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A Study of the Compressibility of Wool, with Special Reference to South African Merino Wool.*

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PART II.

" It is probable ", state Frölich, Spöttel and Tänzer (1929) " that ever since wool has been converted into fabric, it has been recognised that the mechanical properties of wool, besides its fineness, play an important if not decisive role in technical processing ".

It follows that the physical properties of the raw material are of fundamental importance to both producer and manufacturer, although the points of view of the two differ in many respects.

In production practice the estimation of fleece characteristics depends on the senses of sight and touch, and consequently suffers from the errors inherent in human estimation. With the advent of research, exact methods of measurement have been developed, and the results being obtained are laying stress on the necessity of employing such exact methods in breeding practice.

Of the properties of wool, the dimensional attributes of the fibre, and its behaviour under longitudinal stress have received most attention. On the one hand, the factors which influence the production of these properties are being investigated, and on the other hand, the knowledge of the structure of the wool fibre has advanced to a remarkable extent. Other properties, though admitted to be of great importance, have received less attention. One of these is the elastic behaviour of wool in bulk, and its neglect must be considered somewhat surprising in view of the extent to which both producer and manufacturer rely on tactual examination in wool judgment, and the stress generally laid in production on various characteristics known as " quality ", " substance ", " handle ", " harshness ", and others which may be expected to involve the compressibility of wool.

The present study is concerned with the resistance offered by the fibre mass to compression, and it is considered mainly from the point of view of the producer. Since the first essential to the study of any property is its expression in arithmetical terms, the first part of the investigation involves a study of the method of measurement, next the elastic behaviour of wool under compression, and finally the arithmetical expression of resistance to compression.

In the second part a study has been undertaken of the relation between the compressional characteristics and the more obvious fleece and fibre attributes, and of the factors which influence the compressibility. In view of the lack of knowledge of the subject, it has been the aim to investigate several such factors and attributes, rather than to study a few of these exhaustively, for by this method a better foundation for future experimentation could be laid. In the discussion of the results, an attempt has been made to show how the compressibility can influence breeding practice.

PART I.

THE MEASUREMENT OF COMPRESSIBILITY, AND THE ELASTIC BEHAVIOUR OF WOOL IN BULK.

A. The Measurement of Compressibility.

(*a*) *Historical.*

In practice both sheep breeder and wool manufacturer resort to tactual examination for estimating those characteristics of a wool sample which depend upon its compressibility and resilience. The resistance offered by the sample to compression by hand, and its mode of recovery to its original form when released, are taken as the bases of judgment. On the live animal a staple is often separated from the rest of the fleece, and its ability to remain erect is observed.

A method in use for comparing the pliability of single fibres, according to Heyne (1924), consists in blowing gently towards them and noting the extent to which individual fibres respond. Frölich, Spöttel and Tänzer (1929) state that in sheep breeding practice the pliability of a fibre is estimated by holding a 2 cm. length erect between the fingers and blowing towards it until it lies horizontally. From the force necessary, and the extent to which the fibre rises after cessation of the blowing, the pliability of the fibre is estimated.

Such methods are, however, subjective, and the results are dependent on individual opinion and cannot be expressed in arithmetical terms. From time to time, therefore, methods have been devised for studying the elastic behaviour of wool under compression more precisely.

Herzog (1916) studied the properties of materials utilised for upholstery, particularly horsehair. He regarded compressibility and resilience as of supreme importance, and devised an apparatus for their measurement. The sample was compressed by a falling weight, and the compressibility and resilience were estimated from the successive minimum and maximum heights of the material. Sommer (1936) employed the same method, using different coefficients for expressing the relevant characteristics. His determinations were extended to include mixtures of fibre types, and the effect of impregnation with rubber.

A method of comparing the pliability (Schmiegsamkeit) of tops was devised by Krais (1922). A certain length and weight of top was subjected to a torsional couple while under a longitudinal tension of 75 gm. The number of turns in the top and the resulting reduction in length were noted.

M. and J. Eggert (1925) enclosed the sample to be tested in a small rubber balloon. The balloon was in turn enclosed in a glass vessel with the neck protruding. After the spaces inside and outside the balloon had been partially exhausted to the same pressure, compression was effected by allowing small quantities of air into the outer glass vessel. Manometers indicated the pressures inside and outside the balloon, and the difference between them was taken to represent the pressure acting on the wool. The volume of the balloon was determined at one pressure, and since the quantity of air in the system containing the wool remained constant, the volume at other pressures was calculated by Boyle's Law. An equation containing two constants was assumed to represent the relation between pressure and volume. One of the constants was regarded as a measure of the pliability of the wool, and the other as a measure of its softness.

The possibilities of the balloon method as a method of measuring resilience were later investigated by Winson (1932). He took as a measure of the resilience of the wool the area of the hysteresis loop formed between the compression and the release curves, since his results indicated that this quantity corresponded to the trade impression of " springiness ".

The balloon method, in the form employed by the Eggerts and by Winson, suffered from the serious defect that the relative humidity of the air surrounding the wool was uncontrolled and variable. This defect was overcome by Pidgeon and van Winsen (1934). The balloon was compressed by liquid pressure, the neck of the balloon was left open and the enclosed sample could be maintained in equilibrium with any desired relative humidity. While the primary object of the investigation was to study the effect of sorbed water on the compressibility of asbestos, a wool sample and a cotton sample were included for comparison.

The relation between the harshness of two yarns and their compressibility was investigated by Larose (1934). Both the balloon method, as employed by Winson, and a method devised by him were used. In his method the sample was placed in a steel cylinder and compressed by a piston attached to a calibrated spring. Both methods led to the conclusion that the harsher of the two yarns offered the greater resistance to compression, and the effect of dyeing was in each case to increase both the harshness and the resistance to compression.

An instrument, known as the " Pendultex ", designed by Henning (1934, 1935), was based on a different principle. A swinging pendulum was made to compress the sample under test, and the compressional resistance of the sample was measured by the consequent damping of the pendulum. The number of swings during which the amplitude decreased from one value to another was recorded on a counter, and the greater the resistance of the sample to compression, the smaller was the number of swings recorded. The instrument was stated to be suitable for the study of compressibility at any stage from the raw material to the finished product, and was shown to be valuable for following the changes produced by the various processes.

In connection with his researches on felting, Schofield (1938) made a compression test on wool in the form of a sliver, by filling a wooden box with the sliver and placing weights on a lid which fitted into the box exactly.

The methods and instruments described above were all employed or were suitable for determinations on the raw material. To them may be added those of a number of investigators who studied the elastic properties of the finished product, particularly the resistance to bending and the resistance to compression. The work of these authors will not, however, be here considered, since the present investigation is confined to a study of the raw product.

A compressional method of estimating the clean yield of fleeces was developed by Burns and Johnston (1936, 1937) and Johnston and Gray (1939). A high correlation between the percentage of clean wool and the volume occupied by a greasy sample under a high pressure was obtained.

The present study is based on results obtained with the "Pendultex" instrument, but certain aspects of the elastic behaviour of wool in bulk have been investigated by a static cylinder and piston method.

(b) *The "Pendultex" method of determining the compressibility of textiles.*

(i) *Description of the instrument.*

Figure 1 illustrates the essential features of the "Pendultex" instrument designed by Henning (1934) for determining the compressibility of wool and other textiles.

A heavy pendulum, A, is suspended at B and moves over a scale C, graduated in degrees. The pendulum is extended upwards and carries two levers, D and E, which are held in position by the springs F. A light bar G is suspended at its midpoint on the same axis B, but is free to move independently of the pendulum. To one end of the bar G is attached an arm carrying a piston, H, which fits into the stationary compression compartment J. The other end of the bar carries two balancing nuts M.

The wool to be tested is placed in the compression compartment J, and the pendulum is raised to a pre-determined angle and then released. At the lowest position of the pendulum, the lever E engages the piston, which is forced into the compartment J where it compresses the sample. When the lever D reaches the stop K it is forced outwards and draws the lever E off the piston. The piston thus released jumps back to its original position under pressure by the sample. On the return half of the swing the pendulum moves freely.

The process is repeated during each successive swing. The extent to which the sample is compressed is determined solely by the position of the stop K, and is independent of the amplitude, above a certain value of the latter. The position of the stop K can be varied in order to produce different degrees of compression.

