UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Using GeoGebra in transformation geometry: an investigation based on the Van Hiele model

by

Grace Ramatsimele Kekana

Submitted in partial fulfilment of the requirements for the degree

Master of Education

in the Faculty of Education,

University of Pretoria

Supervisor: Dr J.N. Mwambakana

Co-Supervisor: Prof G.H. Stols

31 August 2016

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

DECLARATION

I declare that the dissertation, which I hereby submit for the degree Master of Education at the University of Pretoria, is my own work and has not previously been submitted by me for a degree at this or any other tertiary institution.

....

G.R. Kekana Date

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA
Faculty of Education

RESEARCH ETHICS COMMITTEE

CLEARANCE CERTIFICATE	CLEARANCE NUMBER: SM 14/04/01
DEGREE AND PROJECT	MEd Using GeoGebra in transformation geometry: an investigation based on the Van Hiele model
INVESTIGATORS	Grace Ramatsimele Kekana
DEPARTMENT	Science, Mathematics and Technology Education
APPROVAL TO COMMENCE STUDY	16 September 2014
DATE OF CLEARANCE CERTIFICATE	05 September 2016

Please note:
For Master's application, Ethics Clearance is valid for 2 years
For PhD application, Ethics Clearance is valid for 3 years
CHAIRPERSON OF ETHICS COMMITTEE: Prof Liesel Ebersöhn

_______________ ____________

CC Bronwynne Swarts
Jeanine Mwambakana
Gerrit Stols

This Ethics Clearance Certificate if issued subject to the following conditions:

1. A signed personal declaration of responsibility
2. If the research question changes significantly so as to alter the nature of the study, a new application of ethical clearance must be submitted
3. It remains the student's responsibility to ensure that all the necessary forms for informed consent are kept for future queries

Please quote the clearance number in all enquiries

ABSTRACT

This study investigated the use of an advanced technological development (free GeoGebra software) within the secondary educational setting in four relatively under-resourced schools in the Gauteng Province of South Africa. This advancement is viewed as having the potential to promote the teaching and learning of complex ideas in mathematics, even within traditionally deprived communities. The focus in this study was on the teaching and learning of transformation geometry at Grade 9 and attainment was reflected in terms of the van Hieles' levels of geometrical thinking. A mixed methods approach was followed, where data was collected through lesson observations, written tests and semi-structured interviews. Four Grade 9 teachers from four schools were purposively selected, while twenty-four mathematics learners (six from each school) in the Tshwane metropolitan region were randomly selected. The teachers' lesson observations and interview outcomes were coded and categorised into themes, and the learners' test scripts were marked and captured. The analysis of test scores was structured according to the van Hieles' levels of geometric thought development. As far as the use of GeoGebra is concerned, it was found that teachers used the program in preparation for, as well as during lessons; learners who had access to computers or android technology, used GeoGebra to help them with practice and exercises. As far as the effect of the use of GeoGebra is concerned, improved performance in transformation geometry was demonstrated.

Keywords: GeoGebra; transformation geometry; van Hieles' levels; free mathematics software; Grade 9 mathematics; rotation; translation; reflection; enlargement; reduction.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABBREVIATIONS

ACE	Advanced Certificate in Education
ANA	Annual National Assessment
ANOVA	Analysis of Variance
APA	Angles and Polygons Achievement
CAPS	Curriculum and Assessment Policy Statement
CAS	Computer Algebra System
CCA	Circle and Cylinder Achievement
CPGT	Conjecturing in Plane Achievement
DBE	Department of Basic Education
DGS	Dynamic Geometry System
DIPIP	Data Informed Practice Improvement Project
FET	Further Education and Training
GDE	Gauteng Department of Education
GIST	Geometric Item Sorting Test
GSP	Geometers' Sketchpad
HED	Higher Education Diploma
HSRC	Human Science Research Council
ICAS	International Competitions and Assessment for Schools

ICT	Information and Communications Technology
IGI	International GeoGebra Institute
KSA	Kingdom of Saudi Arabia
LSEN	Learners with Special Educational Needs
LTSM	Learner Teacher Support Material
MANCOVA	Multivariate Analysis of Covariance
NCS	National Curriculum Statement
NCTM	National Council for Teachers of Mathematics
NGO	Non-Governmental Organisation
NPPPR	National Policy Pertaining to the Programme and Promotion Requirements of the National Curriculum Statement Grades R – 12
NRC	National Research Council
SGAT	Solid Geometry Achievement Test
TG	Transformation geometry
TIMSS	Trends in International Mathematics and Science Study
VHGT	Van Hieles' Geometry Test
VPG	Video Pembelarajan Geometri
2D	Two dimensional
3D	Three dimensional

ACKNOWLEDGEMENTS

I wish to express my appreciation to the following organisations and individuals for making this study possible:

- To the NRF for funding towards my tuition.
- To the District Director and principals of participating schools for giving me permission to conduct my research.
- To Dr J.N. Mwambakana, my supervisor, and Prof G.H. Stols, my co-supervisor for their guidance and support throughout the study.
- To Dr M.C. du Plooy for being there when I was about to give up.
- To David, Lesego and Mahlogonolo Kekana, my family, for their support and for the encouragement that I needed.
- To Agnes Mamombe for her assistance with the table of contents.
- To the mathematics teachers and learners who participated in this study and their parents.
- To the University of Pretoria for giving me the opportunity to be a better person academically.
- To the Lord Almighty for giving me strength throughout this academic journey.

CHAPTER 1 - INTRODUCTION AND OVERVIEW

1.1 Orientation and Background

Trends in Mathematics and Science Study (TIMSS) conducted in 1995, 1999, and 2011 indicate the low performances of South African learners as compared to other countries (Bleeker, 2011; Spaull, 2011). 45 countries participated in the TIMSS 2011 study, of which 11969 Grade 9 learners participated. The findings revealed that South African learners achieved on average 352 points out of a total of 600. According to the report this performance is low as compared to other countries that participated. In 2002 the score was at 285 points out of 600. Comparing 2002 and 2011 there is a slight improvement in the South African score, however there is still reason for concern.

According to Spaull (2013), South Africa's education system is poor compared to other countries with the same socio-economic status. Out of every 100 learners who start school together, only 50 enter Grade 12, and out of that 50; only 40 learners pass Grade 12 (Spaull, 2013). South Africa began a process of national learner assessment in 2011 known as the Annual National Assessment (ANA) to assess the success of the educational system at intervals of Grade 3, 6 and 9. The Human Science Research Council (HSRC) verified in 2011 the poor performance levels of the South African learners. The 2011 ANA tests for South African learners also confirmed this poor performance.

Learners' poor performance may be attributed to a mismatch between the requirements of the curriculum and learners' levels of thinking as indicated by Bleeker (2011). The implication is that the learners and their teachers, as the implementers of the curriculum, operate at different levels during their interactions in the classroom. Sometimes the mismatch happens because the teacher is not aware of the level at which his/her learners operate cognitively. As will become clear later on, the implication is that, at least as far as geometry is concerned, the teacher may not be aware of the van Hieles' theory. Learners might seem to understand the topic, while the reality is that learners might have memorised the answers and then reproduced them without the ability to explain in detail how they obtained the solution.

According to Brodie (2011), the International Competitions and Assessments for Schools (ICAS) study done in 2006 and 2008 by the Department of Education also shows the low levels of achievement of learners generally in mathematics. The ICAS was done in partnership with Wits School of Education in a project called the Data Informed Practice Improvement Project (DIPIP), to inform teaching and learning practices in Gauteng schools. The findings from this project indicated the low levels achievement of learners in mathematics generally; such results are not encouraging.

Learners' poor performance in mathematics may also be linked to the phenomenon of mathematics anxiety. Mathematics' anxiety is the feeling of helplessness in an individual in relation to doing mathematics (Uusimaki & Nason, 2004). This anxiety may occur in both teachers and learners alike. Teachers who suffered from mathematics' anxiety during their in-service training may still lack confidence in their own abilities. On the other hand, learners who are underperforming as mentioned above lose confidence in their own abilities and as a result are prone to fall victim to mathematics' anxiety.

In a study by Uusimaki and Nason (2004) the reasons behind pre-service teachers' negative beliefs as well as their fears about mathematics, were investigated. This study confirmed that pre-service teachers certainly have undesirable beliefs about mathematics, which have a definite influence on their careers later in their lives. The mismatch (between learner and teacher understanding) can therefore be brought about by the fact that teachers themselves do not have the confidence to handle complex topics with learners. As a result, they spend most of their time on teaching simple topics at the cost of the more complicated topics (Stols, 2013). This situation calls for remedial interventions targeted at teachers, which may empower them to understand how the mathematics content is actually divided in the Curriculum and Assessment Policy Statement (CAPS). Then they can focus their teaching time on sections that carry more weight. If more time is spent on more complex topics as per Grade 9 mathematics content, maybe learners will understand the topics at higher levels. Mathematics content knowledge in Grade 9 is divided into five sections; namely; Numbers, Operations and Relationships (15%); Patterns, Function and Algebra (35%); Space and Shape (geometry) (30%); Measurement (10%) and Data Handling (10%) as indicated in mathematics CAPS document for the Senior Phase (Grades 7-9).

The focus of this study is on transformation geometry at Grade 9, which is allocated 9 hours teaching time in Term 4. Transformation geometry is traditionally viewed as a challenging topic and the fact that it is placed at a late stage in the year; often results in teachers neglecting it for various reasons. Transformation geometry carries a significant weight within the fourth term; as a result there might be serious implications if learners do not grasp the concepts involved.

1.2 Problem Statement

Along with many other reports, Spaull (2011) and Bleeker (2011) reported in their research the low levels of mathematics achievement in South Africa. South Africa has been graded at the end of the list of achievement in mathematics in TIMSS reports of 1995, 1999 and 2011. Furthermore, the systemic assessments (ANA) done internally, confirm the dilemma of South African mathematics education.

In a study by Shadaan and Leong (2013), it was established that learners have difficulty in understanding aspects that are linked to transformation geometry. There exists proof that learners who are between the ages of 11 and 15 have farfetched misconceptions with regard to geometric concepts (Evbuomwam, 2013). In transformation geometry learners are expected to translate, enlarge/reduce, rotate and reflect object(s). If there is either a mismatch between teacher and learner understanding, or teachers do not use effective teaching approaches, learners will develop misconceptions which will lead to poor performance (Evbuomwam, 2013).

A possibility exists that, given teachers' perceived low levels of confidence to teach this complex topic, they have perhaps not accessed the available technological resources to support the teaching of transformation geometry. For the purpose of this study, a specific technological advancement (GeoGebra) will be investigated.

Although considerable research has been conducted regarding the general use of technology in the teaching and learning of mathematics, there is still little research about the use of GeoGebra in developing countries such as South Africa (Bansilal, 2015). Furthermore, the literature revealed that research conducted on GeoGebra, focused on geometry in general and not on transformation geometry specifically. This study is aimed at investigating the potential of the use of GeoGebra in transformation geometry based on van Hieles' model.

1.3 Aim

The aim of this study was to investigate, on a small scale, the potential of the use of GeoGebra in teaching and learning of transformation geometry to Grade 9 learners.

1.4 Rationale for the Study

According to Lu (2008), algebra and geometry are two mathematics' content areas in which technology has been used for teaching and learning purposes. The researcher was introduced to Computer Science (now known as Information Technology) at the University of Limpopo (MASTEC Campus) in 1997 when enrolled for Higher Diploma in Education (HED). Interest in the advantages of technology, specifically GeoGebra, in mathematics teaching was instilled by a module known as Computers in Maths at the University of Pretoria when the researcher enrolled for an Advanced Certificate in Education (ACE) in 2008.

Transformation geometry content at Grade 9 requires learners to do transformations, in which learners have to recognise, define and do transformations with points, line segments and simple geometric figures on a co-ordinate plane, focusing on the following: reflection in the x- or y axis; translation within and across quadrants; reflection in the line y = x. Learners have to identify what the transformation of a point is, if the co-ordinates of its image are known.

Learners are also required to do extensions and reductions whereby they use ratio to describe the effect of enlargement or reduction on area and perimeter of geometric figures; examine the co-ordinates of the vertices of figures that have been enlarged or reduced by a given scale factor as stipulated in CAPS for senior phase.

The CAPS document for the Senior Phase mathematics indicates that learners must be taught mathematics 4.5 hours per week for each term (two and a half months). The implication in CAPS is that learners in Grade 9 have 180 hours per year notional teaching time for mathematics. The CAPS curriculum is supported by the National Policy Pertaining to the Programme and Promotion Requirements (NPPPR), which requires learners in Grade 9 to obtain a minimum of 40% in mathematics. This target, in my years of experience in teaching Grade 9 mathematics, has been very difficult for learners to achieve.

In analysing the progression of the concepts in transformation geometry from Grade 4-9, it has become clear that the volume and complexity of the topic increases to such an extent that it requires not only a huge cognitive leap for learners, but it also requires greater support for teachers to bridge the progression. Table 1 shows the progression of transformation and transformation geometry learning from Grade 4 to Grade 9.

Table 1

Progression in Transformations in Geometry from Grade 4-9 (Grades 4-6 Mathematics CAPS 2011, Grades 7-9 Mathematics CAPS 2011)

Level	Skills and Transformations	Descriptions
Grade 4: Term 3 (3h); Term 4 (3h)	Tessellate shapes with line symmetry Build composite 2D shapes	Describe patterns in the environment
Grade 5: Term 3 (3h); Term 4 (4h)	Tessellate by rotating, translating and reflecting shapes with line symmetry Physically rotate, translate and reflect to build composite 2D shapes	Describe patterns in the environment in relation to rotation, translation and reflection
Grade 6: Term 3 (3h); Term 4 (3h)	Enlarge and reduce triangles and quadrilaterals	Describe patterns in environment in relation to rotation, translation and reflection
Grade 7: Term 3 (9h)	Identify and draw lines of symmetry in geometric figures Enlarge, reduce, translate, rotate and reflect geometric figures on *quad paper*	Describe geometric figures in relation to rotation, translation and reflection
Grade 8: Term 4 (6h)	Reflect a point (in Y- and X-axis) and translate a point on a *coordinate plane* Reflect a triangle (in Y- and X-axis) and translate a triangle (within and across quadrants) on a *coordinate plane* Rotate a triangle around the origin of a *coordinate plane* Enlarge and reduce area and perimeter of geometric figures on squared paper using proportions	Describe triangle transformations in relation to changes in coordinates of vertices
Grade 9: Term 4 (9h)	Identify the nature of transformation of a point given the co-ordinates of its image Investigate the co-ordinates of vertices of geometric figures which have been enlarged or reduced by a scale factor Identify the effect of enlargement and reduction on area and perimeter of geometric figures in relation to proportions	* Describe reflections with points, line segments and geometric figures in relation to X- and Y- axes and in the line x=y * Describe translation with points, line segments and geometric figures within and across quadrants

1.5 Objectives of the Study

The objectives of the study are as follows:

1.5.1 To evaluate how teachers utilise GeoGebra in teaching transformation geometry.

1.5.2 To measure the effect of using GeoGebra on learners' understanding of transformation geometry.

1.5.3 To establish the teachers' and learners' perceptions on the use of GeoGebra in teaching and learning of transformation geometry.

1.6 Research Questions

The intention of the study was to conduct a small scale investigation based on van Hieles' model on the use of GeoGebra in teaching and learning transformation geometry to Grade 9 learners. As a result, the research questions were formulated as follows:

1.6.1 Primary Research Question

The primary research question guiding the broad investigation was formulated as follows: "How is GeoGebra currently utilised and its effect in the teaching and learning of transformation geometry in selected schools at Grade 9?"

1.6.2 Secondary Research Questions

In order to fully explore the primary question the following components, serve as specific guidelines towards the investigation, were formulated as follows:

1. How do teachers utilise GeoGebra in the teaching of transformation geometry?
2. How does the use of GeoGebra affect learners' understanding of transformation geometry?

3. What are the teachers' and learners' experiences about the use of GeoGebra in the teaching and learning of transformation geometry?

1.7 Research Methodology

In order to enable the investigation into the primary and secondary research questions, the following research methodology was adopted. The specific aspects of the research methodology are discussed in detail in Chapter 3.

1.7.1 Mixed Methods Approach

Following the diverse nature of the secondary research questions, quantitative and qualitative methods were used in this study.

Qualitative research is a research approach that is "generally concerned with interpretation and meaning" (Morgan & Sklar, 2012, p. 72). The interest is in understanding the participants' worlds, their understanding and the attribution to their experiences. The researcher plays a pivotal role in collecting the data. There are various tenets in respect of qualitative paradigm; a researcher can go with interpretivism, constructivism, phenomenology, the humanistic philosophies and postmodernism. Qualitative research designs are phenomenology, grounded theory, ethnography, action research and case study research. Researchers may choose from interviews, observations, documents in respect of techniques for data collection. In this study, semi-structured interviews and classroom observations were used to collect data.

As far as the quantitative research is concerned, "an investigator relies on numerical data to test the relationships between the variables" (Ivankova, Creswell & Plano Clark, 2007, p. 257). An experimental, quasi-experimental, correlational and survey are said to be typical types of research designs to be employed with quantitative research approach. The sample is randomly selected from the population. In this study a sample of learners and teachers were selected using a simple random sampling. Tests, surveys, scales and behavioural check-lists may be used as data collection techniques. In this study, a paper and pencil test was used to collect the data.

Mixed methods research involves the combination of both quantitative and qualitative research approaches. In mixed methods research, "the researcher constructs knowledge about real-world issues based on pragmatism, which places more emphasis on finding the answers to research questions than on the methods used" (Ivankova et al, 2007, p. 262). In this study, both quantitative and qualitative data were collected and analysed in order to answer the primary question about the use of GeoGebra in the teaching and learning of transformation geometry in Grade 9.

1.7.2 Instruments for Data Collection

In this mixed methods research, quantitative and approach were adopted. Semi-structured interviews; lesson observations; paper and pencil test; as well as documents were used to collect data. Additionally, field notes were taken to support the analysis of classroom observations and interviews.

1.7.3 Research Site and Participants

This research was undertaken in one school district in Gauteng Province, within four secondary township schools. Four teachers from four schools and twenty-four Grade 9 learners (six from each school) were sampled. One hour classroom observations were conducted during schools hours, while twenty minutes semi-structured interviews and one hour paper and pencil test were conducted after school hours.

1.7.4 Analysis of Data

The outcomes of classroom observations and semi-structured interviews were coded and categorised into themes enabling some inferences about the use of GeoGebra. The tests were marked and the scores were captured into Excel as percentages. This enabled a general view of the effect of GeoGebra on the learners' levels of understanding.

1.8 Limitations of the Study

The sample size was small and as a result generalisations based on the data resulting from this study cannot be justified. Although the study opened up some avenues for further research, the inferences and conclusions reached must be viewed in the limited context within which the study was conducted. All perceived limitations, together with suggestions for further research, are listed in Chapter 5.

1.9 Clarification of Terms

The terms below are used frequently in the study and therefore are defined for a richer and brief meaning:

- Transformation geometry: the study of figures and the properties of figures conserved by reflections, translations, rotations and their groupings.
- GeoGebra: dynamic free mathematics software.
- Baseline assessment: assessment given to learners by teachers to investigate their current knowledge.
- FET: Further Education and Training (Grade ten to twelve in schools).

1.10 Layout of the Study

This dissertation is outlined as follows:

- Chapter 1 deals with the problem statement, rationale and research questions, educational significance, research methodology, research limitations as well as clarifications of terms.

- Chapter 2 offers a review of some of the current literature in terms of geometry and transformation geometry as well as the usage of GeoGebra in mathematics. The van Hieles' model and related studies to the model are explained in detail.

- Chapter 3 affords a comprehensive description of the research design, methods for data collection as well as instruments, their legitimacy and trustworthiness, analysis of data as well as ethical considerations.

- Chapter 4 deals with the findings followed by a brief discussion of these findings.

- Chapter 5 presents conclusions and recommendations relative to the objectives.

The next chapter presents the literature review.

CHAPTER 2 - LITERATURE REVIEW

In order to review the scholarly literature with regards to transformation geometry, the van Hieles' model for the development of geometrical thinking and the use of GeoGebra to support learners' understanding are investigated.

2.1 Curriculum and Assessment Policy Statement (CAPS)

As defined by Curriculum and Assessment Policy Statement (CAPS) document for senior phase (Department of Basic Education, 2011, p. 10) "mathematics is a language that makes use of symbols and notations to describe numerical, geometric and graphical relationships". The study of mathematics is said to contribute to the learners' talent to think rationally, to work precisely, and to answer everyday difficulties (Muyeghu, 2008). The implication is that mathematics is one of the vital subjects which should be done by all learners in all grades. The Department of Basic Education CAPS document (2011, p. 10) for mathematics in Grades 7-9 (Senior Phase) spells out clearly that the precise targets of mathematics are to cultivate the following:

- A precarious consciousness of how mathematical relations are used in communal, environmental, cultural and economic relations.
- Assurance and proficiency to deal with any mathematical state of affairs without being slowed down by a panic of mathematics.
- Gratefulness for the attractiveness and sophistication of mathematics.
- An essence of inquisitiveness and an adoration for mathematics;
- Acknowledgement that mathematics is an inventive part of human activity;
- Profound theoretical understanding in order to make logic of mathematics;
- Attainment of detailed knowledge and expertise essential for:

- the application of mathematics to physical, communal and mathematical problems;
- the learning of related subject matter; and
- advance study in mathematics.

The implications of these aims are that, if learners lack mathematics knowledge, they might not be able to cope with the complexities and the challenges of this world. For example, learners may relate to real life examples of reflections such as the mass production of shoes, people, spectacle frames; flipping images on a computer; the mirror images of the chemical structure of the sugar molecules such as glucose in sugarcane. With regard to rotation, learners are able to relate it with strumming a guitar; turning of a wheel; picking up a cup, etc. As for translation, learners relate it with the movement of an aircraft as it moves across the sky; sewing with a sewing machine; throwing a shot-put, etc. As far as enlargements or reductions are concerned, learners can relate it with the scaled house plans and machinery parts' designs; enlarging or reducing images on a computer, etc.

It should be noted that mathematics serves as a foundation in a subject such as Physical Sciences. Learners need mathematical skills such as deriving the subject of the formula, solving quadratic equations, etcetera, for example in Physical Sciences. CAPS Grades 7-9 mathematics (2011, p. 11) highlights skills which learners will develop when taking mathematics as one of their subjects. To develop specific skills needed, learners should:

- Propagate the exact use of the linguistic of mathematics;
- Advance number terminology and concepts, calculation and application skills;
- Acquire to listen, connect, contemplate, motivate sensibly and put on the mathematical knowledge gained;
- Be in a position to explore, analyze, exemplify and construe information;

- Cram to pose and resolve problems; and
- Shape a mindfulness of the significant role that mathematics plays in day-to-day situations including the personal improvement of the learner.

2.1.1 Geometry as a Content Area within Mathematics

Grade 9 mathematics learning covers the following content areas: numbers, operations and relationships (in a 15% proportion), patterns, functions and algebra (35%), space and shape or geometry (30%), measurement (10%) and data handling (10%). The percentages following the content areas refer specifically to the relative weighting and subsequent time allocation for Grade 9 (CAPS, 2011, p. 13). The focus of this study is specifically on geometry. Kin-Wai (2005) defined geometry as one of the branches of mathematics which deals with the understanding of axioms and shapes. The word geometry has Greek origins, "geo" meaning earth and "meter" meaning to measure (Jones, 2002).

Geometry is the oldest branch of mathematics, with its origins from ancient cultures such as Indian, Babylonian, Egyptian, Chinese and Greek (Jones, 2002). According to Al-Shehri, Al-Zoubi and Rahman (2011), geometry is one of the core topics included in the modern mathematics syllabus that was approved by National Council for Teachers of Mathematics (NCTM). In addition, Jones (2002) argues that geometry is a brilliant area of mathematics to teach, which consists of theorems, and many applied problems to solve.

Geometry is an integral part of life as it has a bearing on aspects of life such as architecture design, art, building, construction, interior design, etc. It is connected to the visual, aesthetic and intuitive human faculties (Jones, 2002). In every day and job-related careers, many geometrical theories and methods are reassigned from the teaching space to the outside world, for example, in woodworking concepts (Knight, 2006). In forestry the concept of similar triangles is used in determining heights of trees (Knight, 2006). Furthermore, Boyraz (2008) states that geometry is utilised as an application in extra topics in mathematics and also to prepare learners for higher courses in certain subjects. Geometry has reportedly played a pivotal role in people's lives since 2000 BC and is still important in peoples' lives post 2000 AC. Geometry affects not only a few individuals but the universe at large (Boyraz, 2008).

> *Throughout the centuries, great progress was made with regards to mapping the Earth and also with regards to the mathematics associated with these actions. This allowed us to consider mapping the universe and it assisted with the understanding of the images that resulted from such activities. Mapping makes it possible to "see" or perceive such vast phenomena as the galaxies. "…but you can't put them all together without the geometry to analyse what it is that you are seeing" (p. 5).*

The indication by Brodie (2008) is that objects or things that are observed with naked eye are always easy to be remembered. Similarly, the utilisation of GeoGebra might play a pivotal role in ensuring learners' understanding of complex topics.

The development of geometry was not restricted to the study of its foundations (Robinson, 1976). In the nineteenth century, there was further development with regards to geometry: Felix Klein (in Robinson) presented a definition of geometry in the second half of that century, as "the study of properties which remain invariant under groups of transformations". Geometry has not always been viewed as important as arithmetic in everyday life (Robinson, 1976). However, geometry currently constitutes an honourable branch of mathematics and has received due recognition in the school curriculum.

Jones (2002, p. 124) summarised the aims of teaching geometry as follows:

- To develop spatial awareness, geometric intuition and the ability to visualise;

- To provide broad geometrical experiences in two- and three dimensions;
- To develop knowledge and understanding of the geometrical properties and theorems, as well as their use;
- To encourage the developing and use of conjectures, deductive reasoning and proofs;
- To apply geometry through modeling and problem solving in real life contexts;
- To stimulate a positive attitude towards mathematics; and
- To develop an awareness of the historical and cultural heritage of geometry in society.

Congruent with these aims, the NCTM (2000, p. 4) lists a range of skills (through the teaching and learning of geometry) that enable learners to:

- Recognise, define, liken, model, draw and categorize geometric figures in two and three dimensions;
- Cultivate spatial sense;
- Discover effects of transforming, merging, splitting, changing geometric figures;
- Comprehend, apply, and construe properties of and relationship between geometric figures, including congruence and similarity;
- Develop a gratitude of geometry as a means of describing the physical world;
- Sightsee synthetic, transformational and coordinate methodologies to geometry; and
- Discover a vector approach to certain aspects of geometry.

The above list of skills obtained through the study of geometry, provides the parameters within which geometric thinking is systematically developed. Within the present study, specific attention is paid to the transformational aspect of geometry learning, which is one of the most dynamic aspects of this mathematics content area.

It is apparent that geometry enables the development of a whole range of skills that are important to everyday life. Kin-Wai (2005) mentions these skills as,

> *"1) identify, describe, compare, model, draw and classify geometric figures in two and three dimensions; 2) develop spatial sense; 3) explore the effects of transforming, combining, subdividing, and changing geometric figures; 4) understand, apply, and deduce properties of and relationship between geometric figures, including congruence and similarity; 5) develop an appreciation of geometry as a means of describing and modelling and the physical world; 6) explore synthetic, transformational and coordinate approaches to geometry, with college-bound students also require to develop an understanding of an axiomatic system through investigating and comparing various geometric systems; and 7) explore a vector approach to certain aspects of geometry, p. 3".*

However, reports such as TIMSS 1999 and TIMSS 2003 have revealed poor performance in geometry. Furthermore, it is contended that learners show minimal improvement in geometry performance within each consecutive year (Kin-Wai, 2005). As a result, NCTM recommended that teachers use new approaches that will help learners overcome difficulties in learning mathematics (Aydin & Halat, 2009).

There has been a renewed interest in certain aspects of geometry teaching and learning since the 1980s (Aydin & Halat, 2009). Geometry is, for example, considered important for its ability to provide and improve spatial-visual skills (Pleet, 1990). Spatial-visual skills are vital because the knowledge of the world is influenced by perceptions of visual stimuli (Pleet, 1990). In mathematics and science careers, spatial perception and visualisation abilities have become increasingly recognised for scholastic and occupational purposes (Pleet, 1990). Pleet (1990) acclaims spatial visualisation as follows:

> *Spatial visualising ability has been recognised as a distinct and important dimension of human abilities for over half a century. Although spatial ability has never attained the level of recognition afforded verbal and numeric abilities, it has long been*

> *recognised as important for success in many occupations and school subjects, such as engineering, drafting, and art. As a result, educational research in these subject areas often includes the spatial variable.* (Pleet, 1990, p. 6)

The implication in respect of above is that, if learners lack spatial visualisation ability, they will not do well in transformation geometry, as well as in occupations and school subjects, such as engineering, drafting, and art.

2.1.2 Transformation Geometry

Transformation geometry, the focus area for this study, comprises of motions such as translation, reflection and rotation (Akay, 2011) and also with dilations or shrinking and enlargement (Bennie et al., 1999). Pleet (1990) explains transformation geometry as the study of figures and the properties of figures conserved by reflections, translations, rotations, and their groupings. It provides the opportunity for learners to recognise and perform changes in the coordinates of the image of objects regarding their position, orientation, direction and size within a coordinate plane.

2.1.2.1 Within the South African school curriculum.

The geometry content is divided into five sub-topics, namely:

- Geometry of 2D shapes which deals with properties and definitions of triangles (equilateral, isosceles and right-angled) in terms of their sides and angles, as well as definitions of parallelograms with a focus on edges, angles and diagonals;
- Geometry of 3D objects which deals with classification (spheres and cylinders) and building of 3D models;
- Straight lines geometry which deal with angle relationships created by perpendicular lines, intersecting lines and parallel lines;

- Construction of geometric figures which deals with accurate construction of 30^0, 45^0 and 60^0 using a compass, straight edge and a protractor; and
- Transformation geometry where changes with points, line sections and easy geometric figures on a coordinate plane are recognised; described and executed aiming at rotation, reflection, translation, enlargements and reductions.

In a categorisation of geometry by Robinson (1976), a distinction is made within topology between Euclidean and non-Euclidean projections. Reference is made to elliptic, affine (Euclidean or Minkowskian) or hyperbolic projections. Topological transformations can be classified as linear transformations, which are either similarity transformations or procrustean stretch and are either isometric (reflections, rotations, and translations) or dilating. In essence, this categorisation is mirrored in the South African curriculum for transformation geometry. Table 2 has been extracted from the Grade 9 CAPS learners' book (2013). This table gives an overview of what transformation geometry entails, as well as the geometrical context and practical examples to which learners should be exposed.

Table 2

Transformation and the Geometrical Context in Grade 9 (CAPS Platinum Mathematics Learner's Book, 2013)

Transformation	*Invariants*	*What changes*	*Examples*	*Geometrical Context*
Isometric (translation, reflection, rotation)	Shape, distance	Position, Orientation	Tessellations, symmetry	Metric
Dilation (shrinking or enlargement)	Shape	Size	Maps, plans, toys, models, trigonometry, shadow geometry from a point light source	Similarity
Shears/stretches	Parallelism	Shape	Shadow geometry using sun as a light source	Affine
Oblique projection	Cross ratio/ straightness	Parallelism	Perspective drawings, photographs	Perspective
Transformations that do not tear space	Closeness of points	Straightness of lines	Networks, underground maps	Topology

In the first row of Table 2 there are three transformations, explicitly, translation, reflection and rotation. Specifically, when translation, reflection and rotation occurs, the shape of the image after transformation always remain unchanged; while the position of the image is different from the original object. Examples thereof are tessellations and symmetry; while the geometrical context is metric. Within rotation, a plane and all its points are rotated (Wesslen & Fernadez, 2005). Two aspects to be known with regard to rotation are the angle the plane is being rotated through and the point it is being rotated about (also known as the centre of rotation). A reflection on the other hand, mirrors the whole plane and its points where the important concept is the line of reflection (also known as the mirror line). A translation moves the plane and all its points for the same distance and in a particular direction.

Row two shows dilation (shrinking or enlargement) as another form of transformation; in which the size of the image of the object after it has been translated changes. In terms of shrinking, the size of all the dimensions of the image becomes smaller while with enlargement the size of the image becomes bigger by a specific factor. Therefore, with dilations the shape remains unchanged. Examples of dilation include amongst others, trigonometry; shadow geometry from a point light source; enlarging or reducing images. These dilations are very important as they are used in everyday life to help solve problems. A small vein or artery can be enlarged to have a clear image so as to heal an injured person or animal. The geometrical context for dilation is similarity, in which objects are similar in shape but not in size.

The third row shows shears/stretches as another form of translation in which the shape of the image after transformation changes. An example of shears/stretches is the shadow geometry using the sun as a source of light during the day when there are shadows of objects. Light travels in straight lines, until reflected by objects in its path. Learners will also notice that the shape of the image can also change with certain transformations. The shape of a person or object becomes smaller and also changes when the time approaches twelve o'clock during the day.

The fourth row shows oblique projection as a form of transformation in which the parallelism changes. Examples of oblique projection are perspective drawings and photographs.

The last row shows transformations that do not tear space in which straightness of lines changes. In this transformation, properties of space are preserved under continuous deformations such as stretching and bending, however, no tearing or gluing occurs. These can be done by using a collection of subsets, called open sets. Examples in this regard are networks and underground maps.

One of the principles of the National Curriculum Statement Grades R-12 is to create learners who can utilise science and technology successfully and critically (Department of Basic Education, 2011). According to Holzl and Schafer (2013), the new curriculum (Curriculum and Assessment Policy Statement) emphasises the use of spatial skills and properties of shape and objects to identify and solve problems creatively and critically. Our humanity is presently more reliant on technology, as a consequence of employment shifts from physical to academic activity (Verhoef, Coenders, Pieters, Smaalen & Tall, 2015). The implication is that it is paramount that society shifts from doing things in the ordinary way, to doing the same thing in a more technologically driven way, because we need to be able to compete locally, internationally and globally. In addition, the use of technology can support teaching in a positive way.

According to CAPS (2011), transformation geometry is only taught in the fourth term at Grade 9 (see Table 1). For Term 4, there are 33 notional hours allocated to the teaching of six mathematics topics in Grade 9. Of the six topics, transformation geometry and geometry of 3D objects are each assigned 9 hours. The percentages per topic show that transformation geometry has a bigger portion (27.27%) among the topics in Term 4 for Grade 9. It might mean that if learners do not grasp the concept of transformation, mathematics results might be negatively affected. The requirement for passing mathematics at the Senior Phase (Grades 7-9) is that a learner has to achieve a minimum of 40%.

Within modern school curricula, there is an emphasis on coordinate-, vector- and transformation geometry (Dindyaul, 2007). Transformation geometry is an important concept in school curricula as per the international perspective, as is reflected in the International Competitions and Assessments for Schools (ICAS) tests, which contain 28 transformation geometry items out of 82 geometry test questions (Brodie, 2011).

2.1.2.2 Rationale for inclusion in the school curriculum.

There have been debates by researchers with regard to the inclusion of transformation geometry into the school curriculum (Evbuomwam, 2013).

TIMSS reports for 1995, 1999 and 2003 revealed that learners had problems with regard to conceptualising proofs and as a result could not solve geometric problems. However, transformation geometry serves as a tangible reason why learners have early conceptualisation of vectors and can therefore afford an outstanding example of comparing mathematics with the physical world through the notion of isometric transformation. For example, when learners look at themselves in the mirror, they need to understand that the image formed is due to reflection through the mirror. They will also need to know that the object and its image are the same in terms of shape and size. In addition, tessellations are an example of decorative art that occur around us in nature and which are the products of Islamic civilization (Akay, 2011). The making of patterns and the movement of objects are examples of transformation geometry; hence the inclusion of transformation geometry in the school curriculum becomes vital.

According to Lesh (1976), there are several reasons why people must take interest in studying transformations. These reasons are both practical and theoretical. The first reason is that elementary textbooks are now changing from a traditional- to a transformational approach. The second reason for focusing on geometric notions is that most of the models (e.g. number lines, arrays of counters, etc) and diagrams that are used during teaching have the ability to presume certain spatial concepts. The third reason for interest in transformations is to isolate general principles for anticipating the relative difficulty of mathematical ideas.

2.1.2.3 Some characteristics of transformation geometry.

Wesslen and Fernandez (2005) point out that although learners encountered transformation geometry in primary school, there are secondary school learners who still struggle with the concept of transformation. In view of the significant conceptual leap in transformation geometry between Grade 7 and 8 (see Table 1), it may be understandable that new concepts have not yet being solidified at the Grade 9 level. Perhaps it is time that teachers look into possible factors that negatively affect the performance of transformation geometry (Wesslen & Fernandez, 2005).

According to Knight (2006), transformation geometry is perceived in graphic arts and incorporated into archaeology in the making of pottery and other artefacts through various generations and cultures. Archaeologists use translation, reflection, rotation and glide reflection to create symmetry classifications and drawing systems to identify patterns (Knight, 2006). This illustrates the role which transformation geometry plays in people's lives and how important it is to include it in school curriculum.

Robinson (1976) indicates that transformation lends itself to illustration within an experimental setting where learners do hands-on activities. Rotations can be done by rotating a sheet of paper about a point, and projective transformations by casting shadows. A topological transformation may be illustrated by using a stretched rubber band where deformation indicates a continuous transformation. Learners in Grades 1-3 may learn transformations using clay, balloons and elastic threads (Robinson, 1976). A shadow of a straight stick will always be straight, that is an example of projective transformation where straightness is invariant.

Ada and Kurtulus (2010) conducted a comparative study to investigate learners' misconceptions and errors concerning two aspects of transformation geometry, namely, translation and rotation. With regards to translation, the results of the study revealed that 75% of the learners who took part in the study obtained correct answers; 3% of the learners got the wrong answers; whereas 9% made technical mistakes. With regards to rotation, 35% of the learners got the correct answer to the technical question; nonetheless, 55% of them used true rotation transformation. Comparing learners' percentages in terms of translation and rotation, the indication is that learners understood translation better than they understood rotation (Ada & Kurtulus, 2010). The results indicate the need for improvement in the ways and means transformation geometry has been taught so to improve learners' performance in this topic.

