(*c*) *The relation between the work done and the mass of the sample.*

Larose (1934) compressed successively 2·20 gm., 3·26 gm., and 4·34 gm. of the same yarn and found that the ratio of mass to volume at the same pressure was constant. The same result was obtained in the present study by the static method, and it was taken for granted in the employment of equation (11). Assuming the pressure to be a linear function of the inverse cube of the volume, the pressure and work done may be expected to bear a linear relationship to the cube of the mass of the sample.

For comparative purposes it would be sufficient to specify that a certain mass (5 gm. in the present study), be used for a determination. Cases occur, however, where a smaller quantity only is available, and in any case it is far more rapid and convenient to weigh out approximately 5 gm. The relation between the work done, as determined by the dynamic method, and the mass of the sample was, therefore, investigated.

Owing to the variation within a sample, a careful sytem of sampling had to be employed. Accordingly 6 gm. of two samples of high and low compressibility were weighed out. Weights from 3 gm. to 6 gm. at intervals of 0·5 gm. were allotted the numbers 1 to 7, and placed five times in random order by means of tables of random numbers. The various weights were then compressed in these orders in the instrument. After each determination the whole sample of 6 gm. was placed together, so that the next weight was selected from the whole sample. This procedure ensured that no bias occurred in the matter of sampling, while the repeated randomisation of the order ensured that any changes produced in the wool as a result of the extensive handling should be distributed throughout the various masses employed.

The results are given in Table 15.

In Figure 10 the work done is plotted as a function of the cube of the mass. Beyond a certain value of the mass the relation is linear, but the straight lines do not pass through the origin of axes. They cut the y-axis at points which differ for the two samples but are independent of the degree to which each sample is compressed.

An approximation similar to that made when the same sample is compressed to different volumes must obviously be made, probably by the exclusion of certain terms from the general equation, and it is necessary to determine how to fit the results obtained into equation (9), and to show how the coefficient a may be calculated when a mass other than 5 gm. is employed.

It is found that if for v in equation (9) a quantity u is substituted, such that

$$\frac{1}{u^2} = \frac{m^3}{5^3}\left(\frac{1}{v^2} - \frac{1}{v^2_1}\right) + \frac{1}{v^2_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

the value of a as given by equation (9) can be calculated for the different masses. The last two columns of Table 15 give the value of $\frac{1}{u^2}$ as calculated from equation (12), and the corresponding value of a as calculated from equation (9). The calculations are seen to be valid in the range 4 to 6 gm., and an adjustment can be made to 5 gm. for any quantity of wool between these limits.

TABLE 15.

The work done in compressing different masses of two samples from 103·3 c.c. to two different volumes.

Sample.	Final Volume (v). c.c.	Mass (m) gm.	Work done. (Kg. cm.).	$\frac{1}{u^2}$	a (Equation 9).
1....................	55·0	3·04	0·88	0·000143	—
		3·58	1·34	178	13·7 × 10³
		4·05	1·95	217	14·8
		4·54	2·77	270	15·2
		5·04	3·81	337	15·3
		5·53	4·93	416	14·9
		6·08	6·50	522	14·9
	62·4	3·05	0·59	0·000127	—
		3·60	0·96	152	12·2 × 10³
		4·04	1·36	177	14·7
		4·55	1·94	215	15·3
		5·06	2·62	262	14·8
		5·55	3·50	318	15·2
		6·09	4·56	391	15·0
2....................	55·3	3·04	0·59	0·000146	6·0 × 10³
		3·59	0·83	180	6·3
		4·05	1·13	218	6·8
		4·56	1·36	270	6·1
		5·06	1·80	336	6·3
		5·55	2·27	413	6·2
		6·03	2·79	502	6·1
	62·1	3·04	0·44	0·000131	5·2 × 10³
		3·59	0·64	155	6·0
		4·05	0·86	182	6·5
		4·56	1·05	219	6·2
		5·06	1·40	265	6·5
		5·54	1·72	320	6·4
		6·03	2·06	384	6·1

Besides the fact that an approximation is probably made in assuming linearity between the work done and the cube of the mass of the sample, it must be pointed out that quantities less than about 3 gm. do not fill the compression compartment completely, so that v_1 is not the same for masses above and below 3 gm.

Taking the calculated values of a from 4 to 6 gm., the standard deviation of the differences from the mean is found to be $0{\cdot}23 \times 10^3$. For compression to various volumes, as given in Table 7, the standard deviation is $0{\cdot}26 \times 10^3$. The two values are in close agreement, and if a difference between them exists, it may be presumed to be due to the difficulty of determining the volume as accurately as the mass.

(*d*) *Comparison of results obtained by the static and the dynamic methods.*

It has been shown that the coefficient a as given by equation (9) can be regarded as an approximation to the coefficient $\frac{A}{2}$ of equation (6), and it was estimated to be too low by about 7 per cent. It is interesting to see how the coefficient a obtained by the dynamic method agrees with the coefficient $\frac{A}{2}$ as given by the static cylinder and piston method

The result of testing five wools by the two methods is given in Table 16, where the results refer to 5 gm. of material and the pressure is measured in Kg./cm.^{2}

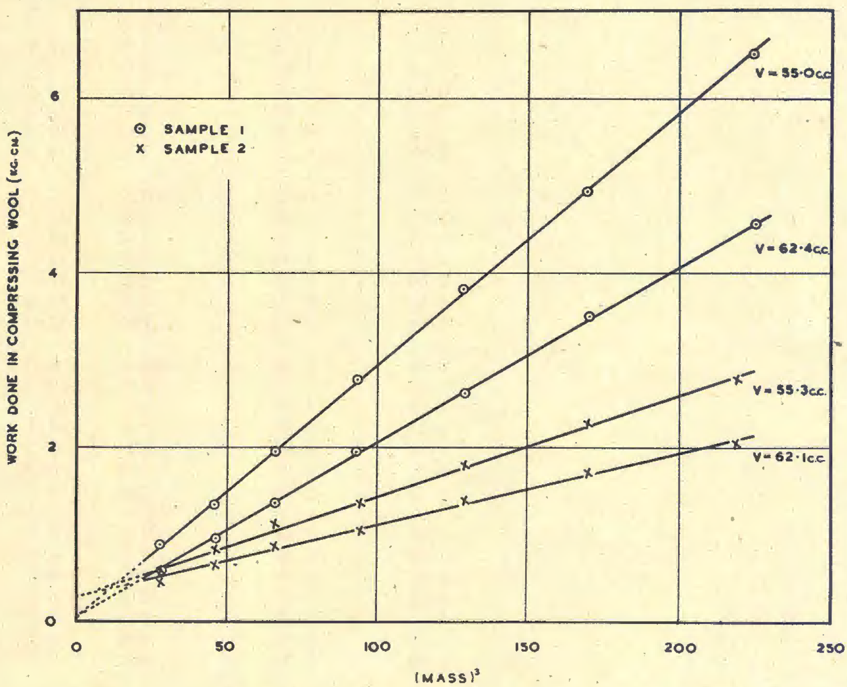


FIGURE 10.—The work done in compressing different masses of wool by the dynamic method, as a function of the cube of the mass.

TABLE 16.

A comparison of the coefficients obtained by the static and dynamic methods.

Sample.	a (Dynamic Method).	$\frac{A}{2}$ (Static Method).	Ratio.	Difference.
1	$5 \cdot 8 \times 10^3$	$3 \cdot 2 \times 10^3$	1·81	$2 \cdot 6 \times 10^3$
2	9·4	5·2	1·81	4·2
3	11·5	6·0	1·92	5·5
4	15·0	9·8	1·53	5·2
5	15·5	11·7	1·32	3·8

While the order of magnitude is the same in the two cases, it is evident that the coefficients differ in two respects. The coefficient a of the dynamic method is in all cases the greater, and the differences do not follow any law consistently, as shown by the ratios and differences.

Several reasons may be advanced as being responsible for the differences. In the first place, the value of a was obtained from the first compression of a sample, while the coefficient $\frac{A}{2}$ was calculated from the final constant cycle of compression by the static method. The question immediately arises as to whether the coefficients obtained by the initial rapid compression and the final slow compression are comparable. Evidence occurred during the investigation which indicated that with the dynamic method the differences between successive cycles lay, not in the coefficient a itself, but in the constants of equation (9) from which the coefficient a was calculated, since the difference in the work done between the initial and the final compressions bore a linear relationship to a^2, as calculated from the first compression. The evidence could not be regarded as conclusive, however, and the point was not investigated further, but it is significant that the inverse cube law was found to hold for the first compression by the dynamic method, and only for the final constant cycle of compression by the static method.

In the second place, the difference in the rate of compression must be regarded as one cause of the difference obtained by the two methods. In the static method, the compression was performed extremely slowly, and an attempt was made to overcome the effects of friction as far as possible. The rapidity of compression by the dynamic method, on the other hand, would include frictional effects in the work done during compression. Assuming the friction to be proportional to the pressure, this would result in an increase in the coefficient a.

A third cause, especially of the irregularity of the differences, lies in the state of the sample. The effect of lumpiness on the coefficient Q in the case of sample 3 has already been considered. By making determinations on the sample by both methods before and after removal of the lumps, it was found that the effect of the lumps had been to increase the coefficient $\frac{A}{2}$ (static method) from $6 \cdot 0 \times 10^3$ to $7 \cdot 9 \times 10^3$, an increase of 32 per cent. The coefficient a (dynamic method) had been increased from $11 \cdot 5 \times 10^3$ to $12 \cdot 4 \times 10^3$, an increase of 8 per cent. The static method may therefore be regarded as being more sensitive to the state of the sample than the dynamic method. A part of the discrepancy between the two coefficients may be attributed to this cause, for in spite of careful separation of the fibres, a small amount of residual lumpiness seemed unavoidable.

It is of interest to note that the percentage difference between the work done during compression and release by the static method was 55 per cent. for the sample in the lumpy state and 56 per cent. after it had been teased out. The difference was insignificant, in spite of the 32 per cent. increase in the value of the coefficient $\frac{A}{2}$.

(e) The arithmetical expression of compressibility.

The present study has been based on results obtained with the "Pendultex" instrument. It has been shown that when a 5 gm. sample is compressed from a volume of 103·3 c.c. to a volume v, where v is less than 75 c.c., the relation between the work done, W, in Kg. cm., and the volume v can be expressed by the equation

$$W = \frac{a}{v^2} - 5{\cdot}344 \times 10^{-9} \cdot a^2 - 0{\cdot}0947 \ldots\ldots\ldots\ldots\ldots (9)$$

for values of a from $4{\cdot}6 \times 10^3$ to $15{\cdot}0 \times 10^3$.

