

**Common cycles and common trends in the stock and oil markets: Evidence from more than 150 years of data\***

Mehmet Balcilar  
Eastern Mediterranean University, Turkey  
and  
University of Pretoria, South Africa  
[mehmet@mbalcilar.net](mailto:mehmet@mbalcilar.net)

Rangan Gupta  
University of Pretoria, South Africa  
[Rangan.Gupta@up.ac.za](mailto:Rangan.Gupta@up.ac.za)

Mark E. Wohar  
University of Nebraska-Omaha, USA  
[mwohar@mail.unomaha.edu](mailto:mwohar@mail.unomaha.edu)  
and  
Loughborough University, UK

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## **ABSTRACT**

This paper investigates the role of permanent and transitory shocks, within the framework of common cycles and common trends, in explaining stock and oil prices. We perform a multivariate variance decomposition analysis of monthly data on the West Texas Intermediate (WTI) oil price and the S&P500. The dataset used in the study spans a long period of 150 years and therefore contains a rich history to examine both the short- and long-run comovement properties of oil and stock prices. Given that the oil and stock markets might comove both in the short- and long-run, it is of interest to see the relative impacts of transitory and permanent shocks on both variables. We find that (log) oil price and (log) S&P 500 share a common stochastic trend for our full sample of September 1859 to July 2015, but a common cycle only exists during the post-WW II period. Full and post-WW II samples have quite different common feature estimates in terms of the impact of permanent and transitory shocks as measured by the impulse responses and forecast error variance decompositions. We also find that in the short-run oil is driven mostly by cycles (transitory shocks) and stock market is mostly driven by permanent shocks. But, permanent shocks dominate in the long-run.

## 1. Introduction

Following the early works of Rasche and Tatom (1977), Mork and Hall (1980), Hamilton (1983), and Hickman et al. (1987), a large literature exists that connects movements in oil returns and its volatility with recessions and inflationary episodes in the US economy (see for example, Balke et al., (2002, 2010), Brown and Yücel (2002), Barsky and Kilian (2004), Jones et al. (2004), Kilian (2008a,b, 2009a,b), Elder and Serletis (2010), Nakov and Pescatori (2010), Baumeister and Peersman (2013a,b), Kang and Ratti (2013a, b), Antonakakis et al., (2014a), and Bjørnland and Larsen (2015), and references cited therein). According to Hamilton (2008), nine of ten recessions in the US since World War II have been preceded by an increase in oil price. In fact, Hamilton (2009) even goes so far as to argue that a large proportion of the recent downturn in the US during the “Great Recession” can also be attributed to the oil price shock of 2007-2008. An extensive analysis of oil price shocks over the history of the oil industry, and its negative association with economic activity has also been recently depicted in Hamilton (2011), and Baumeister and Kilian (2015).

In the same vein, there is also a large literature that relates to the link between stock prices and economic activity, dating as far back as Mitchell and Bums (1938). More recent studies followed, and includes that of of Fischer and Merton (1984), Barro (1990), Fama (1981, 1990), Harvey (1989), Stock and Watson (1989), Choi *et al* (1999), Schwert (1990), Estrella and Mishkin (1998), Colombage (2009), Nyberg (2010), Mili *et al* (2012), and Erdogan et al., (2015). Unlike the empirical literature investigating oil, the impact of stock returns on US recessions is however, mixed, with more recent studies, based on sophisticated econometric techniques, pointing towards its role in predicting economic activity. Interestingly, Campbell et al (2001) proposes that the variance of stock returns, rather than the returns themselves,

have predictive content for output growth. Campbell (2001) finds evidence of high equity price volatility in one quarter to signal low macroeconomic growth in the next.

While, the literature discussed above has been primarily based on in-sample analysis, Stock and Watson (2003), and Rapach and Weber (2004), Kilian and Vigfusson (2011, 2013) provide evidence of the role of oil and the stock market in forecasting output growth and inflation of the US economy. Further, financial historians, like Ahamed (2009) and Ferguson (2008), have suggested that financial crises are often preceded by bubbles in the asset and commodity markets, which, in turn is vindicated by Phillips and Yu (2011) based on formal tests of bubble detection in real-time.

So overall, whether it is based on in-sample or out-of-sample evidence, the importance of the role played by the stock and oil market for the US economy is, in general, undeniable. Hence, from the perspective of a policy maker, it is of paramount importance to determine what types of shocks drives these two markets. In other words, detecting whether these shocks are temporary (namely, aggregate demand shocks, such as those resulting from changes in interest rate, inflation, fiscal policy, taste, velocity, and autonomous investment) or permanent in nature (aggregate supply shocks, such as technology shocks), helps in better policy design. Against this backdrop, the objective of this paper is to investigate the role of permanent and transitory shocks, within the framework of common cycles and common trends, in explaining stock and oil prices. While, there exists a huge literature that relates short and long-run movements in these two markets (for a detailed literature review in this regard, see for example, Kilian and Park (2009), Apergis and Miller (2009), Balcilar and Ozdemir (2012), Antonakakis and Filis (2013), Kang and Ratti (2013b), Antonakakis et al., (2014a, b), Broadstock and Filis (2014), Balcilar et al., (2015), Narayan and Gupta (2015),

Angelidis et al., (forthcoming), and references cited therein), to the best of our knowledge, this is the first study to determine the nature of shocks that drives these two markets simultaneously.<sup>1</sup> For our purpose, we perform a multivariate variance decomposition analysis of monthly data on the West Texas Intermediate (WTI) oil price and the S&P500 starting in September, 1859 (1859:09) to July, 2015 (2015:07), using Vahid and Engle's (1993) approach to identify common trends and cycles in the context of the multivariate Beveridge and Nelson (1981) decomposition technique.<sup>2</sup> Another unique feature of our analysis is that sample period runs from the beginning of the modern era of the petroleum industry with the drilling of the first oil well on August 27, 1859 in Titusville, Pennsylvania.

Note that, a multivariate approach introduces the possibility that trends and cycles may be common among variables. Engle and Kozicki (1993) defines a common feature to exist if a linear combination of the series fails to have the feature even though each of the series individually has the feature. While, cointegration is a common feature, another common feature of interest is the presence of common serial correlation patterns, i.e., common cycles. Issler and Vahid (2001) pointed out that the joint modeling of common-trend and common-cycle restrictions to identify permanent and transitory shocks has a clear advantage over the use of common-trend restrictions only. Understandably, if common-cycle restrictions are correctly imposed, estimates of the dynamic model (traditionally, a vector autoregression (VAR) model) are more precise, and leads to more accurate measurement of the relative importance of permanent and transitory shocks.

