concepts of what an integer number is and how integers are presented on a number line to first introduce learners to adding and subtracting integers, then doing three examples on the whiteboard, namely, $5 + -7$, $-2 - (-3 + 8)$ and $-52 + \square = -2$; then the learners did a task on calculations of adding and subtracting integers (see Figure 4.4) of different levels of difficulty such as calculate $-7 + 7$ and complete $5\frac{1}{4} + \left(-\frac{1}{2}\right) = \square$. The teacher's goal in Lesson 2 was to teach learners how to multiply and divide with integers. She kept to her goal by starting with an investigation (see Figure 4.6) referring to previous concepts on multiplying and dividing positive and negative numbers to first introduce learners to multiplying and dividing integers, then doing three examples on the whiteboard, namely, -5×-4, $-2\times-3\times-8$ and determine the value of $2(3-x)$ when $x=-3$, then the learners did a task on multiplying and dividing integers (see Figure 4.5) on different levels of difficulty such as calculate $-4(2)(-2)$ and complete $-6(\square+7)=12$.

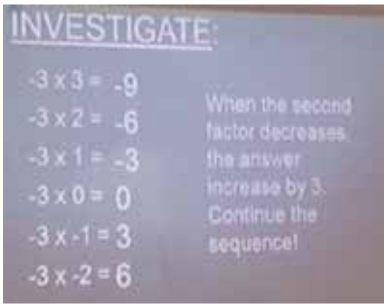

Figure 4.6: Investigation in Lesson 2

The content of the lessons was presented in a logical sequence. The content showed a progression of learning as the content in both lessons escalated from easy to more difficult, as well as from known to unknown. In Lesson 1, the content of the lesson progressed from simplifying basic integer expressions involving adding and subtracting, such as $3-7$, to solving more advanced equations involving adding and subtracting integers, such as $\square-55=-100$, and lastly an investigation was completed by adding or subtracting four consecutive negative integers. In Lesson 2, the content of the lesson progressed from simplifying basic integer expressions involving multiplying and subtracting integers such as 8×-3 to solving more advanced equations involving adding, subtracting, multiplying and dividing integers such as $-5-\square=3(-4)$ and lastly substituting integers into algebraic expressions such as determine the value of $3(n-5)$ when $n=-7$.

Building meaningful connections

Meaningful connections were built between procedures and concepts in Lessons 1 and 2, as the teacher introduced the topic of each lesson by doing examples on the whiteboard of problems similar to what the learners could expect in the task. Teacher B built new knowledge on prior knowledge in Lessons 1 and 2 when she referred to concepts which had been discussed in the previous lessons. She started by recapping on previous concepts learned. For example, she asked: *Who can tell me what did we do yesterday?* Learners had to think what they had learnt the previous day in class in order to link it to today's content. In Lesson 1, the teacher said: *So that is what we did yesterday. We are not going to do that today, but I wanted to go through that again so that you understand the logic behind the rule. Remember in mathematics, we had lots of rules, but there is always logic behind it. There is always someone who worked out and find prove of why this can always be the case or why is this rule always applicable.* In Lesson 2 she said: *Grade 8s, yesterday you wrote down in the summary that a negative number times a negative number gives you a positive number. We are going to use this concept to multiply and divide with integers. And yesterday we proved that when you multiply a positive number by a negative number the answer is a negative number.* She also asked the following: *Quickly tell me what we did yesterday?* And then wrote the following down: $+\times+=+, -\times-=+, -\times+=-$.

Lessons 1 and 2 built understanding around big ideas as topics integrated with other topics. In Lesson 1, the teacher indicated to learners how integers on a number line integrate with doing calculations with them such as adding and subtracting integers. For example, the teacher explained how learners could use a number line to calculate an expression involving integers. In Lesson 2, the teacher indicated to learners how substitution integrates with algebraic expressions. For example, the teacher explained that learners should substitute the variable with the integer in the algebraic expression. The teacher showed the following on the whiteboard: $2(3-b)$ when $b=-3 \rightarrow 2(3-(-3))$.

Engagement in key practices

The teacher taught important content, namely, adding and subtracting integers, in Lesson 1. The first key practice was to understanding the practical principle of positive and negative numbers by drawing the integers on a number line or using the standard rule: $-(-a) = +a$. The standard rule shows that the opposite of a is $-a$ on the number line and the opposite of $-a$ is a; therefore, the rule states $-(-a)=a$. The next key practice was to visualise positive and negative integers with the use of a number line. The last key practice was to study and understand what happens to consecutive negative numbers. The teacher taught important content of multiplying and dividing with integers in Lesson 2. The first key practice was to understand the rules of multiplication and division, such as the product or quotient of a positive integer and a negative integer is a negative integer, and the product or quotient of two negative integers or two positive integers is a positive integer. The second key practice was to apply the rules of multiplication and division. The last key practice was to support learners in developing their substitution skills.

There were no opportunities for learners to apply integers in real-life situations. The teacher also did not give examples where equations or geometry are applicable in real-life situations. Teacher B did not make the mathematics relevant to learners as she did not emphasise the association between situation and context.

4.6.2.2 Cognitive demand

Productive struggling

In Lesson 1, the teacher encouraged learners to learn procedural rules as she gave the learners steps to follow for adding and subtracting integers. Learners had to memorise these steps and apply them in the correct way. For example, in Lesson 1, the teacher mentioned: *You can use the number line as a model to help you visualise adding and subtracting of positive and negative integers. To add integers having the same sign, keep the same sign and add the value of each number. To add integers with different signs, keep the sign of the number with the largest value and subtract the smallest value from the largest. Subtract an integer by adding its opposite.* The teacher repeated the following several times during Lesson 2: *Positive times a negative is a negative.* Learners were required to be intellectually active during Lesson 2 as they were asked to explain their reasoning for solving the problem and to justify their mathematical thinking. Lesson 2 started with an investigation (see Figure 4.6) where the teacher showed: $-3 \times 3, -3 \times 2, -3 \times 1$ and -3×0 to the learners. The teacher then asked learners what they noticed. The class agreed that there was a visible pattern between the answers. When the second factor decreases with one whole number, the product of the answer increases with three as you multiply with three more than the last time. The learners were then asked to determine the next answers of: $-3 \times -1; -3 \times -2$. The class came to the conclusion that a negative number times a negative number equals a positive number and a negative number times a positive number equals a negative number.

Scaffolding

The teacher broke problems up into manageable parts in both lessons. For example, in Lesson 1, she said the following: *Observe what happens with the four consecutive numbers. Notice the difference between these two questions. Remember to use the number line.* In Lesson 2, she said the following: *Remember to use the order of operations. Check if the left side is equal to the right side when completing the equations. Compare the two questions. Explain to me what you see.* The teacher had a good balance between providing assistance and leaving the learners in both lessons as she left learners to discover on their own. In Lesson 1, she mentioned the following: *You need to do it on your own. I can't give you all the answers.* The teacher offered support throughout the lessons and guided learners through the tasks. Learners completed the tasks on their own and could ask for help when they got stuck. The teacher did not give straightforward answers; instead, she used questioning to guide learners through the problem. For example, in Lesson 1 she said: What *else can you tell me? What can you see? Does it increase or decrease?* In Lesson 2 she said: *What can you remember? And then I multiply by what? What will the next one be? What assumption can we make?*

4.6.2.3 Access to mathematical content

Active participation

There was participation in class discussions by learners in Lessons 1 and 2. Learners could interact with the teacher, but not with each other. Discussions were initiated by the teacher and then she invited the learners to contribute. The teacher maintained discussions and allowed learners to become sources of information. The teacher asked all the questions and learners answered. This is illustrated in the following examples in Lesson 1: *Grade 8s, is there a difference between them? Do they all say the same thing?* It is also illustrated in the following examples in Lesson 2: *Can you tell us why you disagree with that?* In Lesson 2, the teacher asked the following: *Can you see that I am multiplying with integers, 3, 2, 1 and they get smaller 0, -1, -2? I start to multiply a negative number with a positive, so what will the answer be?* Learner answered: *The answer here will be negative nine.* The teacher asked: *And then the next one?* Learner answered: *Three times negative two is negative six.* The teacher then said the following: *Before we continue here, let's first look at the pattern. What pattern do you see here? -9, -6, -3. Susan?* Learner answered: *They are all getting bigger by three. What will the next one be then? 0,* *3, 6. When you look at this. A negative number multiplied by a negative number gives you a positive number.* Learners were not afraid to ask or answer questions. Learners answered voluntarily when the teacher asked questions.

Equal access

The teacher walked through the class continuously and answered any questions that the learners may have had. All learners had the opportunity to ask questions and to discuss problems they struggled with. The teacher provided individual assistance to any learner who struggled with a concept in both lessons. When learners asked for help by putting up their hand or calling the teacher, the teacher immediately helped them without any hesitation. This is illustrated in the following examples from Lesson 1: *What's wrong? Are you stuck? Can I help?* It is also illustrated in the following examples from Lesson 2: *Yes, I will look at your answer. What did you do here? Anyone else?* The teacher selected learners to give their answers and she selected a different learner each time. The teacher gave all learners the opportunity to answer questions during discussions. For example, in Lesson 1, she said: *No, someone else? You already had a turn to give your answer.* She called learners out by name and gave everyone a fair chance to participate. Teacher B made sure that every learner concentrated while she explained. For example, in Lesson 2, she asked several times: *Are you with me? Grade 8s, are you managing? Who is still busy?*

4.6.2.4 Agency, authority and identity

Agency

The teacher created opportunities for learners to be involved, act on and be responsible for all aspects of their own learning by guiding them through problems. Teacher B gave learners the opportunity to tackle problems individually through the following discourse. For example, in Lesson 1 she said: C*ompare answers. Predict some of the answers when you work with a different set of four consecutive negative integers. Choose a new set of four consecutive integers and test conjectures.* In Lesson 2 she said: *Can you explain and justify your observation? Will your rule always work, sometimes work or only for special cases? Do the four consecutive integers all must be negative? Explore more.* Teacher B gave learners the opportunity to think about the problems as she paused a few seconds after she asked a question. She also gave learners the opportunity to answer her questions. There was no visible form of self-assessment in either of the lessons.

Authority

Learners got the opportunity to demonstrate how knowledgeable they were by sharing their ideas at any time throughout the lessons. The teacher gave learners the opportunity to debate each other's ideas, solutions and strategies. When learners expressed their ideas to the teacher, she asked the other learners' opinion on that. This is evident in the following examples: *Who agrees?* (Lessons 1 and 2) *Who says his answer is correct?* (Lesson 2). The teacher gave recognition for learners' ideas. This is evident in the following examples from Lesson 1: *Does anybody see what Jacob said? Who said it will be equal to negative nine? Yes, Tracey gave the correct answer*. It is also evident in the following examples from Lesson 2: *Grace is that what you said? Okay, so Ethan asked a very important question? Did you hear what he said? Who can give me another example where this is also true?*

Mathematical identity

Learners were promoted to act independently as the teacher encouraged them to decide for themselves how to tackle problems and which methods to use. This is illustrated in the following examples from Lesson 1: *Yes, it is true. No, think again. Yes, that's right.* Examples from Lesson 2 are: *You don't have to do it exactly like I did. Yes, you are allowed to multiply the integers first and then determine the sign before the answer.* Learners appeared self-confident as they did not ask for assistance throughout the lessons. Learners were motivated and eager to work on the task. The teacher did not praise the learners' ideas throughout any of the lessons.

4.6.2.5 Uses of assessment

Soliciting learner thinking

The teacher used questioning to solicit learner thinking during both lessons. For example, in Lesson 1 she asked: *What did you get? What else did you get? What do I need to do next? Do you know how to calculate that?* In Lesson 2 she asked: *How many can you see here? What did you get for number g? If this is the problem,* *what will the answer be? Can I multiply? What operation I am I doing here? Let's have a quick look here. What can you see? What is the same? Aden?* Learner monitoring was done during each lesson as the teacher moved around in the classroom to check up on learners' work and to monitor the individual progress of each learner. The teacher marked every learner's book during the lessons at the learners' tables. She went through their previous work and commented on that. The teacher used questioning and show of hands to estimate whether the learners understood the classwork. *Who else got minus 48? Yes, it is right.* All learners put up their hands to show if they got the correct answer. Teacher B listened to learners' thinking and responded to it. This was evident in Lesson 1: The teacher asked: *Do you understand?* The learner answered: *Yes, to get the middle value, you have to subtract 18 from 36.* The teacher responded with: *Are you sure? Did you test it?* Learner said: *No, I didn't test it yet.* The teacher said: *Okay, quickly test it and let me know what you got.*

Building on learner ideas and misunderstandings

At the beginning of each lesson, the teacher marked the previous day's homework. When the teacher saw that everyone struggled with a concept, she stopped the class and explained again. For example, in Lesson 2 she said: *Grade 8s just be careful with the homework. In between the multiplication sums they also put in one or two pluses. So they mix multiplication and addition. What's the most important? You're going to do the multiplication first and then add.* Misconceptions were addressed in both lessons. This is illustrated in the following examples: In Lesson 1, the teacher asked a specific learner the following, referring to a learner's answer in her workbook: *Five plus negative five gives you*? The teacher paused and waited until the learner answered. The learner answered: *Zero.* The teacher responded to the learner's answer: *Yes, zero and not negative 10.* In Lesson 2, the teacher asked a specific learner the following, referring to a learner's answer in her workbook: *Three times negative three is equal to what?* The learner answered: *Negative six.* The teacher responded to the learner's answer: *No, added the numbers instead of multiplying them. Try again. Three times negative three is equal to what? The learner answered: Oh, it is equal to negative nine.* In the introduction of Lesson 2, the teacher showed different ways to multiply with integers which helped learners to prevent any misunderstandings. For example, she converted the following: *Do you* *have to multiple or subtract at this example?* $(-2) - 8$. *Look at the difference. If I had* $-2 - 8$, *it is the same as* $(-2) - 8$. *Those brackets are useless because there is a negative number in between the numbers. But if it was* $(-2)(-8)$ *then it is multiplication. Can you see the difference?*

Table 4.9 below presents the summary of Lessons 1 and 2 of Teacher B.

Table 4.9: Teacher B (Summary of Lessons 1 and 2)

TRU Math dimensions	Lesson 1	Lesson 2
1. The mathematics		
1.1 Focused and coherent	Yes	Yes
1.2 Building meaningful connections	Yes	Yes
1.3 Engagement in key practices	No	No
2. Cognitive demand		
2.1 Productive struggling	No	Yes
2.2 Scaffolding	Yes	Yes
3 Access to mathematical content		
3.1 Active participation	Yes	Yes
3.2 Equal access	Yes	Yes
4. Agency, authority and identity		
4.1 Agency	Yes	Yes
4.2 Authority	Yes	Yes
4.3 Mathematical identity	Yes	Yes
5. Uses of assessment		
5.1 Soliciting learner thinking	Yes	Yes
5.2 Building on learner ideas and misunderstandings	Yes	Yes

4.6.3 Post-observation interview

Through the interview, the teacher had the opportunity to make comments about her instructional practice. The teacher's words during the post-observation interview are given in italics. The data from the post-observation interview is now presented.

Table 4.10 below presents Teacher B's comments about her instructional practice. The interview data are, as with the observations, presented based on the TRU Math dimensions and dimension indicators as inclusion criteria for coding the data (see Table 4.1).

Table 4.10: Teacher B (Presentation of the data from the post-observation interview)

TRU Math dimensions	Teacher B's comments about her instructional practice
The mathematics	
Focused and coherent	Teacher B believed that *content should be presented in such a way that it makes sense for the learners.* Therefore, *if the content is presented in a logical order it will make more sense to them.* She also mentioned that *the content in a lesson should continuously focus on the main teaching*

	goal. Moreover, *before you start with a lesson you should know what the goal is and stick to it.*
Building meaningful connections	Teacher B did not mention anything about building new knowledge on prior knowledge and experience or building understanding around big ideas.
Engagement in key practices	Teacher B believed that she *encourages critical thinking and problem solving skills.* She added that *to apply mathematics in the real world is problem solving and you must think critically to achieve this most of the time.*
Cognitive demand	
Productive struggling	Teacher B stated that the essence of learning mathematics successfully was *understanding the logic behind the rule. It's important to give learners enough opportunities to improve and better their understanding.* She said that she did not believe in spoon-feeding. *This is the only way they can master mathematics, but the teacher must lead them and help them.* Teacher B stated that the essence of learning mathematics successfully was *learning for understanding.*
Scaffolding	Teacher B believed that she *explains the work properly and move around the classroom to give guidance and share ideas on how to find solutions.* For ZPD, the teacher mentioned that *you need to start with easy concepts.* She further mentioned that *it is important that learners must discover and learn for themselves.* She added that *this is the only way they can master mathematics, but the teacher must lead them and help them.*
Access to mathematical content	
Active participation	Teacher B mentioned that it was important for learners to ask questions. She believed that *discussion is a good way to get the ball rolling or to start a new lesson with.* She did not mention anything about learners participating actively in class discussions.
Equal access	Teacher B mentioned that she *wants to help the weaker learners as well.* She further said that *everyone should be able to feel like they are able to do mathematics.*
Agency, authority and identity	
Agency	According to Teacher B, *it is important that learners must discover and learn for* themselves. Teacher B's view of teaching was *to always try to let learners discover for themselves*. She mentioned that she could encourage learners more to explore, experiment and discover mathematics on their own. She added that she *thinks it is important for learners to justify their ideas, because that is how you know learners make sense of it, completely understands it and can't be forgotten.*
Authority	Teacher B's view was that *learners have to be curious of their friends.* She further said that *when learners work together with others they lead each other to better ideas and understanding.* Moreover, *learners can also sometimes explain things better to each other.* She felt that *she always encourages questions.*
Mathematical identity	Teacher B mentioned that *learners need to have confidence in order to do their best.* Therefore, *I want all learners in the class to have self-confidence.* She said that *the main goal of providing feedback to learners is to motivate and encourage them.* She further believed *that many methods are good for learners.* She further believed that *you need to teach all the possible methods to the learners.* Teacher B's view was that *learners have to be proud of their own method.*
Uses of assessment	
Soliciting learner thinking	Teacher B mentioned that she *tries to figure out what the learners understand and not understand by asking questions.*
Building on learner ideas and misunderstandings	Teacher B did not mention anything about building on learners' ideas or addressing emerging misunderstandings.

Table 4.11 below presents the summary of the post-observation interview of Teacher B.