Haak (1976) indicates that in recent years the name of M. C. Escher, the Dutch artist has been gaining fame in many mathematics classrooms. According to Haak (1976), any person who desires to succeed in symmetry on a flat surface should take into cognisance three vital principles of crystallography (the experimental science of determining the arrangement of atoms in the crystalline solids), explicitly repeated shifting (i.e, translation), turning about the axis (i.e, rotation) and gliding mirror image (i.e, reflection). Transformation geometry can be integrated effectively into an art work. Little effort is needed to identify symmetries in Escher's work as indicated by Haak (1976). If learners do well in transformation geometry, they may do well in jobs related to it.

The work done using Escher's concepts concerning transformation included symmetrical figures, with different configurations of reflection-, rotation-, translation- and glide-reflection symmetry; symmetry drawings including figures derived from polygons and plane tessellations; the design of original symmetry patterns; and symmetry patterns based on other polygons. Figure 1 indicates three of the artworks of Esher with intricate and complex configurations of three-dimensional shapes from various perspectives.

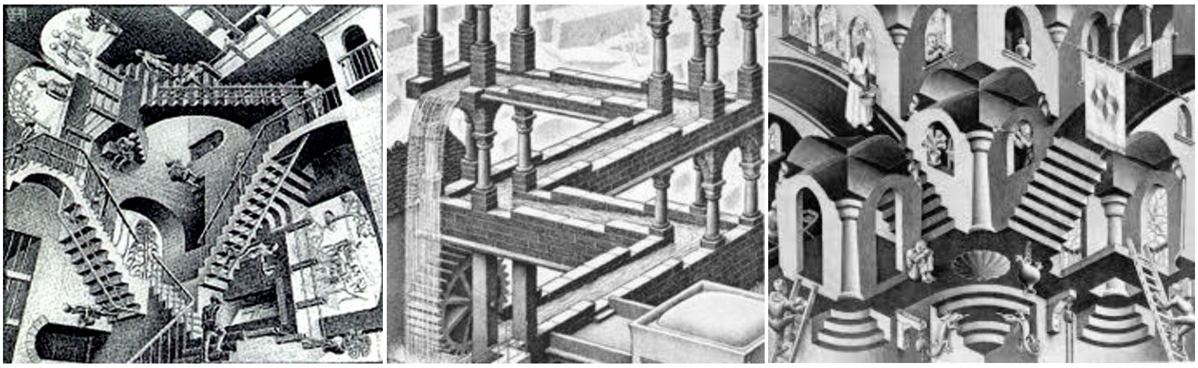


Figure 1. *Three of Escher's famous artworks and optical illusions.*

2.2 The Use of ICT within Transformation Geometry Teaching and Learning

Transformation geometry is a visual topic, therefore the use of computer software which aid visualisation is expected to have a positive effect on learners' learning and attitudes (Hoong & Khoh, 2003). However, the use of technology needs to be firmly related to the instructional programme.

Hoong and Khoh (2003) conducted a quasi-experimental research with 122 secondary learners from an average school in Singapore to investigate the effects of Geometers' Sketchpad (GSP) on learners' ability and attainment in transformation geometry. According to Hoong and Khoh (2003), in class A the teacher used a guided inquiry where learners discovered ideas with practical experience using GSP; class C adopted a teacher-centred approach with the teacher playing a predominant role using the software to demonstrate transformations; on the other hand class B was the 'middle class' which used a directed investigation with the teacher controlling objects on the white screen. The outcomes of the study showed that learners in class A and B achieved meaningfully better than class C in the test that was written. This therefore implies that the use of mathematical software does not guarantee that learners will perform better; the way the software is used plays a pivotal role in determining the desired outcomes.

2.2.1 Rationale for Exploring the Use of ICT in Transformation Geometry

The need to use technology in the classroom is advocated by Bansilal (2015, p. 1), stating that "rapid global technological developments have affected all facets of life, including the teaching and learning of mathematics". Technology can be used to facilitate learning by using dynamic software in the teaching and learning of mathematics (Cullen, Hertel & John, 2013). Merits of technology during the interaction in mathematics embrace abrupt graphing and calculation (Lu, 2008). However, it is probably the case that the use of technology and other software have not been fully utilised in the classroom (Bu & Schoen, 2011).

There are two considerations for ensuring effective teaching when new approaches for teaching are used. Firstly, it is important that teachers need to be given proper training to give quality lesson presentations. Secondly, the new apparatuses or technologies ought to be utilised in a way to support the things that teachers already know. In other words the use of technology and new approaches do not replace the content or knowledge possessed by both teachers and learners, but rather enhance learning in the classrooms. The usage of Information and Communication Technology (ICT), to be exact, mathematical software, provide learners with improved graphical and vibrant demonstrations of mental concepts and phenomena (Lu, 2008). ICT has the ability to link theory with practice such that learners can quickly understand the topic taught. Diagrams and figures may be projected on the screen for better visualisation and understanding. This therefore implies that teachers as facilitator of learning need not use mathematical software without proper preparation and consultation, the use of software does not guarantee effective learning.

In the present study, there is a focus on the use of ICT in advancing learners' levels of understanding of transformation geometry theories in the South African context at the Grade 9 level. The rationale for this focus is found in the assumptions that, (i) many secondary teachers lack either the knowledge or the confidence to guide learners to higher levels of geometric understanding; (ii) learning of geometry at the senior level has to advance from concrete- to abstract conceptualisation, (iii) a significant leap from the primary level; and finally, (iv) that help is available (freely) in the form of ICT software and programs.

Firstly, Uusimaki and Nason (2004) conducted a study to identify the cause essential to pre-service teachers' adverse beliefs and fears around mathematics. Seventeen females and one male, who participated in the study, were selected on the basis of the level of their mathematics anxiety, access to internet, and availability to attend workshops. Semi-structured interviews were used as a means of data collection. The findings revealed that 66% of the teachers' negative beliefs and anxiety emerged from primary school; while 22% of the teachers revealed that their negative beliefs and anxiety originated from secondary school experiences; 11% of the teachers revealed that their mathematics' anxiety started at tertiary level. The indication is that the anxiety was experienced during the writing of tests and examinations, and during verbal classroom discussions. The implication is that while learning to become teachers, the participants lacked understanding of mathematical concepts which led to their perceived underperformance in their own studies, as well as in their ability to guide and direct learners towards conceptual understanding.

Secondly, according to numerous classical learning theory studies, it was found that in order for learners to do well, they have to progress from concrete experiences to abstract generalisations (Bleeker, 2011). The transition from concrete learning of geometrical concepts to abstract generalizations is possible when teachers take into consideration the fact that concrete experiences precede abstract work, and that visualisation is powerful in the conceptualisation of abstract ideas. If the concrete-to-abstract succession in geometrical concept development is ignored, learners may develop deficiencies in their understanding.

Thirdly, Bansilal (2015) states that developing countries may at times be faced with challenges such as poorly managed schools and -education systems, teachers with inadequate support and training, as well as insufficient technological software. This implies that this study is necessary to investigate the use of GeoGebra in sampled South African schools, so as to that the findings may help in the challenges outlined above.

Fourthly, Ogwel (2009) emphasises that there is a tenacious urge to align school curriculum education with technology since society is rapidly changing. Globally, traditional methods are now phased out towards more electronic classrooms that encourage effective learning (Ogwel, 2009). However, since the use of technology in classrooms does not automatically create a comprehension of mathematical concepts, it is imperative that teachers use technology in a proper way to ensure that learners understand the concepts that are taught.

2.2.2 Existing Studies about the Use of ICT in Geometry

Myers (2009) led a study to probe the influence of software, Geometer's Sketchpad (GSP) on the success of Grade 10 geometry learners. He sampled an experimental group where GSP was used in teaching geometry and a control group where it was not used. The test results showed a difference between the two groups of $\alpha = .05$ level of significance, Λ (5.78) = .773 and $p = .001$. The use of GSP was found to have a positive effect on Grade 10 learners' achievement as indicated by the Multivariate Analysis of Covariance (MANCOVA) results with a p-value = .001.

Edwards (1991) conducted a five week study on learners' learning in a computer micro world for transformation geometry. This study was learner oriented and emphasised the understanding of geometrical concepts and their use. Learners were taught transformation geometry in two periods of 50 minutes each on successive days and also had a one hour extra lesson on a weekly basis. Interviews, paper- and-pencil test, as well as a final examination consisting of 24 items were used to collect data. The findings revealed that 70% of the learners were able to pass the final examination. The findings suggested that the micro world and associated activities were effective in assisting learners to construct a working knowledge of transformation geometry (Edwards, 1991). The study implies that the use of a different teaching approach might play a very pivotal role in the teaching and learning of transformation geometry.

In a study with Grade 8 learners, Pleet (1990) conducted a comparative investigation into the effects of using computer graphics (a motions computer program) and a hands-on (Mira) manipulative for the acquisition of transformation geometry. The analysis of data with respect to mathematics achievement of transformation geometry concepts and development of spatial visualisation ability revealed that the Mira (manipulative) approach seemed to favour male learners while the Motions (computer) approach seemed to favour female learners (Pleet, 1990). This study implies that even if a different teaching approach such as the use of computer graphics, there is no guarantee that all learners will pass or understand the concept taught.

In a study conducted by Lupu (2015) with 20 learners during their pedagogical hours, GeoGebra software was used for geometrical illustrations and calculus in resolving problems of plane collinearity. The results confirmed the potential of using GeoGebra as a means of ICT in the teaching and learning of complex concepts (Lupu, 2015). The implication is that the use of GeoGebra enhanced understanding in learners while learning geometry as well as calculus. However, it is imperative to investigate its potential use in transformation geometry.

Gomez-Chacon as cited in Bu & Schoen (2011) conducted a mixed methods study to find out the powers of GeoGebra-integrated mathematics tutoring on learners' attitudes concerning mathematics learning in computer-enhanced settings. The large-scale survey was conducted with 392 learner respondents, a small focus group of 17 learners, as well six individual learners. The intellectual and emotive ways fundamental to learners' attitudes and mathematical behaviours were also scrutinised. The findings of Gomez-Chacon's study were that GeoGebra nurtured learners' determination, curiosity, inductive attitudes and predisposition to search for precision as well as thoroughness in geometric lessons. The implication is that GeoGebra does not only make learners to understand concepts taught, but also brings about their abilities such as determination, curiosity, inductive attitudes and their meticulousness.

2.2.3 Some ICT Software Available in the Market

There are numerous forms of software that can be utilised in the teaching and learning of mathematics, namely, Computer Algebra System (CAS) and Dynamic Geometry Software (DGS); as well as open source software like GeoGebra and others (Lu, 2008). Each type of software is linked to certain aspects of mathematical teaching and knowledge. CAS is associated with algebra, while DGS is associated with geometrical topics. Dynamic software as an example of technology enables learners to visualise a wide range and quality of investigations (Dikovic, 2009; Cullen, Hertel & John, 2013). Computer algebra and dynamic geometry are influential technological tools for mathematics instruction (Dikovic, 2009). Dynamic geometry assisted learners' examination of everyday problems in the intermediate and secondary grades (Bu & Schoen, 2011). In other words learners are able to use what they have learnt in the classroom to solve real-life problems; they do not treat classroom mathematics and real-life problems as two completely separate areas. The use of dynamic modelling may also help in changing the learners' attitudes to mathematics.

2.3 GeoGebra as an ICT Option in Teaching Geometry

GeoGebra, a free software program, is one among other ICT tools which are currently used to change from the conventional methods of teaching geometry to more effective way that ensures effective teaching and learning (Ogwel, 2009). The use of dynamic mathematical software encourages interaction between teachers, learners and mathematics (Dikovic, 2009) and can be used to explore and visualise geometrical properties by dragging objects and transforming figures in ways beyond the scope of traditional paper-and-pencil geometry (Lu, 2008). Learners can visualise and reconnoitre mathematical associations and notions that were challenging to explain prior to software use (Dikovic, 2009). Visualisation is a vehicle for meaningful problem solving in algebra (Presmeg, 2006) and facilitates the effective learning of geometry (Lu, 2008).

Freiman, Martinovic and Karadag (2010) mention that, regardless of the fact that GeoGebra is used by many people; it is paramount that its bearing in the instruction and education of mathematics be investigated. Freiman et al. (2010) further indicates that GeoGebra software is extensively used in middle and high schools, more especially in European countries. However, Hohenwarter, Hohenwarter, Kreis and Lavisca (2008) suggest that more qualitative research results are needed to examine if using GeoGebra is better than using conventional paper and pencil techniques, specifically in transformation geometry. In particular, it is vital that similar studies are done within the South African context at the secondary level, in order to understand GeoGebra's usefulness in the teaching and learning of transformation geometry.

2.3.1 Properties and Characteristics of GeoGebra

GeoGebra is an open-source mathematics software that was created by a mathematician Markus Hohenwarter while doing his Masters at the University of Salzburg in 2002 (Hohenwarter & Lavicza, 2007) with the intention of bridging learners' conceptual gaps in understanding algebra and geometry (Lu, 2008). These authors explain that GeoGebra was planned to combine structures of dynamic geometry software (e.g., Cabri Geometry, Geometer's Sketchpad) and computer algebra systems (e.g., Derive, Maple) in a distinct easy-to-use cohesive system for mathematics instruction and learning. This technological advancement is liberally accessible for online connection from www.geogebra.org/webstart or downloadable for confined installation at www.geogebra.org (Ogwel, 2009).

GeoGebra runs on any platform that supports Java and does not exist in isolation but permits the manufacturing of collaborative web pages with implanted applets (Hohenwarter & Lavicza, 2007). These directed education and demonstration settings are spontaneously shared by mathematics teachers on resolute online platforms like the GeoGebraWiki (Hohenwarter & Lavicza, 2007). Furthermore, the program is multilingual both in its menus and in its commands (Bulut & Bulut, 2011).

GeoGebra as a multi-platform DGS (see Figure 1), has on the left an algebra window which gives algebraic representation of objects; on the right there is the graphics window which gives a default view and drawings of the objects; the spreadsheet view gives a sheet similar to excel where every cell has a specific name; and the input bar at the bottom gives algebraic commands as an alternative to the geometrical tools that are found on the toolbar. An algebra window enables learners to see equations or formulae; while a graphics window projects the figures or diagrams. These two windows enable learners to see the relationship between formulae and graphs for ease of understanding.

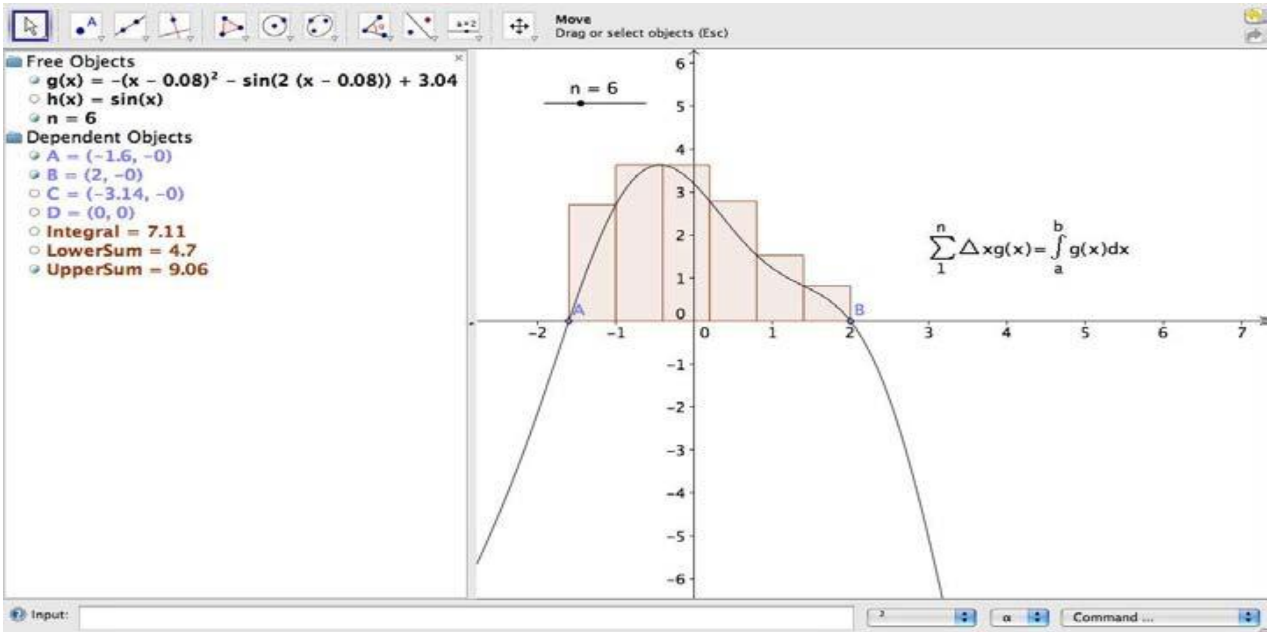


Figure 2. GeoGebra’s screen.

2.3.2 General Advantages and Use of GeoGebra in Mathematics Education

GeoGebra is a most influential piece of mathematics software which can be used to enhance the way learners learn (De Wet, 2011). It was meant specifically for educational purposes, remarkably for the teaching and learning of algebra and geometry; equally, it has tools that can be used for teaching statistics and to calculate probability models (Prodromou, 2012). GeoGebra can also be used in calculus, trigonometry and other topics to facilitate teaching and learning as it is a good tool for visualisation (Hohenwarter et al, 2008; Martinovic, Karadag & McDougall, 2011; Ogwel, 2009). In addition to its ability to be used as a statistical tool, Prodromou (2012) indicates that GeoGebra can be used to help learners perform data analysis and inferences to explore probability models. It further has the ability to help learners comprehend investigational, problem- and exploration-oriented learning of mathematics in the schoolroom and on their own at home (Dikovic, 2009) and can amongst other interesting applications, find roots and extreme values, draw tangents to curves, and perform a full range of transformations on functions and shapes (De Wet, 2011). The software has the ability to integrate graphical, numerical and algebraic expressions simultaneously (Lu, 2008).

GeoGebra has an immense universal user- and inventor community with operators from 190 countries and attracts more than 300 000 downloads per month (Escuder & Furner, 2012). Bu and Schoen (2011) state that GeoGebra has caught the attention of tens of thousands of visitors, together with mathematicians and mathematics teachers. An increasing number of people use GeoGebra as a tool inside the classrooms and during their spare time.

The properties of GeoGebra that contribute to its acclaimed international status are related to changing teacher habits and allowing learners real-time exploration opportunities (Escuder & Furner, 2012). Furthermore, as most mathematical software is commercial, people in lower socioeconomic communities can take-up the free open-source software option such as GeoGebra for teaching and learning, as well as for their own use (Lu, 2008).

In her article about discovering teachers' insights concerning the power of technology in mathematics learning and teaching, Bansilal (2015) found that teachers were keen to use technology in teaching, thus they responded favourably, because technology made the tasks of teaching and learning easier than the more conventional options. Teachers reported that technology provided opportunities for working with different representations; variations in mathematics situations were made available; and concepts could be visualised during teaching and learning.

Regarding the broader advantages of visualisation through technology, a vital research forum report pronounced visualisation as a means for eloquent problem solving in algebra (Presmeg, 2006). Visualisation can be dominant not only in graphic mathematical areas such as geometry and trigonometry, but also in algebra (Saha, Ayub & Tarmizi, 2010). Computer software encourages dynamic visualisation through the use of sliders to show the effect of changing variables in the content.

GeoGebra, as part of dynamic geometry software (DGS), offers a wealth of opportunities for an exploratory style in mathematics education, in particular in space and shape (Holzl & Schafer, 2013). DGS has the ability to directly interact with tools in the system and to manipulate and explore figures while discovering the relationships between multiple representations, making it highly efficient in mathematics manipulation and communication for learning (Lu, 2008). DGS is meant for teachers and learners alike (Lu, 2008) and encourages learners to discover and experiment on their own (Dikovic, 2009).

Mainali and Key (2008) listed the following four salient reasons why mathematics teachers should use technology like DGS:

- DGS gives teachers flexibility in which they acclimatise their lessons and teaching methods more effectually to their learners' prerequisites;
- DGS encourages teachers and learners to become co-learners by engaging in discussions which lead to better understanding of mathematical concepts;
- DGS enables learner-centered approach in which teachers offer learners opportunities to articulate theories and draw their own conclusions;
- DGS develops thinking skills in learners by uncovering them to an inclusive range of mathematical concepts which they can explore for themselves.

Dikovic (2009) states that GeoGebra has (i) an easy-to-use interface; (ii) encourages guided discovery learning; (iii) allows learners to engrave their own formations by adjusting the interface; (iv) makes learners advance an improved understanding of mathematics by controlling variables effortlessly; (v) enables cooperative learning in small groups; and (vi) allows for modifications.

Andraphanova (2015) lists the advantages of a dynamic geometry environment as follows: (i) the attraction of computer tools ensures that learners have accurate thoughts about geometrical creation methods; (ii) the construction of drawings offers a prospect for additional research; (iii) it enables learners to work independently and actively; and (iv) allow them to work at school or conveniently at home. It creates didactical opportunities and develops active mathematical seeing; simulation, or the investigational spotting of the manners of geometric objects and the uncovering of unidentified facts; as well as dynamics which refers to the recognition of the moving consequence of a demonstrative body with calculating means (Andraphanova, 2015). Dynamic geometry environment has the ability to change learners' attitudes towards difficult geometry concepts and to change the traditional process of memorising concepts into clearer concepts that are well understood (Andraphanova, 2015).

Mathematics teaching and learning is a vastly multifaceted practice and needs diverse ways in order to make it more effective, amongst others, through interactive learning technologies such as GeoGebra (Bu, Spector & Haciomeroglu, 2011). Bulut and Bulut (2011); Gulsecen (2012) assert that learners' experience teaching and learning with GeoGebra as being different from face-to-face education, with its many visual features. Thus encouraging the use of mathematical software for effective teaching and learning.

Mainali and Key (2008) however indicate that in order for teachers to use DGS with ease, they require basic skills and knowledge of computer use so that they will navigate the tool bar of GeoGebra without difficulty and with understanding. As a result, teachers need extra support and training in order to gain confidence to use DGS with their learners. Bansilal (2015, p. 1) confirms this need, saying that "the use of technology requires research and careful planning in order for it to achieve potential benefits". Thus implying that the use of technology alone does not necessarily ensure effective learning, teachers should have an expertise in how to utilise the software to ensure the desired outcomes.

The use of GeoGebra might seem to be an easy task however, it requires that teachers pay attention not only to the subject matter, but also to the way in which the software is used in the content (Ljajko, 2013; Fahlberg-Stojanovska & Stojanovska, 2009). For example, teachers can use sliders for learners to see the changes in the different values as a slider is moved left or right, up or down. However, the teacher requires both the knowledge of content and an understanding of how to use the technology for specific content (Escuder & Furner, 2012). Once teachers and learners are conversant with GeoGebra, they will spend less time in drawing diagrams and doing required calculations (Ljajko, 2013).

2.3.3 Some Operational Aspects of GeoGebra as Dynamic Geometry Software

The use of sliders (see Figure 3) within GeoGebra involves learners in their own learning (De Wet, 2011) and stimulates hypothetical questions ("what if?"). As a result, learners are kept engaged as they keep asking questions. This enables teachers to address learners' misconceptions. It is imperative that the use of mathematical software be accompanied by teacher knowledge, both in using the technology, and in the concepts that the technology is illustrated; otherwise the use of such technology may potentially hinder the teaching and learning processes (Lu, 2008).

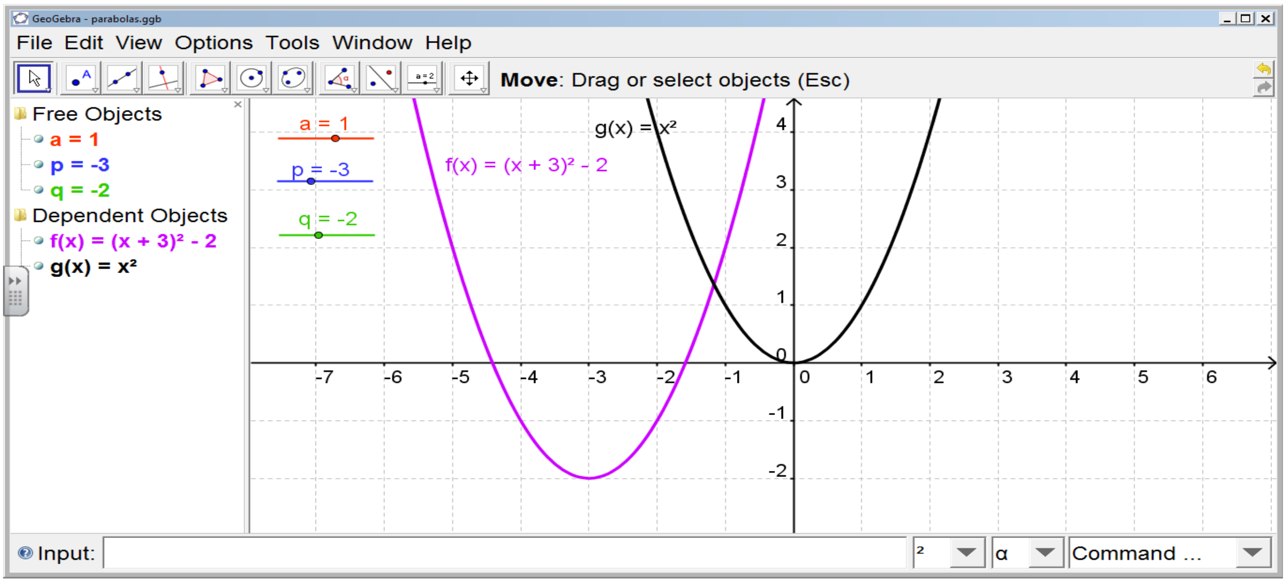


Figure 3. The use of sliders on GeoGebra.

GeoGebra's menu bar consists of "file, edit, view, perspectives, options, tools, window and help", and its faculties are:

- 'move' which moves objects from one position to another;

- ‘rotate around point’ which rotates objects around a given point;
- ‘record to spreadsheet’ which shows spreadsheet;
- ‘points’ for drawing new points, point on object, attaching/detaching point, intersecting two objects, determining midpoint or center on the object, determining complex number;
- ‘lines’ in which perpendicular lines, parallel lines, perpendicular bisector, angle bisector, tangents, polar and diameter lines, best fit line as well as locus can be drawn;
- ‘polygons’ for drawing different polygons, regular polygons, rigid polygons, and vector polygons;
- ‘circles’ for sketching a circle with center through point, a circle with midpoint and radius, compass, a circle through three points, a semicircle through two points, a circular arc with center between two points, a circumcircular arc through three points, a circular sector with center between two points, a circumcircular sector through three points and circular arcs;
- ‘conics’ for drawing ellipse, hyperbola, parabola and conic through five points; ‘measurements’ for measuring angles, angles with given size, distance or length, area, slope, and creating list;
- ‘transformation’ for reflecting a point about a line, reflecting an object about a point, reflecting an object about circle, rotating object around point by a given angle, translating an object by a vector, dilating object from point by factor;
- ‘sliders’ for visualisation on the effect of variables; insert button, insert input box; move graphics view;
- ‘zoom in’ for enlarging the object;
- ‘zoom out’ for reducing the object;

- 'show/hide object' for displaying or making the object to disappear;
- 'show/hide label' for displaying or making the label to disappear;
- 'copy visual style'; and
- 'delete object'.

A further set of commands is available by the 'right click' on the mouse as follows:

- 'axes' can be displayed or the axes made to disappear;
- 'grid' is a function which makes the grid lines available;
- 'zoom' is used for increasing or decreasing the size of objects;
- 'xAxis:yAxis' is utilized for the ratio of the x-axis to the y-axis;
- 'show all objects' is utilised mainly for displaying all objects drawn;
- 'standard view' is used to display a standard graphics window;
- 'graphics' is used to adjust the axis in terms of basic dimensions, colour, line style, and background colour.

A distinction is made between the selection of an object and a label. When right clicking on an object, one can always decide on making the object to disappear or reappear by clicking 'show object'. One can also make the label to disappear or reappear by clicking 'show label'. One can trace on object, record to spreadsheet, copy to input bar, change the name of an object by clicking 'rename', delete the object by clicking 'delete' option, change object properties such as colour, line thickness, line style, filling, as well opacity.

Figure 4 is an illustration of one of the abilities of the GeoGebra with reflections.

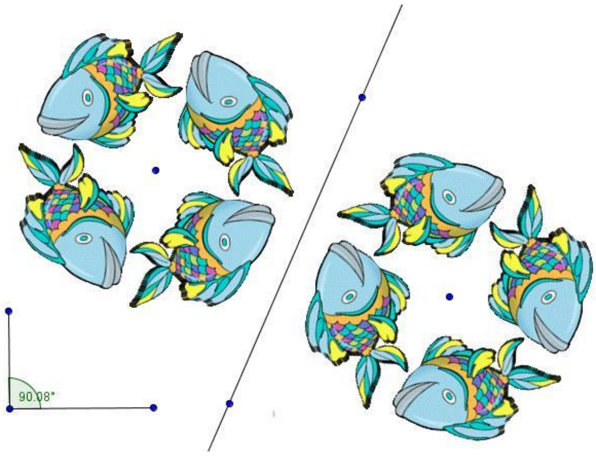


Figure 4. Showing objects reflected using GeoGebra.

The GeoGebra context is user friendly in the way that it can generate graphs of several purposes which can serve as pictorial mediators (Berger, 2013). The different transformations and shapes that the teacher demonstrates are visually accessible to learners, in that raising ease of learning. GeoGebra encourages learners to personalise their own creations by changing font size, language, colour, coordinates, line thickness and line style to mention a few. GeoGebra enables learners to manipulate variables by using sliders, as a result they are able to make generalisations (in words and mathematically) based on their observations.

GeoGebra gives learners an opportunity to either learn individually or cooperatively in small groups. The use of GeoGebra saves teaching time as teachers and learners can use existing formulae in the input bar to make changes and quickly get new information. Existing lesson presentation may be saved for future use, giving teachers more time to focus on some of their other duties. According to Ogwel (2009), GeoGebra has potential, (i) as a presentation tool or as a multiple representation tool (including graphical representation, algebraic representation, dynamic visualisations, making connections); (ii) as a modeling tool (dynamic constructions, experiments and discovery learning); (iii) or as an authoring tool (creation, use, sharing of materials with an online community, research on teaching and learning using GeoGebra).

GeoGebra has the ability to find extreme values, draw tangents to curves, perform a full range of transformations on functions and shapes as shown in Figure 5 (De Wet, 2011).

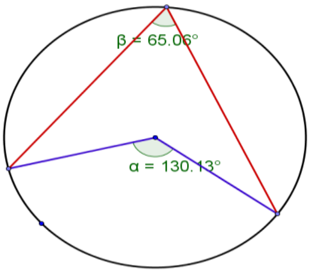


Figure 5. Showing sizes calculated using GeoGebra.

In summary, GeoGebra has a list of computing instruments which embrace an ordinary set of tools which allows the formation of core geometric objects such as a line, a circle, a vector, a polygon, a point, an angle and other apparatuses comprehending extra operations on geometric objects such as fragment division in halves, measurement of segment length and measurement of angle (Andraphanova, 2015). GeoGebra is an inventive kind of scholastic product which is aimed to alter conventional insolences towards the education and knowledge of mathematics content such as transformations (Andraphanova, 2015). This implies that the use of GeoGebra might positively affect learners' understanding of transformation geometry.

2.3.4 Research into the Use and Effect of GeoGebra

Due to the large numbers of people using GeoGebra, it was found to be important that there was a call for a profound research on the instructional plan of GeoGebra-based components and the agreeing influence of its vigorous mathematics resources on teaching and learning. As a result, a non-governmental organisation called the International GeoGebra Institute (IGI) was formed (Hohenwarter & Lavicza, 2007). The main purpose of IGI was to ensure that there is proficient expansion around the unrestricted software. Hohenwarter and Lavicza

(2007) mention that the central goals of IGI were to establish consistent narrow GeoGebra user groups; to develop and share open educational materials; to organise and offer training or workshops for teachers; to advance features related to software GeoGebra; to create and advance research projects on GeoGebra and IGI; as well as to provide presentations at countrywide and worldwide levels. This implies that it is important that teachers network with other mathematics teachers so as to get help with regard to their day to day challenges, more especially during the utilisation of technology.

Following the decision in this study to examine the usage of vibrant geometry software GeoGebra, the literature was reviewed which reported on research studies conducted in this regard. Some of these studies are listed below and briefly discussed in terms of their findings.

Saha, Ayub and Tarmizi (2010) have conducted a quasi-experimental study in which fifty-three secondary mathematics learners from Wilayah Persekutuan Kuala Lumpur were divided into two groups. The investigational group was trained coordinate geometry by means of GeoGebra while the control group endured learning using common ways and means of teaching. The findings of their study displayed a substantial variance in the mean marks of learners' mathematical results (Saha et al., 2010) in favour of the use of Geogebra. This implies that the approach used in facilitating learning is very much important; as a result teachers should look carefully at their methods of teaching.

Shadaan and Leong (2013) conducted a study on the usefulness of utilising GeoGebra on learners' thoughtfulness in learning circles. Fifty-three Grade 10 learners were separated into an investigational and control group. The split up scores showed a significant improvement. The authors also used a survey instrument to investigate learners' perceptions about the use of GeoGebra, which revealed that learners' are optimistic about the utilisation of GeoGebra in learning mathematics. This implies that learners might learn better when Geogebra is used to facilitate learning.

Furner and Marinas (2007) found that GeoGebra could assist in developing a profound understanding of geometric theories in low-grade mathematics teaching space, and could be used easily by both teachers and learners. Learners found answers to given exercises easily when using GeoGebra as linked to traditional teaching methods. Learners spent less time on exercises using GeoGebra because they quickly made conclusions by visualising the concept. Bu and Schoen (2011) state that GeoGebra enabled teachers to detect their individual mathematical notions and to envisage the problem circumstances and obstacles learners experienced. This implies that teachers will be in a position to identify the obstacles and challenges experienced by their learners during teaching and learning.

Ljajko (2013) separated learners into two groups, a control group and an investigational group. The control group was trained on ellipses without the use of technology, while the experimental group was taught ellipses using GeoGebra. The results of tests in this study revealed that learners in the investigational group had better problem solving skills in the concept of ellipse as compared to learners in the control group. This implies that the use of technology ensures that learners gain skills that make them learn better.

Dogan (2011) directed a study about the obligation of the dynamic geometry software GeoGebra in the course of learning about triangles. The participants in the study were 40 Grade 8 learners who were shared into an investigational- and a control group. The investigational group consisted of 9 female and 11 male learners; while the control group entailed 7 girl- and 13 boy-learners. A pre-test which consisted of 13 questions was applied to all participants to establish the learners' attainment level. The investigational group was given activities that were constructed using GeoGebra so as to make the subject dynamic, visual and concrete. The pre-test showed no algebraic variance concerning the control- and experimental-groups, with a mean of 78.00 and 74.00 for the investigational- and the control-group correspondingly. The post-test which comprised of 11 questions was applied to the two groups to ascertain the learners' attainment level. The post-test results showed that there was a numerical variance between the groups. The conclusion based on this study was that computer-based teaching space activities can be successfully used in the learning settings. Learners who participated in the study wished that they could learn more mathematics topics using GeoGebra. The implication is that the use of GeoGebra enhances both learners' motivation to learn, and their understanding of mathematical concepts.

Dikovic (2009) sampled 31 learners to find out whether there is a momentous difference in learners' mathematics accomplishment before and after GeoGebra-based mathematics teaching in calculus. To accomplish the intention of the experiment, Dikovic (2009) embraced two instruments, namely, specially designed GeoGebra applets as well as an equivalent pretest-posttest design, done by teaching staff. The findings of his study indicated that there was a noteworthy variance in the pre-test scores before- and the post-test scores after the GeoGebra workshops. There was a notable discrepancy amid the mean before the workshops which was (M = 22.95) and after the workshops where M = 51.64, implying a positive effect in differential calculus with the use of GeoGebra applets. This implies that the use of GeoGebra has a positive impact on the learners understanding of mathematical concepts.

Lupu (2005) investigated the role of GeoGebra in the teaching and learning of mathematics. His research respondents comprised 2 experimental and 2 control groups from Grade 11. Participants from the pre- and work-related teacher subdivisions steered trial lessons in computer science using GeoGebra for geometrical representations and calculus. The following methods and techniques were used to obtain the data: a portfolio, a feedback form, observation, psycho-pedagogical investigation, dialogue, breakdown of the activity products and research of documents, the method of the assessments, as well as the methods for the mathematical-statistical presentation and dispensation of the research data. The results revealed that 39 learners did very well in the written tests; 38 did well; 14 performed satisfactorily; while 13 learners did not do well in the tests. All the advisors and participants treasured the importance of proficiently using the computer. The results of the questionnaires in the study by Lupu (2005) exposed the constructive and emotive probable of using GeoGebra. This implies that GeoGebra has potential in ensuring that learners understand mathematical concepts.

Martinovic et al. (2011) inspected the assertion that an active learning atmosphere permits the grasping of mathematical concepts. Their interest was to find out whether the preamble of visualisation has a consequence on learners' compelling the base rate and counter misjudgement. The method used in obtaining the results was the design experiment through which the assessment and the intervention were both used. Their study took place in Grade 11 classrooms with 23 participants. Learners were presented with the GeoGebra applet signifying area-proportionate Venn drawings whose magnitude could be influenced using the slider feature of GeoGebra. The results of their study showed that utilising the lively feature of the applet; learners were able to comprehend the association between the base rates and conditional probability. The implication of their study was that an active learning atmosphere such as GeoGebra had the ability to create the means for profound analysis of transformation geometry concepts.