For the purpose of the present investigation, the coefficient a as defined by equation (9) has been taken as *the coefficient of resistance to compression.* The coefficient is derived from the equation

$$p = A \left(\frac{1}{v^3} - \frac{1}{v^3_0}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

assumed to represent the relation between pressure and volume, where a is an approximation to the coefficient $\frac{A}{2}$ of equation (5). In the more symmetrical form

$$p = A' \left(\frac{v^3_0}{v^3} - 1\right) \ldots\ldots$$

the dimension of A' is that of pressure in Kg./cm.^{2} As the dynamic method does not give the value of v_0, the latter form could not be adopted, and the dimension of a is, therefore Kg./cm.^{2}(c.c.)^3. For convenience the unit will simply be designated Kg. cm.^{7} per 5 gm.

If compressibility is given its usual definition of $\frac{1}{v} \cdot \frac{dv}{dp}$, it follows from equation (5) that it is equal to $-\frac{v^3}{3A}$, and at any volume is thus proportional to the reciprocal of the coefficient A, and hence of a. A high value of a indicates that a sample offers a high resistance to compression, and consequently has a low compressibility.

C. Technique Employed for the Measurement of Compressibility.

In the case of a fleece or a similar quantity of wool, the whole bulk was spread out evenly over a table and divided into ten or more zones. Staples were taken at random from each zone in succession until a composite sample of 90 gm. had been collected. The composite sample was divided into two, and from each portion a sub-sample of 15 gm. was made up by removing a few strands from each staple. The remaining 60 gm. was kept for the determination of other properties.

The number of crimps per inch of staple of each of the strands occurring in the two sub-samples was next determined, and a small portion of each of the strands was placed together in a bundle for fibre thickness measurements.

The two sub-samples were then washed in three changes of benzene at 40° C., after which the dust and vegetable matter were removed by teasing out the wool thoroughly. Finally the sub-samples were washed twice in distilled or rain water at 50° C.

The samples were put out to become thoroughly airdry. As a rule the relative humidity was in the region of 50 per cent., but during damp weather the wool was kept until several dry days had passed. When subsequently placed in a room maintained at 65 per cent. relative humidity, the samples therefore attained equilibrium under adsorption conditions. No attempt was made to dry the wool by heating, as this procedure reduces the affinity of wool for water.

The first set of measurements was made with the room maintained at 70 per cent. relative humidity and 70° F. (21·1° C.) temperature, but the conditions were later changed to 65 per cent. relative humidity and 70° F. temperature. A study was made of the influence of adsorbed water on the resistance to compression, and it was found that results obtained at 70 per cent. relative humidity could be converted to those at 65 per cent. relative humidity by multiplying by the factor 1·12. All results given in the present paper, therefore, refer to 65 per cent. relative humidity and 70° F. (21·1° C) temperature.

The samples had to be conditioned for at least fourteen days before constant results were obtained. At the end of this period 5 gm. was weighed out from each sample on a rough balance, and the resistance to compression determined in the "Pendultex" instrument five times in succession. After each determination the sample was removed from the compartment and teased out into as loose a mass as possible. Since the duplicates were compressed alternately, one sample was teased out while the other was being compressed, and no time was wasted. Finally the samples were weighed correct to 0·01 gm.

From the number of swings, the volume to which the sample had been compressed, and the weight of the sample, the coefficient a as defined was calculated.

In order to ensure that no additional damping forces had developed, the pendulum was set swinging with the compression compartment empty, at regular intervals, and the number of swings noted.

D. Discussion.

(a) *Methods of measuring compressibility.*

The determining factor in the choice of a method will be the nature of the information desired. The balloon method, and cylinder and piston methods such as employed by Larose (1934) and by the author give the compression and release curves separately. Consequently when the two curves have to be compared, for example by the work done during compression and the work done during removal of the compression force, any of these methods should prove suitable.

The author prefers his method of applying the load to that of employing a spring, since it obviates the calibration of the spring and makes for greater stability of the piston. By inverting the system and applying the weights to a cord drawn over a pulley, it should be a simple matter to adapt the method for studying the compressibility of wool immersed in various liquids.

Various authors seem to be agreed that the balloon method is more tedious than other methods, but one of the most serious drawbacks must be the fact that the calculation of each volume depends on the measurement of the volume at one pressure. From the author's own limited experience of the method, it was concluded that the accurate determination of the volume at one pressure is a matter of some difficulty, and it must be repeated for every test. The error in the measurement is reflected in the calculated value of all other volumes.

Cylinder and piston methods do not suffer from this disability. On the other hand, exception may be taken to these methods on the ground that the wool is compressed from one side only, whereas in the balloon method the compression takes place over the whole surface of the sample in contact with the balloon. This does not, however, remove the lack of uniformity in the packing of the sample at the lower pressures.

The " Pendultex " method may be regarded as giving the work done during application of the compressive force, although the rapidity with which the wool recovers from compression may influence the result if the recovery is not complete before the subsequent compression commences. The calculations, as developed in the present study, are based on a large number of successive compressions, and the final result may be regarded as being influenced by those compressions, in spite of the fact that it is expressed as the work done during the first compression.

The reduction in work done during successive compressions is smaller than with the static method, since it has been found that with the static method, the final constant cycle is attained the sooner, the more slowly the compression is performed. This effect is also apparent from the fact that the number of compressions made before the pendulum recorded a constant number of swings was of the order of 150, while with the static method constancy was attained after the ninth to the fifteenth cycle.

The accuracy of the " Pendultex " method at least equals that of other methods, according to published results, but its main advantage from the point of view of routine determinations is its rapidity. Each determination requires a few minutes, and several successive determinations can be made with a considerable resulting accuracy. With the static method, on the other hand, the author's own determinations took at least three hours each for a sample.

The advantage is obvious for routine measurements in wool production studies, where large numbers have ordinarily to be dealt with. Once the instrument has been calibrated, tables may be drawn up and the resistance to compression, as defined in the present study, obtained at a glance from the number of swings recorded. The manipulation of the instrument is simple and requires little training.

In this connection it must be pointed out that with any method of determination by far the greatest portion of the time required is taken up by the preparation of the sample for testing. With the method as described, the cleansing of a series of samples for even a small-scale experiment may require several weeks. When measurements of fibre thickness and crimping are made in addition, it is obvious that rapidity is an essential requirement in the method of measuring compressibility, if large-scale determinations are to be practicable.

On the score of reproducibility of results, and the rapidity with which determinations can be repeated, the first compression has been adopted as the basis in the present study, while other workers, employing static methods, have made their observations after repeated compression. With the static method the first compression is ill-defined and not easily reproducible, while with the dynamic method the same value may be obtained for successive determinations of the first compression of a teased sample.

In this connection it is to be noted that the value obtained by any method after constancy has been attained is dependent upon the way in which the sample has been teased out and packed into the compression compartment. It is, therefore, not sufficient to measure successive constant cycles, for reproducibility can be tested only by complete removal of the sample and re-insertion into the compression compartment. No record can be found that this fact has been borne in mind by previous workers.

(b) *The elastic behaviour of wool in bulk.*

From observations of the static method of compressing wool, the conclusion has been reached that initially the density of packing of the fibres is not uniform, but increases with successive cycles. It is therefore doubtful whether an initial cycle of compression by static methods is reproducible, and whether it has any meaning in the case of a loose mass of wool. As the pressure is raised, the packing tends to become more uniform, a fact which accounts for the observation that the volume tends to the same value at a high pressure for any cycle of compression. A part of the uniformity of packing is retained after each cycle until successive cycles are identical. Whether the final constant cycle depends on the maximum pressure to which the sample has been subjected, or upon the degree of compression, is a matter for investigation. If such is the case, the maximum pressure or degree of compression would have to be specified in each case.

Even during the final constant cycle it is clear that results obtained at low pressures should not be considered together with the later values, since on the release of the pressure, that portion of the sample nearest to the piston opens up to a greater extent than the rest of the sample. Such an effect may be expected to occur in the balloon method as well, where the friction of the fibres among themselves will oppose the release of the fibres in the centre of the mass. The result is that the wool acts like a spring at low pressures, a conclusion also reached by Schofield (1938), who stated that his sample obeyed Hooke's Law initially.

In considering the form of the curve relating pressure and volume, the compression of the sample may in the first place be regarded as consisting in the bending of individual fibres. Owing to the contacts between the fibres, the unit which bends is not primarily a whole fibre, but the element between adjacent contacts. Such elements are, however, connected to one another within a fibre, so that a certain amount of straightening, or even stretching, of the fibre may be expected. As the volume diminishes, the number of contacts and the number of elements increase, with a corresponding diminution in the length of the bending elements. The force necessary to bend an element by a certain amount is inversely proportional to the cube of its length. The two effects, viz., the increase in the number of elements and the diminution in their length will rapidly increase the resistance of the sample to compression, and hence the slope of the pressure-volume curve.

Against this a certain amount of slippage of the fibres over one another must be regarded as a possibility, and Pidgeon and van Winsen (1934) go so far as to say that " the pressure-volume relation of a mass of fibre is ultimately dependent on the ease with which they slip over one another ".

Besides bending and stretching of the fibres, some torsion may be expected in view of the twist present in wool fibres. With such a complexity of factors in operation, it is doubtful whether a complete theoretical derivation of the relation between pressure and volume is possible.

Theoretical Considerations.—In spite of the complexity of the factors an attempt will be made to approach the problem with the aid of simpler cases. Two regular geometrical patterns will first be presented and simple bending only will be considered.

When an element dx of a bar is bent into an arc of radius R,

Bending moment......$M = \dfrac{iY}{R}$ ....(13)

Energy in length dx, $dE = \dfrac{iY}{2R^2} \cdot dx$....................(14)

where i is the " moment of inertia of cross-section ", and Y is Young's modulus.

(i) *Closed Solenoid.*—Suppose that a fibre of length l and diameter d forms a *closed* solenoid of radius R. Then if v is the volume of the solenoid,

$$R = \frac{2v}{ld} \qquad \text{(15)}$$

and from (14), since R is constant,

$$E = \frac{iYl}{2R^2} = \frac{iYl^3.d^2}{8v^2}$$

Providing the shape of the solenoid does not alter, a pressure perpendicular to the axis of the solenoid will produce a diminution in volume similar to that caused by an axial torque. Considering the wool sample to consist of a large number of such solenoids, the pressure is given by

$$p = -\frac{dE}{dv} = \frac{iYl^3 \cdot d^2}{4v^3}$$

For a fibre of circular cross-section, $i = \dfrac{\pi . d^4}{64}$, and if m is the mass of the material and ρ the specific gravity

$$m = \frac{\pi d^2 l \rho}{4}$$

Hence

$$p = \frac{Ym^3}{4\pi^2\rho^3 v^3} \qquad \text{(16)}$$

Now equation (16) is identical to equation (5) except for an additive constant, which may be explained by the fact that wool fibres are bent and looped initially without stress, and in addition have in the mass a pressure among themselves in the absence of applied pressure.

It is to be noted that according to equation (16), the relation between pressure and volume depends on the total mass but not on the fibre diameter.

Although the solenoidal form is only a simple approximation to the complicated forms taken by the fibres, it is not altogether an unreasonable one. In this connection the observations of Woods (1935) are of some interest. Woods found that when single fibres were immersed in water, they relaxed and formed loops corresponding in a regular way to the original crimping. In the fibre mass a large number of such loops may be expected, and more will be formed when the mass is compressed and the fibres bend round one another, and the loops may be regarded as elementary solenoids.