Table 4.11: Teacher B (Summary of the post-observation interview)

TRU Math dimensions	Post-observation interview
1. The mathematics	
1.1 Focused and coherent	Yes
1.2 Building meaningful connections	No
1.3 Engagement in key practices	Yes
2. Cognitive demand	
2.1 Productive struggling	Yes
2.2 Scaffolding	Yes
3. Access to mathematical content	
3.1 Active participation	No
3.2 Equal access	Yes
4. Agency, authority and identity	
4.1 Agency	Yes
4.2 Authority	Yes
4.3 Mathematical identity	Yes
5. Uses of assessment	
5.1 Soliciting learner thinking	Yes
5.2 Building on learner ideas and misunderstandings	No

4.7 Teacher C

In this section, I discuss the results from the document analysis, lesson observations and post-observation interview of Teacher C.

4.7.1 Document analysis

The document analysis was based on mathematical tasks from learner textbooks that the teacher planned to use during the lessons. The teacher provided the tasks she planned to use just before each observation. The mathematical tasks of the second lesson were analysed as the first lesson did not include any documentary sources. In Lesson 1, learners were not required to complete any mathematical tasks, they only had to copy the teacher's notes from the PowerPoint presentation. The tasks used in the second lesson were from the textbook Curro Mathematics 8 (Brombacher & Associates, 2015). The textbook was in the form of an e-book on each learner's tablet.

The topic of Teacher C's first lesson was terminology of algebraic concepts and the second lesson was about writing algebraic expressions in shorthand. Task 1 involved recognising and interpreting conventions for writing algebraic expressions, identifying and classifying like and unlike terms in algebraic expressions, and recognising and identifying coefficients and exponents in algebraic expressions. Task 1 consisted of six questions (see Figure 4.7).

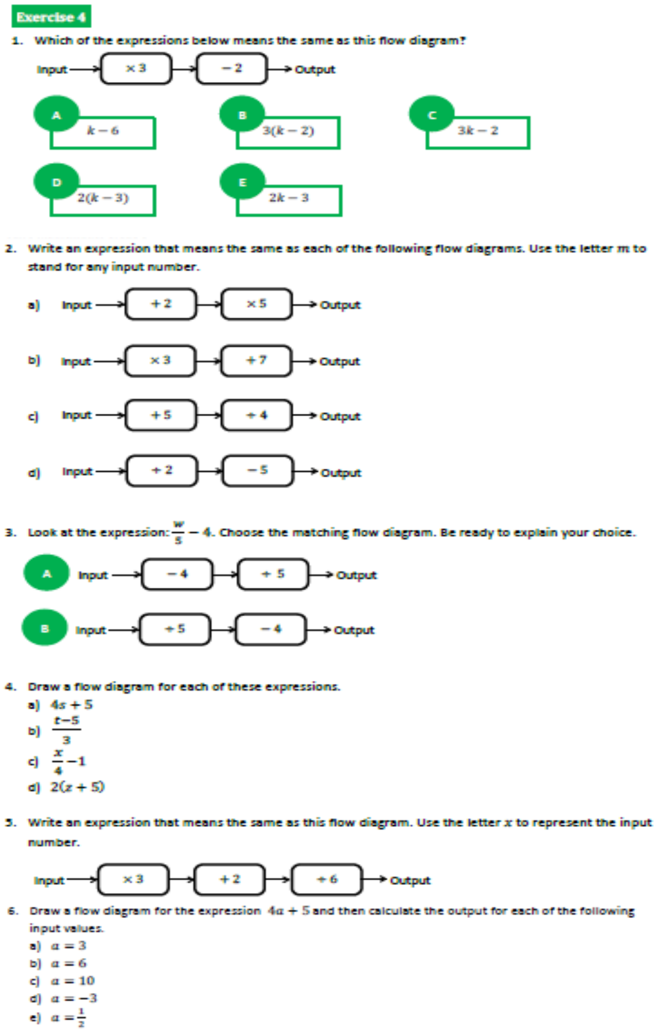

Figure 4.7: Task 1

Table 4.12 below presents the analysis of Task 1 of Teacher C.

Table 4.12: Teacher C (Document analysis of Task 1)

TRU Math dimensions	Task 1
The mathematics	
Focused and coherent	The goal in Task 1 was to teach learners how to write algebraic expressions in shorthand. The goal was kept through all of Task 1 as all of the content focused on developing skills to teach learners how to write algebraic expressions in shorthand (see Figure 4.7). The knowledge was presented in a coherent structure as it was offered in a logical way. The task showed a progression of learning as it escalated from easy to more difficult. For example, Task 1 started with: Which of the expressions below means the same as this flow diagram? and ended with: Draw a flow diagram for the expression $4a + 5$ and then calculate the output for each of the following input values.

TRU Math dimensions	Task 1
Building meaningful connections	Meaningful connections between procedures, concepts and contexts were promoted in Task 1 as learners had to make comparisons, justify their own ideas, make mathematical conjectures and test ideas. For example, Question 3 asked learners to look at the expression $\frac{w}{5} - 4$ and to choose the matching flow diagram. Learners were also required to explain their choice. There was evidence of connecting to prior experience and knowledge in Question 1 as learners had to think about previous concepts they have learned about algebraic expressions. For example: Which of the expressions below means the same as this flow diagram? (see Figure 4.7). The mathematics supported learners to develop mental representations as part of a bigger network as concepts were developed and linked to other concepts. For example, writing the algebraic expressions indicated to learners how algebra integrates with order of operations in algebra.
Engagement in key practices	The tasks included basic algebraic expressions concepts, but did not enhance creative thinking, decision making, or critical and logical thinking skills. No problems involving real-life scenarios were given in any of the questions of Task 1 (see Figure 4.7).
Cognitive demand	
Productive struggling	Task 1 gave learners the opportunity to engage in key practices such as problem solving and reasoning. For example, the first question of Task 1 asked: Which of the expressions below means the same as the flow diagram? The last question in Task 1 asked learners to draw a flow diagram for the expression $4a + 5$ and then to calculate the output for each of the following input values. Learners had to construct and use mathematical models to interpret and solve problems in Questions 4 and 6. Question 4 for example asked the following: Draw a flow diagram for each of these expressions, while Question 6 asked to draw a flow diagram for the expression $4a + 5$ and then calculate the output for each of the following inputs (see Figure 4.7). Task 1 required learners to build understanding and engage in mathematical practices by making connections, analysing information and drawing conclusions. For example, Question 6 asked the following: Draw a flow diagram for the expression $4a + 5$ and then calculate the output for each of the following input values. Question 4 helped learners to develop the meaning of mathematical concepts by the use of visual diagrams and symbols. Learners also had to think conceptually to write an expression from the flow diagram. This higher-level questions required some degree of thinking as learners could not solve it mindlessly. Question 3 gave learners the opportunity to construct viable arguments to support their own reasoning as the question required the learners to explain their choice (see Figure 4.7).

Table 4.13 below presents the summary of Task 1 of Teacher C.

Table 4.13: Teacher C (Summary of Task 1)

TRU Math dimensions	Task 1
1. The mathematics	
1.1 Focused and coherent	Yes
1.2 Building meaningful connections	Yes
1.3 Engagement in key practices	No
2. Cognitive demand	
2.1 Productive struggling	Yes

4.7.2 Lesson observations

Teacher C was observed while teaching the terminology of algebraic concepts and writing algebraic expressions in shorthand. There were 23 learners present in class during the first observation and 15 learners during the second observation. Learners sat in rows of three. The duration of each lesson was 47 minutes. The teacher used the overhead projector to teach in Lesson 1. Lesson 1 was given in the form of a PowerPoint presentation. The teacher used the whiteboard to teach in Lesson 2. Only Lesson 2 required learners to complete a task.

4.7.2.1 The mathematics

Focused and coherent

The content of Lessons 1 and 2 was focused and coherent. The teacher's goal in Lesson 1 was to teach learners the terminology of algebraic concepts. She kept to her goal in Lesson 1 as she went through the terminology and definitions of variables, constants, terms, algebraic expressions, adding and subtracting with integers and multiplying and dividing with integers. For example, she read the following definition on the PowerPoint presentation: *An expression in algebra is a collection of quantities made up of constants and variables, linked by operations and not including an equal sign.* The teacher's goal in Lesson 2 was to teach learners how to write algebraic expressions in shorthand. She kept to her goal by writing three examples of writing an algebraic expression in a flow diagram on the whiteboard, namely, $Input \rightarrow \times 3 \rightarrow +4 \rightarrow output,\ Input \rightarrow +4 \rightarrow \div\, 3 \rightarrow output,$ $Input \rightarrow +4 \rightarrow \times 3 \rightarrow output$ and one example of drawing a flow diagram from an algebraic expression $\frac{2(x+3)}{4}$, and then the learners worked through six questions involving algebraic expressions and flow diagrams (see Figure 4.7) on different levels of difficulty, such as: write an expression that means the same as the following diagram: $input \rightarrow +5 \rightarrow \div\, 4 \rightarrow output$ and: draw a flow diagram for $\frac{x}{4} - 1$. However, there was no referral to any previous knowledge or concepts to first introduce the terminology of algebraic concepts or writing algebraic expressions in shorthand.

The content of Lesson 2 was presented in a logical sequence. The teacher introduced the topic by doing examples on the whiteboard of problems similar to what the learners could expect in the task. The content showed a progression of learning as the content in both lessons escalated from easy to more difficult. The content of the lesson progressed from identifying a flow diagram's algebraic expression to writing an algebraic expression from a flow diagram, $Input \rightarrow \times 3 \rightarrow +4 \rightarrow output$, to drawing a flow diagram from an algebraic expression such as $\frac{2(x+3)}{4}$. After that, learners had the opportunity to apply what they had just learned by completing the task. In Lesson 1, the teacher talked all the time and the learners were not required to complete a task on what was learned about the topic that was presented in the lesson.

Building meaningful connections

Meaningful connections were built between procedures and concepts in Lessons 1 and 2. The teacher continuously recapped on any previous concepts learned throughout the lessons. For example, in Lesson 1 she said the following: *Those variables on the top can change. If I for example, tell you to do the substitution. You have heard of substitution before, we did it in previous exercises. Where I give you a value for x and I tell you, for example x needs to be equal to three. Then everywhere you see a little x in your little sum you put a little three and you recalculate it. That's why they can change to whatever we want them to be.* In Lesson 2, she said the following: *Now I'm going to show you how to use the guys we did yesterday and turning them into an expression. For example, we worked with input, then we had a little calculation that had to be done. Maybe two. And we had an output. Yesterday we did the flow diagrams and we did the formulas we had to compile. Now we are going to use stuff like that to make our own algebraic expression.*

The teacher connected concepts to previous concepts throughout both lessons. She used examples and also reminded learners what they did in primary school, in Grade 7, the first term, the previous week, the previous day and in previous examples in class. For example, in Lesson 2, she said: *Remember here* (pointing to example one) *you could choose your own input.* In Lesson 1, Teacher C made connections between concepts by comparing variables and numbers to each other. For example, when she explained adding variables she said: *In the same way, we can add* *numbers,* $4 + 4 + 4 + 4 + 4 + 4 = 4 \times 6$*, we can add variables. We call* $4 + 4 + 4 + 4 + 4 + 4$ *the longhand and* 4×6 *the shorthand. You did this before. You took four and you wrote in out six times. Therefore,* $x + x + x + x + x + x = x \times 6$ *. Now we got six* x*'s. How many times did we write out the* x*? Six times, because we write it down as six times* x*, we can also just say* $6x$*. that is why it is multiplication when it is stuck together.* Although the teacher connected concepts to previous knowledge, she did not show it visually. She only talked through it. For example, in Lesson 2, she said: *Now you are used to having values that you had to put in the output with your little table. You remember that. The table that had a one, a two, a three, a four or a minus five and you have to pop it into to get the answer. Now instead of having all those little values. You are going to have unknown variables.*

Lessons 1 and 2 built understanding around big ideas as topics integrated with other topics. This is illustrated in the following examples: In Lesson 1 the teacher indicated to learners how adding, subtracting, multiplying and dividing integers are integrated with algebra and exponents. For example, the teacher explained the following: *In the same way, we can multiply numbers we can multiply variables.* For example, $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$. So*, if you multiple a number by itself write is as a power. How many times did this four multiplied it by itself? Six times. Now we can do the same with variables.* $x \times x \times x \times x \times x \times x = x^6$*. How many times did the* x *times it by itself? Six times. So, we write is as the exponent six.* In Lesson 2 the teacher indicated to learners how flow diagrams integrate with algebraic expressions. For example, the teacher explained: *You are going to get an exercise where you have to make up your own formulas given the flow diagram. But they can also reverse it. They can give you a formula and you've going have to figure out how the order was.*

Engagement in key practices

The teacher taught important content of terminology of algebraic concepts in Lesson 1. The key practice was to understand that algebra is the study of mathematical symbols and the rules for manipulating these symbols. The teacher taught important content of algebraic expressions in shorthand in Lesson 2. The first key practice was to understand that algebra is a formal symbolic language which includes numbers, variables and operators. The second key practice was to see algebraic expressions as a generalised form of numeric expressions. The next key practice was to interpret algebraic expressions. The last key practice was to promote a meaningful understanding of algebraic expressions. There were no opportunities for learners to apply terminology of algebraic concepts or algebraic expressions in shorthand in real-life situations. The teacher also did not give examples where equations or geometry are applicable in real-life situations. Teacher C did not make the mathematics relevant to learners as she did not emphasise the association between situation and context.

4.7.2.2 Cognitive demand

Productive struggling

The teacher did not expect anything from the learners in Lesson 1 as she spoon-fed the learners. Learners were not required to be intellectually active. Lesson 1 required learners to sit back and listen. For example, when she explained multiplying with variables, she said the following, without any pauses: *Variables are written in an alphabetical order in a product. Now, what is a product again? When we multiply stuff to each other. If we multiply certain variables or unknown alphabet letters, I will write it in alphabetical order:* $\mathrm{a} \times \mathrm{b} \times \mathrm{c} = \mathrm{abc}$*. It is not wrong when you write* cba*, it will be marked correctly. But you have to write in alphabetical order to be neat and nice. Constants always have to come before the variable if we write it. We are not going to say* $x6$*, we are going to say* $6x$*. For example,* $3 \times b$ *will be* $3b$*.*

Lesson 2 encouraged very little active engagement as learners did not get an opportunity to debate their ideas, solutions and strategies. For example, she asked the following questions continuously: *What is an algebraic expression again? Who can take a guess? In your normal words, what do you remember of what I just taught you about what is algebraic expression? What does it consist of? What do you put together?* Then Teacher C paused for a bit, but no one answered. She answered her own questions with the following: *Constants and variables.* Then she asked: *And what do we do with them?* Then the teacher responded to her own question again: *You put them together. How?* She did not give learners an opportunity to answer. She immediately answered her own question again: *Dividing, multiplying, adding,* *subtracting, putting them together and making a sum.* And then she said: *That is an algebraic expression. Now I want to do a few examples with you.*

Scaffolding

The teacher did not have a good balance between aiding and leaving the learners in either of the lessons as she offered too much support. The teacher was eager to help learners and did not leave learners to discover on their own. Teacher C spoon-fed the learners. For example, in Lesson 1 she said the following without any pauses: *Just to go back, do you see the difference? Six x with the big six is when you are adding and when you are multiplying it is the power. Remember what I told you with exponents. When you multiply with exponents, what do I do with the little exponents? I add them. What was here by each little $\times\ x \times x \times x \times x \times x \times x$. Each x has a one for example x^1. So, I add all the ones and thus it became a six because it was multiplied by itself.* In Lesson 2, she said the following without any pauses: *That is our little formula that we can use for any values of z. So, I can instead of giving you a flow diagram, I could give you this formula and I can tell you okay make z two. What would we then do? We would substitute two in z's place, because I told you z must be two. So, I will pop it into z's place. He is like a guy on the bench. Here the z is playing the game. Now I need to substitute him with someone else that needs to come play. Will z still be on the field? No, z will have to come off and the substitute goes into z's place. And then we calculate. Then BODMAS applies. With the flow diagram BODMAS did not apply. Therefore, it is easier to work with the flow diagrams. Because you knew you had to do this, do this and do that. But here you need to do it in order. What comes first. Brackets. So, we are going to multiply because they are stuck together. Then add four and that will give your ten. Okay let's see what happens if I make z minus five. Then I'm going to have a different answer, but I'm going to use the same formula.*

In both lessons, the teacher told the learners exactly what to do, talked them through the tasks and sometimes even gave the answers. The teacher asked questions, but instead of waiting for an answer, she kept explaining. The teacher used questioning during the lessons. For example, in Lesson 1, she asked and answered the following: *Now algebraic expressions are when we have variables and constants* *together in a formula. Know what makes up a formula? The plusses, the minuses, the brackets, the multiplication and division. There is one example: the three is stuck to the x, so the three is multiplied by the x. Anything that is stuck to or has a multiplication sign or has a little dot between them means multiplying. And there is a two being added. So, two in that one is your constant and x is your variable.* In Lesson 2 she said the following without any pauses: *What would I do first? How many things are happening here? Let's look at that. The multiplying with the two, we adding with a three and we are dividing with a four. How many little things are there going to be in you diagram? Three. So, I'm going to have an input. Then something is gonna happen, something is gonna happen, something is gonna happen, and I'm going to have an output. What is my input in this formula? Listen carefully what I'm asking. What is the input that I had here? In this case my input is x. so I put x in. What did I do first? I first added three. What would I do next? I will times by two. What would I do last? Minus four. Anyone any questions?* No one had a question and she carried on.

4.7.2.3 Access to mathematical content

Active participation

Learners did not have the opportunity to engage in rich discussions with each other. The teacher was the main source of information during both lessons. The teacher rarely asked the learners to discuss or show ideas. Most of the time, the teacher was in charge of all discourse. Teacher C provided the direction and invited the learners to contribute, but did not allow enough time for learners to answer her questions. There was very little active participation by learners in Lessons 1 and 2. The teacher was in control all of the time and gave learners minimum chance to answer questions. Minimum time was given to learners to ask questions about their struggles or to discuss their point of view. Learners rarely responded to any of the questions she asked. When learners responded, they responded with short answers. Following now are examples of some learner responses: The teacher asked in Lesson 1: *How many terms do you think there would be?* Learners (*several*) responded: *Three terms*. Teacher C said: *Yes, three, because there is $3x$ and that is one term. Y is the next one and z is the next one. There are plusses and minuses in-between them splitting them up.* During Lesson 2 she said: *What must I put over three? The four? The a? The whole thing. Remember when we did the flow diagram.*

We would do it step by step. For instance, if my input was two, I would say two plus four. Then it would give me six. Then I take the answer and I would divide it by three which would give me two. Do you agree? So, the same thing counts here. There must be a way to indicate that I must first do that before I divide by three. If I do this, do you think it will be correct? The class responded with an answer "no" and then she continued with the following: W*hat does that mean? All the calculations apply when we work with formulas.*

Equal access

The teacher provided individual assistance to any learner who struggled with a concept in both lessons. When learners asked for help by putting up their hand the teacher immediately helped them without any hesitation. This was illustrated in the following examples: Lesson 1: *I can look at that for you if you want? Yes, Brandon? Why didn't you ask me before?* Lesson 2: *Now you will be okay. Do you need help again? Please ask me.* In Lesson 2, the teacher focused her attention only on one learner in the class that sat right in front. Teacher C ignored the rest of the class when asking questions. When she asked a question, she used this specific learner's answer all the time. For example, in Lesson 2, the teacher said: *Now you choose an unknown for me again.* The same learner said a, where the rest of the class gave different variables. Then the teacher said: *Okay let's go with* a.