The resistance offered by the sample has a damping effect on the pendulum, and is measured by the number of swings during which the

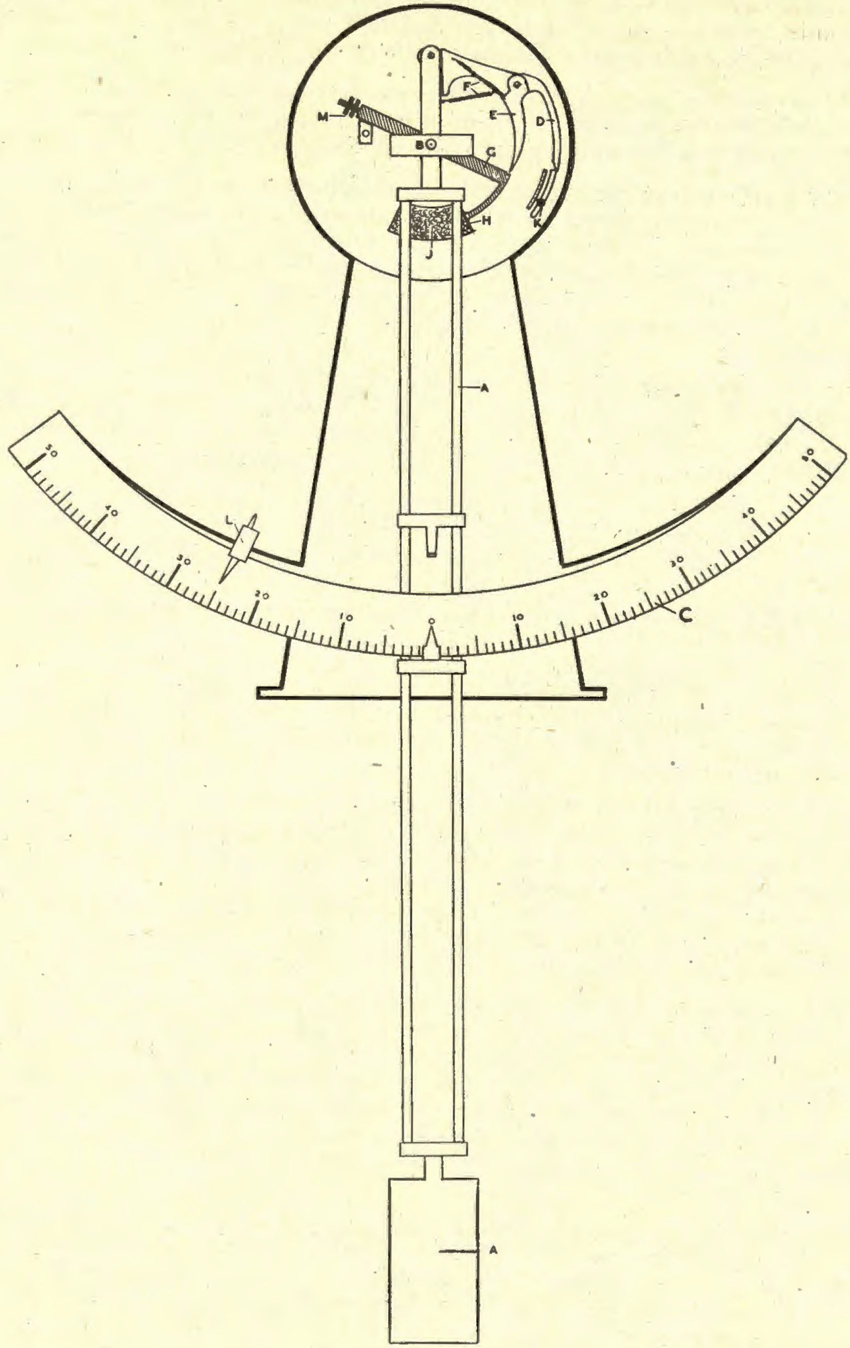

FIGURE I.—The " Pendultex " instrument designed by Henning, for determining the compressibility of wool and other textiles.

amplitude decreases from its initial value to a final fixed value. At the latter angle, a counter L is attached to the scale C and records the number of swings made by the pendulum.

It may be considered that a method of recording the reduction in amplitude during a swing directly is to be preferred to the less direct method of counting the number of swings between two fixed values of the amplitude. It should be borne in mind, however, that the apparatus was designed for routine determinations, and the experience gained during the course of the present study confirmed the designer's view that the method of counting the number of swings is far simpler, requires less manipulation, and gives a more reliable result.

The designer as well as the manufacturer of the instrument state that for comparative purposes the number of swings recorded by the counter is a sufficient indication of the resistance offered by the sample. Since results obtained in this form are dependent upon the constants of the pendulum and the arbitrarily chosen initial and final amplitudes, they cannot be compared with results obtained by other methods. The most serious objection to this method of presenting the results is, however, the non-linear relationship between the number of swings and the work done in compressing the sample, making it a matter of extreme difficulty to assess the true magnitude of the differences between samples.

The first object in the investigation was to find a relation between the work done in compressing a sample and the total number of swings of the pendulum between the two fixed amplitude values. The following is a summary of the procedure adopted:—

The work done in compressing the sample was estimated from the loss of potential energy of the pendulum during one swing. If M is the mass of the pendulum in Kg., and h the distance from the centre of gravity to the point of suspension, the potential energy at amplitude θ is given by $Mh(1-\cos\theta)=2Mh(\sin^2\frac{\theta}{2})$ in Kg.cm.

During the first swing the amplitude is reduced from θ_0 to θ_1, hence the loss of potential energy during the first swing is $2Mh\,(\sin^2\frac{\theta_0}{2}-\sin^2\frac{\theta_1}{2}) = 2Mh\ (A_0^2 - A_1^2)$, where $A=\sin\frac{\theta}{2}$. Since A_0 is fixed, the problem is that of finding A_1.

Instead of determining A_1 directly, the amplitudes after successive swings were noted, and the equation

$$\sin\frac{\theta}{2}=A_0-Bn+Cn^2 \quad \ldots\ldots\ldots \quad (1)$$

where n was the number of swings, was fitted to the observations, whence A_1 could be calculated.

If N was the total number of swings, then

$$A_1=A_0-B+C, \text{ and}$$
$$A_N=A_0-BN+CN^2.$$

Eliminating B,

$$A_1=\frac{A_N+(A_0-CN)\ (N-1)}{N} \quad \ldots\ldots\ldots \quad (3)$$

Now A_0 and A_N were fixed, and C was found experimentally to be independent of the compressibility of the sample and of the degree of compression. By making 62 determinations, and fitting equation (1) to each, a mean value of C was derived and substituted in equation (3). The only unknowns in equation (3) were then A_1 and N, so that A_1 could be calculated for any value of N. Thus the loss of potential energy of the pendulum during the first swing, given by $2Mh(A_0^2 - A_1^2)$, was known from N, the total number of swings between the two fixed amplitudes.

In order to find the work done in compressing the wool, a correction had to be applied for the natural damping of the pendulum. The correction was evaluated in two stages, (1) the loss of potential energy with the pendulum swinging freely, obtained by observing successive amplitudes as before, and (2) increased loss with load on the piston, obtained by determining the loss of potential energy of the pendulum while extending springs whose load-extension relations were known.

On applying the corrections, the work done in compressing the wool sample could be calculated from the number of swings made by the pendulum between the two fixed amplitude values.

The procedure outlined above is elaborated in the following pages.

The relevant constants of the instrument used in the present study were found by measurement to be as follows:—

v_1 = Total volume of compression compartment = 103·3 c.c.

M = Mass of pendulum ... ... = 11·666 Kg.

h = Distance from centre of gravity to point of suspension ... ... = 96.25 cm.

Hence the potential energy of the pendulum at amplitude θ was given by $Mh(1 - \cos\theta) = 1123(1 - \cos\theta) = 2246 \sin^2\frac{\theta}{2}$ Kg. cm.

The amount of wool to be tested was limited to 5 gm., as it was found that the 7 gm. recommended imposed a serious strain on certain parts of the apparatus. The intrument was set up in a room maintained at constant relative humidity and temperature, and all determinations were carried out on samples conditioned in this room.

(ii) *The effect of repeated compression.*

Previous investigators have found that when a wool sample is taken through successive cycles of compression and release, the position of the pressure-volume curve is altered after each cycle, but becomes constant after several cycles. Winson, therefore, made his determinations on the eighth cycle, Pidgeon and van Winsen proceeded to the fourth cycle, while Larose found that constancy was attained after the fifth or sixth cycle.

With the "Pendultex" instrument it was found that the number of swings registered by the counter increased with successive determinations until a constant value was obtained after about the fifth determination, representing at least 150 compressions. The resistance to compression offered by the wool was, therefore, reduced by a diminishing amount. For this reason Henning recommended that successive determinations be made without removal of the wool, until a constant value of the number of swings was obtained.

It was also found that if the wool was removed after each determination and teased out into as loose a mass as possible, exactly the same value could be obtained for a subsequent determination. It was decided, therefore, to base the study throughout on the resistance offered by the wool during the first compression. This quantity was evaluated from the total number of swings, and therefore lost nothing in accuracy compared to the method advocated by Henning. The question will be raised again in the subsequent discussion.

All results which follow must therefore be regarded as having been obtained from the first compression of teased samples.

(iii) *The motion of the pendulum.*

The first step in evaluating the work done in compressing the wool was to express the loss in potential energy during the first swing as a function of the total number of swings between the two fixed values of the amplitude. Two ways were open for accomplishing this object. Either an automatic recorder, such as a strip of waxed paper on the scale and a recording needle on the pendulum, could have been employed for recording the reduction in amplitude during the first swing, or the amplitude after each swing could have been noted, and the loss of amplitude during the first swing calculated by fitting an equation to the observations.