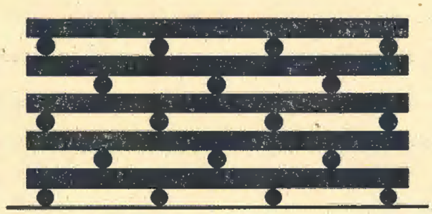
Equation (16) was derived from the condition of equation (15) viz., that the radius of curvature is proportional to the volume. The total length of fibre in a mass is of the order of 10^5 cm. per gm., and in consequence there will be a large number of fibre elements and loops. If it be assumed that for such a large number, the mean radius of curvature is proportional to the volume occupied by the mass, equation (16) giving the pressure as proportional to the inverse cube of the volume may be applied to the fibre mass (taking initial conditions into account).

When equation (15) for the solenoid is applied to a mass of fibres of 20μ diameter occupying 20 c.c. per gm., R becomes $0 \cdot 04$ cm. These values apply to a *closed* solenoid and are only approximate for the wool, but it is interesting to note that Woods gives the values $0 \cdot 029$ cm. and $0 \cdot 040$ cm. for the natural radius of curvature of a dry and a wet fibre of 20μ diameter respectively, values which are of the same order of magnitude as those calculated.

The above reasoning could no doubt be extended to the case of a yarn, where the fibres are twisted round one another, and their form approximates to solenoids with equally spaced turns. Provided the spacing between the turns remains constant, the radius of curvature of such a solenoid is proportional to its volume.

It is to be noted that simple bending only has been considered. In view of the twist already present in the fibres, further twisting must be considered as likely.

(ii) *Pile of rods*.—As a second geometrical pattern consider an arrangement as in the accompanying figure, where a large number of weightless rods have been piled on one another.



Let $n =$ the number of rods,
$N =$ the number of layers,
$L =$ the length of a rod,
$d =$ the diameter of a rod,
$l =$ the total length of the rods,
$= nL$.

The volume occupied by the pile is

$$v_0 = NdL^2 \qquad (17)$$

The distance $2b$ between adjacent contacts of two layers is

$$2b = \frac{NL}{n} \qquad (18)$$

or from (17), $2b = \dfrac{v_0}{ld}$ $\qquad (19)$

Let a uniform pressure be applied to the top of the pile. At each point of contact the rods are bent by an amount y_1. The new volume becomes

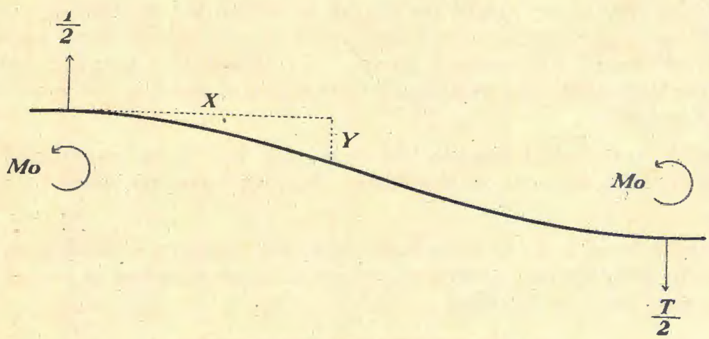
$$v = v_0 - NL^2 y_1.$$

Hence $y_1 = \dfrac{v_0 - v}{NL^2}$

or from (17) $y_1 = \dfrac{d(v_0 - v)}{v_0}$ $\qquad (20)$

Let forces T act at the midpoint and ends of the portion $2b$. Then since the two halves are symmetrical, one half of length b may be considered.

At each point of contact, a force $\frac{T}{2}$ may be considered to be effective on the length b.



The bending moment at a distance x along the rod is given by $(\frac{Tx}{2} - M_0)$, where M_0 is a bending moment caused by the forces on the extension of b beyond the point of contact. But from (13), the bending moment is balanced by a moment $\frac{iY}{R}$.

Hence $\dfrac{iY}{R} = \dfrac{Tx}{2} - M_0.$

At $x = \dfrac{b}{2}$ there exists a point of inflexion where $\dfrac{1}{R} = 0.$

Hence $M_0 = \dfrac{Tb}{4}$, and if R is large

$$\frac{iY}{R} = iY \cdot \frac{d^2y}{dx^2} = \frac{T}{2} \cdot \left(x - \frac{b}{2}\right) \qquad (21)$$

$$\therefore\ iY \cdot \frac{dy}{dx} = \frac{T}{2} \cdot \left(\frac{x^2}{2} - \frac{bx}{2}\right), \text{ since } \frac{dy}{dx} = 0 \text{ when } x = 0,$$

$$iY . \int_o^{y_1} dy = \frac{T}{2} \int_o^b \left(\frac{x^2}{2} - \frac{bx}{2}\right) dx$$

$$\text{i.e. } iYy_1 = -\frac{Tb^3}{24} \qquad \ldots\ldots(22)$$

Now the number of contacts between two layers is equal to the square of the number of rods in a layer, and hence equals $\left(\frac{n}{N}\right)^2 = \frac{L^2}{4b^2}$, from (18).

Hence the total force on a layer $= T . \frac{L^2}{4b^2}$

$$\text{and the pressure } p = \frac{T . L^2}{L^2\ 4b^2}$$

$$= \frac{T}{4b^2}$$

$$= \frac{6iYy_1}{b^5}, \text{ from (22).}$$

Substituting for y_1 from (20) and for b from (19),

$$p = \frac{192 iYd^6l^5}{v^6_0} . (v_0 - v).$$

Putting $i = \frac{\pi d^4}{64}$, and $m = \frac{\pi d^2 l \rho}{4}$, where m is the total mass, and ρ the specific gravity,

$$p = \frac{3072 . Y . m^5}{\pi^4 \rho^5 v^6_0} . (v_0 - v) \qquad \ldots\ldots(23)$$

The same result may be obtained by considering the energy of the system. From (14), the energy in a length dx is

$$dE = \frac{iY}{2R^2} . dx$$

$$= \frac{T^2}{8iY} . \left(x - \frac{b}{2}\right)^2 . dx, \text{ from (21).}$$

The energy in a length b is

$$E = \frac{T^2}{8iY} \cdot \int_o^b \left(x - \frac{b}{2}\right)^2 . dx$$

$$= \frac{T^2 b^3}{96 iY}.$$

Hence the energy in a total length l is

$$E = \frac{T^2 l b^2}{96 i Y}$$

$$= \frac{6 i Y l y^2_1}{b^4}, \text{ from (22)}$$

$$= \frac{6 i Y l d^2}{b^4 . v^2_0} . (v_0 - v)^2, \text{ from (20)}$$

$$= \frac{96 i Y l^5 d^6}{v^6_0} . (v_0 - v)^2, \text{ from (19)}$$

$$\text{Hence } p = -\frac{dE}{dv} = \frac{192 i Y l^5 d^6}{v^6_0} . (v_0 - v)$$

$$= \frac{3072 Y m^5}{\pi^4 \rho^5 v^6_0} . (v_0 - v) \qquad (23)$$

Again, as in the case of the solenoids, the relation between pressure and volume depends on the mass but not on the diameter of the rods.

For small compressions of the pile, the pressure is proportional to the reduction in volume, and it has been pointed out that the same is true for small compressions of a wool sample. It is obvious that the result obtained can be considered only in the light of small compressions, for beside the approximation made, the distance $2b$ between contacts is regarded as constant, while in practice more contacts will be formed as the fibres are bent.

From (19), the distance b is initially given by

$$b = \frac{v_0}{2ld}$$

Assuming that in the case of wool the relation holds for large degrees of compression, and defining an element length as b, the relation may be written

$$b = \frac{v}{2ld} = \frac{v}{m} . \frac{\pi d \rho}{8} = 0{\cdot}51 \frac{v}{m} . d \qquad (24)$$

while from (15)

$$R = \frac{2v}{ld} = \frac{v}{m} . \frac{\pi d \rho}{2} = 2{\cdot}04 \frac{v}{m} . d$$

where m is the mass and ρ the specific gravity of wool. For a given density of packing, i.e., for a given value of $\frac{v}{m}$, the element length and the radius of curvature are proportional to the fibre diameter.

The length of a fibre element between contacts in the case of a sample of 20μ diameter occupying 20 c.c. per gm. is then 0·02 cm. This value is one-quarter of the radius of curvature obtained by analogy with a solenoid. Although an exact comparison of the two quantities is hardly valid in view of the different grounds on which the formulae have been based, the two quantities, element length and radius of curvature, may reasonably be regarded as being of the same order of magnitude. At a density of 20 c.c. per gm. the element length is about ten times the fibre diameter, and at 10 c.c. per gm. about five times the fibre diameter.

The smallest volume which the pile can occupy occurs when $2b = d$. Then

$$v_0 = ld^2 = \frac{m}{\rho} \cdot \frac{4}{\pi}$$

The smallest volume is thus $\frac{4}{\pi}$ times the volume of the material, i.e., the ratio of the area of a square to the area of the inscribed circle. Since wool fibres are arranged in all directions, the limiting volume will be even greater. This should be borne in mind when assigning a value to the constant v' of equation (5a) and equation (11), pages 123 and 124.

(iii) A less regular arrangement is required for the following treatment, in which the numerical factor has not been determined.

Suppose the wool to be compressed in a vertical direction in a container of unit area of cross-section, and let the depth at any instant be v, so that the volume occupied by the wool is v. It will be assumed that the elements of length b, formed between adjacent contacts, are the units which bend when the mass is compressed.

Let c be the vertical height occupied by an element, and consider a layer bounded by two horizontal planes a distance c apart. The layer will be one element thick, and since its area is unity, an increase in pressure dp will be equal to the increase dF in the force on each element, multiplied by the number of elements in the layer. If l is the total length of fibre, the total number of elements is $\frac{l}{b}$, and the number in the layer will be $\frac{c}{v} \cdot \frac{l}{b}$

Hence $dp = \frac{lc}{vb} \cdot dF$

When a bar of length b is bent by an amount dy, then

$$dF = \frac{kiY}{b^3} \cdot dy \qquad (25)$$

where k is a numerical factor depending on the conditions of bending, i is the "moment of inertia of cross-section", and Y is Young's modulus. The relation is commonly employed in the determination of Young's modulus by the bending of a bar, and is similar to equation (22).

Hence $dp = \frac{kiYlc}{vb^4} \cdot dy$.

Each layer of thickness c will be reduced in thickness by an amount equal to dy, so that the reduction in volume dv is given by

$$dv = -\frac{v}{c} \cdot dy$$

and $dy = -\frac{c}{v} \cdot dv$.

Hence $dp = -\frac{kiYlc^2}{v^2.b^4} \cdot dv$.