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<sup>1</sup> Applications of the common cycles and common trends methods to stock markets within and across countries can be found in Narayan (2011) looks at comovements between stock prices of emerging economies: Singapore, Taiwan, and South Korea. Narayan and Thuraisamy (2013) look at comovements between stock prices within the US: S&P 500, Dow Jones and the NASDAQ.

<sup>2</sup> Common trends and cycles can also be introduced into the multivariate structural time series model of Harvey (1989) and Koopman et al. (2000).

Our approach consists of four steps: First, we perform a battery of unit root tests (without and with breaks) on the WTI oil and S&P500 stock prices to establish the integration property of the data series. Second, we investigate the existence of common trends (i.e. co-movement over the long run) by applying the Johansen (1988, 1991) maximum likelihood techniques, and the multivariate cointegration method of Stock and Watson (1988a). Third, we examine the existence of common cycles (i.e. co-movement over the short run), by using the following test statistics: Weak Form (WF) reduced ranked structure test (Hecq et al. 2000, 2002, 2006); Serial Correlation Common Features (SCCF) (Engle and Kozicki, 1993; Vahid and Engle, 1993), and; Polynomial Serial Correlation Common Features (PSCCF) (Cubadda and Hecq, 2001, 2003). Finally, we conduct multivariate variance decomposition and impulse response analyses to examine the relative importance of permanent and transitory innovations in explaining variations in WTI oil and S&P500 stock prices. Note that, though not the primary goal of the paper, but once we identify the common trend and common cycle amongst these two variables, we are also able to decipher the role played by oil and stock prices in driving these two common features.

The remainder of the paper is organized as follows: Section 2 presents the methodology, with Section 3 discussing the data and the results. Section 4 concludes the paper with policy recommendations.

## **2. Methodology**

### ***2.1. Common features and short- and long-run comovements***

The major focus of this study is to examine the short- and long-run comovement of oil and stock prices. The dataset used in the study spans a long period of 150 years and therefore has a quite rich history to examine both the short- and long-run comovement properties of oil and

stock prices. There are various channels that may lead to short-run or cyclical comovement and long-run comovement (co-trending) of these series. A large number of papers discussed in the introduction find evidence that oil price rises may lead to recessions and falling stock prices, resulting in cocycles. The long-run growth trends in the market value of firms will drive both stock prices and oil prices through resulting economic growth.

Long-run comovement among nonstationary time series require these series to be cointegrated and implies that their long-run tendencies are driven by some common stochastic trends. Common stochastic trends lead to the comovement of the low frequency (permanent) components of the series. The long-run comovement of the oil and stock markets then requires oil and stock price be cointegrated.

However, oil and stock markets might also comove in the short-run. This means that deviations of these series from their long-run common trends also comove leading to cocycling behavior. A large number of papers discussed in Section 1 documented this cocycling behavior based on various econometric methods. The short-run comovement behavior relates to the high frequency (temporary) components of the series and thus usually referred as “cocycles”. The cocycling behavior of oil and stock prices might be due to business cycles, wars, common structural breaks (co-breaks), volatility spillovers, etc. Cocycling behavior between oil and stock markets show to what extent transitory shock in one market is transmitted to the other.

## ***2.2. Cointegration and common features in the short- and long-run***

Both the short- and long-run comovements of economic time series involve various challenges in terms of empirical modeling. The existence of common stochastic trends or

cointegration among economic time series is one of the well known stylized facts. In this case, an  $n \times r$ ,  $r < n$ , matrix  $\beta$  exists with the property that  $\beta'Y_t \sim I(0)$ , where  $Y_t$  is  $n \times 1$  vector of time series. Each component of  $Y_t$  is  $I(1)$  and contains a stochastic trend. When the series are cointegrated some of the stochastic trends are common to each series and the linear combinations  $\beta'Y_t$  eliminate  $k = n - r$  common stochastic trends, leaving  $r$  stationary combinations. Thus,  $\beta'Y_t$  does not have the stochastic trend property that each of  $n$  series has since cointegration vectors annul these stochastic trends. Engle and Kozicki (1993) define this as a “common feature”. A feature is a common feature, if each of the series has the feature but a linear combination of the series fails to have this feature.

Analogues to common stochastic trends, Engle and Kozicki (1993) introduced a notion of serial correlation common features (SCCF). A group of time series will have an SCCF structure, if each series has serial correlation, but a linear combination of the series is serially uncorrelated or an innovation process with respect to the past information set,  $\mathcal{I}_{t-1}$ , available at time  $t$ . Serial correlation signifies a (persistent) cyclical feature in the level of each series, but when the cyclical feature is common to each series, we will be able to find linear combinations of the cyclical component of each series that does not have the cyclical feature. For cointegrated time series  $Y_t$ ,  $\Delta Y_t$  will be generally a persistent serially correlated  $I(0)$  series and when there exists some serially correlated processes that are common to each series, then we can find an  $n \times s$ ,  $s < n$ , matrix  $\delta$  such that each element of  $\delta'\Delta Y_t$  is a white noise process.

### ***2.3. Cointegration, common trends, and permanent transitory decomposition***

Our empirical investigation first tests for common-trends and common cycles. Then, we estimate reduced rank regression (RRR) models imposing the common trends and common

cycles features. Common trend and cycle components are also estimated. The estimated restricted models are used to decompose both series into their permanent and transitory (PT) components. Finally, we obtain impulse responses (IR) and forecast error variance decompositions (FEVD) of permanent and transitory shocks.