4.7.2.4 Agency, authority and identity

Agency

The teacher told learners exactly what to do during both lessons and did not leave them to act on their own. Examples of this from Lesson 1 are: *First we are going to go through some terminology on the topic and then you can copy it. Variables we called them unknowns as well. That is when you use a letter from the alphabet,* a, b, c, x, y, z, *any of those. Constant is those little numbers that you sometimes see in a formula or in an equation. Thus, constants are actual values that never change.* Examples of this from Lesson 2 are: *So, I'm going to add the four first and then take the answer of the brackets and multiply it by three. Let's test this formula. So, it would be three. I would put an input of two. So, that will give me three. That will give me six and that will give me 18. My output will be the algebraic expression. Let's say*

a was the input. There was no visible form of self-assessment in either of the lessons.

Authority

Learners sat in rows and were allowed to help each other during the task in Lesson 2. Learners were allowed to ask each other questions. Other than that, learners did not get an opportunity to demonstrate how knowledgeable they were as there were no opportunities to debate their ideas, solutions and strategies and they were not allowed to interact with each other. The teacher oversaw all communications that took place during both lessons. The teacher initiated all conversations and asked most of the questions. Learners' speech turns were short in both lessons. Little opportunity was given to learners to ask questions about their struggles or to discuss their point of view with each other or the teacher. There was no opportunity given in either of the lessons for learners to explain their ideas and reason with each other. There was no evidence of the teacher giving recognition for learners' ideas in either of the lessons. Teacher C asked learners to choose a variable during Lesson 2, but in the end, she used what she wanted to use. For example, she asked: *What do you want me to use?* The learners answered: *Let's use x.* Then the teacher responded with: *Let's use z, we always use x. So, let's use a z.* The teacher did not praise the learners' ideas throughout any of the lessons.

Mathematical identity

Learners did not act independently as they asked continuously for assistance throughout the lessons and were not confident to do the problems on their own. If learners got stuck, they immediately put their hands up and asked the teacher for help. The teacher did not encourage learners to be mathematically independent as she talked all the time and the learners had to sit back and listen or to write her exact words down. This was evident in the following examples: In Lesson 1 she said: *Now what makes one term. You've might have heard of it in primary school as well. If you've got a little sum like this one? They sometimes ask you how many terms are in that little formula. Then we would say there are two terms. Terms are separated with a plus or a minus, but it is made one with a multiplication or a divide. That is why the three and the x is not two terms but is one term that is formed by the multiplication sign. But the plus splits it up into different terms.* In Lesson 2 she said:

Now I need to realise that I need to make the formula to show the people they must first add four then go times with the three. How do I do that? Brackets. In mathematics, we would like to write the number in front of the variables when I multiply. So, a better way to write this would be $3(n + 4)$. *Okay and that will be my formula.* Learners did not have the confidence to share new ideas. Learners did not seem to enjoy the lessons, as the teacher proposed which methods to use. For example, in Lesson 1, she explained: *You don't use a division sign in algebra. When you get older we prefer writing it as a fraction. You will rarely see a division sign. You will only see it if you put it there by yourself. You need to get use to the first guy is on top, that is called your numerator. And the bottom guy is your denominator. Okay so we will write it like that.* When helping learners with the tasks, Teacher C did not explore different methods. During Lesson 1 the teacher said: *So, when you do your homework. You need to think and be careful. What must be first. How will I show this people that's doing this that that must be first? Always with brackets. Dividing everything. Multiplying everything in a bracket.* In Lesson 2 she explained two methods, but did not leave learners to figure it out on their own. For example, she said: *There's two ways. She said the first way. Put everything over three. Remember when we did the fractions I taught you? You first do the top, then the bottom then you do the fraction. So, this indicates to you that you first have to do that. Then divide the whole thing by three. Okay so that will be a correct way. What will be another way? I've got a plus four. How else can I put the three in there? I can put it in brackets and then say divided by three and then write it as a fraction. But remember you are turning into the senior maths now so we prefer a fraction rather than a divide sign.* The teacher did not motivate learners, for example in Lesson 2 she said: *Totally wrong. Do you see that.*

4.7.2.5 Uses of assessment

Soliciting learner thinking

Teacher C used questioning to solicit learner thinking. The only examples of this are: Lesson 1: *How many terms will there be? Who says two? Who says three?* Lesson 2: *What will my expression end up to be? What is the first thing I need to do to a?* When learners completed the task, she walked around in the classroom to check each learner's work and assisted those who had questions. The teacher did not listen to what learners had to say. She asked some questions, but answered them herself. For example, the teacher said the following: In Lesson 1: *All of the calculations tells me what to do first? What must I do first?* She answered: *Brackets.* In Lesson 2: *If I give you this, what is going to be your output?* She answered: *My output will be the algebraic expression. Let's say a was the input.* Teacher C did not give learners enough time to give an answer to her questions.

Building on learner ideas and misunderstandings

The teacher did not build on learners' ideas during instruction as she asked all the questions, answer them herself many times and did most of the talking. For example, in Lesson 2 her pattern of discourse was as follows: *So, your input is going to be a z. What do you think my output will look like?* She immediately answered her own question. She did not pause for learners to think about it or to answer. She answered with: *First of all, with the flow diagram we don't look at order of calculations. We have to do it as the flow diagram goes from the left to right. So, the first thing we are going to do is, is we going to times z with three. What will that answer be?* The teacher just continued with the sum without learners' co-operation. Then she said: $3z$ *or* $z3$. *What do you think?* Teacher C answered immediately again: $3z$. *Remember we said in the previous presentation, you always have a number, then you have the unknown. So, it's going to be* $3z$. *And then what must you do? So, we first get the answer then we add four. So why don't we say? This output ended up being equal to* $3z + 4$. Teacher C did not address any misunderstandings throughout any of the lessons.

Table 4.14 below presents the summary of Lessons 1 and 2 of Teacher C.

Table 4.14: Teacher C (Summary of Lessons 1 and 2)

TRU Math dimensions	Lesson 1	Lesson 2
1. The mathematics		
1.1 Focused and coherent	Yes	Yes
1.2 Building meaningful connections	No	Yes
1.3 Engagement in key practices	No	No
2. Cognitive demand		
2.1 Productive struggling	No	No
2.2 Scaffolding	No	No
3 Access to mathematical content		
3.1 Active participation	No	No
3.2 Equal access	Yes	No
4. Agency, authority and identity		
4.1 Agency	No	No

4.2 Authority	No	No
4.3 Mathematical identity	No	No
5. Uses of assessment		
5.1 Soliciting learner thinking	No	No
5.2 Building on learner ideas and misunderstandings	No	No

4.7.3 Post-observation interview

Through the interview, the teacher had the opportunity to make comments about her instructional practice. The teacher's words during the post-observation interview are given in italics. The data from the post-observation interview are now presented.

Table 4.15 below presents Teacher C's comments about her instructional practice. The interview data are, as with the observations, presented based on the TRU Math dimensions and dimension indicators as inclusion criteria for coding the data (see Table 4.1).

Table 4.15: Teacher C (Presentation of the data from the post-observation interview)

TRU Math dimensions	Teacher C's comments about her instructional practice
The mathematics	
Focused and coherent	Teacher C did not mention anything about the content being focused or presented in a logical and sequenced way.
Building meaningful connections	Teacher C mentioned that *when you connect the knowledge of the learners to the new knowledge they tend to understand the new knowledge better.* She did not mention anything about building understanding around big ideas.
Engagement in key practices	Teacher C said that *there is not really time to do real-life investigations.* She further said, *I go through the curriculum and teach the required content.*
Cognitive demand	
Productive struggling	Teacher C also mentioned that *learners should be given recipes or rhymes to help them to solve complex problems.* She did not mention anything about the development of conceptual understanding through productive struggling.
Scaffolding	Teacher C mentioned that *there is no point teaching above learners' level of understanding.* Therefore, *I try to teach at their level and understanding.* Teacher C did not mention anything about scaffolding to reduce the ZPD.
Access to mathematical content	
Active participation	Teacher C mentioned that *the teacher has the most knowledge and learners can take from that knowledge.* She also added that *I don't like a noisy class, as long as all the learners behave and listen we will go far.*
Equal access	Teacher C said *I try to give all learners equal attention, but sometimes it is difficult.* She added *when learners need the extra support, I will rather help them than someone that understands it already.*
Agency, authority and identity	
Agency	Teacher C said i*f the goal of your teaching is for learners to understand the work, then they have to make sense of the problem*

TRU Math dimensions	Teacher C's comments about her instructional practice
	and how the endurance to work through that problem. She also mentioned that she encouraged learners to *examine their own learning progress by testing their answer with a second method and to look for a mistake in their own work.*
Authority	Teacher C believed that there was *always more than one way to solve a problem.* However, she did not mention anything about opportunities to demonstrate how knowledgeable learners are or being recognised for that by the teacher.
Mathematical identity	Teacher C explained that *confidence and good work ethics are important in a mathematics classroom*. She believed that *it is important to praise and encourage learners.* She did not mention anything about mathematics being enjoyable. She failed to mention the importance of mathematics being enjoyable and learners acting independently.
Uses of assessment	
Soliciting learner thinking	Teacher C said that *when I see a learner needs help, I will ask the learner with what he or she struggle with.*
Building on learner ideas and misunderstandings	Teacher C mentioned *it is important to start from the beginning as some learners forget what they have done previously and then to build on former knowledge.*

Table 4.16 below presents the summary of the post-observation interview of Teacher C.

Table 4.16: Teacher C (Summary of the post-observation interview)

TRU Math dimensions	Post-observation interview
1. The mathematics	
1.1 Focused and coherent	No
1.2 Building meaningful connections	Yes
1.3 Engagement in key practices	No
2. Cognitive demand	
2.1 Productive struggling	No
2.2 Scaffolding	No
3 Access to mathematical content	
3.1 Active participation	No
3.2 Equal access	No
4. Agency, authority and identity	
4.1 Agency	Yes
4.2 Authority	No
4.3 Mathematical identity	No
5. Uses of assessment	
5.1 Soliciting learner thinking	Yes
5.2 Building on learner ideas and misunderstandings	Yes

5. CHAPTER 5: Discussion of findings and cross-case analysis

The purpose of this study was to explore Grade 8 mathematics teachers' creation and utilisation of opportunities for learners to develop mathematical understanding in their classrooms. The possible opportunities teachers could use were based on Schoenfeld et al.'s (2014) TRU Math scheme. The data from the document analyses, lesson observations and post-observation interviews were presented in Chapter 4 and in this chapter I present an analysis of each teacher, followed by a cross-case analysis and summary. This discussion is used to answer the research questions in Chapter 6.

5.1 Analysis of each teacher

The findings from the three data collection instruments are compared for each of the three teachers in the following sections.

5.1.1 Teacher A

5.1.1.1 The mathematics

The tasks which Teacher A planned to use during his lessons corresponded closely to what he taught in the classroom as both tasks were focused and coherent, but did not promote engagement in key practices. Both the tasks and lessons were presented in a focused and coherent way as the content was focused and goal-oriented. However, he did not mention anything about the content being focused or logically sequenced in the post-observation interview. A contradiction is that, although he mentioned in the post-observation interview that he usually provided tasks and lessons that were connected to a real-life application, it was not applied while teaching. None of the tasks connected the mathematical concepts to real-life situations. During Teacher A's instruction, he did not develop flexible and adaptive learning and learners would most probably struggle to transfer knowledge that they had already learned to new settings. Regarding building connections, there was an inconsistency between the two tasks which he planned to use during the lessons. Learners did not have to make meaningful connections between procedures, concepts and contexts to complete Task 1 as it was primarily skills-orientated. Only Task 2 stressed the connections between the learners' pre-knowledge and experience of a task in everyday contexts with the new concept being learned. Learners learnt the mathematics with understanding through Task 2 as they actively built new knowledge on prior experience and knowledge as recommended by Van De Walle et al. (2013) (see section 2.4.1). Task 2 provided more opportunities for learners to engage with rich content and to develop useful mathematical thinking skills by engaging in mathematical practices. Task 2 further required learners to explore and understand the relationship between angles and 2-D shapes.

5.1.1.2 Cognitive demand

Learners had the opportunity to experience some productive struggling, but not enough to do the mathematics with understanding. There were very few opportunities for learners to make sense of and persevere in challenging mathematics. Task 1 promoted mastery of skills and not conceptual understanding about the topic. Furthermore, Task 2 promoted productive struggling as the task developed conceptual understanding. Task 2 gave learners the opportunity to solve geometric problems on different levels. Regarding the learners being intellectually active, there was an inconsistency between the two tasks. Task 1 focused on procedural fluency, whereas Task 2 focused more on solving problems with mathematical reasoning. Task 1 promoted learners mostly to apply memorised procedures or to work routine exercises. Lesson 1 involved recalling data and information, such as procedurally moving a number or variable from the one side to the other by solving linear algebraic equations. During Teacher A’s instruction, he also required learners to listen, duplicate, memorise, drill and calculate. According to Gamoran and Nylstrand (1991), practices like Teacher A’s lead to learners experiencing mathematics as overwhelming with never-ending lists of rules, remote skills, concepts and symbols that must be practised and restored repeatedly. Learners were required to be intellectually active during Lesson 2 as they were asked to explain their reasoning for solving the problem and asked to justify their mathematical thinking.

Teacher A did not provide opportunities for learners to struggle on their own and did not allow the learners to think for themselves. According to Van De Walle et al. (2013), teachers must use probing to promote learners to think critically and understand the mathematics they are exploring. When learners struggled with a task, Teacher A helped them immediately and showed them where they had made a mistake. It appeared as if learners knew they did not need to struggle as he would help them anyway. This resulted in the teacher doing the thinking for the learners, where instead he could have used these opportunities to apply a constructivist approach to his teaching. A similarity between the observation and the post-observation interview was that Teacher A did not mention anything about learners being intellectually active and did not promote learners to be intellectually active during his instruction. Teacher A failed to mention anything about reducing the ZPD and he also did not reduce the ZPD during his instruction.

5.1.1.3 Access to mathematical content

Teacher A used different teaching styles and approaches than what he mentioned in the post-observation interview. Although he said it was important for learners to participate actively, there were limited opportunities for learners to participate in rich discussions in the lessons. Promoting learner participation is a key aspect to a responsive mathematics learning environment (Ontario Ministry of Education, 2011). Based on the observations, one may conclude that Teacher A did not provide opportunities for learners to facilitate meaningful mathematical discussions. There was no discussion around problems in the lessons that could help learners to understand and construct the deeper meaning of concepts. Teacher A used questioning as a technique to initiate class discussions. However, these questions did not invite learners to participate in class discussions as the questions were mainly closed-ended. A link could be found between the information provided in response to the post-observation interview and the actual teaching observed in the classroom regarding equal access. Teacher A believed that it was important for a teacher to be flexible and to accommodate all learners, and it was visible in his lessons. It is essential to create an environment that offers an equal opportunity for all learners to learn (Van De Walle et al., 2013).

5.1.1.4 Agency, authority and identity

It appeared that Teacher A did not encourage learners to make progress on their own as he was quick to tell learners what to do and how to solve a problem. Classroom tasks were strictly performed under clear instructions of the teacher and limited learner individuality was allowed. Learners did not act mathematically independently as the teacher made decisions for them and told them how to tackle the problem. In the lessons, learners were required to follow certain procedures instead of explaining their reasoning for solving the problem or justifying their mathematical thinking. The teacher believed that it was important for learners to make sense of their own mathematical ideas, but it was not visible during his instruction. Learners should validate their solutions themselves and should not guess and then seek confirmation from the teacher (Curro Centre for Educational Excellence, 2012). During his instruction, Teacher A did not require learners to monitor their own thinking and he also did not mention anything about it in the post-observation interview. For example, in Lesson 1 he did not tell learners to test their answers by substituting the answer in and checking whether the left-hand side is equal to the right-hand side when solving algebraic linear equations. Even though he mentioned the importance of authority in the post-observation interview, learners were not given a chance to explain how they thought a problem should be solved or why they had done their calculations in a particular way. The teacher said that learners should be able to explain their thinking; however, he did not give learners the opportunity to explain their thinking in any of the lessons. Teachers should create opportunities for learners to conjecture, explain, make mathematical arguments and build on one another's ideas in ways that contribute to their development (Schoenfeld et al., 2014). It was not visible during Teacher A's instruction that learners enjoyed doing mathematics. Teacher A also failed to mention the importance of mathematics being enjoyable, and it showed during his instruction.

5.1.1.5 Uses of assessment

Teacher A mentioned that he would ask questions to solicit learner thinking, but it was not visible during his instruction. He did not implement any strategies to solicit learner thinking and did not provide an opportunity to hear the learners' voices. Questions that encourage rich mathematical conversation not only give the learners an opportunity to share and expand on their ideas, but also allow the teacher to check for understanding of the content (Manouchehri, 2007). A consistency was that Teacher A did not mention anything about building on learners' ideas and this was also not visible during his instruction. Teacher A made an effort to address misunderstandings during his instruction and he mentioned the importance of addressing misunderstanding in the post-lesson observation. Identifying a misunderstanding as soon as possible is the starting point to treating it (Curro Centre for Educational Excellence, 2012).