Since the elements are arranged in all directions, the mean value of c^2 may be taken as being equal to the mean value of $\frac{b^2}{3}$ so that

$$dp = -\frac{kiYl}{3v^2b^2} \cdot dv.$$

Assuming equation (24) for the pile of rods, i.e. $b = \frac{v}{2ld}$,

$$dp = -\frac{4kiYl^3d^2}{3.v^4} \cdot dv$$

For a bar of circular cross-section, $i = \frac{\pi.d^4}{64}$, hence

$$dp = -\frac{4k\pi Yl^3d^6}{192.v^4} \cdot dv$$

$$= -\frac{4kYm^3}{3\pi^2\rho^3} \cdot \frac{dv}{v^4}$$

where m is the mass, and ρ the specific gravity of wool.

Integrating,

$$p = \frac{4kYm^3}{9\pi^2\rho^3} \cdot \left(\frac{1}{v^3} - \frac{1}{v^3_0}\right) \qquad \text{(5c)}$$

and by comparison with equation (5),

$$A = \frac{4kYm^3}{9\pi^2\rho^3} \qquad \text{(26)}$$

Now the assumptions made may be regarded as reasonable provided there are a sufficiently large number of elements. Taking 1 gm. of wool of 20μ diameter compressed into 10 c.c. per gm., the highest density employed in the present study, the mean element length may be calculated, by analogy with a pile of rods [equation (24)], to be $0 \cdot 010$ cm. Since the total length of fibre is $2 \cdot 45 \times 10^5$ cm., the total number of elements will be $2 \cdot 5 \times 10^7$ per gm. of wool, and the number in the layer of unit area of cross-section and thickness c approximately 10^4.

Besides depending on the bending conditions, the constant k may be regarded as including a number of factors which will be briefly considered.

1. The mean element length has been taken to be equal to $\frac{v}{2ld}$ by analogy with a pile of rods. Since wool fibres are arranged in all directions, it is probable that the expression should be multiplied by some factor. For example, again consider the wool to be enclosed in a container of unit area of cross-section and depth v, and consider a spherical particle of diameter d equal to that of the fibres, moving in a vertical direction. The probability that it will strike a fibre is the area presented by the fibres projected on a horizontal plane, divided by the total area of cross-section of the container, which is unity. Since a collision will occur when the centre of a particle comes within a distance d of the axis of the fibre, the area presented by the fibres will be $2ld$, and the mean square of the projection of this area on a horizontal plane will be $\frac{2}{3} \cdot 4 \cdot l^2 \cdot d^2$. If then in travelling a distance v, the

particle strikes a fibre $\sqrt{\frac{2}{3}}\cdot 2ld$ times, then the root mean square distance between successive collisions will be given by $\sqrt{\frac{3}{2}} \cdot \frac{v}{2ld} = 0.61 \cdot \frac{v}{ld}$. If a fibre is regarded as the path of such a particle, the R.M.S. distance between successive contacts is thus $0\cdot61\cdot\frac{v}{ld}$. (The mean square of the projection of the area has been considered, since the substitution is made for b^2). While this calculation is only approximate, since it considers the mean value of b, it confirms the validity of the expression $\frac{v}{2ld}$ for the calculation of the element length, and shows that the resultant error takes the form of a factor which is dimensionless.

2. In the derivation a mean value for the element length b has been employed, a procedure which is valid only when b is constant. The length b will, however, follow some law of distribution, and a rigorous calculation would have to take the nature of the distribution and the extent of the variation into account. For example, in the expression $\frac{1}{b^3}$ occurring in the relation between force and deflection, it is obvious that the contribution of the shorter elements, especially those approaching d, will be large compared to the contribution of longer elements, and may reach enormous values.

3. The " moment of inertia of cross-section ", i, depends upon the fibre diameter, which varies considerably within a sample, and the variation would have to be taken into account in calculating the deflection of an element. It is further to be observed that the cross-section of a wool fibre more nearly resembles an ellipse than a circle. Assuming the fibre to adjust itself so as to offer the least resistance to bending, an approximate calculation suggests that the value of i calculated by assuming circularity should be multiplied by the ratio of the minor to the major axis of the ellipse.

4. The equation

$$dF = \frac{kiY}{b^3} \cdot dy$$

assumes the deflection dy and the force dF to be perpendicular to the direction of the fibre, which will not in general be the case.

The constant k may, however, on the whole be regarded as dimensionless, and although its theoretical derivation may not be practicable, it may be obtained experimentally if the value of Y can be determined independently of the resistance to compression A. In the case of a straight fibred wool of the coarser type the measurement of Y may be practicable, e.g., by Searle's method, but in the present study no such wools were encountered.

Values of A obtained by the static method (Table 16) ranged from $6\cdot4 \times 10^3$ to $23\cdot4 \times 10^3$ Kg. cm.^{7} per 5 gm. Assuming a value of, say 10 for k, Y is found to range from $2\cdot5 \times 10^8$ dynes/cm.^{2} to $9\cdot2 \times 10^8$ dynes/cm.^{2}.

For Young's modulus as obtained by extension Speakman (1930) gives $3\cdot55 \times 10^{10}$ dynes/cm.^{2} for Cotswold wool at 65 per cent. relative humidity, while the present author has found values ranging from $1\cdot5 \times 10^{10}$ dynes/cm.^{2}

for Merino wool to $3 \cdot 2 \times 10^{10}$ dynes/cm.^{2} for mohair at 70 per cent. relative humidity (van Wyk, 1932). In view of the structure of the fibre, the value of Young's modulus obtained by bending is not likely to be the same as that obtained by stretching, a point which will be considered later in connection with the effect of adsorbed water.

Assuming b to be constant and equal to $\dfrac{v_0}{2ld}$, the relation becomes

$$p = \frac{4kYm^3}{3\pi^2\rho^3v^2_0} \cdot \left(\frac{1}{v} - \frac{1}{v_0}\right) \qquad (27)$$

i.e., for compressions which are so small that no new contacts are formed, the pressure varies as the inverse first power of the volume. This result is in contrast to that obtained by the regular arrangement of rods, in the case of which the pressure was found to vary directly with the reduction in volume.

The above treatment of the problem requires that there shall be no slippage of the fibres over one another, a point to be considered later. It also assumes the density of packing to be uniform throughout the mass, explaining why the inverse cube law does not hold at small degrees of compression.

The coefficient A of equation (5) has been found to contain the mass but not the diameter of the fibres. Hence in all three cases considered, the relation between pressure and volume contained the mass but was independent of the fibre diameter.

It is to be noted that the above considerations take no account of the hysteresis loop formed between the compression and release curves. It may appear remarkable that after successive cycles the wool mass should return to its volume at zero pressure in spite of the considerable hysteresis loop. The same phenomenon occurs in the extension of single fibres, but in the case of the compression of a mass of wool the factor of friction between fibres is added to the lag of the fibres in recovering from strain. The greater part of the recovery of the mass to its original volume takes place on removal of the last traces of the pressure. During this stage the sample opens up rapidly and unevenly, and the density of packing is far from uniform. It is over the same range of low pressures that the compression curve follows Hooke's law.

It has been shown that in practice the pressure may be regarded as a linear function of the inverse cube of the volume. M. and J. Eggert (1925) appear to have found different values for the index of the volume. The author has, unfortunately, not been able to gain access to their publication, and has had to rely for his information on the quotations of other authors. It has to be emphasised, however, that the inverse cube law holds only after repeated compression by the static method, and the results at low pressures do not follow the law. The use of the quantity v_0, suggests that these workers used the results obtained at both low and high degrees of compression. Even assuming that these factors are taken into consideration, however, the exact value 3 may not be the best-fitting one in all cases, but an additional constant is introduced when the index is deduced from the observational data. Now the conviction has already been expressed that compressibility measurements do not justify the use of an equation involving more than two unknown constants, and different values of the index may easily be found as a result of experimental error. The other constants are at the same time affected, so that the resultant error may far exceed any that may arise from the assumption of the inverse cube law.

The Eggerts reduced the number of their unknown constants to two by taking for one constant the value of v_0, the volume at zero applied pressure. If, as the present study has suggested, the results at low pressures do not " fit in " with the results at higher pressures, this procedure cannot be considered as entirely justifiable. In the second place, it is a matter of extreme difficulty to assess when the balloon has been suitably filled. In the present study the volume at zero pressure after repeated compression was found to be at least 30 c.c. per gm. Authors who have employed the balloon method appear to have placed samples of 4 to 5 gm. in balloons having capacities of 80 to 100 c.c. It is almost certain that the wool must have exerted some pressure on the balloon initially and the volume at zero pressure must in such cases be regarded as rather arbitrary.

In conclusion it must be observed that several interesting features of the pressure-volume relation were encountered, which required explanation The scope of the study had, however, to be borne in mind, and only those features relevant to the study were investigated.

(*c*) *The arithmetical expression of compressibility.*

Since the first object in any investigation must be to express the quantity being measured in arithmetical terms, various authors have placed different interpretations on their results. M. and J. Eggert, assuming the relation

$$\phi^{\gamma} (\pi + \pi_0) = \pi_0 \ . \ 10^{\gamma}$$

where π was the pressure, and ϕ ten times the ratio of the volume to the volume at zero pressure, regarded π_0 as an expression of the softness (*Weichheit*) of the wool, and γ as a measure of its pliability (*Geschmeidigkeit*).

Now wool fibres are bent and looped naturally without stress, and the " latent " pressure, π_0, is presumably the pressure which would have been necessary to bring the fibres into this state had they been straight initially. In the fibre mass, however, the fibres are in state of strain owing to the fact that they prevent one another from taking up their normal form. Thus in the absence of applied pressure there nevertheless exists a pressure among the fibres, and this pressure will influence the determination of π_0, the " latent " pressure of the wool. It would appear, therefore, that this " latent " pressure must partly depend on the extent to which the wool was teased out prior to compression.

Subjecting the wool to repeated cycles of compression and release will not improve the position, since it has been found with tests by both the static and dynamic methods that the pressure-volume relation after repeated compression depends as much on the extent to which the wool has been separated and on the method of insertion into the compression comparment as does the relation of the first compression. It has been suggested that owing to the difficulty of assessing when the balloon has been suitably filled, the wool initially exerts some pressure on the balloon. In such a case, the value of the " latent " pressure of the wool obtained by the balloon method must be regarded as rather arbitrary.

When $\gamma = 3$, the Eggert equation becomes equation (5) of the present study, and

$$\pi_0 . v^3_0 = A$$

so that
$$\frac{d\pi_0}{\pi_0} = -\frac{3.dv_0}{v_0}.$$

It is thus seen that π_0 will be very sensitive to errors in v_0.

As regards the quantity γ as a measure of pliability, it has been found that the value may be assumed to be 3 in all cases. The effect of experimental errors on the determination of γ will be to produce a negative correlation between γ and π_0. It is significant that the Eggerts, according to Sommer (1936), actually found a high negative correlation.

It is recommended that the value 3 should be assumed throughout, as in equation (5) or (5a), and that the constant $\frac{A}{m^3}$ be taken as a measure of the resistance to compression of the sample. Or equation (11) may be applied, taking a common value for Q which may be decided upon after the results have been obtained. In that case the coefficient P may be regarded as a measure of the elastic characteristics of the sample.