In the present investigation the latter method was employed. It was found to be a matter of extreme difficulty to start the pendulum in exactly the correct plane in every case, so that the first swing gave rather erratic results, while the value calculated from the observed amplitudes after successive swings gave reproducible results.

A study was therefore made of the reduction in amplitude of the pendulum during successive swings, and Figure 2 illustrates three typical cases during the compression of five gm. of three different wools by approximately 50 per cent.

The amplitudes at the end of equal numbers of successive swings were noted, but the difficulty of observing the angles with sufficient accuracy, and the somewhat inconsistent oscillation of the pendulum made it necessary that some method had to be applied for smoothing the data. The most suitable method was the fitting of an equation, and since the theoretical derivation of the equation of motion was impracticable owing to the large amplitudes and the discontinuous nature of the damping forces, an empirical relation had to be found.

In an attempt to express the number of swings as a function of the amplitude, the simple trigonometric and hyperbolic functions, their logarithms, and a large number of combinations of these were tried. The simplest functions failed to give a satisfactory fit, while the more complicated combinations introduced so many constants that a good fit was obtained, but not the desired smoothing.

The introduction of the second power of the number of swings led to the equation

$$\sin\frac{\theta}{2} = A_0 - Bn + Cn^2 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where θ was the amplitude, n the number of swings and A_0 the initial value of $\sin\frac{\theta}{2}$. The equation was found to fit the observations closely in the range 48° to 25°, and is represented in each case by the full line in Figure 2. The constant C varied little from one sample to another, and bore no relationship to either the compressibility of the samples or the degree of compression. The assumption seemed justified that a simple equation applicable to all cases had been found.

The nature of the equation shows that its range is limited, since in certain cases it predicts no zero value for θ. Replacing $\sin \frac{\theta}{2}$ by its logarithm in equation (1) gave as good results, which in fact differed but little from those of equation (1), but the calculations were more complicated. Equation (1) was used throughout the present study, and it was regarded as conforming to the conditions that it should fit the observations closely, be applicable to all cases, and contain a minimum number of constants.

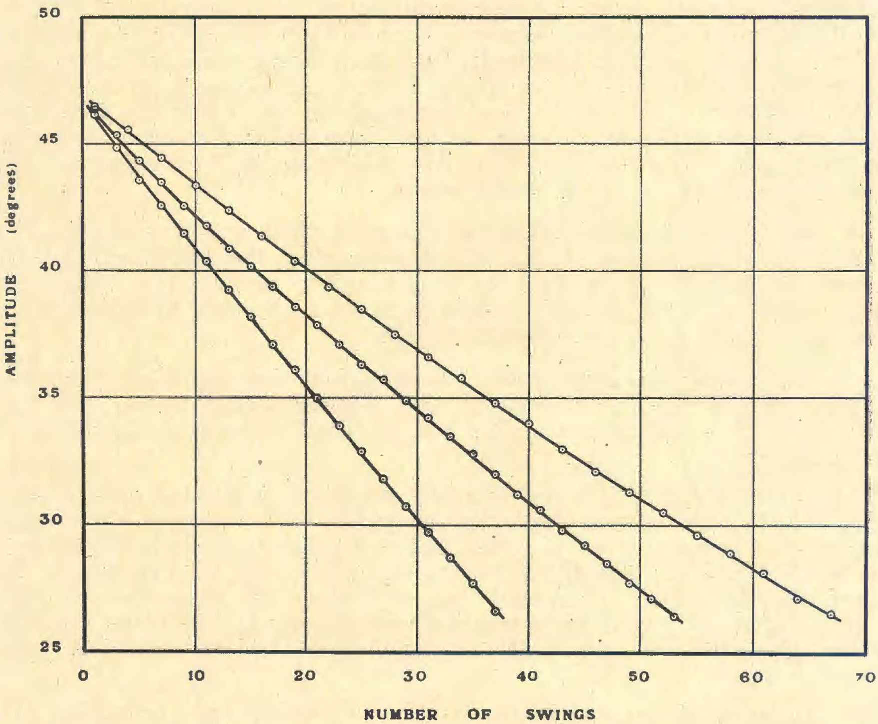

FIGURE 2.—The reduction in amplitude of the pendulum, while compressing 5 gm. of three different wools by 50 per cent.

An added advantage was the ease with which the relevant calculations could be made. For example, the potential energy at an amplitude θ is proportional to $\sin^2 \frac{\theta}{2}$, and the reduction in $\sin \frac{\theta}{2}$ during the swing from θ_n to θ_{n+1} is given by:—

$$\sin \frac{\theta_n}{2} - \sin \frac{\theta_{n+1}}{2} = \sqrt{B^2 - 4A_0C + 4C \sin \frac{\theta_n}{2}} - C \ldots\ldots\ldots (2)$$

Equation (1) was fitted to the observations by the method of least squares, and a method based on that of Gauss was used for solving the normal equations rapidly.

(iv) *The relation between the loss of potential energy during the first swing, and the total number of swings between two fixed amplitudes.*

The next step consisted in expressing the loss in potential energy during the first swing as a function of the total number of swings between the two fixed amplitudes. For this purpose a series of wool samples was selected, and five gm. of each compressed to various degrees in the instrument, by varying the stop K (Figure 1). In each case the amplitudes at the end of every second, third or fourth swing (depending on the total number of swings) were noted, and equation (1) fitted. By evaluating the constants B and C for each degree of compression of each sample, the loss in potential energy of the pendulum during the first swing was calculated in each case. In all, sixty-two independent determinations were made, no sample being compressed to the same volume more often than once. The circles in Figure 3 represent the values so obtained.

The average relation was found by substituting 1 and N for n in equation (1), and eliminating B from the two equations thus obtained, giving

$$A_1 = \frac{A_N + (A_0 - CN)\,(N-1)}{N} \quad \ldots\ldots\ldots\ldots(3)$$

Now A_0 and A_N were kept constant, but small variations were found in the values calculated from the sixty-two determinations, variations in A_0 being mainly due to errors in starting the pendulum, and those in A_N being due to the fact that the counter recorded only complete swings. The means of the calculated values of A_0, A_N and C were

$$A_0 = 0{\cdot}39769$$
$$A_N = 0{\cdot}22666$$
$$C = 4{\cdot}81515 \times 10^{-6}.$$

The employment of a mean value for C was justified by the experimentally found independence of C on either the compressibility or the degree of compression. Substituting in (3), the only unknowns were A_1 and N, so that A_1 could be calculated for any value of N.

The loss in potential energy during the first swing was then given by $2Mh(A_0^2 - A_1^2) = 2246(A_0^2 - A_1^2)$ in Kg. cm. Values so calculated for various values of N are given in Table 1.

TABLE 1.

The loss of potential energy of the pendulum (in Kg. cm.) during the first swing, as a function of the total number of swings, N.

N.	0	1	2	3	4	5	6	7	8	9
30	10·31	9·99	9·70	9·43	9·17	8·93	8·70	8·48	8·28	8·09
40	7·90	7·73	7·57	7·41	7·26	7·12	6·98	6·85	6·73	6·61
50	6·50	6·39	6·28	6·18	6·09	6·00	5·91	5·82	5·74	5·66
60	5·59	5·52	5·44	5·38	5·31	5·25	5·19	5·13	5·07	5·02
70	4·96	4·91	4·86	4·82	4·77	4·72	4·68	4·64	4·60	4·56
80	4·52	4·48	4·45	4·41	4·38	4·35	4·31	4·27	4·25	4·22
90	4·20	4·17	4·14	4·12	4·09	4·07	4·04	4·02	4·00	3·97

Table 1 was used for evaluating the loss in potential energy of the pendulum during all subsequent investigations made on the first compression of a teased sample.

The agreement with the loss of potential energy calculated for each of the sixty-two determinations by means of equation (1) may be judged from Table 2. In Figure 3 the full line represents Table 1, while the circles represent the individual calculations.

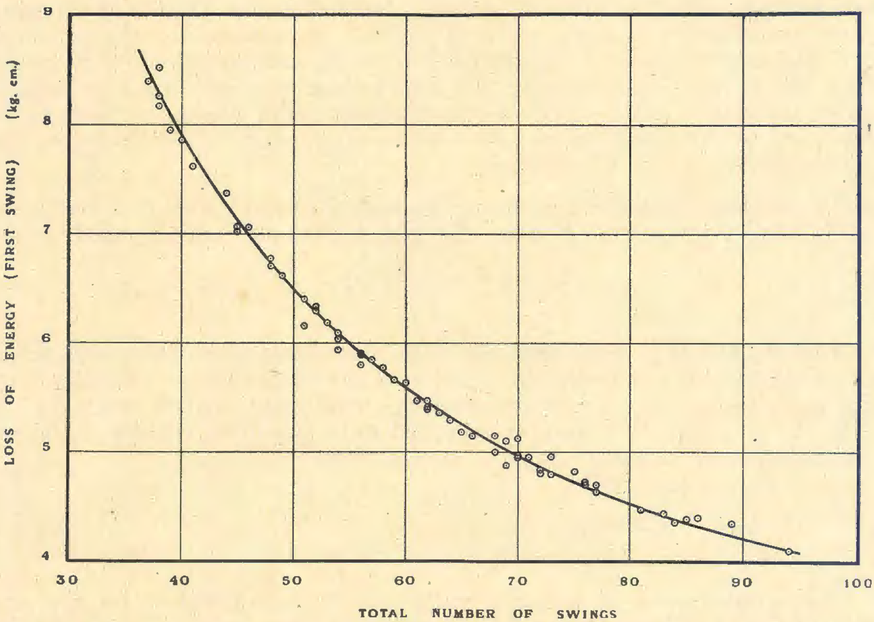

FIGURE 3.—The relation between the loss in potential energy of the pendulum during the first swing and the total number of swings between the two fixed amplitudes.