Cointegrated time series following a vector autoregressive process of order  $p$ , VAR( $p$ ), can be written as a vector error correction model (VECM) (see, e.g., Engle and Granger, 1987; Johansen, 1988, 1991):<sup>3</sup>

$$\begin{aligned}\Delta Y_t &= \Pi Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \varepsilon_t \\ \Gamma(L)\Delta Y_t &= \alpha\beta'Y_{t-1} + \varepsilon_t\end{aligned}\tag{1}$$

where  $\varepsilon_t$  is a multivariate Gaussian white noise process with 0 mean and nonsingular covariance matrix  $\Sigma$ ,  $\varepsilon_t \sim N(0, \Sigma)$ ,  $\Pi$  is  $n \times n$  matrix,  $\Gamma(L) = I_n - \sum_{i=1}^{p-1} \Gamma_i$  with  $n \times n$  coefficient matrices  $\Gamma_i$ ,  $L$  is the lag operator defined as  $L^k x_t = x_{t-k}$ , and  $\Delta = 1 - L$  is the difference operator. Cointegration requires  $\Pi$  to be reduced rank,  $\text{rank}(\Pi) = r < n$ . In this case, there exist  $n \times r$  matrices  $\alpha$  and  $\beta$  such that  $\Pi = \alpha\beta'$ , since  $\Pi$  has a reduced rank. Here the matrix  $\alpha'_\perp \Pi \beta'_\perp$  has full rank, where  $\alpha'_\perp$  and  $\beta'_\perp$  are  $n \times (n - r)$  matrices with rank  $n - r$  which are orthogonal complements of  $\alpha$  and  $\beta$ , respectively, i.e.,  $\alpha'_\perp \alpha = 0$  and  $\beta'_\perp \beta = 0$ . Here,  $\beta$  is the cointegration matrix and the elements  $\alpha$  are the adjustment speeds which measures to speed of adjacent to the equilibrium.

Alternatively, we can focus on the Wold representation of the VECM as in Engle and Granger (1987). As  $\Delta Y_t$  is a stationary process, it admits the following Wold representation:

$$\Delta Y_t = C(L)\varepsilon_t\tag{2}$$

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<sup>3</sup> We leave out any deterministic variable from the model for easy exposition, but deterministic variables can be introduced without any loss of generality.

where  $C(L) = I_n + \sum_{i=1}^{\infty} C_i L^i$  and  $\sum_{i=1}^{\infty} i|C_i| < \infty$  so that the coefficients  $C_i$  have exponential decay (Johansen, 1996). The Wold representation in Equation (2) is the infinite order vector moving average representation,  $VMA(\infty)$ , of the VECM in Equation (1). We note that the matrix  $C(1) = I_n + \sum_{i=1}^{\infty} C_i$  has the reduced rank property with rank  $k = n - r$  and satisfy the condition  $\beta' C(1) = 0$ . The VECM (1) imposes  $k = n - r$  unit roots in the VAR by including stationary first differences of all variables and  $r = n - k$  stationary combinations  $Z_t = \beta' Y_t$ , where  $Z_t$  is know as the error correction term. The matrix  $\beta$  annuls  $k = n - r$  common stochastic trends shared by all variables in  $Y_t$  and obtains  $r$  stationary combinations  $\beta' Y_t$  (see, e.g., Johansen, 1996).

### ***2.3.1. Permanent transitory decomposition of cointegrated time series***

Testing for cointegration, maximum likelihood estimation of parameters, and other statistical inference for VECM are now well established (see, e.g., Engle and Granger, 1987; Johansen, 1988, 1991, 1996). We will thus not discuss testing and estimation of cointegration and focus on inference which is more relevant to this paper. The polynomial  $C(L)$  in Equation (2) can be factorized as:

$$C(L) = C(1) + C^*(L)\Delta \quad (3)$$

where  $C^*(L) = (1 - L)^{-1}[C(L) - C(1)] = \sum_{i=0}^{\infty} C_i^* L^i$  with  $C_i^* = -\sum_{j=i+1}^{\infty} C_j$ . By substituting Equation (3) into Equation (2) and assuming that the initial values are zero,  $Y_0 = 0$ , and solving backwards for the levels of  $Y_t$ , we obtain the well known common trends representation of Stock and Watson (1988b):

$$Y_t = C(1)\xi_t + C^*(L)\varepsilon_t \quad (4)$$

where  $\xi_t = \sum_{s=1}^t \varepsilon_s$  and  $\varepsilon_0 = 0$  for  $i \leq 0$ . Equation (4) is the multivariate Beveridge-Nelson (1981) (BN) decomposition of  $Y_t$ . It decomposes  $Y_t$  into its permanent component  $C(1)\xi_t$ ,

which arises from the  $k$  stochastic trends, and the transitory or cyclical components  $C^*(L)\varepsilon_t$ , which can be interpreted as the temporary deviation from the long-run trends. The transitory or cyclical component  $\psi_t = C^*(L)\varepsilon_t$  is a stationary and serially correlated multivariate process and accepts a Wold representation, and hence can be represented by a general multivariate autoregressive moving average (ARMA) process. Some of the stationary components may be common to all series, and these are called common cyclical features, analogous to the definition of common stochastic trends.

### 2.3.2. Common stochastic trends and permanent component

In order to show the relationship of the permanent component to the common stochastic trends, note that  $C(1)$  has rank  $k = n - r$ . In this case, a nonsingular matrix  $\Lambda$  exists such that  $C(1)\Lambda = [H \ 0_{n \times r}]$ , where  $H$  is an  $n \times k$  matrix with full column rank (Stock and Watson, 1988b).<sup>4</sup> Using the matrix  $\Lambda$  we have  $C(1)\xi_t = C(1)\Lambda\Lambda^{-1}\xi_t$  and thus

$$\begin{aligned} Y_t &= H\tilde{\tau}_t + \psi_t \\ &= \tau_t + \psi_t \end{aligned} \quad (5)$$

where  $\tau_t = H\tilde{\tau}_t$  and  $\tilde{\tau}_t$  are the first  $k$  components of  $\Lambda^{-1}\xi_t = \Lambda^{-1}\sum_{s=1}^t \varepsilon_s$ . An equivalent representation of the common stochastic trends can be obtained noting the property that  $C(1)$  is rank  $k = n - r$ :

$$C(1) = \beta_{\perp}(\alpha'_{\perp}(I_n - \sum_{i=1}^{p-1} \Gamma_i)\beta_{\perp})^{-1}\alpha'_{\perp} \quad (6)$$

Thus,  $k$  common stochastic trends can be set as

$$\tilde{\tau}_t = \alpha'_{\perp} \sum_{s=1}^t \varepsilon_s \quad (7)$$

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<sup>4</sup> The matrix  $\Lambda$  is not unique and can be constructed from the eigenvalues of  $H$ . The first  $r$  columns of  $\Lambda$  will be eigenvectors corresponding to the nonzero eigenvalues of  $H$ , while last  $n - r$  columns will be the remaining eigenvectors corresponding to zero eigenvalues of  $H$ .

and by defining  $H = \beta_{\perp}(\alpha'_{\perp}(I_n - \sum_{i=1}^{p-1} \Gamma_i)\beta_{\perp})^{-1}$ , we obtain Equation (5). This last expression shows that  $\alpha_{\perp}$  is the matrix that extracts common stochastic trend components in a VECM.