In a study by Mainali and Key (2008), 15 mathematics teachers participated in a four-day preliminary workshop in Nepal to afford a synopsis of GeoGebra and its probable amalgamation into the teaching and learning of secondary school mathematics. Teachers' misrepresentations and opinions regarding both the training and the software were inspected in the context of appropriateness in Nepalese schools. In Nepal, traditional teaching methods are still superseding, while the usage of information and communication technology (ICT) is slight to non-existence in secondary schools. Reasons cited are that schools lack resources and that teachers lack skills to use ICT in the teaching and learning of mathematics. However, the use of ICT is encouraged by the ministry of education by the initiation of a school reform project. Analysis of the data showed that all participants had affirmative impressions of using GeoGebra and they also appeared very attracted to the software as a group. All participants indicated that they would continue using the software. One teacher, who had the lowest experience assumed that GeoGebra generated an easy teaching and learning environment for teachers as well as learners. This implies that participating learners and teachers will be encouraged by the use of GeoGebra in the teaching and learning of transformation geometry.

Mehanovic (2011) investigated the use of dynamic software GeoGebra in upper secondary level in Sweden in order to investigate some of its aspects into mathematics classroom from both teachers' and learners' perspectives, with a focus on the way learners and teachers utilised GeoGebra. Learners who used GeoGebra in their mathematics learning found the software to be helpful and having advantages in their mathematical work. Learners and teachers were found to be passionate about the use of GeoGebra in their classrooms. Sweden established a new curriculum for mathematics in which the use of technology was emphasised in all upper secondary courses. However, Mehanovic (2008) indicates that there is a need that teachers be supported in order for GeoGebra to be used to its maximum potential. This implies that the use of GeoGebra in the teaching and learning of transformation geometry will arouse learners' and teachers' interest.

Verhoef, Coenders, Pieters, van Smaalen and Tall (2015) sampled seven teachers from secondary schools in different regional Dutch schools and five staff members of the University of Twente made the study crew. Their study intended to detect teachers' proficient improvement in a lesson study group incorporating a sensible approach to calculus by using GeoGebra. Verhoef et al. (2005) defined a sensible approach as the approach that takes into cognisance the structures of mathematics. Research instruments used to gather data comprised lesson preparation forms, teachers' anticipated intermediations (GeoGebra) and questionnaires with learners' activities, field notes of learners' observations which were documented during the lessons, accounts of the deliberations and the contemplative meeting to uncover teachers' professional development, as well as exit-interviews which were used to corroborate teachers' proficient development in relation to data from other instruments mentioned. The positive effect on teachers' professional development was confirmed. The implication towards this study is that, teachers will move from their current level to the next level as they will be developed by the use of software GeoGebra.

Xistouri and Pitta-Pantazi (2013) found that instructional activities with GeoGebra for teaching reflection at the primary school level were helpful. Their study aimed at demonstrating the way in which GeoGebra can be used to design an instructional program based on the stages of the so-called 5E's instructional model (engagement, exploration, explanation, elaboration and evaluation). Learners were asked to use GeoGebra to reflect objects. The findings of their study revealed that GeoGebra was a valuable tool for learners' active learning of geometric reflection concepts and properties at primary school and that GeoGebra offered a rich environment for designing a variety of activities with different cognitive requirements and objectives. This implies that the use of GeoGebra will offer a different environment for learners such that they learn to the best of their ability.

Bulut and Bulut (2011) established how pre-service teachers used the dynamic mathematics software (GeoGebra) during the instruction and education of mathematics concepts. Their study revealed that some pre-service teachers used GeoGebra for establishing real life examples looking at the world at large (Bulut & Bulut, 2010). For example, CENSUS or Statistics South Africa can use GeoGebra to calculate measures of central tendency of a particular population. The data about a specific population can be easily and quickly analysed for information purposes. GeoGebra offered web-publishing opportunities for mathematics education, could be used for writing examinations questions, and to create multiple representations of mathematical concepts in geometric, algebraic and spreadsheet formats (Bulut & Bulut, 2010). Thus, saving teachers' time for preparation as the tools are readily available in the software.

In their one week study with Grades 10 and 11 learners from four schools in Grahamstown, Holzl and Schafer (2013) discovered that the use of GeoGebra made learners understand concepts such as reflection with ease (Holzl & Schafer, 2013). The use of GeoGebra encouraged learners to be engaged enthusiastically with the task given to them. Learners were able to create sophisticated and interesting figures relating to reflection of an object. For example, learners reflected pictures of fish, animals, trees and etc. GeoGebra incorporates conceptual understanding which is learner-centred, as long as teachers use the appropriate pedagogy (Holzl & Schafer, 2013). This implies that learners will be able to transform different objects they can think of, thus erasing the picture that transformation geometry is only concerned with diagrams such as triangles, parallelograms, etc.

The literature that was reviewed for the purpose of this study revealed that many studies conducted about the use of GeoGebra were with regard to geometry in general. Only a limited number of studies to investigate the use of GeoGebra in transformation geometry were done. In order to add to the body of knowledge and to fill the gap that was identified, this study was conducted. In addition, transformation geometry was chosen because it is was identified to be a topic that was less researched.

It appears that GeoGebra is a valuable tool for learners' active learning of transformation geometry. It offers a rich environment for designing a variety of activities with different cognitive requirements and objectives. In addition to this, it can support functions of the planned sequence of instructional stages based on the engagement, exploration, explanation, elaboration, and evaluation. This makes GeoGebra a valuable tool for designing a structured and effective instructional program for transformation geometry. It appears that GeoGebra has countless likelihood to support learners to get a distinctive feeling and to picture adequate mathematical processes.

2.4 The Theoretical Framework

Having reviewed the research studies above, one specific theory of the development of geometric thinking was recurring to an extent that the researcher could not avoid engaging with this theory in-depth. The van Hieles' theory postulated about the levels according to which geometric teaching and learning should take place proved invaluable in any study with a focus of improving geometrical thinking. Not only that, but the van Hieles' theory came to use in the assessment of geometric attainment. As a result therefore, an in-depth study was conducted about this theoretical basis for geometrical development and its principles were subsequently applied within the data collection instruments designed for the present study.

"The van Hieles' theory developed by Pierre Marie van Hiele and Dina van Hiele-Geldof in the 1950s had been internationally recognised and affected the teaching of geometry in schools significantly" (Abu & Abidin, 2013, p. 17). The van Hieles' were disappointed with learners' low level of knowledge in geometry and were also concerned by their own failure to communicate ideas successfully during their time as mathematics teachers (Kin-Wai, 2005). They were Dutch teachers who experienced challenges with regard to their learners' lack of understanding of geometric concepts, which explains their interest in investigating ways that could help learners to understand geometric concepts better (Pusey, 2003).

In 1957 the van Hieles' proposed a five level scale according to which learners could be assisted to progress from one level to the next and they described the geometric thinking at each level. Pierre and Dina took different angles in their respective research studies: Pierre designed the base model from a learning perspective and described in detail five ascending levels of geometric understanding, thought or development. These levels were originally numbered from zero to five (Meng & Idris, 2012; Pusey, 2003) and were described using abstract nouns: Level 0 is visualisation or recognition; Level 1 is analysis; Level 2 is informal deduction or abstraction; Level 3 is formal deduction; and Level 5 is rigor (Knight, 2006). Dina's research was done from a teaching perspective and focused on the process of helping learners to progress by, in that describing five teaching phases, the first phase is inquiry, the second phase is direct orientation, the third phase is explication, the fourth phase is free orientation, and the fifth phase is integration (Pusey, 2003). These teaching phases each corresponded with a learning level, or a level of development in geometric thinking. The combined model would later become known as "the van Hieles' levels" for short. Teaching based on the van Hieles' model is widely acclaimed as being effective to motivate learners and to create a better environment for the teaching and learning of geometry (Abu & Abidin, 2013).

The teaching of school mathematics, geometry and arithmetic, has been a source of many misunderstandings (van Hiele, 1999). A gap in learners' prerequisite understanding of geometrical concepts causes learners not to grasp further concepts. It is important that learners master their current level in order for them to proceed and master the next level of geometric thought (van Hiele, 1999). As far as transformation geometry is concerned, it is imperative that learners master reflection, translation, rotation and enlargement/reduction of a point before they can master transformation geometry in respect of figures.

Abdullah and Zakaria (2013) asserts that the aim of the van Hieles' model as assisting learners to progress from one level to the next, which does not only reinforce their present understanding of geometry theories, but also helps them to progress to further levels. The core assertion of the theory is that learners progress by sequentially from one level to the next by working through instructional activities that are applicable in terms of dialectal and mission for their level of understanding (Connoly, 2010).

2.4.1 Analysis of the van Hieles' Model

Knight (2006) analyses the van Hieles' theory as having three main components, namely, the existence of levels of understanding, the actuality of properties that are innate to individual level, and the requirement that movement to a subsequent level is preceded by the previous level. The implications are that each level is distinguished by a different understanding; that each level has different characteristics; and that, in order to proceed to the next level, one must have mastered the previous lower level.

Knight (2006, p. 6) identifies and describes five common properties of the five van Hieles' levels, namely fixed sequence, adjacency, distinction, separation, and attainment, as follows:

(i) "Fixed sequence", implying that learners have to master the lower Level *n* before they can proceed to the next higher Level *n+1*. This property is in accordance with the Vygotskian notion that learners have to be taught spontaneous concepts before scientific concepts.

(ii) "Adjacency", meaning that the properties of entity(s) well-read at a certain Level *n* are important for the properties of object(s) at Level *n+1*. Once learners master a level, their zone of proximal development (also a Vygotskian idea) now encompasses the next higher level.

(iii) "Distinction", which is defined as the ability of a learner to distinguish between figures or objects that appear to be similar.

(iv) "Separation", implying that if two people are at dissimilar levels of understanding, they will not comprehend each other.

(v) "Attainment" or "advancement", denoting that the learning progression that directs to comprehensive understanding at the next higher van Hieles' level.

Knight (2006) asserts that it is the teacher's obligation to offer the right materials, relevant instruction and support that will facilitate learners to master a level. Memorising of concepts in mathematics or geometric understanding is discouraged, while the recommendation is that teachers need to facilitate learning such that learners have deeper understanding of the reasons for geometric concepts (Knight, 2006). Therefore, the use of mathematical software might encourage learners not to be memorise concepts, but rather understand concepts taught by their teachers.

The indication is that learners who are taught at a higher level than the one they possess or are ready for, will not make it in high school as well as in further studies that deal with geometry (Knight, 2006). Knight (2006) gives an example of a learner at Level 1, who may recognise that a rectangle is a rectangle because of the way it appears or because of its shape; however, at Level 2 a rectangle seen as a such because it has two pairs of parallel sides that are equal in length as well as four right angles; though at Level 3 learners recognise that a rectangle has opposite sides which are parallel due to the fact that it has four 90° angles. An illustration of "distinction" is given by Knight (2006), when he indicates that a learner at Level 1 will notice that a square and a rectangle are two different figures, nonetheless he will not recognise that a square is a rectangle since they do not as yet know the properties of each. However, a learner at Level 2 will realise that a square is a rectangle but that does not mean a rectangle is a square, since a square has all properties that make up a rectangle.

"Separation" in this scenario, occurs when a teacher is operating at Level 2 while teaching learners who are at Level 1 of geometric understanding. She might talk about properties of figures which learners operating at Level 1 do not understand. There is separation or a difference between the levels at which teacher and learner operates, which may explain learners' failure in geometry (Knight, 2006, p. 6). When the two are not operating at the same level, learners may think they understand the concept while the teacher facilitates learning, however, when they attempt an exercise alone, they cannot perform.

The "attainment" property develops along the five phases of teaching (Dina van Hiele's construct), namely, inquiry, direct orientation, explication, free orientation and integration in conjunction with the five levels of geometric thinking (Yazdani, 2007). These phases lead to complete understanding at further van Hieles' levels. If instruction is developed according to the sequence of phases of thinking, higher levels of thinking may be attained.

During Phase 1 ("inquiry") teachers and learners discuss the topic. The teacher as a facilitator of learning asks questions in order to establish what learners already know and what they do not know (prior knowledge) and also to identify gifted and slow learners in the classroom (Muyeghu, 2008). Learners can subsequently be given differentiated support: more activities are given to gifted learners, while the teacher gives slow learners individual attention to assist them to better understand the concept being taught.

In Phase 2 ("direct orientation") learners investigate the properties of figures by mentioning them one by one as they observe them. The teacher provides learners with materials in an order that will direct them to develop steadily about the awareness of the properties under investigation (Knight, 2006). Muyeghu (2008) elucidated that the teacher leads the learners along a path to make sure that learners become conversant with definite key ideas that are connected to the topic. The main aim is ensuring that learners are actively engaged throughout the lesson while the teacher only guides the learners in the right direction.

During Phase 3 ("explication") learners begin to clarify the relationships within the topic studied. Besides helping with the right vocabulary, the teacher does not provide learners with the expected observations and explanations. She guides the learners in an appropriate way while the learners work much more independently (Muyeghu, 2008). At this level learners explain concepts, using their own words, about what they have learned for the day.

In Phase 4 ("free orientation") learners work individually and independently on more complex problems to investigate rules and conjectures. Learners are free to use any method or heuristics to reach answers or solutions (Knight, 2006). The tasks, according to Muyeghu (2008) are of such a nature that the elucidations need blend and utilisation of concepts and associations formerly elaborated. Muyeghu (2008) identified three roles of the teacher at this phase, firstly, to select an appropriate geometry concept to teach together with activities and unambiguous instructions; secondly, to motivate learners to clarify the geometric problems; and thirdly, to make known the geometrical terms and concepts applicable to the lesson of the day.

In Phase 5 ("integration") learners make summaries and generate conclusions, both in words and mathematically, in other words, they evaluate and replicate on their learning and understanding (Muyeghu, 2008). The teacher helps with the summary and make certain that no fresh information is presented at this point (Knight, 2006). At the conclusion of this phase, learners would have mastered the level and are ready to proceed to the next higher level (Knight, 2006). Clements and Battista (1992, cited in Muyeghu, 2008) confirmed the part of the teacher at this period as a facilitator who motivates learners to reflect and consolidate learners' geometric knowledge and by so doing, developing geometric understanding (Muyeghu, 2008). The van Hieles' model can be graphically represented as in Figure 6.

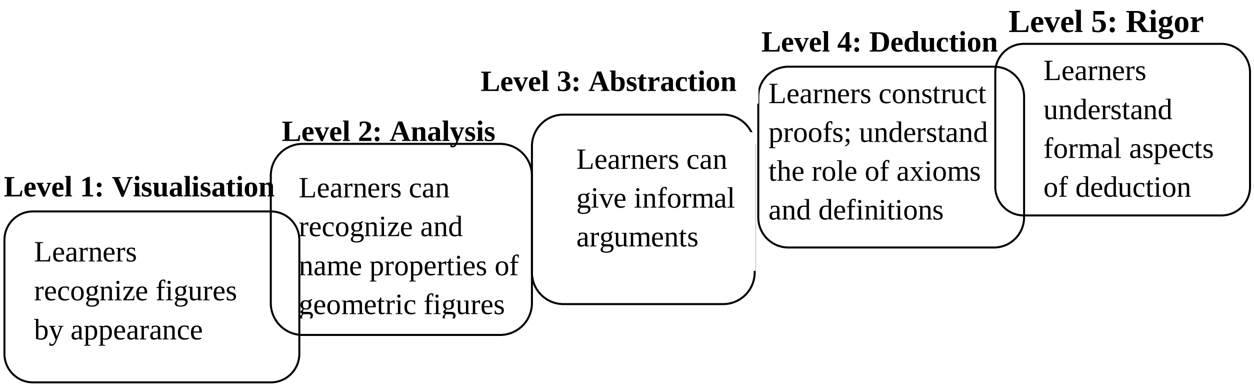


Figure 6. van Hieles' theory (adapted from Mason, 2002).

Figure 6 can be explained as follows:

2.4.1.1 Level 1 (visualisation).

At this level learners are expected to informally recognise a difference in the shapes of objects, but they may not formally differentiate between the objects based on their properties or sizes. They can identify only visual characteristics of the shapes, but may for example not make a distinction relating to a rhombus and a parallelogram (Muyeghu, 2008). In other words, they identify geometric figures by physical appearance, and not through partial characteristics (Lee & Kim, 2012).

At this level, the identification of parallel lines and right angles, for example, are not possible. Therefore, learners may generally identify a rectangular box as having a square shape; a triangular ruler as having a triangle shape; and a pizza or a ball as a having a circular shape. Muyeghu (2008) advocates that learners at Level 1 should be given tasks that are clear, simple and without ambiguity of any kind. The tasks given to learners ought not to be more intricate than the concrete concepts taught; the undertaking should permit sovereignty of elucidation; and they should be as truthful as possible (Muyeghu, 2008).

2.4.1.2 Level 2 (analysis).

Learners at level 2 are expected to spot and label properties of geometric figures, but do not grasp the connection between these properties. Learners can mention all properties of geometric objects though they may not discern those that are sufficient to describe the object, for example they may describe a rectangle as having four sides, however, may not be able to differentiate between a square, a rectangle, a rhombus and a parallelogram, as they all have four sides. Learners may for example, recognise that a parallelogram has parallel sides and are equal; however, they are unable to identify a quadrangle with equal parallel lines as a parallelogram (Lee & Kim, 2012).

2.4.1.3 Level 3 (abstraction/informal deduction).

At this level learners are expected to distinguish relationships between properties within and between shapes, learners realise that the properties of a square correspondence with the properties of a rectangle in some respects, but not all: properties of a square are present in a rectangle; therefore a square is a rectangle. They may also tell which property is not found in another object, but the rationale behind the difference in the properties is not yet understood, for example a rectangle is not a square (Muyeghu, 2008). Also, they realise that a parallelogram has opposite sides that are equivalent and parallel, hence opposite angles that are equal. According to Malloy (2002), learners at this level are able to formulate generalisations and informal arguments about what they have learned previously to substantiate rules derived.

2.4.1.4 Level 4 (deduction).

Learners at level 4 learners are expected to build proofs and comprehend the starring role of axioms and definitions. Learners advance orders of proclamations that soundly substantiate conclusions. According to Malloy (2002), learners at Level 4 prove theorems deductively and understand the structure of the geometric system. Given an isosceles triangle, for example, learners are able to prove that base angles are equal (Lee & Kim, 2012). However, learners do not yet fully understand how important it is to be precise with regard to their answers; neither do they understand the thinking transition relation when doing deductions (Lee & Kim, 2012).

2.4.1.5 Level 5 (rigor).

Learners are expected to deduce or make clear, valid and correct conclusions when given exercises to solve. Learners can also use geometric proofs and appreciate geometry as a whole (Muyeghu, 2008). According to Malloy (2002), learners at Level 5 institute theorems in dissimilar systems of assertions and match and scrutinise deductive systems. Learners at this level are expected not to confuse theorems, but rather to use their knowledge in making deductions. Furthermore, at this level, learners understand the postulate system's characteristics, namely, non-contradiction, independence and completeness (Lee & Kim, 2012). With this insight, learners may use axioms without proving them and therefore become efficient and flexible in geometric reasoning.

Each level involves an improvement over the reasoning abilities of the previous one. There is a strong hierarchy implied in the levels of geometric thought (Choi-Koh, 2000). The implication is that learners at a subordinate level cannot be required to understand lessons offered to them at an upper level of thought. If learners are expected to understand a higher level before mastering their current level; they will underperform in mathematics (Choi-Koh, 2000). Therefore, learners must be introduced to simpler aspects of transformation, for example, transformation of a point before transformation of a figure or object.

2.4.2 Educational Implications of the van Hieles' Model

Knight (2006) analyses some implications of the van Hieles' model on content included in the teaching and learning of mathematics and geometry in particular. According to the expectations set by the National Council of Teachers of Mathematics (NCTM), learners in Grades R-2 are anticipated to deal mostly with visualisation and detection (Level 1) and then be introduced to identification of parts (Level 2). A Grade R-2 learner has to be able to identify objects because of the way they appear, that is, a learner can recognise a triangle, a rectangle, a circle, etc, because of their appearance. Learners in Grades 3-5 are expected to deal entirely with recognition and description (Level 2) and ought to be able to tell that a square has four sides, a triangle has three sides, and so on. Grades 6-8 learners are expected to deal mainly with precision of descriptions, using manipulatives (objects which are designed so that a learner can perceive some mathematical concept by manipulating it), to generate casual inferences about transformation rules and to resolve problems (Level 3). Learners are expected to be able to give explanations for specific transformations. For example, a learner may list and compare the properties of two figures, noticing that the properties of a square are present in a rectangle, and that a square is a rectangle though a rectangle is not a square. Grades 9-12 learners are expected to transact solely with making, trying and ascertaining conjectures (Level 4). Learners who did not attain a van Hieles' Level 2 of geometry understanding prior to taking secondary school geometry programme have inadequate levels of understanding to achieve or pass (Knight, 2006; Crowley, 1987). This implies that research into ways and means of ensuring that learners understand transformation geometry concept is vital to help learners to achieve in mathematics.

Lee and Kim (2012) advocate for the facilitation of instruction in such a way that learners will proceed through the grade with the correct levels of expectation. Learners should therefore be given the right instruction at the right time and be tested according to the corresponding expectations, thus ensuring that what learners learn, how they learn it, and how they demonstrate what they have learned is a match for their present level.

There are some slight indications of alignment with the van Hieles' levels within the South African CAPS document for mathematics at the Senior Phase (Grades 7-9). At Grade 9, straight recall of mathematical facts (the knowledge level of cognition) bears a weight of 25% where for example a learner is expected to remember that the addition of the inner angles of a triangle equals to 180°. Routine procedures, for example calculating the total exterior surface area of a square based pyramid, is weighted 45%. Complex procedures like proving congruency or similarity is weighted at 20% where a learner is expected to make significant connections between different sets of information. The fourth cognitive level in CAPS is problem solving, where learners are expected to solve non-routine problems and to have higher order understanding, a level that shows similarities with the van Hieles' Level 5.

2.4.3 Classroom Applications of the van Hieles' Theory

Kin-Wai (2005) asserts that after the van Hieles' model has been publicly adopted by many researchers, teachers increasingly implemented it in their practices of teaching and learning geometry. However, in the literature review for this study, only a few articles were encountered of which the research was done with the van Hieles' theory on transformation geometry; more studies on van Hieles' theory was based on the broader field of geometry.

In Figure 7 (an adaptation of Figure 6) the practical application of the van Hieles' theory inside the teaching and learning of transformation geometry is illustrated and explicated afterwards.

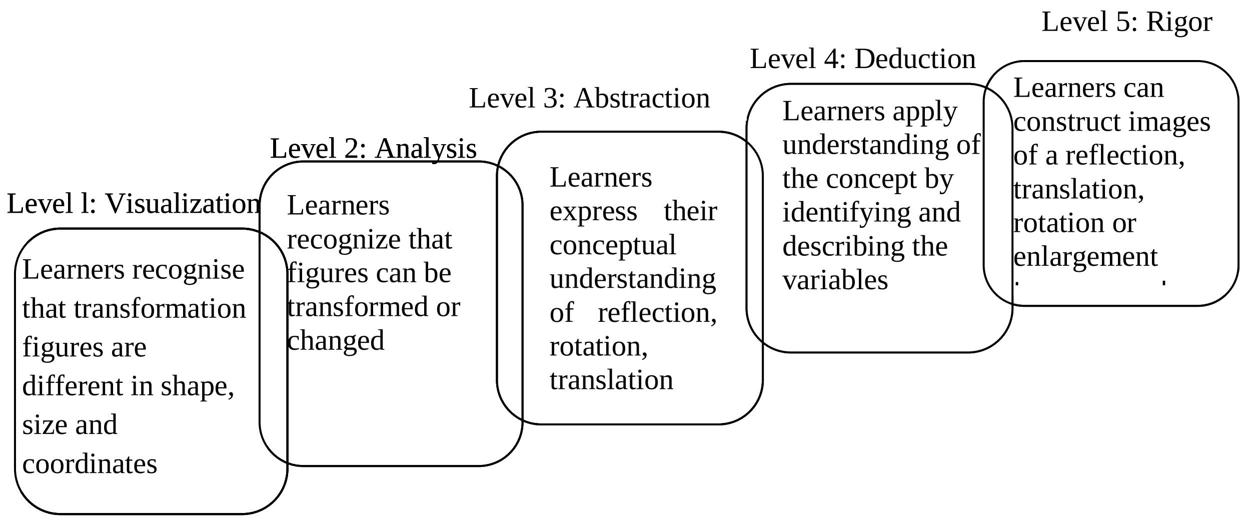


Figure 7. Van Hieles' theory applied to transformation geometry.

2.4.3.1 Level 1 (visualisation).

This level is labelled "recognition" and is also known as "visualisation" (Pandiscio & Knight, 2011). Visualisation is regarded as a basic and holistic and level (Abu & Abidin, 2013). The teacher accesses learners' prior-knowledge through exercises and then engages them in a new concept through additional exercises that arouse their interest in the topic (Xistouri & Pitta-Pantazi, 2013). At this level learners may for example be given a triangle ABC and its image A'B'C' that is reflected along the y-axis. Learners are then asked to check how the coordinates of the object relate to the coordinates of its image. Suppose the learners are asked to reflect triangle ABC along the y-axis with coordinates as follows: A (-3; 1); B (- 2; 2) and C (-1; 1). The image A'B'C' will have coordinates A' (3; 1); B' (2; 2) and C' (1; 1). It is visually possible for learners to work out the pattern of the positive x coordinates that become negative in the reflected triangle, as it is reflected along the y-axis. The observation (visualisation) of this change may (or may not) at this stage, stimulate a primitive rule for reflection along the y-axis.

2.4.3.2 Level 2 (analysis).

At this level, learners may complete investigative activities that help them to practise what they already acquainted with, in order to derive new ideas, explore questions and possibilities, and then conduct a preliminary investigation (Xistouri & Pitta-Pantazi, 2013). Activities should be geared towards helping the learner to understand the concept of exploring the direction of the image's position with respect to the pre-image. In this way, learners may discover the relationships between the reflection line and the images (Xistouri & Pitta-Pantazi, 2013). Suppose the learners are asked to reflect rectangle PQRS (see Figure 8) along a line x = y with coordinates as follows: P (-2; 2); Q (-1; 2); R (-1; 1) and S (-2; 1). The image P'Q'R'S' will have coordinates P' (2; -2); Q' (2; -1); R' (1; -1) and S' (1; -2). By analysis, it is possible for learners to figure out the relationship between the reflection line x = y, and the change in coordinate values within the original rectangle and its reflected image.

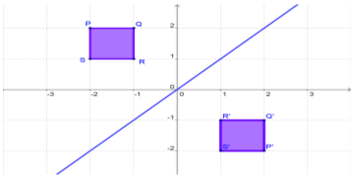


Figure 8. Reflection along the line y = x.

2.4.3.3 Level 3 (abstraction/informal deduction).

This level is known as the level of abstract/rational-, theoretical-, co-relational- and informal deduction (Abu & Abidin, 2013). These words imply that learners begin to make simple deductions in the form of words to explain a specific transformation. At this level learners' focus on an individual facet of their engagement and familiarities with geometrical concepts that may lead to conceptual understanding (Xistouri & Pitta-Pantazi, 2013). Suppose learners are asked to identify and describe the role of the line of reflection as a variable distance between the image and the object. Learners are expected to abstract that, within reflection for example, the distance between every point of the image is always equal to the corresponding distance of the object point (Xistouri & Pitta-Pantazi, 2013). Examples can be drawn from the triangle ABC (see Figure 9) that was reflected along the y-axis with coordinates A (-3; 1);

B (- 2; 2) and C (-1; 1), and A'B'C' with coordinates A' (3; 1); B' (2; 2) and C' (1; 1); and also in the case of the rectangle PQRS (see Figure 9) along the x = y with coordinates P (-2; 2); Q (-1; 2); R (-1; 1) and S (-2; 1); and P'Q'R'S' with coordinates P' (2; -2); Q' (2; -1); R' (1; -1) and S' (1; -2). A is at an equal distance from the y-axis than A' and the P is at an equal distance from the line x = y than P', proved by the fact that x = y.

At this level of geometry thinking, activities should be geared towards helping the learner understand the concept of the position between the reflecting line and the object; as well as the distance between the reflecting line and the image. This will enable them to draw the image at the correct position when the object is drawn for them.

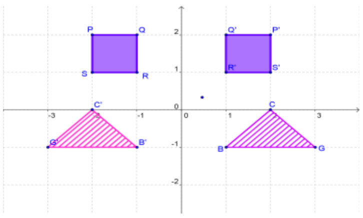


Figure 9. Reflection along the y-axis.

2.4.3.4 Level 4 (deduction).

At this level learners apply their existing understanding of the concept by engaging with the given exercises (Xistouri & Pitta-Pantazi, 2013). Learners apply their knowledge in the coordinate plane by identifying and describing the variables that affect reflection, for example in a vertical line (Xistouri & Pitta-Pantazi, 2013). Based on previous understanding, learners have observed, analysed and abstracted the pattern (and the rule for the patterns) that occur in various scenarios of reflection, including reflection along the x-axis, the y-axis and along a line x = y. The progression at this level, is that learners can generalise and predict, based on their empirical knowledge, what the position of the image of any shape, line or point will be when reflected along any of the mentioned lines.

At this level activities should be geared towards helping the learner understand the concept of generalisation and describing a transformation both in words and mathematically.

2.4.3.5 Level 5 (rigor).

This stage of geometrical understanding encourages learners to assess their understanding and abilities while providing opportunities for teachers to check the progress of the learners (Xistouri & Pitta-Pantazi, 2013). Learners can check their progress as they engage with different types of exercises in order to evaluate their understanding of the topic. Following on our example(s), the aim is to construct the image of a reflection (see Figure 10).

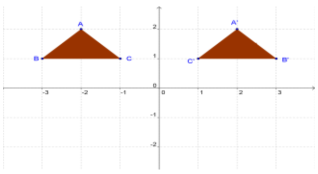


Figure 10. Figure ABC reflected along the y-axis.

At this level of geometry thinking, activities should be geared towards helping the learner to be creative and to finding alternative methods of solving a given exercise.

2.5 Research Regarding the van Hieles' Theory in Geometry Teaching and Learning

Extensive research studies have been conducted across the globe over a period of six decades regarding various applications of the van Hieles' theory within the teaching and learning of geometry at different school levels, as well as at the tertiary- and professional teacher's levels. These applications pertain to the classroom use of the model, (i) to various types of assessments (diagnostic-, formative- and summative assessments), and (ii) to the effects on the learning of geometry and also to the use of the model in tandem with ICT software programmes in support of geometry understanding.

2.5.1 The Influence of the van Hieles' Theory on Teaching and Learning Geometry

Siew, Chong and Abdullah (2013) investigated the effects of the van Hieles' phases of learning on Grade 3 learners' understanding. A pretest-posttest single group experimental design was used. 221 participants were categorised into high, moderate and low ability. The learners were taught two-dimensional geometry as well as symmetry using a tangram. A geometric test was given to learners prior and after the intermediation. In order to establish the treatment effect, paired sample t-tests using a one-way multivariate analysis of variance were done in order to compare the learners' pre- and post-test mean marks. The findings revealed that there was a noteworthy dissimilarity between pre- and post-test in the geometric thinking of the learners. The findings of this study also revealed that there was a much greater improvement in low ability learners' scores as compared to moderate and high ability learners' scores.

In a study about the upshots of research-based approaches for projecting the van Hieles' levels of geometric thinking, Najdi (not dated) sampled 207 mathematics teachers at Alquds Open University in Palestine. The intentions of the investigation were to find out both the van Hieles' levels of geometric thinking on teaching (with the sample of 207 teachers), and the consequences of these strategies on learning (with a sample of 18 learners). The test comprised two fragments: fragment one (true-false) contained levels 1 and 2 of the van Hieles' theory. Fragment two consisted of subjective questions and covered van Hieles' levels 3, 4 and 5. The pre-test mean was 2.67 as compared to the post-test mean of 3.28, resulting in a statistically significant mean score difference with $t = 3.051$ and $p = 0{,}007 < 0.05$. The following findings which are learners' levels according to van Hieles' theory were recorded:

- Level 0: 1.9%
- Level 1: 14.5%
- Level 2: 28.5%
- Level 3: 25.7%
- Level 4:19.8%
- Level 5: 9.6%

These findings reveal that teaching geometry while applying research-oriented methods for increasing the van Hieles' levels of geometric thinking meaningfully increased learners' understanding of geometrical concepts. This implies that research on GeoGebra for transformation geometry with van Hieles' theory might reveal vital information for teachers as facilitators during the teaching process.

In their study to test the helpfulness of van Hieles' phase-oriented learning, Abdullah and Zakaria (2013) divided 94 learners into two clusters; one control cluster and the one investigational cluster. The control group was taught transformation topics using conventional methods; while conversely the treatment group comprehended the same subject matter based on van Hieles' phases. Both groups were given a van Hieles' Geometry Test (VGHT) before and after the treatment. The analysis of results which were obtained using Wilcoxon-t tests uncovered that there was a substantial difference between learners' final levels. However, they indicated that the majority of learners in both groups reached the first van Hieles' levels with complete acquisition, a low acquisition at Level 2 and no acquisition at Level 3. Learners in the control group showed a boost of geometric thinking from the first to the second level; without any one achieving Level 3. In contrast, almost all learners in the experimental group completely achieved Level 1 to 2, and one learner achieved at Level 3. Learners in the experimental group showed a high level of acquisition, according to Abdullah and Zakaria (2013). Similarly with the present study, learners were given a paper and pencil test that was marked and analysed according to the van Hieles' levels in order to conclude the findings.

In their study to investigate South African Grade 10 learners' geometric thinking, Alex and Mammen (2012) purposively selected 191 mathematics Grade 10 learners from 5 senior secondary schools in one education district in the Eastern Cape. Learners were given a van Hieles' geometry test which entailed items questions 1-5, questions 6-10, questions 11-15, questions 16-20, for identifying van Hieles' levels 1 up to 5 correspondingly. The test was administered by mathematics teachers from all participating schools during school hours in the respective classrooms. The data collected was analysed using Microsoft Excel. In their findings, the researchers revealed that 48% of the learners were accomplishing at level 1, learners at level 2 made 29%, learners at level 3 made 14%, learners at level 4 made 18% and no learner was at level 5 of van Hieles' geometric thinking. Alex and Mammen (2012) point out that while mathematics teachers are perplexed about learners' poor performance in geometry, studies based on van Hieles' geometric thinking shed light with regard to factors that may affect the performance of learners. Teachers might not be taking into consideration learners' levels of geometric thinking before they start engaging learners with geometry concepts. Through understanding the van Hieles' (1986) levels, teachers may gain insight into learners' readiness to learn geometry. Learners will not be able to perform at level n without mastering a lower level n-1; in other words levels of thinking have a hierarchical arrangement. Therefore, it is important that in order for teachers to ensure that learners are successful in geometry, delivery of instruction in line with their levels of geometric thinking is an imperative. To a great extent, learners' failure in mathematics is generally associated to teachers' failure to deliver instruction that is relevant and appropriate to learners' level of geometry thinking (Alex & Mammen, 2012).

Brodie (2011) investigated the geometric thinking of Grade 6 teachers. The rationale behind the study was based on the low performance of Gauteng learners in the International Competitions and Assessments for Schools (ICAS) tests which took place in 2006. Documents, questionnaires and interviews were used at tools for data collection. Questionnaires were administered to 40 ACE teachers; while interviews were conducted with six teachers. Interviews comprised five ICAS questions which contained van Hieles' levels 1 up to 3 (Brodie, 2011) that the expectation was that Intermediate Phase learners would perform up to van Hieles' Level 3. The findings revealed that the majority of teachers were not at the required level of geometric thinking as required by the NCS assessment standards

(Brodie, 2011). Teachers were therefore unable to understand their learners' geometric errors, as a result they could not develop learners' problem solving skills.

Kin-Wai (2005) has successfully investigated the effectiveness of Van Hieles' based instruction with 132 mathematics learners and two teachers. All learners who took part in the study were from four diverse classes and aged between 14 and 15 years. A quasi-experimental design with four consecutive lessons and a pre-test were used to answer the research question. The control group consisted of 67 learners from class 3A and 3B; while the investigational group entailed 65 learners from 3C and 3D. The indication is that learners were evenly distributed into the two groups. Outdated methods of teaching were used in the control group and the textbook, without any innovation. On the other hand, the investigational group was taught by means of van Hieles'-based instruction. Lesson observation was conducted in the control group to ensure that the abstract method of teaching was used. Only three van Hieles' levels were included in the tests as studies had already highlighted that the middle Grade learners cannot reach the van Hieles' level 4. The findings of the pre-test revealed that no learner in the control group was at van Hieles' level 0; however 41.79% of the learners functioned at van Hieles' level 1; 29,85% functioned at van Hieles' level 2, 25.37% of the learners functioned at van Hieles' level 3; while 2.99% functioned at level 4. The results of pre-test in the experimental group revealed that 1.54% of the learners were at van Hieles' initial level; 35.38% were at van Hieles' second; 26.15% were at van Hieles' third; 30.77% were at van Hieles' fourth; 6.15% were at van Hieles' last level. However, the post-test in the control group revealed that there were 1.49% of the learners at van Hieles' level 0; while there was no learner at van Hieles' level 0 in the experimental group. The findings revealed that most learners were at van Hieles' level 1-3. The analysis further revealed that learners in the experimental group performed at an upper van Hieles' level as associated to the control group. The indication is that the use of van Hieles'-based instruction work towards a higher understanding of geometry and can help learners who are at a level 1 to perform at a higher level. Therefore, it might also help in respect of this study to use van Hieles' model in the teaching and learning of transformation geometry.

