Joint design of QC-LDPC codes for coded cooperation system with joint iterative decoding

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In this paper, we investigate joint design of quasi-cyclic low-density-parity-check (QC-LDPC) codes for coded cooperation system with joint iterative decoding in the destination. First, QC-LDPC codes based on the base matrix and exponent matrix are introduced, and then we describe two types of girth-4 cycles in QC-LDPC codes employed by the source and relay. In the equivalent parity-check matrix corresponding to the jointly designed QC-LDPC codes employed by the source and relay, all girth-4 cycles including both type I and type II are cancelled. Theoretical analysis and numerical simulations show that the jointly designed QC-LDPC coded cooperation well combines cooperation gain and channel coding gain, and outperforms the coded non-cooperation under the same conditions. Furthermore, the bit error rate performance of the coded cooperation employing jointly designed QC-LDPC codes is better than those of random LDPC codes and separately designed QC-LDPC codes over AWGN channels.

Keywords: QC-LDPC codes; coded cooperation; equivalent parity-check matrix; girth-4 cycles; joint design

1. Introduction

The basic idea of cooperative communications is that the mobile users equipped with single antenna share their antennas and form virtual antenna arrays to emulate multiple-input multiple-output (MIMO) system. Recent studies have shown that cooperation diversity gain can be achieved via cooperative communications (Laneman, Tse, & Wornell, 2004; Nosratinia, Hunter, & Hedayat, 2004; Sendonaris, Erkip, & Aazhang, 2003a, 2003b). Three main categories of protocols that support cooperative communications have been introduced, that is, amplify-and-forward (Laneman, Wornell, & Tse, 2001), detect-and-forward (Cover & Gamal, 1979) and coded cooperation (Sendonaris et al., 2003a) protocols. Because of the much higher transmission reliability, many previous studies have been concentrated on the coded cooperation protocol (Chakrabarti, De Baynast, Sabharwal, & Aazhang, 2007; Li, Yue, Khojastepour, Wang, & Madihian, 2008; Li, Yue, Wang, & Khojastepour, 2008; Razaghi & Yu, 2007; Zhang & Duman, 2005), in particular, using various low-density-parity-check (LDPC) codes (Gallager, 1963).

To further improve the performance of LDPC coded cooperation systems, many researchers have done the works on LDPC codes design for relay channels or coded cooperation. In Chakrabarti et al. (2007), LDPC codes are designed for half-duplex relay channels, their designs are based on the information theoretic random coding schemes. The exact relationships that the component LDPC code profiles must satisfy in coded relay cooperation have been derived. Based on these relationships, the density evolution algorithm is used to search for good relay code profiles. To design LDPC codes for coded cooperation, Li, Yue, Khojastepour et al. (2008) consider an efficient analysis framework, which decouples the factor graph into successive partial factor graph. They develop design methods to find the optimum code ensemble for the partial factor graph. In Razaghi and Yu (2007), bilayer-expurgated and bilayer-lengthened LDPC codes are devised to approach the theoretically promised rate of the relay cooperation strategy. The proposed bilayer LDPC codes are capable of working at two different channel parameters and two different rates. A novel physical layer network coded LDPC code structure is proposed in Li, Chen, Lin, and Vucetic (2013) for a non-orthogonal multiple access relay channel, and the code profile is optimised to approach the system achievable rate by utilising the extrinsic mutual information transfer chart. Those designed LDPC codes in Chakrabarti et al. (2007), Li et al. (2013), Li, Yue, Khojastepour et al. (2008), and Razaghi and Yu (2007) have better bit error rate (BER) or achievable rate performance in the coded cooperations; however, the encoding complexity of these designed LDPC codes is high. Meanwhile, these designed LDPC codes require much sum of memory when implemented in the hardware.

Quasi-cyclic LDPC (QC-LDPC) codes (Karimi & Banihashemi, 2012; Li, Chen, Zeng, Lin, & Fong, 2006), as a subclass of LDPC codes, are proposed in Fossorier (2004). Compared with other types of LDPC codes, these codes have linear encoding complexity and require much less memory due to the quasi-cyclic structure of their parity-check matrices. For the point-to-point (non-cooperation) system, Chen, Xu, Djurdjevic, and Lin (2004), Jiang and Lee (2009), Kang, Huang, Zhang, Zhou, and Lin (2010), and Zhang, Sun, and Wang (2013) propose various approaches to QC-LDPC codes design. In Zhang et al. (2013), girth-8 QC-LDPC codes with any block length above a lower bound are constructed via a simple inequality in terms of greatest common divisor. A class of QC-LDPC codes are constructed in Kang et al, (2010) by array dispersions of row-distance constrained matrices formed based on additive subgroups of finite fields. These codes have large minimum distances comparable to finite geometry LDPC codes. Based on the multiplicative inverses in finite fields, Shen, Fei, Liu, and Kuang (2011) design QC-LDPC codes in the coded cooperation; however, the parity-check matrices corresponding to those designed QC-LDPC codes are limited to be low triangular. It is well known that the short girth cycles decrease the BER performance. In this paper, by cancelling short girth cycles, we jointly design QC-LDPC codes employed by the source and relay in coded cooperation system. The contributions of this paper can be summarised as follows: (1) we investigate a kind of QC-LDPC codes which are constructed based on the base matrix and exponent matrix. These QC-LDPC codes have much more flexible code rate and code length compared with the formal QC-LDPC codes in Fossorier (2004). We result the joint equivalent parity-check matrix and the corresponding bilayer Tanner graph for QC-LDPC coded cooperation. (2) We jointly design the QC-LDPC codes employed by the source and relay in the coded cooperation. Short girth cycles in the joint equivalent parity-check matrix are cancelled and the BER performance is improved.

The rest of this paper is organised as follows. In Section 2, the general fundamental principle of LDPC-coded cooperation is presented. Section 3 mainly deals with joint design of QC-LDPC codes for coded cooperation. Section 4 offers a joint iterative decoding algorithm in the destination. Simulation results are given in Section 5. Finally, Section 6 concludes the whole paper.

2. System description

2.1. *LDPC-coded cooperation system*

An LDPC-coded cooperation system is depicted in Figure 1, where a code word c_1 conveying information bits encoded by the first LDPC encoder (LDPC-1) is sent simultaneously to the relay node (R) and destination node (D) over a broadcast channel, respectively. The relay decodes the incoming signal to recover the bits, which are again encoded into another distinct code word c_2 by the second LDPC encoder (LDPC-2). As both code words c_1 and c_2 are functions of their common information bits, R only sends its check bits to D over R-D channel so as to retain a high-efficient coded transmission.

Assume that the overall signals arriving in the destination can be distinguished by proper means, such as frequency/time/code division multiplexing techniques. The decoder in D performs the joint iterative decoding algorithm with respect to the two incoming signals. We also assume that the fading or noises regarding the S-D, S-R and R-D channels are independent of each other. The relay can correctly decode the received signal from the source, and an ideal coded cooperation is formed in this paper.

2.2. *Joint equivalent parity-check matrix and bilayer tanner graph*

Suppose that the source and relay employ two different LDPC codes LDPC-1 and LDPC-2 defined by the sparse parity-check matrices $(\boldsymbol{H}_1)_{M_1\times N}$ and $(\boldsymbol{H}_2)_{M_2\times(N+M_2)} = [\boldsymbol{A}_{M_2\times N} \quad \boldsymbol{B}_{M_2\times M_2}]$, respectively. We assume that the last M_2 columns of $(\boldsymbol{H}_2)_{M_2\times(N+M_2)}$ are linearly independent, whose corresponding code word bits can be equivalently viewed as "redundant bits" of LDPC-2. The relay only sends the parity-check bits to the destination.

From the viewpoint of the destination, the joint equivalent parity-check matrix $\tilde{\boldsymbol{H}}$ of the LDPC-coded cooperation satisfies

$$\tilde{\boldsymbol{H}}\boldsymbol{c} = 0, \tag{1}$$

where

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} (\boldsymbol{H}_1)_{M_1\times N} & \boldsymbol{0}_{M_1\times M_2} \\ \boldsymbol{A}_{M_2\times N} & \boldsymbol{B}_{M_2\times M_2} \end{bmatrix}, \tag{2}$$

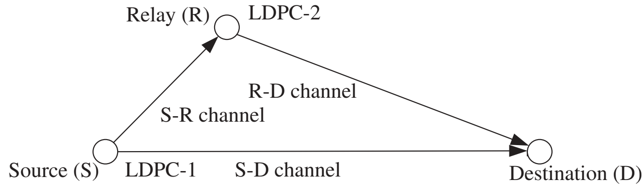

Figure 1. LDPC-coded cooperation system model.