We showed above that the presence of  $n - r$  stochastic trends in Equation (5) is equivalent to cointegration, since all series comove in the long-run according to the path determined by the  $k$  stochastic trends. In this case, the cointegration vectors eliminate the stochastic trends, i.e.,  $\beta'\tau_t = 0$  (Engle and Granger, 1987) and, thus, leading to the existence of  $n - r$  common stochastic trends, which determines the long-run comovement of the series.

### ***2.3.3. Transitory component***

We now relate the cointegration property to the stationary transitory components  $\psi_t$ . The cointegration matrix  $\beta$  annuls  $n - r$  common stochastic trends in Equation (5), i.e.,  $\beta'\tau_t = 0$  and leaves  $r$  linear stationary components  $\beta'Y_t = \beta'\psi_t$ . Therefore, linear cointegration combinations are linear combinations of the transitory or cyclical components. This shows the relationship between the multivariate BN decomposition, common trends, and cyclical components arising from a VECM.

It is interesting to examine whether the reduced rank restrictions exists that can be imposed on the short-run cyclical component  $\psi_t$  in Equation (5). These restrictions will naturally relate to the transitory component of the multivariate BN decomposition. Motivated from the common trends restrictions, we can search for reduced rank restrictions that reduce the number of elements (and also number of parameters needed to capture the short-run dynamics) in the cyclical component  $\psi_t$ .

We will examine three notions of common cycles which all imply that some form of reduced rank restriction can be imposed on the VECM (1) leading to fewer number of terms in  $C^*(L)\varepsilon_t$  and therefore each of the  $n$  transitory components in  $\psi_t$  can be obtained as a linear combination of less than  $n$  cyclical components.

The first of such common cycle notion is the serial correlation common features (SCCF) of Engle and Kozicki (1993) and Vahid and Engle (1993).<sup>5</sup> The SCCF imposes the reduced rank restrictions on  $\Delta Y_t$ , because they describe the short-run movements all the terms relating to the  $\Delta Y_t$  given in VECM in Equation (1). A second notion of common cycles we consider is the polynomial serial correlation common features (PSCCF) of Cubadda and Hecq (2001, 2003). The PSCCF specification does not require contemporaneous synchronization among the cycles in order to reduce to number of cyclical components and allows some cycles to lead or lag the others. The third is the weak form serial correlation common features (WF) of Hecq *et al.* (2000, 2002, 2006). The WF applies to  $\Delta Y_t$  in deviations from the error correction terms  $\alpha\beta'Y_{t-1}$ , i.e.,  $\Delta Y_t - \alpha\beta'Y_{t-1}$ , so it requires both  $\Delta Y_t$  and  $\alpha\beta'Y_{t-1}$  have the same serial correlation structure.

#### **2.3.4 Serial Correlation Common Features (SCCF).**

Engle and Kozicki (1993) introduced the notion of SCCF and Vahid and Engle (1993) further generalized and presented the statistical inference. The notion of SCCF implies that a vector of serially correlated stationary time series move together in such a way that some linear combinations of these series exists, which are serially uncorrelated vector white noise processes. The same linear combination that eliminates serial correlation when applied to the

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<sup>5</sup> Hecq *et al.* (2000, 2002, 2006) also define a similar notion with less restrictive conditions and, therefore, refer the SCCF as the strong form (SF).

first differences eliminates the common cycles when applied to the levels. The vector that eliminates the common cycles (or common serial correlations) is then a common feature vector. Naturally, SCCF only applies to VAR models with Granger causality. In the absence of Granger causality there cannot be any comovement between the series and the common features cannot exist.

Applied to the VECM (1), the series  $\Delta Y_t$  has SSCF, if there exists an  $n \times s$  matrix  $\delta_S$  such that

$$\delta_S' \Delta Y_t = \delta_S' \varepsilon_t \quad (8)$$

which imposes the restrictions (1)  $\delta_S' \alpha = 0$  and (2)  $\delta_S' \Gamma_i, i = 1, 2, \dots, p - 1$ , on the VECM (1). The  $n \times s$  matrix is called the cofeature matrix and has similar properties as the cointegration matrix  $\beta$  of the VECM (1). The cofeature matrix eliminates the serial correlation in  $\Delta Y_t$  arising from both the stationary error correction term  $\beta' Y_{t-1}$  and first differences  $\Delta Y_{t-1}, i = 1, 2, \dots, p - 1$ .

In terms of the BN decomposition in Equation (5), SCCF implies that  $\delta_S$  is a matrix such that the linear combinations of  $Y_t$  formed by the columns of  $\delta_S$  do not contain the cycles  $\psi_t$  since  $\delta_S$  eliminates serial correlation in  $\Delta Y_t$  or equivalently the cycles in  $Y_t$ , i.e.,  $\delta_S' \psi_t = 0$ . Applied to the equivalent form of the BN decomposition in Equation (4), SCCCf implies that

$$\delta_S' Y_t = \delta_S' C(1) \xi_t + \delta_S' C^*(L) \varepsilon_t = \delta_S' \xi_t = \tau_t \quad (9)$$

which implies that  $\delta_S' C(1) = \delta_S'$  and  $\delta_S' C^*(L) = 0$ . Therefore, existence of common cycles implies unit roots for the matrix  $C(1)$  and the cofeature vectors or the columns of  $\delta_S$  are the eigenvectors corresponding to the unit eigenvalues of  $C(1)$ . Recall that the cointegration vectors in  $\beta$  are the eigenvectors corresponding zero eigenvalues of  $C(1)$ , then  $\delta_S$  and  $\beta$  must

be linearly independent and  $s \leq (n - r)$ . Therefore, there can be at most  $n - r$  cofeature vectors with minimum of  $r$  common cycles. The transformation  $\delta_S$  when applied to  $\Delta Y_t$  in Equation (2) eliminates all positive powers of the lag polynomial  $C(L)$ , i.e., all the serial correlations of the first differences. This, in turn, implies that the same transformation when applied to the levels eliminates all the cycles.<sup>6</sup>

A drawback of the SCCF is the requirement that the common cycles must be contemporaneously synchronized. In this case, there will be no lead or lag relationship among the common cyclical components. The following variant of the SCCF notion of common cycles relaxes this assumption and allows common serial correlations among the non-contemporaneous elements.