Unlike other researchers who looked at more than one levels of van Hieles' theory, Wu and Ma (2005) focused their research on the investigation of the geometric conceptions of basic school learners at van Hieles' first level. 5581 elementary school learners (2717 girls and 2864 boys) who were randomly selected from 23 cities in China participated in their investigation. The main objective of their study was to investigate the passing degree of each geometric shape and type. The main focus was based at level 1 which is a level where learners are able to visualise objects and can therefore differentiate them because of their appearance and shape. Wu's geometry test which consisted of 25 multiple-choice questions was designed specifically for the study as they could not find suitable Chinese instruments. The 25 van Hieles' level 1 questions were branded into nine categories grounded on their geometric designations. The first question type were based on recognition of open and closed numbers; the second type were recognition of convex and concave figures; the third type involved recognition of straight line and curve line; the fourth type involved recognition of rotate figure; the fifth type involved recognition of figures of unlike sizes; the sixth type involved recognition of exceedingly obtuse figures; the seventh type involved recognition of wide and slim figures; the eighth type involved recognition on the width of contour line; the ninth type involved recognition on occupied and hollow figures. The outcomes of their study revealed that the overall performance was 77.5% with 85.14% for the circle concept; followed by 75.88% for the triangle; and 71.49% for the quadrilateral. The results indicated that there were concepts which seemed easy and those that seemed difficult for the learners (Wu & Ma, 2005). The implication for the current study is that teachers need to consider that the van Hieles' levels of the learners need to be taken into consideration during teaching and learning of transformation geometry.

Shaughnessy and Burger (1986) conducted a study with 45 learners from five schools in three states. The 45 learners were obtained from a grade of seven categories (0 to 6). The first was early primary- , the second category was Grades 2-3, the third was Grades 4-8, the fourth was algebra 1, the fifth was geometry, the sixth was algebra 2, and the seventh was college mathematics majors. In their report researchers indicated that the initial year of the study was a changing phase in which the investigational duties, a dialogue script, and a protocol scrutiny coding packet were written and reviewed three times. The experimental tasks were administered to all learners using an audio tape clinical interview. Learners were provided with all the necessary Learner Teacher Support Material (LTSM). The interview asked further clarity seeking questions in order to be certain that learners meant what they said. Each interview which lasted for about 40 to 90 minutes was conducted in a noise free environment without disturbance. The tasks were designed in such a manner they mirrored the van Hieles' geometry understanding. The approved reasoning tasks were aimed to gain data about van Hieles' second and third level. The tasks did not embrace van Hieles' level's fourth level. The discoveries of the investigation hinted that the van Hieles' levels are expedient in unfolding the thinking processes of learners on polygon tasks. The implication in respect of the current study is that learners should be given an instruction that will enable them to perform even at van Hieles' level 5 (rigor). The use of GeoGebra as a mathematical software will add value in learners' understanding of transformation geometry.

2.5.2 The Use of the van Hieles' Hierarchical Attribute in Learner Assessment

The van Hieles' levels of understanding provide a valuable aid in the assessment of learner performance. Atebe and Schafer (2010) probed the ranked property of the van Hieles' levels and their relationships with learners' performance in high school mathematics. These researchers conducted their study, based on an experimental design. 24 learners from each grade were selected using both stratified and purposive sampling techniques. Five paper and pencil and hands-on activity tests were used to obtain the data. These tests included the van Hieles' Geometry Test (VHGT), Geometric Item Sorting Test (GIST), Conjecturing in Plane Geometry Test (CPGT), School Examination in Mathematics (SEM) and the Terminology in Plane Test (TPGT). The test-retest reliability coefficients in their study were found to be 0.87 and 0.52 for the TPGT and the VGHT respectively. The findings of their study indicated that there was no learner at van Hieles' Level 5 (rigor); however 53% of the learners were at recognition level (Level 1); 22% were at analysis level (Level 2); 24% were at abstraction level (Level 3); while 1% of the learners were at deduction level (Level 4). The implication of the analysis above is that very few learners performed at van Hieles' level 5. Therefore the use of GeoGebra as a way to increase the learners' levels to the maximum might be helpful.

Gunhan (2014) investigated a small scale study with six Grade 8 learners (3 girls and 3 boys); getting their training at a community basic school that was indiscriminately nominated from among schools of a reasonable socioeconomic prominence (Gunhan, 2014). The results enabled the classification of performance according to the van Hieles' levels of thinking, and in doing so, created an improved understanding between teachers and learners. Generally, such information may support teachers better comprehend the manner in which their learners contemplate when they are unravelling geometry problems, and can help them to see their learners' reasoning skills (Gunhan, 2014). Thus the use of van Hieles' model might enhance teaching and learning of transformation geometry.

Halat (2006) compared the attainment of the van Hieles' levels of learners in Grade 6, in which 273 learners were separated into two sets: 123 learners were classified as the control group were the instruction was facilitated using traditional methods and 150 learners were the treatment group was taught according to the reform-based curriculum. The participants were given a multiple-choice test which was administered to all participants. The findings revealed the following:

- Level 1: 27,6% (control group) vs 17.3% (treatment group)
- Level 2: 52% (control group) vs 65.3% (treatment group)
- Level 3: 20.4% (control group) vs 17.4% (treatment group)

The indication from the results is that learners from both groups progressed by the levels; however, learners did not reach the fourth (abstraction) and the fifth level (rigor).

Al-Shehri, Al-zoubi and Rahman (2011) aimed at investigating the effectiveness of gifted learners' centres in developing geometric thinking. The sample of the study consisted of 60 gifted learners who were divided into two equal groups. The sample was classified as gifted by the criteria set out by the ministry of education in Kingdom of Saudi Arabia (KSA). The control group comprised 30 learners who were studying at Giftedness Resource Rooms in KSA; while the experimental group comprised 30 learners who were studying at Gifted Centre in Najran. All participants were given a van Hieles' test as a pre- and post-test so as to collect data. The test which was validated by mathematics specialists consisted of 35 multiple-choice and true or false questions which covered all the five levels of the van Hieles' theory. A grade zero was given to the wrong answer, while a grade one was given to the correct answer. Test-retest with a measured factor of 0.84 was obtained to ensure reliability of the study, as well as using KR-20 formula with a measured reliability factor of 0.89. The findings of their study revealed that centres for gifted learners had the ability to develop geometric thinking of the learners. Two-Way Analysis of Variance (ANOVA) was utilised to institute the variance in the learners' levels of geometric thinking. The outcomes depicted the role of the gifted centres in developing geometric thinking skills in learners.

Aydin and Halat (2009) aimed at investigating probable effects of unlike college level mathematics courses on college participants. In their study, the emphasis was also to inspect whether there were variations in van Hieles' levels of learners who had commenced with non-geometry courses that accentuated rationality and proofs, that is, category 1 courses and those that did not openly use or teach rationality or proof writing, that is, category 2 courses. In their inquest, academics used an expediency sampling procedure to select 149 learners. Group 1 was made of 41 learners from Category 1 courses, while group 2 included 108 learners from Category 2 courses. The VGHT was directed to all the partakers during a one class period, adjacent to the end of the semester, with the aim of collecting data. The VGHT consisted of 25 multiple choice geometry questions. Independent samples t-tests were run to associate the two groups' levels of van Hieles' and to institute the effects of the courses from both categories. The descriptive indicators showed that the mean mark of the first group (4.02) was more than the mean mark of the second group (2.64). The autonomous samples t-test showed that there was a considerable higher variance ($p < .001$, momentous at the $\alpha/2 = .025$ utilising a crucial value of $t\ \alpha/2 = 1.96$), which favoured learners in the Category 1 course. Students' average at the van Hieles' levels fell between Level 2 and Level 3.

In his research, Dindyal (2007) conducted a qualitative study over a three-month period in two rural schools in the United States. The class from the first school comprised 21 learners;

while the class from the second school comprised 18 learners. Algebra tests constructed by the researcher; as well as the VHGT test which was obtained from Usiskin (1982) study were directed to these two classes. The algebraic test was authenticated by mathematics teachers from participating schools and three other experts in the field. Three learners from each school were selected for interviews; their selection was based on their performance from the two tests which were administered. Anton, Mary and Beth from the first school were found to be at van Hieles' level fifth, second and second level in that order; while Kelly, Ashley and Phil from the second school were at van Hieles' fourth, second and second level in that order. The focus learners were interviewed for 40 minutes each; while two participating teachers were interviewed for 30 minutes. Documents from the interviewed learners were also analysed for data collection. Participating classrooms were also observed 12 times in three months' time. The findings revealed that there is a hierarchical development of geometric thinking in learners; and also the fact that levels in geometry cannot overlook the important link between geometry and algebra (Dindyal, 2007). Conversely, the learners' levels of geometry may also be positively affected by technology, for instance dynamic geometry software which may be used to explore geometric concepts.

Duatepe and Ubuz (2009) investigated the special effects of drama-orientated geometry tuition on learner achievement, attitudes and intellectual levels with 102 Grade 7 learners from a public school in Balgat district of Ankara, Turkey. The indication by Duatepe and Ubuz (2009) is that quite a number of researchers agree that learners are not learning geometry to satisfactory levels. A quasi-experimental approach with 34 learners randomly assigned to a control group and 68 learners in the experimental group. The two tests, namely, the Angles and Polygons Achievement (APA) test and the Circle and Cylinder Achievement (CCA) test consisting of 24 and 25 questions respectively were developed by the researchers.

Questions were validated by mathematics lecturer and a graduate in mathematics. The revised questions were steered to 129 and 153 Grade 8 learners from two public elementary schools. The outcomes of the average and standard deviation of these groups were analysed; they revealed that mean score of van Hieles' levels of the experimental group was 6.15 and 7.41 for pre- and post-test correspondingly; whereas the control group had a mean score of 7.40 and 6.16 for pre and post-test in that order. The analysis of the findings was unsuccessful to detect a statistically weighty difference in mathematics mind-sets and van Hieles' geometric thinking levels between groups (Duatepe & Ubuz, 2009).

A case study on the exploration of intellectual skills in geometry was conducted by Gunham (2014) with six Grade 8 learners. The learners were selected on the basis that they were composed of two high; two medium and two low achievers. The participants were selected by their mathematics teacher who had 23 years of teaching experience. However, the participating school was selected randomly among schools that belong to moderate socioeconomic level. Qualitative research methods were utilised so as to answer research question. Multiple choice questions which were prepared by experts were used to collect data from the learners. The results of the Gunham (2014) study revealed that school curriculum places a reduced amount of emphasis on reasoning skills. The implication is that if teachers do not take into consideration learners' level of geometric thinking, there will be lack of understanding of geometrical concepts.

Gutierrez et al. (1991) conducted a study on unconventional paradigm to assess the acquirement of the van Hieles' levels with 50 learners. The 50 learners represented 20 future primary school teachers in their third year; 13 future primary school teachers specialising in foundation phase; eight primary school teachers specialising in languages; as well as nine Grade 8 learners in the same classroom of a state primary school. The spatial Geometry Test with 9 items was intended to value the van Hieles' of learners in three-dimensional geometry. The test was administered to pilot groups of primary school learners and to future primary school teachers. The result of their study revealed that there was no acquisition of level 4; however, there was intermediate acquisition of level 3 (weight = 50), a complete acquisition of level 2 (weight = 100). The findings revealed a relationship between the levels and the degree of acquisition. The findings showed that the higher the level, the lower the degree of acquisition, which supports the hierarchical structure of the van Hieles' levels. Learners are not cognisant of the presence of thinking means specific to a new level, but once they begin to be aware of methods of thinking at a given level, they try to utilise them. However, they might fail because of their lack of experience (Gutierrez et al., 1991). As the learners use the methods more often at a particular level, they start to have an intermediate degree of acquisition. Nevertheless, due to the fact that they do not master the methods, they are likely to fall back to a lower level. Learners' skills are strengthened by experience; that is when they acquire a high degree of acquisition at that level. Once learners master the skills and methods at a particular level, they therefore have a complete acquisition of the new level.

Gutierrez and Jaime (1998) steered a study on the assessment of the van Hieles' levels in which they showed that since the beginning of application of van Hieles' model, researchers always deemed it necessary to use instruments to measure learners' geometric thinking levels. The indication is that some researchers have also attempted to use clinical interviews to assess learners' levels of geometric thinking as interviews as they provide more information about the learner's way of reasoning that other procedures. However, the use of interviews is not feasible when many learners are involved because it is time consuming (Gutierrez & Jaime, 1998). In their study, 179 Grade six learners (aged 11 to 14) and 130 Grades 10 to 11 learners (aged 14 to 18) from four secondary schools in Spain were sampled.

The tests were validated by using several pilot studies; as well as involving experts in the field. The findings revealed that a high percentage of learners with the exception of Grade Six learners had a complete acquisition of level 1, and the learners were progressing in the acquisition of level 2. 23% of the learners did not answer the last question; this had a negative influence in the results for other levels. Gutierrez and Jamie (1998) highlighted the fact that the van Hieles' levels are assimilated via a number of abilities which learners must master. The implication is that a learner might master some abilities but not all; as a result a learner may progress to the next higher level without all the cognitive requirements of the previous level. The reality in the teaching of geometry and mathematics is that learners are taught a level that is higher than the level which they possess (Gutierres & Jamie, 1998). Learners are therefore forced to think at a higher level, resulting in learners' underperformance.

In his thesis, Knight (2006) investigated the level of thoughtful geometry of pre-service basic and secondary mathematics instructors at the Maine University. The main aim was to investigate whether there was statistically significant evidence that learners have a level of thoughtful geometry at or beyond their anticipated audience. The data was gathered prior and after the learners' finishing of the prerequisite course and evaluated with respect to van Hieles' level beforehand and after the finishing point of the course for which they registered. Participants were requested to finalise a profile survey that escorted the pre-test. The van Hieles' test which consisted of 25 questions was utilised as a pre- and post-test that was patented in 1982 by Usiskin (Knight, 2006). The piloted test used in the study was given as an oral examination administered in three states of Maine. The device was then directed to four schools to all-inclusive classes. The questions were organised in order in sets of five so that questions 1 to 5 measured learners' understanding at van Hieles' level 1; on the other hand, questions 6 to 10 measured learners' understanding at van Hieles' level 2; questions 11 to 15 measured learners' understanding at van Hieles' level 3; there was also questions 16 to 20 measured learners' understanding at van Hieles' level 4; while questions 21 to 25 were included with the purpose of measuring learners' understanding at van Hieles' level's last level. The discoveries publicised that learners' level of geometry understanding was below level 3. The expectation was that learners who are undergraduates or graduates were expected to perform at van Hieles' level 3 or above (Knight, 2006).

Pandiscio and Knight (2011) examined the van Hieles' levels of pre-service basic and secondary mathematics teachers. The VHGT test was directed to accumulate data about the geometric thinking of learners. Pre- and post-tests were directed towards all participating learners. The results of the first sample consisting of all learners taking the 400 level geometry course revealed that out of 18 learners who wrote the pre-test, the mean was found to be 2.895, standard deviation of 0.658 and t = -7.324. The post-test scores revealed that out of 12 learners who wrote the test, their mean was 3.077, standard deviation of 0.862 and t = -3.860. These results show that the van Hieles' level of learners in the 400-level course was statistically lower than the level 4 (deduction) that was expected (Pandiscio & Knight, 2011). The results are worrying because learners are performing at lower level of geometric thinking. The important question that comes to mind is how to break the cycle of limited geometric understanding. The findings suggest that instruction can enhance content knowledge (Pandiscio & Knight, 2011).

2.5.3 The van Hieles' Theory Combined with ICT in Geometry

So as to have a richer understanding of the usage of van Hieles' model as a theoretical framework to assess the progression of geometrical thinking while using supportive software, several studies have been reviewed to understand the role of this model.

Meng (2009) interacted with Grade 8 learners in the education of solid geometry in a phase-based instructional setting bestowing to the van Hieles' theory, using Geometer's Sketchpad (GSP), a popular commercial interactive geometry software program. The outcomes exposed that learners' preliminary van Hieles' levels oscillated from level 0 to level 2, and over phase-based intercession in their levels of thinking were raised.

Later, Meng and Idris (2012) conducted a case study with mixed-ability Grade 6 learners to explore whether learners' intellectual accomplishments in solid geometry may perhaps be enhanced through phase-based instruction as stated by the van Hieles' theory. In one instance, learners used manipulatives (objects which are designed so that a learner can perceive some mathematical concept by manipulating it), and in another using GSP. An interview instrument was established to measure the participants' van Hieles' levels grounded in Mayberry's interview tool. The motivation for picking Mayberry's interview tool was that it was premeditated to evaluate learners' van Hieles' levels vis-à-vis clear geometric concepts. A Solid Geometry Achievement Test (SGAT), comprising of two parts (part A with 15 multiple choice questions and part B with 10 short-answer items) was also constructed based on the Grade 6 syllabus. An analysis of their results revealed that participants advanced from level 1 to level 3 on behalf of cubes and cuboids. Partakers also displayed understanding of delineations, class invasions, likeness, congruence and insinuations concerning cubes and cuboids.

Meng and Idris (2012) found that phase-based instruction (as per van Hieles'), by means of manipulatives (objects which are designed so that a learner can perceive some mathematical concept by manipulating it), possibly will augment the van Hieles' levels of secondary learners; however, the results obtained with regard to the effectiveness of phase-based instructions using the computer, were inconclusive. They speculated about the possible explanations for the progress in participants' understanding, and amongst others, considered the conducive effect of the use of instructional undertakings centred on the van Hieles' theory.

In a further quasi-experimental study by Abdullah & Zakaria (2013), 94 secondary schools learners and two teachers were sampled in order to exam the efficiency of van Hieles' phase-based learning on learners' level of geometric conception. Half of the group was the control group (taught conventionally) while the other half was the treatment group (taught through the use of GSP). The intention of the study was to exam how operative the van Hieles' phases in the erudition of geometry through GSP were. The van Hieles' geometry test was administered to both groups. The findings revealed that learners in the treatment assemblage attained improved levels of geometric thinking in comparison to the control group (Abdullah & Zakaria, 2013).

Based on their study, Olkun, Sinoplu and Deryakulu (2009) presented an article on geometry activities for elementary school learners grounded on the van Hieles' geometric intellectual levels by means of a dynamic geometry application, Geometers' Sketchpad. The instructional activities emphasised learners' learning through explorations instead of teaching a specific mathematical content (Olkun, Sinoplu, Deryakulu, 2009). The reported evidence based on classroom visits indicated that learners both enjoyed, and learned much from the lesson(s), as they participated actively. The results of classroom-tested geometry events founded on the van Hieles' geometric applications made evident that learners raised their level of geometric thinking by building on their current geometric understanding (Olkun, Sinoplu & Deryakulu, 2009). Learners criticised the way they were taught previously, as they indicated that the use of Geometers' Sketchpad assisted them to understand mathematics better.

In a study with 30 Grade 9 learners, Abdulla and Zakaria (2011) conducted a two day pilot study using GSP. The aim of their study was to investigate learners' discernments in the direction of the van Hieles' phases of studying geometry via Geometer's Sketchpad. Transformation geometry events grounded in van Hieles' phases of learning were established. In order to obtain data about learners' opinion with regard to the actions centred on van Hieles' phases of geometry education; surveys were employed. The discoveries of their investigation were that 86.66% of the participants conveyed that the utilisation of the activities given to them, made them learn geometry topics easily. 90% of the learners also indicated that the use of GSP software which is a technological tool, boosted their confidence in geometry learning. Figure 10 shows an example of the activities that were set for the purpose of the study. Based on the data obtained, Abdulla and Zakaria (2011) came to the conclusion that van Hieles' levels are credible and can be employed as a strategy for learning geometry in tandem with their use of GSP (see Figure 11).

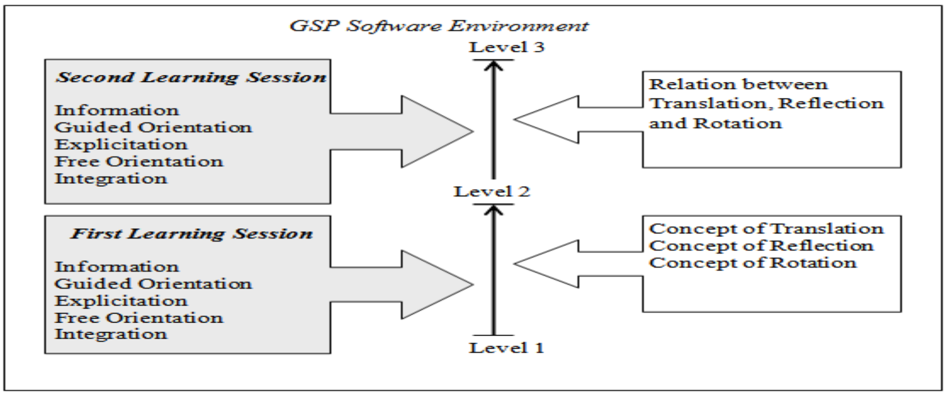


Figure 11. Van Hieles' phases of learning in GSP software environment.

An educational video, called Video Pembelajaran Geometri (VPG) was developed in a study by Abu and Abidin (2013) with the aim of helping Grade 9 learners to improve their levels of van Hieles' geometric thinking. VPG was developed as an alternative aid to overcome the perceived limitations of information- and communication technology. 180 participating learners were categorised into 90 learners attaining (van Hieles') Level 1 of understanding, 60 learners at Level 2, and 30 learners at Level 3. A van Hieles' geometry test was used to collect the data. Researchers found that pre- and post the use of VPG, a significant difference was demonstrated in learners' mean test scores. The results also revealed that there was a significant improvement in learners geometric thinking; that is, the number of learners at Level 1 decreased from 90 to 30, while at Level 2 the number increased from 60 to 70 and from 30 to 62 at Level 3. The indications are that the usage of the van Hieles' theory could to shift learners from a junior level to the next upper level, as 75% of the learners improved either between the levels or within a level.

Idris (2007) investigated the upshot of using Geometers' Sketchpad on enactment in geometry accomplishment and van Hieles' levels between Grade seven learners in one secondary institution in Kuala Lumpur. A quasi-experimental design was sought to be the best option in order to obtain the data. An experimental group with 32 learners was taught using Geometers' Sketchpad; though the control group with 33 learners was trained using conventional methods. The van Hieles' geometry study used pre- and post-test examinations with directed, questionnaires and checklists were used as data collection techniques. Descriptive statistics were obtained in order to answer the research question on learners' van Hieles' levels. The upshots revealed that there was no substantial change in the pre-achievement test between the control and investigational sets ($p<0.05$). However, the post-test revealed that there was a substantial variance amongst the control and investigational group (t-value of 2.78 and $p=0.02$). Van Hieles'-based instructional resources and the usage of Geometers' Sketchpad seem to be good tools that assisted learners to progress from a junior level to the next advanced level (Idris, 2007).

In another study Idris (2009) conducted a similar study investigating the power of utilising Geometers' Sketchpad on Malaysian learners' attainment and geometric thinking of van Hieles' with 65 Grade 10 learners in Perak. The treatment group encompassed 32 learners who underwent geometry lessons using Geometers' Sketchpad, while 33 learners in the control were taught similar lessons using more traditional methods. The learners in each class were of mixed ability; for that reason the single class was allotted to be a control cluster, while the other was allocated to be an experimental set. Pre- and post-geometry tests were directed so as to assemble data and to compare learners' geometry achievement and van Hieles' levels. Questionnaires were also given to the learners in order to explore their reaction concerning the use of Geometers' Sketchpad. In analysing the change in van Hieles' levels by group, the results revealed that the experimental group had sum scores of 378.95, standard deviance of 45.72 and a mean score of 4.80. On the other hand, the control group had total scores of 309.17, standard deviance of 345.85 and an average score 3.63. Van Hieles'-based instructional tools and the usage of Geometers' Sketchpad have an indispensable role to play in assisting learners advance from individual level of geometric thinking to the next upper level (Idris, 2009). Thus the indication is that the use of ICT can impact positively on learners' understanding of mathematical concepts, similarly to the current study on the use of GeoGebra.

An inquiry on the influence of web project-based learning on basic school learners' growth of van Hieles' geometric thought was conducted by Huang et al. (2013) in Taiwan. These researchers indicate that advancement through the levels is more reliant on the training received and not on the age or adulthood of the learners. That means the way instruction is conducted plays a pivotal role in the progression from a solitary level to the subsequent upper level of geometric thinking. Convenience sampling was used to select two Grade 5 classes from one elementary school in Taipel (Taiwan). Learners were alienated into a control set (29 learners) and an investigational set (31 learners). Participants received a score for each van Hieles' level according to Usiskin's grading system. The findings revealed that the experimental group had a pre-test of 71% at level 1, 55% at level 2, no acquisition of level 3. The control group had pre-test scores of 75% at level 1, 54% at level 2 and no acquisition of level 3. However, the post-test scores revealed an interesting phenomenon. The experimental group had post-test scores of 90%, 90% and 3% for the van Hieles' first, second third level in that order. On the other hand, the control group had post-test scores of 89%, 75% and 11% of van Hieles' first, second and third level correspondingly. The results displayed an escalation per level in the pre-test scores in comparison with the post-test scores. The results implied that the use of web project-based learning impacted positively on learners' levels of geometric thinking (Huang et al., 2013). Therefore it is imperative that research on the use of GeoGebra be done so as to maybe raise the learners' level of geometric thinking to van Hieles' level 5.

Kutluca (2013) investigated the consequence of geometry teaching with dynamic geometry software (GeoGebra) on van Hieles' geometry indulgence of Grade 11 learners. A quasi-experimental means with pre- and post-tests was utilised. 24 learners in the experimental group received lessons with the help of computer-assisted instruction with dynamic software GeoGebra; while 18 learners in the control group received their lessons using traditional methods. All participating learners were computer literate even though there are no computer literacy courses in the curriculum (Kutluca, 2013). In the experimental group, the teacher played the role of a facilitator by encouraging learners to work on the worksheets in their small groups of two learners per group; thereafter sharing their findings with the rest of the class. Inversely, the control set was not given any therapy; the teacher dominated the lessons by teaching using traditional methods such as explaining and asking questions. Mann-Whitney U-test revealed that there was on no account noteworthy variance among the pre-test scores of experimental and control group ($u = 128.00$, $p > .05$). However the post-test scores showed that there was numerical variance between the investigational and control group where the Wilcoxon signed rank test was used ($z = -3.655$; $p < 0.01$). The outcomes of the investigation implied that the usage of dynamic software for instance GeoGebra has the ability to increase learners' van Hieles' geometry thinking levels.

2.6 Responses to the van Hieles' Model

Over six decades, the seminal work of the van Hieles' has attracted numerous opinions, research studies and also suggested additions and alterations of the original (see Section 2.6.4).

2.6.1 Support for the Theory

There is widespread support amongst researchers (Pleet, 1990; Malloy, 2002; Lee & Kim, 2012; Knight, 2006) for the van Hieles' theory as a background in the teaching and learning of geometry, with its identifiable hierarchical development of cognition (Dindyal, 2007). Pleet (1990), in his study about the effects of computer graphics and Mira on the acquirement of transformation geometry conceptions and psychological rotation abilities at Grade 8, summarises his impressions of the model as follows:

> *Van Hieles' showed that in order for a learner to function adequately at one of the advanced levels, he or she must have mastered the prior levels. Thus, it is no surprise that learners have had difficulty in high school geometry- level 4- when they are entering the course with only level 1 proficiency. It is clear that elementary school*

and junior high school experience are insufficient background for learners who are expected to perform at level 4. (p. 40).

It is therefore important that during the teaching and learning, teachers take into cognisance the fact that learners are at different van Hieles' levels; hence, learners need to be given the instruction that will enable them to move from one van Hieles' level to the next higher level. The use of GeoGebra in the teaching and learning of transformation might help to achieve the desired goal of taking learners up the level of geometric thinking.

2.6.2 Parallel Conceptualisations of Developmental Progression in Geometry

Battista (2011) investigated conceptualisations and questions correlated to learning advancements, learning curves and levels of intricacy. Battista (2011) indicates that the National Research Council (NRC) pronounces learning advancements as depictions of the sequentially more intellectual way of thinking about a subject from simple to complex topics. However, learning advancements is not developmentally inexorable, but somewhat depends on the instruction given to learners. The indication is that the way instruction is given to the learners play a pivotal role in ensuring that learners understand the topic taught. In order for learners to learn, they need not memorise concepts or bypass levels. Learners are at a certain van Hieles' level when their inclusive perceptive structures and dispensation cause them to be dexterous of thinking about a topic in a specific way (Battista, 2011). There is a clear link between van Hieles' theory and Battista's learning progressions as they both advocate that learners should progress from one level of thinking to the next higher level. Van Hieles' and Battista might have used different terminologies with the same meaning.

2.6.3 Numbering Structure of the van Hieles' Levels

As far as the numbering of the van Hieles' levels is concerned, Muyeghu (2008) for example, condensed the van Hieles' levels to four, and described them in terms of adjectives, namely, visual, descriptive/analytic, abstract/relational and proof. Knight (2006), asserts that if mathematics teachers realised that learners did not have a complete conception even at the primitive level (recognition), they might need to identify a level prior to Level 0 (recognition). A numbering system ranging from zero to five, would accommodate learners who had not become proficient at van Hieles' initial Level 0 (recognition) and the new Level 0 would be referred to as pre-recognition (Muyeghu, 2008). Thus van Hieles' numbered their levels of geometric thinking as Level 1 to Level 5; this numbering system is used in the current study.

Teppo (1991) re-examined the van Hieles' theory of levels of geometric thinking. In his study Teppo (1991) mentions three van Hieles' levels instead of five levels as indicated by researchers such as Duatepe & Ubuz (2009) and Gunham (2014) to mention a few. Teppo (1991) indicates that van Hieles' currently characterises his model in terms of three levels of geometry understanding: the first level known as visualisation), the second level known as descriptive and the third level known as theoretical. These levels are said to be achieved by passing through different learning periods where learners examine applicable objects of inquiry, cultivate precise language linked to the objects, and participate in collaborative learning activities. In level 1 (visual), learners are expected to recognise objects globally and go through integration, free orientation, explication, bound orientation as well as information as phase of learning. In level 2 (descriptive) learners are expected to recognise objects by their geometric properties and go through all the phases mentioned in level 1. The last level,

level 3 (theoretical) is the level of deductive reasoning in which learners prove geometric relationships.

For the purpose of this study, the numbering system 1-5 was used for the levels as adapted by Mason (2002), namely level 1 (visualisation); level 2 (analysis); level 3 (abstraction); level 4 (deduction); and level 5 (rigor).

In connection with the hands-on application spectrum of the van Hieles' model, Huang, Liu and Kuo (2013) point out that these five levels of geometric thought could be stages "that any learner could be at, and the level of development was related to teaching factors, it was not influenced by the children's age and maturity" (Huang, Liu & Kuo, 2013, p. 51).

2.6.4 Perceived Shortcomings of the van Hieles' Model

Concerns exist about the perceived lack of generality pertaining to the van Hieles' theory of geometric thinking. Guitierrez, Jaime and Fortuny (1991) argued that subject to the complication of the task at hand, a learner might develop two successive van Hieles' levels simultaneously. Guitierrez et al. (1991) did however not reject the van Hieles' theory but suggested that it be amended to cater for such instances. Guitierrez et al. (1991) indicate that a human being might not operate in a simple, linear manner by moving from Level 1-5 in sequence as the van Hieles' indicate in their model.

According to Dindyal (2007), some researchers suggested that an alternative framework that merges the van Hieles' theory with additional theories such as the Structure of the Observed Learning Outcomes taxonomy (SOLO) and Skemp's (1987) model, addressed the identified shortfalls of the van Hieles' theory. Pandiscio and Orton (1998) as cited in Dindyal (2007), argue for an amalgamation of the van Hieles' and Piaget models as a framework for the teaching and learning of geometry. Pegg and Davey (1998) as cited in Dindyal (2007), argue for a combination of the van Hieles' and SOLO genres for exploration in geometry.

2.7 The Application of the van Hieles' Theory within the Present Study

This investigation, in which both mathematics teachers and learners participated, aimed to investigating the usage of GeoGebra in the teaching and learning of transformation geometry. The secondary research questions were formulated in such a way as to direct the investigation into how teachers used GeoGebra in the teaching of transformation geometry. This with, respect to the consequence of using GeoGebra in the teaching and learning of transformations and what the experiences and perceptions of teachers and learners were about the use of GeoGebra. The ultimate concern that has led to this research endeavour is the perceived inability of learners of Grade 9 learners to successfully progress in their geometric thinking to the level required for deep conceptual understanding of the ideas and dynamics, in this case, of transformations. This concern required the understanding of geometrical thinking, a matter that is addressed to great depth by the acclaimed van Hieles' theory of the development in geometric understanding. This theory underpins the present study at a theoretical level.

2.8 Summary of Literature Review and Theoretical Framework

The literature review conducted as the theoretical underpinning for the study, was arranged according to the two main foci of the study, namely, transformation geometry and the levels of development of geometrical thinking. The overarching theoretical model presented by van Hieles' (1996) directed, at a theoretical level, the course of the present study. As far as the secondary research questions are concerned, the same theory served as a guideline towards the development of the research instruments. The theory of a mixed methods research (Creswell, 2010) was applied within the methodology of the present study and is discussed in Chapter 3.

Some of the anchor theories upon which the study was based, were as follows:

- *Linking Geometry and Algebra: A Multiple-case study of Upper Secondary Mathematics Teachers' Conceptions and Practices of GeoGebra in England and Taiwan* (Lu, 2008)
- *Structure and Insight: A Theory of Mathematics* (van Hiele,1986)
- *An Investigation into the Difficulties Faced by Form C Students in the Learning of Transformation Geometry in Lesotho Secondary Schools* (Evbuowam, 2013).

2.9 Conceptual Framework

In the present investigation, the conceptual framework emanated in the identified problem situation as discussed in Chapter 1, and was developed, mainly through a review of the literature containing theories and research outcomes pertaining to the aim and objectives of this research. The need for the present study emerged, not only from the identified problem within mathematics, but also from the fact that there is limited research on the specific topic in focus, namely transformation geometry. The focus on GeoGebra as a possible contributor to a solution within the South African mathematics education context, specifically as far as transformation geometry at Grade 9 is concerned, is at the core of the conceptual framework presented in Figure 12. The framework progresses towards the conceptualisation of how the intended investigation would address the problem situation in tandem with the suggested solution, as depicted in Figure 12:

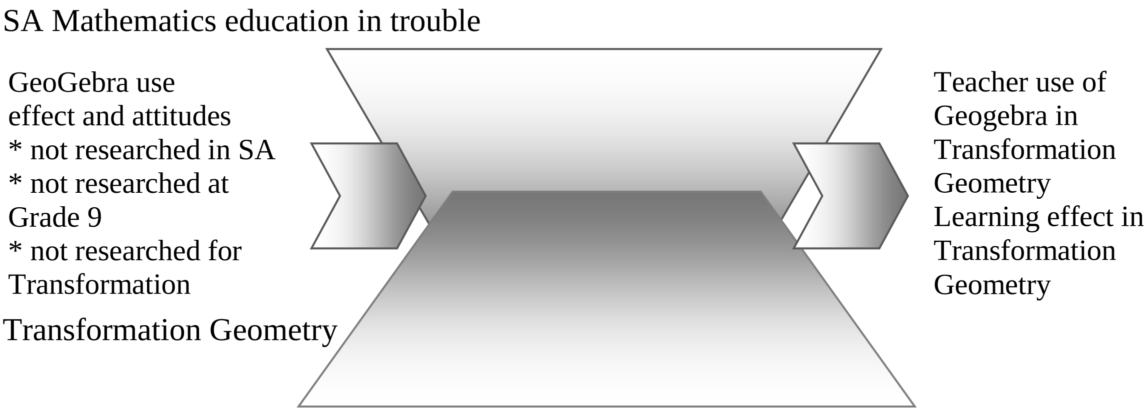


Figure 12. Conceptual framework for the use of GeoGebra.

2.10 Conclusion

This chapter highlighted what GeoGebra is and its use in the teaching and learning of transformation geometry. Also explained in detail is the van Hieles' model as well as the theoretical framework used in the study. The next chapter presents the research design for this study.

CHAPTER 3 – RESEARCH DESIGN

This chapter presents the methods used in order to explore the usage of GeoGebra in the teaching and learning of transformation geometry. The focus was on mathematics teachers and learners who use GeoGebra in the classrooms. The research was planned to respond to the following questions:

1. How do teachers utilise GeoGebra in the teaching of transformation geometry?
2. How does the use of GeoGebra affect learners' understanding of transformation geometry?
3. What are the teachers' and learners' experiences in the teaching and learning of GeoGebra?

With the purpose of providing answers to the questions specified above, this subdivision provides the following: research framework for the study, analysis of the research paradigm, the methodological perspective, sample and participants, data collection strategies and instruments, data analysis, research norms and ethical considerations.

Guidance in this study was given by the five phases of research indicated by Muyeghu (2008) which demonstrated the collaborative processes of sampling and collection, data documenting, scrutiny and exhibition, as well elucidations founded on the analysis of data. The five phases are

- Preparation,
- Commencing data collection,
- Elementary data collection,
- Final data collection, and
- Conclusion.

Chronologically, the actual research unfolded in the following way:

3.1 Research Framework

Academic research studies intended to contribute to the existing body of knowledge need to be grounded in the relevant theory, in order to guide the design in a meaningful way (Greene, Caracelli & Graham, 1989). It was paramount in this study to look how literature relates to the research problem and the subsequent research questions, as well as to the research instruments and methodology. This study works within a literature-research-evaluation structure, of which the framework may be represented schematically and chronologically as follows:

Table 3

Research Framework and Chronological Planning of the Study

Planning a	**Planning b**	**Data collection**	**Closing data collection**	**Completion**
2013a	**2013b**	**2014**	**2015**	**2016**
Needs analysis	Ethical clearance	Lesson observation	Coding, Excel, Inferences	Teacher use of GeoGebra in TG
Literature review	Arrangements for access & consents	Written tests	Van Hieles' levels	Effect of Geogebra on learning TG
Curriculum review	Lesson observation protocol	Learner and teacher interviews	Transcribing, coding, Themes, Inferences	Teacher- and learner attitudes wrt GeoGebra
Research questions formulation	Questionnaire for semi-structured interviews	Documents	Excel, Inferences	
Research proposal	Test design	Field notes		
Proposal defence				
Identification of anchor theories	*Assessment of TG:van Hiele (1986)* *GeoGebra software: Lu (2008)* *Methodology: Creswell(2010)*			

Following the literature review and the needs analysis (Planning a, 2013a), the next planning phase (Planning b, 2013b) involved seeking permission to conduct research, description of the type of setting where the research would unfold, and selecting participants or respondents for the proposed research. Permission was granted by the ethics department of the University or Pretoria, the district director (see Appendix A) and the principals of selected schools (see Appendix B). After this, the site where the study would take place was visited to establish a relationship with the respondents or the participants. This activity included an introduction and seeking consent from the participating teachers (see Appendix D), parents/guardians of learners (see Appendix C) and learners (see Appendix J). The purpose of the study was well explained to teachers and learners who participated in the study. The procedures to be followed were also highlighted to participating teachers and learners so that they were fully informed when the study commenced. These actions were taken in as far as the planning for data collection was concerned.