An an illustration, consider the effect of the dye on the two samples of yarn tested by Larose (1934). Table 17 gives the coefficient A of equation (5a) and the coefficient P of equation (11).

TABLE 17.

The effect of the dye on the two samples of yarn tested by Larose.

Sample.	A	Ratio.	P	Ratio.	Ratio $\frac{A}{P}$
1 (undyed)............ 1 (dyed)..............	$10 \cdot 71 \times 10^6$ $22 \cdot 78 \times 10^6$	2·13	210·0 401·9	1·91	$5 \cdot 10 \times 10^4$ $5 \cdot 67 \times 10^4$
2 (undyed)............ 2 (dyed)..............	$3 \cdot 13 \times 10^6$ $7 \cdot 59 \times 10^6$	2·43	60·7 152·9	2·52	$5 \cdot 16 \times 10^4$ $4 \cdot 96 \times 10^4$
Mean..................		2·28		2·22	$5 \cdot 22 \times 10^4$

According to both coefficients, the effect of the dye has been practically the same for the two samples, in spite of the fact that the compressibility of one sample is three times that of the other. The table clearly illustrates the value of applying equations (5a) or (11) for finding the effect of the treatment, and for assessing the relative degrees of compressibility of the two samples.

Another illustration of the value of the equations is given by Larose's measurements on different masses of the same sample. The samples of 3·26 gm. and 4·34 gm. showed good agreement in the ratio of mass to volume at each pressure but the 2·20 gm sample showed agreement at the higher pressures only. Fitting equations (5a) and (11) to the data, the results shown in Table 18 are obtained. The constant B is equivalent to $\frac{A}{(v_0 - v')^3}$ of equation (5a).

TABLE 18.

The coefficients of equations (5a) *and* (11) *when fitted to Larose's determinations on different masses of the same sample.*

Mass.	Equation (5*a*).		Equation (11).		
	$\frac{A}{m^3}$	B	P	Q	R
2·20 gm................	$1 \cdot 98 \times 10^5$	77	138	13·1	397
3·26 gm................	$2 \cdot 23 \times 10^5$	177	144	13·6	528
4·34 gm................	$2 \cdot 15 \times 10^5$	168	142	13·5	521

The discrepancy in the smallest sample has reduced the coefficient $\frac{A}{m^3}$ by about 10 per cent. and the coefficient P by about 4 per cent. These differences cannot be considered serious, and can be attributed to the difficulty of handling small samples. A large discrepancy occurs, however, in the coefficients B and R of the 2·20 gm. sample, explaining why Larose found agreement only at the higher pressures. The difference points to either a systematic error in the measurement of the pressure, or to a difference in the packing into the compression compartment. Since the results given are the mean of several determinations, a systematic error in the measurement of the pressure could hardly have escaped notice, and the second of the two causes suggested may therefore be regarded as probable.

The main problem in the arithmetical expression of compressibility lies in the basis on which different wools have to be compared. Thus, samples may be compared on the basis of equal masses, of equal lengths of fibre, or of equal values of v_0, the volume at zero applied pressure. Now the coefficient A of equation (5) includes the cube of the mass, and the methods of equal masses and of equal lengths of fibre will probably yield coefficients differing in the sixth power of the fibre diameter, so that the fibre diameter can be eliminated and the two coefficients reduced to a common value.

Had the volume v_0 at zero applied pressure been well-defined and reproducible, this would undoubtedly have been the best basis for comparing different wools, at least theoretically. It is the basis adopted by the Eggerts (1925), and it will be shown to be the most suitable for evaluating the work done during compression. Where the density of packing of the fibres has already been increased, as in the case of felt, cloth and even yarn, such a method is practicable, but the lack of reproducible initial conditions in the case of a loose mass, and the difference between the pressure-volume relation at low and high degrees of compression, as has been stressed, precludes the accurate comparison of wools on the basis of equal values of v_0. On such a basis the most satisfactory method would probably be to calculate v_0 from the relation fitted to the later values.

With the dynamic method employed in the present study, samples can be compared only on the basis of equal masses, and this basis has been adopted in the present study. When the compressibility of a sample is estimated by hand, it is probable that the judge will grasp a constant volume, i.e., a constant value of v_0. On the other hand, the manufacturer employs the weight as a fundamental quantity. Which of the two methods of comparison conforms to manufacturing practice remains a matter for investigation.

The method of expressing compressibility by means of the work done during compression.—Some previous investigators have attempted to express the compressional characteristics of a sample by means of the work done during compression, and have compared different samples by the work done in compressing the samples between the same limits of pressure. Since the work done is an integration of the function *p.dv* between volume limits, the question arises as to whether the limits employed are justifiable.

When different masses of the *same* sample are to be compared it is clearly necessary that the limits should be so chosen that the work done is proportional to the mass. The work done per unit mass will then be the same whatever the mass chosen for the determination. Since the ratio of mass to volume is the same for all masses of the same sample at any pressure, and therefore at zero pressure, the above condition will be fulfilled when the limits chosen are v_0, the volume at zero pressure, and some fraction of v_0 which is the same for all masses. For the same sample, such volume limits will also be given by equal pressure limits.

When two different samples are to be compared on the basis of equal masses, the ratio of mass to volume is not the same for the two samples, and the theoretical and practical significance of a comparison of the work done between volume limits given by equal pressure limits is not quite clear. When the lower limits are v_0 and the upper limits the same fraction of v_0 for the two samples, the ratio of the work done per unit mass is the ratio of the coefficients of resistance to compression [e.g., A in equation (5)], multiplied by a factor involving the respective values of v_0. Thus on the basis of equation (5), the ratio of the work done per unit mass is given by

$$\frac{W_1}{W_2} = \frac{A_1}{A_2} \cdot \left\{\frac{v_{0\,(2)}}{v_{0\,(1)}}\right\}^2 \qquad \text{(28)}$$

As a practical illustration, the data of Larose may again be utilised. When equations are fitted to the data, the work done may be evaluated between any limits of volume. The results are given in Table 19.

TABLE 19.

The work done, in arbitrary units, in compressing four samples of yarn at 50 per cent. relative humidity, as calculated for various limits of volume from data by Larose.

Sample.	Volume limits given by— Equal Limits of Pressure. W	Equal Limits of Pressure. $\frac{W}{(v_0-v)}$	Equal Limits of $\frac{v}{v_0}$ W	Equal Limits of $\frac{v}{v_0}$ Ratio Dyed/Undyed.
1 (undyed)	4,016	109	4,658	1·67
1 (dyed)	4,196	99	7,770	
2 (undyed)	3,288	112	2,282	1·62
2 (dyed)	3,780	108	3,700	

Limits of volume given by the same limits of pressure give the work done in the same order as the coefficients of resistance to compression, but not in the same relative magnitude (Table 17). The ratio $\frac{W}{v_0}$ or $\frac{W}{v_0 - v}$, is the equivalent result obtained from the area of the curve when pressure is plotted as a function of $\frac{v}{v_0}$, a method employed by Winson. This procedure is clearly not sufficient for comparing the samples, when equal pressure limits are employed, as the values are practically the same.

When the integration is performed between the same limits of $\frac{v}{v_0}$ the ratio of the dyed to the undyed values is the same for the two samples, although somewhat smaller than the ratios given by coefficients A and P on account of the smaller volume occupied by the undyed sample at zero pressure in each case.

When the samples are compared on the basis of the same limits of volume, the ratio of the work done, assuming the samples to follow equation (5), is equal to the ratio of the coefficients A, multiplied by a factor which contains the cubes of the volume at zero pressure. This is the method employed in the " Pendultex " apparatus, but it is not satisfactory in the case of static methods, since the volume occupied by one sample at the highest pressure employed may still be greater than the volume occupied by another sample of equal mass at zero pressure.

When it is desired to compare samples by the work done in compressing them by static methods, the most profitable limits of volume appear to be those given by the same limits of $\frac{v}{v_0}$. It is recommended that the calculated value of v_0 should be employed for this purpose.

The method of expressing resilience.—Larose considered that the compression curve alone was sufficient to specify the degree of harshness of his samples. The coefficient P, for example, as given in Table 17, had the values 60·7, 152·9, 210·0 and 401·9 when the samples were placed in increasing order of harshness as tactually estimated.

Henning regarded the compression curve as of supreme importance, arguing on the basis that the wool having a superior compression curve would also have a superior release curve. This argument may be valid in the case of untreated wools, but the possibility exists that the extent of the hysteresis may be altered by chemical treatment. In this connection attention must be drawn to the work of Speakman, Stott and Chang (1933) on the load-extension curves of wool fibres immersed in acids and alkalis and in water at various temperatures.

While the compression curve may readily be represented by an analytical equation, this is hardly practicable in the case of the release curve, and the only convenient method of studying the release conditions and the hysteresis effect is by means of the work done. Thus Winson took the area of the hysteresis loop as a measure of resilience, stating that this quantity appeared to correspond to the trade impression of " springiness ". As other authors have pointed out, the use of the term " resilience " cannot be justified in this sense, nor can its use by Schiefer (1933) as the difference in the work done between the compression and release operations expressed as a ratio of the work done during compression. Resilience is usually defined as the energy stored in a strained elastic body. Granting the importance of the hysteresis, attention must, however, again be drawn to the questionable practice of evaluating the work done between volume limits given by equal limits of pressure.

In this connection consider the work done during compression and release in the case of four yarns tested by Larose, as given in Table 20.

TABLE 20.

The work done, in arbitrary units, in compressing and releasing the four samples of yarn tested by Larose at 50 *per cent. relative humidity.*

Sample.	Work Done.			
	Compression.	Release.	Difference.	Percentage Difference.
				Per Cent.
1 (undyed)........................	4,016	1,872	2,144	53
1 (dyed)........................	4,196	1,956	2,240	53
2 (undyed)........................	3,288	1,744	1,544	47
2 (dyed)........................	3,780	1,612	2,168	57

The work done as given in Table 20 was evaluated between volume limits given by equal pressure limits, the form in which the data were given. The values for difference and percentage difference are too close together to be of practical value, and the results suggest that no difference exists between the samples as regards percentage hysteresis. According to Winson, three of the samples would have to be classed as having the same springiness. It has been demonstrated that the most profitable method of evaluating the work done during compression is by means of volume limits given by the same limits of $\frac{v}{v_0}$. It is reasonable to expect that the same applies to the hysteresis. Such a method corresponds to that successfully employed by Speakman, Stott and Chang (1933) in their work on the extension of single fibres, since each fibre was extended by 30 per cent. of its original length.

The value of the compression curve has been demonstrated, and applications will be found in Part II of this paper, but the practical significance of the hysteresis effect needs investigation. Such determinations as have been made can be regarded as of little value owing to the failure of the investigators concerned to employ the correct limits of volume in evaluating the work done.

APPENDIX A.

Method employed for fitting the equation

$$p = Pe^{\frac{Qm}{(v-v')}} - R \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Fitting was accomplished in two stages. First approximate values of the constants P, Q and R were derived, and these were next improved upon by successive approximation.