### ***2.3.5 Polynomial Serial Correlation Common Features (PSCCF).***

Introduced by Cubadda and Hecq (2001, 2003), the notion of PSCCF states that the series  $\Delta Y_t$  have  $s$  PSCCFs if a first order polynomial  $\delta(L) = \delta_p - \delta_1 L$  exists such that

$$\delta(L)' \Delta Y_t = \delta_p' \varepsilon_t \quad (10)$$

In terms of the VECM (1), PSCCF imposes the restrictions (1)  $\delta_p' \alpha = 0$ , (2)  $\delta_p' \Gamma_1 = \delta_1$ , and (3)  $\delta_p' \Gamma_i = 0$ ,  $i = 2, 3, \dots, p - 1$ . The matrix  $\delta_p$  has the same properties as the matrix  $\delta_S$  of the SCCF notion. One significant difference arises between the SCCF and PSCCF in terms of the BN decomposition in Equations (4)-(5). The PSCCF restrictions imply that  $\delta(L)' \psi_t = -\delta_1 C(1) \varepsilon_t$ . Therefore, if  $\Delta Y_t$  have the PSCCF structure than the BN cycles of  $Y_t$ ,  $\psi_t$ , have the same PSCCF. In this case, the same PSCCF relationship will cancel the dependence from the past of both  $\Delta Y_t$  and  $\psi_t$ .

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<sup>6</sup> Under these restrictions on the matrices  $C_i$  and  $C_i^*$ , the impulse response functions of both  $Y_t$  and  $\Delta Y_t$  are collinear since all  $C_i$  and  $C_i^*$  are reduced rank and their columns are linearly dependent.

Two important features of SCCF and PSCCF are: (1) the matrices  $\delta_s$  and  $\delta_p$  must lie in the left null space of the loading matrix  $\alpha$  containing the adjustment speed coefficients due to restrictions  $\delta'_s \alpha = 0$  and  $\delta'_p \alpha = 0$ , respectively, hence, the number of the SCCFs and PSCCFs cannot exceed  $n - r$ , i.e.,  $s \leq (n - r)$  and (2) the long-run dynamics and short-run dynamics are related due to presence of the error-correction terms on the right hand side of the VECM model. Hecq *et al.* (2000, 2002, 2006) introduces a weak form (WF) serial correlation common features notion that relaxes these restrictions.

### **2.3.6 Weak Form Serial Correlation Common Features (WF).**

Under the WF notion of Hecq *et al.* (2000, 2002, 2006), there exist an  $n \times s$  full column matrix  $\delta_W$  such that

$$\delta'_W (\Delta Y_t - \alpha \beta' Y_{t-1}) = \delta'_W \varepsilon_t \quad (11)$$

The restrictions imposed by the WF on the VECM (1) are, therefore,  $\delta'_W \Gamma_i = 0$ ,  $i = 1, 2, \dots, p - 1$ . The WF notion has the property that  $\delta'_W \alpha \neq 0$ .

From Equation (11), we can interpret the WF as the existence of linear combinations of the series  $\Delta Y_t - \alpha \beta' Y_{t-1}$  that are white noise. Therefore, the WF restrictions require the series  $\Delta Y_t - \alpha \beta' Y_{t-1}$  have the same common serial correlations restricting the short-run dynamics only. As the orthogonality of  $\delta_W$  and  $\alpha$  is not required  $\delta_W$  does not need to be in the left null space of  $\alpha$  and number of common cyclical features might be greater than  $n - r$ , but still needs to be less than  $n$ , i.e.,  $s \leq (n - 1)$ . Under the WF assumption,  $\Delta Y_t$  and  $\alpha \beta' Y_{t-1}$  have the same serial correlation pattern, their impulse responses are collinear, and they have similar dynamics. This means that the short- and long-run dynamics of  $Y_t$  are unrelated.

#### 2.4. Estimation and testing for common features

Testing for cointegration and estimation of cointegration vectors are well established (see, e.g., Johansen, 1996). In this section, we briefly explain testing for common cyclical features and estimation of the cofeature matrix as well as the other parameters of interest.

Following Johansen (1988, 1991), the maximum likelihood (ML) inference can be based on canonical correlations (CanCor). From Johansen (1996), it follows that it is impossible to find an explicit solution for the likelihood equations when both the cointegration rank,  $r$ , and the number of common cycle restrictions,  $s$ , are unknown. Therefore, we rest on the two-step approach of Vahid and Engle (1993). In the first step, the cointegration rank  $r$  can be determined using the approach of Johansen (1988, 1991) and cointegration parameters are estimated using the ML. The restrictions on the short-run parameters that arise for the common cycles restrictions are ignored in the first step. In the second step, both  $r$  and  $\beta$  are fixed at the values obtained in the first step and  $s$  is determined and cofeature matrix is estimated using again the ML approach.

The likelihood ratio test for the  $s$  common feature restrictions is based on the Maximum Likelihood (ML) inference and given in Hecq *et al.* (2000, 2002, 2006) and Cubadda and Hecq (2001, 2003). The ML inference is equivalent to canonical correlations  $CanCor(\Delta Y_t, X_t | Z_t)$ , which denotes the partial canonical correlations between  $\Delta Y_t$  and  $X_t$  after removing their linear dependence on the vector of variables  $Z_t$ . The specifications of  $X_t$  and  $Z_t$  and number of restrictions ( $v$ ) arising from the common cycle reduced rank structure for the three cases we examined above are as follows:

$$SCCF: X_t = (\Delta Y'_{t-1}, \dots, \Delta Y'_{t-p+1}, Y'_{t-1}\beta)' , Z_t = 1, v = s(n(p-2) + r + s)$$

PSCCF:  $X_t = (\Delta Y'_{t-2}, \dots, \Delta Y'_{t-p+1}, Y'_{t-1}\beta)'$ ,  $Z_t = (1, \Delta Y'_{t-1})'$ ,  $v = s(n(p-2) + s)$

WF:  $X_t = (\Delta Y'_{t-1}, \dots, \Delta Y'_{t-p+1})'$ ,  $Z_t = (1, Y'_{t-1}\beta)'$ ,  $v = s(n(p-3) + r + s)$

The canonical correlation problem can then be solved using the proper  $X_t$  and  $Z_t$ . Let  $\hat{\lambda}_j$ ,  $j = 1, 2, \dots, n$ , be the  $i$ th smallest squared partial canonical correlations (estimated eigenvalues), then the likelihood ratio (LR) test statistics for the null hypothesis that  $H_0: s \geq s_0$  for  $s_0 = 1, 2, \dots, n$ , i.e., a minimum of  $s_0$  common features exists is given by:

$$LR = -T \sum_{j=1}^{s_0} \ln(1 - \hat{\lambda}_j), \quad s_0 = 1, 2, \dots, n \quad (12)$$

and asymptotical distributed as  $\chi^2(v)$  with  $v$  degrees of freedom.