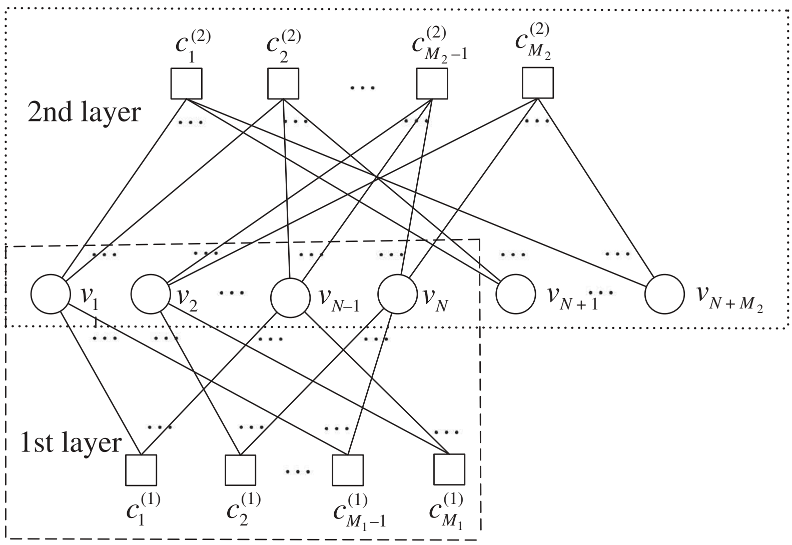

Figure 2. The bilayer Tanner graph used to characterise the joint equivalent parity-check relationship of one-relay LDPC-coded cooperation.

$$\boldsymbol{c} = \begin{bmatrix} \boldsymbol{c}_1 \\ \boldsymbol{p}_2 \end{bmatrix}. \tag{3}$$

$\boldsymbol{c}_1$ and $\boldsymbol{p}_2$ correspond to the code word of LDPC-1 and the "redundant bits" of LDPC-2.

Figure 2 illustrates the bilayer Tanner graph (Razaghi & Yu, 2007; Zhang et al., 2013) corresponding to the joint equivalent parity-check matrix $\tilde{\boldsymbol{H}}$.

The first layer of the bilayer Tanner graph associated with $(\boldsymbol{H}_1)_{M_1 \times N}$ consists of variable nodes $v_n(n = 1, \cdots, N)$ and check nodes $c_m^{(1)}(m = 1, \cdots, M_1)$. The second layer with respect to $(\boldsymbol{H}_2)_{M_2 \times (N+M_2)}$ contains variable nodes $v_n(n = 1, \cdots, N + M_2)$ and check nodes $c_m^{(2)}(m = 1, \cdots, M_2)$. It is seen in Figure 2 that $v_n(n = 1, \cdots, N)$ participate in all the check equations given by $(\boldsymbol{H}_1)_{M_1 \times N}$ and $(\boldsymbol{H}_2)_{M_2 \times (N+M_2)}$; however, $v_n(n = N + 1, \cdots, N + M_2)$ only attend the check equations given by $(\boldsymbol{H}_2)_{M_2 \times (N+M_2)}$.

2.3. *General design goal of larger girth QC-LDPC codes for coded cooperation*

Compared with random LDPC codes, QC-LDPC codes have advantages in hardware implementation of encoding and decoding. Encoding of a QC-LDPC code can be efficiently implemented using simple shift registers with complexity linearly proportional to its length. In hardware implementation of its decoder, the quasi-cyclic structure of the code simplifies the wire routing for message passing and allows partially parallel decoding.

Definition 1: A cycle is a sequence of connected variable nodes and check nodes that starts and ends at the same node in the Tanner graph and contains no vertices more than once. The girth of a cycle is the number of edges it contains.

It is known that the iterative belief propagation (BP) decoding algorithm converges to the optimal solution if the Tanner graph is free of cycles, and the short girth cycles degrade the performance of an LDPC code.

Considering the advantages of QC-LDPC codes and the influence of the short girth cycles, we adopt QC-LDPC codes in the coded cooperation and design QC-LDPC codes to cancel the short girth cycles in the corresponding bilayer Tanner graph, which will further improve the performance of the QC-LDPC coded cooperation system.

3. Joint design of QC-LDPC codes for coded cooperation

We investigate a kind of QC-LDPC codes which are constructed based on the base matrix and exponent matrix and jointly design this kind of QC-LDPC codes for the source and relay in coded cooperation system.

3.1. *QC-LDPC codes description based on the base matrix and exponent matrix*

The parity-check matrix $\boldsymbol{H}$ of a QC-LDPC code can be presented by Equation (4):

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{I}(p_{1,1}) & \boldsymbol{I}(p_{1,2}) & \cdots & \boldsymbol{I}(p_{1,L}) \\ \boldsymbol{I}(p_{2,1}) & \boldsymbol{I}(p_{2,1}) & \cdots & \boldsymbol{I}(p_{2,L}) \\ \vdots & \ddots & \vdots & \\ \boldsymbol{I}(p_{J,1}) & \boldsymbol{I}(p_{J,2}) & \cdots & \boldsymbol{I}(p_{J,L}) \end{bmatrix}, \tag{4}$$

where

$$\boldsymbol{I}(p_{j,l}) = \begin{cases} \boldsymbol{0}_{B\times B} & \text{if } p_{j,l} = 0 \\ \boldsymbol{I}_{B\times B}^{(p_{j,l})} & \text{if } 0 < p_{j,l} \le B \end{cases}, \tag{5}$$

$\boldsymbol{I}_{B\times B}^{(p_{j,l})}$ is an identity matrix $\boldsymbol{I}_{B\times B}$ with $p_{j,l}$-right-cyclic-shift.

Definition 2: Let two important matrices, associated with the parity-check matrix $\boldsymbol{H}$ in Equation (4), be defined as follows:

$$\boldsymbol{M}(\boldsymbol{H}) = \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots\cdot & b_{1,L} \\ b_{2,1} & b_{2,2} & \cdots\cdot & b_{2,L} \\ \vdots & \vdots & \ddots & \vdots \\ b_{J,1} & b_{J,2} & \cdots\cdot & b_{J,L} \end{bmatrix}, \boldsymbol{E}(\boldsymbol{H}) = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots\cdot & p_{1,L} \\ p_{2,1} & p_{2,2} & \cdots\cdot & p_{2,L} \\ \vdots & \vdots & \ddots & \vdots \\ p_{J,1} & p_{J,2} & \cdots\cdot & p_{J,L} \end{bmatrix}, \tag{6}$$

where

$$b_{j,l} = \begin{cases} 0 & \text{if } p_{j,l} = 0 \\ 1 & \text{if } p_{j,l} \ne 0 \end{cases}, \quad 1 \le j \le J, 1 \le l \le L. \tag{7}$$

The entry $p_{j,l}$ ($1 \le j \le J, 1 \le l \le L$) is given in parity-check matrix $\boldsymbol{H}$.

$\boldsymbol{M}(\boldsymbol{H})$ and $\boldsymbol{E}(\boldsymbol{H})$ in Equation (6) are respectively called the base matrix and exponent matrix of $\boldsymbol{H}$. Any QC-LDPC codes can be fully represented by its base matrix and exponent matrix. QC-LDPC codes in Fossorier (2004) can be achieved by assigning all $p_{j,l}$ ($1 \le j \le J, 1 \le l \le L$) to be non-zero.

Example 1: Assume the parity-check matrix $\boldsymbol{H}$ of a QC-LDPC code is

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{I}(1) & \boldsymbol{I}(0) & \boldsymbol{I}(0) & \boldsymbol{I}(2) \\ \boldsymbol{I}(0) & \boldsymbol{I}(3) & \boldsymbol{I}(1) & \boldsymbol{I}(0) \end{bmatrix}, \tag{8}$$

where $B = 4$.