The data collection phase (2014) involved collecting data using semi-structured interviews, lesson observations, paper-and-pencil test, documents, as well as field notes. The preparation of these instruments for data collection and the proceedings of the actual field work are discussed later in this chapter.

In closing the data collection phase (2015), possible interpretations and verification of emerging findings were conducted. This process is discussed in detail in Chapter 4. During this period, the analysis of data involved seeking an all-inclusive sense of the relationship between the diverse parts of the investigation.

In closing, the study was written up (2016) as presented in the chapters of this dissertation.

In the next section is the research paradigm for the present study.

3.2 Research Paradigm

Paradigms are a set of propositions that explain how the world is perceived; they provide assumptions that help researchers to describe reality (CSS Forums, 2007). A paradigm is a whole system of thinking and thus different types of paradigms exist.

This study followed a pragmatic paradigm with a combined “Qual + quan” approach (Creswel, 2010). Although the main approach is qualitative which focused on teachers’ and learners’ interviews as well as lesson observations; quantitative components were included as is evidenced by the results of paper and pencil test.

Pragmatism is generally considered to be the most appropriate paradigm for a mixed methods approach of research, because it includes a set of assumptions about knowledge and review that underpropped the mixed methods approach (Denscombe, 2008). Pragmatism is considered to be the third alternative approach to social academics if they choose that neither quantitative nor qualitative research singlehandedly will afford satisfactory conclusions for the specific section of research they propose (Denscombe, 2008). However, there can sometimes be misconceptions associated with the word pragmatic. Some might think the term pragmatic denotes a definite deficiency of principles fundamental to a course of action and for that reason there is vulnerability in associating a mixed methods tactic with this understanding that anything is tolerable (Denscombe, 2008). It ought to be accentuated that this is not the metaphysical connotation of pragmatism and it is not a denotation that should be connected with the mixed methods approach.

In her study, Feilzer (2010, p. 14) mentions that “pragmatism can be used as a guide not only for top-down deductive research design but also for grounded inductive or deductive research”. Hence, a pragmatic approach serves as the most appropriate approach in answering the research question as mentioned earlier in the dissertation.

3.3 Research Approach: A Mixed Methods Research

Maree and van der Westhuizen (2007) point out that the mode of inquiry informs the research design, and also what happens to the subjects and the methods used in collecting data. The authors further note that the researcher may adopt a qualitative, a quantitative or a mixed mode of inquiry (Maree & van der Westhuizen, 2007). As mentioned before, this study followed a mixed methods research approach in that both qualitative and quantitative aspects were investigated.

According to Ivankova, Creswell and Clark (2007), the mixed methods approach evolved in psychology in the 1950s. The authors indicate that researchers collect multiple quantitative measures and thereafter assess them with a separate method in one psychological construct. Information systems researchers suggest that there was a necessity for an added mixed method research (Petter & Gallivan, 2004). Conversely, the approach for applying mixed methods to research is associated with a number of names in different fields, such as multimethods, integrated, combined, multimethodology, mixed methodology and triangulation (Ivankova et al., 2007). Ivankova et al. (2007) defined a mixed method research as a procedure for accumulating, scrutinising and merging together quantitative and qualitative data at some point so as to comprehend a problem even better. The important thing to note is that mixed method research does not look at research from one angle; it tends to investigate the knowledge of both what is happening and how or why things happen (Lu, 2008). Therefore, a mixed methods approach can be well thought-out for both an expansive (large-scale) project as well as a profound (small scale) study, as research is conducted from all angles. However, the present study focused on a small scale project where participants came from one education district of Gauteng Province.

There are four main reasons why researchers might want to do a mixed method research, according to Ivankova et al. (2007). The first reason might be that the researchers want to explain quantitative outcomes with successive qualitative outcomes. The second reason might be that researchers want to use qualitative records to acquire a first-hand theory that is consequently quantitatively verified. The third reason might be that researchers want to compare quantitative and qualitative data scenarios that will be trustworthy. The fourth reason might be that researchers want to augment their research with a complementary data set; it may be quantitative or qualitative. For this study, the researcher used both quantitative and qualitative outcomes to answer the primary question.

According to Ivankova et al., (2007), there are four basic methodological designs within the mixed method approach that are utilised by investigators. The first design is known as explanatory design, the second is exploratory design, the third is triangulation design and the fourth is embedded design (Ivankova et al., 2007). The purpose of the explanatory design is to utilise qualitative discoveries to support shed light on the quantitative outcomes. The exploratory design is applied when the researcher desires to explore a matter utilising qualitative data before trying to examine it quantitatively. The third, the triangulation design, is a well-known and popular design. In this design the researcher uses both the quantitative and qualitative methods in order to understand the situation better. Triangulation design is utilised when the researcher aims to accumulate both quantitative and qualitative data simultaneously in order to associate and make a distinction of the findings. The fourth design is embedded mixed method, in this design a researcher desires to respond to a secondary research question that is different from the primary research question (Ivankova et al., 2007).

For the purpose of this study, a triangulation mixed method design was regarded as most suitable option as the researcher intended to collect both qualitative and quantitative data at the same time. However, it must be noted that more focus was on the qualitative aspect than on the quantitative part, as the greater part of the data collected is of a qualitative nature. The qualitative aspect of the research used semi-structured interviews and lesson observations; while the quantitative aspect used a paper and pencil test. Triangulation strives to mend the correctness of the outcomes over the assemblage and breakdown of diverse methods (Petter & Gallivan, 2004).

3.4 Data Collection: A Triangulation Approach

This section describes the various data collections that were used to triangulate findings. The way teachers used GeoGebra in the actual teaching of transformation geometry was investigated. Learners' performance in a one hour paper and pencil test was also investigated. Teachers were observed while teaching transformation geometry using GeoGebra. Observations were video-taped in order to answer the research question. Both teachers and learners were interviewed after school hours in order to find out their experiences and how they perceived the utilisation of GeoGebra in the teaching and learning of transformation geometry. In this unit, the minutiae of the data collection process are discussed.

3.4.1 Sample and Participants

The research site for this study was ordinary public secondary schools were learners were taught on a daily basis from Monday to Friday. The study took place during normal school hours for purposes of interviews and lesson observations; however, paper and pencil test as well as semi-structured interviews were conducted after school hours in the library where there was less disturbance and noise.

3.4.1.1 Purposive sampling of schools and teachers.

Purposive sampling was used in this study to select four secondary schools from one district in Gauteng province. These schools are located in one district (Tshwane-West) within the province. Purposive sampling techniques may be defined as choosing individuals or institutions based on a specific purpose in answering the research question (Teddlie & Yu, 2007). Purposive sampling is a sampling in which the researcher deliberately selects settings or persons because they are perceived as being capable of answering the research question (Muyeghu, 2008). In addition, purposive sampling is explained as the type of sampling where researchers choose the representation on the foundation of their verdict of their characteristic (Cohen et al., 2000). According to this criterion for purposive sampling, four Grade 9 mathematics teachers (one teacher per school from four schools) who were willing to participate in this study were selected on the basis that they were using GeoGebra for the teaching of transformation geometry as well as other topics in mathematics. These teachers were using GeoGebra in mathematics for some time resulting in them having expertise in the use of the program. They understood its applications as well as its limits. The teachers also had several years teaching experience in mathematics.

3.4.1.2 Random sampling of learners.

The population in this study was Grade 9 learners who are taught mathematics on daily basis by their teachers. The entire population of learners is taught for a maximum of one hour period per week by their teachers. Six Grade 9 mathematics learners per school were sampled from the said population. They were randomly selected by assigning each learner a number. Numbers were put in a box in order to select six learners per school. Learners who picked numbers from 1-6 were selected to represent participating learners per school. Random sampling is done when processing the entire dataset is unnecessary and too expensive in terms of response time or usage of resources (Olken, 1993). Random sampling provides all persons or events with an equal opportunity of being chosen (Onwuegbuzie & Collins, 2007). It was found to be convenient and less time consuming in choosing learners randomly. A total of twenty-four participating learners who were taught transformation geometry using GeoGebra in their everyday settings, were the participants of this study.

3.4.2 Instruments for Data Collection

There are numerous data collection techniques that can be used in research; namely, interviews, questionnaires, observations, surveys, etc (Maree & van der Westhuizen, 2007). In this section, the sources and instruments used to gather data are elaborated in detail.

In order to answer the first research sub-question about how teachers utilise GeoGebra in the teaching of transformation geometry, lesson observations (see Appendix F) and semi-structured interviews (see Appendix E) were conducted with participating mathematics teachers.

In addressing the second sub-question regarding how the use of GeoGebra affect learners' understanding of transformation geometry, a one hour paper and pencil test (see Appendix L) was administered to participating Grade 9 learners.

For the third Sub-question about teachers' and learners' experiences about the use of GeoGebra, semi-structured conversations were steered at the finalisation of the lessons in a quiet place in the school premises, namely the library.

3.4.2.1 Face-to-face interviews.

An interview as indicated by Cohen (2000); is an exchange of thoughts or sentiments between two or more individuals concerning the topic investigated. In addition, McMillan and Schumacher (2001) referred to interviews as a data collection method that enables researchers to scrutinise their interpretations of how they see the creation and the things in it. Interviews are utilised in this study with a view to understand how GeoGebra was used in the teaching and learning of transformation geometry. When correctly directed, interviews provide the greatest response to a qualitative question (Cutis, Andy, Tery & Michelynn, 2000). Interviews also necessitate a fundamental significance in communal research for the reason that they can elicit people's experiences and observe their social worlds (Lu, 2008). The tangible advantage of interviews is that the researcher is physically present with the interviewees, therefore any misapprehensions may be speedily clarified (Muyeghu, 2008).

This study used semi-structured interviews which often take the form of a conversation with the intention that the researcher explores the participant's view (Nieuwenhuis, 2007). Semi-structured interviews were seen as the best interview type in this study as they are well suited for the exploration of the perceptions and opinions of teachers and learners regarding GeoGebra and allowing inquisitiveness for more information and explanation of responses

(Barriball & White, 1994). The framework, according to which the teacher interviews were conducted, is reflected in Table 4 below.

Table 4

The Themes for Teacher Interviews (adapted from Lu, 2008)

Key Ideas	Illustrations of interview questions
Participant background.	Tell me briefly about your background. How do you teach challenging topics in Grade 9?
Opinion of GeoGebra	How long have you been using GeoGebra in your teaching and learning of transformation geometry? How do you feel about the utilisation of GeoGebra in the teaching and learning of mathematics?
Software evaluation	What do you like most when using GeoGebra? Which part(s) or aspect of GeoGebra do you think requires enhancement?
GeoGebra usage	How repeatedly do you utilise GeoGebra? For what intention (s) do utilize GeoGebra for? For which topics do you use GeoGebra?

Keeping in mind the requirements of rigorous research, it was found that a standardised interview schedule provided exactly the same wording and sequence of questions for each respondent so as to be certain that differences in the answers were due to differences among the respondents rather than in the questions asked. Due to the same wording in the questions asked, reliability and validity rested on passing on resemblance of meaning to questions asked (Barriball & White, 1994).

On the practical side, the advantage of using interviews is the fact that they are easy to administer and does not require the participants to read and write. As a result, willingness to participate is more likely (Maree, 2007). All tapes were kept in a locked cabinet in the natural science building of the University of Pretoria.

Teachers and learners were interviewed face-to-face inside the school premises in a quiet library classroom. A set of semi-structured questions were used for guidance and both teachers and learners were encouraged to share their individual experiences regarding the use of GeoGebra in the teaching and learning of mathematics and transformation geometry in particular. Each interview was audio-recorded for later transcription.

Transcripts (see Appendix M and N) were read closely in order to identify emerging themes as discussed in Chapter 4. Identified themes were then tabulated in order to answer the third research sub-question about teachers' and learners' perceptions and experiences on the usage of GeoGebra in the teaching and learning of mathematics.

The use of face-to-face interviews helped in understanding both how teachers and learners used GeoGebra; and also how they felt about the use of GeoGebra in the teaching and learning of transformation geometry. Teachers and learners illustrated their responses with examples of the topics used with GeoGebra and the reasons why they chose GeoGebra for topics indicated.

3.4.2.2 Lesson observations.

Observations are precise record of what the contributors perform and articulate in their normal settings (Muyeghu, 2008). Observations are an essential data gathering technique as they gives the researcher an opportunity to see, hear and experience reality as participants do (Niewenhuis, 2007). In addition, observations are said to be a chance to collect live data from locations as they materialise (Cohen et al, 2000). By being on site with participants, the researcher may also employ other data gathering techniques with ease as she is familiar to the participants.

The purpose of the lesson observations was to shed light on how teachers used GeoGebra to teach transformation geometry, how they applied it to the concepts of transformation geometry and also how learners responded to questions asked by the teacher. A peculiar phenomenon was observed, namely, almost all learners had smartphones or tablets. These devices were used to download GeoGebra as it is free software, so that they can use it to enhance effective learning of transformation geometry and other topics in Mathematics.

According to Niewenhuis (2007), there are four types of observations. The initial type is whole observer where the investigator is not participating and looks at the state of affairs from a distance. Secondly, the participant is viewed as an onlooker where the investigator comes to be a portion of the investigation process by working with the participants. Thirdly, the observer is seen as a contributor where the investigator gets immersed in the state of affairs but focuses mainly on his or her role as an observer without influencing the details of the situation. And fourthly, the researcher is a complete contributor as the investigator becomes completely immersed in the setting to an extent that those observed are not aware that they are the subjects of the observation.

Looking carefully at the four types of observations mentioned, an observer as a 'complete observer' seemed to be the best option for this study as the researcher intended to focus on her role as an observer without influencing the study. The researcher watched and video-taped all lessons without distracting teachers and learners. Niewenhuis (2007) suggested three ways in which observations may be recorded, firstly, anecdotal records which are short descriptions of basic actions observed capturing key phrases without self-reflective notes; secondly, running records which are more detailed and indicate sequential accounts of what is observed; and thirdly, structured observations which use predetermined categories of behaviour of what is likely to be observed. In this study, a structured observation protocol was designed by the researcher, and was then used to record all lessons (see Appendix F). The following were observed: the way mathematics teachers taught the learners; their interaction with the learners; the utilisation of GeoGebra during the teaching of transformation geometry; and the activities taught.

In analysing the observation reports (see Chapter 4); the use of lesson observations enhanced the body of data pertaining to teachers' practise of GeoGebra in the teaching and learning of transformation geometry. Observations helped with understanding teachers' levels of expertise with regard to transformation geometry as well as the use of GeoGebra for teaching. Each individual teacher displayed a unique skill in using GeoGebra for teaching transformation geometry.

3.4.2.3 Paper and pencil test.

The paper and pencil test was used with the intention of answering the second sub-question about how GeoGebra affected learners' understanding of transformation geometry. The researcher designed the test in such a way that it accommodated all of the van Hieles' levels. The Grade 9 learners' CAPS textbook was used so as to ensure that the questions were standardised and suitable for the grade. All learner participants were present during the day of the test administration in their respective schools. Selected learners were given a one hour paper and pencil test (see Appendix G) after the lessons at their schools. The one hour test was written in the school's library classroom as it was found to be a conducive environment without much noise or disturbances. The test was invigilated by the researcher in order to ensure that the responses provided in the test belonged to learners. Learners were able to finish the test within the apportioned time of one hour.

The test was marked and analysed by the researcher in order to establish the van Hieles' levels of understanding at which participating learners performed. All written test scripts are kept in a locked cabinet in the Natural Sciences building of the University of Pretoria.

Paper and pencil results were helpful in benchmarking learners' performance, and most importantly, in finding out the learners' level according to the van Hieles' model.

3.4.2.4 Documents.

Document examination and accounts are valuable foundations of information that are expeditiously obtainable and steady with ease of access (Muyeghu, 2008). This method of data collection is said to serve as an element of triangulation. When documents are used as a data collecting procedure, attention ought to be paid to all printed communiqués that may shed light on the study (Nieuwenhuis, 2007). Written sources may include published and unpublished documents, company reports, memoranda, agendas, administrative documents, letters or any other document that is connected to the investigation (Nieuwenhuis, 2007). Documents provide a rich source of information and may be used in quantitative or qualitative studies (Seabi, 2012). The advantages of using documents are that they are economical, relatively quick and easy to access (Seabi, 2012). There are three different types of documents, namely, personal documents such as diaries, confessions, autobiographies or life histories; public documents such as the mass media, statistical year books and school records; archival records such as service records of organisations, hospitals and social workers (Seabi, 2012). CAPS document for mathematics Grade 7-9; mathematics grade 9 textbook; learners' classwork books; as well as teachers' lesson plans were used in this study.

3.4.2.5 Field notes.

Field notes are recorded observations and interactions of what the researcher observes in the field of study (Seabi, 2012). The researcher using field notes should not record only what she/he perceives as important, but should rather record everything observed in the field.

In summary, Table 5 shows each research question, indicator, data source and the instrument used for each research question. The first row contains a question about how teachers utilise GeoGebra in the teaching and learning of transformation geometry. The practise of GeoGebra is an indicator in this regard; while observations and interviews were used as data sources to answer the question. The second row contains a question on how GeoGebra affect learners' understanding of transformation geometry; with the use of paper and pencil test as data source. The analysis of the written test will give an indication of how learners performed; as well as their level of achievement in the test. The third row contains teachers' and learners' experiences bout the use of GeoGebra. Experiences served as indicator, while open-ended questions served as data source.

Table 5

Summary of Instrumentation

Research question	Indicator	Instrument
1. How do teachers utilise GeoGebra in the teaching and learning of transformation geometry?	GeoGebra practice in transformation geometry	Observation schedule Open-ended questions
2. How does the use of GeoGebra affect learners' understanding of transformation geometry?	Levels of achievement	Paper and pencil test
3. What are teachers' and learners' experiences about the use of GeoGebra?	Experiences and views about GeoGebra	Open-ended questions

3.5 Data Analysis

Data analysis is not a distinct stage which stands on its own after or during a singular time within the research period (Cohen et al., 2000); rather it is an on-going process. Muyeghu (2008) defined data analysis as a stage of describing data in significant terms. This stage of research requires researcher to be open to both positive and negative explanations as indicated by Muyeghu (2008). Data analysis in mixed methods investigation takes place within both the quantitative and qualitative data collection processes. The present investigation employed both quantitative and qualitative methods, therefore, both types of data were interpreted together (triangulation) once all the data collection was completed and processed.

Qualitative data analysis is a process of checking for patterns in the data, constructing and testing conjectures, asking questions and seeking more data (Mouton, 2001). For the qualitative aspect of the study pertaining to the third research question, audio-taped interviews were transcribed, coded and categorised as the responses to particular questions. This was with the view to identify structures, patterns and trends, divergence and possible explanations. General themes, subthemes and categories were identified. A template style of data analysis was used to apply themes to the data. Themes were tabulated and inferences made so as to address the third research question.

For the purpose of answering the first research question concerning teachers' utilisation of GeoGebra, the lessons for individual teachers were video-taped and later analysed. The instrument shed light on how teachers used GeoGebra and how they interacted with learners inside the classrooms. Learners' behaviour and responses were also captured.

For the quantitative aspect of the study, a descriptive data analysis approach was followed, according to which learners' paper and pencil scripts were marked by the researcher. The memorandum according to which the scripts were marked indicated the van Hieles' level of attainment (see Appendix L). The marks were categorised, tabled and graphed per level on the van Hieles' model so as to address the second research question. Each level was then calculated as a percentage to check how learners performed at each level of the van Hieles' model. This stage of the analysis was crucial as it led to the conclusions and answering of the research questions. The transcripts were revisited numerous times so as to create sagacity of them and then compared and contrasted the data in order to categorise it into themes. The findings are discussed in detail in the next chapter.

3.6 Methodological Norms

According to Gronlund (1998), there are two important questions which need to be asked when it comes to a study, firstly, to what extent will the results be appropriate and meaningful (validity). Secondly, 'to what extent will the results be free from errors?' which is reliability.

3.6.1 Reliability

Cresswell (2010) explained the reliability of an instrument as the degree to which the instrument measures accurately and consistently what it was intended to measure.

3.6.1.1 Reliability of the interviews.

To ensure reliability in the interview data collected, interview questions were approved by the study supervisors and two mathematics teachers (not participants) who were knowledgeable about the use of GeoGebra. Changes were made to the way questions were initially structured in order for them to elicit the necessary responses. Similar wording were used in order to be certain that all participants understood the questions in the same way without any bias (see Appendix E and H). The researcher read word for word to ensure that no participant was given an unfair advantage over other participants. No follow up questions were asked so as to ensure consistency in the questioning. The same audio-tape was used for all participants in order to ensure fairness in the quality of the instrument used. All interviews took place at the end of the lessons in the schools' libraries to avoid noise and disturbance of any kind. Participants were free in answering the questions as the interview was one on one without other people in the library who might make the participant (teachers or learners) uneasy or anxious. Transcripts were assessed by the study supervisors in order to ensure that the research questions were answered. For interviews, some questions were adapted from similar studies testing the use of GeoGebra and the van Hieles' theory.

3.6.1.2 Reliability of classroom observations.

In order to ensure reliability during lesson observations, the same video-tape was used with all teachers and learners. This measure ensured that no data was distorted due to the use of selective memory. The researcher remained uninvolved in all cases so as to avoid interfering with the study and avoiding any element of bias. The researcher focused on her role as an observer in all lessons observed.

3.6.2 Validity

Validity denotes the magnitude to which a computing instrument calculates what it is meant to calculate (Di Fabio & Maree, 2012; Muyeghu, 2008). Validity may also be defined as the appropriateness, meaningfulness, correctness and usefulness of any deductions that are obtained through the use of an instrument (Lu, 2008).

3.6.2.1 Validity of the paper and pencil test.

The test was approved by Grade 9 mathematics teachers in order to check whether it was appropriate for the learners' level. The study supervisors also made inputs before the test could be administered to the learners. A Grade 9 learners' CAPS textbook was used for questions to make certain that the questions were consistent and applicable to the level of the learners.

3.6.3 Triangulation

Triangulation is indispensable in establishing that the composed data is trustworthy and the process necessitates investigators to check the degree to which inferences founded on qualitative sources are reinforced by a quantitative viewpoint or vice-versa (Maree & van der Westhuizen, 2007). Muyeghu (2008) defines triangulation as a process of using different data collection methods to cross validate data and the findings. Several sources and methods were used for data collection in this study, namely, interviews, documents, lesson observations, paper and pencil test and field notes, thus ensuring triangulation.

3.7 Ethical Considerations

It is essential in any research, more especially when human participants are involved to adhere to strict ethical requirements (Maree, 2007). This means that all relevant legislation and guidelines have to be followed. Muyeghu (2008) explains ethics as the study of morals and values that have to be taken into consideration when conducting any study. In addition, Cohen et al. (2000) focused ethics on the basis that the data collected should be kept confidential and only be used for the purpose for which it is intended. This study did not involve any experimental methods, neither was it intrusive in the sense that no sensitive personal information was required. All steps were taken to ensure that ethical considerations were adhered to as is explained below.

3.7.1 Permission to Conduct the Research

Letters requesting permission to conduct research were sent to Gauteng Department of Education (GDE) and to the District Director. Further letters were sent to principals of the sampled schools in order to secure participants' consent. Letters were referred to teachers and parents of learners before the study commenced.

3.7.2 Ethical Clearance from the University of Pretoria

Prior to the commencement of the field work, the research proposal for this study was directed to the ethics committee of the University of Pretoria for authorisation. On completion of the study, the final ethical clearance was obtained from the university according to standard regulations.

3.7.3 Confidentiality

In order to maintain the confidentiality of participating schools, they were referred to as school A, B, C and D. Similarly, teachers were allocated pseudo names while learners were referred to as L1, L2 up to L24 to protect their identities. The results were presented anonymously and all audio- and video tapes are kept safely locked-up at the University of Pretoria according to the institution's regulations.

3.7.4 Safety in Participation

Within the letters of consent, participants were reassured that no harm would be inflicted as a result of involvement in the research project. This undertaking was honoured throughout the executing of the field work. Honesty, sensitivity, sympathy and respect were maintained throughout the study.

3.7.5 Voluntary Participation

In the consent letters, the purpose of the study was clearly explained as well as the voluntary nature of participation, and participants' freedom to withdraw at any time during the study was clarified. Flexibility during the study was of utmost importance and participants' rights were in no way undermined.

3.8 Conclusion

The chapter described the research design of the investigation that was conducted in four secondary schools in Gauteng Province. The research paradigm, methodological perspective, sample and participants, data collection and instruments, data analysis, triangulation, reliability and validity were presented in detail. The chapter closes with ethical considerations that were taken into account in the study. The next chapter presents the results of this study.

CHAPTER 4 – RESULTS

The prospects of all the cases investigated are presented in this section. The objective of the investigation was to investigate the use of GeoGebra in the teaching and learning of transformation geometry. Detailed results from teacher interviews, learner interviews, lesson observations, paper and pencil test and documents are presented. The results are presented in accordance with pre-determined themes as mentioned earlier in the dissertation, which are teachers' and learners' backgrounds, their views on the use of GeoGebra, usage of GeoGebra, as well as their evaluation of GeoGebra.

In pursuit of the primary objective of the research, namely, to inspect the usage and influence of GeoGebra within the teaching and learning of transformation geometry in relation to the van Hieles' levels of geometric understanding, the questions that guided the investigation are:

1. How do teachers utilise GeoGebra in the teaching of transformation geometry?

2. How does the use of GeoGebra affect learners' understanding of transformation geometry?

3. What are the teachers' and learners' experiences about the use of GeoGebra in the teaching and learning of transformation geometry?

The subsequent outcomes and data were obtained.

4.1 Results Pertaining to Sub-question One

This section begins by detailing the observations that were made during classroom visits to four classrooms of four different schools. The researcher visited each school during the appropriate time allocations for mathematics periods as pre-arranged with participating teachers. A video-tape was used to record the lessons which were later captured on the observation instrument in order to answer the research question.

4.1.1 Classroom Observations

Teachers gave learners examples that helped learners to understand the conceptbeing taught. Below are some of the examples given by the teachers during lesson observations that were done while the teaching and learning of transformation geometry took place.

4.1.1.1 John's lesson (reflection along the x-axis). John gave learners an example as a way of explaining the concept of reflection. He gave an example of reflection on the x-axis as shown in Figure 13. He indicated to the learners that the x-axis is also known as y = 0 since on the x-axis y values equal to zero.

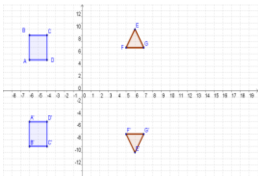


Figure 13. Example of reflection along the x-axis given by John.

John wanted the learners to look at the connection or the relationship between the coordinates. This is what he said, "Let us talk about object ABCD and A'B'C'D', what are the coordinates of each vertex?" The coordinates were: A (-6; 5); A' (-6; -5); B (-6; 9); B' (-6; -9); C (-4; 9); C' (-4; -9); D (-4; 5); D' (-4; -5).

He then said, Let us now compare the x coordinate of object ABCD with the x coordinate of its image A'B'C'D', also the y coordinate of ABCD with the y coordinate of its image A'B'C'D'".

He gave a conclusion for reflecting figures on the x-axis as follows: "In words: x coordinate remains the same while y coordinate changes the sign; that is if it was positive it changes to negative and if negative it becomes positive. Mathematically: (x; y) $\longrightarrow$(x; -y)".

Then he gave learners an exercise as a way of assessing whether they understood reflection. He said, "do the same for object EFG and its image E'F'G'. Write the coordinates of the object and its image; then the conclusion in words and mathematically".

John presented his lesson in an interesting way. He introduced the lesson by explaining the topic (reflection) to the learners. He gave learners examples of reflection in order to get them to visualise reflection along the x-axis. John used GeoGebra at van Hieles' level 1 (visualisation) for the greater part of the lesson. The learners remained active and participated by answering questions asked by John.

Apart from the lessons observed, John uses GeoGebra on a daily basis for the lessons which require learners to see diagrams clearly. The program used more especially for concepts such as transformation geometry which requires visualisation. He uses it for the teaching of graphs, in statistics and in transformation geometry. John said that he uses GeoGebra for demonstrating reflection. Figure 14 is an example of reflection in the x-axis that was given to learners during the classroom observation.

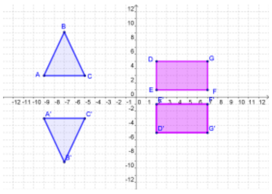


Figure 14. More examples of reflection in the x-axis given by John.

In order to represent the neatness and visualisation ability of GeoGebra, John presented learners with examples of reflection along the y-axis (see Figure 15).

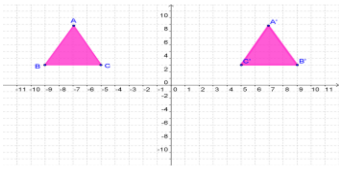


Figure 15. Example of reflection in the y-axis given by John.

In Table 6, a summary of John's lesson is provided.

Table 6

Summary of John's Lesson

Activity	Comment
What type of manipulations does the teacher do in class?	John gave different examples on transformation geometry.
At what level does the teacher use GeoGebra in the classroom?	John used GeoGebra at van Hieles' initial and second level.
Are the learners participating during the lesson?	The learners were very active and participated throughout the lesson.
Are the learners able to answer questions asked by the teacher?	The learners answered questions given orally and in the form of a class work
What type of questions does the teacher asks?	The teacher asked questions mostly visualisation questions.
Are the diagrams clear and legible when using GeoGebra?	The diagrams were very clear and neat.
Is the teacher able to use GeoGebra without consulting the manuals or seeking help?	The teacher knows how to use GeoGebra as he has been using it for years.
Does the teacher use GeoGebra throughout the lesson?	The teacher used GeoGebra throughout the lesson for example and exerxises.
Are the learners asking questions with regard to transformation geometry?	Learners constantly asked questions where they do not grasp the concept.
What type of questions do learners ask?	Learners asked questions at different levels between 1 and 3.
Is transformation geometry explained logically and clearly to learners?	Learners seemed to have grasped the concept taught.

4.1.1.2 Patrick's lesson (reflection along the y axis).

Patrick taught reflection on the y axis as an example of reflection. He used GeoGebra to show learners examples and told learners y-axis can also be identified as x = 0 since the x values on the y axis equals to zero. He said, "Let us reflect objects in the y-axis" and figures were reflected using GeoGebra (Figure 16).

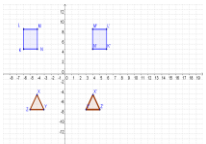


Figure 16. Example of reflection in the y-axis given by Patrick.

He then asked learners to look at the coordinates of figure KLMN and its image K'L'M'N'. Learners identified the coordinates as K (-6; 5); K' (6; 5); L (-6; 9); L' (6; 9); M (-4; 9); M' (4; 9); N (-4; 5); N' (4; 5). He then asked learners to "Let us now compare the x coordinate of object KLMN with the x coordinate of its image K'L'M'N'; also the y coordinate of KLMN with the y coordinate of its image K'L'M'N'".

Patrick concluded the lesson by saying, "Y coordinate remains the same while x coordinate changes the sign; that is if it was positive it changes to negative and if negative it becomes positive. Mathematically the conclusion can be written as: (x; y) $\longrightarrow$ (-x; y)". He gave learners an exercise in order to assess their understanding of reflection in the y axis. He said, "Do the same for object XYZ and its image X'Y'Z'. Write the coordinates of the object and its image; then the conclusion in words and mathematically".

Patrick demonstrated to the learners reflection in the line y = x using GeoGebra, as depicted in Figure 17.

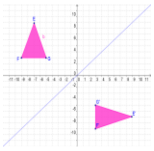


Figure 17. Example of reflection in the line y = x given by Patrick.

Patrick with his years of experience gave a well-structured lesson presentation which kept his learners active throughout the lesson. He used GeoGebra to provide examples and exercises on reflection along the y axis. GeoGebra was used to facilitate lessons at van Hieles' level 1 (visualisation), and to some extend at level 2 (analysis). The learners were able to answer questions posed by their teacher by show of hands (verbally) as well as through written classwork. Patrick asked questions at van Hieles' level 1 to 3. Patrick mastered the use of GeoGebra in order to present the concept of reflection.

In Table 7, a summary of Patrick's lesson is provided.

Table 7

Summary of Patrick's Lesson

Activity	Comment
What type of manipulations does the teacher do in class?	The teacher gave a variety of examples related to transformation geometry.
At what level does the teacher use GeoGebra in the classroom?	The teacher used GeoGebra at van Hieles' level1 (visualisation) and to some extent at level 2.
Are the learners participating during the lesson?	The learners were very active during the learning process. They answered questions asked by the teacher.
Are the learners able to answer questions asked by the teacher?	Learners were able to answer most questions in their classwork books.
What type of questions does the teacher asks?	Questions from van Hieles' level 1 to level 3. Learners were asked about whether the shape of the object changed after being transformation. Some questions included writing a rule for a particular transformation.
Are the diagrams clear and legible when using GeoGebra?	Diagrams were clear, legible and neat.
Is the teacher able to use GeoGebra without consulting the manuals or seeking help?	The teacher knew how GeoGebra works. The teacher used GeoGebra without seeking help.
Does the teacher use GeoGebra throughout the lesson?	The teacher used GeoGebra throughout the lesson.
Are the learners asking questions with regard to transformation geometry?	Learners asked clarity seeking questions related to the topic. For example, why the shape remained the same while the size changed with enlargement.
What type of questions do learners ask?	Questions that enable them to understand the topic better. For example, does transformation mean change of some sort.
Is transformation geometry explained logically and clearly to learners?	The teacher explained using different examples and shapes to do different transformations. However, few learners did not grasping the concept quickly.

4.1.1.3 Suzan's lesson (reflection along y = x and translation).

Suzan gave learners an example of reflection in the line y = x using GeoGebra (see Figure 18)

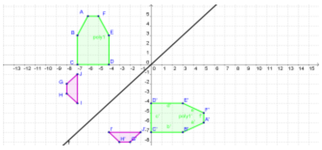


Figure 18. Example of reflection in the line y = x.

Suzan asked learners to look at the vertices of the hexagon ABCDEF and its image A'B'C'D'E'F' that were drawn. Learners gave the following coordinates: A (-6, 5), A' (5, -6), B (-7, 3), B' (3, -7), C (-7, 0), C' (0, -7), D (-4, 0), D' (0, -4), E (-4, 3), E' (3, -4), F (-5, 5), F' (5, -5). She then asked learners to compare the x coordinates of object ABCDEF with the x coordinates of its image A'B'C'D'E'F' ; as well as the y coordinates of object ABCDEF with the y coordinates of its image A'B'C'D'E'F.

Suzan helped the learners to come up with the conclusion both in words and mathematically as follows: x and y coordinate of the object 'swaps' in its image. Mathematically: (x; y)$\rightarrow$(y; x). She gave learners an exercise to test whether they have understood the concept of reflection in the line y = x. She said to learners, "Do the same for object GHIJ and its image G'H'I'J'. Write the coordinates of the object and its image, then the conclusion in words and mathematically".

*Translation***.** Suzan introduced translation to learners by making the following points:

- Translation is a rigid transformation which moves the object either vertically, horizontally or both.
- The shape and size does not change; only the position changes.

Suzan gave learners an example on translation (see Figure 19).

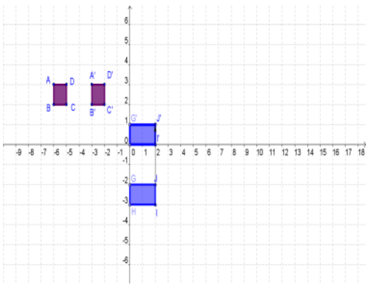


Figure 19. Example of translation given by Suzan.

She said, “Let’s talk about object ABCD and A’B’C’D’, what are the coordinates of each vertex?” Learners could give the coordinates of figure ABCD and its copy after it was translated. A (-6, 3), A’ (-3, 3), B (-6, 2), B’ (-3, 2), C (-5, 2), C’ (-2, 2), D (-5, 3), D’ (-2, 3). “Let’s now compare the X coordinate of object ABCD with the X coordinate of its copy A’B’C’D’ ; also the Y coordinate of object ABCD with the Y coordinate of its copy A’B’C’D’”. Suzan concluded by telling learners what happened to figure ABCD after translation. She said, “For translation of object ABCD to A’B’C’D’ we can see that y coordinate remains unchanged, but x coordinate changed. Let us scrutinize how x coordinate changed”.

In order to come up with a rule that was used for the translation, Suzan asked learners questions. She asked, "-6 became -3; what can we do to -6 so that it becomes -3? -5 became -2; what can we do to -5 so that it becomes -2?"

Suzan then said, "We can say -6 + 3 = -3. -5 + 3 = 2 as well, therefore generally 3 is added to x values. The figure is translated 3 units to the right (horizontally). Mathematically we can write it as follows: $(x; y) \longrightarrow (x + 3; y)$".

Teachers generally give their learners an exercise at the end of every activity in order to assess levels of understanding. Suzan gave learners this exercise by saying, "do the same for the figure GHIJ and its image G'H'I'J'. Write the coordinates of the object and its image; then the conclusion in words and mathematically".

In Table 8 below, a summary of Suzan's lesson is provided.