5.1.2 Teacher B

5.1.2.1 The mathematics

A link could be found between the document analysis, the actual teaching observed in the lesson and information provided in response to the post-observation interview regarding the mathematics being focused and coherent. A coherent framework for mathematical ideas was visible in both tasks and the tasks flowed in a logical sequence. This way of presenting mathematics has a positive impact on understanding mathematics (Hiebert & Carpenter, 1992; NCTM, 2000a). Furthermore, the content of the lessons was focused and goal-oriented. The way Teacher B described her teaching did not always correspond with how she taught in the classroom. She built mostly meaningful connections in her teaching of both lessons, but she did not mention anything about building new knowledge on prior knowledge and experience, or building understanding around big ideas in the post-observation interview. The content of the lessons did not include enough examples of how operations with integers are connected to the big idea. The mathematical content did not show learners the reason why a negative number times a positive number equals a negative number. For example, in Lesson 2, the content of the lesson did not provide an opportunity for learners to understand the proof of: -3×3 equals 3×-3 (commutative property) and multiplication is repeated addition: $-3+-3+-3=-9$. Task 2 further required learners to explore and understand the relationship between adding and subtracting integers and multiplying and dividing integers. Learners also learnt mathematics with understanding through the tasks as learners actively built new knowledge on prior experience and knowledge. The tasks promoted engagement in key practices such as problem solving and reasoning. However, there were no questions in either of the tasks that required learners to understand the relationship between concepts of integers and real-life situations. No real-life problems were discussed in either of the lessons. A contradiction to that is that Teacher B believed that she encouraged problem solving skills so that learners could apply mathematics in the real world. Teachers may possibly avoid solving real-life problems because of time constraints, lack of resources or a feeling of inadequacy (Fan, 2008).

5.1.2.2 Cognitive demand

There was a contradiction between the two lesson observations regarding productive struggling. During Teacher B's instruction of Lesson 1, she did not promote productive struggling, but she provided an opportunity for learners to struggle on their own during Lesson 2. Findings from the document analysis showed some similarities to her response in the post-observation interview regarding productive struggling. The tasks gave learners the opportunity to experience productive struggling as the tasks maintained learner engagement. The value of such learner engagement is stressed by various researchers (Hyland, 2003; McRae, 2007; Rypisi et al., 2009). These higher-level tasks required some degree of thinking as learners could not solve the problems mindlessly. Teacher B did not believe in spoon-feeding and that was confirmed by her choice of tasks. A link could be found between the actual teaching observed in the lesson and information provided in her response during the post-observation interview regarding scaffolding. She kept assisting learners and asking them questions to guide them towards the next step. The teacher's hints or scaffolds supported learners in productive struggling in building understanding and engaging in mathematical practices. The teacher asked many open-ended questions, which encouraged learners to think about the problems. The teacher mentioned in the post-observation interview that it was important that learners must discover and learn for themselves which were evident during her instruction. Teachers should create classroom environments where there is a good balance between aiding and leaving the learners (Schoenfeld et al., 2014).

5.1.2.3 Access to mathematical content

There was an inconsistency between the actual teaching observed in the lesson and information provided in response to the post-observation interview regarding learners' active participation. The teacher actively supported, and to some degree achieved, broad and meaningful mathematical participation. Based on the observations, one may conclude that Teacher B provided opportunities for learners to facilitate meaningful mathematical discourse. There was active participation by learners in the lessons. Learners were not afraid to ask questions. The teacher facilitated active classroom discourse as emphasised by Alpert (1987) (see section 2.4.3.1). Learners had the opportunity to engage in richer class discussions. However, she did not mention anything during the interview about learners participating actively in class. A link could be found between the actual teaching observed in the lesson and information provided in response to the post-observation interview regarding equal access. The classroom environment invited and maintained active participation for all learners and Teacher B mentioned that all learners should feel like they can do mathematics.

5.1.2.4 Agency, authority and identity

A link could be found between the actual teaching observed in the lessons and information provided in response to the post-observation interview regarding agency, authority and identity. Teacher B mentioned that learners should discover and learn independently, which reflected in her teaching. Both tasks allowed learners to make their own sense of some mathematical ideas. She gave learners the opportunity to work independently on their tasks. Learners got the opportunity to demonstrate how knowledgeable they were by sharing their ideas at any time throughout the lessons. She asked learners to comment on each other’s ideas. Schoenfeld et al. (2014) maintain that teachers should create opportunities for learners to conjecture, explain, make mathematical arguments and build on one another’s ideas in ways that contribute to their development. The investigation in the beginning of Lesson 1 gave learners the opportunity to construct viable arguments to support their own reasoning and to critique the reasoning of others. Teacher B’s desire was to create independent learners who could solve problems on their own and it was clear during her instructions. A consistency was that she mentioned the importance of learners being motivated and having self-confidence – aspects that were observed during her instruction. Another consistency was that she did not mention anything about praising learners’ efforts during the interview and did not do it during her instruction.

5.1.2.5 Uses of assessment

A link could be found between the actual teaching observed in the lessons and information provided in response to the post-observation interview regarding soliciting learner thinking. In all the lessons, Teacher B monitored learner progress by asking open-ended questions. She asked questions to determine learners' level of understanding. The lessons started by marking the previous day's work. By doing this, she solicited learner thinking. The teacher provided opportunities for learners to talk about what they have done and why. During her instruction, she assisted learners individually when they struggled to complete the task. The teacher further solicited learner thinking and used the learners' ideas in subsequent instruction. The teacher asked the learners to put up their hands to show if they got the correct answer. This is a good way for the teacher to see whether she needed to explain the answer (Curro Centre for Educational Excellence, 2012). Teacher B also mentioned in the post-observation interview that she normally asked questions during her instruction to determine what learners understod. Even though Teacher B monitored the learners closely and corrected their mistakes immediately, she did not mention anything about addressing emerging misunderstandings in the post-observation interview.

5.1.3 Teacher C

5.1.3.1 The mathematics

There was a consistency between the findings in the document analysis and Teacher C's instruction during the observed lessons. In Lesson 1, learners were not required to complete any mathematical tasks; they only had to copy the teacher's notes from the PowerPoint presentation. A coherent framework for mathematical ideas was visible in the task in Lesson 2, the content flowed in a logical sequence and the content of the lessons was presented in a focused and coherent structure. However, she did not mention anything about the content being focused or coherent in the post-observation interview. Teacher C used words such as 'guys', 'things' and 'them' many times during her instruction, which is neither good practice nor meaningful. A link could be found between the document analysis, the actual teaching observed in the lesson and information provided in response to the post-observation interview regarding building meaningful connections and engaging in key practices. The way Teacher C described her teaching corresponded closely with how she taught in the classroom. During her instruction, she stimulated learners' connections to prior knowledge and learners learnt mathematics with understanding through the task as learners had to actively build new knowledge on prior experience and knowledge (Van De Walle et al., 2013). However, in Lesson 2, Teacher C did not mention the principle of multiplication which is repeated addition, nor the principle of exponents which is repeated multiplication. The teacher had the opportunity in Lesson 1 to remind learners of the commutative property of integers, but she did not. No real-life problems were discussed in either of the lessons. The lessons only targeted content and did not offer multiple opportunities for learners to apply content, construct meaning or practise higher-order thinking skills. There were no questions in the tasks that required learners to understand the relationship between the mathematical concept and real-life situations. The teacher did not encourage the learners to use ideas outside the planned curriculum.

5.1.3.2 Cognitive demand

Even though the task promoted productive struggling, Teacher C did not apply productive struggling during her instruction nor did she mention the importance thereof in the post-observation interview. The task gave learners the opportunity to experience productive struggling as the tasks maintained learner engagement. According to Schoenfeld et al. (2014), effective instructions consist of an ongoing challenge of finding the right balance between supporting learners to understand the difficulties in the mathematical classroom tasks and giving them the opportunity to make their own progress. The task engaged learners intellectually, but learners did not get the opportunity in the lessons to actively think about the mathematical concepts involved. A consistency was found between instruction during the lessons and the response in the post-observation interview. Teacher C believed that learners needed much guidance and assistance, and this was confirmed by how she taught. There were few opportunities for learners to make sense of and persevere in challenging mathematics. Another similarity between the lesson observed and the post-observation interview was to give learners recipes or rhymes to help them to solve problems.

5.1.3.3 Access to mathematical content

A link could be found between the actual teaching observed in the lessons and information provided in response to the post-observation interview regarding active learner participation. Teacher C was the main source of knowledge when she taught and no discussions took place in either of the lessons. The teacher talked all the time during her instruction and the learners had to listen. The teacher knew what questions to pose to encourage active participation, but instead of letting the learners answer, she answered the questions herself. Based on the observations, one may conclude that Teacher C did not provide opportunities for learners to facilitate meaningful mathematical discourse. There was no discussion around problems in the lessons that could help learners to understand and construct deeper meaning of concepts. Ontario Ministry of Education (2011) maintains that promoting learner participation is a key aspect to a responsive mathematics learning environment and that means that all learners interact with the teacher and with each other. There was no evidence of active participation by learners in the lessons. Teacher C controlled the learning as she did all the explaining. She also mentioned that she does not like a noisy class and said that, “as long as all the learners behave and listen we will go far”. The classroom environment did not invite and maintain active participation for all learners as she did not encourage all learners to participate equally. The teacher did not respond to the learners’ answers and she divided her attention unfairly in the class as she focused her attention on only a few learners in the class. This was supported in her response in the post-observation interview as she mentioned that she tried to give all learners equal opportunities to engage with mathematical tasks, but struggled at times.

5.1.3.4 Agency, authority and identity

A contradiction between the lessons observed and the post-observation interview was that Teacher C said that learners need to be taught to think for themselves, but instead, she only used rote learning. Even though Teacher C mentioned the importance of agency, she struggled to apply it during her instruction. Learners did not have opportunities to explore concepts on their own or to validate their solutions. There were no opportunities recommended by the DBE (2011) given to estimate, explain and justify the soundness of the learners’ answers. A link could be found between the actual teaching observed in the lesson and information provided in her responses during the post-observation interview regarding authority. Teacher C did not encourage learners to share their answers and she did not mention anything about opportunities to demonstrate how knowledgeable learners are or being recognised for that by the teacher in the post-observation interview. Even though Teacher C mentioned the importance of learners having self-confidence, she did not promote it during her instruction. The teacher did not praise learners or take their ideas into account.

5.1.3.5 Uses of assessment

Even though Teacher C mentioned the importance of soliciting learner thinking and addressing emerging misunderstandings, she struggled to apply it during her instruction. The teacher referred to learner thinking, perhaps even to common mistakes, but specific learners' ideas were not built on or used to address misunderstandings. She did not implement a variety of strategies to monitor learner learning. Teachers can use techniques such as think-pair-share, wait time, cold calling, sharing learner-generated solutions, and learner response systems such as mini whiteboards and exit cards to check for understanding during or right after a lesson (Wiliam, 2008). Teacher C just carried on explaining things without checking whether the learners understood what she explained. There were some similarities between the lessons observed and the post-observation interview. The teacher mentioned in the post-observation interview that it was important to start from the beginning as some learners forget what they have done previously and she applied it in her teaching. She added that she preferred to start with easy examples and slowly make them more difficult. Lastly, she said that she allowed learners to ask other learners to explain to them if they get stuck.

5.2 Cross-case analysis

In this section, a comparison is drawn between the three data collection instruments, followed by an integration of findings for each of the three teachers.

5.2.1 Comparison of the document analyses

In this study, two of the three teachers chose the most powerful mathematical tasks. There was, however, no teacher that provided a task which engaged learners in key practices such as real-life application through critical and reflective thinking. Curro Centre for Educational Excellence (2012) maintains that real-life problems that make sense to learners are used as point of departure for new work and provide the contexts wherein learners can experience mathematical concepts and procedures as something that makes sense. Teacher B and Teacher C adhered to the same number of TRU Math dimension indicators. Table 5.1 contains a summary of the categories addressed in the data analysis of each teacher.

Table 5.1 indicates that Teacher B and Teacher C provided tasks that included most of the indicators of the TRU Math dimensions, whereas Teacher A provided tasks that included the least of the indicators of the TRU Math dimensions. A possible reason for this can be that different schools use different textbooks and the choice of textbook is sometimes made by the head of the mathematics department.

Table 5.1: Cross-case analysis of document analyses

	Teacher A		Teacher B		Teacher C
	Task 1	Task 2	Task 1	Task 2	Task 1
1. The mathematics					
1.1 Focused and coherent	Yes	Yes	Yes	Yes	Yes
1.2 Building meaningful connections	No	Yes	Yes	Yes	Yes
1.3 Engagement in key practices	No	No	No	No	No
2. Cognitive demand					
2.1 Productive struggling	No	Yes	Yes	Yes	Yes

5.2.2 Comparison of the observations

Interpreting the data and comparing the teachers should be done with caution. The underpinning philosophy of this study is constructivism, which implies that each teacher created his or her own experiences in the classroom. Table 5.2 contains a summary of the categories addressed in the observations by each teacher.

The content of all the lessons of all the teachers was focused and coherent. None of the teachers engaged learners in key practices such as solving problems related to real-life situations. There are various reasons why teachers choose to avoid solving real-life problems, such as time constraints, lack of resources or a feeling of inadequacy (Fan, 2008). The content of most of the lessons built meaningful connections. Teacher A and Teacher B built meaningful connections in all their lessons, while Teacher C built meaningful connections in only one of her lessons.

Teacher A and Teacher C did not promote productive struggling and scaffolding during their instruction as much as Teacher B. In terms of the development of learners' higher order thinking skills, it is necessary for learners to experience productive struggling (Schoenfeld et al., 2014). Teacher B had a good balance between aiding and holding back assistance as learners solved problems.

Teacher A and Teacher C did not encourage active participation in either of their lessons, while Teacher B established a classroom culture that enhanced, supported, facilitated and maintained high-level discussion. Classroom discussions develop learners' ability to reason mathematically and their ability to communicate that reasoning, aspects of a lesson that are highly valued by NCTM (2013). Teacher A and Teacher B provided more mathematical access to content than Teacher C.

Teacher B promoted agency, authority an identity more than Teacher A and Teacher C. Teacher A and Teacher C allocated little time to working together and allowing learners to do work on their own and in their own way. Teacher A and Teacher C told their learners how to approach a problem by providing them with formulae or procedures that could be used to solve specific problems. However, this approach does not ensure teaching for understanding, as it enables learners to simply follow the procedures to solve a problem without any real understanding. This is not conducive to building understanding and becoming independent learners. One of the aims in the CAPS document is to develop deep conceptual understanding to make sense of mathematics (DBE, 2011), an aspect that was therefore only partially achieved.

Teacher B solicited learner thinking, built on learner thinking, and built on learner ideas and misunderstandings more than Teacher A and Teacher C. As suggested by Curro Centre for Educational Excellence (2012), all the teachers used questioning as a strategy to gather information about learner thinking. Effective teachers will listen to their learners to learn about their learners' understanding and thought processes instead of listening only for the correct answer. However, only Teacher B used the information she gathered about learner thinking to build further on her instruction, and in the process, she built on the learners' existing knowledge and addressed misunderstandings.

From the above discussion, one can conclude that Teacher B applied nearly all the dimensions of the TRU Math scheme during her lessons. The intensity and duration of the application of the dimensions were also considerably higher than in the case of the other teachers. It can also be concluded that Teacher A and Teacher C promoted almost the same number of dimensions. The observations of Teacher A and Teacher C revealed that there was a slight difference between the intensity and duration of the promotion of these dimensions as Teacher A promoted the dimensions more often and with more intensity during the lessons than Teacher C.

Table 5.2: Cross-case analysis of the observations

	Teacher A		Teacher B		Teacher C	
	Lesson 1	Lesson 2	Lesson 1	Lesson 2	Lesson 1	Lesson 2
1. The mathematics						
1.1 Focused and coherent	Yes	Yes	Yes	Yes	Yes	Yes
1.2 Building meaningful connections	Yes	Yes	Yes	Yes	No	Yes
1.3 Engagement in key practices	No	No	No	No	No	No
2. Cognitive demand						
2.1 Productive struggling	No	No	No	Yes	No	No
2.2 Scaffolding	No	No	Yes	Yes	No	No
3 Access to mathematical content						
3.1 Active participation	No	No	Yes	Yes	No	No
3.2 Equal access	Yes	Yes	Yes	Yes	Yes	No
4. Agency, authority and identity						
4.1 Agency	No	No	Yes	Yes	No	No
4.2 Authority	No	No	Yes	Yes	No	No
4.3 Mathematical identity	No	No	Yes	Yes	No	No
5. Uses of assessment						
5.1 Soliciting learner thinking	No	No	Yes	Yes	No	No
5.2 Building on learner ideas and misunderstandings	No	No	Yes	Yes	No	No

5.2.3 Comparison of the post-observation interview

Table 5.3 contains a summary of the participants' beliefs regarding how Grade 8 mathematics should be taught. Any comparison and interpretation of the data should be done with caution as the way in which the teachers taught did not reflect the answers given in the interviews. For instance, Teacher A mentioned various TRU Math dimension indicators in the post-observation interview that he did not use during his teaching. On the other hand, the other two teachers omitted many of the TRU Math dimension indicators in the post-observation interview, but did apply them while teaching. It should therefore be clear that it would not be appropriate to draw any conclusions and make comparisons between the different teachers' post-observation interviews. The way teachers' views and actions contradicted each other suggests that they did not reflect their instructional practices.

Table 5.3: Cross-case analysis of the post-observation interviews

	Teacher A	Teacher B	Teacher C
1. The mathematics			
1.1 Focused and coherent	No	Yes	No
1.2 Building meaningful connections	No	No	Yes
1.3 Engagement in key practices	Yes	Yes	No
2. Cognitive demand			
2.1 Productive struggling	No	Yes	No
2.2 Scaffolding	No	Yes	No
3 Access to mathematical content			
3.1 Active participation	Yes	No	No
3.2 Equal access	Yes	Yes	No
4. Agency, authority and identity			
4.1 Agency	Yes	Yes	Yes
4.2 Authority	Yes	Yes	No
4.3 Mathematical identity	No	Yes	No
5. Uses of assessment			
5.1 Soliciting learner thinking	Yes	Yes	Yes
5.2 Building on learner ideas and misunderstandings	No	No	Yes

5.2.4 Integration of findings

In this section, the findings from the three teachers are compared and integrated.

5.2.4.1 The mathematics

The content of the tasks and the lessons of all three teachers was focused and coherent. A similarity was found between Teacher A and Teacher C; neither teacher indicated that they thought their content was properly structured, cohesive or clearly directed. Most of the content in the tasks built meaningful connections and all three teachers provided opportunities for learners to build meaningful connections in their lessons. There were only minor differences between the content of the teachers' tasks regarding building meaningful connections. The content of the tasks of Teacher B and Teacher C built new knowledge on prior knowledge and experience, and built understanding around big ideas, whereas the content of only one task of Teacher A built meaningful connections (see Table 5.1). For example, the mathematics of Task 1 of Teacher A did not entirely support learners to develop mental representations as part of a bigger network of algebraic equations, as the task had only one main question, namely, to solve x. This task did not require learners to make valuable connections, comparisons, justifications or conjectures, or testing ideas or developing sense-making skills by comparing to the other tasks.