The base matrix $\boldsymbol{M}(\boldsymbol{H})$ and exponent matrix $\boldsymbol{E}(\boldsymbol{H})$ of $\boldsymbol{H}$ in Equation (8) are

$$\boldsymbol{M}(\boldsymbol{H}) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \boldsymbol{E}(\boldsymbol{H}) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 3 & 1 & 0 \end{bmatrix}. \tag{9}$$

As $\boldsymbol{H}$ is achieved by replacing the “1”s and “0”s in $\boldsymbol{M}(\boldsymbol{H})$ with right-cyclic-shift identity matrices and zero matrices, the degree distributions for the variable and check nodes of $\boldsymbol{H}$ are the same as its base matrix $\boldsymbol{M}(\boldsymbol{H})$. Assume that the numbers of “1”s in each column and row of $\boldsymbol{M}(\boldsymbol{H})$ are d_v and d_c, respectively. The degree distributions for the variable and check nodes of $\boldsymbol{H}$ are as follows:

$$\boldsymbol{\lambda}(x) = x^{d_v - 1}, \ \boldsymbol{\rho}(x) = x^{d_c - 1}. \tag{10}$$

3.2. *Cancellation of girth-4 cycles in QC-LDPC codes*

Based on the base matrix and exponent matrix, we design the QC-LDPC codes with girth-4 cycles cancelled by two steps.

(a) The construction of the base matrix $\boldsymbol{M}(\boldsymbol{H})$
$\boldsymbol{M}(\boldsymbol{H})$ is constructed based on the requirements of row and column weights for a QC-LDPC code. The girth-4 cycles in $\boldsymbol{M}(\boldsymbol{H})$ can be completely or partially removed in the process of $\boldsymbol{M}(\boldsymbol{H})$design. Bit filling method (Campello, Modha, & Rajagopalan, 2001) can be adopted in this step. If there are no girth-4 cycles in $\boldsymbol{M}(\boldsymbol{H})$, there are no girth-4 cycles in $\boldsymbol{H}$.

(b) The construction of the exponent matrix $\boldsymbol{E}(\boldsymbol{H})$
If there are girth-4 cycles in $\boldsymbol{M}(\boldsymbol{H})$, $\boldsymbol{E}(\boldsymbol{H})$ should be designed carefully to avoid girth-4 cycles in $\boldsymbol{H}$. Assume there are unavoidable girth-4 cycles in $\boldsymbol{M}(\boldsymbol{H})$, we present Theorem 1 to design $\boldsymbol{E}(\boldsymbol{H})$ to cancel girth-4 cycles in $\boldsymbol{H}$.

Example 2: Assume a QC-LDPC code with $B = 4$, whose base matrix and exponent matrix are as follows:

$$\boldsymbol{M}(\boldsymbol{H}) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \quad \boldsymbol{E}(\boldsymbol{H}) = \begin{bmatrix} 0 & 0 & p_{1,3} & 0 \\ p_{2,1} & p_{2,2} & 0 & p_{2,4} \\ p_{3,1} & p_{3,2} & 0 & 0 \end{bmatrix}. \tag{11}$$

It is seen that there is a girth-4 cycle in $\boldsymbol{M}(\boldsymbol{H})$ and we should design $\boldsymbol{E}(\boldsymbol{H})$ to cancel the girth-4 cycles in $\boldsymbol{H}$. Assume $p_{2,1} = 1$, $p_{2,2} = 3$, $p_{3,1} = 3$, how can we choose $p_{3,2}$?

For $p_{3,2} = 1$,

$$\begin{bmatrix} \boldsymbol{I}(p_{2,1}) & \boldsymbol{I}(p_{2,2}) \\ \boldsymbol{I}(p_{3,1}) & \boldsymbol{I}(p_{3,2}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}(1) & \boldsymbol{I}(3) \\ \boldsymbol{I}(3) & \boldsymbol{I}(1) \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \end{bmatrix}. \tag{12}$$

There are girth-4 cycles in the submatrix.
For $p_{3,2} = 2$,

$$\begin{aligned} \begin{bmatrix} \boldsymbol{I}(p_{2,1}) & \boldsymbol{I}(p_{2,2}) \\ \boldsymbol{I}(p_{3,1}) & \boldsymbol{I}(p_{3,2}) \end{bmatrix} &= \begin{bmatrix} \boldsymbol{I}(1) & \boldsymbol{I}(3) \\ \boldsymbol{I}(3) & \boldsymbol{I}(2) \end{bmatrix} \\ &= \begin{bmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{bmatrix}. \end{aligned} \tag{13}$$

There are no girth-4 cycles in the submatrix.
For $p_{3,2} = 3$,

$$\begin{aligned} \begin{bmatrix} \boldsymbol{I}(p_{2,1}) & \boldsymbol{I}(p_{2,2}) \\ \boldsymbol{I}(p_{3,1}) & \boldsymbol{I}(p_{3,2}) \end{bmatrix} &= \begin{bmatrix} \boldsymbol{I}(1) & \boldsymbol{I}(3) \\ \boldsymbol{I}(3) & \boldsymbol{I}(3) \end{bmatrix} \\ &= \begin{bmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{bmatrix}. \end{aligned} \tag{14}$$

There are no girth-4 cycles in the submatrix.

For $p_{3,2} = 4$,

$$\begin{bmatrix} \boldsymbol{I}(p_{2,1}) & \boldsymbol{I}(p_{2,2}) \\ \boldsymbol{I}(p_{3,1}) & \boldsymbol{I}(p_{3,2}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}(1) & \boldsymbol{I}(3) \\ \boldsymbol{I}(3) & \boldsymbol{I}(4) \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{bmatrix}. \tag{15}$$

There are no girth-4 cycles in the submatrix.

It is seen that to cancel the girth-4 cycles in H, $p_{3,2}$ cannot be randomly selected.

$$\begin{cases} (p_{2,1} - p_{3,1}) + (p_{3,2} - p_{2,2}) = (1-3) + (1-3) = -4 \ , & -4 \mod 4 \ = 0 & \text{for } p_{3,2} = 1 \\ (p_{2,1} - p_{3,1}) + (p_{3,2} - p_{2,2}) = (1-3) + (2-3) = -3 \ , & -3 \mod 4 \ = 1 \neq 0 & \text{for } p_{3,2} = 2 \\ (p_{2,1} - p_{3,1}) + (p_{3,2} - p_{2,2}) = (1-3) + (3-3) = -2 \ , & -2 \mod 4 \ = 2 \neq 0 & \text{for } p_{3,2} = 3 \\ (p_{2,1} - p_{3,1}) + (p_{3,2} - p_{2,2}) = (1-3) + (4-3) = -1 \ , & -1 \mod 4 \ = 3 \neq 0 & \text{for } p_{3,2} = 4 \end{cases}. \tag{16}$$

It should satisfy

$$(p_{2,1} - p_{3,1}) + (p_{3,2} - p_{2,2}) \mod 4 \ \neq \ 0. \tag{17}$$

Furthermore, we deduce and prove Theorem 1 to cancel girth-4 cycles in QC-LDPC codes.

Theorem 1: Assume b_{j_k,l_k}, b_{j_{k+1},l_k}, $b_{j_k,l_{k+1}}$ and $b_{j_{k+1},l_{k+1}}$ $(j_k \neq j_{k+1}, l_k \neq l_{k+1})$ in $\boldsymbol{M}(\boldsymbol{H})$ are equal to 1, which form a girth-4 cycle. Their corresponding right-cyclic-shift values in $\boldsymbol{E}(\boldsymbol{H})$ are p_{j_k,l_k}, p_{j_{k+1},l_k}, $p_{j_k,l_{k+1}}$ and $p_{j_{k+1},l_{k+1}}$. To avoid girth-4 cycles in $\boldsymbol{H}$, a necessary and sufficient condition that should be satisfied is

$$\left(p_{j_k,l_k} - p_{j_{k+1},l_k}\right) + \left(p_{j_{k+1},l_{k+1}} - p_{j_k,l_{k+1}}\right) \neq 0 \mod B. \tag{18}$$

Please see the proof in Appendix.