The differences in Table 2 vary in absolute magnitude from zero to 0·24, and can be attributed to several causes. Part may be due to errors in the observation of the amplitudes, and part to differences in the ability of different wools to recover sufficiently rapidly from compression. Another source of error is no doubt the fact that N, the number of swings, was recorded in complete swings, and according to Table 1, one swing meant a difference of from 0·02 to 0·20 Kg. cm. Thus, for example, one sample might just have been able to record the fortieth swing on the counter, while another may just have failed to record the forty-first. According to Table 1, both samples would have been regarded as having caused a loss of potential energy of 7·90 Kg. cm., while actually a true difference in the number of swings may have been 0·8 of a swing, representing a difference of 0·13 Kg. cm. In routine practice, errors due to this cause were reduced by taking for N the average of at least five determinations.

(v) *The natural damping of the pendulum.*

The values given in Table 1 represent the total loss of potential energy of the pendulum during the first swing. In order to obtain that portion which was due to the work done in compressing the wool, a correction had to be applied for the natural damping of the pendulum.

TABLE 2.

The loss of potential energy of the pendulum (in Kg. cm.) during the first swing as calculated from equation (1) for each of the sixty-two determinations, compared with the values given by Table 1.

N.	Loss of Energy from Equation (1).	Loss of Energy (from Table 1).	Difference.	N.	Loss of Energy from Equation (1).	Loss of Energy (from Table 1).	Difference.
37	8·39	8·48	— 0·09	62	5·42	5·44	— 0·02
38	8·52	8·28	+ 0·24	62	5·47	5·44	+ 0·03
38	8·26	8·28	— 0·02	62	5·41	5·44	— 0·03
38	8·17	8·28	— 0·11	63	5·36	5·37	— 0·01
39	7·95	8·09	— 0·14	64	5·29	5·31	— 0·02
40	7·86	7·90	— 0·04	65	5·18	5·25	— 0·07
41	7·62	7·73	— 0·11	66	5·15	5·19	— 0·04
44	7·37	7·26	+ 0·11	68	5·15	5·07	+ 0·08
45	7·02	7·12	— 0·10	68	5·00	5·07	— 0·07
45	7·07	7·12	— 0·05	69	4·88	5·02	— 0·14
46	7·06	6·98	+ 0·08	69	5·11	5·02	+ 0·09
48	6·78	6·73	+ 0·05	70	5·12	4·96	+ 0·16
48	6·71	6·73	— 0·02	70	4·95	4·96	— 0·01
49	6·62	6·61	+ 0·01	70	4·97	4·96	+ 0·01
51	6·16	6·39	— 0·23	71	4·96	4·91	+ 0·05
51	6·41	6·39	+ 0·02	72	4·80	4·86	— 0·06
52	6·34	6·28	+ 0·06	72	4·83	4·86	— 0·03
52	6·30	6·28	+ 0·02	73	4·79	4·82	— 0·03
53	6·19	6·18	+ 0·01	73	4·96	4·82	+ 0·14
54	6·04	6·09	— 0·05	75	4·82	4·72	+ 0·10
54	5·94	6·09	— 0·15	76	4·70	4·68	+ 0·02
54	6·10	6·09	+ 0·01	76	4·72	4·68	+ 0·04
56	5·89	5·91	— 0·02	77	4·63	4·64	— 0·01
56	5·91	5·91	0	77	4·70	4·64	+ 0·06
56	5·80	5·91	— 0·11	81	4·46	4·48	— 0·02
57	5·85	5·82	+ 0·03	83	4·43	4·41	+ 0·02
58	5·77	5·74	+ 0·03	84	4·35	4·38	— 0·03
59	5·66	5·66	0	85	4·38	4·35	+ 0·03
60	5·64	5·59	+ 0·05	86	4·39	4·31	+ 0·08
61	5·47	5·52	— 0·05	89	4·34	4·22	+ 0·12
62	5·40	5·44	— 0·04	94	4·09	4·09	0

Mean difference.. = — 0·003 Kg. cm.
Standard deviation of differences.......................... = 0·080 Kg. cm.

By observing successive amplitudes as before, the loss of potential energy with the compression compartment empty was determined with the release stop K (Figure 1) set at six different positions corresponding to different degrees of compression. The mean of ten independent determinations at each position is given in Table 3.

The loss increased as the instrument was set for lower degrees of compression, as a result of increased extension of the springs F (Figure 1), and increased friction of the lever D against the stop K. Since no tension acted on the piston while the results of Table 3 were obtained, a further study was made of the motion of the pendulum while extending a spiral spring attached between the balancing nuts M and the base of the instrument.

TABLE 3.

The loss of potential energy of the pendulum during the first swing, with the compression compartment empty.

	Loss of Potential Energy during first swing.
Compression from 103·3 c.c. to—	Kg. cm.
55 c.c.	3·11
61 c.c.	3·15
67 c.c.	3·18
73 c.c.	3·21
79 c.c.	3·25
86 c.c.	3·28

From the load-extension curve of the spring, the work done in extending it could be determined, and from the initial length and the length at maximum extension, that portion of the loss of potential energy which was due to extension of the springs could be determined. The reduction in amplitude was observed as before, and the total loss in potential energy of the pendulum calculated by fitting equation (1). (The total loss could not be evaluated from the number of swings, as given in Table 1, since Table 1 was applicable only to the case of wool samples which offered a diminishing resistance to compression).

Each determination was repeated five times. Three springs of mean strengths of 63·5 gm./cm., 45·4 gm./cm., and 27·0 gm./cm. respectively were used, and each was stretched by six different amounts. The results are given in Table 4, where E is the total loss in potential energy of the pendulum, e is the loss with the compression compartment empty, as given in Table 3, and W is the work done in extending the springs, as calculated from the load-extension curves of the springs.

As shown in Figure 4, a linear relationship existed between W, the work done in extending the springs, and $(E-e)$, the excess of the total loss of potential energy over the loss with the compression compartment empty. The relationship could be expressed by the equation

$$W = 0\cdot674(E-e) - 0\cdot027 \quad \ldots\ldots\ldots \quad (4)$$

obtained by the method of least squares. Theoretically the line should pass through the origin of axes, but a small error is to be expected in view of the fact that no correction was applied for the work done in raising and lowering the springs, and in slightly displacing the connecting rods, as the magnitude of these factors appeared rather doubtful. The true value of the work done may possibly be given by the right hand side of equation (4) with the second term omitted. In the present investigation the equation as given was, however, employed, and the possible resulting discrepancy may be regarded as extremely small.

It appears that of the total loss of potential energy of the pendulum, just over 3 Kg. cm. was due to friction and air resistance, while of the remainder, two-thirds was due to extension of the springs. The remaining one-third must, therefore, be accounted for by an increase in the natural damping as a result of pressure on the piston. In addition, play of the levers with pressure on the piston may result in a slight further extension of the springs F (Figure 1).

TABLE 4.

The potential energy lost by the pendulum, compared with the work done in extending the springs.

Spring.	Extension (cm.).	Work done (W)	E (Kg. cm.).	e (Kg. cm.).	$E-e$ (Kg. cm.).
1........................	3·8	2·14	6·29	3·11	3·18
	3·4	1·86	5·95	3·15	2·80
	3·0	1·60	5·63	3·18	2·45
	2·7	1·40	5·32	3·21	2·11
	2·3	1·16	5·02	3·25	1·77
	2·0	1·00	4·79	3·28	1·51
2........................	3·8	1·70	5·66	3·11	2·55
	3·4	1·49	5·42	3·15	2·27
	3·1	1·34	5·20	3·18	2·02
	2·7	1·14	4·93	3·21	1·72
	2·2	0·90	4·58	3·25	1·53
	1·8	0·72	4·38	3·28	1·10
3........................	3·8	1·34	5·16	3·11	2·05
	3·4	1·18	4·98	3·15	1·83
	3·1	1·06	4·81	3·18	1·63
	2·8	0·94	4·66	3·21	1·45
	2·5	0·84	4·52	3·25	1·27
	1·6	0·52	4·10	3·28	0·82

E = total loss of potential energy of the pendulum.
e = loss of P.E. with the compression compartment empty.
W = work done in extending springs.