Neither Teacher A nor Teacher B mentioned anything in the post-observation interview about building meaningful connections. A similarity was found between all three teachers regarding the engagement in key practices. The content of all tasks and lessons did not promote the application of real-life problems. However, Teacher A and Teacher C mentioned the importance of solving real-life problems in their interviews.

5.2.4.2 Cognitive demand

Similarities were found between all the teachers regarding learners' productive struggling. Teacher A and Teacher C, as well as Lesson 1 of Teacher B, did not promote productive struggling. Moreover, Teacher A and Teacher C did not use scaffolding during their lessons. These two teachers also did not mention anything about productive struggling or scaffolding in their interviews, whereas Teacher B mentioned the importance of productive struggling and scaffolding during her interview.

5.2.4.3 Access to mathematical content

A similarity was found between Teacher A and Teacher C as there was no visible active participation by the learners in either of their lessons. Another similarity was found between Teacher B and Teacher C as neither teacher mentioned the importance of active participation in their interviews. A link was found between Teacher A and Teacher B regarding equal access as both invited all learners to participate equally and both mentioned the importance of equal access to mathematical content in their interviews. An inconsistency was found between Teacher C and the other two teachers as she did not invite all learners to participate equally in all her lessons and she did not mention the importance of equal access during the interview.

5.2.4.4 Agency, authority and identity

A similarity was found between Teacher A and Teacher C. Neither of these teachers promoted agency, authority or mathematical identity in either of their lessons. However, Teacher B promoted agency, authority and mathematical identity in her lessons. A consistency was found between the three teachers regarding agency as all three teachers mentioned in their interviews the importance of learners' capacity and willingness to engage mathematically. Teacher A and Teacher B both mentioned the importance of creating opportunities to demonstrate how knowledgeable learners are and being recognised for that by the teacher. Teacher B and Teacher C mentioned the importance of promoting learners' mathematical identity to make them positive doers of mathematics.

5.2.4.5 Uses of assessment

A similarity was found between Teacher A and Teacher B regarding uses of assessment as neither of them gathered information about learner thinking, built on learners' ideas or addressed emerging misunderstandings in their lessons. Another similarity was found between all three teachers regarding soliciting learner thinking as all three mentioned the importance of gathering information about learner thinking in their interviews. Lastly, a similarity was found between Teacher A and Teacher B regarding building on learner ideas and misunderstandings as neither of them mentioned the importance of it.

5.3 Summary

In this chapter, the collected data were discussed and interpreted according to the five dimensions set out in the theoretical framework. Data were collected from three teachers by means of document analysis, lesson observations and post-observation interviews. A cross-case analysis was done by comparing the different cases to each other. In the next chapter, the final conclusions and implications are discussed.

6. CHAPTER 6: Conclusions and implications

In this chapter, the research questions are answered, conclusions are drawn, implications and recommendations are stated, limitations of this study are given, and lastly, a final reflection is offered. Each research question is discussed in light of the findings and the relevant literature.

6.1 Discussion of the research questions

The aim of this study was to explore how Grade 8 mathematics teachers create and utilise opportunities to develop mathematical understanding in their classrooms. The five secondary questions, based on the five TRU Math dimensions of Schoenfeld et al.'s (2014) TRU Math scheme answer the main research question.

6.1.1 Sub-question 1: The mathematics

Sub-question 1 is about the mathematics: How focused and coherent is the discussion of the mathematics and how are the connections between procedures, concepts and contexts (where appropriate) addressed and explained? The three TRU Math dimension indicators used to answer this question regarding the mathematics are: focused and coherent, building meaningful connections and engagement in key practices (see Table 4.1). To answer this question, a document analysis schedule (see Addendum C), observation schedule (see Addendum D) and post-observation interview schedule (see Addendum E) were used.

The content of the lessons and tasks of all three teachers was focused and coherent (see Table 5.1 and Table 5.2). However, the content of the chosen tasks being focused and coherent could have influenced the teachers' lessons to also be focused and coherent. The goals were kept through all tasks as all the content focused on developing the required skills. The knowledge was presented in a coherent structure and was offered in a logical way. The tasks showed a progression of learning as the questions in all tasks escalated from easy to more difficult. Studies have revealed (Hiebert et al., 2003; Redeker, 2000; Stigler & Perry, 1998) that focused and coherent mathematics lessons may help learners to understand mathematics better and to learn mathematics conceptually.

The content of all the lessons built meaningful connections, except for Lesson 1 of Teacher C (see Table 5.2). It was, however, interesting to note that it was only Teacher C who deemed the building of meaningful connections valuable during the interview (see Table 5.3). According to Van De Walle et al. (2013), it is vital to build understanding around big ideas as it will support learners to see that mathematics is integrated and not a collection of remote parts and sections (Van De Walle et al., 2013).

In all the tasks of all the observed lessons (see Table 5.1 and Table 5.2), the content did not engage learners in important mathematical content and none of the tasks provided opportunities for learners to apply the content to solve real-life problems. One of the TRU Math dimension indicators is engagement in key practices. Table 5.2 indicates that no instance of engagement in key practices was found in any of the observed lessons. This finding was unexpected since the DBE (2011) explains that instructional practices must allow learners to understand the physical world and to solve problems connected to the world. It is important to incorporate everyday-life problems, such as economic, social, cultural, political health, scientific and environmental issues, into everyday mathematics lessons. Although all teachers neglected the engagement in key practices in their lessons, Teachers A and C acknowledged the importance thereof in their interviews (see Table 5.3).

6.1.2 Sub-question 2: Cognitive demand

Sub-question 2 is about cognitive demand: What opportunities do learners have to make their own sense of mathematical ideas? The two TRU Math dimension indicators used to answer this question regarding cognitive demand are productive struggling and scaffolding (see Table 4.1). To answer this question, a document analysis schedule (see Addendum C), observation schedule (see Addendum D) and post-observation interview schedule (see Addendum E) were used.

The document analysis of the tasks showed that, contrary to expectations, most of the tasks created opportunities for learners to experience productive struggling (see Table 5.1), as the tasks gave learners the opportunity to engage in key practices such as problem solving and reasoning. For example, one of the questions asked learners what they noticed about the answers when adding or subtract four consecutive negative integers. However, the lesson observations showed that Teacher B was the only teacher who provided opportunities for learners to be both intellectually active and develop conceptual understanding through productive struggling during a lesson. Learners were required to be intellectually active during her lessons as they were asked to explain their reasoning for solving the problem and to justify their mathematical thinking. Teachers A and C on the other hand did not provide opportunities for learners to struggle on their own and did not allow the learners to think for themselves (Table 5.2). Teacher A spoon-fed the learners as he told learners exactly what to do and when to do it and he used very few probing questions to promote productive struggling during both lessons. Teacher C encouraged very little active engagement as learners did not get an opportunity to debate their ideas, strategies and final solutions.

Teacher B was the only teacher who applied scaffolding to reduce the ZPD and was also the only teacher who acknowledged the use of scaffolding during her interview. According to Goos (2004), a way to ensure that the challenges are not too demanding, causing learners to become discouraged, is to reduce the ZPD by using scaffolding. These findings confirm the association between productive struggling and ZPD. However, this also leaves many questions about ZPD versus productive struggling. Productive struggling is not necessarily opposed to scaffolding, but can be promoted by asking probing questions and breaking up questions in smaller steps. Teacher B broke problems up into manageable parts and had a good balance between providing support and leaving the learners on their own to discover. For example, in Lesson 1, she said the following: *Observe what happens with the four consecutive numbers. Notice the difference between these two questions. Remember to use the number line.* In Lesson 2, she said the following: *Remember to use the order of operations. Check if the left side is equal to the right side when completing the equations. Compare the two questions. Explain to me what you see.* The teacher did not give straightforward answers; instead, she used questioning to guide learners through the problem. For example, in Lesson 1 she said: What *else can you tell me? What can you see? Does it increase or decrease?* In Lesson 2 she said: *What can you remember? And then I multiply by what? What will the next one be? What assumption can we make?*

According to the NCTM (2014), teachers sometimes see learner frustration or absence of instant accomplishment as indicators that they have by some means failed their learners. Consequently, they solve the problems themselves by breaking down the task and guiding learners step by step through the difficulties or giving answers too early. Teachers sometimes think that breaking down a task and guiding learners step-by-step through the difficulties are scaffolding and therefore misapply it. Although their intentions are good, they undermine the hard work of learners, decrease the cognitive demand of the task and remove the opportunities from learners to engage fully in making sense of mathematics. Star (2015) highlights the need to create a classroom environment that promotes productive struggling as a natural part of the learning process and allows learners to see the potential in persevering. Silver (2011) suggests that productive struggling can be promoted by using the following scaffolding methods: assess the learners' current knowledge and experience; relate content to the learner's prior knowledge; break a task into smaller; more manageable parts with opportunities for recurrent feedback and use prompting to assist the learner. Teacher B confirmed that productive struggling was associated with more than only breaking down a task into smaller, more manageable parts. The implication of only breaking down a task into smaller, more manageable parts and guiding learners step-by-step through the difficulties are that the best learners in South Africa are less proficient than an average performing learner in top performing countries like Finland, Singapore, Chinese Taipei, Republic of Korea and Japan (HSRC, 2011). According to the TIMSS (2011), South African learners received poor results with problem-solving skills and higher-level cognitive abilities involving understanding (Spaull, 2013).

To summarise the cognitive demand of the tasks, although the tasks promoted productive struggling, teachers did not provide sufficient opportunities for learners to make their own sense of mathematical ideas during the observed lessons. However, Teacher B allowed for productive struggling by breaking problems up into manageable parts and using questioning to guide learners through the problem. Teacher B was also the only teacher who applied scaffolding to reduce the ZPD. Moreover, Teacher B also deemed the productive struggling and scaffolding to be valuable during the interview.

6.1.3 Sub-question 3: Access to mathematical content

Sub-question 3 is about access to mathematical content: How do teachers invite and support the active engagement of all the learners in the classroom with the core mathematics being addressed in the lesson? The two TRU Math dimension indicators used to answer this question regarding access to mathematical content are active participation and equal access (see Table 4.1). To answer this question, an observation schedule (see Addendum D) and post-observation interview schedule (see Addendum E) were used.

Teacher B was the only teacher who promoted active participation in class discussions (see Table 5.2). She used all five teaching strategies as described by Chapin et al. (2009) for improving class discussions, namely: talk interchanges that engage learners in discussion; the art of questioning; using learner thinking to propel discussions; setting up a supportive environment; and orchestrating the discussions. Although Teacher A, acknowledging the value of active participation, initiated classroom discussions, he did not maintain them. Teacher C, on the other hand, was the main source of information during her lessons (see Table 5.3).

Most of the teachers provided equal access to mathematical content to a wide range of learners (see Table 5.2). Teacher B allowed all learners to participate in discussions while she facilitated learning. Teacher C seemed to act as the expert as she did not value the learners' opinions. Teacher C also rarely asked questions and talked all the time during her instruction. Regarding equal opportunities for learners to learn, in one of her lessons, Teacher C focused her attention only on one learner in the class who sat right in front of her. Moreover, Teachers A and B acknowledged the importance of equal access during their interviews (see Table 5.3).

To summarise, regarding access to mathematical content, Teachers A and B invited the active engagement of all the learners in the classroom with the core mathematics being addressed in the lessons. However, only Teacher B supported the active engagement of all the learners in the classroom. Teacher C promoted active participation and equal access to mathematical content rarely or not at all.

6.1.4 Sub-question 4: Agency, authority and identity

Sub-question 4 is about agency, authority and identity: What opportunities do learners have to explain their own and respond to each other's mathematical ideas? The three TRU Math dimension indicators used to answer this question regarding agency, authority and identity are agency, authority and mathematical identity (see Table 4.2). To answer this question, an observation schedule (see Addendum D) and post-observation interview schedule (see Addendum E) were used.

Although all three teachers acknowledged the importance of agency during their interviews (see Table 5.3), Teacher B was the only teacher who developed the learners' capacity and willingness to engage mathematically during the lessons (see Table 5.2). Teacher B provided opportunities for learners to validate their solutions themselves, which resulted in learners not guessing the answers, nor seeking confirmation from the teacher (Curro Centre for Educational Excellence, 2012; Meyer, 2013). Teacher B created opportunities for learners to be involved, act on and be responsible for all aspects of their own learning by guiding them with questions through solving the problems. Teachers A and C, on the other hand, told learners exactly what to do during both lessons and did not leave them to act on their own. However, Teacher A was the only teacher who implemented self-assessment during the lessons.

Only Teacher B created opportunities for learners to demonstrate how knowledgeable they were and recognised it during her instruction (see Table 5.2). Teacher B promoted authority in the following ways: learners got the opportunity to demonstrate how knowledgeable they were by sharing their ideas at any time throughout the lessons, she gave learners the opportunity to debate each other's ideas, solutions and strategies, and when learners expressed their ideas to the teacher, she asked the other learners' opinion on that. All the teachers except for Teacher C acknowledged the importance of authority during the interview (see Table 5.3).

Teacher B was again the only teacher who promoted learners to be positive doers of mathematics (see Table 5.2) and was the only teacher who acknowledged the importance of mathematical identity during her interview (see Table 5.3). Teacher B encouraged learners to act independently by allowing them to make their own decisions on how to tackle problems and to decide which methods, including computational methods, are the most effective to use. Teachers A and C acknowledged the importance of learners having self-confidence, but did not mention anything about mathematics being enjoyable or learners acting independently.

To summarise, regarding agency, authority and identity, Teacher B was the only teacher who promoted learners' capacity and willingness to engage mathematically, provided opportunities for learners to demonstrate how knowledgeable they are and being recognised for that by the teacher; and encouraged learners to be positive doers of mathematics during her lessons. The intensity and duration of the use of this dimension by Teacher B also differed significantly from how it was used by the other two teachers, as she used them constantly and more meaningfully. Although Teachers A and C mentioned two of the three indicators during their interviews, neither promoted it to the extent that Teacher B applied it in her lessons.

6.1.5 Sub-question 5: Uses of assessment

Sub-question 5 is about uses of assessment: How does instruction build on learners' ideas or address emerging misunderstandings? The two TRU Math dimension indicators used to answer this question regarding uses of assessment are: soliciting learner thinking and building on learner ideas and misunderstandings (see Table 4.2). To answer this question, an observation schedule (see Addendum D) and post-observation interview schedule (see Addendum E) were used.

Although all teachers acknowledged the importance of soliciting learner thinking in their interviews (see Table 5.3), only Teacher B successfully gathered information about learner thinking during the observed lessons. Teacher B used techniques such as waiting time and sharing learner-generated solutions to check for understanding during the observed lessons.

Only Teacher B built on learners' ideas and addressed emerging misunderstandings (see Table 5.2). Teacher B used specific references to learner ideas, such as talking about a specific learner's idea and leading the class to build upon it. It was interesting to note that, although Teacher B built on learner ideas and misunderstandings during her instruction, she was the least informed about them during the interview. A possible reason for this may be that she was not entirely informed about what it means to build on learner ideas and misunderstandings. Furthermore, Teacher C was the best informed and acknowledged the importance of building on learners' ideas and addressing emerging misunderstandings (see Table 5.3). Teacher A was aware of addressing misunderstandings, but did not mention anything about building on learners' ideas. According to Curro Centre for Educational Excellence (2012), one learner's misunderstanding is frequently a communal misunderstanding in the class and the best way to treat misunderstandings is to address all emerging misunderstandings, however insignificant they may seem.

To summarise, regarding the uses of assessment, Teacher B built on learners' ideas and addressed emerging misunderstandings during her instruction. Although Teacher C acknowledged both indicators of using assessments during her interview, she did not apply them in any of her observed lessons. Teacher A mentioned one of the indicators during his interview and applied none during his instruction.

6.1.6 Summary

The primary research question is: How do Grade 8 mathematics teachers create and utilise opportunities to develop mathematical understanding in their classrooms? It has emerged that only one of the three teachers created and utilised sufficient opportunities to develop mathematical understanding in her classroom. Schoenfeld (2013) maintains that the five TRU Math dimensions of mathematically productive classrooms are the best way to promote learner understanding. Although no new dimensions or dimension criteria emerged from the study, the study clearly showed which dimensions were applied by teachers and which dimensions were neglected by teachers. It also indicated the extent to which these dimensions were applied by the teachers to create and utilise opportunities to develop mathematical understanding in their classrooms. Teacher A applied the TRU Math dimensions the second most of all the teachers. Teacher A applied the mathematics well, access to mathematical content moderately, and cognitive demand, agency, authority and identity and uses of assessment poorly during the observed lessons. Teacher B applied the TRU Math dimensions the most of all the teachers. Teacher B applied all TRU Math dimensions well during the observed lessons. Teacher C applied the TRU Math dimensions the least of all the teachers. Teacher C applied the mathematics moderately, and she applied cognitive demand, access to mathematical content, agency, authority and identity and uses of assessment poorly during the observed lessons.

To summarise, these were the opportunities created and utilised by the teachers to develop mathematical understanding in their classrooms:

- Teacher A: the mathematics; access to mathematical content.
- Teacher B: the mathematics; cognitive demand; access to mathematical content; agency, authority and identity; uses of assessment.
- Teacher C: the mathematics.

6.2 Conclusions

Having analysed the data and having reflected on the study, I have found that four major conclusions can be drawn from this research.

1. **Most of the teachers in this study did not have the sufficient skills to apply all Schoenfeld et al.'s (2014) TRU Math dimensions highly.**

Schoenfeld et al.'s (2014) TRU Math scheme was used to analyse three Grade 8 mathematics teachers to explore how they create and utilise opportunities to develop mathematical understanding in their classrooms. Table 6.1 contains a visual representation of how the teachers created and utilised opportunities to develop mathematical understanding in their classrooms. Despite its exploratory nature, this study offers some insight into which TRU Math dimensions are highly, moderately and poorly used. Table 6.1 indicates that that only one of the three teachers had the sufficient skills to apply all Schoenfeld et al.'s (2014) TRU Math dimensions highly. The results of this study indicate that two of the three teachers did not use all Schoenfeld et al.'s (2014) TRU Math dimensions to create and utilise opportunities to develop mathematical understanding in their classrooms. The dimension identified which the teachers applied highly in their classrooms was: the mathematics. The dimensions identified in which teachers still lack skills the most were: cognitive demand, access to mathematical content, agency, authority and identity and uses of assessment.