The above QC-LDPC codes design method can be targeted to a specific code length related to the application. The LDPC code in DVB-S2 has normal frame with code length of 64,800 and short frame with code length of 16,200, whose code rates range from 1/4 to 9/10 and from 1/4 to 8/9, respectively. For example, we will achieve a QC-LDPC code with length of 64,800 and rate of 1/2, whose parity-check matrix is $\boldsymbol{H}_{32400\times64800}$. First, we construct the base matrix $\boldsymbol{M}(\boldsymbol{H})$ with size of 162×324 and 3 "1"s and 6 "1"s in each column and row. Second, according to Theorem 1, we design the exponent matrix $\boldsymbol{E}(\boldsymbol{H})$ with size of 162×324, whose element values are selected from $0, 1, 2 \ldots B$. $B = 200$. Finally, based on $\boldsymbol{M}(\boldsymbol{H})$ and $\boldsymbol{E}(\boldsymbol{H})$, we obtain the parity-check matrix $H_{32400\times64800}$ with girth-4 cycles cancelled.

3.3. *Cancellation of girth-4 cycles in QC-LDPC codes for coded cooperation*

To improve the performance of the coded cooperation employing QC-LDPC codes, the girth-4 cycles should be cancelled. We jointly design QC-LDPC codes for coded cooperation by two parts as follows.

(a) **The construction of $\boldsymbol{M}(\tilde{\boldsymbol{H}})$**

According to the restricted condition such as the code rate and code length in the source or relay, the based matrices corresponding to codes employed by the source and relay are independently constructed as $\boldsymbol{M}(\boldsymbol{H}_1)$ and $\boldsymbol{M}(\boldsymbol{H}_2)$. Furthermore, the base matrix of the joint equivalent parity-check matrix $\tilde{\boldsymbol{H}}$ is resulted as

$$\boldsymbol{M}(\tilde{\boldsymbol{H}}) = \begin{bmatrix} \boldsymbol{M}(\boldsymbol{H}_1) & 0 \\ \boldsymbol{M}(\boldsymbol{A}) & \boldsymbol{M}(\boldsymbol{B}) \end{bmatrix}. \tag{19}$$

There are two types of girth-4 cycles in $\boldsymbol{M}(\tilde{\boldsymbol{H}})$. One type is in $\boldsymbol{M}(\boldsymbol{H}_1)$ or $\boldsymbol{M}(\boldsymbol{H}_2)$, the other type is between $\boldsymbol{M}(\boldsymbol{H}_1)$ and $\boldsymbol{M}(\boldsymbol{H}_2)$

Example 3: Two types of girth-4 cycles in $\boldsymbol{M}(\tilde{\boldsymbol{H}})$ are shown as follows:

$$\boldsymbol{M}(\tilde{\boldsymbol{H}}) = \begin{bmatrix} \boldsymbol{M}(\boldsymbol{H}_1) & \boldsymbol{0} \\ \boldsymbol{M}(\boldsymbol{A}) & \boldsymbol{M}(\boldsymbol{B}) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

- - - Type-I: girth-4 cycles in $\boldsymbol{M}(\boldsymbol{H}_1)$ or $\boldsymbol{M}(\boldsymbol{H}_2)$

—— Type-II: girth-4 cycles between $\boldsymbol{M}(\boldsymbol{H}_1)$ and $\boldsymbol{M}(\boldsymbol{H}_2)$

We propose an algorithm to jointly design QC-LDPC codes for the source and relay, where there are neither type I nor type II girth-4 cycles in $\tilde{\boldsymbol{H}}$.

Similar to the base matrix of $\tilde{\boldsymbol{H}}$, the exponent matrix of $\tilde{\boldsymbol{H}}$ is achieved as

$$\boldsymbol{E}(\tilde{\boldsymbol{H}}) = \begin{bmatrix} \boldsymbol{E}(\boldsymbol{H}_1) & \boldsymbol{0} \\ \boldsymbol{E}(\boldsymbol{A}) & \boldsymbol{E}(\boldsymbol{B}) \end{bmatrix}. \tag{21}$$

(b) **The construction of $\boldsymbol{E}(\tilde{\boldsymbol{H}})$.**

In this part, we propose Algorithm 1 to jointly construct $\boldsymbol{E}(\tilde{\boldsymbol{H}})$

Algorithm 1 Jointly design QC-LDPC codes by cancelling girth-4 cycles for coded cooperation

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Jointly design $\boldsymbol{E}(\boldsymbol{H_1})$ and $\boldsymbol{E}(\boldsymbol{A})$

Step 1. Generate the first column of $\boldsymbol{E}(\tilde{\boldsymbol{H}})$. Based on $\boldsymbol{M}(\tilde{\boldsymbol{H}})$, replace the "1"s in the first column of $\boldsymbol{M}(\tilde{\boldsymbol{H}})$ by some integers randomly selected from $[1, 2, \cdots, B]$.

Step 2. Generate the second column of $\boldsymbol{E}(\tilde{\boldsymbol{H}})$.

- **2.1** Replace the first "1" in the second column of $\boldsymbol{M}(\tilde{\boldsymbol{H}})$ by one integer p_{j_1,l_2} randomly selected from $[1, 2, \cdots, B]$.
- **2.2** Replace the second "1" in the second column of $\boldsymbol{M}(\tilde{\boldsymbol{H}})$. That "1" is not replaced by p_{j_2,l_2} randomly selected from $[1, 2, \cdots, B]$ anymore. It should satisfy Theorem 1.
- **2.3** Replace all the "1"s in the second column of $\boldsymbol{M}(\tilde{\boldsymbol{H}})$ using the same method.

Step 3. Repeat step 2 to generate from the third to the last columns of $\boldsymbol{E}(\boldsymbol{H_1})$ and $\boldsymbol{E}(\boldsymbol{A})$ according to Theorem 1.

By jointly designing $\boldsymbol{E}(\boldsymbol{H_1})$ and $\boldsymbol{E}(\boldsymbol{A})$, type I girth-4 cycles in $\boldsymbol{H}_1$ and type II girth-4 cycles between $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ are cancelled.

Jointly design $\boldsymbol{E}(\boldsymbol{A})$ and $\boldsymbol{E}(\boldsymbol{B})$

For $\boldsymbol{E}(\boldsymbol{A})$ have been constructed, now we construct $\boldsymbol{E}(\boldsymbol{B})$ to cancel type I girth-4 cycles in $\boldsymbol{H}_2$. which are between $\boldsymbol{A}$ and $\boldsymbol{B}$. According to Theorem 1, $\boldsymbol{E}(\boldsymbol{B})$ can be constructed column by column via considering $\boldsymbol{E}(\boldsymbol{A})$ as the former part which has been constructed.

Based on $\boldsymbol{M}(\tilde{\boldsymbol{H}})$ and $\boldsymbol{E}(\tilde{\boldsymbol{H}})$ respectively designed in parts (a) and (b), we obtain the joint equivalent parity-check matrix $\tilde{\boldsymbol{H}}$ with both type I and type II girth-4 cycles cancelled.

The joint equivalent parity-check matrix corresponding to QC-LDPC codes for the source and relay in the coded cooperation is intuitively depicted in Figure 3.