1. *Approximate values.*

Pressure was plotted as a function of $\dfrac{m}{(v - v')}$, and a smooth free-hand curve was drawn through the points. From the curve, pressure was read off at equal intervals of $\dfrac{m}{(v - v')}$, and designated p_0, p_1, p_2 . . . For the observations to fit the equation a linear relation had to exist between p_n and p_{n+1}, a relation expressible in the form

$$p_n + R = e^{Qx} . (p_{n+1} + R),$$

where x was the interval between successive values of $\dfrac{m}{(v - v')}$. The constants Q and R could thus be determined, and P could be calculated from the original observations.

2. The constants so obtained were regarded as approximations P', Q' and R' such that

$$P = P' + a$$
$$Q = Q' + \beta$$
$$R = R' + \gamma$$

Now if $p = f\left(P,Q,R, \dfrac{m}{(v - v')}\right)$ where p was the observed value and p' was the value of p given by the approximate constants, the values of a, β and γ could be obtained by making $\Sigma\left(a . \dfrac{df}{dP} + \beta . \dfrac{df}{dQ} + \gamma . \dfrac{df}{dR} + p' - p\right)^2$ a minimum. (See Scarborough, 1930).

It was found necessary to repeat the calculation at least three times before sufficiently small values for the corrections a, β and γ were obtained.

With the static cylinder and piston method employed in the present study, the pressures were given by weights and were therefore determined with negligible error, The equation was consequently fitted by regarding p as the independent variable, and taking

$$\frac{m}{(v - v')} = f(P,Q,R,p).$$

For the satisfactory fitting of any equation it is necessary that one of the variables should have been measured with negligible error. This condition was not fulfilled by the results published by some previous investigators, since certain measurements entered into the evaluation of both pressure and volume. Both variables must therefore be considered as having been measured with sensibly the same error.

APPENDIX B.

Table of the indefinite integral $\int e^{\frac{1}{x}} . dx$, for values of x from $0 \cdot 4$ to $2 \cdot 5$ at intervals of $0 \cdot 001$, employed for evaluating the work done in compressing a wool sample, when the relation between pressure and volume is given by

$$p = Pe^{\frac{Qm}{(v - v')}} - R \text{.................................} (11)$$

x	0	·001	·002	·003	·004	·005	·006	·007	·008	·009
0·40	− 2·6236	− 2·6114	− 2·5993	− 2·5873	− 2·5754	− 2·5636	− 2·5518	− 2·5401	− 2·5285	− 2·5169
0·41	− 2·5054	− 2·4940	− 2·4826	− 2·4713	− 2·4601	− 2·4489	− 2·4378	− 2·4268	− 2·4158	− 2·4049
0·42	− 2·3941	− 2·3833	− 2·3726	− 2·3619	− 2·3513	− 2·3407	− 2·3303	− 2·3198	− 2·3095	− 2·2991
0·43	− 2·2889	− 2·2787	− 2·2685	− 2·2584	− 2·2484	− 2·2384	− 2·2285	− 2·2186	− 2·2087	− 2·1990
0·44	− 2·1892	− 2·1795	− 2·1699	− 2·1603	− 2·1508	− 2·1413	− 2·1319	− 2·1225	− 2·1131	− 2·1039
0·45	− 2·0946	− 2·0854	− 2·0762	− 2·0671	− 2·0580	− 2·0490	− 2·0400	− 2·0311	− 2·0222	− 2·0133
0·46	− 2·0045	− 1·9958	− 1·9870	− 1·9783	− 1·9697	− 1·9611	− 1·9525	− 1·9440	− 1·9355	− 1·9270
0·47	− 1·9186	− 1·9102	− 1·9019	− 1·8936	− 1·8853	− 1·8771	− 1·8689	− 1·8608	− 1·8526	− 1·8445
0·48	− 1·8365	− 1·8285	− 1·8205	− 1·8126	− 1·8047	− 1·7968	− 1·7889	− 1·7811	− 1·7733	− 1·7656
0·49	− 1·7579	− 1·7502	− 1·7426	− 1·7350	− 1·7274	− 1·7198	− 1·7123	− 1·7048	− 1·6974	− 1·6899
0·50	− 1·6825	− 1·6751	− 1·6678	− 1·6605	− 1·6532	− 1·6459	− 1·6387	− 1·6315	− 1·6243	− 1·6171
0·51	− 1·6100	− 1·6029	− 1·5959	− 1·5888	− 1·5818	− 1·5748	− 1·5679	− 1·5609	− 1·5540	− 1·5472
0·52	− 1·5403	− 1·5335	− 1·5267	− 1·5199	− 1·5131	− 1·5064	− 1·4997	− 1·4930	− 1·4864	− 1·4797
0·53	− 1·4731	− 1·4665	− 1·4600	− 1·4534	− 1·4469	− 1·4404	− 1·4339	− 1·4275	− 1·4211	− 1·4147
0·54	− 1·4083	− 1·4019	− 1·3956	− 1·3893	− 1·3830	− 1·3767	− 1·3704	− 1·3642	− 1·3580	− 1·3518
0·55	− 1·3457	− 1·3395	− 1·3334	− 1·3273	− 1·3212	− 1·3151	− 1·3090	− 1·3030	− 1·2970	− 1·2910
0·56	− 1·2850	− 1·2791	− 1·2732	− 1·2672	− 1·2613	− 1·2555	− 1·2496	− 1·2437	− 1·2379	− 1·2321
0·57	− 1·2263	− 1·2206	− 1·2148	− 1·2091	− 1·2034	− 1·1976	− 1·1920	− 1·1863	− 1·1806	− 1·1750
0·58	− 1·1694	− 1·1638	− 1·1582	− 1·1527	− 1·1471	− 1·1416	− 1·1361	− 1·1306	− 1·1251	− 1·1196
0·59	− 1·1141	− 1·1087	− 1·1033	− 1·0979	− 1·0925	− 1·0871	− 1·0817	− 1·0764	− 1·0711	− 1·0657
0·60	− 1·0604	− 1·0552	− 1·0499	− 1·0446	− 1·0394	− 1·0342	− 1·0289	− 1·0237	− 1·0185	− 1·0134
0·61	− 1·0082	− 1·0031	− 0·9979	− 0·9928	− 0·9877	− 0·9826	− 0·9776	− 0·9725	− 0·9674	− 0·9624
0·62	− 0·9574	− 0·9524	− 0·9474	− 0·9424	− 0·9374	− 0·9324	− 0·9275	− 0·9226	− 0·9177	− 0·9128
0·63	− 0·9079	− 0·9030	− 0·8981	− 0·8932	− 0·8884	− 0·8835	− 0·8787	− 0·8739	− 0·8691	− 0·8643
0·64	− 0·8595	− 0·8548	− 0·8500	− 0·8453	− 0·8406	− 0·8358	− 0·8311	− 0·8264	− 0·8217	− 0·8171
0·65	− 0·8124	− 0·8078	− 0·8031	− 0·7985	− 0·7939	− 0·7893	− 0·7847	− 0·7801	− 0·7755	− 0·7709
0·66	− 0·7664	− 0·7618	− 0·7573	− 0·7528	− 0·7483	− 0·7438	− 0·7393	− 0·7348	− 0·7303	− 0·7258
0·67	− 0·7214	− 0·7169	− 0·7125	− 0·7081	− 0·7037	− 0·6993	− 0·6949	− 0·6905	− 0·6861	− 0·6817
0·68	− 0·6774	− 0·6730	− 0·6687	− 0·6644	− 0·6601	− 0·6558	− 0·6515	− 0·6472	− 0·6429	− 0·6386
0·69	− 0·6343	− 0·6301	− 0·6258	− 0·6216	− 0·6174	− 0·6132	− 0·6089	− 0·6047	− 0·6005	− 0·5964
0·70	− 0·5922	− 0·5880	− 0·5839	− 0·5797	− 0·5756	− 0·5714	− 0·5673	− 0·5632	− 0·5591	− 0·5550
0·71	− 0·5509	− 0·5468	− 0·5427	− 0·5386	− 0·5346	− 0·5305	− 0·5265	− 0·5224	− 0·5184	− 0·5144
0·72	− 0·5104	− 0·5064	− 0·5024	− 0·4984	− 0·4944	− 0·4904	− 0·4864	− 0·4825	− 0·4785	− 0·4746
0·73	− 0·4707	− 0·4667	− 0·4628	− 0·4589	− 0·4550	− 0·4551	− 0·4472	− 0·4433	− 0·4394	− 0·4355
0·74	− 0·4317	− 0·4278	− 0·4240	− 0·4201	− 0·4163	− 0·4124	− 0·4086	− 0·4048	− 0·4010	− 0·3972
0·75	− 0·3934	− 0·3896	− 0·3858	− 0·3820	− 0·3783	− 0·3745	− 0·3707	− 0·3670	− 0·3632	− 0·3595
0·76	− 0·3558	− 0·3521	− 0·3483	− 0·3446	− 0·3409	− 0·3372	− 0·3335	− 0·3298	− 0·3262	− 0·3225
0·77	− 0·3188	− 0·3152	− 0·3115	− 0·3079	− 0·3042	− 0·3006	− 0·2969	− 0·2933	− 0·2897	− 0·2861
0·78	− 0·2825	− 0·2789	− 0·2753	− 0·2717	− 0·2681	− 0·2645	− 0·2610	− 0·2574	− 0·2538	− 0·2503
0·79	− 0·2467	− 0·2432	− 0·2397	− 0·2361	− 0·2326	− 0·2291	− 0·2256	− 0·2220	− 0·2185	− 0·2150
0·80	− 0·2116	− 0·2081	− 0·2046	− 0·2011	− 0·1976	− 0·1942	− 0·1907	− 0·1873	− 0·1838	− 0·1804
0·81	− 0·1769	− 0·1735	− 0·1701	− 0·1666	− 0·1632	− 0·1598	− 0·1564	− 0·1530	− 0·1496	− 0·1462
0·82	− 0·1428	− 0·1394	− 0·1361	− 0·1327	− 0·1293	− 0·1259	− 0·1226	− 0·1192	− 0·1159	− 0·1125