Table 6.1: Visual representations of how the teachers created and utilised opportunities to develop mathematical understanding in their classrooms

Teacher A	Teacher B	Teacher C
The mathematics	The mathematics	The mathematics
Cognitive demand	Cognitive demand	Cognitive demand
Access to mathematical content	Access to mathematical content	Access to mathematical content
Agency, authority and identity	Agency, authority and identity	Agency, authority and identity
Uses of assessment	Uses of assessment	Uses of assessment

HIGHLY	MODERATELY	POORLY

2. **Most of the teachers in this study still controlled the learning in class and made use of direct instruction.**

Schoenfeld et al.'s (2014) dimensions of a mathematically productive classroom are linked to a learner-centred approach towards teaching, as a learner-centred approach creates opportunities to develop mathematical understanding in teachers' classrooms. As indicated in Chapter 2, South Africa's education system has moved away from a traditional teacher-centred approach towards a learner-centred approach (DBE, 2001). However, the study's findings revealed that only one of the three teachers from the private schools in Mpumalanga who took part in this study applied a learner-centred approach. The other two teachers applied a traditional teacher-centred approach, not providing sufficient opportunities for learners to develop conceptual understanding. From the results of the study, one may conclude that most teachers still control the learning in class and make use of direct instruction.

3. **The three teachers in this study were still lacking in developing learners' understanding and problem-solving skills.**

From the literature, it has emerged that teaching should focus on the key development of concepts rather than on artificial facts and processes (McTighe & Seif, 2003). As discussed in Chapter 1, South African learners were ranked highly with their ability to recall facts and answer questions involving procedural knowledge. The problem, however, lies in learners' lack of problem-solving skills and higher cognitive abilities involving conceptual understanding (Spaull, 2013). This study's findings confirm previous findings of Spaull and Venkatakrishan (2014) and Maree and Van der Walt (2007) and contribute additional evidence that suggests that the three teachers of this study were still lacking in developing learners' understanding and problem-solving skills as none of the teachers encouraged learners to apply knowledge to unfamiliar problems.

4. **Although the tasks had potential for productive struggling, most teachers did not have sufficient skills to create opportunities for learners to make their own sense of mathematical ideas during the observed lessons.**

This study has shown the tasks created opportunities for learners to experience productive struggling as the tasks gave learners the opportunity to engage in key practices such as problem solving and reasoning. However, the lesson observations showed that only one of the three teachers used these tasks to provide opportunities for learners to be intellectually active and develop conceptual understanding through productive struggling. Although most of the tasks created opportunities for learners to experience productive struggling, two of the three teachers did not provide opportunities for learners to struggle on their own and did not allow the learners to think for themselves. The findings of the current study are consistent with those of the NCTM (2014), which found that teachers sometimes remove productive struggling by breaking down the task and guiding learners step by step through the difficulties or giving answers too early. Consequently, they undermine the hard work of learners, decrease the cognitive demand of the task and remove the opportunities from learners to engage fully in making sense of mathematics (NCTM, 2014). The teacher can also use probing questions to keep learners engaged in productive struggling until they solve the problem (Hiebert & Grouws, 2007).

6.3 Implications and recommendations for future research

The findings of this study add to a growing body of literature on Schoenfeld et al.'s (2014) TRU Math scheme. The teachers acknowledged the value of participating in the study as they reflected on their practices, especially during and after the interviews. The study not only enhanced the teachers' understanding of what it means to create and utilise opportunities to develop mathematical understanding in Grade 8 mathematics classrooms, but also my own understanding as researcher. The ultimate purpose of the study was to make a possible contribution towards improving the quality of teaching and learning in South African mathematics classes.

The findings of this study have several important implications. Having only three participants, the research findings cannot be generalised, but may provide insight into some Grade 8 mathematics teachers' instructional practices in a way that can inform other teachers' practices as to how opportunities can be created to enhance learners' understanding. The scholarly community can be informed of the significance the study has for educational policy. Teacher training institutions can also benefit from this study's findings in the manner that lecturers provide opportunities for learners to develop conceptual understanding in their mathematics courses, as well as in their methodology courses where they prepare future mathematics teachers. The findings can further be used to adjust the framework to be used again as a tool to analyse mathematics teachers' instruction.

Several aspects of the TRU Math dimensions require further research for teachers to create and utilise opportunities to develop mathematical understanding in their classroom. The following are possible studies that can be conducted:

- Focusing on only one of the five dimensions for a more in-depth investigation.
- In light of the fact that the fourth secondary research question was difficult to answer, this question could possibly be researched on its own to probe learners' knowledge of and beliefs concerning agency, authority and identity by making use of questionnaires and interviews.
- Focusing on the learners and investigating how learners learn with understanding using TRU Math dimensions.
- Assessing the impact of teaching mathematics in primary school, as it is especially in learners' earlier years where the foundation is laid and where teachers should also can create opportunities for the development of learners' mathematical understanding.

- Examining how the choice of documentary sources influence the application of the TRU Math dimensions during instruction.
- Replicating a study of this nature on a larger scale to investigate how teachers create and utilise opportunities to develop mathematical understanding in their classroom.
- Observing various teachers' instructional practices could give an indication of where the problems and challenges lie in teachers' attempts to develop learners' understanding.
- It would be interesting to compare tasks regarding promoting productive struggling and the implementation of it during lessons.

6.4 Limitations of this study

Several limitations to this case study need to be mentioned. Firstly, the findings of this research and the generalisability of the results were subject to certain limitations related to the participants. Using the constructivist paradigm, a case study was done with only three teachers who taught Grade 8 mathematics. Secondly, purposive and convenient sampling was used as I used private schools in Mpumalanga and I also chose schools in Mpumalanga because I lived in Mpumalanga at the time of data collection. Thirdly, all three teachers were white, two of the teachers were women, and one teacher was a man. The results could have been different if more male teachers or teachers from different racial backgrounds, or even teachers from government schools, had been included.

Furthermore, I was the only researcher in the study, which could have influenced the reliability of the collected data. Finally, due to time constraints, only two lessons presented by each teacher were observed. Although, at the time, two observed lessons were enough to give me a fair indication of how the teachers taught, it is possible that the results might have been different if more lessons had been observed or if lessons about other topics were covered as well.

6.5 Final reflections

From the time when I decided to complete a master's degree, I was determined to find a topic that interested me. It was my interest in this topic that kept me focused on my studies. I have learnt so much about myself and have grown personally, both in my work as a mathematics teacher and academically. I always felt that private schools were being neglected by researchers. This opinion was substantiated when I started doing research for the literature review and discovered how little research had been done in South Africa on teaching mathematics regarding the TRU Math dimensions in private schools. Private schools are a vital part of the education system as many South African learners attend private schools. Noting private schools in South Africa are often considered as models which public schools aspire to emulate in terms the equity of teaching, the findings of the current study regarding the teaching and learning of mathematics is a cause of concern.

The three teachers that participated in my research were open to sharing their experiences and time with me, and for that I am extremely grateful. The observations were challenging for me as I had to remain focused on what I had to observe and had to guard against becoming distracted. I experienced that the designing of data collection instruments is complex and that much thought has to go into how to formulate the questions for the questionnaires and how to record the data while observing. During the data analysis and presentation of the data, I became mindful of the complications involved in analysing the TRU Math dimensions. I realised that the boundaries between these dimensions are very vague. I also realised that not all stated beliefs are true beliefs.

It is my hope that this study will contribute to mathematics teachers' instructional practices in both private and government schools, as well as to pre- and in-service teacher training.

7. LIST OF REFERENCES

Adams, S. (2012). Understanding maths 'more important than learning by rote'. *The Telegraph*. Retrieved from http://www.telegraph.co.uk/education/educationnews/9415280/Understanding-maths-more-important-than-learning-by-rote.html

Aiken, L. R. (1970). Attitudes toward mathematics*. Review of Educational Research, 40*(4), 551-596.

Allsopp, D. H., Kyger, M. M., & Lovin, L. H. (2007). *Teaching mathematics meaningfully: Solutions for reaching struggling learners.* Baltimore: Brookes Publishing.

Andrews, T. (2012). What is social constructionism? *Grounded Theory Review*, *11*(1), 39-46.

Alpert, B.R. (1987). Active, silent, and controlled discussions: Explaining variations in classroom conversation. *Teaching and Teacher Education, 3*(1), 29–40.

Andreassen, R., & Braten, I. (2011). Implementation and effects of explicit reading comprehension instruction in fifth-grade classrooms. *Learning and Instruction, 21*, 520-537.

Anthony, G., & Walshaw, M. (2009a). Characteristics of effective teaching of mathematics: A view from the west. *Journal of Mathematics Education, 2*(2), 147-164.

Anthony, G., & Walshaw, M. (2009b). Effective pedagogy in mathematics. *Educational Practices Series, 19*. Brussels: International Academy of Education.

Arends, F. (2008). *Understanding teacher demand in South Africa*. Teacher Education Series. Cape Town: HSRC Press.

Armstrong, J. S. (2012). *Natural learning in higher education*. Encyclopedia of the Sciences of Learning. Heidelberg: Springer.

Ball, D. L. & Cohen, D. K. (1999). Developing practice, developing practitioners: toward a practice-based theory of professional development. In L. Darling-

Hammond & G. Skyes (Eds.), *Teaching as the learning professional: Handbook of policy and practice*. (pp. 3-32). San Francisco: Jossey-Bass.

Baroody, A. J., Feil, Y., & Johnson, A. R. (2007). An alternative reconceptualization of procedural and conceptual knowledge. *Journal for Research in Mathematics Education, 38*, 115-131.

Baxter, P., & Jack, S. (2008). Qualitative case study methodology: Study design and implementation for novice researchers. *The Qualitative Report, 13*(4), 544-559.

Bay-Williams, J. M. (2010). Influences on student outcomes: Teachers' classroom practices. In D. Lambdin (Ed.) *Teaching and learning mathematics: Translating Research for Elementary Teachers* (pp. 31-36). Reston, VA: NCTM.

Berniker, E., & McNabb, D. E. (2006). Dialectical inquiry: A structured qualitative research method. *The Qualitative Report*, *11*(4), 643-664. Retrieved from http://nsuworks.nova.edu/tqr/vol11/iss4/2

Bishop, A. J. (1988). *Mathematical enculturation a cultural perspective on mathematics education*. Dordrecht: Kluwer Academic Publishers.

Bisman, J. E. & Highfield, C. (2012). The road less travelled: An overview and example of constructivist research in accounting, *Australasian Accounting, Business and Finance Journal,* *6*(5), 3-22. Retrieved from http://ro.uow.edu.au/aabfj/vol6/iss5/2

Boaler, J. (2015). *Fluency without fear: Research evidence on the best ways to learn math facts*. Retrieved from http://youcubed.stanford.edu/fluency-without-fear.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.). (2000). *How people learn: Brain, mind, experience, and school*. Washington, D.C.: National Academy Press.

Brombacher & Associates. (2015). Curro mathematics 8. Curro Centre for Educational Excellence.

Brooks, J. G. & Brooks, M. G. (1993). *In search of understanding: The case for constructivist classrooms.* Alexandria, VA: Association for Supervision and Curriculum Development.

Burr, V. (2003). *Social constructionism* (2nd ed.). London: Routledge.

Bursal, M., & Paznokas, L. (2006). Mathematics anxiety and preservice elementary teachers' confidence to teach mathematics and science. *School Science and Mathematics, 104(*6), 173-180.

Cakir, M. (2008). Constructivist approaches to learning in science and their implications for science pedagogy: A literature review. *International Journal of Environmental & Science Education, 3*(4), 193-206.

Carpenter, T. P., Fennema, E., Franke, M., Levi, L. & Empson, S. B. (2000). *Cognitively guided instruction: A research-based teacher professional development program for elementary mathematics.* Research Report 003. Madison, WI: National Center for Improving Student Learning and Achievement in Mathematics and Science.

Carter, J. (2008). Visualization and understanding in mathematics. In B. Sriraman, C. Michelsen, A. Beckman, & V. Freiman: *Proceedings of the 2nd International Symposium on Mathematics and its Connections to the Arts and Sciences, Odense*. Centre for Science and Mathematics Education, University of Southern Denmark. (Vol 4).

Cazden, C. B. (2001). *Classroom discourse: The language of teaching and learning* (2nd ed.). Portsmouth, NH: Heineman.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, grades K–6* (2nd ed.). Sausalito, CA: Math Solutions.

Cobb, P., Gravemeijer, K., Yackel, E., McClain, K., & Whitenack, J. (1997). Mathematizing and symbolizing: The emergence of chains of signification in one first-grade classroom. In D. Kirschner & S. J. A. Whitson (Eds.), *Situated cognition: Social, semiotic and psychological perspectives* (pp. 151-233). Mahwah, NJ: Erlbaum.

Cohen, J. H., & Amidon, E. J. (2004). Reward and punishment histories: A way of predicting teaching style? *Journal of Educational Research, 97*(5), 269-277.

Cohen, L., Manion, L., & Morrison, K. (2011). *Research methods in education (7th ed.).* London: Routledge.

Cohen, E. G. & Lotan, R. A. (Eds.). (1997). *Working for equity in heterogeneous classrooms: Sociological theory in practice*. New York: Teachers College Press.

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, D.C.: National Governors Association Center for Best Practices and the Council of Chief State School Officers. Retrieved from http://www.corestandards.org/the-standards.

Creswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches* (4th ed.). USA: SAGE publications, Inc.

Curro Centre for Educational Excellence. (2012). *Curriculum and assessment overview: Senior phase, grades 7-9, Mathematics*. South Africa: Curro Centre for Educational Excellence.

Darling-Hammond, L. (Ed.). (2008). *Powerful learning: What we know about teaching for understanding*. San Francisco: Jossey-Bass.

Davis, R.B. (1992). Understanding "understanding". *Journal of mathematical behaviour, 11*, 225-241.

De Corte, E., Mason, L., Depaepe, F., & Verschaffel, L. (2011). Self-regulation of mathematical knowledge and skills. In B. J. Zimmerman, & D. H. Schunk (Eds.), *Handbook of self-regulation of learning and performance* (pp. 155–172). New York: Routledge.

Department of Basic Education (DBE). (2011). *National curriculum statement, curriculum and assessment policy statement, senior phase, grades 7-9, Mathematics*. Pretoria and Cape Town: Government Printers.

Department of Basic Education (DBE). (2012). *National curriculum statement, curriculum and assessment policy statement, senior phase, grades 7-9, Mathematics.* Pretoria and Cape Town: Government Printers.

Department of Education (DoE). (2000). *A South African curriculum for the 21st century. Report of the Review Committee on Curriculum 2005*. Pretoria: Government Printer.

Department of Education (DoE). (2002). *Revised national curriculum statement, grades R-9 (schools) policy*. *Overview, Mathematics.* Pretoria: Government Printer.

Engle, R.A. (2011). The productive disciplinary engagement framework: Origins, key concepts, and continuing developments. In D. Y. Dai (Ed.), *Design research on learning and thinking in educational settings: enhancing intellectual growth and functioning* (pp. 161-200). Londen: Taylor & Francis.

Erasmus, J., & Mda, T. (2008). *Scarce and critical skills: Educators*. Unpublished Report prepared for the Department of Labour project on Scarce and Critical Skills. Pretoria: HSRC.

Fakudze, C. G. (2004). Learning of Science concepts within a traditional socio-cultural environment. *South Africa Journal of Education, 24*(4), 270-277.

Fan, L. (Ed.). (2008, September). *Performance assessment in mathematics: Concepts, methods, and examples from research and practice in Singapore classrooms* [Resource booklet]. Singapore: National Institute of Education.

Fernandez, C., Yoshida, M., & Stigle, J. (1992). Learning mathematics from classroom instruction: On relating lessons to pupils' interpretations. *The Journal of Learning Sciences, 2*, 333–365.

Fillingim, J. G., & Barlow, A. T. (2010). From the inside out. *Teaching Children Mathematics, 17*, 80-88.

Flyvbjerg, B. (2011). Case study. In N. K. Denzin, & Y. S. Lincoln (Eds.), *The SAGE handbook of qualitative research* (4th ed.) (pp. 301-316). Los Angeles: SAGE.

Friedman, T. L. (2007). *The world is flat: A brief history of the twenty-first century*. New York: Farrar, Straus and Giroux.

Gamoran, A., & Nystrand, M. (1991). Background and instructional effects on achievement in eighth-grade English and social studies. *Journal of Research on Adolescence* (Lawrence Erlbaum), *1*(3), 277-300. doi:10.1111/1532-7795.ep11298224

Ginsburg, H. (1977). *Children's arithmetic: The learning process.* New York: Van Nostrand.

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education, 35*(4), 258-291.

Grasha, A. (2001). An integrated model of teaching and learning. In A. Grasha (Ph.D.) *Teaching with style: Enhancing learning by understanding teaching and learning styles*. (pp. 149-205). San Bernadino, CA: Alliance Publishers.

Gresalfi, M., & Cobb, P. (2006). Cultivating students' discipline-specific dispositions as a critical goal for pedagogy and equity. *Pedagogies: An International Journal, 1*(1), 49–57.

Gifford, S., & Latham, P. (2013). Removing the glass ceiling on mathematical achievement in primary classrooms: Engaging all pupils in mathematical learning. *Mathematics Teaching, 232*, 31-34.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D.A. Grouws (Ed.), *Handbook of research in mathematics teaching and learning* (pp. 65-97). New York: Macmillan.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., Stigler, J.W. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study (NCES 2003-013).* Washington, DC: U.S. Department of Education.

Hiebert, J., Gallimore, R., & Stigler, J. W. (2002). A knowledge base for the teaching profession. *Educational Researcher, 31*(5), 3–15.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F.K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Charlotte, NC: Information Age Publishers.

Hiebert, J. & Stigler, J. W. (2004). Improving mathematics teaching. *Educational Leadership, 61*(5), 12-17.

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's Mathematics Professional Development Institutes. *Journal for Research in Mathematics Education, 35*(5), 330-351.

Hinton, C., Fischer, K. & Glennon, C. (2013). Applying the science of how we learn. In R. Wolfe, A. Steinberg & N. Hoffman (Eds.) *Anytime, anywhere: student-*

centered learning for schools and teachers (153-170). Cambridge, MA: Harvard Education Press.

Hoffman, B. L., Breyfogle, M. L., & Dressler, J. A. (2009). The power of incorrect answers. *Mathematics Teaching in the Middle School, 15*(4), 232-238.

Hogan, J., Dolan, P. & Donnelly, P. (2009). Introduction. In J. Hogan, P. Dolan & P. Donnelly (Eds.), *Approaches to qualitative research: Theory and its practical application - A guide for dissertation students*, (pp. 1-18). Ireland: Oak Tree Press.

Human Science Research Council (HSRC). (2011). *Highlights from TIMSS 2011: The South African Perspective,* 5-12. Pretoria: HSRC.

Hyland, F. (2003). Focusing on form: Student engagement with teacher feedback. *System, 31*(2), 217-230.