The degree distributions for the variable and check nodes of $\tilde{\boldsymbol{H}}$ are the same as its base matrix $\boldsymbol{M}(\tilde{\boldsymbol{H}})$. Assume that $\boldsymbol{M}(\boldsymbol{H}_1)$ is an $m_1 \times n_1$ matrix and the numbers of "1"s in each

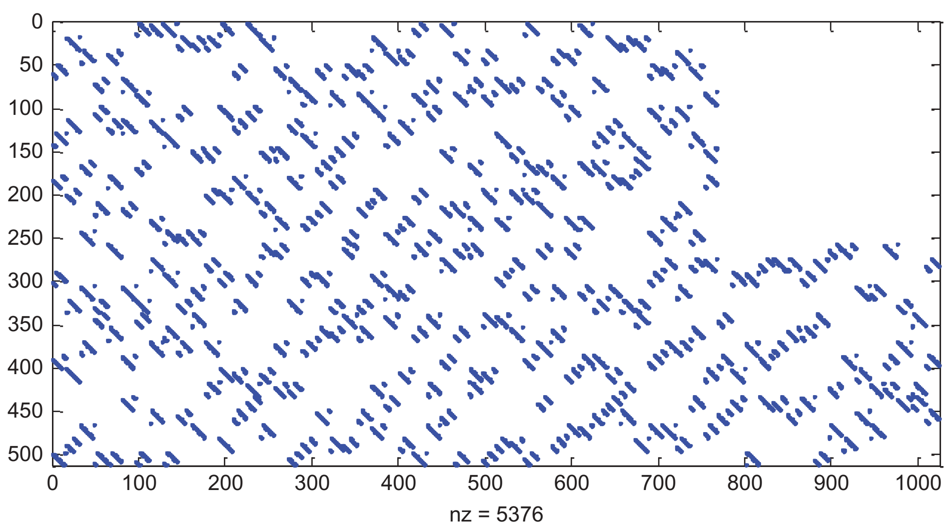

Figure 3. Illustration of joint equivalent parity-check matrix corresponding to QC-LDPC codes for the source and relay in the coded cooperation.

column and row are d_{v1} and d_{c1}. Similarly, $\boldsymbol{M}(\boldsymbol{H}_2)$ is an $m_2 \times (n_1 + m_2)$ matrix and the numbers of "1"s in each column and row are d_{v2} and d_{c2}. The degree distributions for the variable and check nodes of $\tilde{\boldsymbol{H}}$(or $\boldsymbol{M}(\tilde{\boldsymbol{H}})$) are achieved as follows:

$$\lambda(x) = \frac{(d_{v1} + d_{v2})n_1}{(d_{v1} + d_{v2})n_1 + d_{v2}m_2} x^{d_{v1}+d_{v2}-1} + \frac{d_{v2}m_2}{(d_{v1} + d_{v2})n_1 + d_{v2}m_2} x^{d_{v2}-1} \tag{22}$$

$$\rho(x) = \frac{d_{c1}m_1}{d_{c1}m_1 + d_{c2}m_2} x^{d_{c1}-1} + \frac{d_{c2}m_2}{d_{c1}m_1 + d_{c2}m_2} x^{d_{c2}-1}. \tag{23}$$

3.4. *Cancellation of large girth cycles in QC-LDPC codes for coded cooperation*

We have described the cancellation of girth-4 cycles of QC-LDPC codes for coded cooperation. In this part, we extend to cancel larger girth (e.g. girth-6, girth-8 or girth-10) cycles. The algorithm is similar to Algorithm 1; however, it should be according to the following condition (Fossorier, 2004) rather than Theorem 1.

Assuming a girth-G (G is larger than 4) cycle in $\boldsymbol{M}(\tilde{\boldsymbol{H}})$, the corresponding right-cyclic-shift values in the exponent matrix $\boldsymbol{E}(\tilde{\boldsymbol{H}})$ are $p_{j_1,l_1}, p_{j_2,l_1}, p_{j_2,l_2}, \cdots, p_{j_{G/2},l_{G/2}}, p_{j_1,l_{G/2}}$ To avoid them forming a girth-G cycles in $\tilde{\boldsymbol{H}}$, a necessary and sufficient condition that should be satisfied is

$$\sum_{g=1}^{G/2} \left(p_{j_g,l_g} - p_{j_{g+1},l_g}\right) \neq 0 \mod \mathrm{B} \tag{24}$$

where $p_{j_{G/2+1},l_{G/2}}$ is defined as $p_{j_1,l_{G/2}}$.

3.5. *Cancellation of girth-4 cycles in QC-LDPC codes for multi-relay coded cooperation*

In this part, we extend the cancellation of girth-4 cycles in QC-LDPC codes for single relay coded cooperation to multiple relay coded cooperation. Assume there are W relays noted as $\mathrm{R}_1, \cdots, \mathrm{R}_W$. Relays encode the messages from the source by LDPC-R_1, $\cdots$, LDPC-R_W to generate additional parity-check bits, which are finally transmitted to D by TDMA manner. The parity-check matrix of LDPC-R_w is given as

$$\boldsymbol{H}_{\mathrm{R}_w} = [\boldsymbol{A}_w \quad \boldsymbol{B}_w]. \tag{25}$$

As in Section 2.2, the joint equivalent parity-check matrix is resulted as (Zhang, Yang, & Tang, 2013a)

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{A}_1 & \boldsymbol{B}_1 & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{A}_2 & \boldsymbol{0} & \boldsymbol{B}_2 & \cdots & \boldsymbol{0} \\ \cdots & \cdots & \cdots & \cdots & \\ \boldsymbol{A}_W & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{B}_W \end{bmatrix}. \tag{26}$$

Furthermore, the base matrix and the exponent matrix of the joint equivalent parity-check matrix $\tilde{\boldsymbol{H}}$ are resulted as

$$\boldsymbol{M}(\tilde{\boldsymbol{H}}) = \begin{bmatrix} \boldsymbol{M}(\boldsymbol{H}_1) & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{M}(\boldsymbol{A}_1) & \boldsymbol{M}(\boldsymbol{B}_1) & \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{M}(\boldsymbol{A}_2) & \mathbf{0} & \boldsymbol{M}(\boldsymbol{B}_2) & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \boldsymbol{M}(\boldsymbol{A}_W) & \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{M}(\boldsymbol{B}_W) \end{bmatrix} \tag{27}$$

$$\boldsymbol{E}(\tilde{\boldsymbol{H}}) = \begin{bmatrix} \boldsymbol{E}(\boldsymbol{H}_1) & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{E}(\boldsymbol{A}_1) & \boldsymbol{E}(\boldsymbol{B}_1) & \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{E}(\boldsymbol{A}_2) & \mathbf{0} & \boldsymbol{E}(\boldsymbol{B}_2) & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \boldsymbol{E}(\boldsymbol{A}_W) & \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{E}(\boldsymbol{B}_W) \end{bmatrix}. \tag{28}$$

The base matrices corresponding to QC-LDPC codes employed by the source and relays are independently constructed as $\boldsymbol{M}(\boldsymbol{H}_1)$ and $\boldsymbol{M}(\boldsymbol{H}_{\mathrm{R}_w}) = [\boldsymbol{M}(\boldsymbol{A}_w)\ \ \boldsymbol{M}(\boldsymbol{B}_w)]$ $(w = 1, \cdots, W)$. $\boldsymbol{M}(\tilde{\boldsymbol{H}})$ is obtain as in Equation (27), and we further construct the corresponding $\boldsymbol{E}(\tilde{\boldsymbol{H}})$ in Equation (28). There are two kinds of girth-4 cycles, that is, type I girth-4 cycles in $\boldsymbol{H}_1$ or $\boldsymbol{H}_{\mathrm{R}_w}(w = 1, \cdots, W)$, and type II girth-4 cycles between any two of $\boldsymbol{H}_1, \boldsymbol{H}_{\mathrm{R}_1}, \cdots, \boldsymbol{H}_{\mathrm{R}_W}$.

As described in Algorithm 1, $\boldsymbol{E}(\tilde{\boldsymbol{H}})$ is jointly constructed by the following two steps: (1) we jointly design $\boldsymbol{E}(\boldsymbol{H}_1), \boldsymbol{E}(\boldsymbol{A}_w)$ $(w = 1, \cdots, W)$ to cancel type I girth-4 cycles in $\boldsymbol{H}_1$ or $\boldsymbol{H}_{\mathrm{R}_w}(w = 1, \cdots, W)$ and type II girth-4 cycles between any two of them. (2) We jointly design $\boldsymbol{E}(\boldsymbol{A}_w)$,$\boldsymbol{E}(\boldsymbol{B}_w)$ $(w = 1, \cdots, W)$ to cancel type I girth-4 cycles in $\boldsymbol{H}_{\mathrm{R}_w}(w = 1, \cdots, W)$. According to Theorem 1, $\boldsymbol{E}(\boldsymbol{B}_w)$ $(w = 1, \cdots, W)$ can be constructed column by column via considering $\boldsymbol{E}(\boldsymbol{A}_w)$ $(w = 1, \cdots, W)$ as the former parts which have been constructed. Finally, we obtain the joint equivalent parity-check matrix $\tilde{\boldsymbol{H}}$ with both type I and type II girth-4 cycles cancelled.