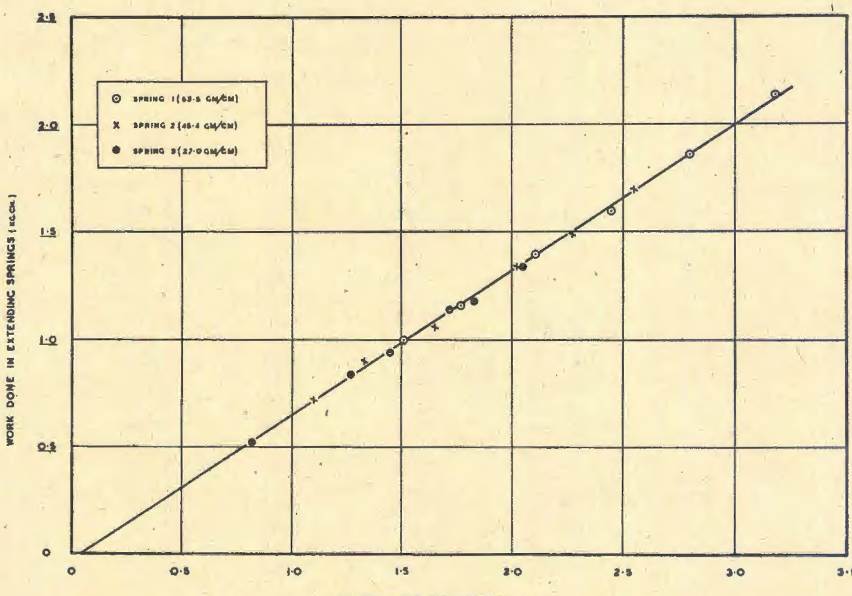

FIGURE 4.—The relation between the work done in extending the springs and the loss in potential energy of the pendulum.

Since the correction was found to be directly related to the *work done* in extending the springs, the relation as found was assumed to hold for the compression of wool samples, although it is to be noted that the pressure-volume relation of wool is totally different from the load-extension relation of the springs.

(vi) *Air resistance within the compression compartment.*

Besides offering a diminishing resistance to compression, wool differs in another important respect from a spring. While a spring is being extended, the compression compartment is empty, and the enclosed air is expelled through the space between the piston and the walls of the compartment. With five gm. of wool, however, the air has in addition to be expelled through the spaces between the fibres, and the surface area of five gm. of of wool of 20μ diameter is approximately 8×10^3 sq. cm. The more rapid the compression, therefore, the greater should be the work done by the pendulum, if the expulsion of the air offers any appreciable resistance, and since the rate of compression is dependent on the amplitude, it is to be expected that the work done should increase with the amplitude of the pendulum.

The point was investigated by starting the pendulum in succession at 48°, 45°, 42°, 39° and 36°, while compressing three wools of high, medium and low compressibility respectively. As before, the amplitudes after successive swings were noted, and the loss of potential energy of the pendulum calculated by fitting equation (1) to the observations. Each determination was carried out four times, and after the necessary corrections had been applied, the results shown in Table 5 were obtained.

TABLE 5.

The effect of the amplitude on the work done (in Kg. cm.) in compressing wool samples.

Sample.	Initial Amplitude.				
	48°	45°	42°	39°	36°
1	4·06	3·87	3·85	3·68	3·95
	3·63	3·73	3·70	3·83	3·75
	3·86	3·89	3·79	3·78	3·83
	3·78	3·78	3·66	3·77	3·75
Mean	3·83	3·82	3·75	3·77	3·82
2	2·28	2·24	2·27	2·11	2·09
	2·10	2·17	2·19	2·24	2·32
	2·21	2·26	2·32	2·26	2·27
	2·24	2·17	2·20	2·15	2·19
Mean	2·21	2·21	2·25	2·19	2·22
3	1·50	1·37	1·37	1·37	1·39
	1·52	1·46	1·41	1·40	1·36
	1·48	1·37	1·43	1·40	1·38
	1·38	1·29	1·34	1·38	1·30
Mean	1·47	1·37	1·39	1·34	1·36

In Table 6 the variation between the amplitudes is compared with the variation between the values within each amplitude, by means of the standard deviation.

TABLE 6.

Analysis of variance.

Variance.	D.F.	Standard Deviation.		
		Sample 1.	Sample 2.	Sample 3.
Between amplitudes....................	4	0·0738	0·0401	0·0875 } $z = 0{\cdot}574$.
Within amplitudes......................	15	0·1077	0·0737	0·0493 }

According to Table 6, the variation between amplitudes is less than that within amplitudes in the case of samples 1 and 2, and the variation between the amplitudes may, therefore, be directly attributed to the variation among individual determinations. In the case of sample 3, the variation between amplitudes exceeds that within amplitudes, the value of z (i.e., the natural logarithm of the ratio of the two standard deviations) being 0·574. This value suggests significance at the 5 per cent. probability level, and an examination of Table 5 shows that this is due to the high value at 48°. Now the velocity of the pendulum when started at 48° is approximately $^4/_3$ that when started at 36°, and as there is no tendency for the work done to alter in the same ratio, it can be concluded that the effect of air resistance in the compression compartment is negligible over the range of velocities examined. This conclusion is confirmed by experiments carried out later on the rate of flow of air through a plug of wool fibres, in an investigation of a method developed by Cassie (1942) for determining fibre diameter. The resistance offered by wool to the passage of air at a density of 10 c.c. per gm. (the highest density employed in the present study) is negligibly small compared to the resistance offered by a sample to compression.

As for the effect of the rate of compression on the resistance of the wool, this must also be considered negligible over the range examined, and the results obtained may be described as those applying under conditions of rapid loading as opposed to those obtained by static methods.

Table 6 shows that the standard deviation of the observations increases as the resistance to compression increases. The average standard deviation of the three samples is 0·081 Kg. cm., in good agreement with the standard deviation of the errors found in Table 2, viz., 0·080 Kg. cm.

(*c*) *Additional cylinder and piston method employed.*

While the present study has been based on results obtained with the "Pendultex" instrument, certain aspects of the elastic behaviour of wool were investigated by a method giving pressures directly.

For this purpose the simple apparatus illustrated in Figure 5 was constructed. A thick-walled glass cylinder A was let into a base B and closed at the lower end by a steel disc. The base B was firmly attached to a bench with the cylinder projecting over the edge. A steel disc C acted as the piston for compressing the wool in the cylinder, and pressure was applied by placing slotted 100 gm.weights W on a platform carried by a stiff bronze wire attached to the piston and passing through the wool along the axis of the cylinder.

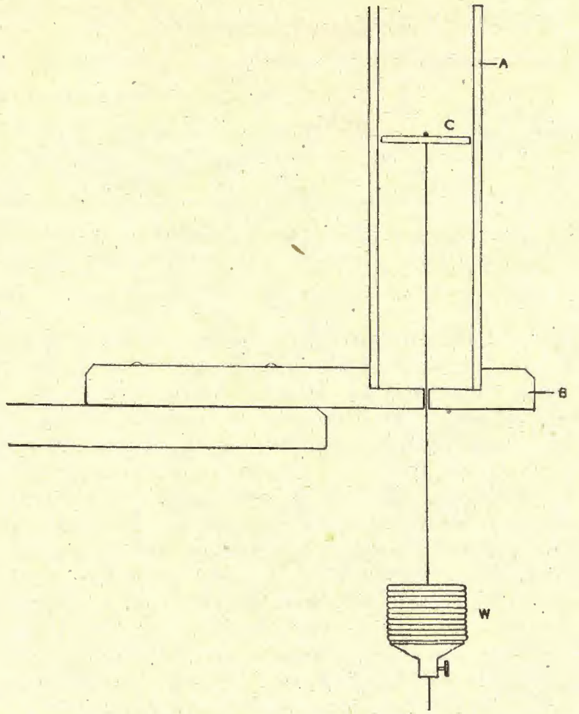

FIGURE 5.—Static cylinder and piston method employed.

For determining the volume of wool at any pressure, the height from the top of the piston to the top of the cylinder was measured at four points situated at the ends of two diameters at right angles to each other. From the area of cross-section of the cylinder and its total length, the volume occupied by the wool could be calculated. The necessary corrections were applied for the thickness of the piston and the variable volume of the wire.

The piston was so attached to the wire that it was capable of a swivel action. This gave an indication of the uniformity of packing of the wool in the cylinder, and determinations were made only when the piston lay horizontal and showed no tendency to slip sideways and bear against the walls of the cylinder. Some such method for indicating the uniformity of packing of the wool is highly desirable, whatever method is employed.

After each weight had been placed on the platform, it was found necessary to tap the base of the instrument fairly vigorously for about five minutes before the volume became constant. Previous investigators, employing other static methods, have noted the same lag in taking up the final volume. It is probably due to friction of the fibres against the walls of the containing vessel, and of the fibres among themselves.

B. The Elastic Behaviour of Wool in Bulk.

(a) *Historical.*

It has already been stated that previous investigators have found that when wool samples are taken through successive cycles of compression and release, the position of the pressure-volume curve is altered, at first rapidly and then more slowly until the curve appears to reach a constant position. This was found to be the case when the wool was compressed in the apparatus shown in Figure 5, and was shown in the case of the "Pendultex" instrument by an increase in the number of swings recorded.