International Association for the Evaluation of Educational Achievement (IEA). (2011). *IEA studies*. Retrieved from http://www.iea.nl

Karp, K. S. (1991). Elementary school teachers' attitudes toward mathematics: The impact on students' autonomous learning skills. *School Science and Mathematics, 91*(1), 265-270.

Kazeni, M. & Onwu, G. (2013). Comparative effectiveness of and context-based and traditional approaches in genetics: Student views and achievement. *African Journal of Research in Mathematics, Science and Technology Education, 17*(1), 50-62.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Labinowicz, E. (1985). *Learning from children: New beginnings for teaching numerical thinking*. Menlo Park: Addison Wesley.

Lauckner, H., Paterson, M. & Krupa, T. (2012). Using constructivist case study methodology to understand community development processes: Proposed methodological questions to guide the research process. *The Qualitative Report, 17*, 1-22.

Leahy, S., Lyon, C., Thompson, M., & Wiliam, D. (2005). Classroom assessment: Minute by minute, day by day. *Educational Leadership, 63*(3), 19–24.

Lerner, J. W., & Johns, B. (2009). *Learning disabilities and related mild disabilities* (11th ed.). Belmont: Wadsworth Cengage Learning.

Lester Jr, F. K., & Cai, J. (2016). Can Mathematical Problem Solving Be Taught? Preliminary Answers from 30 Years of Research. In *Posing and Solving Mathematical Problems* (pp. 117-135). Springer International Publishing.

Lim, D. H., & Morris, M. L. (2009). Learner and instructional factors influencing outcomes within a blended learning environment. *Educational Technology & Society, 12*(4), 282-293.

Lincoln, Y. S., Lynham, S. A., & Guba, E. G. (2011). Paradigmatic controversies, contradictions, and emerging confluences, revisited. In N. K. Denzin, & Y. S. Lincoln (Eds.). *The SAGE handbook of qualitative research* (pp. 97-128). Los Angeles: SAGE.

Longfield, J. (2009). Discrepant teaching events: Using an inquiry stance to address students' misconceptions. *International Journal of Teaching and Learning in Higher Education, 21*(2), 266-271.

Loveless, T. (2003). Trends in math achievement: The importance of basic skills. *Presentation at the Secretary's Summit on Mathematics, Washington, DC.*

Loyens, S., Magda, J., & Rikers R. (2008). Self-directed learning in problem-based learning and its relationships with self-regulated learning. *Educational Psychology Review, 20*, 411-427.

Ma, X. (1999). A meta-analysis of the relationship between anxiety toward mathematics and achievement in mathematics. *Journal for Research in Mathematics Education, 30*(5), 520-540.

Mack, N., Woodsong, C., MacQueen, K. M., Guest, G., & Namey, E. (2005). *Qualitative research methods: A data collector's field guide.* Research Triangle Park, North Carolina: Family Health International.

Manouchehri, A. (2007). Inquiry-discourse mathematics instruction. *The Mathematics Teacher, 101(4),* 290-300.

Maree, K. (2012). *First steps in research*. Pretoria: Van Schaik.

Maree, K., & Van der Walt, M. (2007). Do mathematics learning facilitators implement metacognitive strategies? *South African Journal of Education, 27*(2), 223-241.

McMahon, B. (2000). Scaffolding: A suitable teaching characteristic in one-to one teaching in maths recovery. In J. Bana & A. Chapman (Eds.), Mathematics education beyond 2000. *Proceedings of the twenty-third annual conference of the mathematics education research group of Australasia incorporated held at Fremantle, Western Australia, 5-9 July, 2000* (pp. 417-423). Fremantle: MERGA.

McRae, D. (2007). *Student engagement: Attendance, participation and belonging*. Canberra, Australia: Department of Education, Science and Training (DEST).

McTighe, J. & Seif, A. (2003). Teaching for meaning and understanding: A summary of underlying theory and research. *Pennsylvania Educational Leadership, 24*(1), 6-14.

Mercer, C. D., Mercer, A. R., & Pullen, P. (2014). *Teaching students with learning problems* (8th ed.). Harlow, Essex: Pearson Education.

Mertens, D. M. (2015). *Research and evaluation in education and psychology: Integrating diversity with quantitative, qualitative, and mixed methods* (3rd Ed.). London: SAGE Publications, Inc.

Meyer, B. (2013). *Mathematics and/of ourselves*. San Diego, CA: Create Space Independent Publishing Platform.

Miller, A. & Cunningham, K. (2011). *Classroom environment*. Retrieved from http://www.education.com/reference/article/classroom-environment/

Moller, A., Deci, E., & Ryan, R. (2006). Choice and ego-depletion: *The moderating role of autonomy. Personality and Social Psychology Bulletin, 32*, 1024-1036.

Mueller, M. & Maher, C. (2009). Convincing and justifying through reasoning. *Mathematics Teaching in the Middle School, 15*(2), 108-115.

National Mathematics Advisory Panel. (2008). *Reports of the task groups and subcommittees*. Washington, DC: Author.

National Council of Teachers of Mathematics (NCTM). (2000a). *Executive summary: Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics (NCTM). (2000b). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). (2007). *Effective strategies for teaching students with difficulties in mathematics.* Reston, VA.

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to action: Ensuring mathematics success for all*. Reston, VA: National Council of Teachers of Mathematics.

Nelissen, J. M. C., & Tomic, W. (1993). Learning and thought processes in realistic mathematics instruction. *Curriculum and Teaching, 8*(1), 19-38.

Nieuwenhuis, J. (2007). Introducing qualitative research. In J. G. Maree (Ed.), *First steps in research* (pp. 48-68). Pretoria: Van Schaik Publishers.

Oakes, J., Joseph, R., & Muir, K. (2001). Access and achievement in mathematics and science. In J. A. Banks & C .A. McGee Banks (Eds.), *Handbook of research on multicultural education* (pp.69-90). San Francisco: Jossey-Bass.

Ontario Ministry of Education, (2011). *Paying attention to mathematics education: Seven principles for improvement in mathematics K-12*. Retrieved from https://www.edu.gov.on.ca/eng/teachers/studentsuccess/FoundationPrincipals.pdf

Organisation for Economic Co-operation and Development (OECD). (2005). *Formative assessment: Improving learning in secondary classrooms*. Paris: OECD Publishing.

Parish, S. (2014). *Number talks: Helping children build mental math and computation strategies, grades K-5*, Updated with Common Core Connections. CA, USA: Math Solutions.

Patton, M.Q. (1990). *Qualitative evaluation and research methods* (2nd ed.). Newbury Park, CA: Sage.

Protheroe, N. (2007). What does good math instruction look like? *Principal*, *7*(1), 51-54.

Reddy, C. (2004). Assessment principles and approaches. In J. G. Maree, & W. J. Fraser (Eds.), *Outcomes-based assessment* (pp. 29-44). Sandown: Heinemann.

Redeker, G. (2000). Coherence and structure in text and discourse. In W. Black & H. Bunt (Eds.), *Abduction, belief and context in dialogue: Studies in computational pragmatics* (pp. 233-263). Amsterdam: Benjamins.

Reys, R. E., Suydam M. N., Lindquist M. M., & Smith N. L. (1999). *Helping children learn mathematics* (5th ed). New York: John Wiley and Sons.

Rule, P., & John, V. (2011). *Your guide to case study research.* Pretoria: Van Schaik.

Rypisi, C., Malcom, L. & Kim, H .S. (2009). Environmental and Developmental Approaches to Supporting Women's Success in STEM Fields. In S. R. Harper & S. J. Quaye. (eds.) *Student engagement in higher education: Theoretical perspectives and practical approaches for diverse populations* (pp. 117-135). New York and London: Routledge.

Sankaram, S., & Bui, T. (2001). Impact of learning strategies and motivation on performance: A study Web-based instruction. *Journal of Instructional Psychology, 28*(3), 191-198.

Saskatchewan Education. (1991). *Instructional approaches: A framework for professional practice*. Regina, SK: Saskatchewan Education.

Saunders, M and Tosey, P. (2013). *The Layers of Research Design*. Retrieved from https://www.academia.edu/4107831/The_Layers_of_Research_Design.

Sawyer, R. Keith. (2006). *The Cambridge handbook of the learning sciences*. New York: Cambridge University Press.

Schmidt, W. H. (2008). What's missing from maths standards? *Focus, rigor, and coherence. American Educator, 32*(1), 22–24.

Schnepper, L. C., & McCoy, L. P. (2013). Analysis of Misconceptions in High School Mathematics. *Networks, 15*(1).

Schoenfeld, A. H. (2013). Classroom observations in theory and practice. *ZDM, 45*(4), 607-621.

Schoenfeld, A. H., & Floden, R. E. the Algebra Teaching Study and Mathematics Assessment Project. (2014). *An introduction to the TRU Math dimensions*. Retrieved from http://map.mathshell.org/trumath/trumath_dimensions_alpha.pdf

Setati, M. (2002). The power of mathematics. City Press. Retrieved from http://www.amesa.org.za/Gethi2.htm

Sherman, H. J., Richardson, L. I., & Yard, G. J. (2013). *Teaching learners who struggle with mathematics* (3rd ed.). Upper Saddle River, NJ: Pearson Education, Inc.

Silver, D. (2011). Using the 'zone' to help reach every learner. *Kappa Delta Pi Record, 47*(1), 28-31.

Skemp, R. R. (1989). *Mathematics in the primary school*. London: Routledge Chapman & Hall.

Smith., S. (1996, November 27). Study rates Singapore best in math, science. *Boston College Chronicle*. Retrieved from http://www.bc.edu/bc_org/rvp/pubaf/chronicle/v5/N27/timss.html

Spaull, N. (2013). *South Africa's education crisis: The quality of education in South Africa 1994-2011.* Johannesburg, Pretoria: Report Commissioned by the CDE.

Spaull, N., & Venkatakrishnan, H. (2014). What do we know about primary teachers' mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. *International Journal of Educational Development, 41,* 121-130.

Spencer, L., Ritchie, J., Lewis, J., & Dillon, L. (2003). *Quality in qualitative evaluation: A framework for assessing research evidence*. London, UK: National Centre for Social Research, Government Chief Social Researcher's Office.

Star, J. R. (2015). *When not to persevere – Nuances related to perseverance in mathematical problem solving.* Chicago, IL: Spencer Foundation. Retrieved from http://hub.mspnet.org/index.cfm/28127/

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap*. New York: Free Press.

Stigler, J., & Perry, M. (1998). Mathematics learning in Japanese, Chinese, and American classrooms. *New Directions for Child Development, 41*, 27–54.

Stols, G. (2013). SA Journal of Education, Vol 33, No 1 (2013). *SA Journal of Education, 33(1).*

Swars, S. L., Daane, C. J., & Giesen, J. (2006). Mathematics anxiety and mathematics teacher efficacy: What is the relationship in elementary pre-service teachers? *School Science and Mathematics, 106*(7), 306-315.

Takahashi, A. (2006). Characteristics of Japanese mathematics lessons. *Paper presented at APEC-Tsukuba International Conference, January 2006, Tokyo, Japan.*

Taylor, R. (1990). Teacher expectations of students enrolled in an algebra course. In E.L. Edwards (Ed.). *Algebra for everyone*. Reston, VA: National Council of Teachers of Mathematics, Inc.

Terre Blanche, M., & Durrheim, K. (2006). Histories of the present: Social science research in context. In M. Terre Blanche, K. Durrheim, & D. Painter (Eds.), *Research in practice* (2nd ed.) (pp. 2-17). Cape Town: UCT Press.

Trilling, B. & Fadel, C. (2009). *21st Century Skills: Learning for life in our times*. San Francisco, CA: Jossey-Bass.

Van De Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2013). *Elementary and middle school mathematics teaching developmentally* (8th ed.). Pearson Education, Inc.

Verenikina, I. (2003). Understanding scaffolding and the ZPD in educational research. *Proceedings of The Joint AARE/NZARE Conference*. Retrieved from http://www.aare.edu.au/03pap/ver03682.pdf

Verenikina, I. (2008). Scaffolding and learning: Its role in nurturing new learners. In P. Kell, W. Vialle, D. Konza, & G. Vogl (Eds.), *Learning and the learner: Exploring learning for new times* (pp. 161–180). Wollongong: University of Wollongong, Australia.

Vygotsky, L. (1978). Interaction between learning and development. In *Mind and Society* (pp. 79-91). Cambridge, MA: Harvard University Press.

Walters, K., Smith, T., Leinwand, S., Surr, W., Stein, A., & Bailey, P. (2014). *An up-close look at student-centered math teaching*. Washington, DC: American Institutes for Research.

Wang, T., & Murphy, J. (2004). An examination of coherence in a Chinese mathematics classroom In K. Fan, J. Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics* (pp. 107–123). Davers: World Scientific Publication.

Wayne, A. J., & Youngs, P. (2003). Teacher characteristics and student achievement gains: A Review. *Review of Educational Research, 73*(1), 89-122.

Weber, E. (2008). *Educational change in South Africa: Reflections on local realities, practices, and reform*. Rotterdam. Sense Publishers.

Weiss, I. R., Pasley, J. D., Smith, P. S., Banilower, E. R., & Heck, D. J. (2003). *Looking inside the classroom: A study of K-12 mathematics and science education in the United States*. Chapel Hill, NC: Horizon Research, Inc.

Wiliam, D. (2008). Changing classroom practice. *Educational Leadership, 65*(4), 36-42.

Wilson, S. M., & Peterson, P. L. (2006). *Theories of learning and teaching: What do they mean for educators?* Washington, D.C.: National Education Association.

Writer, S. (2015). South Africa's school kids rank second-to-last in the world in maths and science. *My broadband*. Retrieved from http://mybroadband.co.za/news/general/126236-south-africas-schoolkids-rank-second-to-last-in-the-world-in-maths-and-science.html

Yin, R. K. (2009). *Case study research: Design and methods* (4th ed.). Los Angeles: SAGE.

Zull, J. E. (2011). *From brain to mind: Using neuroscience to guide change in education*. Sterling, VA: Stylus Publishing.

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

8. ADDENDUMS

8.1 Addendum A: Letters of permission and consent

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

FACULTY OF EDUCATION
Mrs G de Jager
Natural Science Building
Groenkloof campus
University of Pretoria
Gerdi.d01@curro.co.za
Cell: 076 476 4407

4 August 2015

Dear Mr Dawie van Emmenis

Letter of consent to the Regional Head of Curro Holdings

I hereby request permission to use Curro Schools for my research. I would like to invite Grade 8 **mathematics** teachers to participate in this research aimed at investigating opportunities for the development of understanding in Grade 8 mathematics classrooms. This research will be reported upon in my Master's dissertation at the University of Pretoria.

Before the start of the research I need to select my sample (3 teachers). For this I need Curro Holdings to select participating teachers according to the pre-determined criteria and willingness to participate in this research project. The three teachers will be selected based on the following criteria: The Grade 8 mathematics teachers should come from different Curro Schools and should have a minimum of four years teaching experience.

I intend to observe a mathematics teacher teaching a class for about 2 lessons. I would like to video record the lessons if consent can be obtained from all parties involved (principal, teacher and parents). This will allow clear and accurate record of the teacher's classroom practice. If the all the relevant parties do not give permission for the video recording of the lessons, then, I will position myself in such a way that those learners will not be part of the observation.

The process will be as follows: during the last term of this year and the first term of next year, should you look favourably upon my request, I would like to observe the lessons taught by a willing mathematics teacher for about 2 lessons on different days. After the second observation the teachers will be interviewed. Interviews will be used to explore teachers' thoughts about how they create and utilize opportunities to develop mathematical understanding in their classrooms. The interview will be scheduled at a time and place convenient to the teacher. Document analysis will only be conducted for clarification of the observations and interview if deemed necessary.

The learners will not take part in the research but will be in attendance of the class together with the researcher. The learners will receive a letter to inform them about the research that will be conducted. The parents/guardians will receive a letter of informed consent for the video recording of the lessons.

All participation is voluntary and once committed to the research the teacher(s) or school may still withdraw at any time. Confidentiality and anonymity will be guaranteed at all times by using a letter of the alphabet for the school and the teacher. Your school and the teacher will therefore not be identifiable in the findings of my research and only my supervisor and I will have access to the video/audio recordings which will be password protected. The data collected will only be used for academic purposes.

After the successful completion of my Master's degree I will give feedback to the school in the form of a written report and if the school is willing I would like to do a presentation of my findings to all mathematics teachers at your school.

For any questions before or during the research, please feel free to contact me. If you are willing to allow members of your staff to participate in this study, please sign this letter as a declaration of your consent.

Yours sincerely

________________________ 04/08/2015
Researcher: Mrs G de Jager Date

Botha 04/08/2015
Supervisor: Dr H Botha Date

I the undersigned, hereby grant consent to Mrs G de Jager to conduct her research in Curro Schools for her Master's research. I the undersigned, hereby also grand consent to Mrs G de Jager to video record the lessons.

Regional Head of Curro Holdings' name: ____________________

Regional Head of Curro Holdings' signature: _________________ Date: ________________

E-mail address: ____________________________________

Contact number: ___________________________________

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

FACULTY OF EDUCATION
Mrs G de Jager
Natural Science Building
Groenkloof campus
University of Pretoria
Gerdi.d01@curro.co.za
Cell: 076 476 4407

2 November 2015

Dear Sir / Madam

Letter of consent to the Principal

I hereby request permission to use your school for my research. I would like to invite one Grade 8 **mathematics** teacher to participate in this research aimed at investigating opportunities for the development of understanding in Grade 8 mathematics classrooms. This research will be reported upon in my Master's dissertation at the University of Pretoria.

If consent can be obtained from all parties involved (you, teacher and parents), the data collection process will be as follows: during the first term of 2016, the teacher will be asked to provide documentary sources such as mathematical tasks before each observation. I will observe the lessons taught by the mathematics teacher for about two lessons on different days in one week on different topics and done at a time convenient to the teacher and should not disrupt the teacher's timetable. The observations will be video recorded. This will allow for a clear and accurate record of the teacher's classroom practice. In cases where parents do not give permission for the video recording of the lessons, I will position myself in such a way that those learners will not be part of the observation. Apart from the two observations and document analysis, I would like to interview the teacher once individually after the two observed lessons. The interview should be conducted at a time and place convenient to the teacher and should not take longer than one hour. The interview will be audio-taped by me in order to have a clear and accurate record of all the communication that took place. The purpose of the interview is to explore teachers' views and opinions about how they create and utilize opportunities to develop mathematical understanding in their classrooms.

The learners will not take part in the research except for being present in class together with the researcher. The learners will receive a letter to inform them about the research that will be conducted. The parents/guardians will receive a letter of informed consent for the video recording of the lessons.