4. Joint iterative decoding for QC-LDPC coded cooperation

LDPC codes deliver excellent performance when decoded by standard BP; however, it has high implementation complexity and is not suitable to be used in the hardware. Fortunately, a so-called simplified BP-based known as "Min-Sum" algorithm (Fossorier, Mihaljevic, & Imai, 1999) is introduced. It greatly reduces the implementation complexity without much degradation in decoding performance, and it is much easier to be implemented in the hardware. We employ the joint "Min-Sum" iterative decoding (Yang, Chen, Zong, Zhang, & Zhang, 2011; Zhang et al. 2013a; Zhang, Yang, Tang, & Maharaj, 2013b) based on the bilayer Tanner graph, which is associated with double QC-LDPC codes used by the source and relay.

In the bilayer Tanner graph shown in Figure 2, the set $C(v_n)$ contains all the check nodes in both layers related to the variable node v_n. $V(c_m^{(i)})$ $(i = 1,\ 2)$ is the set of all variable nodes in both layers associated with $c_m^{(i)}$. Let the output sequences of the received signals after the matched filter in the destination associated with the source and relay be $\boldsymbol{y}^{(1)} = (y_1, \ldots,\ y_N)$ and $\boldsymbol{y}^{(2)} = (y_{N+1}, \ldots,\ y_{N+M_2})$, respectively. Let $\boldsymbol{y} = (Re(\boldsymbol{y}^{(1)}),$

$Re(\boldsymbol{y}^{(2)})$), where $Re(\ )$ is the e real part function. $\boldsymbol{y}$ is directly applied to the joint iterative decoding. The entire code word related to $\boldsymbol{y}$ is $\boldsymbol{d} = (d_1, \ \ldots, \ d_{N+M_2})$. The code word is modulated by binary phase shift keying (BPSK) in the following.

Define $Lq_{m,n}^{(i)}$ as the extrinsic information from a variable node v_n to an incident check node $c_m^{(i)}$ and $Lr_{m,n}^{(i)}$ as the extrinsic information from a check node $c_m^{(i)}$ to an incident variable node v_n. Based on the bilayer Tanner graph, the joint "Min-Sum" iterative decoding algorithm is summarised as follows.

Preparations Initially, the decoder in the destination only obtains the received signals and does not have any prior information from the check nodes. Each bit n is assigned a log-likelihood ratio (LLR)

$$Lp_n = \log \frac{P(d_n = 0|y_n)}{P(d_n = 1|y_n)} \quad (n = 1, \ \cdots, \ N + M_2). \tag{29}$$

Notes: For the joint "Min-Sum" algorithm, Lp_n in Equation (29) can be further evaluated as (Zhang et al., 2013a)

$$Lp_n = y_n. \tag{30}$$

Step 1 (*Initialisation*): Before commencing the iterative decoding, $Lq_{m,n}^{(i)}$ can be initialised as Lp_n in Equation (30).

Step 2 (*Horizontal process*): The extrinsic information $Lr_{m,n}^{(i)}$ sent from a check node $c_m^{(i)}$ to an incident variable node v_n is evaluated as

$$Lr_{m,n}^{(i)} = \left(\prod_{v_{n'} \in V(c_m^{(i)}) \setminus v_n} sign(Lq_{m,n'}^{(i)}) \right) \times \min_{v_{n'} \in V(c_m^{(i)}) \setminus v_n} (|Lq_{m,n'}^{(i)}|) \Bigg). \tag{31}$$

where $sign(\ )$ and $min(\ )$ are the sign function and minimal function, respectively. The updated extrinsic information $Lr_{m,n}^{(i)}$ from the check nodes $c_m^{(i)}$ in the first $(i = 1)$ or second $(i = 2)$ layer of the bilayer Tanner graph is resulted.

Step 3 (*Vertical process*): Update the extrinsic information $Lq_{m,n}^{(i)}$ sent from a variable node v_n to an incident check node $c_m^{(i)}$.

(a) For $i = 1$, this implies that the extrinsic information $Lq_{m,n}^{(1)}$ is sent from a variable node v_n to an incident check node in the first layer of the bilayer Tanner graph.

$$Lq_{m,n}^{(1)} = Lp_n + \sum_{c_k^{(1)} \in C(v_n) \setminus c_m^{(1)}} Lr_{k,n}^{(1)} + \sum_{c_l^{(2)} \in C(v_n)} Lr_{l,n}^{(2)}. \tag{32}$$

(b) For $i = 2$, this means that the extrinsic information $Lq_{m,n}^{(2)}$ is sent from a variable node v_n to an incident check node in the second layer of the bilayer Tanner graph. Similarly,

$$Lq_{m,n}^{(2)} = Lp_n + \sum_{c_k^{(1)} \in C(v_n)} Lr_{k,n}^{(1)} + \sum_{c_l^{(2)} \in C(v_n) \backslash c_m^{(2)}} Lr_{l,n}^{(2)}. \tag{33}$$

Step 4 (*Final decision*): Repeat steps 2 and 3 until the maximum number of decoding iterations is reached. The *a posterior* LLR concerning each code word bit is calculated as

$$\mathrm{R}_n = Lp_n + \sum_{c_k^{(1)} \in C(v_n)} Lr_{k,n}^{(1)} + \sum_{c_l^{(2)} \in C(v_n)} Lr_{l,n}^{(2)} \quad (n = 1, \ \cdots, \ N + M_2). \tag{34}$$

Therefore, the final decoded block of $N + M_2$ bits is resulted as

$$\hat{d}_n = \begin{cases} 0, & \mathrm{R}_n \geq 0 \\ 1, & \mathrm{R}_n < 0 \end{cases} \quad (n = 1, \ \ldots, N + M_2). \tag{35}$$

The joint “Min-Sum” iterative decoding algorithm for the jointly designed QC-LDPC coded cooperation can greatly accelerate the decoding convergence for the following two reasons: (1) compared to the traditional decoding algorithm (Chakrabarti et al., 2007), the extrinsic information in the bilayer Tanner graph is exchanged sufficiently in the joint “Min-Sum” iterative decoding algorithm, hence it can accelerate the decoding convergence. (2) In the equivalent parity-check matrix corresponding to the jointly designed QC-LDPC codes employed by the source and relay, all girth-4 cycles including both type I and type II are cancelled. It can also accelerate the convergence of decoding in the destination.

The traditional decoding algorithm in the destination has to employ two decoders for the received signals from S and R, first, one decoder decodes signal from R, and then the other decodes signal from S with the help of the decoding result from the first one. In the joint “Min-Sum” iterative decoding algorithm, just one decoder is needed in the destination on basis of bilayer Tanner graph. It greatly reduces the computation complexity of the decoding in the destination. Furthermore, the joint “Min-Sum” iterative decoding algorithm greatly accelerates the decoding convergence which reduces the iteration times and further reduces the decoding complexity.

5. Simulation results

In this section, numerical simulations are performed to investigate the performance of ideal coded cooperation employing jointly designed QC-LDPC codes. The joint “Min-Sum” iterative decoding algorithm and BPSK modulation are assumed in the destination. Two different channel models are considered in our simulations: (a) the S-D and R-D links are independent additive white Gaussian noise (AWGN) channels with different signal-to-noise ratios (SNR). Furthermore, a reasonable assumption that SNR of the signal from the source is 1 dB less than that of the signal from the relay is made in the simulation. (b) Rayleigh fading scenario is assumed. The S-D and R-D links are independent Rayleigh block fading channels. The fading coefficient for each channel remains constant over a code word, and the average received SNRs of the signals from S and R are the same. QC-LDPC codes employed by the source and relay are given in Table 1.

Table 1. QC-LDPC codes employed in simulations.

	LDPC codes for source	LDPC codes for relay
QC-LDPC coded cooperation	$\boldsymbol{M}(\boldsymbol{H}_1)$: 16×48, $B = 16$. $d_v = 3$, $d_c = 9$	$\boldsymbol{M}(\boldsymbol{H}_2)$: 16×64, $B = 16$. $d_v = 3$, $d_c = 12$
Random LDPC coded cooperation	Code length $N_1 = 768$, code rate $r_1 = 2/3$, $d_v = 3$, $d_c = 9$.	Code length $N_2 = 1024$, code rate $r_2 = 3/4$, $d_v = 3$, $d_c = 12$.