The optimal estimates of both the common features vectors and partial RRR coefficients can be obtained from the  $CanCor(\Delta Y_t, X_t | Z_t)$  problem. Let  $\hat{\delta}_i = (\hat{\delta}_{i,1}^{\Delta Y}, \hat{\delta}_{i,2}^{\Delta Y}, \dots, \hat{\delta}_{i,s}^{\Delta Y})'$  and  $\hat{\phi}_i = (\hat{\phi}_{i,s+1}^X, \hat{\phi}_{i,s+2}^X, \dots, \hat{\phi}_{i,n}^X)'$ ,  $i = S, P, W$ , denote the partial canonical correlations coefficients for of  $\Delta Y_t$  and  $X_t$ , respectively, where  $\hat{\delta}_{i,j}^{\Delta Y}$  and  $\hat{\phi}_{i,j}^X$  corresponds to  $j$ th squared partial canonical correlation  $\hat{\lambda}_j$ . Then, using the appropriate definition of  $X_t$  and  $Z_t$  we can estimate the associated common features vectors  $\hat{\delta}_i$   $i = S, P, W$ , and other parameters  $\hat{\phi}_i$ . Rest of the parameters can then be estimated using OLS after fixing the various matrices  $\hat{\phi}_i$  at their estimated values.

In addition to establishing the existence of the cointegration and common features Cubbada (2007) suggests a canonical correlations approach for the estimations of the VECM models under common features restrictions. Consider the following RRR model:

$$W_t = \delta_{\perp} \Phi' X_t + \tilde{\varepsilon}_t \quad (13)$$

where  $X_t = (\Delta Y'_{t-1}, \dots, \Delta Y'_{t-p+1}, Y'_{t-1} \beta)'$ ,  $W_t = (\Delta Y'_t, Y'_{t-1} \beta, \Delta Y'_{t-1})$ ,  $\tilde{\varepsilon}_t = (\varepsilon'_t, 0_{1 \times (r+n)})'$ ,  $\delta$  is an  $(2n + r) \times s$  matrix containing the cofeature matrix with  $s < n$ , and  $\Phi$  is an  $(r + np - n) \times (2n + r - s)$  matrix that contains the short run parameters such that

$$\delta_{\perp} \Phi' = \begin{pmatrix} (\alpha, \delta_1) & (\Gamma_1, \dots, \Gamma_{p-1}) \\ I_{r+n} & 0_{((r+n) \times (np-2n))} \end{pmatrix} \quad (14)$$

The SCCF, PSCCF, and WF models are nested within Equations (13)-(14) and statistical inference can be conducted by appropriate restrictions on the solution of the canonical correlations  $CanCor(W_t, X_t|1)$ . With proper restrictions, the solution of  $CanCor(W_t, X_t|1)$  is equivalent to  $s$  common common features, in the form of SCCF, PSCCF, or WF.<sup>7</sup>

## 2.5. PT Decomposition, IR, and FEVD

Given that the oil and stock markets might comove both in the short- and long-run, it is of interest to see the relative impacts of transitory and permanent shocks on both variables. The analysis might reveal valuable information on what type of shocks drives these variables in the short- and long-run. The information might also help to disentangle the likely sources of transitory and permanent shocks, some of which are common to each variable.

In a VECM framework, the statistical theory for decomposition of variables in the model into permanent and transitory components (the multivariate BN decomposition), and IR and FEVD analysis are well established. Our analysis considers both common stochastic trends and common cycles. Therefore, not only the common stochastic trend restrictions but also common cycles restrictions apply in our case.

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<sup>7</sup> The statistical inference should be based on the  $n$  smallest eigenvalues since the  $n + r$  largest eigenvalues will be equal to exactly one as both  $W_t$  and  $X_t$  include the terms  $\Delta Y'_{t-1}$  and  $Y'_{t-1} \beta$ .

Assume that the cointegration and common features are imposed on the model as in Equations (13)-(14) and the short-run parameter estimates are obtained. Let  $V$  be an  $n \times n$  nonsingular matrix, chosen such that permanent ( $u_t^P$ ) and transitory ( $u_t^T$ ) shocks are obtained as  $u_t = (u_t^P, u_t^T)' = V\varepsilon_t$ , and the permanent and transitory components of  $Y_t$ ,  $Y_t = \tau_t + \psi_t$ , satisfy  $\Delta\tau_t = P(L)u_t^P$  and  $\Delta\psi_t = T(L)u_t^T$ , with the properties  $T(1) = 0$  and  $E[u_t^T u_t^{P'}] = 0$ . Here,  $u_t^P$  is an  $(n-r) \times 1$  vector of permanent shocks,  $u_t^T$  is an  $r \times 1$  vector of transitory shocks, and  $P(L) = \sum_{i=0}^{\infty} P_i L^i$  and  $T(L) = \sum_{i=0}^{\infty} T_i L^i$  are lag polynomials with  $n \times (n-r)$  and  $n \times r$  coefficient matrices  $P_i$  and  $T_i$ , respectively. The condition  $T(1) = 0$  ensures that the transitory shocks do not have permanent effect on the levels of the series. The condition  $E[u_t^T u_t^{P'}] = 0$  ensures that the permanent component  $\tau_t$  and the transitory component  $\psi_t$  are uncorrelated at all lags and leads. In order to see the long run impacts, the Granger representation theorem result in Equation (4) can be written as

$$Y_{t+h} = C(1) \sum_{s=1}^{t+h} \varepsilon_s + \sum_{s=0}^{\infty} C_s \varepsilon_{t+h-s} \quad (15)$$