Moreover, at any value of the pressure, the volume occupied during removal of the compressing force was lower than that during its application, but when the position of the curve became constant, the sample returned to its original volume on complete removal of the pressure.

While some investigators were content to draw their conclusions from the plotted curves, others measured the work done during compression and release, while two obtained a relation between pressure and volume.

M. and J. Eggert (1925) used the relation

$$\phi^{\gamma}(\pi + \pi_0) = \pi_0 \cdot 10^{\gamma}$$

where π was the pressure and ϕ ten times the ratio of the volume to the volume at zero applied pressure. The constant π_0 was regarded as a measure of the softness (Weichheit) of the wool, while γ was taken to indicate the pliability (Geschmeidigkeit).

Schofield (1938) stated that for the later points, the equation

$$\text{Pile thickness} = 12{\cdot}9\ (\text{Load})^{-0{\cdot}3}$$

very nearly fitted his experimental results.

(b) *The relation between pressure and volume.*

The Eggert equation, together with Schofield's results, suggested that the pressure should bear a linear relation to the inverse cube of the volume. The author accordingly plotted pressure as a function of the inverse cube of the volume for data given by Pidgeon and van Winsen (1934), Larose (1934) and Schofield (1938). Except for the points representing small degrees of compression, the relations were found to be linear in all cases. Typical examples are illustrated in Figure 6, where the units have been plotted arbitrarily, since the different authors used different units. It is to be noted that the curves shown were obtained by three different methods, with wool under widely different conditions.

With the cylinder and piston method illustrated in Figure 5, it was found that initially the pressure varied nearly as the inverse first power of the volume, the index increasing (negatively) with successive cycles until during the final constant cycle, the index became -3, in complete agreement with the curves illustrated in Figure 6. Observation of the wool during compression showed that initially most of the reduction in volume was taken up by that portion of the wool which was nearest to the piston, and the sample could then be regarded as behaving like a spring. With increasing pressure more and more of the wool was compressed, until the whole mass showed the same density of packing.

On the release of the pressure, it was evident that a certain amount of inter-locking of the fibres had taken place, and the friction against the walls of the cylinder prevented that portion of the wool furthest removed from the piston from opening up completely. The result was that the sample did not reach its initial volume, and the volumes at the same pressures during the following cycle of compression were considerably lower than those during the first cycle. This process was repeated during successive cycles, the reduction in initial volume becoming smaller with each cycle until the initial volume became constant. Even at this stage, however, it could be clearly observed that on removal of the pressure, the wool nearest to the piston opened up to a greater extent than that portion furthest removed from the piston.

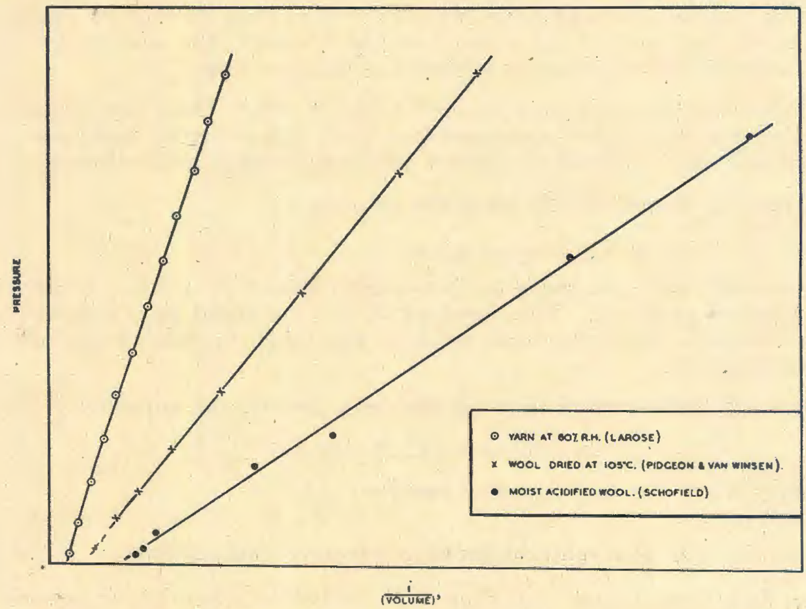

FIGURE 6.—The pressure as a function of the inverse cube of the volume for data given by Schofield (1938), Pidgeon and van Winsen (1934) and Larose (1934). (Arbitrary units and origin.)

The results of the observations on the wool during compression may be summed up as follows:—

1. The compression during the initial cycle is not uniform, since the density of packing is not uniform throughout the sample.
2. The curves obtained from successive cycles exhibit a tendency to coincide at high pressures.
3. In the final constant cycle, the results obtained at low pressures should not be considered together with those obtained at higher pressures, since the density of packing the wool is not uniform when the compressive force is completely removed, and the subsequent compression resembles that of a spring initially.
4. In the final constant cycle, the relation between pressure and volume is such that the pressure bears a linear relation to the inverse cube of the volume.

(i) *The inverse cube equation.*

The relation between pressure and volume may therefore be expressed by the equation

$$p = A \left(\frac{1}{v^3} - \frac{1}{v^3_0}\right) \ldots\ldots\ldots(5)$$

where p is the pressure at volume v, v_0 is the volume at zero applied pressure and A is a constant. Equation (5) is that of the Eggerts with $\gamma = 3$.

Hence the work done in compressing a sample from a volume v_1 to a volume v should be given by

$$W = \frac{A}{2} \left(\frac{1}{v^2} - \frac{1}{v^2_1}\right) + \frac{A}{v^3_0} (v - v_1) \ldots\ldots\ldots(6)$$

With the "Pendultex" instrument the relation was studied by selecting 5 gm. samples of Merino wools, A, B, C and D and one sample of Romney wool, E. The samples were compressed to various volumes and the work done calculated from the number of swings recorded. The means of five determinations each are given in Table 7, and illustrated in Figure 7.

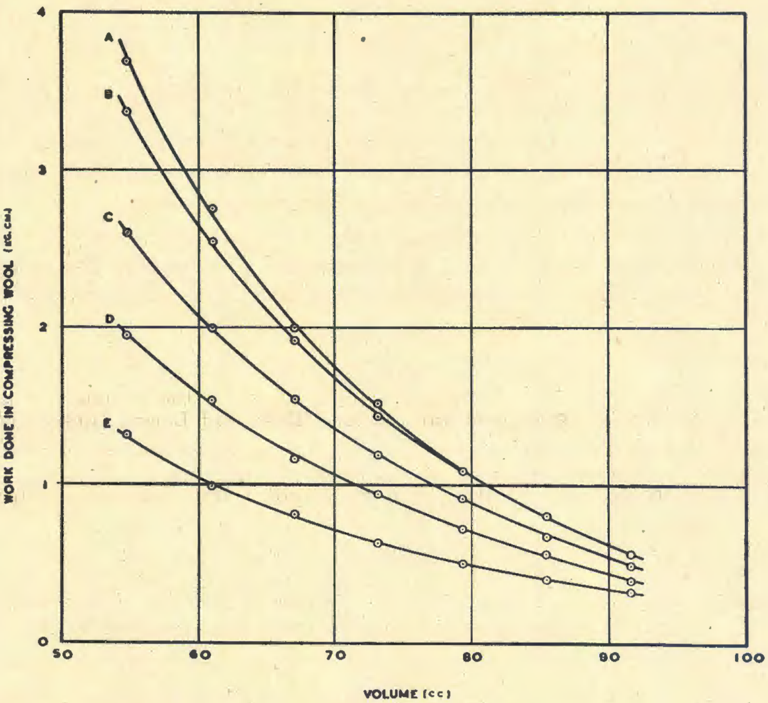

FIGURE 7.—The work done in compressing 5 gm. of different wools by the dynamic method from 103·3 c.c. to various volumes, as a function of the volume.

The result of applying equation (6) to the data of Table 7 was not satisfactory. It was obvious that the wool did not behave as if it was being compressed from the volume v_1, and when v_1 was derived from the experimental results the coefficient of the second term was positive in some cases and negative in others. As in the case of wool compressed by static methods, small compressions were not transmitted uniformly throughout the bulk of the material owing to the frictional forces in operation. The effect was greater in the case of the "Pendultex" instrument, owing to the short duration of the compression.

TABLE 7.

The work done (in Kg. cm.) in compressing 5 *gm. of different wools from* 103·3 *c.c. to various volumes in the "Pendultex" instrument.*

Final Volume. (c.c.).	Sample.				
	A.	B.	C.	D.	E.
54·8	3·69	3·37	2·60	1·95	1·32
61·0	2·76	2·55	1·98	1·54	1·00
67·1	2·00	1·92	1·55	1·17	0·82
73·2	1·52	1·44	1·20	0·95	0·64
79·4	1·09	1·09	0·92	0·73	0·51
85·5	0·81	0·81	0·68	0·57	0·41
91·7	0·57	0·58	0·50	0·40	0·33

Approximation.—In Figure 8 the work done is plotted as a function of the inverse square of the volume. It is evident that for compressions to volumes below about 75 c.c., the relation becomes linear, showing that the wool is being compressed practically uniformly, and the term containing $\frac{1}{v^2}$ in equation 6 becomes predominant.