Note: d_v is the number of ones in each column; d_c is the number of ones in each row.

5.1. *BER performance of QC-LDPC coded cooperation versus that of the non-cooperation over AWGN channels*

Jointly designed QC-LDPC codes with girth-4 cycles cancelled are employed in the coded cooperation. From the viewpoint of the destination, equal QC-LDPC codes as the coded cooperation are employed by the non-cooperation.

Figure 4 shows that the BER performance of the coded cooperation clearly outperforms the non-cooperation over AWGN channels. The significant gain can be owned to the deployment of relay that can substantially increase the SNR of the partially received signal from the relay and the high-efficient joint "Min-Sum" iterative decoding algorithm.

5.2. *Comparison of coded cooperation employing jointly designed QC-LDPC codes and other LDPC codes over AWGN channels*

In AWGN channels scenario, we compare the BER performance of coded cooperation employing jointly designed QC-LDPC codes with that of coded cooperation employing

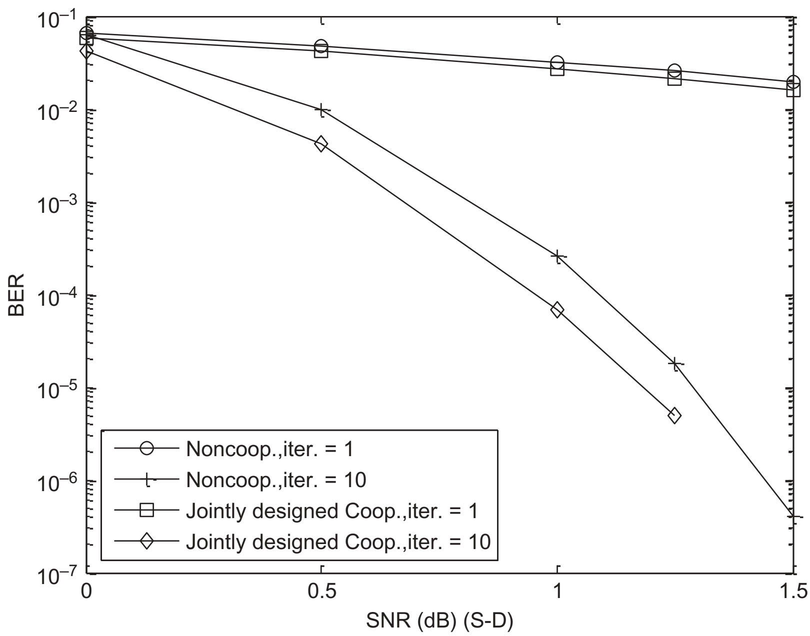

Figure 4. BER performance of jointly designed QC-LDPC coded cooperation and non-cooperation over AWGN channels with 1 or 10 decoding iterations in the destination.

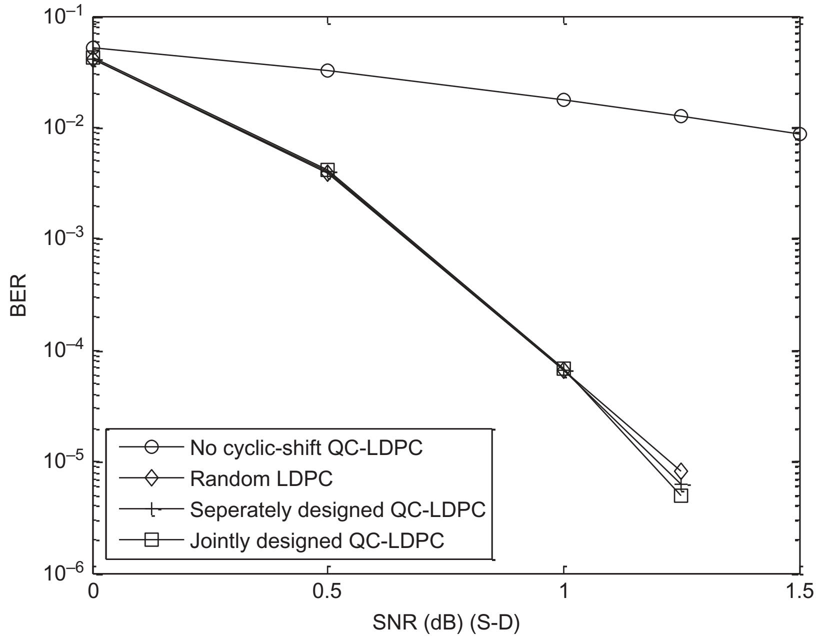

Figure 5. BER comparison of coded cooperation employing jointly designed QC-LDPC codes, no cyclic-shift QC-LDPC code and random LDPC codes over AWGN channels with 10 decoding iterations in the destination.

separately designed QC-LDPC codes, no cyclic-shift QC-LDPC codes or random LDPC codes. Here, the separately designed QC-LDPC codes for the source and relay have the same base matrices as the jointly designed ones; however, their exponent matrices are designed separately, which means that only type I girth-4 cycles are cancelled. The no cyclic-shift QC-LDPC codes for the source and relay also have the same base matrices as the jointly designed ones; however, all the ones in the base matrices are replaced by the identity matrices without cyclic-shift, which result in many girth-4 cycles. Random LDPC codes for the source or relay have the same length and code rate as their corresponding QC-LDPC codes.

As shown in Figure 5, we can see that the BER performance of the coded cooperation employing no cyclic-shift QC-LDPC codes is much worse than that of other LDPC codes. This is because of too many girth-4 cycles in this kind of codes. In relatively high SNR range, the BER performance of separately designed QC-LDPC codes is even slightly better than that of random LDPC codes because type I girth-4 cycles are cancelled for the separate designed QC-LDPC codes. Due to the cancellation of type I and type II girth-4 cycles, the BER performance of the jointly designed QC-LDPC codes is the best.

5.3. *BER performance of jointly designed QC-LDPC coded cooperation versus that of non-cooperation over Rayleigh fading channels*

In this part, Rayleigh fading scenario is assumed. We investigate the BER performance of jointly designed QC-LDPC coded cooperation versus that of the non-cooperation over Rayleigh fading channels. The jointly designed QC-LDPC codes are the same as in

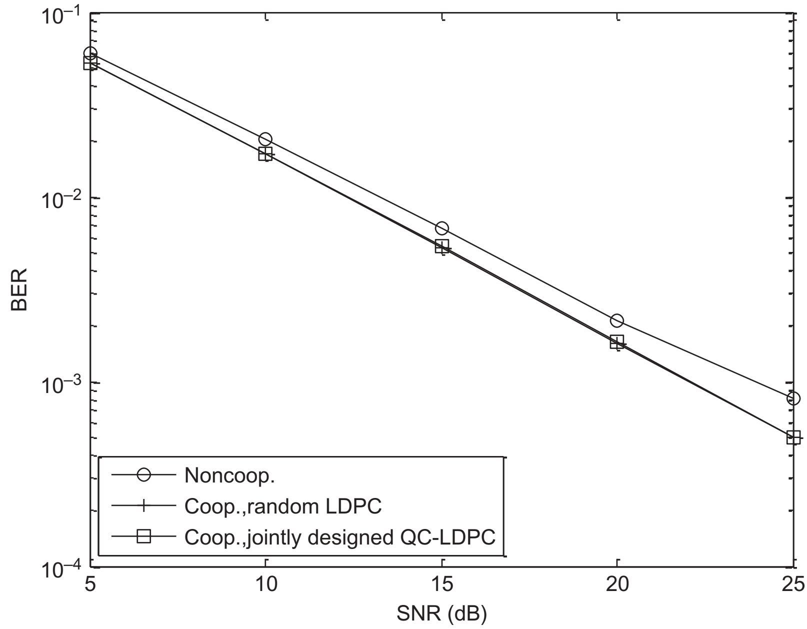

Figure 6. BER performance of non-cooperation, coded cooperation employing jointly designed QC-LDPC codes or employing random LDPC codes over Rayleigh fading channels.