The permanent and transitory shocks respectively require that  $\lim_{h \rightarrow \infty} E[\partial Y_{t+h} / \partial u_t^P] \neq 0$  and  $\lim_{h \rightarrow \infty} E[\partial Y_{t+h} / \partial u_t^T] = 0$ . Note from Equation (15), that a change in  $\varepsilon_t$  at time  $t$  equal to  $c$  implies

$$\lim_{h \rightarrow \infty} E \left( \frac{\partial Y_{t+h}}{\partial \varepsilon_t} c \right) = \lim_{h \rightarrow \infty} [C(1)c + C_h c] = C(1)c \quad (16)$$

Therefore, a shock of size  $c$  at time  $t$  propagates through the system and its total impact becomes  $C(1)c$  in the long-run, thus, the permanent effect is  $C(1)c$ . We see that the permanent effect of a the shock  $\varepsilon_t$  is  $C(1)\varepsilon_t = \beta_{\perp} (\alpha'_{\perp} (I_n - \sum_{i=1}^{p-1} \Gamma_i) \beta_{\perp})^{-1} \alpha'_{\perp} \varepsilon_t$  from Equation (6). Therefore, part of the shocks that has a permanent effect in the long-run is  $\alpha'_{\perp} \varepsilon_t$  and, thus, we define the permanent shocks as  $u_t^P = \alpha'_{\perp} \varepsilon_t$ . A definition of transitory shock satisfying the condition  $E[u_t^T u_t^{P'}] = 0$  is then obtained as  $u_t^T = \alpha' \Sigma^{-1} \varepsilon_t$ . The matrix  $V$  is

then  $V = [\alpha'_\perp \alpha' \Sigma^{-1}]'$ , which gives  $u_t = V \varepsilon_t = [\alpha'_\perp \varepsilon_t \alpha' \Sigma^{-1} \varepsilon_t]'$ . Then, using the identity  $\Sigma \alpha_\perp (\alpha'_\perp \Sigma \alpha_\perp)^{-1} \alpha'_\perp + \alpha (\alpha' \Sigma^{-1} \alpha)^{-1} \alpha' \Sigma^{-1} = I_n$ , the permanent ( $\varepsilon_t^P$ ) and transitory ( $\varepsilon_t^T$ ) components of  $\varepsilon_t$ ,  $\varepsilon_t = \varepsilon_t^P + \varepsilon_t^T$ , can be written in terms of the permanent and transitory shocks  $u_t^P = \alpha'_\perp \varepsilon_t$  and  $u_t^T = \alpha' \Sigma^{-1} \varepsilon_t$  as (see, e.g., Johansen, 2010):

$$\varepsilon_t = \Sigma \alpha_\perp (\alpha'_\perp \Sigma \alpha_\perp)^{-1} \alpha'_\perp \varepsilon_t + \alpha (\alpha' \Sigma^{-1} \alpha)^{-1} \alpha' \Sigma^{-1} \varepsilon_t \quad (17)$$

From the decomposition in Equation (17) we see that, defining the contemporaneous effect of  $\varepsilon_t$  as  $I_n$ , the contemporaneous effect of the permanent shock  $u_t^P$  and the transitory shock  $u_t^T$  are  $\Sigma \alpha_\perp (\alpha'_\perp \Sigma \alpha_\perp)^{-1}$  and  $\alpha (\alpha' \Sigma^{-1} \alpha)^{-1}$ , respectively, while their long-run effects are  $C(1) \Sigma \alpha_\perp (\alpha'_\perp \Sigma \alpha_\perp)^{-1} = \beta_\perp (\alpha'_\perp (I_n - \sum_{i=1}^{p-1} \Gamma_i) \beta_\perp)^{-1}$  and  $C(1) \alpha (\alpha' \Sigma^{-1} \alpha)^{-1} = 0$ . These are easily verified by substituting the definition in Equation (17) into the Wold representation of the VECM in Equation (2) and noting that the polynomials  $P(L)$  and  $T(L)$  are, respectively, given by  $P(L) = C(L) \Sigma \alpha_\perp (\alpha'_\perp \Sigma \alpha_\perp)^{-1}$  and  $T(L) = C(L) \alpha (\alpha' \Sigma^{-1} \alpha)^{-1}$ . These polynomials can, then, be used to obtain the IRs of the variables  $\Delta Y_t$  as well as the FEVDs. The IRs of the levels can be recovered by the cumulative sums of the IRs of  $\Delta Y_t$ . The multivariate BN decomposition can be then be obtained numerically from the polynomials  $P(L)$  and  $T(L)$ . The explicit formula for the BN decomposition that only involves observable variables, parameters estimates from the VECM (1), cointegration vectors, and common features vectors can be found in Hecq *et al.* (2000).

Summarizing, our estimation procedure involves the following steps:

1. Decide the number of cointegration vectors  $r$  and estimate the VECM (1) with  $p - 1$  order polynomial  $\Gamma(L)$ . This step will obtain consistent estimates of  $\beta$ ,  $\hat{\beta}$ , which can be used to construct  $\hat{\beta}_\perp$ .

2. Fix  $\beta$  at estimated values  $\hat{\beta}$ , determine  $s$  and estimate the parameters under common features restrictions from Equations (13)-(14). This step will yield consistent estimates of  $\alpha$ ,  $\Sigma$ ,  $\varepsilon_t$ , and  $\Gamma(L)$ , i.e.,  $\hat{\alpha}$ ,  $\hat{\Sigma}$ ,  $\hat{\varepsilon}_t$ , and  $\hat{\Gamma}(L)$ . These can then be used to construct  $\hat{\alpha}_\perp$ ,  $\hat{\Sigma}^{-1}$ ,  $\hat{C}(L)$ ,  $\hat{P}(L)$ , and  $\hat{T}(L)$ .
3. Construct  $\hat{u}_t = (\hat{u}_t^P, \hat{u}_t^T)' = \hat{V}\hat{\varepsilon}_t = [\hat{\alpha}'_\perp \hat{\varepsilon}_t \quad \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\varepsilon}_t]'$ .
4. Obtain a lower block triangular matrix  $\hat{M}$ , e.g., by applying Choleski decomposition to  $\hat{\Psi} = \text{cov}(\hat{V}\hat{\varepsilon}_t) = \hat{M}\hat{M}'$ , and construct  $\hat{\eta}_t = \hat{M}^{-1}\hat{u}_t$ . Elements of  $\hat{\eta}_t$  are mutually uncorrelated and have unit variances.
5. Transform  $\hat{C}(L)$  as  $\hat{R}(L) = \hat{C}(L)\hat{V}^{-1}\hat{M}$ , which obtains  $\Delta Y_t = \hat{R}(L)\hat{\eta}_t$ .