The samples designated A and E represented very nearly the extremes in compressibility found in the present investigation, hence it was assumed that the equation

$$W = \frac{a_1}{v^2} - a_2 \qquad (7)$$

where v was the final volume, was applicable to all 5 gm. samples tested, when compressed to volumes below 75 c.c. Applying equation (7) to the five samples under consideration, and neglecting volumes above 75 c.c., the constants shown in Table 8 were obtained.

TABLE 8.

The constants a_1 and a_2 of equation (7) *evaluated for the five wools,* A, B, C, D *and* E *compressed to various volumes (see Table* 7).

Sample.	a_1	a_2	a_2 (calculated from Equation 8).	Difference.
A	14,970	1·289	1·292	+ 0·003
B	13,200	0·989	0·981	− 0·008
C	9,529	0·575	0·580	+ 0·005
D	6,927	0·347	0·351	+ 0·004
E	4,578	0·211	0·207	− 0·004

As shown by Figure 9, a simple relation held between the constants a_1 and a_2 of equation (7), viz.,

$$a_2 = 5\cdot344 \times 10^{-9} \cdot a_1^2 + 0\cdot0947 \dots\dots\dots(8)$$

The values of a_2 calculated from equation (8) are given in Table 8.

Combining (7) and (8), and writing a for a_1, the work done in compressing a five gm. sample to a volume v becomes

$$W = \frac{a}{v^2} - 5\cdot344 \times 10^{-9} \cdot a^2 - 0\cdot0947 \dots\dots\dots(9)$$

Equation (9) contains one constant, a, which is readily calculated for any values of W and v. The relation (8) between a_1 and a_2 is an approximation, but it points to some connection between a_1 and v_0.

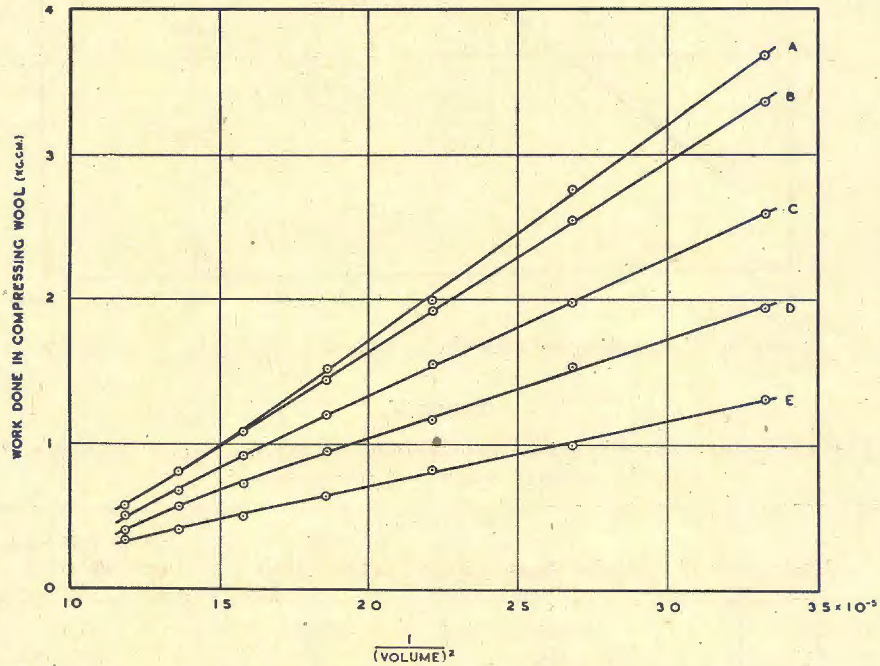

FIGURE 8.—The work done in compressing 5 gm. of different wools by the dynamic method from 103·3 c.c. to various volumes, as a function of the inverse square of the volume.

Accuracy of the approximation.—The extent to which it is possible to fit equation (7) to results which in reality must be regarded as following equation (6) may be judged from the case of sample A. The results of this sample when applied to equation (6) yield the equation

$$W = \frac{15943}{v^2} + 0\cdot007491 \cdot v - 2\cdot016 \dots\dots\dots(10)$$

Values of W calculated from equation (10) at equal intervals of $\frac{1}{v^2}$ are given in Table 9.

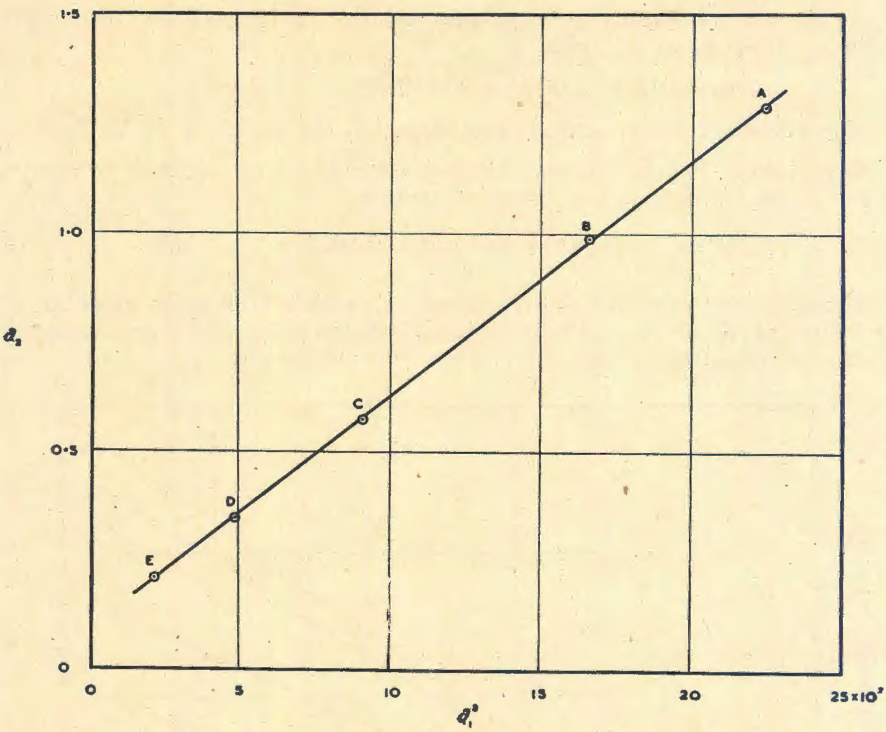

FIGURE 9—The relation between the constants a_1 and a_2 of equation 7.

TABLE 9.

The work done as calculated from equation (10) *for equal intervals of the inverse square of the volume.*

v (c.c.).	$\frac{1}{v^2}$	W (Kg. cm.).	First Difference. (Kg. cm.).
100·0	0·00010	0·33	0·25
91·3	12	0·58	0·27
84·5	14	0·85	0·27
77·5	16	1·12	0·29
74·5	18	1·41	0·29
70·7	20	1·70	0·30
67·4	22	2·00	0·29
64·6	24	2·29	0·30
62·0	26	2·59	0·31
59·8	28	2·90	0·30
57·7	30	3·20	0·30
55·9	32	3·50	0·31
54·2	34	3·81	

For compression to volumes below 75 c.c., the first differences are very nearly constant, and a linear relation between work done and the inverse square of the volume may be assumed for purposes of calculation.

Regarding the constant a as calculated from equation (9) as an approximation to the constant $\frac{A}{2}$ of equation (6), the relation between them may be obtained by differentiating both expressions for W with respect to $\frac{1}{v^2}$, and equating the results. Thus—

$$a = \frac{A}{2}(1 - \frac{v^3}{v^3_0})$$

The value of v_0, the volume of a 5 gm. sample at zero pressure, was estimated to be in the region of 160 c.c., by observing what portion of a 5 gm. sample appeared to fill the compression compartment completely without applied pressure. Equation (9) was based on values of v between 73·2 and 54·8 cc.. For these limits the factor $(1 - \frac{v^3}{v^3_0})$ has the values 0·904 and 0·960 respectively. The mean value of 0·932 suggests that the approximation gives the value of $\frac{A}{2}$ too low by about 7 per cent.

Similarly equation (10) with $\frac{A}{2} = 15943$ gives the pressure at 55 c.c. as 184 gm./sq. cm., while equation (9) with $a = 14970$ gives 170 gm./sq. cm., a difference of 7 per cent.

The differences between the observed values given in Table 7 and those calculated after fitting equation (7) have a standard deviation of 0·018 Kg. cm., which is far below those found for the errors in Tables 2 and 6, viz., 0·080 Kg. cm. The use of Table 1, after each observation had been repeated five times, consequently reduced the error considerably.