Section 5.1, and the number of joint decoding iteration is 10. As shown in Figure 6, the performance of the coded cooperation clearly outperforms the non-cooperation over Rayleigh fading channels. For instance, at a BER of 10^{-3}, the coded cooperation achieves about 2 dB gain over its respective non-cooperation. The significant gain can be owned to the fact as follows: two signals from the source and relay, which are through independent fading channels, are jointly decoded by the high-efficient joint "Min-Sum" iterative decoding algorithm, hence it can dramatically overcome the signals fading to achieve the diversity gain.

In Figure 6, we also compare BER performance of coded cooperation employing the jointly designed QC-LDPC codes with that of random LDPC codes over Rayleigh fading channels. It is shown that the system employing jointly designed QC-LDPC codes can achieve similar BER to the system employing random LDPC codes. However, the jointly designed QC-LDPC codes have advantages in hardware implementation of encoding and decoding, and require much less memory than random LDPC codes.

5.4. *BER performance of jointly designed QC-LDPC multi-relay coded cooperation*

In AWGN channels scenario, we compare the BER performance of multi-relay coded cooperation employing jointly designed QC-LDPC codes with that of multi-relay coded cooperation employing random LDPC codes. For simplicity, assume there are two relays. QC-LDPC codes employed by the source and relays are given in Table 2.

Figure 7 depicts the BER curves of two-relay coded cooperation employing jointly designed QC-LDPC codes and random LDPC codes. When the number of joint

Table 2. QC-LDPC codes employed by source and two relays.

	LDPC codes for source (S)	LDPC codes for relays $(\mathrm{R}_1, \mathrm{R}_2)$
QC-LDPC multi-relay coded cooperation	$\boldsymbol{M}(\boldsymbol{H}_1)$: 5×10, $B = 20$ $d_v = 3$, $d_c = 6$	$\boldsymbol{M}(\boldsymbol{H}_{\mathrm{R}_1}), \boldsymbol{M}(\boldsymbol{H}_{\mathrm{R}_2})$: 5×15, $B = 20$ $d_v = 3$, $d_c = 9$
Random LDPC multi-relay coded cooperation	Code length $N_1 = 200$, code rate $r_1 = 1/2$, $d_v = 3$, $d_c = 6$.	Code length $N_2 = 300$, code rate $r_2 = 2/3$, $d_v = 3$, $d_c = 9$.

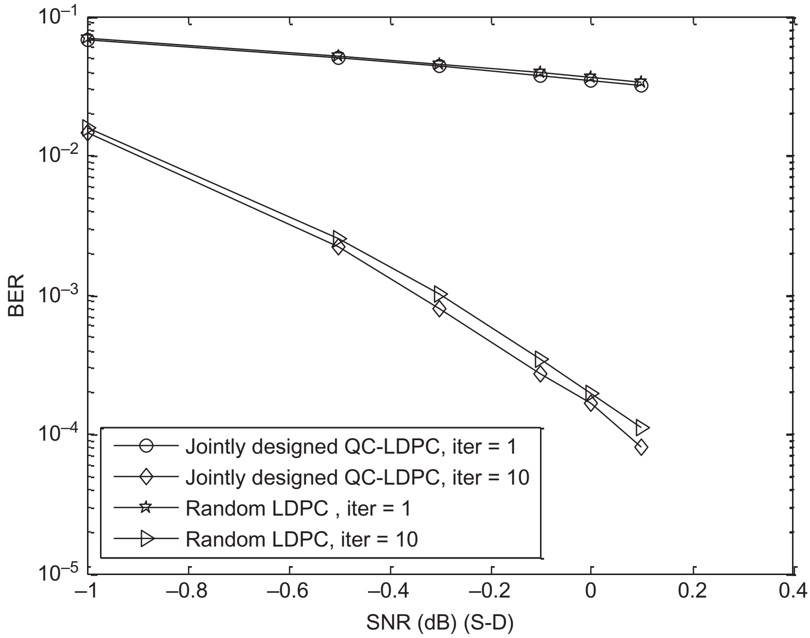

Figure 7. BER comparison of two-relay coded cooperation employing jointly designed QC-LDPC codes and random LDPC codes over AWGN channels.

decoding iteration is one, the BER performance of these two codes is similar. It is because the extrinsic information is not exchanged sufficiently, and the influence of cycles does not appear obviously. When the number of joint decoding iteration increases to 10, it is shown that the BER performance of jointly designed QC-LDPC codes clearly outperform that of random LDPC codes, which is attributed to the fact that both type I and type II girth-4 are cancelled in jointly designed QC-LDPC codes. After 10 joint iterations, the superiority of jointly designed QC-LDPC codes with girth-4 cycles cancelled is revealed.

6. Conclusion

We have studied coded cooperation system employing jointly designed QC-LDPC codes. The joint equivalent parity-check matrix corresponding to codes employed by the source and relay is resulted. By proposing an algorithm which cancels all girth-4 cycles including both type I and type II, we jointly design QC-LDPC codes for coded cooperation system. A joint "Min-Sum" iterative decoding is implemented in the destination to achieve the cooperation gain and coding gain. Simulations results show that the jointly designed QC-LDPC coded cooperation clearly outperforms the coded non-cooperation under the same conditions. In AWGN channels scenario, the BER performance of the coded cooperation employing jointly designed QC-LDPC codes is better than that of random LDPC codes and separately designed QC-LDPC codes. In Rayleigh fading channels scenario, the BER performance of the coded cooperation employing jointly designed QC-LDPC codes is as good as that of random LDPC codes.

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Appendix

Proof of Theorem 1

Proof: Assume there is a girth-4 cycle in the base matrix $\boldsymbol{M}(\boldsymbol{H})$. For simplicity, their corresponding right-cyclic-shift values in the exponent matrix $\boldsymbol{E}(\boldsymbol{H})$ are p_1, p_2, p_3, and p_4. For their corresponding submatrix in $\boldsymbol{H}$ as shown in (A.1), we deduce the necessary and sufficient condition for no girth-4 cycles.

(i,j) $\quad$ (i,k_1) $\quad$ (k_2,j) $\quad$ (k_2,k_1)

$$\begin{bmatrix} \boldsymbol{I}(p_1) & \boldsymbol{I}(p_2) \\ \boldsymbol{I}(p_3) & \boldsymbol{I}(p_4) \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} & \begin{pmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & ? \\ \times & \times & \times & \times \end{pmatrix} \end{bmatrix} \tag{A.1}$$

Without loss of generality, we assume the "1" in $\boldsymbol{I}(p_1)$ with coordinate (i, j). We can find a "1" in $\boldsymbol{I}(p_2)$ with coordinate (i, k_1) and a "1" in $\boldsymbol{I}(p_3)$ with coordinate (k_2, j). Hence, we can locate the coordinate in $\boldsymbol{I}(p_4)$: (x, y). If the value of (x, y) is also "1", there are girth-4 cycles in the submatrix, otherwise, there are no girth-4 cycles.

$$\begin{aligned} (x,y) &= (i,j) + [(i,k_1) - (i,j)] + [(k_2,j) - (i,j)] \\ &= (k_2, k_1) \end{aligned}. \tag{A.2}$$

Hence, the necessary and sufficient condition for the submatrix has no girth-4 cycles is

$$\text{The value of}\,(k_2,k_1)\,\text{in}\ \boldsymbol{I}(p_4)\ \text{is not "1".} \tag{A.3}$$

As $\boldsymbol{I}(p_4)$ is an identity matrix $\boldsymbol{I}_{B\times B}$ with p_4-right-cyclic-shift of each row. Equation (A.3) is equivalent to

$$k_1 \neq (k_2 + p_4) \bmod B. \tag{A.4}$$

According to the property of the right-cyclic-shift of identity matrix $\boldsymbol{I}_{B\times B}$, we have

$$j = (i + p_1) \bmod B, \tag{A.5a}$$

$$k_1 = (i + p_2) \bmod B, \tag{A.5b}$$

$$k_2 = (j - p_3) \bmod B = (i + p_1 - p_3) \bmod B. \tag{A.5c}$$

Note: if $u = (v) \bmod B$ is 0, u is redefined as B.
By (A.5), Equation (A.4) is equivalent to

$$(i + p_2) \bmod B \neq (i + p_1 - p_3 + p_4) \bmod B, \tag{A.6}$$

which can be further rewritten as

$$(p_1 - p_3) + (p_4 - p_2) \neq 0 \bmod B. \tag{A.7}$$

The end of proof.