x	0	·001	·002	·003	·004	·005	·006	·007	·008	·009
0·83	− 0·1092	− 0·1059	− 0·1025	− 0·0992	− 0·0959	− 0·0926	− 0·0893	− 0·0860	− 0·0827	− 0·0794
0·84	− 0·0761	− 0·0728	− 0·0695	− 0·0662	− 0·0630	− 0·0597	− 0·0564	− 0·0532	− 0·0499	− 0·0467
0·85	− 0·0434	− 0·0402	− 0·0369	− 0·0337	− 0·0305	− 0·0273	− 0·0240	− 0·0208	− 0·0176	− 0·0144
0·86	− 0·0112	− 0·0080	− 0·0048	− 0·0016	+ 0·0015	+ 0·0047	+ 0·0079	+ 0·0111	+ 0·0142	+ 0·0174
0·87	0·0206	0·0237	0·0269	0·0300	0·0331	0·0363	0·0394	0·0425	0·0457	0·0488
0·88	0·0519	0·0500	0·0581	0·0612	0·0643	0·0674	0·0705	0·0736	0·0767	0·0798
0·89	0·0829	0·0859	0·0890	0·0921	0·0951	0·0982	0·1013	0·1043	0·1074	0·1104
0·90	0·1134	0·1165	0·1195	0·1225	0·1256	0·1286	0·1316	0·1346	0·1376	0·1406
0·91	0·1436	0·1466	0·1496	0·1526	0·1556	0·1586	0·1616	0·1645	0·1675	0·1705
0·92	0·1735	0·1764	0·1794	0·1823	0·1853	0·1882	0·1912	0·1941	0·1971	0·2000
0·93	0·2029	0·2059	0·2088	0·2117	0·2146	0·2175	0·2205	0·2234	0·2263	0·2292
0·94	0·2321	0·2350	0·2379	0·2408	0·2436	0·2465	0·2494	0·2523	0·2552	0·2580
0·95	0·2609	0·2638	0·2666	0·2695	0·2723	0·2752	0·2780	0·2809	0·2837	0·2866
0·96	0·2894	0·2922	0·2950	0·2979	0·3007	0·3035	0·3063	0·3091	0·3120	0·3148
0·97	0·3176	0·3204	0·3232	0·3260	0·3288	0·3315	0·3343	0·3371	0·3399	0·3427
0·98	0·3455	0·3482	0·3510	0·3538	0·3565	0·3593	0·3621	0·3648	0·3676	0·3703
0·99	0·3731	0·3758	0·3785	0·3813	0·3840	0·3868	0·3895	0·3922	0·3949	0·3977
1·00	0·4004	0·4031	0·4058	0·4085	0·4112	0·4139	0·4166	0·4193	0·4220	0·4247
1·01	0·4274	0·4301	0·4328	0·4355	0·4382	0·4409	0·4435	0·4462	0·4489	0·4515
1·02	0·4542	0·4569	0·4595	0·4622	0·4649	0·4675	0·4702	0·4728	0·4755	0·4781
1·03	0·4807	0·4834	0·4860	0·4887	0·4913	0·4939	0·4965	0·4992	0·5018	0·5044
1·04	0·5070	0·5096	0·5122	0·5149	0·5175	0·5201	0·5227	0·5253	0·5279	0·5305
1·05	0·5331	0·5356	0·5382	0·5408	0·5434	0·5460	0·5486	0·5511	0·5537	0·5563
1·06	0·5589	0·5614	0·5640	0·5666	0·5691	0·5717	0·5742	0·5768	0·5793	0·5819
1·07	0·5844	0·5870	0·5895	0·5921	0·5946	0·5971	0·5997	0·6022	0·6047	0·6073
1·08	0·6098	0·6123	0·6148	0·6173	0·6199	0·6224	0·6249	0·6274	0·6299	0·6324
1·09	0·6349	0·6374	0·6399	0·6424	0·6449	0·6474	0·6499	0·6524	0·6549	0·6574
1·10	0·6598	0·6623	0·6648	0·6673	0·6697	0·6722	0·6747	0·6771	0·6796	0·6821
1·11	0·6846	0·6870	0·6895	0·6919	0·6944	0·6968	0·6993	0·7017	0·7042	0·7066
1·12	0·7091	0·7115	0·7139	0·7164	0·7188	0·7212	0·7237	0·7261	0·7285	0·7310
1·13	0·7334	0·7358	0·7382	0·7406	0·7431	0·7455	0·7479	0·7503	0·7527	0·7551
1·14	0·7575	0·7599	0·7623	0·7647	0·7671	0·7695	0·7719	0·7743	0·7767	0·7791
1·15	0·7815	0·7839	0·7862	0·7886	0·7910	0·7934	0·7958	0·7981	0·8005	0·8029
1·16	0·8053	0·8076	0·8100	0·8123	0·8147	0·8171	0·8194	0·8218	0·8241	0·8265
1·17	0·8289	0·8312	0·8335	0·8359	0·8382	0·8406	0·8429	0·8452	0·8476	0·8499
1·18	0·8523	0·8546	0·8569	0·8593	0·8616	0·8639	0·8662	0·8686	0·8709	0·8732
1·19	0·8755	0·8778	0·8801	0·8825	0·8848	0·8871	0·8894	0·8917	0·8940	0·8963
1·20	0·8986	0·9009	0·9032	0·9055	0·9078	0·9101	0·9124	0·9147	0·9170	0·9193
1·21	0·9215	0·9238	0·9261	0·9284	0·9307	0·9329	0·9352	0·9375	0·9398	0·9420
1·22	0·9443	0·9466	0·9488	0·9511	0·9534	0·9556	0·9579	0·9602	0·9624	0·9647
1·23	0·9669	0·9692	0·9714	0·9737	0·9759	0·9782	0·9804	0·9827	0·9849	0·9872
1·24	0·9894	0·9917	0·9939	0·9961	0·9983	1·0006	1·0028	1·0050	1·0073	1·0095
1·25	1·0117	1·0140	1·0162	1·0184	1·0206	1·0228	1·0251	1·0273	1·0295	1·0317

x	0	·001	·002	·003	·004	·005	·006	·007	·008	·009
1·26	1·0339	1·0361	1·0383	1·0405	1·0427	1·0450	1·0472	1·0494	1·0516	1·0538
1·27	1·0560	1·0582	1·0604	1·0625	1·0647	1·0669	1·0691	1·0713	1·0735	1·0757
1·28	1·0779	1·0801	1·0822	1·0844	1·0866	1·0888	1·0910	1·0931	1·0953	1·0975
1·29	1·0997	1·1018	1·1040	1·1062	1·1083	1·1105	1·1126	1·1148	1·1170	1·1191
1·30	1·1213	1·1235	1·1256	1·1278	1·1299	1·1321	1·1342	1·1364	1·1385	1·1407
1·31	1·1428	1·1450	1·1471	1·1492	1·1514	1·1535	1·1557	1·1578	1·1599	1·1621
1·32	1·1642	1·1663	1·1685	1·1706	1·1727	1·1749	1·1770	1·1791	1·1812	1·1834
1·33	1·1855	1·1876	1·1897	1·1918	1·1939	1·1961	1·1982	1·2003	1·2024	1·2045
1·34	1·2066	1·2087	1·2108	1·2129	1·2151	1·2172	1·2193	1·2214	1·2235	1·2256
1·35	1·2277	1·2298	1·2319	1·2339	1·2360	1·2381	1·2402	1·2423	1·2444	1·2465
1·36	1·2486	1·2507	1·2527	1·2548	1·2569	1·2590	1·2611	1·2632	1·2652	1·2673
1·37	1·2693	1·2715	1·2735	1·2756	1·2777	1·2797	1·2818	1·2839	1·2859	1·2880
1·38	1·2901	1·2921	1·2942	1·2963	1·2983	1·3004	1·3024	1·3045	1·3066	1·3086
1·39	1·3107	1·3127	1·3148	1·3168	1·3189	1·3209	1·3230	1·3250	1·3271	1·3291
1·40	1·3312	1·3332	1·3352	1·3373	1·3393	1·3413	1·3434	1·3454	1·3475	1·3495
1·41	1·3515	1·3536	1·3556	1·3576	1·3596	1·3617	1·3637	1·3657	1·3677	1·3698
1·42	1·3718	1·3738	1·3758	1·3778	1·3799	1·3819	1·3839	1·3859	1·3879	1·3900
1·43	1·3920	1·3940	1·3960	1·3980	1·4000	1·4020	1·4040	1·4060	1·4080	1·4100
1·44	1·4120	1·4140	1·4160	1·4180	1·4200	1·4220	1·4240	1·4260	1·4280	1·4300
1·45	1·4320	1·4340	1·4360	1·4380	1·4400	1·4420	1·4440	1·4459	1·4479	1·4499
1·46	1·4519	1·4539	1·4559	1·4578	1·4598	1·4618	1·4638	1·4658	1·4677	1·4697
1·47	1·4717	1·4737	1·4756	1·4776	1·4796	1·4815	1·4835	1·4855	1·4875	1·4894
1·48	1·4914	1·4934	1·4953	1·4973	1·4992	1·5012	1·5032	1·5051	1·5071	1·5090
1·49	1·5110	1·5130	1·5149	1·5169	1·5188	1·5208	1·5227	1·5247	1·5266	1·5286
1·50	1·5305	1·5325	1·5344	1·5364	1·5383	1·5402	1·5422	1·5441	1·5461	1·5480
1·51	1·5500	1·5519	1·5538	1·5558	1·5577	1·5596	1·5616	1·5635	1·5654	1·5674
1·52	1·5693	1·5712	1·5732	1·5751	1·5770	1·5789	1·5809	1·5828	1·5847	1·5867
1·53	1·5886	1·5905	1·5924	1·5943	1·5963	1·5982	1·6001	1·6020	1·6039	1·6058
1·54	1·6078	1·6097	1·6116	1·6135	1·6154	1·6173	1·6192	1·6211	1·6230	1·6250
1·55	1·6269	1·6288	1·6307	1·6326	1·6345	1·6364	1·6383	1·6402	1·6421	1·6440
1·56	1·6459	1·6478	1·6497	1·6516	1·6535	1·6554	1·6573	1·6591	1·6610	1·6629
1·57	1·6648	1·6667	1·6686	1·6705	1·6724	1·6743	1·6762	1·6780	1·6799	1·6818
1·58	1·6837	1·6856	1·6875	1·6893	1·6912	1·6931	1·6950	1·6969	1·6987	1·7006
1·59	1·7025	1·7044	1·7062	1·7081	1·7100	1·7119	1·7137	1·7156	1·7175	1·7193
1·60	1·7212	1·7231	1·7249	1·7268	1·7287	1·7305	1·7324	1·7343	1·7361	1·7380
1·61	1·7399	1·7417	1·7436	1·7454	1·7473	1·7491	1·7510	1·7529	1·7547	1·7566
1·62	1·7584	1·7603	1·7621	1·7640	1·7658	1·7677	1·7695	1·7714	1·7732	1·7751
1·63	1·7769	1·7788	1·7806	1·7825	1·7843	1·7862	1·7880	1·7898	1·7917	1·7935
1·64	1·7954	1·7972	1·7990	1·8009	1·8027	1·8046	1·8064	1·8082	1·8101	1·8119
1·65	1·8137	1·8156	1·8174	1·8192	1·8211	1·8229	1·8247	1·8265	1·8284	1·8302
1·66	1·8320	1·8339	1·8357	1·8375	1·8393	1·8412	1·8430	1·8448	1·8466	1·8484
1·67	1·8503	1·8521	1·8539	1·8557	1·8575	1·8594	1·8612	1·8630	1·8648	1·8666
1·68	1·8684	1·8702	1·8721	1·8739	1·8757	1·8775	1·8793	1·8810	1·8829	1·8847