Table 8

Summary of Suzan's Lesson

Activity	Comment
What type of manipulations does the teacher do in class?	She did different examples and exercises.
At what level does the teacher use GeoGebra in the classroom?	She used it at level 1 (visualisation) most of the time.
Are the learners participating during the lesson?	Learners were actively involved in the learning process. They answered all questions.
Are the learners able to answer questions asked by the teacher?	Learners were able to answer most questions.
What type of questions does the teacher asks?	Questions that made learners to think critically. Learners were encouraged to perform at level 3.
Are the diagrams clear and legible when using GeoGebra?	Diagrams were very legible and neat.
Is the teacher able to use GeoGebra without consulting the manuals or seeking help?	The teacher used GeoGebra without consulting the manual.
Does the teacher use GeoGebra throughout the lesson?	GeoGebra was used throughout the lesson.
Are the learners asking questions with regard to transformation geometry?	Learners asked why the shape of a rectangle did not change when it was reflected, translated and rotated.
What type of questions do learners ask?	Learners asked questions which indicated that they are van Hieles' level 1. They could notice that the shape and size remained the same with reflection, rotation and translation.
Is transformation geometry explained logically and clearly to learners?	The topic was explained in detail with examples by the teacher. The teacher explained that reflection, rotation and translation have a similarity because the shape and size did not change; whereas with enlargement/reduction the size changed.

4.1.1.4 Lerado's lesson (enlargement, dilation and rotation).

Lerado introduced the lesson by explaining that "enlargement involves increasing the original object by a certain scale factor. Scale factor is the value of the multiplier used to make an enlargement".

Lerado gave learners an example of enlargement using GeoGebra. Example below was given (see Figure 20).

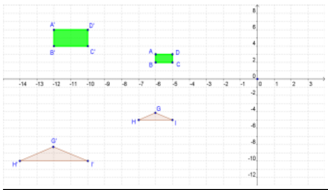


Figure 20. Example of enlargement given by Lerado.

Lerado then asked the learners to discuss the coordinates of figure ABCD and A'B'C'D'. Learners were able to give the coordinates of figure ABCD and its image after it was enlarged as follows: A (-6, 3), A' (-12, 6), B (-6, 2), B' (-12, 4), C (-5, 2), C' (-10, 4), D (-5, 3), D' (-10, 6). Lerado asked learners to compare the x coordinates of object ABCD with the x coordinate of its copy A'B'C'D' ; also the y coordinate of object ABCD with the y coordinate of its copy A'B'C'D'.

In trying to determine the enlargement factor, the following points were discussed:

- In point A: -6 changed to -12; 3 changed to 6. In point B: -6 changed to -12; 2 changed to 4. In point C: -5 changed to -10; 2 changed to 4. In point D: -5 changed to -10; 3 changed to 6.
- Each and every coordinate has been multiplied by 2.
- In words: An image has been enlarged by a scale factor of 2.
- Mathematically: (x; y) $\longrightarrow$ (2x; 2y)

Lerado gave learners exercise to assess their level of understanding the concept of enlargement. She said, "now do the same for object GHI and its image G'H'I'. Write the coordinates of the object and its image; then the conclusion in words and mathematically".

Dilation. Lerado started dilation sub-topic by saying, "Dilation involves reducing/decreasing the original object by a certain scale factor. Scale factor is the value of the multiplier used to make the dilation".

Lerado gave learners an example on dilation in order for then to get a clearer picture of how dilation differs with enlargement (see Figure 21).

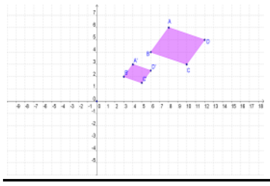


Figure 21. Example of reduction given by Lerado.

*Rotation***.** Lerado seemed to be a ‘fast’ teacher; as a result she was able to cover three aspects of transformation, namely, enlargement, dilation and rotation.

Lerado said, “Rotation is a rigid transformation that involves turning the object clockwise or anti-clockwise through a certain angle”, and then she gave the example in Figure 22.

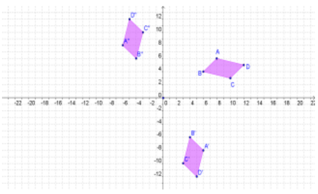


Figure 22. Example of rotation.

Lerado said, “Let us talk about object ABCD and A’B’C’D’, what are the coordinates of each vertex?” Learners then responded by giving the following answers: A (8, 6), A’ (6, -8), B (6, 4), B’ (4, -6), C (10, 3), C’ (3, -10), D (12, 5), D’ (5, -12). She then asked learners to compare the x coordinate of object ABCD with the x coordinate of its copy A’B’C’D’ ; also the y coordinate of object ABCD with the y coordinate of its copy A’B’C’D’. Lerado stated the following two points in order to conclude the concept of dilation:

- In words: x value changed the sign x and y values ‘swapped’.
- Mathematically: (x; y) $\longrightarrow$ (-x, y) $\longrightarrow$ (y, -x)

Learners were given an exercise from the textbook in order to test their understanding of rotation. She said to her learners, "Now do the same for object ABCD and its image A''B''C''D''. Write the coordinates of the object and its image; then the conclusion in words and mathematically". She made extra comments with regard to rotation of a figure(s):

- 180^{o} clockwise rotation is the same as 180^{o} anti-clockwise rotation about the origin.
- 270^{o} rotation clockwise is the same as 90^{o} anticlockwise rotation about the origin.

Lerado is a patient teacher who is passionate about mathematics and the use of GeoGebra. She was keen to show learners different aspects of transformation geometry using GeoGebra. Learners remained active and answered the questions she asked them. Learners seemed to want to do transformation geometry for the entire day because they understood. All her learners performed at van Hieles' level 1 as they indicated during interviews that GeoGebra made them to see figures as well as coordinates clearly. Lerado asked questions that led them to make conjectures with regard to the given sub-topic. Different questions were asked in order to test their ability and thinking level.

The questions asked were as follows:

(i). Enlarge object GHI which is on the board by a scale factor of 2. Write the coordinates of the image and also write a mathematical rule for this transformation.

(ii). Now do the same in terms of rotation for object ABCD that is on the board and its image A''B''C''D''. Write the coordinates of the object and its image; then the conclusion in words and mathematically".

In Table 9 below, a summary of Lerado's lesson is provided.

Table 9

Summary of Lerado's Lesson

Activity	Comment
What type of manipulations does the teacher do in class?	Manipulations which kept learners active throughout the lesson.
At what level does the teacher use GeoGebra in the classroom?	The teacher used GeoGebra such that learners are performing at van Hieles' level 1.
Are the learners participating during the lesson?	Learners were very active, learners were willing to try the answers before they were corrected by their teacher.
Are the learners able to answer questions asked by the teacher?	Learners answered most questions correctly.
What type of questions does the teacher asks?	Questions that lead them to general formulae for different transformations. Van Hieles' level 3.
Are the diagrams clear and legible when using GeoGebra?	Diagrams were clear and big enough for all learners to visualise.
Is the teacher able to use GeoGebra without consulting the manuals or seeking help?	The teacher used GeoGebra in an outstanding manner by using different shapes and objects.
Does the teacher use GeoGebra throughout the lesson?	GeoGebra was used throughout the lesson with limit use of the whiteboard to write the coordinates.
Are the learners asking questions with regard to transformation geometry?	Learners asked few questions as they are clear with the concept taught. For example, can transformation be done with any shape.
What type of questions do learners ask?	Questions related to the coordinates. For example, they asked whether reflection involves swapping of coordinates.
Is transformation geometry explained logically and clearly to learners?	The teacher clearly explained the concept with excellent examples.

4.1.2 Field Notes

Field notes were utilised in conjunction with a reflective diary during the research period as it would be difficult to remember all actions if not recorded at the time. According to Theron and Malindi (2012), it is important to write down the date, time, site, participants' names or participants' pseudonyms as well as the participants' non-verbal aspects of the interview when collecting data during research (Theron & Malindi, 2012).

Reflections regarding conversations, interviews, intuitions and other relevant reactions were noted. Participants were nodding during the questions as an indication that they understood the questions clearly. Participants also raised their eyebrows when the question asked was not well understood, enabling the researcher to repeat the question. The schools were visited on the 17th September 2014 in order to check that everything is in order before the actual data collection takes place. Interviews were conducted in schools A, B, C and D on the 18th, 19th, 22nd and the 23rd September 2014 respectively. The lengths of the interviews which was 20 minutes and anonymous information about the participants gender, participants' non-verbal actions such as nodding or raising eyebrows, and the factors that could have affected the research in a positive or negative such as a hot or rainy day were noted; so as to consider them during conclusion and recommendations.

4.2 Results Pertaining to Sub-question Two

Class work books and paper and pencil test were analysed to answer the second research question regarding how the use of GeoGebra affected learners' understanding of transformation geometry.

4.2.1 Documents

The use of learners' test shed light on how the use of GeoGebra affected learners' understanding of transformation geometry.

4.2.2 Paper and Pencil Test

Appendices O, P and Q are marked script of the test for learner 1 (L1). The findings from learners' test are presented in tabular form (see Table 10). The data is also presented graphically (see Figure 23) in order to depict the discrepancy in the learners' levels as described by van Hieles'. An indication of how learners answered the paper and pencil test is presented in Table 10. The findings are presented and discussed in order to answer research sub-question 2.

Key: A = achieved at that level, NA = not achieved at that level.

Table 10

A Summary of Learners' Achievement

Learner	*1st Level*	*2nd Level*	*3rd Level*	*4th Level*	*5th Level*
L1	A	A	A	A	A
L2	A	A	A	A	A
L3	A	A	A	A	A
L 4	A	A	A	A	A
L5	A	A	A	A	A
L6	A	A	A	NA	NA
L 7	A	A	A	NA	NA
L 8	A	A	A	NA	NA
L9	A	A	A	NA	NA
L10	A	A	A	NA	NA
L11	A	A	A	A	A
L12	A	A	A	A	NA
L 13	A	A	A	A	NA
L14	A	A	A	NA	NA
L15	A	A	A	A	NA
L16	A	A	A	NA	NA
L17	A	A	A	A	NA
L18	A	A	A	NA	NA
L19	A	A	A	A	NA
L20	A	A	A	A	NA
L21	A	A	A	A	NA
L22	A	A	A	A	NA
L23	A	A	A	A	NA
L24	A	A	A	A	NA

Based on the data in Table 10 above, the following observations can be made:

Some learners were able to answer questions at van Hieles' level 1 to 5. Three categories of learners emerged from the data. The first category was that of learners who progressed through all the van Hieles' levels (L1, L2, L3, L4, L5 and L11), implying that they understood transformation geometry as it was taught by their teachers in their different classrooms. Perhaps the use of GeoGebra had a positive effect in ensuring that learners grasped the concepts taught.

The second category was that of learners who correctly answered the questions at van Hieles' level 1 to 4 (L12, L13, L15, L17, L19, L20, L21, L22, L23 and L24); they were unable to get the last question which was testing the ability of learners to do formal deductions. They did however attempt to answer the last question on formal deduction to the best of their ability.

The last category is that of learners who answered questions at van Hieles' level 1 to 3 (L6, L7, L8, L9, L10, L14, L16 and L18). These learners were eager to answer all questions; however, their responses for questions at van Hielels' level 4 and 5 were totally incorrect. The indication is that more time should be spent with these learners in order to help them progress through all the levels.

The percentage of learner achievement per van Hieles' level is presented in Table 11. The percentage was calculated out of a total of 24 learners who participated in the study. 24 learners all achieved van Hieles' level 1 to 3 questions, which mean 100% achievement for level 1 to 3. However, only 6 learners managed to answer questions at van Hieles' level 5, which is 25% achievement at the upper level.

Table 11

Percentage of Learners' Achievement

Van Hieles' level	*Number wrote*	*Number achieving each level*	*% of learners achieving*	*Number of learners not achieving*	*% of learners not achieving*
Level 1	24	24	100.0	0.0	0.0
Level 2	24	24	100.0	0.0	0.0
Level 3	24	24	100.0	0.0	0.0
Level 4	24	16	66.7	8.0	33.3
Level 5	24	6	25.0	18.0	75.0

Table 11 is a summary of learner performance at each level of van Hieles' theory in percentage. 24 learners all wrote the paper and pencil test under strict supervision of the researcher. All 24 learners answered questions at van Hieles' level 1 to 3, obtaining a 100% achievement. 16 learners performed at van Hieles' level 4, which is 66.7% of achievement, while 8 learners who did not achieve at level 4 make 33.3 % of the participating learners. 6 learners performed at van Hieles' level 5 making 25.0 % of achievement. The indication in Table 11 is that learners' percentage of achievement decreases from the first to the last level of van Hieles' level. However, it should be commended that 100% of the participating learners can perform up to level 3.

24 learners wrote a one hour paper and pencil test on transformation geometry. The total mark of the test was 38. The mean mark of the test scores is 26, the mode of the scores was found to be 33. The range of the scores is 13. This is an indication that most learners performed satisfactorily in the paper and pencil that was administered.

The percentage of achievement per van Hieles' level is clearly notable when presented in the form of a bar graph in Figure 23. The bar graph shows a visual image of the discrepancy between level 3 to 5.

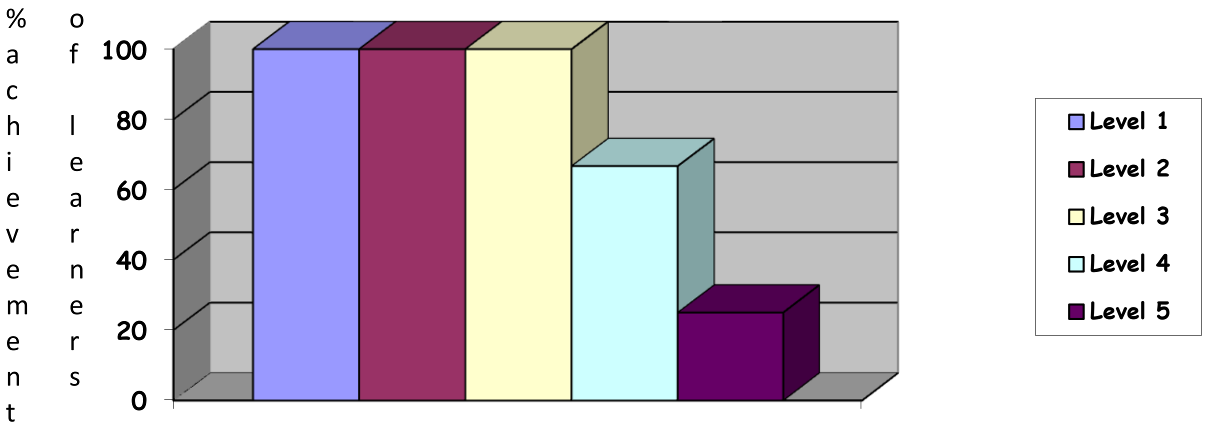


Figure 23. Graphic representation of learners' test results (in percentage).

The graphic representation of the percentage of achievement shows that all learners performed at van Hieles' level 1 to 3. However, the indication is that there is a decrease in performance of 33.3 % between van Hieles' level 3 and level 4 which was calculated by subtracting the percentage achievement of learners at level 3 from level 4. Similarly, there is also a decrease in performance of 75 % between van Hieles' level 4 and level 5.

4.3 Results Pertaining to Sub-question Three

Semi-structured interviews were conducted to answer the third research question about teachers' and learners' perceptions and experiences on the use of GeoGebra.

4.3.1 Teacher interviews

Following below are the transcripts of the interviews conducted with the participating teachers according to their pseudo names.

4.3.1.1 John

John has been a mathematics teacher in School A for the past nine years in secondary school. Due to his experience, he seemed to be knowledgeable in terms of Grade 9 mathematics teaching as evident from the way he explained to the learners; using different ways of finding the solution. The researcher asked John: "How many years have you been teaching mathematics in general and transformation geometry in particular?". Here is his response: "I have been teaching Grade 9 for nine years now. It has been five years now teaching transformation geometry". He attends short courses organised by the Department of Education in order to keep up to date with the changes in education.

John's opinions on the usage of GeoGebra in mathematics teaching. John has always liked the use of technology ever since he was a student. As an ICT coordinator in his school, he made sure that the school has computers, laptops and projectors to be used in the classrooms by talking to the school governing body. He was introduced to GeoGebra by his lecturer when he was doing an ACE course. He has eight years of experience in using GeoGebra in the teaching and learning of mathematics. The researcher asked John a question about learner performance when GeoGebra is used to facilitate learning, she asked: “How is the performance of the learners, now that GeoGebra is used during the teaching and learning of transformation geometry?”. John responded as follows: “Learners have been performing poorly, since they could not see some of the objects drawn on the plain chalkboard available at our school. Now there is an improvement in performance in mathematics, more especially transformation geometry”. According to John, learners’ performance in mathematics, especially transformation geometry has improved since the use of GeoGebra in the teaching and learning of our topic.

John’s GeoGebra usage. John uses GeoGebra for his lesson preparations and presentations. The researcher asked John the purposes for which he uses GeoGebra and this question was asked as follows: “How do you GeoGebra? For preparation of the lesson only or while teaching the lesson?”. He then said: “I use GeoGebra for my lesson preparation and lesson presentation. More especially for concepts such as transformation geometry which needs learners to see objects clearly. But most of my lesson presentations, I use it”. According to John, he uses GeoGebra not only during lesson presentation, but also at home while preparing for the lessons.

John’s GeoGebra evaluation. John perceives GeoGebra as mathematical software that is user friendly. He sees this software as a good visualisation tool. The researcher asked the following question to find out whether John likes GeoGebra or not: “Would you recommend GeoGebra to colleagues in other schools”. John’s response was: “It is user friendly, it makes objects very clear and neat, unlike on the chalkboard. I would definitely recommend it to all mathematics teachers and also learners.” John feels that mathematics teachers and learners must use it in their tutorial rooms for operative teaching and learning.

Besides its ability to produce clear and neat objects, John also likes the fact that GeoGebra is free software which can be used without internet connection, making it easier for needy learners who would not have money for data. The researcher asked John: “What do you like most when using GeoGebra”. John said: “I like the fact that GeoGebra is free software; therefore people can use it without worrying about the Internet”.

4.3.1.2 Patrick

Patrick has been teaching for the past twelve years. However, he has only four years’ experience in teaching mathematics in Grade 9 at School B. He is one of the few teachers in his school who use computer more often in teaching. He also facilitates computer afternoon lessons to empower interested learners and teachers. The researcher asked Patrick, “How many years of experience do you have teaching Grade 9?”. Patrick’s response was as follows: “I have been teaching mathematics, and this is my twelfth year teaching mathematics. I have been teaching transformation geometry for four years. This is my fourth year using GeoGebra in my teaching”.

Patrick’s opinions on the usage of GeoGebra in mathematics teaching. Patrick views GeoGebra as mathematical software that saves time and as a good visualisation tool. He views GeoGebra as a tool that enhances learner performance. The researcher asked Patrick: “What do you like most when using GeoGebra?”. His response was as follows: “It saves time; learners see what I have to teach them clearly. They are able to visualise what is needed out of them.”

Patrick’s GeoGebra usage. Patrick uses GeoGebra for both lesson preparation and also during the presentation of his lessons; he uses it in topics such as transformation geometry. The researcher asked Patrick: “How do you use GeoGebra? For preparation of the lesson only or while teaching the lesson?”. His response was: “I use it for both. When I prepare my lessons I use GeoGebra, when I am teaching, I also use GeoGebra”.

Patrick's GeoGebra evaluation. Patrick views GeoGebra as a good tool because it is free, therefore anyone can use it anytime without worrying about paying. The researcher asked Patrick this question: "How is the performance of your learners, now that GeoGebra is used during the teaching and learning of transformation geometry?". He responded by saying: "It was bad, but since I started using GeoGebra I see improvement. Like I said in the previous question, since I started using GeoGebra I see good results, performance of the learners in terms of analytical, in terms of transformation geometry is good".

4.3.1.3 Lerado

Lerado has been teaching Grade 9 for eleven years at School C with six of those years teaching mathematics. Lerado was introduced to GeoGebra two years ago by a colleague from another school. The researchers asked Lerado: "How many years have you been teaching mathematics in general and transformation geometry in particular?". Her response was: "I have been teaching for eleven years now. Ever since I started teaching, I have been teaching mathematics. But I have about six years of teaching transformation geometry".

Lerado's opinions on the usage of GeoGebra in mathematics teaching. Lerado views GeoGebra as a tool that improves learner performance. The researcher asked her: "How is the performance of your learners, now that GeoGebra is used during the teaching and learning of transformation geometry?". Lerado answered: "The performance was really bad. Most learners achieved level 1"(0-29%) "and only a few could manage a level 2 "(30-39%)". The learners are now achieving the minimum level 3 "(40-49%)" which is satisfactory. Only a few are still struggling at level 1. And there are brilliant learners who achieve even level 5 "(60-65%)".

Lerado indicated that she would recommend the use of GeoGebra to other teachers so that they can see its advantages. "Would you recommend the use of GeoGebra to colleagues in other schools?", asked the researcher. She said confidently: "I would definitely recommend GeoGebra to my colleagues and other teachers in other schools and provinces. I would also advise learners who have access to a computer to use it for their practice and for their homework".

Lerado's GeoGebra evaluation. Lerado sees GeoGebra as excellent mathematics software accessible to all, as it is free. The researcher asked Lerado: "What do you like most when using GeoGebra?". She answered: "Firstly, I like the fact that it is free software. Secondly, it makes drawing of graphs easier, neat and legible to most learners if not all. Lastly, I would say it also saves times as compared to the old way of chalk and board".

Lerado's GeoGebra usage. "How do you use GeoGebra? For preparation of the lesson only or while teaching the lesson?", asked the researcher. Lerado then said, "I use it when I prepare my lessons and during the presentation of mathematics lesson. I use it in other grades for the teaching of trigonometric graphs as they come out and clear and because you can use sliders to show learners the effect of variables in the general formulae". Lerado uses GeoGebra for lesson preparations as well as for presenting her lessons. She uses GeoGebra to teach transformation geometry as well as trigonometric graphs.
Lerado uses GeoGebra not only in transformation, but also in other grades for different topics: "I use it in other grades for the teaching of trigonometric graphs as they come out and clear and because you can use sliders to show learners the effect of variables in the general formulae."

4.3.1.4 Suzan

The researcher asked Suzan: "How many years have you been teaching mathematics in general and transformation geometry in particular?". Suzan responded: "I have been teaching for eight years. I have eight years of teaching mathematics in general, as well as well as in Grade 9". Suzan has been teaching mathematics for her entire eight year career at School D. She likes using technology in her everyday mathematics' teaching. She normally uses a laptop and a projector in her class. She uses software programs such as GeoGebra in teaching mathematics. She has been using GeoGebra for the past seven years.

Suzan's opinions on the usage of GeoGebra in mathematics teaching. Suzan views GeoGebra as a software that makes the drawing of figures as quick and neat as possible. Suzan feels figures drawn enhance learners' conceptual understanding. The researcher asked her: "What do you like most when using GeoGebra?”. The response from Suzan was: “GeoGebra makes the drawing of objects to be as quick and neat as possible. The objects also come out clear and visible”.

Suzan’s GeoGebra evaluation. Suzan views GeoGebra as a visualisation tool. She explains that learners are able to see objects clearly when drawn with GeoGebra, unlike when drawn with a chalk on the board without grid lines. The researcher asked the question about learner performance by saying: “How is the performance of your learners, now that GeoGebra is used during the teaching and learning of transformation geometry?”. The response from Suzan was as follows: “Well, it wasn’t satisfactory. Even though there were exceptional learners who will always perform well. But majority of the learners were underperforming. They have improved, most of them managed to obtain a minimum of level 3” (40-49%). The researcher asked a question on recommendation to other teachers and said, “Would you recommend the use of GeoGebra to colleagues in other schools?”. Suzan indicated that she would recommend the use of GeoGebra to other teachers as well: “I would recommend the use of GeoGebra to colleagues in other schools”.

Suzan’s GeoGebra usage. The researcher asked Suzan, “How do you use GeoGebra? For preparation of the lesson only or while teaching the lesson?”. The response from Suzan was, “Most of the time I use it for my lesson presentations. I like the use of GeoGebra in my mathematics classrooms”. Suzan uses GeoGebra for most of her lesson presentations, and a few times during lesson preparations. She also uses it for her own assignments for her postgraduate studies.

4.3.2 Learner interviews

Following below is an overview of learner responses during the face-to-face interviews following the classroom observations. Presented in Appendix N are the learners’ interview transcripts from Schools A, B, C and D.

4.3.2.1 Background

Twenty-three of the learners who participated in this study were doing Grade 9 for the first time; only one learner (L9) was in Grade 9 for the second time. Learners were between the ages of fourteen and seventeen. They generally seemed to be positive and encouraged by the use of GeoGebra in the teaching and learning of mathematics, in particular in the studying about transformation geometry.

4.3.2.2 Opinions on the use of GeoGebra

All the learners who participated in this study were excited about the use of GeoGebra in their mathematics classes. Learners are able to do their tasks in the school computer center after hours and during their free periods. GeoGebra keep them engaged in their school. It motivated them to do their schoolwork. A selection of direct quotations is presented below to reflect learners' views and their experiences about the use of GeoGebra in their learning.

The researcher asked learners questions individually and said, "How do you find the use of GeoGebra in mathematics in general and transformation geometry in particular?". The learners responded individually as indicated below.

Learner 1 (L1). She seemed to be a shy girl; however, she was able to show passion and was positive about the use of GeoGebra. She said, "It is so interesting because it is so simple".

Learner 2 (L2). He was outspoken and indicated that using GeoGebra in teaching transformation geometry made him understand better. He said, "I like it because it makes me understand".

Learner3 (L3). She said that GeoGebra helps her to understand mathematics better. She indicated that prior to the use of GeoGebra she could not understand mathematics. She said with confidence, “GeoGebra is very simple to use in order to understand mathematics. I like it because it is understandable.”

4.3.2.3 GeoGebra evaluation.

All Grade 9 mathematics learners were taught by their teachers at their different schools. All learners received letters of assent because a video-tape was used during lesson observations. However, only six learners per school were interviewed for the purposes of the study. Learners liked the point that GeoGebra is unrestricted; they can make use of it anytime as long as they have a computer or a laptop. Learners indicated that during the use of chalkboard, diagrams were not neat and legible and they could not see the points clearly. However, the use of GeoGebra enabled teachers to project or illustrates clear, legible and neat diagrams. The researcher asked learners another question and said, “Does the use of GeoGebra make you understand transformation geometry better?”.

The learners responded by saying,

L1: “ Yes mam”.

L2 : “ Yes, everything was clear”.

L3: “Yes”.

L4 : “Yes”.

4.3.2.4 GeoGebra usage.

As indicated above, all Grade 9 mathematics learners were taught by their teachers at their different schools. Nevertheless, only six learners per school were sampled from the population of Grade 9 mathematics learners. Learners had access to computers at their schools; therefore they use GeoGebra for exercises given to them by their teachers and also for practice. The sampled learners could draw different shapes and different graphs using GeoGebra, it was very easy for them to learn how to transform objects in the classroom and even for their home works by using GeoGebra.

All 24 learners indicated that they liked GeoGebra so much that they would recommend it to other mathematics learners. The researcher asked learners individually, “Will you recommend the use of GeoGebra to other learners?" This is the direct quotations of some of the learners: L1 said that "I like it", while learner 4 (L4) said, "I will tell them [other learners] to use GeoGebra".

Table 12 below presents a summary of the relationship between the learners and their teachers. The table will be discussed in Chapter 5.

Table 12

The Relationship between Participating Learners and Their Teachers

	John	Suzan	Patrick	Lerado
Teaching with GeoGebra	8 years	7 years	4 years	2 years
About GeoGebra	Very passionate	Passionate	Passionate	Passionate
GeoGebra usage	Daily basis for teaching	Daily basis for teaching	For teaching	For teaching
Learners	L1, L2, L3, L23, L21, L20	L4, L5, L11, L14, L13, L22	L6,L15,L17, L19, L7, L8	L18, L12, L16, L9, L10, L24
% of learners achieving Van Hieles' level 4	100.00	83.33	50.00	33.33

Table 12 shows the relationship between the van Hieles' level of learners and their teachers' experience in the use of GeoGebra. John has been using GeoGebra for eight years, with 100% of his learners achieving van Hieles' level 4. Suzan has been operating GeoGebra for seven years, with 83.33% of her learners achieving van Hieles' level 4. Patrick has been operating GeoGebra for four years, with 50 % of his learners obtaining van Hieles' level 4. On the other hand, Suzan has the lowest number of years of experience, with only 33.33% of her learners achieving van Hieles' level 4.

4.4 Summary

Teachers' lesson observations, paper and pencil test and teacher and learner interviews, gave an indication about the usage of GeoGebra in the teaching and learning of transformation geometry.

Lesson observations gave a clearer picture of how teachers interacted with their learners using GeoGebra. Teachers mastered the use of GeoGebra as they have been using it for quite a number of years. They needed no assistance while using it to facilitate their lessons. The classroom atmosphere seemed conducive for effective teaching and learning as learners concentrated on the screen. Learners remained active throughout the lessons by answering questions asked by their teachers.

The results of the paper and pencil test revealed that learners performed at different levels of van Hieles' theory. 100 % of the learners managed to perform up to level 3 while only 25 % performed up to the last level of van Hieles' theory. This shows that for learners to perform at a higher level, they first need to master the lower level as depicted in Table 10 above.

From the interviews, one can say with confidence that both teachers and learners were passionate about the use of GeoGebra in transformation geometry. Learners had an opportunity to do exercises during lessons in their different schools. Both teachers and learners liked the way GeoGebra showed figures and their coordinates. They also seemed to like how quickly GeoGebra can reflect, translate, rotate, enlarge or dilate figures. The use of GeoGebra saved teachers time; they could always save the presentations for use with other learners.

The next chapter presents a discussion of the findings, conclusions and recommendations that emanated from the data.

CHAPTER 5 – SUMMARY OF THE FINDINGS

5.1 Background and Aim of the Study

The intention of this research was to examine the role that GeoGebra played in the teaching and learning of transformation geometry in Grade 9 in a sample of schools. The literature review (Chapter 2) illustrated that most of the research done on GeoGebra was outside the South African context where schools are generally well resourced. This is different in South Africa where only a few schools have functional computer laboratories where learners can access computers for their use during the teaching and learning of mathematics. The main difference between the international literature and this South African study is the context in which the research was conducted. As mentioned previously, the need for the present study emerged, not only from the identified problem within the mathematics, but also from limited perspective on the specific topic in focus.

Qualitative and quantitative data were collected using multiple sources. In Chapter 4, the results were reported and a descriptive analysis of the data was presented in various forms. In this chapter, some inferences will be made based on the statistical information as well as on the anecdotal and observational information obtained during the research.

5.2 Interpretation of Findings According to van Hieles' Levels

In an interpretation of the various findings as reported in Chapter 4, the following connections could be made between the use of GeoGebra and its effect on learning of transformation geometry within the samples used in this study. As indicated in this dissertation, the study aimed at investigating the use of GeoGebra in transformation geometry in Grade 9.

5.2.1 Visualisation

The requirement that learners should be able to informally recognise a difference in the shapes of objects is the first level of geometrical understanding according to van Hieles' (1986). This means that learners may not yet able to formally differentiate between the objects based on their properties or sizes. Only the visual identification of the characteristics of the shape can be done. Geometric figures at this stage are identified by physical appearance, and not through partial characteristics.

During observations of teachers using GeoGebra, great emphasis was laid on visualisation. For example, teachers used the drag mode and sliders while the images were made visible on the screen. Learners reported that it was easy for them to see the change in the size, position, orientation and direction of shapes being transformed through using GeoGebra.

The results of the paper and pencil test revealed that 100% of the learners attained van Hieles' level 1. Although it is impossible to discern whether this attainment can be attributed to the use of GeoGebra, to the teaching style of the teachers, or to previous learning, it can at the very least be inferred that, due to the emphasis of visualisation in GeoGebra, it is highly possible that visualisation is reinforced by this software.

5.2.2 Analysis

Learners can identify and label characteristics of geometric figures at this level; however, they do not comprehend the association between these properties. They may not yet be able to discern the properties that are sufficient to describe the object.

It was revealed during lesson observations where teachers used GeoGebra that their learners reached the analysis level. Learners were able to give the coordinates of the object and that of its image after it was transformed. Learners could clearly write the corresponding coordinates next to one another.

Evidence of learners' ability to analyse was revealed by the results of the paper and pencil test where all learners attained level 2. Having mentioned above with visualisation, it cannot be confirmed for certain that it was the use of GeoGebra that enabled the learners to achieve a 100% at level 2, but perhaps it could be said that it had a positive effect on these results.

5.2.3 Abstraction

Abstraction implies that learners are able to distinguish relationships between properties within and between shapes. They may also tell which property is not found in another object, but the rationale behind the differences in the properties is not yet understood. Learners at this level are able to formulate generalisations and informal arguments about what they have learned previously to substantiate rules derived.

It was also observed in this research that learners were able to reach an abstraction level. These learners were able to tell the relationship amid the x coordinate of the figure and the x coordinate of the copy once transformed. The same was observed by the learners for the y coordinate of the object and the y coordinate of the image after the object was transformed.

The attainment of 100% from the test written by the learners served as evidence that learners understood what they were taught during the teaching of transformation geometry. Their participation confirmed that they liked transformation geometry, and the usage of GeoGebra in the teaching and learning of transformation geometry.

5.2.4 Deduction

The building of proofs and a well-defined understanding of the part of axioms and meanings are accomplished at this level of understanding. Learners advance successions of proclamations that logically justify conclusions.

Learners at van Hieles' level 4 are able to derive some general rules that they observe during teaching of transformation geometry. It was also observed that 66.7% of the learners who participated in this study could derive the general rule for a particular transformation. Learners were able to say in words that, for example, the x coordinate changed the sign after it was reflected on the line x=0 (y-axis), while the y coordinate remained unchanged.

5.2.5 Rigor

At the final level of understanding, learners fully comprehend formal aspects of deduction and can also use geometric proofs in appreciation of geometry as a whole. At this ultimate level, learners finally understand the characteristics of the postulate system, namely, non-contradiction, independence and completeness. It becomes possible now to use axioms without the need of proving them. Flexible geometric reasoning becomes possible.

Even though not all learners reached this level, 25% of the learners were able to fully grasp the formal aspect of deduction (rigor). These learners could write the general rule for a particular transformation, both in words as well as mathematically (in symbols). They could also draw the object and the image when given the mathematical rule that was used for transformation. However, it cannot be confirmed that it was the use of GeoGebra that enabled these learners to attain level 5, however the use of GeoGebra could have made it possible for them to attain this level of thinking.

5.2.6 Discussion

During the analysis and summary of findings, an interesting observation was made in terms of teachers and their learners, and the level of attainment that the groups (by teacher) reached.

There is a remarkable correspondence between teachers' years of experience and the learners' level of achievement. For John with the most experience, all the learners reached van Hieles' level 3, with some learners at van Hieles' level 4. Yet Lerado with the least number of years of experience, had learners who while also achieving van Hieles' level 3, could not achieve level 4. The indication is that the use of GeoGebra coupled with teachers' level of experience was an indicator for learners to achieve a high level on the van Hieles' model. Even though the findings in this study are based on a small sample size, the observations made here warrant a further research within a bigger cohort of participants, both teachers and learners.

Furthermore, it appeared from the findings as indicated in Table 12 that there is also a correspondence between teachers' enthusiasm and the learners' level of achievement. John's learners had achieved level 4 because he was very passionate about the use of GeoGebra, therefore it seemed his passion affected the way he used GeoGebra to teach transformation geometry. Additionally, it seems completely valid to argue that his passion was contagious to the learners.

5.3 Reflection on Achievement of the Objectives of this Study

At the onset of the study (Chapter 1), the objective for the study was spelled out as aiming to make a contribution with regard to the use of GeoGebra within a South African context in the teaching and learning of transformation geometry at Grade 9. To this end the main research question was formulated to guide the investigation, followed by the research outcomes which contributed to the existing body of knowledge in the field.

5.3.1 Response to the Main Research Question

Through the analysis of data (Chapter 4), the secondary research questions had been addressed. In this chapter, a conclusion will be drawn as to whether the main research question has been duly addressed. This question as mentioned before is: "How is GeoGebra currently utilised in the teaching and learning of transformation geometry in selected schools at Grade 9?"

The response to this question is now discussed in terms of the contribution that the study made towards the existing body of knowledge in the specific field of research.

5.3.2 Contribution of the Study

This study contributed to the current literature as it indicated that for the sampled participants in South Africa, the use of software was effective in the teaching and learning of mathematics and transformation geometry in particular. As discussed in the previous paragraph, however, it seems that teacher experience and enthusiasm are contributing factors to improved learner achievement.

5.3.2.1 School resources.

From this study, it was evident that the use of GeoGebra was effective even though the type of schools where the study took place were Quintile 1 (where no fee is required for teaching). This implies that because of the learners' low socio-economic backgrounds, learners are exempted from paying school fees. Learners in these groups of schools use scholar transport that is funded by the Department of Education. There are no computer laboratories in many schools and where schools have computer laboratories, they are not functional. Some teachers at these schools do not have laptops that they can use to facilitate teaching in their classrooms; however, those who have a passion for ICT in the classrooms use their personal computers to use during lessons.

The research schools had little access to the internet or to free Wi-Fi connectivity; therefore, it is not possible for the teachers to work online with programs that could enhance teaching and learning. However, GeoGebra being free software is ideal for these schools where teachers made use of the limited resources, such as personal laptops, i-pads, tablets, smartphones, etc.

Teachers mostly used a data projector (school's equipment) connected to a personal laptop to project the GeoGebra construction on the whiteboard. The topic of the day was written on the white board by the teachers while learners copied it in their class work books.

5.3.2.2 Teacher experience.

Teachers who took part in this study had a range of mathematics' teaching experience in terms of years. It seems that even the learners of a teacher with the least experienced teacher, for example Lerado, could still perform to at least van Hieles' level 3 as indicated in Table 10 where GeoGebra was used to facilitate teaching. In other words, even though the teacher had the least experience, GeoGebra served as a compensatory mechanism in this regard.

The findings revealed that teachers, irrespective of the level of experience, were knowledgeable about the content and the use of GeoGebra. Therefore, it was easy for the teachers to teach transformation geometry using GeoGebra. Teachers were able to answer all questions posed by the learners, which made it easier for them to proceed to the next question and to finish the planned lesson.