It has been shown that the results of compressibility determinations agree with the assumption that the pressure varies in a linear manner with the inverse cube of the volume. The further refinement of reducing the volume by a quantity v', the volume of wool substance, may be added, on the basis that the limiting value of the volume will theoretically be the volume of the wool substance itself, athough such a condition may not be attainable in practice. The equation

$$p = A\left(\frac{1}{(v - v')^3} - \frac{1}{(v_0 - v')^3}\right) \qquad \text{..........(5a)}$$

is then obtained. As regards agreement with experimental data there appeared to be little to choose between the two equations (5) and (5a), but for the comparison of different wools it is essential that either one or the other be adhered to.

(ii) *Exponential equation.*

A large number of empirical relations have been investigated, and an equation which provides an excellent fit will be considered in some detail, since it exhibits some interesting features.

Equations of the class

$$p = Pe^{Q\left(\frac{m}{v - v'}\right)^n} - R$$

where m is the mass, and P, Q and R, are constants, fit the observational results over a range of several values of n. For simplicity the case $n=1$ has been considered, giving the equation

$$p = Pe^{\frac{Qm}{(v - v')}} - R \quad \ldots\ldots\ldots\ldots(11)$$

Equation (11) gives a better fit than equation (5), owing to the fact that it contains three unknown constants. As will appear later, the accuracy of compressibility measurements does not justify the use of an equation containing more than two constants, and erroneous conclusions may therefore be drawn from the constants of equation (11). In this connection it is to be noted that in the present study the quantity v_0 has in no case been adopted as an observed constant in an equation, since it has been affirmed that results at higher pressures should not be considered together with those obtained at low pressures. Equation (5) is, therefore, regarded as a two-constant equation which may be written as

$$p = \frac{A}{v^3} - B \quad \ldots\ldots\ldots\ldots(5b)$$

Similarly equation (11) is regarded as containing three unknown constants, although the constant R is made up of the constants P, Q and v_0.

Applying equation (11) to the pressure-volume results of five wools as obtained by the static method, the values for the constants P, Q and R shown in Table 10 were obtained. (In fitting the equation, by the method given in Appendix A, p was taken as the independent variable, as this quantity was measured with negligible error, and the quantity $\frac{m}{(v - v')}$ as the dependent variable.

TABLE 10.

The constants P, Q and R obtained by fitting equation (11) to the results of compressing five samples by the static method.

Sample.	P	Q	R
1	179	20·0	301
2	447	17·1	798
3	526	16·8	1,048
4	473	21·2	1,060
5	613	20·3	1,270

The five samples are given in increasing order of resistance to compression, as given by the position of the pressure-volume curves and the coefficient A of equation (5). The results of Table 10 suggest that the constant Q may be independent of the sample, and that the variation found is due to experimental error.

The possibility must be considered, however, that the constancy of Q may be due to the fact that the results actually follow equation (5). A direct comparison of the constants of the two equations is made impracticable by the non-integrability of the function $e^{\frac{1}{x}}$ in finite terms. Accordingly the best-fitting values of the constants of equation (5) were calculated from the results as employed for the evaluation of the constants of equation (11) given in Table 10. From the five equations so obtained, values of $\frac{m}{(v - v')}$ for the same values of p as the experimental observations were calculated. Equation (11) was then fitted to the values so obtained. The values of P, Q and R given in Table 11 are therefore those obtained by fitting equation 11 to data which rigorously follow equation (5).

TABLE 11.

The constants P, Q and R obtained when equation (11) *is fitted to results which rigorously follow equation* (5).

Sample.	P	Q	R
1..................................	247	17·9	453
2..................................	230	22·0	436
3..................................	240	22·6	492
4..................................	262	26·4	646
5..................................	249	28·6	596

According to Table 11 the coefficient Q increases with the resistance to compression, and the supposed constancy of this coefficient in Table 10 cannot be attributed to the fact that the results follow equation (5). The coefficient P shows a tendency to be constant.

It appears reasonable to conclude that the coefficient Q is constant and independent of the sample. Taking the value 20 for Q, the values of P and R as calculated from the experimental observations are given in Table 12. The coefficient A of equation (5) is included for comparison.

TABLE 12.

The values of P and R on the assumption that Q has the value 20 *in all cases.*

Sample.	P	Q	R	A	$\frac{A}{P}$
1..............	177·2	20	297·1	$1{\cdot}198 \times 10^6$	$6{\cdot}76 \times 10^3$
2..............	300·8	20	536·4	1·971	6·55
3..............	337·1	20	712·3	2·221	6·59
4..............	545·0	20	1,186·3	3·702	6·79
5..............	631·6	20	1,303·6	4·429	7·01

When Q is given the constant value 20, the coefficient P is practically proportional to the coefficient A of equation (5) as appears from the last column of Table 12. Either of the two coefficients may therefore be regarded as suitable for expressing the compressibility of a sample. This is not the case for the coefficient P as given in Table 10, showing that a three-constant equation is not to be justified on practical grounds. Taking a fixed value for Q reduces the number of unknown constants to two.

The next point to be considered is how the value 20 for Q compares with that calculated from the results of previous investigators. The available data are those of Larose (1934) and Pidgeon and van Winsen (1934), and the coefficient Q as calculated from their results by the present author is given in Table 13.

TABLE 13.

The coefficient Q calculated from the results of previous investigators.

Author.	Sample.	Mass of Sample.	Q
		Gm.	
Larose (1934)................	Yarns at 50 per cent. relative humidity.	3·22	
	No. 1 Undyed.....................		17·0
	No. 1 Dyed........................		16·5
	No. 2 Undyed.....................		12·3
	No. 2 Dyed........................		12·5
Larose (1934)................	Yarns at 60 per cent. relative humidity.	3·25	—
	No. 1 Undyed.....................		12·2
	No. 1 Dyed........................		17·5
	No. 2 Undyed.....................		9·8
	No. 2 Dyed........................		7·7
Larose (1934)................	Yarn (different weights)..............	2·20	13·1
		3·26	13·6
		4·34	13·5
Pidgeon and van Winsen (1934)	Loose wool—		
	Dry..............................	3·5	12·8
	95 per cent. relative humidity.......	5·0	13·9
	Mean........................	....	13·2

The results of other workers tend to give considerably lower values than those obtained in the present investigation, and it is of interest to consider the possible factors which may influence the value of the coefficient Q.

1. *State of the sample.*—Sample 3 was a short-stapled wool, and after washing, the staples were found to have formed small compact lumps. A determination on the sample in this form gave the value 23·0 for Q, while after the lumps had been carefully removed, the value 16·8 was obtained. The lumps, therefore, caused an increase in the value of Q, and it may be supposed that the yarn form would do the same, but the values calculated from Larose's results on yarn are lower than those obtained here.

2. *Successive cycles.*—The values obtained for successive cycles are given in Table 14.

While the experimental error is larger, since each value except the last is based on one determination only, the trend is unmistakable. The value of Q increases with successive cycles of compression. This result does not, however, explain why the results of previous authors are lower, since Larose proceeded to the fifth or sixth cycle, while Pidgeon and van Winsen's values are based on the fourth cycle. These authors considered that the wool had by then attained a final steady condition, though it is to be noted that in the present investigation constancy was obtained at from the ninth to the sixteenth cycle.

TABLE 14.

The coefficient Q obtained for successive cycles of compression.

Cycle No.	Sample 1.	Sample 4.
1	13·7	9·0
3	—	9·9
5	16·9	17·4
7	21·5	18·4
Final	20·0	21·2

3. *Rate of loading.*—An uncontrolled factor which may influence the value of Q is the rate of loading. While the static method employed in the present study did not permit of the accurate control of the rate of loading, initial experiments in which determinations were carried out more rapidly gave the lower value of 15 for Q in the case of sample 1. The determinations from which the results of Table 10 were obtained, were performed extremely slowly with vigorous tapping of the base of the instrument, and approximately five minutes were allowed to elapse before a reading of the volume was taken at each pressure. In this way frictional effects were to some extent overcome, and allowance was made for a lag due to any other cause.

Application to dynamic method.—The pendulum method gives the work done in compressing a sample, so that the application of equation (11) involves the integration of the function

$$e^{\frac{Qm}{(v - v')}}$$

with respect to v, and this can only be done by means of an infinite series. The small number of observations also precludes the accurate evaluation of the coefficient Q. The best fitting value of Q is not, however, a critical one, as judged by the closeness of fit of the equation when both the values 20·0 and 13·3 are assumed for Q. When the value 13·2 is assumed, the ratio of the constants R and P is approximately the same for the five samples, with a mean value of 1·725. Taking $Q = 13{\cdot}2$ and $\frac{R}{P} = 1{\cdot}725$, the ratio of the coefficient a of equation (9) to the coefficient P of equation (11) is exactly the same for the five wools, viz., $1{\cdot}65 \times 10^5$. It is evident that for comparing different wools, it is immaterial which of the two equations is employed, provided the coefficient Q is assumed to be the same for all wools. This conclusion is confirmed by Table 12.

The exponential equation provides an interesting field of investigation. Its further study was, however, considered to fall outside the scope of the present investigation. Assuming the equation to fit experimental observations, the work done in compressing a sample may be evaluated by means of the tables in Appendix B.