x	0	·001	·002	·003	·004	·005	·006	·007	·008	·009
1·69	1·8865	1·8883	1·8901	1·8920	1·8938	1·8956	1·8974	1·8992	1·9010	1·9028
1·70	1·9046	1·9064	1·9082	1·9100	1·9118	1·9136	1·9154	1·9172	1·9190	1·9208
1·71	1·9226	1·9243	1·9261	1·9279	1·9297	1·9315	1·9333	1·9351	1·9369	1·9387
1·72	1·9405	1·9423	1·9440	1·9458	1·9476	1·9494	1·9512	1·9530	1·9548	1·9565
1·73	1·9583	1·9601	1·9619	1·9637	1·9654	1·9672	1·9690	1·9708	1·9726	1·9743
1·74	1·9761	1·9779	1·9797	1·9814	1·9832	1·9850	1·9868	1·9785	1·9903	1·9921
1·75	1·9939	1·9956	1·9974	1·9992	2·0009	2·0027	2·0045	2·0062	2·0080	2·0098
1·76	2·0115	2·0133	2·0151	2·0168	2·0186	2·0203	2·0221	2·0239	2·0256	2·0274
1·77	2·0292	2·0309	2·0327	2·0344	2·0362	2·0379	2·0397	2·0415	2·0432	2·0450
1·78	2·0467	2·0485	2·0502	2·0520	2·0538	2·0555	2·0573	2·0590	2·0608	2·0626
1·79	2·0642	2·0660	2·0677	2·0695	2·0712	2·0730	2·0747	2·0765	2·0782	2·0799
1·80	2·0817	2·0834	2·0852	2·0869	2·0887	2·0904	2·0921	2·0939	2·0956	2·0974
1·81	2·0991	2·1008	2·1026	2·1043	2·1060	2·1078	2·1095	2·1112	2·1130	2·1147
1·82	2·1164	2·1182	2·1199	2·1216	2·1234	2·1251	2·1268	2·1286	2·1303	2·1320
1·83	2·1337	2·1355	2·1372	2·1389	2·1406	2·1424	2·1441	2·1458	2·1475	2·1493
1·84	2·1510	2·1527	2·1544	2·1561	2·1579	2·1596	2·1613	2·1630	2·1647	2·1665
1·85	2·1682	2·1699	2·1716	2·1733	2·1750	2·1768	2·1785	2·1802	2·1819	2·1836
1·86	2·1853	2·1870	2·1887	2·1905	2·1922	2·1939	2·1956	2·1973	2·1990	2·2007
1·87	2·2024	2·2041	2·2058	2·2075	2·2092	2·2109	2·2126	2·2144	2·2161	2·2178
1·88	2·2195	2·2212	2·2229	2·2246	2·2263	2·2280	2·2297	2·2314	2·2331	2·2348
1·89	2·2365	2·2382	2·2399	2·2416	2·2432	2·2449	2·2466	2·2483	2·2500	2·2517
1·90	2·2534	2·2551	2·2568	2·2585	2·2602	2·2619	2·2636	2·2652	2·2669	2·2686
1·91	2·2703	2·2720	2·2737	2·2754	2·2771	2·2787	2·2804	2·2821	2·2838	2·2855
1·92	2·2872	2·2889	2·2905	2·2922	2·2939	2·2956	2·2973	2·2989	2·3006	2·3023
1·93	2·3040	2·3057	2·3073	2·3090	2·3107	2·3124	2·3140	2·3157	2·3174	2·3191
1·94	2·3208	2·3224	2·3241	2·3258	2·3274	2·3291	2·3308	2·3325	2·3341	2·3358
1·95	2·3375	2·3391	2·3408	2·3425	2·3441	2·3458	2·3475	2·3492	2·3508	2·3525
1·96	2·3542	2·3558	2·3575	2·3591	2·3608	2·3625	2·3641	2·3658	2·3675	2·3691
1·97	2·3708	2·3724	2·3741	2·3758	2·3774	2·3791	2·3807	2·3824	2·3841	2·3857
1·98	2·3874	2·3890	2·3907	2·3923	2·3940	2·3957	2·3973	2·3990	2·4006	2·4023
1·99	2·4039	2·4056	2·4072	2·4089	2·4105	2·4122	2·4138	2·4155	2·4171	2·4188
2·00	2·4204	2·4221	2·4237	2·4254	2·4270	2·4287	2·4303	2·4320	2·4336	2·4353
2·01	2·4369	2·4385	2·4402	2·4418	2·4435	2·4451	2·4468	2·4484	2·4500	2·4517
2·02	2·4533	2·4550	2·4566	2·4582	2·4599	2·4615	2·4632	2·4648	2·4664	2·4681
2·03	2·4697	2·4714	2·4730	2·4746	2·4763	2·4779	2·4795	2·4812	2·4828	2·4844
2·04	2·4861	2·4877	2·4893	2·4910	2·4926	2·4942	2·4958	2·4975	2·4991	2·5007
2·05	2·5024	2·5040	2·5056	2·5073	2·5089	2·5105	2·5121	2·5138	2·5154	2·5170
2·06	2·5186	2·5203	2·5219	2·5235	2·5251	2·5268	2·5284	2·5300	2·5316	2·5332
2·07	2·5349	2·5365	2·5381	2·5397	2·5413	2·5430	2·5446	2·5462	2·5478	2·5494
2·08	2·5511	2·5527	2·5543	2·5559	2·5575	2·5591	2·5608	2·5624	2·5640	2·5656
2·09	2·5672	2·5688	2·5704	2·5721	2·5737	2·5753	2·5769	2·5785	2·5801	2·5817
2·10	2·5833	2·5849	2·5865	2·5882	2·5898	2·5914	2·5930	2·5946	2·5962	2·5978
2·11	2·5994	2·6010	2·6026	2·6042	2·6058	2·6074	2·6090	2·6106	2·6122	2·6139

x	0	·001	·002	·003	·004	·005	·006	·007	·008	·009
2·12	2·6155	2·6171	2·6187	2·6203	2·6219	2·6235	2·6251	2·6267	2·6283	2·6299
2·13	2·6315	2·6331	2·6347	2·6363	2·6379	2·6395	2·6411	2·6426	2·6442	2·6458
2·14	2·6474	2·6490	2·6506	2·6522	2·6538	2·6554	2·6570	2·6586	2·6602	2·6618
2·15	2·6634	2·6650	2·6666	2·6682	2·6697	2·6713	2·6729	2·6745	2·6761	2·6777
2·16	2·6793	2·6809	2·6825	2·6840	2·6856	2·6872	2·6888	2·6904	2·6920	2·6936
2·17	2·6952	2·6967	2·6983	2·6999	2·7015	2·7031	2·7047	2·7062	2·7078	2·7094
2·18	2·7110	2·7126	2·7142	2·7157	2·7173	2·7189	2·7205	2·7221	2·7236	2·7252
2·19	2·7268	2·7284	2·7300	2·7315	2·7331	2·7347	2·7363	2·7378	2·7394	2·7410
2·20	2·7426	2·7441	2·7457	2·7473	2·7489	2·7504	2·7520	2·7536	2·7552	2·7567
2·21	2·7583	2·7599	2·7614	2·7630	2·7646	2·7662	2·7677	2·7693	2·7709	2·7724
2·22	2·7740	2·7756	2·7771	2·7787	2·7803	2·7819	2·7834	2·7850	2·7866	2·7881
2·23	2·7897	2·7913	2·7928	2·7944	2·7959	2·7975	2·7991	2·8006	2·8022	2·8038
2·24	2·8053	2·8069	2·8085	2·8100	2·8116	2·8131	2·8147	2·8163	2·8178	2·8194
2·25	2·8209	2·8225	2·8241	2·8256	2·8272	2·8287	2·8303	2·8318	2·8334	2·8350
2·26	2·8365	2·8381	2·8396	2·8412	2·8427	2·8443	2·8459	2·8474	2·8490	2·8505
2·27	2·8521	2·8536	2·8552	2·8567	2·8583	2·8598	2·8614	2·8629	2·8645	2·8660
2·28	2·8676	2·8691	2·8707	2·8722	2·8738	2·8753	2·8769	2·8784	2·8800	2·8815
2·29	2·8831	2·8846	2·8862	2·8877	2·8893	2·8908	2·8924	2·8939	2·8955	2·8970
2·30	2·8985	2·9001	2·9016	2·9032	2·9047	2·9063	2·9078	2·9093	2·9109	2·9124
2·31	2·9140	2·9155	2·9171	2·9186	2·9201	2·9217	2·9232	2·9248	2·9263	2·9278
2·32	2·9294	2·9309	2·9325	2·9340	2·9355	2·9371	2·9386	2·9401	2·9417	2·9432
2·33	2·9448	2·9463	2·9478	2·9494	2·9509	2·9524	2·9540	2·9555	2·9570	2·9586
2·34	2·9601	2·9616	2·9632	2·9647	2·9662	2·9678	2·9693	2·9708	2·9724	2·9739
2·35	2·9754	2·9769	2·9785	2·9800	2·9815	2·9831	2·9846	2·9861	2·9876	2·9892
2·36	2·9907	2·9922	2·9938	2·9953	2·9968	2·9983	2·9999	3·0014	3·0029	3·0044
2·37	3·0060	3·0075	3·0090	3·0105	3·0121	3·0136	3·0151	3·0166	3·0182	3·0197
2·38	3·0212	3·0227	3·0242	3·0258	3·0273	3·0288	3·0303	3·0319	3·0334	3·0349
2·39	3·0364	3·0379	3·0395	3·0410	3·0425	3·0440	3·0455	3·0470	3·0486	3·0501
2·40	3·0516	3·0531	3·0546	3·0561	3·0577	3·0592	3·0607	3·0662	3·0637	3·0652
2·41	3·0668	3·0683	3·0698	3·0713	3·0728	3·0743	3·0758	3·0773	3·0789	3·0804
2·42	3·0819	3·0834	3·0849	3·0864	3·0879	3·0894	3·0909	3·0925	3·0940	3·0955
2·43	3·0970	3·0985	3·1000	3·1015	3·1030	3·1045	3·1060	3·1075	3·1090	3·1106
2·44	3·1121	3·1136	3·1151	3·1166	3·1181	3·1196	3·1211	3·1226	3·1241	3·1256
2·45	3·1271	3·1286	3·1301	3·1316	3·1331	3·1346	3·1361	3·1376	3·1391	3·1406
2·46	3·1421	3·1436	3·1451	3·1466	3·1481	3·1496	3·1511	3·1526	3·1541	3·1556
2·47	3·1571	3·1586	3·1601	3·1616	3·1631	3·1646	3·1661	3·1676	3·1691	3·1706
2·48	3·1721	3·1736	3·1751	3·1766	3·1781	3·1796	3·1811	3·1826	3·1841	3·1856
2·49	3·1871	3·1886	3·1901	3·1916	3·1931	3·1945	3·1960	3·1975	3·1990	3·2005
2·50	3·2020	3·2035	3·2050	3·2065	3·2080	3·2095	3·2110	3·2124	3·2139	3·2154