5.3.2.3 Learner access and opinions.

The study was conducted within an environment where it could be assumed that learners did not have access to computers or laptops because most of them are coming from poor backgrounds as discussed in the previous paragraph. Some of the learners come from child headed families where there is no access to resources that could enhance their learning.

Learners were keen to go to the board to solve given exercises and to share their methods with the rest of the class. Learners attempted all questions given to them by their teachers during their interaction.

Learners liked the fact that GeoGebra made it easier for them to see the coordinates clearly, to see the correct shapes of the figures and to see the correct position after transformation took place. Learners indicated that it was difficult for them to give the correct coordinates when diagrams were drawn for them on the chalkboard by their teachers. Learners used GeoGebra in their smartphones or tablets to do their homework.

It seems that even with limited resources in schools, inexperienced teachers, lack of access to technological support, GeoGebra could still be used to raise the learners' level of thinking. This narrow window of opportunity as described above can be utilised to the benefit of learners and teachers alike.

5.4 Discussion and Summary of Findings

In this section, a short synopsis is provided of the characteristics and advantages of GeoGebra itself, as well as its use and effect within low resourced school settings.

5.4.1 Synopsis of Characteristics and Advantages of GeoGebra

The availability of the algebra- and graphic window in GeoGebra made learners to see the relationship between the two windows as discussed in Chapter 2. A further advantage is the promotion of independent learning as learners were willing to work individually to work on exercises given to them. Learners were willing to discover relationship between objects given to them by their teachers.

It was evident in this study that GeoGebra was found to be an effective tool in the teaching and learning of transformation geometry. Both teachers and learners indicated that they would recommend GeoGebra to other teachers and learners in other schools as it made visualisation easy.

Learners were able to give the coordinates of the figures that were translated. Teachers constantly referred to diagrams that were projected on the screen when asking learners questions. Teachers made the figures to look appealing by changing the colour, line style, line thickness and the opaqueness of the figures. Learners used their class workbooks to write notes and complete exercises given to them by their teachers during the lessons. At the end of each sub-topic (e.g. reflection, translation, rotation, enlargement and dilation), learners were given exercises in order to assess their level of understanding of the sub-topic. Teachers gave all learners an opportunity to attempt all the given exercises in specified period of time. Teachers marked learners' exercises and gave them an opportunity to write their solution on the board while explaining to the rest of the class how they arrived at their answer. All learners had an opportunity to learn transformation geometry during their interaction.

5.4.2 The Practical use of GeoGebra within a Classroom Setting

As far as the teaching process is concerned, the use of GeoGebra positively affected the understanding of learners as they correctly completed the exercises given. Learners were able to answer questions posed by their teachers. Learners were actively involved throughout the lessons as they raised their hands to give their solutions. Learners were very active as they all wanted to write their answers on the board. Learners wrote all the given exercises on their own so that teachers could identify their mistakes in case they did not get the method correct.

As far as the effect of GeoGebra on learning transformation geometry, it is evident from the paper and pencil test that was written by the learners that GeoGebra had a positive effect on their understanding.

5.5 Reflection and Discussion of the Research Process

The study was based on the van Hieles' theory as a framework which served to analyse the hierarchy of reasoning on which the sampled Grade 9 learners operated, as outlined by the results of the paper and pencil test.

The methodological approach and the data collection instruments used in this study were simple to administer and were helpful in answering the research question. The results were obtained in a direct manner and provided unambiguous data.

Lesson observations were interesting as they revealed the manner in which teachers and learners interacted with one another during the teaching and learning of transformation geometry. The decision to use lesson observation was appropriate, since the researcher could understand how teachers and learners interacted during transformation geometry lessons.

The use of interviews helped with in-depth understanding of how teachers and learners perceived GeoGebra; as well as their experiences of GeoGebra. Both teachers and learners expressed how effective GeoGebra was during teaching and learning, as well as its positive effect in the learners' understanding of transformation geometry.

Learners' paper and pencil test was straight forward to administer and the results of the test helped to rank the participating learners according to the levels in van Hieles' model. The results from the test showed that 67% of learners performed only up to the third level of the van Hieles' theory.

5.6 Primary Conclusions

The findings in this study do not contradict the current literature. However, this study encourages teachers, learners, Heads of Departments or subject heads, curriculum implementers in South Africa and countries within the continent to facilitate learning using software; particularly GeoGebra even in the absence of advanced resources.

5.6.1 It is Beneficial for Teachers to Use GeoGebra

It can be suggested that mathematics teachers acquire skills about how to utilise GeoGebra for effective teaching and learning of transformation geometry, as well as for mathematics lessons in general.

5.6.2 It Would Enhance the Effect if Teachers Received Training

If teachers are able to use GeoGebra effectively, then their learners will also be able to use the tool at their own pace for learning and homework given to them by their teachers.

5.6.3 It is Imperative that Teachers Become Acquainted with van Hieles' Levels

It is paramount that teachers consider all the five levels when planning their mathematics lessons, transformation geometry lessons in particular. If a learner is at level 2 of van Hieles' scale, it is not viable to expect him/her to perform at a higher level. Therefore it implies that mathematics teachers who do not use GeoGebra need to be trained in the form of a workshop about its utilisation as well as on van Hieles' theory. It would be beneficial to learners if teachers considered the van Hieles' theory when planning their geometry lessons even without assistance from GeoGebra.

5.6.4 It is Essential that Education Authorities Endorse the Use of GeoGebra

Endorsement of GeoGebra by the Department of Basic Education and school governing bodies might ensure that classrooms have computers; white boards for projection and functional computer centres for successful teaching and learning of mathematics.

5.6.5 It is Advocated that Differentiated Learning be Applied

Learner grouping should be considered at the beginning of each academic year as it will enable teachers to keep rack with the official work schedule; as learners would be in the same level of van Hieles' level.

5.7 Limitations and Recommendations for Further Research

This study has some limitations. These are discussed below as are recommendations for further research.

5.7.1 Sample Size

This investigation has been conducted with a small sample; wherein only four teachers were purposively selected in a district not conversant with the use of GeoGebra. However, it should be noted that the intension of the study was not to generalise, but rather to understand the usage of GeoGebra in the teaching and learning of transformation geometry. This study therefore presents an opportunity for future study at a larger scale, with larger groups of teachers and learners.

5.7.2 The Area of Research

The study was conducted in one area within one district of Tshwane West. It is recommended that future research be conducted within a bigger area in different districts or as well as other provinces in South Africa.

5.7.3 Type of school

The involvement of only Quintile 1 (no fee) schools could also be a limiting factor as learners from other quintiles might give new information to this type of research. The recommendation is that further research be conducted with learners from different types of schools so as to generalise to all learners.

5.7.4 The Grade Investigated

One grade (Grade 9) was used in this study, therefore it is impossible to generalise that the use of GeoGebra is effective in teaching and learning of transformation geometry in other grades. Thus it is recommended that additional grades be involved in future studies so as to generalise for all the grades.

5.7.5 Topic of Instruction

The topic in this study was limited to transformation geometry, whereas GeoGebra can be explored for other topics.

5.7.6 Time Constraints

The study was conducted in five weeks; it is recommended that a comparative longitudinal study be conducted over a longer period of time so as to observe the long term effects of teaching with GeoGebra.

5.7.7 Limitation of a Single Researcher

The study was conducted by a single researcher; hence the study was done within a small scale with few participants. Involving a team of researchers could yield additional information that would add value to the existing body of knowledge.

5.8 Final Word

This study has been an eventful journey in investigating, on a small scale, the potential of the use of GeoGebra in the teaching and learning of transformation geometry to Grade 9 learners. It would be a positive step to encourage all mathematics teachers in different phases to use GeoGebra for their lesson presentations as its effect has been found to yield positive learner results. It would also be advantageous if mathematics learners had access to GeoGebra so that they could use it to practice mathematics and to help them with their homework. The use of GeoGebra may break the negative attitude that some learners are having towards mathematics as many learners perceive mathematics as a difficult subject. Researchers have already alluded to the fact that transformation geometry as a component of geometry is worth teaching as it helps learners to perceive it not only as a subject to be taught in the classroom, but as a subject that will help learners to integrate every day encounters with what they have learned in the classroom.

This research used van Hieles' theory by way of a theoretical outline which encouraged learning to progress through different cognitive phases of learning, and was found to have helped learners to advance from a lesser level to an upper level of comprehension. The implication is that if learners moved from a lesser level of geometric understanding to an upper level, they would thoroughly understand the topic taught and as a result they would perform better in the subject.

It is hoped that teachers, curriculum developers, the Department of Basic Education, non-governmental organisations, and other stakeholders in education take into consideration the effectiveness of van Hieles' theory of geometric understanding in order to help learners to learn better.

REFERENCES

Abdullah, A. H., & Zakaria, E. (2011). Students' Perceptions Towards the Van Hieles' Phases of Learning Geometry Using Geometers' Sketchpad Software. *Australian Journal of Basic and Applied Sciences, 5(7)*, 787-792.

Abdullah, A. H., & Zakaria, E. (2013). Enhancing Students' Level of Geometric Thinking Through Van Hieles' Phase-Based Learning. *Indian Journal of Science and Technology, 6(5)*, 4432-4446.

Abu, M. S. (2013). Improving the Levels of Geometric Thinking of Secondary School Students Using Geometry Learning Video Based on Van Hieles' Theory . *International Journal of Education and Research in Education, 2(1)*, 16-22.

Ada, T., & Kurtulus, A. (2010). Students' Misconceptions and Errors in Transformation Geometry. *International Journal of Science and Technology, 41(7),*, 901-909.

Akay, G. (2011). The effect of Peer Instruction Method on 8th Grade Students' Mathematics Achievement in Transformation Geometry and Attitudes Towards Mathematics. *Middle East University*.

Alex, J. K., & Mammen, K. J. (2012). A survey of South AFrican Grade 10 Learners' Geometric Thinking Levels in Terms of Van Hieles' Theory. *Anthropologist, 14(2)*, 123-129.

Al-Sheri, M. A., Al-Zoubi, S., & Rahman, M. B. (2011). The Effectiveness of Gifted Students Centers in Developing Geometric Thinking. *International Research Journals*, 1676-1684.

Andraphanova, N. V. (2015). Geometrical Similarity Transformations in Dynamic Geometry Environment GeoGebra,. *European Journal of Contemporary Educattion, 12(2),*, 116-128.

Arzarello, F. (2005). Technology and Mathematics in the Classroom: Lights and Shadows. *CIEAEM 57 Oral Presentations in Working Groups*, 23-27.

Atebe, H. U. (2010). Research Evidence on Geometric Thinking Levels Hierarchies and Their Relationships with Students' Mathematical Performance. *Journal of the Science Teachers Association of Nigeria, 40*, 75-84.

Aydin, N. &. (2010). The Impacts of Undergraduate Mathematics Courses on College Studens' Geometric Reasoning Stages . *Faculty Publications, 6(1-2)*, 150-163.

Bakar, K. A. (2002). Exploring Secondary School Students' Motivation Using Technologies inTeaching and Learning Mathematics. *Procedia Social and Behavioral Sciences, 14*, 1-8.

Bansilal, S. (2015). Exploring Student Teachers' Perceptions of the Influence of Technology in Learning Mathematics . *South African Journal of Education, 35(4)*, 1-8.

Barriball, K. L. (1994). Collecting Data Using a Semi-Structured Interview: A discussion Paper. *Journal of Advanced Nursing, 19*, 328-335.

Battista, M. T. (2011). Conceptualisations and Issues Related to Learning Progressions, Learning Trajectories, and Levels of Sophistication. *The Mathematics Enthusiast, 8(3)*, 507-569.

Benni, K. J. (1999). MALATI and the Open Society Fiundation for South Africa.

Berger, M. (2013). Examining Mathematical Discourse to Understand In-Service Teachers' Mathematical Activities. *Pythagoras, 34(1)*, 1-10.

Berstein, A. M. (2013). Maths Teaching in SA Adds to Multiplying Class Divisions. *Mail & Guardian*.

Bleeker, C. (2011). The relationship between teachers' Instructional Practices and Learners' Levels of Geometry Thinking. *University of Pretoria, South Africa*.

Boyraz, S. (2008). A Thesis Submitted to Graduate School of Social Sciences of Middle East Technincal University. *Middle East Technical University.*

Brodie, K. (2011). An Investigation into Grade 6 Teachers' Understanding of Geometry According to the Van Hieles' Leve of Geometric Thought. *University of Witwatersrand, South Africa*.

Bruce, D. M. (2011). Significant Teaching and Learning Moments Using Interactive Whiteboards in Mathematics. In D, Martinovic., Z, Karadag., & D, McDougall (Eds). In *Second North American GeoGebra Conference: Where Mathematics, Education and Technology Meet.*

Bu, L. M. (2011). GeoGebra As Pedagogical Tool: A Preliminary Taxonomy.

Bu, L. S. (2011). Toward Model-Centered Mathematics Learning and Instruction Using GeoGebra. . In *L, Bu & R, Schoen (Eds). Model-Centered Learning: Pathways to Mathematical Understanding Using GeoGebra* (pp. 7-12). Sense Publishers.

Bu, L., & SChoen, R. (2011). Model-Centerd Learning: Pathways to Mathematical Understanding Using GeoGebra. *Sense Publishers*, 7 - 12.

Bulut, M. &. (2011). Pre-Service Teachers' Usage of Dynamic Mathematics Software. *The Turkish Online Journal of Educational Technology, 10(4)*, 294-299.

Choi-Khoi, S. S. (2000). The Activities Based on Van Hieles' Model Using Computer As a Tool. *Journal of the Korea Society of Mathematical Education, 4(2)*, 63-77.

Clements, D. H. (1992). Geometry and Spatial Reasoning. In *D. Grows (Ed.). Handbook of Research on Mathematics Teaching and Learning.* New York, Macmillan.

Cohen, L. M. (2000). *Research Methods in Education.* London: Routledger Falmer.

Connoly, S. (2010). The Impact of Van Hieles'-Based Geometry on Student Understanding. *St John Fisher College, 4(1)*, 1-88.

Creswell, J. W. (2010). *Educatioanal Research: Planning, Conducting and Evaluating Quantitative and Qualitative Research.* New Jersey: Pearson Prentice Hall.

Crowley, M. L. (1987). The Van Hieles' Modell of the Development of Geometry Thought. In *M. Montgomery (Ed.). Learning and Teaching Geometry, K-12.* The National Council of Teachers of Mathematics.

CSSForums. (2007). *Theory and Paradigms and Different Types of Paradigm.* Retrieved from http://www. cssforums.com.pk/css.

Cullen, C. J. (2013). Investigating Extrema with GeoGebra . *The National Council of Teachers of Mathematics, Inc, 107(1)*, 68-73.

Cutis, M. A. (2000). Mathematics Education Research, A guide for the Research Mathematician. *American Mathematical Society*.

De Wet, P. (2011). Learning and Teaching Mathematics. *Hilton College, 9*, 40-41.

Denscombe, M. (2008). A Research Paradigm for the Mixed Methods Approach. *De Montfort University, Leicester: United Kingdom*.

Department of Basic Education. (2011). Curriculum and Assessment Policy Statement Grade 7 - 9 : Mathematics. Pretoria: Government Printer.

Di Fabio, A., & Maree, J. G. (2012). Ensuring Quality in Scholarly Wrinting. In *J. G, Maree(Ed.). Complete your Thesis or Dissertation Successfully: Practical Guidelines.* Juta and Company.

Dikovic, L. (2009). Applications GeoGebra into Teaching Some Topics of Mathematics at the College Level. *6(2), Retrieved from http://www.Doiserbia.nb.rs*, 191-203.

Dindyal, A. (2007). The Need for an Inclusive Framework for Students' Thining in School Geometry. *The Montana Mathematics Enthusiast, 4(1)*, 73-83.

Dogan, M. (2011). The role of Dynamic Geometry Software in the Process of Learning: GeoGebra Example about Triangles. *The Scientific Research Fund of Selcuk University*.

Duatepe, A., & Ubuz, B. (2009). Effects of Drama-Based Geometry Instruction on Student Achievement and Thinking Levels. *Journal of Educational Research*, 122-137.

Du Plooy, M. C. (2016).

Edwards, L. D. (199). Children's Learning in a Computer Microworld for Transformation Geometry . *Journal for Research in Mathematics Education, 22(2)*, 122-137.

Escuder, A., & Furner, J. M. (2012). The Impact of GeoGebra in Math Teachers' Professional Development . *Florida Atlantic University*.

Evbuomwam, D. (2013). An Investigation into the Difficulties Faced by Form C Students in the Learning of Transformation Geometry in Lesotho Secondary Schools.

Fahlberg-Stojanovska, L., & Stojanovska, V. (2009). GeoGebra-Freedom to Explore and Learn.

Feilzer, M. Y. (2010). Doing Mixed Methods Research Pragmatically: Implications for the Rediscovery of Pragmatism . *SAGE Publications, 4(1)*, 6-16.

Feza, N., & Webb, P. (2005). Assessment Standards , Van Hieles' Levels, and Grade Seven Learners' Understanding of Geometry. *University of Port Elizabeth*.

Freinman, V., Martinovic, D., & Karadag, Z. (2010). A Snapshot of GeoGebra Community Endeavours: Building a Research Agenda. *Ithaca College: USA*.

Furner, J. M., & Marinas, C. A. (2007). Geoboards to GeoGebra: Moving from the Concrete to the Abstract Geometry. *Florida & Barry University*.

Fuys, D., Geddes, D., Lovett, C. J., & Tischler, R. (1988). The Van Hieles' Model of Thinking in Geometry among Adolescents . *Journal for Research in Mathematics Education*.

Greene, J. C. (1989). Toward a Conceptual Framework for Mixed-Methods Evaluation Designs. *Educational and Policy Analysis, 11(3)*, 255-274.

Gronluund, N. E. (1998). Assessment of Student Achievement . *Massachusetts: Allyn & Bacon*.

Guay, R. (1980). Spatial Ability Measurement: A Critique and an Alternative. *Boston April*.

Gulsecen, S. (2012). Can GeoGebra Make Easier the Understanding of Cartesian Co-ordinates? A Quantitative Study in Turkey. *International Journal on New Trends in Educationand Their Implication, 3(4)*, 309-624.

Gunhan, B. C. (2014). A Case Study on the Investigation of Reasoning Skills in Geometry. *South African Journal of Education, 34(2)*, 1-19.

Gutierrez, A., Jaime, A., & Fortuny, J. (1991). Alternative Paradigm to Evaluate the Acquisition of the Van Hieles' Levels. *Journal of Research in Mathematics Education, 21(3)*, 31-47.

Haak, S. (1976). Transformation Geometry and the Artwork of M. C. Escher. *Mathematics Teacher*.

Halat, E. (2006). Reform-Based Curriculum & Acquisition of the Levels . *Eurasia Journal of Mathematics, Science & Technology, 3(1)*, 41-49.

Halat, E. (2008). In-Service Middle and HIgh School Mathematics Teacher. *The Mathematics Educator, 18(1)*, 8-14.

Henning, R. D., Rensberg, v., & WSmit, B. (2004). Finding Your Way in Qualitative Research. *van Schaik, Pretoria*.

Hohenwarter, M., & Lavicza, Z. (2007). Mathematics Teacher Development with ICT: Towards an International GeoGebra Institute. In *D, Kuchemann (Ed.). Proceedings of the British Society for Research into Learning Mathematics, 27(3).*

Hohenwarter, M., Hohenwarter, J., Kreis, Y., & Lavicza, Z. (2008). Teaching and Learning Calculus with Free Dynamic Mathematics Software GeoGebra. *11th International Congress on Mathematical Education*.

Holzl, R., & Schafer, M. (2013). Dynamic geometry Software as a Dynamic Tool for Spatial Exploration. *Rhodes University, Grahamstown*.

Hoong, L. C., & Khoh, L. S. (2003). Effects of Geometers' Sketchpad on Spatial and Achievement in Transformation Geometry among Ssecondary Two Students in Singapore. *Association of Mathematics Teacher, 7(1)*, 32-48.

Huang, T., Liu, Y., & Kuo, R. (2013). The Impact of Web Project-Based Learning on Elementary School Students' Development of Van Hieles' Geometric Thought in Taiwan . *International Journal of Evaluation and Research in Education, 9(20*, 50-59.

Idris, N. (2007). The Effects of Geometers' Sketchpad on the Performance in Geometry of Malaysian Students' Achievement and Van Hieles' Geometric THinking. *Malaysian Journal of Mathematical Sciences, 1(2)*, 169-180.

Idris, N. (2009). The Impact of Using Geometers' Sketchpad on Malaysian Students' Achievement and Van Hieles’ Geometric Thinking. *Journal of Mathematics Education, 2(2)*, 94-107.

Ivankova, N. V., Creswell, J. W., & Plano Clark, V. L. (2007). Foundations and Approaches to Mixed Methods Research. In *K, Maree (Ed.). First Steps in Research.* van Schaik, Pretoria.

Jansen, J. D. (2007). The Language of Research. In *K, Maree (Ed.). First Steps in Research.* van Schaik, Pretoria.

Jarvis, D. (2001). GeoGebra and the Global Village-Theatre: Realising the 21st Century Potential of Dynamic Mathematics For/By All.

Jones, K. (2002). Issues in the Teaching and Learning of Geometry. In *L. Haggarty (Ed.). Aspects of Teaching Secondary Mathematics: Perspectives on Practice.* (pp. 121-139). London, RoutledgerFalmer.

Kin-Wai, L. (2005). The Effectiveness of Van Hieles’-Based Instruction. *The University of Hong Kong*.

Kutluca, T. (2013). The Effect of Geometry Instruction with Dynamic Geometry Software GeoGebra on Van Hieles’ Geometry Understanding Levels of Students. *Dicle University, College of Education, 8(17)*, 1509-1518.

Lee, J., & Kim, M. (2012). Development of Web-Based Courseware for Van Hieles’ Visualisation Level. *International Journal of Modern Engineering Research, 2(6)*, 4086-4090.

Lesh, R. (1976). Transformation Geometry in Elementary School: Some Research Issues. In *J. L, Martin., & D. A, Bradbard (Eds.). Space and Geometry.*

Ljajko, E. (2013). Development of Ideas in a GeoGebra-Aided Mathematics Instruction. *University of Pristina, Serbia*.

Lu, Y. A. (2008). English and Taiwaneses Upper Secondary Teachers' Approaches to the Use of GeoGebra. *University of Cambridge, 10(2)*, 38-55.

Lupu, C. (2015). The Role of Technology in the Process of Learning Mathematics . *American Journal of Operations Research, 5*, 89-94.

Mainali, B. R., & Key, M. B. (2008). Using Dynamic Software GeoGebra in Developing Countries: A Case Study of Impressions of Mathematics Teachers in Nepal.

Malloy, C. (2002). The Van Hieles' Framework. *The National Council of Teachers*.

Maree, K. (2007). First Steps in Research. van Schaik, Pretoria.

Maree, K., & van der Westhuizen, C. (2007). Planning a Research Proposal. In *K, Maree. First Steps in Research.* van Schaik, Pretoria.

Martinovic, D., Karadag, Z., & Freiman, V. (2012). First Decade of GeoGebra: Looking Back Through Socio-Cognitive Lenses. *Ontario Institute for Studies in Education*.

Martinovic, D., Karadag, Z., & McDougall, D. (2011). GeoGebra-NA 2011 Proceedings of the Second North American GeoGebra Conference: Where Mathematics, Education and Technology Meet. *University of Windscor, Canada*.

Mason, M. (2002). The Van Hieles' Levels of Geometric Understanding. *University of Vrginia, Charlottesville*, 4-8.

McMillan, J. H. (2001). Research in Education: A Conceptual Introduction. *London, Evans.*

Mehanovic, S. (2011). The Potential and Challenges of the Use of Dynamic Software in Upper Secondary Mathematics. *Linkoping Studies in Science and Technology*.

Meng, C. C., & Idris, N. (2009). Enhancing Students' Geometric Thinking Through Phase-Based Instruction Using Geometers' Sketchpad: A Case Study. *School of Education Studies*.

Merriam, S. B. (1998). *Qualitative Research and Case Study Applications in Esducation.* San Francisco: Jossey-Bass Publishers.

Morgan, B., & Sklar, R. H. (2012). Sampling and Research Paradigms. In *J. G, Maree(Ed.). Complete your Thesis or Dissertation Successfully: Practical Guidelines.* Juta and Company

Mouton, J. (2001). *How to Succeed in Your Master's and Doctoral Studies.* Pretoria: van Schaik.

Muyeghu, A. (2008). The Use of Van Hieles' Theory in Investigating the Teaching Strategies Used by Grade 10 Geometry Teacchers in Namibia. *Rhodes University*.

Myers, R. Y. (2009). *The Effects of the Use of Technology in Mathematics Instruction on Student Achievement* . Florida International University.

Najdi, R. (n.d.). *The Effects of Research-Based Strategies on Raising the Van Hieles' Levels of Geometric Thinking.* Alquds Open University.

National Council of Teachers Mathematics NCTM (2000). *Principle and Standards for School Mathematics.*Retrieved from http://www.nctm.org

Niewenhuis, J. (2007). Introducing Qualitative Research . In *K, Maree (Ed.). First Steps in Research.* Pretoria: van Schaik.

Niewenhuis, J. (2007). Qualitative Research Design and Data Gathering Techniques. In *K, Maree (Ed.). First Steps in Research.* Pretoria: van Schaik.

Ogwel, A. (2009). Using GeoGebra in Secondary School Mathematics Teaching. *Center for Mathematics , Science and Technology Education in Africa*.

Olken, F. (1993). *Random Sampling from Databases.* Berkeley: University of California.

Olkun, S., Sinoplu, B., & Deryakulu, D. (2009). Geometric Explorations with Dynamic Geometry Applications Based on Van Hieles' Levels. *International Journal for Mathemtics Teaching and Learning*.

Onwuegbuzie, A. J., & Collins, K. M. (2007). A Topology of Mixed Methods Sampling Designs in Social Science Research. The Qualitative Report. *Springer Science + Business Media, 12(2)*, 281-316.

Pandiscio, E. A., & Knight, K. C. (2011). An Investigation into Van Hieles' Levels of Understanding Geometry of Preservice Mathematics Teachers. *Journal of Research in Education, 21(1)*, 45-53.

Pandiscio, E., & Orton, R. E. (1998). Geometry and Metacognition: An Analysis of Piaget's and Van Hieles' Perspectives . *Focus on Learning Problems in Mathematics, 20(2-3)*, 78-87.

Pegg, J., & Davey, G. (1998). Interpreting Student Understanding of Geometry: A Synthesis of Two Models. In *K, Lehre., & D, Chazan (Eds.). Designing Learning Environments for Developing Understanding of Geometry and Space.* Mahwah, NJ: Lawrence Erlbaum Associates.

Petter, S. C., & Gallivan, M. J. (2004). *Toward a Framework for Classifying and Guiding Mixed Research in Information Systems.* Georgia State University.

Pleet, L. J. (1990). *The Effects of Computer Graphics and Mira on Acquisition of Transformation Geometry Concepts and Development of Mental Rotation Skills in Grade Eight.* Los Angeles: Oregon State University.

Presmeg, N. (2006). Research on Visualisation in Learning and Teaching Mathematics. In *A. Gutierrez., & P. Boero (Eds). Handbook of Research on the Psychology of Mathematics Education.* Rotterdam: Sense Publishers.

Prodromou, T. (2012). *GeoGebra in Teaching and Learning Introductory Statistics.* Australia: University of New England.

Pusey, E. L. (2003). *The Van Hieles' Model of Reasoning in Geometry: A Literature Review.* North Carolina State University.

Robinson, E. (1976). Mathematical Foundations of the Development of Spatial and Geometrical Concepts. In *J. L. Martin., & D. A. Bradbard (Eds.). Space and Geometry. Pages From a Research Workshop.*

Rynhart, P. (2012). The Van Hieles' Model of Geometric Thinking. Proactive Play.

Saha, R. A., Ayub, A. F., & Tarmizi, R. A. (2010). The Effects of GeoGebra on Mathematics Achievement: Enlightning Coordinate Geometry Learning. *Procedia Social and Behavioral Sciences, 8*, 686-693.

Seabi, J. (2012). Research Designs and Data Collection Techniques. In *J. G. Maree (Ed.). Complete Your Thesis or Dissertaion Succesfully: Practical Guidelines.* Cape Town: Juta & Company Ltd.

Shadaan, P., & Leong, K. E. (2013). Effectiveness of Using GeoGebra on Students' Understanding in Learning Circles. *The Malaysian Online Journal of Education*.

Siew, N. M., Chong, C. L., & Abdullah, M. R. (2013). Thinking Through Van Hieles' Phase-Based Learning Using Tangram. *Science Publications Malaysia Sabah*.

Spaull, N. (2013). South Africa's Education Crisis: The Quality of Education in South Africa. 1994-2011.

Stols, G., & Kriek, J. (2011). Why Do not All Maths Teachers Use Dynamic Geometry Software inTheir Classrooms? *Australian Journal of Educational Technology, 27(1)*, 137-151.

Teddlie, C., & Yu, F. (2007). Mixed Methods Sampling. *SAGE Publications*.

Teppo, A. (1991). Van Hieles' Levels of Geometric Thought Revisited. *The Mathematics Teacher, 84(3)*, 210-221.

Theron, L. C., & Malindi, M. J. (2012). Conducting Qualitative Research: Practical Guidelines on Fieldwork. In *J. G. Maree (Ed.). Complete Your Thesis or Dissertation Successfully: Practical Guidelines.* Cape Town: Juta & Company Ltd.

Usiskin, Z. (1982). *Van Hieles' Levels of Achievement in Secondary School Geometry*. USA: Columbus.

Uusimaki, L., & Nason, R. (2004). Causes Underlying Pre-Service Teachers' Negative Beliefs and Anxieties About Mathematics. *Group for the Psychology of Mathematics Education, 4*, 369-376.

Van de Walle, J. A. (2001). *Geometric Thinking and Concepts. In Elementary and Middle School Mathematics: Teaching Developmentally*. Boston: Allyn & Bacon.

Van Hieles', P. M. (1986). *Structure and Insight: A Theory of Mathematics.* New York: Academic Press.

Van Hieles', P. M. (1999). Developing Geometric Thinking Through Activities That Begin With Play. *Teaching Children Mathematics, 5(6)*, 310-316.

Van Putten, S. (2008). *Content Knowledge of Pre- Service Mathematics Educators in Euclidian Geometry*. Pretoria: University of Pretoria.

Verhoef, N. C., Coenders, F., Pieters, J. M., van Smaalen, D., & Tall, D. O. (2015). Professional Development Through Lesson Study: Teaching Derivative Using GeoGebra. *Professional Development in Education, 41(1)*, 109-126.

Way, J. (1997). The Development of Spatial and Geeometric Thinking: The Importance of Instruction.

Wesslen, M., & Fernandez, A. (2005). *Transormation Geometry*. London: King's College.

Xistouri, X., & Pitta-Pantanzi, D. (2013). Using Geogebra to Develop Primary School Students' Understanding of Reflection. *North American GeoGebra Journal, 2(1)*, 19-23.

Yazdani, M. (2007). Correlation Between Students' Level of Understanding Geometry According to the Van Hieles's' Model and Students Achievement in Plane Geometry. *Journal of Mathematical Sciences & Mathematics Education, 2(2)*, 40-45.

Zengin, Y., Furkan, H., & Kutluca, T. (2011). The Effect of Dynamic Mathematics Software GeoGebra on Student Achievement in Teaching of Trigonometry. *Procedia Social and Behavioral Sciences, 31*, 183-187.

Appendix A Letter for district director

UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
DEPARTMENT OF SCIENCE,
MATHEMATICS AND TECHNOLOGY
EDUCATION
Groenkloof Campus
PRETORIA
0002
Republic of South Africa

The director
Tshwane-West district
P/BAG X38
ROSSLYN
200

Dear Sir/Madam

RE: Request for permission to carry out a research project in your district

I am a student currently studying through the University of Pretoria. I am currently enrolled for my Master's degree in the Faculty of education. I intend to carry out a research project as part of the requirements towards the fulfilment of the degree. The title of my study is: Using GeoGebra in transformation geometry: an Investigation based on the van Hieles' model.

I hereby request your permission to work with four schools within your district. As part of my research, I wish to observe lessons on transformation geometry taught by two teachers per school, who are using GeoGebra. A video tape will be used to capture the lessons and I will conduct a semi-structured interview with participating teachers. Six randomly selected mathematics learners per school will be given a one hour paper and pencil test on transformation geometry after school set and marked by the researcher. The purpose of the test is to analyse the learners' level of achievement according to the van Hieles' theory and these learners will be interviewed. All interviews will be tape recorded.

Both teachers and learners will receive letters through the principal's office requesting their consent/assent to participate in the study. Letters of assent will be given to all the learners in the class where the study will be taking place. The names of the participants, the school and the province from which the data will be collected will not be mentioned in the final report. Participation is voluntary and participants may withdraw from the study at any time without reasons. The participants have the right to decline the invitation without giving reasons and their withdrawal from the study will not be used against them. All video and audio tapes will be kept safely at the University of Pretoria and these materials will be accessible only to the researcher and the supervisors.

I sincerely hope your office will respond promptly and positively to my request.

Yours Sincerely, Kekana G.R

------------------------------------- Date-----------------

In case you have any questions, please feel free to contact me or my supervisor at the contact details provided below:

Name of the student: Kekana, GR	Name of the supervisor: Dr Mwambakana, JN
Signature of the student:	Signature of the supervisor:
Cell-phone: 0837207828	Cell-phone: 0827743113
e-mail of the student: kekanagrace@webmail.co.za	e-mail of the supervisor: Jeanine.mwambakana@up.ac.za

Appendix B Letter for school principal

UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
DEPARTMENT OF SCIENCE,
MATHEMATICS AND TECHNOLOGY
EDUCATION
Groenkloof Campus
PRETORIA
0002
Republic of South Africa

The Principal
…………………..
…………………..
Dear Sir/Madam

RE: Request for permission to carry out a research project at your school

I am a student currently studying through the University of Pretoria. I am currently enrolled for my Master's degree in the Faculty of education. I intend to carry out a research project as part of the requirements towards the fulfilment of the degree. The title of my study is: Using GeoGebra in transformation geometry: an Investigation based on the van Hieles' model.

For the purpose of my study, I wish to observe lessons on transformation geometry taught by two teachers in your school, who are using GeoGebra.

A video tape will be used to capture the lessons and I will conduct a semi-structured interview with participating teachers. Six randomly selected mathematics learners, in their class, will be given a one hour paper and pencil test on transformation geometry after school set and marked by the researcher. The purpose of the test is to analyse the learners' level of achievement according to the van Hieles' theory and these learners will be interviewed. All interviews will be tape recorded.

Both teachers and learners will receive letters through the principal's office requesting their consent/assent to participate in the study. Letters of assent will be given to all the learners in the class where study will be taking place. The names of the participants, the school and the province from which the data will be collected will not be mentioned in the final report. Participation is voluntary and participants may withdraw from the study at any time without reasons. The participants have the right to decline the invitation without giving reasons and their withdrawal from the study will not be used against them. All video and audio tapes will be kept safely at the University of Pretoria and these materials will be accessible only to the researcher and the supervisors.

I sincerely hope your office will respond promptly and positively to my request.

Yours Sincerely, Kekana G.R

-------------------------------------   Date----------------

In case you have any questions, please feel free to contact me or my supervisor at the contact details provided below:

Name of the student: Kekana, GR	Name of the supervisor: Dr Mwambakana, JN
Signature of the student:	Signature of the supervisor:
Cell-phone: 0837207828	Cell-phone: 0827743113
e-mail of the student: kekanagrace@webmail.co.za	e-mail of the supervisor: Jeanine.mwambakana@up.ac.za

Appendix C Letter of consent

UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
DEPARTMENT OF SCIENCE,
MATHEMATICS AND TECHNOLOGY
EDUCATION
Groenkloof Campus
PRETORIA
0002
Republic of South Africa

Dear parent/guardian

RE: Informed consent

I am a student currently studying through the University of Pretoria. I am currently enrolled for my Master's degree in the Faculty of education. I intend to carry out a research project as part of the requirements towards the fulfilment of the degree. The title of my study is: Using GeoGebra in transformation geometry: an Investigation based on the van Hieles' model.

I request your child to be a participant in the proposed study.

For the purpose of my study, I wish to observe lessons on transformation geometry taught by your child's teacher at school. A video-tape will be used to capture the lesson. On the day of the classroom observation, your child will be interviewed during the break and this interview will be tape recorded so that I do not lose all the information collected.

Your child will be requested to write a one hour paper and pencil test on transformation geometry after school on this day………..... The test will be set and marked by the researcher in order to analyse the level of achievement obtained by your child. Your child's name will not be used anywhere in the final report.

Kindly note that your child's participation is voluntary; and that he/she has the right to decline this invitation without giving reasons. Your child may withdraw from the study at any time without giving reasons and his/her withdrawal from the study will not be used against him/her. I do not perceive any harm in the study. The collected information will be solely used for the purposes of this research.

I sincerely hope that you will respond promptly and positively to my request.

Yours Sincerely, Kekana G.R

------------------------------------- Date-----------------

In case you have any questions, please feel free to contact me or my supervisor at the contact details provided below:

Name of the student: Kekana, GR	Name of the supervisor: Dr Mwambakana, JN
Signature of the student:	Signature of the supervisor:
Cell-phone: 0837207828	Cell-phone: 0827743113
e-mail of the student: kekanagrace@webmail.co.za	e-mail of the supervisor: Jeanine.mwambakana@up.ac.za

If you grant me permission, please fill in the consent form provided.

Please complete this form and send it back to school as soon as possible. Thank you.

I, ___________________________, parent/guardian of __________________________, in Grade, _________ give permission for my child to participate in this study. I understand that should he/she wish to withdraw from the project, he/she may do so, and that his/her identity will not be disclosed in any written report.