All participation is voluntary and once committed to the research, the teacher or school may still withdraw at any time. Confidentiality and anonymity will be guaranteed at all times by using a letter of the alphabet for the school and the teacher. The school and the teacher will therefore not be identifiable in the findings of my research and only my supervisor and I will have access to the video/audio recordings which will be password protected. The data collected will only be used for academic purposes. All data collected with public funding may be made available in an open repository for public and scientific use.

After the successful completion of my Master's degree, I will give feedback to the school in the form of a written report and if the school is willing I would like to do a presentation of my findings to all mathematics teachers at that school.

For any questions before or during the research, please feel free to contact me. If you are willing to allow a member of your staff to participate in this study, please sign this letter as a declaration of your consent.

Yours sincerely

____________________ 07/09/2015

Researcher: Mrs G de Jager Date

Botha 07/09/2015

Supervisor: Dr H Botha Date

I the undersigned, hereby grant consent to Mrs G de Jager to conduct her research in this school for her Master's degree research. I the undersigned, hereby also grand consent to Mrs G de Jager to analyse the teacher's documents, video record the lessons and audio record the interview.

School principal's name: ___________________________

School principal's signature: ________________________ Date: ________________

E-mail address: ___________________________________

Contact number: __________________________________

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

FACULTY OF EDUCATION
Mrs G de Jager
Natural Science Building
Groenkloof campus
University of Pretoria
Gerdi.d01@curro.co.za
Cell: 076 476 4407

30 October 2015

Dear Ms/Mr

Letter of consent to the mathematics teacher

You are invited to participate in a research project aimed at investigating opportunities for the development of understanding in Grade 8 mathematics classrooms. This research will be reported upon in my Master's dissertation at the University of Pretoria. It is proposed that you form part of this study's data collection phase by being observed teaching one mathematics class for about two lessons on different days in one week on different topics, providing documentary sources and being interviewed once.

The process will be as follows: Before the observation you will be asked to provide documentary sources such as mathematical tasks (15 minutes). Next you will present your lesson (40 minutes), and note that you are not required to do anything extra than what you normally do during teaching; no extra preparation is needed. The observations will be done at a time convenient to you and should not disrupt your timetable. During my observation of the lessons, I will make field notes on an observation sheet that has been prepared in advance based on the research questions to be answered. I would like to video record the lessons if consent can be obtained from all parties involved (principal, you and parents). This will allow for a clear and accurate record of your instructional practice. If permission is not granted from parents, then those learners may sit in the back of the classroom where they will not be video recorded. Apart from the two observations and document analysis, an interview will be done afterwards in order to receive feedback and to have a clear and accurate record of all the communication that took place (1 hour). The interview will be done outside school hours at a time and place convenient to you. The interviews will be audio-taped by me in order to have a clear and accurate record of all the communication that took place.

Should you declare yourself willing to participate in this research, you will be one of three teachers that form part of my research project. Your participation in this research is voluntary and confidential and anonymity will be guaranteed at all times. This will be done by allocating a letter of the alphabet to you and the school. You may decide to withdraw at any time without giving any reasons for doing so. You and your school will not be identifiable in the findings of my research and only my supervisor and I will have access to the video/audio recordings which will be password protected. You will have access to the transcription of the interview should you wish to do so. The data collected will only be used for academic purposes. All data collected with public funding may be made available in an open repository for public and scientific use.

After the successful completion of my Master's degree, I will give feedback of my findings to the school in the form of a written report and if the school is willing I would like to do a presentation of my findings to all mathematics teachers at your school.

If you are willing to participate in this study, please sign this letter as a declaration of your consent, i.e. that you participate willingly and that you understand that you may withdraw at any time.

Yours sincerely

______________________ 07/09/2015
Researcher: Mrs G de Jager Date

Botha 07/09/2015
Supervisor: Dr H Botha Date

I the undersigned, hereby grant consent to Mrs G de Jager to observe my Grade 8 class twice, have access to my documentary sources such as mathematical tasks and to conduct an interview with me for her Master's degree research. I grand consent to Mrs G de Jager to analyse my documents, video record the lessons and audio record the interview.

Teacher's name: ____________________________

Teacher's signature: _________________________ Date: ________________

E-mail address: _____________________________

Contact number: ____________________________

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

FACULTY OF EDUCATION
Mrs G de Jager
Natural Science Building
Groenkloof campus
University of Pretoria
Gerdi.d01@curro.co.za
Cell: 076 476 4407

7 Septmeber 2015

Dear Parent/Guardian

Letter of consent to the parents/guardians

I am currently enrolled for a Master's degree at the University of Pretoria. My research is focused on the way Grade 8 mathematics teachers create and utilise opportunities to develop mathematical understanding in their classrooms. In order to do this I will be observing two lessons in order to determine how your child's mathematics teacher teaches mathematics. I would like to video record these lessons as it will help me to have an accurate record of the teacher's teaching practice.

The focus of my research is on the teacher and not the learners in class. Your child does therefore not form part of my research, but will be present in the class while I observe and video record the teacher. The video recordings will be taken from the back of the class and I will, as far as possible, only film the teacher. Your child's confidentiality and anonymity will be protected at all times and only my supervisor and I will have access to the recordings. The video recordings will be password protected and will only be used for the completion of my Master's degree and not for any other purpose. All data collected with public funding may be made available in an open repository for public and scientific use.

If you have any questions or concerns, please do not hesitate to contact me. If you are willing for your child to be present during the video recorded lessons, please sign this letter as a declaration of your consent.

Yours sincerely

Researcher: Mrs G de Jager	07/09/2015 Date
Supervisor: Dr H Botha	07/09/2015 Date

I the undersigned, hereby grant consent to Mrs G de Jager to video record the lessons where my child will be present, for her Master's degree research. I am aware that my child will remain anonymous and that the findings of this research will be used to promote teaching and learning.

Parent's/guardian's name: ______________________________

Parent's/guardian's signature: ___________________________ Date: _____________________

Child's name: ____

Grade (e.g. 8E): ______________________________________

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

OPVOEDKUNDE FAKULTEIT
Mev. G de Jager
Aldoel gebou 1-136
Groenkloof kampus
Universiteit van Pretoria
Gerdi.d01@curro.co.za
Sel: 076 476 4407

30 Oktober 2015

Beste Ouer/Voog

Toestemmingsbrief aan die ouers/voog

Ek is tans ingeskryf vir 'n Meestersgraad aan die Universiteit van Pretoria. My navorsing is gefokus op die wyse waarop Graad 8 wiskunde-onderwysers geleenthede skep en benut om wiskundige begrip in hul klaskamers te ontwikkel. Om dit te doen, sal ek twee lesse waarneem om te bepaal hoe die wiskunde onderwyser wiskunde onderrig. Ek wil graag hierdie lesse met ‘n video kamera opneem om ’n akkurate rekord te hê van die onderwyser se onderrigpraktyk.

Die fokus van my navorsing is op die onderwyser en nie op die leerders in die klas nie. U kind sal dus nie deel vorm van my navorsing nie, maar sal teenwoordig wees in die klas terwyl ek die onderwyser waarneem en die les met ‘n video kamera opneem. Die video-opnames sal van agterin die klas geneem word en ek sal, sover moontlik, net die onderwyser afneem. Jou kind se vertroulikheid en anonimiteit sal te alle tye beskerm word en slegs ek en my studieleier sal toegang hê tot die opnames. Die video-opnames sal met ‘n wagwoord beskerm word en sal slegs gebruik word vir die voltooiing van my Meestersgraad en nie vir enige ander doeleindes nie. Alle data wat ingesamel is met openbare befondsing kan in 'n oop bron vir die publiek en wetenskaplike gebruik beskikbaar gestel word.

Indien u enige vrae of kommentaar het, moet asseblief nie huiwer om my te kontak nie. As u gewillig is dat u kind teenwoordig mag wees in die lesse waartydens die video opnames gemaak word, teken asseblief hierdie brief as ‘n bewys van toestemming.

Beste wense

Navorser: Mev G de Jager	07/09/2015 Datum
Botha Studieleier: Dr H Botha	07/09/2015 Datum

Ek, die ondergetekende, gee hiermee toestemming aan Mev G de Jager om 'n video-opname van die lesse te maak waar my kind teenwoordig gaan wees vir haar studies vir haar Meestersgraad. Ek is bewus daarvan dat my kind anoniem sal bly en dat die bevindinge van die navorsing gebruik sal word om onderrig en leer te bevorder.

Ouer/voog se naam: ________________________________

Ouer/voog se handtekening: __________________________ Datum: ___________________

Kind se naam en van: _______________________________

Graad van kind (bv. 8A): ____________________________

8.2 Addendum B: Letter of assent to learners

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

FACULTY OF EDUCATION
Mrs G de Jager
Natural Science Building
Groenkloof campus
University of Pretoria
Gerdi.d01@curro.co.za
Cell: 076 476 4407

2 November 2015

Dear Learner

Letter of assent to the learners

I am enrolled for a Master's degree at the University of Pretoria and want to determine how mathematics teachers teach mathematics. This implies that I will not be teaching you. When coming to observe two of your mathematics lessons, I will have a video camera as I want to film your teacher while s/he is teaching mathematics. I will be standing at the back of the classroom and the camera will be focused on your teacher and not you. The video will be used for my studies and no one will see the video recording but my supervisor and me.

You will not be involved in any way and you do not have to do anything except what your teacher expects you to do. If you have any questions you may contact me at any time.

Yours sincerely

____________________________ 07/09/2015
Researcher: Mrs G de Jager Date

Botha
____________________________ 07/09/2015
Supervisor: Dr H Botha Date

I, the undersigned, hereby grant assent to be present at the mathematics lessons that will be video recorded by Mrs G de Jager.

Learner's name: ____________________________

Learner's signature: __________________________ Date: ____________

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

OPVOEDKUNDE FAKULTEIT
Mev. G de Jager
Aldoel gebou 1-136
Groenkloof kampus
Universiteit van Pretoria
Gerdi.d01@curro.co.za
Sel: 076 476 4407

30 Oktober 2015

Beste Leerder

Brief van instemming aan die leerders

Ek is ‘n Meesters student van die Universiteit van Pretoria en wil graag bepaal hoe wiskunde-onderwysers klas gee. Dit impliseer dat ek nie vir julle sal klas gee nie. Wanneer ek twee van jou wiskundelesse kom waarneem, sal ek ‘n video kamera hê waar ek julle onderwyser gaan afneem terwyl hy/sy klasgee. Ek gaan agter in die klas staan en die kamera gaan hoofsaaklik net op julle onderwyser(es) gefokus wees. Die video gaan slegs gebruik word vir my studies en niemand anders behalwe ek en my studieleier gaan na die video kyk nie.

Jy gaan geensins betrokke wees nie en hoef niks anders te doen behalwe wat deur die onderwyser van jou verwag word nie. As jy enige vrae het, kan jy my enige tyd kontak.

Beste wense

______________________ 07/09/2015
Navorser: Mev G de Jager Datum

Botha ______________________ 07/09/2015
Studieleier: Dr H Botha Datum

Ek, die ondergetekende, stem hiermee in om teenwoordig te wees by die wiskunde lesse wat met ‘n video kamera opgeneem gaan word deur mev G de Jager.

Leerder se naam: ______________________________

Leerder se handtekening: ______________________ Datum: ____________________

Graad (bv. 8A): ______________________________

8.3 Addendum C: Document analysis: Mathematical tasks

Document Analysis: 1 2

Teacher in school: A B C Date: ………………………….

Mathematical tasks: ……………………………………………………………..………………………

Textbook: ………………………………………………………………………………………………

Research Question 1: The Mathematics

1.1: The mathematics discussed

1.1.1 How accurate is the content (aligned with the curriculum)?

Extremely ☐ Very ☐ Moderately ☐ Slightly ☐ Not at all ☐

Comments: ..……………………………………………………………………………

Examples:

1.2: Coherence

1.2.1 How coherent is the content (logical)?

Extremely ☐ Very ☐ Moderately ☐ Slightly ☐ Not at all ☐

Comments: ………………………………………………………………………..…………..

Examples:

1.3: Connections

1.3.1 Connection between concepts are properly addressed and explained (The mathematics supports learners to develop mental representations as part of bigger networks and builds on learners' connections between concepts, instead of letting them memorise concepts).

Extremely ☐ Very ☐ Moderately ☐ Slightly ☐ Not at all ☐

Comments: ………………………………………………………………………………

Examples:

Research Question 2: Cognitive demmand

2.1: Productive struggling

2.1.1 Learners have the opportunity to experience productive struggling.

Extremely ☐ Very ☐ Moderately ☐ Slightly ☐ Not at all ☐

Comments: ………………………………………………………………………………

Examples:

Research Question 3: Access to Mathematical Content

The document analysis is not applicable to Research Question 3: *How does the teacher support access to the content of the lesson for all learners?*

Research Question 4: Agency, Authority and Identity

The document analysis is not applicable to Research Question 4: *What opportunities do learners have to explain their own and respond to each other's mathematical ideas?*

Research Question 5: Uses of Assessment

The document analysis is not applicable to Research Question 5: How does instruction build on learners' ideas or address emerging misunderstandings?

8.4 Addendum D: Observation schedule of teachers' instructional practice

Observation: 1 2

Date: ……………………………….. Teacher in school: A B C

Time: ………………………………. Period: ………………………

Background information

Section A: The lesson and class observed

A1. Type of lesson: New / Revision / Other	
A2. Size of class	
A3. Organisation of the learners in the classroom (Rows, groups, pairs)	

Specifics about the teaching and learning in this lesson

Research Question 1: The Mathematics	***1.1: The mathematics discussed*** 1.1.1 When learners suggest new ideas, the teacher encourages them to use these ideas outside the planned curriculum and let the learners be part of the process of discovering
	1.2: Coherence 1.2.1 How coherent is the content (logical)?
	1.3: Connections 1.3.1 Building meaningful connections between concepts (Connect to previous knowledge and experiences, real-life examples and generalisation)
Research Question 2: Cognitive Demand	***2.1: Productive struggling*** 2.1.1 Learners have the opportunity to experience productive struggling (Sustain productive struggling and sense making)
	2.2: Zone of proximal development (ZPD) 2.2.1 Stress the connections between the learners' pre-knowledge and experience of a task in everyday contexts with the new task or concept being learned

Research Question 3: Access to Mathematical Content	***3.1: Active participation*** 3.1.1 There was evidence of active participation by learners during the lesson
	3.2: Equal access 3.3.1 Even access or participation and providing mathematical access to a wide range of learners
Research Question 4: Agency, Authority and Identity	***4.1: Agency*** 4.1.1 Learners have the opportunity to estimate, explain and justify the soundness of their answers
	4.2: Authority 4.2.1 Learners have self-confidence to do mathematics and develop their skills to solve various problems
	4.3: Mathematical identity 4.3.1 Learners explore new knowledge and share their new ideas
Research Question 5: Uses of Assessment	***5.1: Solicits learner thinking*** 5.1.1 Gathers information about learner thinking
	5.2: Builds on learner ideas and misunderstandings 5.2.1 Building on learners' ideas and addressing emerging misunderstandings

8.5 Addendum E: Teacher post-observation interview schedule

Date: ……………………………….. Time: ……………………….

Teacher in school: A B C Period: ……………………….

Introduction

I appreciate you allowing me to observe your classes. I have some questions I would like to ask you related to the lessons you presented. May I please audio record the interview? It will help me stay focused on our conversation and it will ensure I have an accurate record of what we discussed.

Background information

Section B: The teachers' personal information

B.1 Gender: Male Female

B.2 How old are you?

B.3 For how long have you been teaching secondary school learners?

B.4 What is your position at your school (teacher, principal, etc.)?

B.5 What is your qualifications?

Section C: The teacher's classes

C.1 I am currently teaching

a) Mathematics to grades: 8 9 10 11 12

b) Mathematical Literacy to grades: 10 11 12

C.2 More information:

Subject	Grade	Number of classes	Number of years of teaching experience	Number of learners in class	Learners per class (avg)
Mathematics	8				
	9				
	10				
	11				
	12				

The questions are based on the topics you taught in the observed lessons:

Research Question 1: The Mathematics

1.1: The Mathematics discussed

Questions about *the mathematics* in the observed lesson (e.g., accurate, coherent, topics in depth and problem solving).

Question 1.1.1: ………………………………………………………………………………

1.2: Coherence

Questions about the *coherence* in the observed lesson (e.g., how logical is the content?).

Question 1.2.1: ..……………………………………………………………………………

1.3: Connections

Questions about the *connections* in the observed lesson (e.g., mental representations, builds on learners' connections between concepts, connection between concepts addressed and opportunities for learners to engage with rich content).

Question 1.3.1: ………………………………………………………………………………

Research Question 2: Cognitive Demand

2.1: Productive struggling

Questions about the *productive struggling* in the observed lesson (e.g., make their own sense of mathematical ideas).

Question 2.1.1: ………………………………………………………………………………

2.2: Scaffolding

Questions about the *scaffolding* in the observed lesson (e.g., extent to which teacher provides scaffolding to reduce the zone of proximal development).

Question 2.2.1: ………………………………………………………………………………

Research Question 3: Access to Mathematical Content

3.1: Active participation

Questions about the *active participation* in class discussions in the observed lesson (e.g., who asks questions and who answers).

Question 3.1.1: ………………………………………………………………………………

3.2: Equal access

Questions about the *equal access* in the observed lesson (e.g., provide mathematical access to a wide range of learners through equity).

Question 3.2.1: ……………………………………………………………………………………

Research Question 4: Agency, Authority and Identity

4.1: Agency

Questions about the *agency* in the observed lesson (e.g., capacity and willingness to engage and learners' perceptions).

Question 4.1.1: ……………………………………………………………………………………

4.2: Authority

Questions about the *authority* in the observed lesson (e.g., learners' opinions are recognized by others).

Question 4.2.1: ……………………………………………………………………………………

4.3: Mathematical identity

Questions about the *mathematical identity* in the observed lesson (e.g., faith in their ability to understand).

Question 4.3.1: ……………………………………………………………………………………

Research Question 5: Uses of Assessment

5.1: Solicits learner thinking

Questions about gathering information about learner thinking (e.g., discussions or probing).

Question 5.1.1: ……………………………………………………………………………………

5.2: Builds on learner ideas and misunderstandings

Questions about *building on learners' ideas and addressing emerging misunderstandings* in the observed lesson (e.g., instruction builds on learners' ideas or addresses emerging misunderstandings).

Question 5.2.1: ……………………………………………………………………………………

Thank you for your time. If I have any additional questions or need clarification, how and when is it best to contact you?