Parent/guardian signature: ________________ Date: __________________

Appendix D Letter of consent

UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
DEPARTMENT OF SCIENCE,
MATHEMATICS AND TECHNOLOGY
EDUCATION
Groenkloof Campus
PRETORIA
0002
Republic of South Africa

Dear colleague

RE: Informed consent

I am a student currently studying through the University of Pretoria. I am currently enrolled for my Master's degree in the Faculty of education. I intend to carry out a research project as part of the requirements towards the fulfilment of the degree. The title of my study is: Using GeoGebra in transformation geometry: an Investigation based on the van Hieles' model.

I hereby request you to participate in the proposed study. As part of my data collection, I wish to observe lessons that you will teach using GeoGebra on transformation geometry. A video-tape will be used to capture the lessons and I will conduct a tape recorded semi-structured interview with you on your experience of teaching geometry using GeoGebra. The interview will last for approximately ten minutes.

Three randomly selected learners from your class will write a one hour paper and pencil test on transformation geometry after school set and marked by the researcher.

Your participation in this research is voluntary and you have the right to decline this invitation without giving reasons. You may withdraw from the study at anytime without reasons and your withdrawal from the study will not be used against you. Your name will not be mentioned anywhere in the final project report. The collected information will be solely used in my Thesis and a journal article. All video and audio tapes will be kept safely at the

University of Pretoria and these materials will be accessible only to the researcher and the supervisors.

I sincerely hope you will respond promptly and positively to my request.

Yours sincerely,

Kekana G.R

------------------------------------ Date----------------

In case you have any questions, please feel free to contact me or my supervisor at the contact details provided below:

Name of the student: Kekana, GR	Name of the supervisor: Dr Mwambakana, JN
Signature of the student:	Signature of the supervisor:
Cell-phone: 0837207828	Cell-phone: 0827743113
e-mail of the student: kekanagrace@webmail.co.za	e-mail of the supervisor: Jeanine.mwambakana@up.ac.za

If you grant me permission, please fill in the consent form provided.

Please complete this form and send it back to school as soon as possible. Thank you.

I, ___________________________________, am willing to participate in the above mentioned research project. I understand that should I wish to withdraw from the project, I may do so, and that my identity will not be disclosed in any written report.

Participant's signature: _______________ Date: __________________

Appendix E Teachers' interview protocol

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education
Fakulteit Opvoedkunde
Lefapha la Thuto

UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
DEPARTMENT OF SCIENCE,
MATHEMATICS AND TECHNOLOGY
EDUCATION
Groenkloof Campus
PRETORIA
0002
Republic of South Africa

Feel free in answering the following questions:

Topic: Using GeoGebra in transformation geometry: an Investigation based on the van Hieles' model.

1.1 How many years of experience do you have teaching Grade 9?

1.2 How many years have you been teaching mathematics in general and transformation geometry in particular?

1.3 How long have you been using GeoGebra in your teaching?

1.4 How do you use GeoGebra? For preparation of the lesson only or while teaching the lesson?

1.5 I would like you to think back, how was the performance of your learners in transformation geometry prior to the use of GeoGebra?

1.6 How is the performance of your learners, now that GeoGebra is used during the teaching and learning of transformation geometry?

1.7 What do you like most when using GeoGebra?

1.8 Would you recommend the use of GeoGebra to colleagues in other schools?

1.9 To your opinion, does GeoGebra improve :

1.9.1 Visualisation skills of the students?

1.9.2 Analysis skills of the students?

1.9.3 Deduction skills of the students?

1.9.4 Rigorous thinking skills of the students?

1.10 Is there anything you would like to make me aware of with regard to GeoGebra?

Thank you for taking part in this study.

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Appendix F Lesson observation schedule

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education
Fakulteit Opvoedkunde
Lefapha la Thuto

UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
DEPARTMENT OF SCIENCE,
MATHEMATICS AND TECHNOLOGY
EDUCATION
Groenkloof Campus
PRETORIA
0002
Republic of South Africa

Topic : Using GeoGebra in transformation geometry: an Investigation based on the van Hieles' model.

Activity	*Comment*
What type of manipulations does the teacher do in class?	
At what level does the teacher use GeoGebra in the classroom?	
Are the learners participating during the lesson?	
Are the learners able to answer questions asked by the teacher?	
What type of questions does the teacher asks?	
Are the diagrams clear and legible when using GeoGebra?	
Is the teacher able to use GeoGebra without consulting the manuals or seeking help?	
Does the teacher use GeoGebra throughout the lesson?	
Are the learners asking questions with regard to transformation geometry?	
What type of questions do learners ask?	
Is transformation geometry explained logically and clearly to learners?	

Appendix G Written test

Faculty of Education
Fakulteit Opvoedkunde
Lefapha la Thuto

UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
DEPARTMENT OF SCIENCE,
MATHEMATICS AND TECHNOLOGY
EDUCATION
Groenkloof Campus
PRETORIA
0002
Republic of South Africa

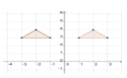
Learner code: …… School code: ……

Paper and pencil test Duration: 1Hour

INSTRUCTIONS

- Answer all questions.
- Shows all steps you used to arrive to the answer.
- Write neatly and legibly.

<u>Question 1</u>


A


B


C

1.1 Each of the above diagrams represents a different transformation of triangle ABC, which among the transformations represents an enlargement? Answer-------------------

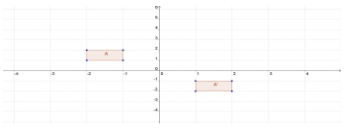
(2)

1.2 The other figure(s) represent which transformation(s)? (2)

Question 2

2.1.1 A figure and its image after transformation is given below draw or locate the following:

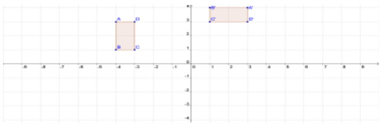
(a) Center of rotation (b) angle of rotation (4)



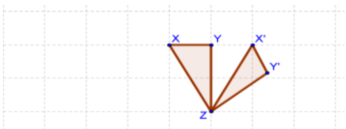
2.1.2. What can you say about the image? --- (2)

Question 3

3.1. Describe the transformation below in which rectangle ABCD is mapped onto rectangle A'B'C'D'---(3)



Question 4



The triangle XYZ has been rotated through $+70^0$ about the y-axis. X'Y'Z' is the image of XYZ after rotation.

4.1.1 What type of triangle is XYZ? -- (2)

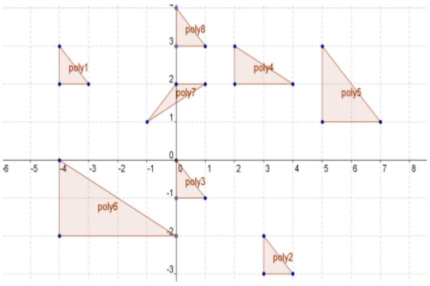
4.1.2 Give a reason for your answer to question 4.1.1------------------------------------- (2)

4.1.3 Use transformation to prove that triangle XYZ is congruent to triangle X'Y'Z

-- (4)

4.2. Do you agree that when an object is rotated, its image is always congruent to the object? Explain why you say so using a diagram. (4)

Question 5



5.1 Name figures that are congruent to poly 1. ---------------------------------- (3)

5.2 Name the transformation from poly 1 to poly 8 in words and mathematically. (4)

5.3 What are the similarities and the differences between poly 2 and poly 6? (4)

5.4 State the coordinates of poly3. (2)

Appendix H Learners' interview protocol

UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
DEPARTMENT OF SCIENCE,
MATHEMATICS AND TECHNOLOGY
EDUCATION
Groenkloof Campus
PRETORIA
0002
Republic of South Africa

Mathematics and GeoGebra

1. Do you have access to a computer at school?

2. Do you have access to a computer at home?

3. How do you find the use of GeoGebra in mathematics in general and transformation geometry in particular?

4. Which part of transformation geometry do you like and why?

5. Does the use of GeoGebra make you understands transformation geometry better?

6. Will you recommend the use of GeoGebra to other learners?

7. Is there anything you would like to bring to my attention with regard to GeoGebra?

Thank you for participating in this study.

Appendix I Participating learner

UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
DEPARTMENT OF SCIENCE,
MATHEMATICS AND TECHNOLOGY
EDUCATION
Groenkloof Campus
PRETORIA
0002
Republic of South Africa

Dear learner

RE: Letter of assent

I am a student currently studying through the University of Pretoria. I am currently enrolled for my Master's degree in the Faculty of education. I intend to carry out a research project as part of the requirements towards the fulfilment of the degree. The title of my study is: Using GeoGebra in transformation geometry: an Investigation based on the van Hieles' model. I request you to be a participant in the proposed study.

For the purposes of my study, I wish to observe lessons on transformation geometry taught by your teacher in your class. A video-tape will be used to capture the lesson. On the day of the classroom observation, you will be interviewed during break and this interview will be tape recorded so that I do not lose all the information thus collected. The interview will last for approximately ten minutes.

You will be requested to write a one hour paper and pencil test on transformation geometry <u>after school</u> on this day: ……………………….The test will be set and marked by the researcher in order to analyse your level of achievement. Your name will not be used anywhere in the final report.

Kindly note that your participation is voluntary and you have the right to decline this invitation without giving reasons. Your withdrawal from the study will not be used against you. I do not perceive any harm in the study. The collected information will be solely used for the purposes of this research.

All video and audio tapes will be kept safely at the University of Pretoria and these materials will be accessible only to the researcher and the supervisors.

I sincerely hope that this letter serves the purpose it is intended for.

Yours Sincerely, Kekana G.R

-------------------------------------   Date-----------------

In case you have any questions, please feel free to contact me or my supervisor at the contact details provided below:

Name of the student: Kekana, GR	Name of the supervisor: DrMwambakana, JN
Signature of the student:	Signature of the supervisor:
Cell-phone: 0837207828	Cell-phone: 0827743113
e-mail of the student: kekanagrace@webmail.co.za	e-mail of the supervisor: Jeanine.mwambakana@up.ac.za

If you agree with the contents of this letter, please fill in the consent form provided.

Please complete this form and send it back to school as soon as possible. Thank you.

I, _________________________, in Grade, _________ give my consent to participate in this study. I understand that should I wish to withdraw from the project, I may do so, and that my identity will not be disclosed either at school or in any report.

Signature: _____________   Date: _________________

Appendix J Learner in the classroom to be observed

UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
DEPARTMENT OF SCIENCE,
MATHEMATICS AND TECHNOLOGY
EDUCATION
Groenkloof Campus
PRETORIA
0002
Republic of South Africa

Dear learner

RE: Letter of assent

I am a student currently studying through the University of Pretoria. I am currently enrolled for my Master's degree in the Faculty of education. I intend to carry out a research project as part of the requirements towards the fulfilment of the degree. The title of my study is: Using GeoGebra in transformation geometry: an Investigation based on the van Hieles' model. For the purposes of my study, I wish to observe lessons on transformation geometry taught by your teacher in your class. A video-tape will be used to capture the lesson. This letter serves to request that a video-tape be used while I am conducting classroom observation. The collected information will be solely used for the purposes of my research and your identity will not be disclosed in any report.

Kindly note that your participation is voluntary and you have the right to decline this invitation without giving reasons. Your withdrawal from the study will not be used against you. I do not perceive any harm in the study. All video and audio tapes will be kept safely at the University of Pretoria and these materials will be accessible only to the researcher and the supervisors.

I sincerely hope that this letter serves the purpose it is intended for.

Yours Sincerely, Kekana G.R

------------------------------------- Date-----------------

In case you have any questions, please feel free to contact me or my supervisor at the contact details provided below:

Name of the student: Kekana, GR	Name of the supervisor: Dr Mwambakana, JN
Signature of the student:	Signature of the supervisor:
Cell-phone: 0837207828	Cell-phone: 0827743113
e-mail of the student: kekanagrace@webmail.co.za	e-mail of the supervisor: Jeanine.mwambakana@up.ac.za

If you agree with the contents of this letter, please fill in the form provided.

Please complete this form and send it back to school as soon as possible. Thank you.

I, ________________, in Grade, _________ give my assent to be part of the proposed study. I understand that should I wish to withdraw from the project, I may do so, and that my identity will not be disclosed either at school or in any report.

Signature: _____________ Date: ________________

Appendix K Letter of consent

UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
DEPARTMENT OF SCIENCE,
MATHEMATICS AND TECHNOLOGY
EDUCATION
Groenkloof Campus
PRETORIA
0002
Republic of South Africa

Dear parent/guardian

RE: Letter of consent

I am a student currently studying through the University of Pretoria. I am currently enrolled for my Master's degree in the Faculty of education. I intend to carry out a research project as part of the requirements towards the fulfilment of the degree. The title of my study is: Using GeoGebra in transformation geometry: an Investigation based on the van Hieles' model.

This letter serves to request your permission to video-tape your child while classroom observation is taking place when your child and other learners are taught transformation geometry using GeoGebra. The collected information will be solely used for the purposes of this research and will not be communicated to anyone who is not involved in this project.

Kindly note that your child's participation is voluntary and that he/she has the right to decline this invitation without giving reasons and his/her withdrawal from the study will not be used against him/her. I do not perceive any harm in this study. All video and audio tapes will be kept safely at the University of Pretoria and these materials will be accessible only to the researcher and the supervisors.

I hereby request your informed consent in this matter.

I sincerely hope that you will respond promptly and positively to my request.

Yours Sincerely, Kekana G.R

------------------------------------- Date----------------

In case you have any questions, please feel free to contact me or my supervisor at the contact details provided below:

Name of the student: Kekana, GR	Name of the supervisor: DrMwambakana, JN
Signature of the student:	Signature of the supervisor:
Cell-phone: 0837207828	Cell-phone: 0827743113
e-mail of the student: kekanagrace@webmail.co.za	e-mail of the supervisor: Jeanine.mwambakana@up.ac.za

If you agree with the contents of this letter, please fill in the form provided.

Please complete this form and send it back to school as soon as possible. Thank you.

I, ___________________________, parent/guardian of __________________________, in Grade, __________ give permission for my child to be part of this study. I understand that should he/she wish to withdraw from the project, he/she may do so, and that his/her identity will not be disclosed in any written report.

Parent/guardian signature: ________________ Date: __________________

Appendix L Written test memorandum

UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
DEPARTMENT OF SCIENCE,
MATHEMATICS AND TECHNOLOGY
EDUCATION
Groenkloof Campus
PRETORIA
0002
Republic of South Africa

PAPER AND PENCIL TEST MEMORANDUM TOTAL: 40 MARKS

Question 1 (Level 1)

1.1 C √√

1.2 Figure A: reflection along the y-axis √

Figure B: reflection along the x-axis√

Question 2 (Level 2)

2.1.1 a) (0; 0) √√

b) 180^0 clockwise√√

2.1.2 The image is the same distance from the origin as the object. The object is the same size as the image. √√

Question 3 (Level 3)

3.1 x changes the sign; then x and y values ‘swaps’. √√√

Question 4 (Level 4)

4.1.1 A scalene triangle. √√

4.1.2. All sides not equal. √√

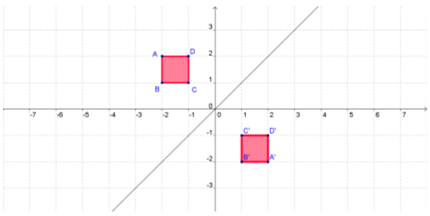
4.1.3 XY = X’Y’; √

XZ = X’Z’ √

YZ = Y’Z’ √

XYZ Ξ X’Y’Z’ √

4.2



(4)

<u>Question 5</u> (Level 5)

5.1 Poly 2; poly 3 & poly 8 √√√

5.2 Poly 1 has been translated 4 units to the right; then 1 unit up. √√

$(x; y) \longrightarrow (x + 4; y + 1)$ √√

5.3 <u>Similarities</u>

- Same shape √

- Same angles √

Differences

- Different sizes √

- Corresponding sides not equal √

5.4 (0; -1); (0; 0) & (1; -1) √√

Appendix M Teachers' transcripts

John

Interviewer (I): Sir, you are teacher number one and I will call you John. Please note that you may withdraw anytime during our interview.

Interviewer (I): How many years of experience do you have teaching Grade 9?

John: I have been teaching Grade 9 for nine years now.

Interviewer (I): How many years have you been teaching mathematics in general and transformation geometry in particular?

John: Eh, its five years of teaching transformation geometry.

Interviewer (I): How long have you been using GeoGebra in your teaching?

John: This is my third year since I have been using GeoGebra. I was introduced to GeoGebra in 2012 when doing Ace at the University of Pretoria.

Interviewer (I): How do you use GeoGebra? For preparation of the lesson only, or while teaching the lesson?

John: I use for my lesson preparation and lesson presentation. More especially for concepts such as transformation geometry which needs learners to see objects clearly. But most of my lesson presentation, I use it. I use it in the teaching of graphs, trigonometric graphs as well as functions. Also in presenting statistics in Grade 9 and other Grades.

Interviewer (I): I would like you to think back, how was the performance of your learners in transformation geometry prior to the use of GeoGebra?

John: Learners have been performing poorly, since they could not see some of the objects drawn on the plain chalkboard available at our school. But after I started presenting my lessons with GeoGebra, I have seen them changing their attitude to the better. Also there is an improvement in performance in mathematics, more especially transformation geometry.

Interviewer (I): How is the performance of your learners, now that GeoGebra is used during the teaching and learning of transformation geometry?

John: The performance has honestly improved. Learners like technology, as a results, they remain focused throughout the lesson.

Interviewer (I): What do you like most when using GeoGebra?

John: It is user friendly, it makes objects very clear and neat, unlike on the chalkboard.

Interviewer (I): Would you recommend the use of GeoGebra to colleagues in other schools?

John: Definitely, I would recommend it to all mathematics teachers and also learners.

Interviewer (I): To your opinion, does GeoGebra improve:

a) Visualisation skills of the students?

b) Analysis skills of the students?

c) Deduction skills of the students?

d) Rigorous thinking skills of the students?

John: I would say, 'pause', visualisation as pictures or objects come out clearly. Also analysis skills.

Interviewer (I): Is there anything you would like to make me aware of with regard to GeoGebra?

John: I like the fact that GeoGebra is free software; therefore people can use it without worrying about the Internet.

Interviewer (I): Thank you for taking part in this study.

John: It's my pleasure.

Patrick

Interviewer (I): Sir, you are teacher number two and I will call you Patrick. Please note that you may withdraw anytime during our interview.

Interviewer (I): How many years of experience do you have teaching Grade 9?

Patrick: I have got five years of teaching Grade 9 mathematics.

Interviewer (I): How many years have you been teaching mathematics in general and transformation geometry in particular?

Patrick: I have been teaching mathematics, 'stammering', this is my twelfth year teaching mathematics. I have been teaching transformation geometry for, for four years.

Interviewer (I): How long have you been using GeoGebra in your teaching?

Patrick: Ah, this is my fourth year using GeoGebra in my teaching.

Interviewer (I): How do you use GeoGebra? For preparation of the lesson only, or while teaching the lesson?

Patrick: I use it for both. When I prepare my lessons I use GeoGebra, when I am teaching, I also use GeoGebra.

Interviewer (I): I would like you to think back, how was the performance of your learners in transformation geometry prior to the use of GeoGebra?

Patrick: Aah, it was bad, but since I started using GeoGebra I see improvement.

Interviewer (I): How is the performance of your learners, now that GeoGebra is used during the teaching and learning of transformation geometry?

Patrick: Ah, like I said in the previous question, since I started using GeoGebra I see good results. Performance of the learners in terms of analytical, in terms of transformation geometry is good.

Interviewer (I): What do you like most when using GeoGebra?

Patrick: It saves time, I do not have…, and learners see what I have to teach them clearly. They are able to visualise what is needed out of them.

Interviewer (I): Would you recommend the use of GeoGebra to colleagues in other schools?

Patrick: Definitely so, because this is a good program. Thank you.

Interviewer (I): To your opinion, does GeoGebra improve:

a) Visualisation skills of the students?

b) Analysis skills of the students?

c) Deduction skills of the students?

d) Rigorous thinking skills of the students?

Patrick: Definitely is going to be number (a), it improves visualisation of the learners. When it comes to this, ah, when it comes to transformation geometry figures. Learners are able to see what, when you say translation they are able to see that. When you say rotation they are able to see that. When you say reduction they are able to see that. Thank you.

Interviewer (I): Is there anything you would like to make me aware of with regard to GeoGebra?

Patrick: Yaa. I think GeoGebra is so good because you do not need internet to use it. Once downloaded, you can use it every day. You do not need internet.

Interviewer (I): Thank you for taking part in this study.

Patrick: You are welcome ma'm.

Lerado

Interviewer (I): How many years of experience do you have teaching Grade 9?

Lerado: I have been teaching for eleven years now.

Interviewer (I): How many years have you been teaching mathematics in general and transformation geometry in particular?
Lerado: Ever since I started teaching, I have been teaching mathematics. But I have about six years of teaching transformation geometry.

Interviewer (I): How long have you been using GeoGebra in your teaching?
Lerado: Mm, I was introduced to GeoGebra in 2010, so I have five years of using GeoGebra in the teaching of mathematics.

Interviewer (I): How do you use GeoGebra? For preparation of the lesson only or while teaching the lesson?
Lerado: I use it when I prepare my lessons and during the presentation of mathematics lesson. I use it in other Grades for the teaching of trigonometric graphs as they come out and clear and because you can use sliders to show learners the effect of variables in the general formulae.

Interviewer (I): I would like you to think back, how was the performance of your learners in transformation geometry prior to the use of GeoGebra?
Lerado: Oh, the performance was really bad. Most learners achieved level1 (0-29%) and only a few could manage a level 2 (30-39%). Remember the minimum pass requirement in mathematics CAPS Grade 9 is level 3 (40-49%).

Interviewer (I): How is the performance of your learners, now that GeoGebra is used during the teaching and learning of transformation geometry?
Lerado: Wao! The learners are achieving the minimum level 3 (40-49%) which is satisfactory. Only a few are still struggling at level 1. And there are brilliant learners who achieve even level 5(60-65%).

Interviewer (I): What do you like most when using GeoGebra?

Lerado: Firstly, I like the fact that it is free software, unlike most programs such as PHET and other mathematics software that need internet connection. Secondly, it makes drawing of graphs easier, neat and legible to most learners if not all. Lastly, I would say it also saves times as compared to the old way of chalk and board.

Interviewer (I): Would you recommend the use of GeoGebra to colleagues in other schools?

Lerado: I would definitely recommend GeoGebra to my colleagues and other teachers in other schools and provinces. I would also advise learners who have access to a computer to use it for their practice and for their homework.

Interviewer (I): To your opinion, does GeoGebra improve:

a) Visualisation skills of the students?

b) Analysis skills of the students?

c) Deduction skills of the students?

d) Rigorous thinking skills of the students?

Lerado: It improves learners' visualisation skills.

Interviewer (I): Is there anything you would like to make me aware of with regard to GeoGebra?

Lerado: Well, I think GeoGebra is one of the most interesting programs which teachers can utilize. In case teachers do not have the knowledge of the program, the department of education can organise a person who can workshop them. By so doing they will be investing in the future of our children.

Interviewer (I): Thank you for taking part in this study.

Lerado: You are welcome.

Suzan

Interviewer (I): How many years of experience do you have teaching Grade 9?

Suzan: I have been teaching for eight years.

Interviewer (I): How many years have you been teaching mathematics in general and transformation geometry in particular?

Suzan: I have eight years of teaching mathematics in general, as well as well as in Grade 9.

Interviewer (I): How long have you been using GeoGebra in your teaching?

Suzan: I have been using GeoGebra for four years now.

Interviewer (I): How do you use GeoGebra? For preparation of lesson only or while teaching the lesson?

Suzan: Most of the time I use for my lesson presentation. I like the use of GeoGebra in my mathematics classrooms.

Interviewer (I): I would like you to think back, how was the performance of your learners in transformation geometry prior to the use of GeoGebra?

Suzan: Well. It wasn't satisfactory. Even though there were exceptional learners who will always perform well. But majority of the learners were underperforming.

Interviewer (I): How is the performance of your learners, now that GeoGebra is used during the teaching and learning of transformation geometry?

Suzan: They have improved, most of them managed to obtain a minimum of level 3 (40-49%).

Interviewer (I): What do you like most when using GeoGebra?

Suzan: GeoGebra makes the drawing of objects to be as quick and neat as possible. The objects also come out clear and visible.

Interviewer (I): Would you recommend the use of GeoGebra to colleagues in other schools?

Suzan: I would recommend the use of GeoGebra to colleagues in other schools.

Interviewer (I): To your opinion, does GeoGebra improve:

a) Visualisation skills of the students?

b) Analysis skills of the students?

c) Deduction skills of the students?

d) Rigorous thinking skills of the students?

Suzan: It improves visualisation skills of the students.

Interviewer (I): Is there anything you would like to make me aware of with regard to GeoGebra?

Suzan: I can only say, if our school had enough infrastructures, we can move away from the traditional ways of teaching to new environments that are technology inclined. (I): Thank you

Appendix N Learners' transcripts

Learner 1

Interviewer (I): Are you doing Grade 9 for the first time this year?

Learner 1 (L1): Yes ma'm.

Interviewer (I): Which subject(s) do you like most and why?

L1: Accounting because I have interest on it.

Interviewer (I): Do you know the pass requirements for mathematics in Curriculum Assessment Policy Statement (CAPS) in Grade 9?

L1: Yes ma'm. Eh, level 4.

Interviewer (I): Do you have access to a computer at school?

L1: No.

Interviewer (I): Do you have access to a computer at home?

L1: Yes ma'm.

Interviewer (I): How do you find the use of GeoGebra in mathematics in general and transformation geometry in particular?

L1: It is so interesting because it is so simple.

Interviewer (I): Which part of transformation geometry do you like and why?

L1: Eh, square reflection.

Interviewer (I): Does the use of GeoGebra make you understands transformation geometry better?

L1: Yes ma'm.

Interviewer (I): Will you recommend the use of GeoGebra to other learners?

L1: Yes.

Interviewer (I): Is there anything you would like to bring to my attention with regard to GeoGebra?

L1: No.

Interviewer (I): Thank you for participating in this study.

L1: Thank you.

Learner 2

Interviewer (I): Are you doing Grade 9 for the first time this year?

Learner 2 (L2): Yes ma'm.

Interviewer (I): Which subject(s) do you like most and why?

L2: English and Setswana because they are languages.

Interviewer (I): Do you know the pass requirements for mathematics in Curriculum Assessment Policy Statement (CAPS) in Grade 9?

L2: Level 3.

Interviewer (I): Do you have access to a computer at school?

L2: Yes.

Interviewer (I): Do you have access to a computer at home?

L2:No.

Interviewer (I): How do you find the use of GeoGebra in mathematics in general and

transformation geometry in particular?

L2: I like it because it makes me understand.

Interviewer (I): Which part of transformation geometry do you like and why?

L2: Mmm, many things because I understood.

Interviewer (I): Does the use of GeoGebra make you understands transformation geometry better?

L2: Yes, because everything was clear.

Interviewer (I): Will you recommend the use of GeoGebra to other learners?

L2: Yes they must use it so that they can understand.

Interviewer (I): Is there anything you would like to bring to my attention with regard to GeoGebra?

L2:'Laughs', eh no.

Interviewer (I): Thank you for participating in this study.

L2: My pleasure.

Learner 3

Interviewer (I): Are you doing Grade 9 for the first time this year?

Learner 3 (L3): Yes.

Interviewer (I): Which subject(s) do you like most and why?

L3: Setswana, English and Mathematics because I want to be a pilot.

Interviewer (I): Do you know the pass requirements for mathematics in Curriculum Assessment Policy Statement (CAPS) in Grade 9?

L3: Level 4.

Interviewer (I): Do you have access to a computer at school?

L3: No.

Interviewer (I): Do you have access to a computer at home?

L3: No.

Interviewer (I): How do you find the use of GeoGebra in mathematics in general and transformation geometry in particular?

L3: I like it.

Interviewer (I): Which part of transformation geometry do you like and why?

L3: Reflection.

Interviewer (I): Does the use of GeoGebra make you understands transformation geometry better?

L3: Yes.

Interviewer (I): Will you recommend the use of GeoGebra to other learners?

L3: Yes.

Interviewer (I): Is there anything you would like to bring to my attention with regard to GeoGebra?

L3: GeoGebra is very simple to use in order to understand mathematics.

Interviewer (I): Thank you for participating in this study.

L3: You are welcome.

Learner 4

Interviewer (I): Are you doing Grade 9 for the first time this year?

Learner 4 (L4): No, for the third.

Interviewer (I): Which subject(s) do you like most and why?

L4: Maths, Natural Science and Setswana.

Interviewer (I): Do you know the pass requirements for mathematics in Curriculum Assessment Policy Statement (CAPS) in Grade 9?

L4: This one, I do not know.

Interviewer (I): Do you have access to a computer at school?

L4: Yes.

Interviewer (I): Do you have access to a computer at home?

L4: No ma'm.

Interviewer (I): How do you find the use of GeoGebra in mathematics in general and transformation geometry in particular?

L4: <u>I like it</u> because it is understandable.

Interviewer (I): Which part of transformation geometry do you like and why?

L4: Writing the general rule.

Interviewer (I): Does the use of GeoGebra make you understands transformation geometry better?

L4: Yes.

Interviewer (I): Will you recommend the use of GeoGebra to other learners?

L4: I will tell them to use GeoGebra.

Interviewer (I): Is there anything you would like to bring to my attention with regard to GeoGebra?

L4: The program is very much interesting.

Interviewer (I): Thank you for participating in this study.

Appendix O Learner 1 paper and pencil script (page 1)

Appendix VII

Learner code: …… School code: ……

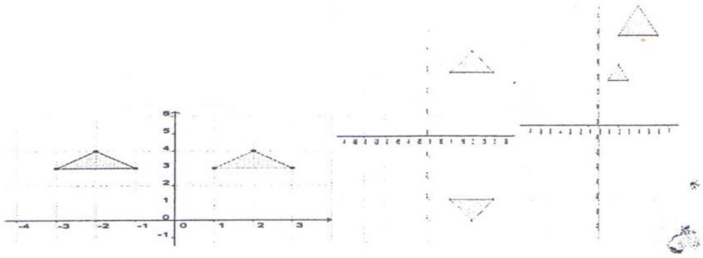
Paper and pencil test Duration: 1Hour

30/38

INSTRUCTIONS

- Answer all questions.
- Shows all steps you used to arrive to the answer.
- Write neatly and legibly.

QUESTION 1 [Level 1]



A B C

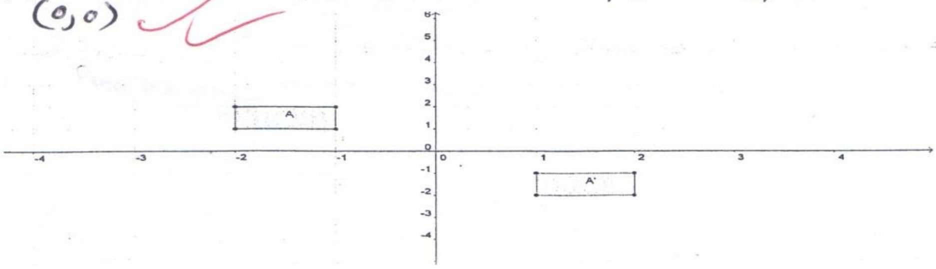
1.1 Each of the above diagrams represents a different transformation of triangle ABC, which among the transformations represents an enlargement? Answer---------- C
-------- 2 (2)

1.2 The other figure(s) represent which transformation(s)? (2)
(a) Reflection along y-axis
(b) Reflection along x-axis
(c) Enlargment
2

Question 2 [Level 2]

2.1.1 A figure and its image after transformation is given below draw or locate the following:

(a) Centre of rotation (b) angle of rotation 180° Clockwise 4 (4)
(0,0)

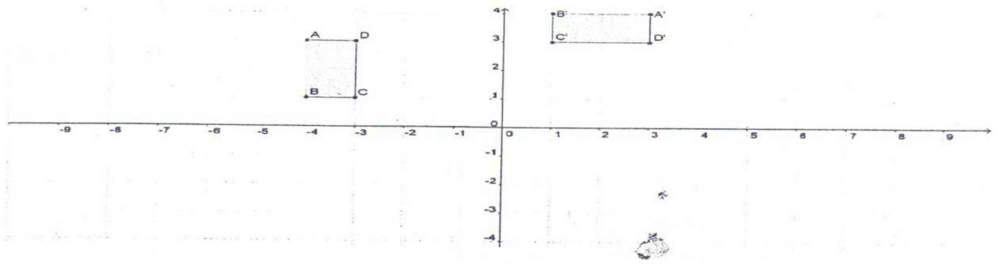


Appendix P Learner 1’s paper and pencil script (page 2)

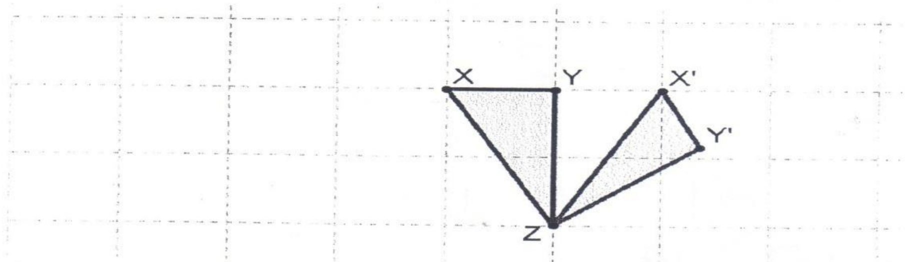
2.1.2. What can you say about the image? ---The Image is the same distance from the origin as the figure--- (2)

QUESTION 3 [Level 3]

3.1. Describe the transformation below in which rectangle ABCD is mapped onto rectangle A’B’C’D’---They swap and x changes it's sign--- (3)



Question 4 [Level 4]



The triangle XYZ has been rotated through +70 degree about X. X’Y’Z’ is the image of XYZ after rotation.

4.1.1 What type of triangle is XYZ? ---Isosceles--- (2)

4.1.2 Why? ---Two sides are equal--- (2)

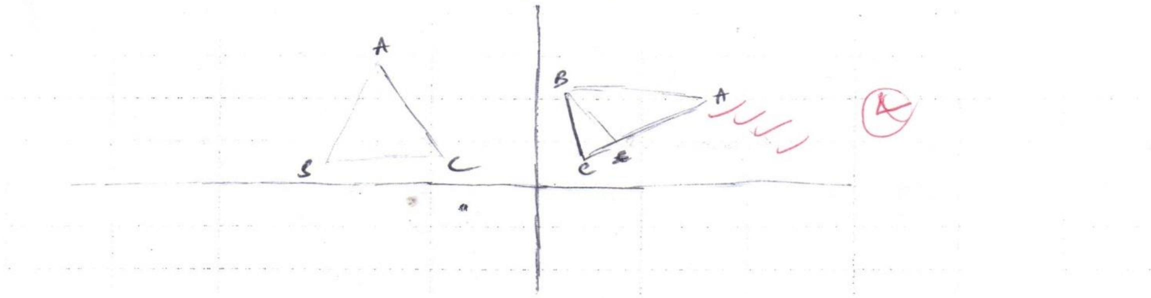
4.1.3 Use transformation to prove that triangle XYZ is congruent to triangle X’Y’Z

xy = x'y'
yz = y'z'
xz = x'z'
S,S,S

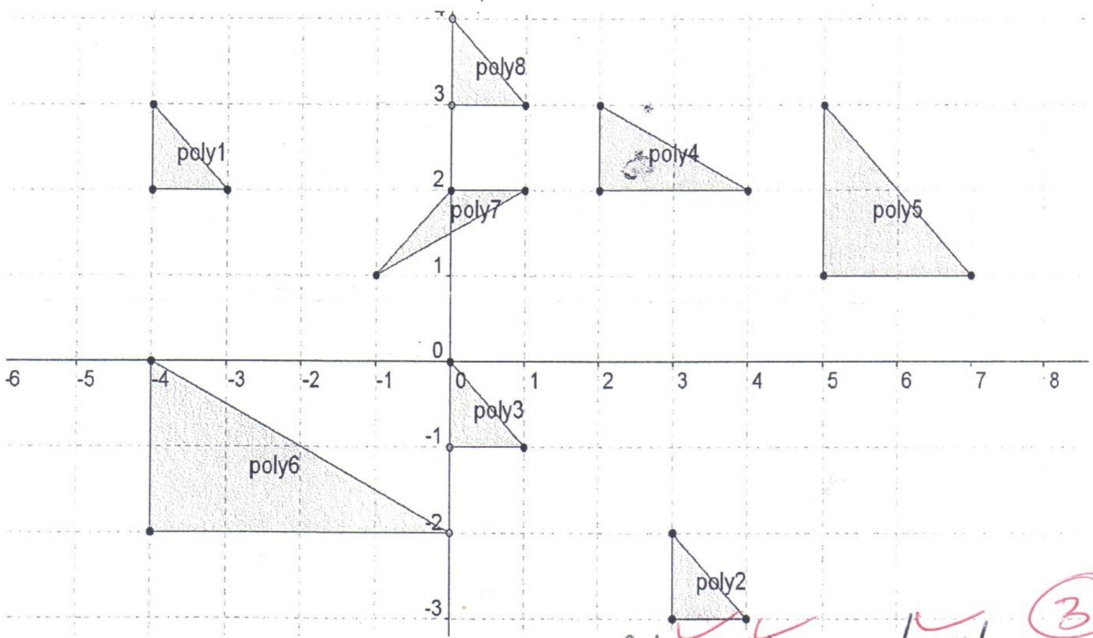
--- (4)

Appendix Q Learner's paper and pencil script (page 3)

4.2. Do you agree that when an object is rotated, its image is always congruent to the object? Explain why you say so using a diagram. (4)



Question 5 [Level 5]



5.1 Name figures that are congruent to poly 1. Poly 8, Poly 3 and poly 2 (3)

5.2 Name the transformation from poly 1 to poly 8 in words and mathematically. (4)

1 unit up 4 units right

5.3 What are the similarities and the differences between poly 2 and poly 6? (4)

L's are equal, shaps are the same and differens

5.4 State the coordinates of poly3. (2)

(0,0) (0;-1) (1;-1)

The end