The IRs, FEVDs, and PT decompositions can then be obtained from the canned routines available in software packages using  $\hat{R}(L)$ , with the difference that the Choleski decompositions is applied to  $\text{cov}(\hat{V}\hat{\varepsilon}_t)$  rather than to  $\text{cov}(\hat{\varepsilon}_t)$ .

### 3. Data and Results

We begin our empirical investigation by examining Figure 1 which plots the (log) of S&P 500 and (log) WTI Crude oil price. Our full sample period extends from September 1859 to July 2015, with the data coming from the Global Financial database. We seasonally adjust the data using the X-13 procedure of the US Census Bureau. We can see that the series move together over time. The spread between the two series increases over time suggesting that the cointegration with a deterministic trend may characterize the series. Along with the tendency of co-movement in the two series another feature that appears are structural breaks and regime shifts. Short-run changes seem to be harmonized as well.

We turn next to a discussion of descriptive statistics presented in Table 1 which reports metrics for the log level of S&P 500 and the log WTI oil price as well as log returns of the two series. The average return for the (log) SP500 is 7.8 times higher than the average return for (log) WTI oil price. The WTI return is more volatile than the SP500 return, evidenced by the fact that the coefficient of variation (CV) for WTI returns is 15 times the coefficient of variation (CV) for the SP500 return. Not surprisingly, normality is rejected for both level and return series. Furthermore, returns are negatively skewed, and fat tailed. Both series also have ARCH effects.

Table 2 reports unit root tests, including ADF (Dickey and Fuller, 1981), , KPSS (Kwiatkowski et al., 1992), Phillips and Perron (1988), Ng and Perron (2001), DF-GLS (Elliott et al., 1996), and Zivot and Andrews (2002) structural break in mean and trend. We conduct these tests over our full sample period, September 1859 to July 2015, as well as the post-WW II sample, October 1945 to July 2015 (owing to a structural break found in 1945 discussed later). All tests indicate uniformly that the (log) level of S&P 500 and (log) level of WTI oil price are I(1) series for both sample periods.

Table 3 reports Johansen (1988, 1991) and Stock and Watson (1988a) cointegration tests. Panel A of Table 3 reports information criteria used to select the lag length in the VAR. We use the AIC criteria and select a lag of 6. The information criteria of BIC and HQ select a lag of 2. We believe it is preferred to have a longer lag length than lag length that is too short. Panel B reports the lambda-max and trace statistics when we allow for a deterministic trend in the cointegrating vector. Results indicate evidence of one cointegrating vector (reported in Panel B). The trend in the cointegrating vector is employed as we discussed the visual inspection of the two series in figure 1. We also conduct cointegration tests without a trend

in the cointegrating vector and find evidence of cointegration as well under that specification. Both tests show that log SP500 and log WTI are cointegrated both in the full sample and the post-WW II sample. Panel C reports result for the Stock and Watson (1988a) cointegration tests. The results support the Johansen tests in finding evidence of one cointegrating vector.

We also conducted a likelihood ratio test for the constancy of the cointegrating vector. The results (not reported here) indicate that a structural break is most likely to have occurred in 1945. For this reason we also report cointegration tests on a post-WW II sample period, October 1945 to July 2015, in Table 4. For this sample period we employ a lag length of 2 in the VAR and find that both the Johansen and Stock and Watson (1988a) tests indicate evidence of one cointegrating vector. The cointegrating vector for the full sample and the post-WW II period are different reflecting the finding of a structural break.

Figure 2 (for the full sample period) plots permanent components of both (log) S&P500 and (log) WTI oil price as well as the common trend and common cycle series. The upper panel reports the permanent components (in dashed lines) and the actual series (in solid lines). The lower panel shows the common cycle and common trend components of the two series. Figure 3 reports comparable results for the post-WW II period. From the figures we can see that the stock market is driven primarily by the long-run stochastic trend, as actual stock price closely follows the permanent component (see Figures 2(b), 3(b)). The common trend component is mostly driven by the changes in the stock price (see Figures 2(d), 3(d)). It is the oil price that deviates mostly from this long-run common trend. However, this deviation feeds back to the system as the cycle is common in the post-WW II model. So, the deviation of oil price should impact the stock market in the short-run as well. This impact is quite large in the short-run (44% at step 1, see Table 7). Notice that the full sample estimates are quite

different from the full sample results. For the full sample a common cycle is not found and the permanent shock dominates (see Table 6). Figures 2(b) and 3(b) show that the stock price is always close to its permanent component, while the oil price significantly deviates from its permanent component in several periods. So the common cycle is driven by the oil price deviating from the long-run trend.

Figure 4 reports (for the full sample) impulse response functions for a one standard deviation to a permanent shock for the level of S&P 500 and the level of oil price (panel a and b) and for S&P 500 returns and oil returns (panel c and d). Figure 5 reports impulse responses to a transitory shock for the full period. We see a positive and significant response to permanent shocks from both (log) S&P 500 and (log) oil price. The stock market responds faster and its response is relatively higher than the response of the oil market. For the full sample, the responses of oil and stock price to transitory shocks are asymmetric. Oil responds negatively and significantly, while stock price responds positively but insignificantly. Initial response of oil is very large and negative with some undershooting.

Figure 6 and Figure 7 report comparable results to Figure 4 and figure 5. In the post-WW II sample, responses to permanent shocks are quite different for the stock market than in the full sample period. The response of oil price in the post-WW II period is negative and significant. The response of oil price in the full sample and post-WW II samples are similar. It is the response of stock market that is largely affected by the structural break. The responses of both oil and stock markets to transitory shocks in the post-WW II sample (Figure 7) vary significantly compared to the full sample model. First, the response of oil price now becomes positive and exponentially decaying as well as the response of the stock market. In the post-

WW II period we see more changes for oil price's response to transitory shocks. The stock market's response also becomes larger and significant in the post-WWW II sample.

Table 5 reports three common cyclical features tests: i) Weak Form (WF) reduced rank structure (Hecq et al. 2000, 2002, 2006); ii) Serial Correlation Common Features (SCCF) (Engle and Kozicki, 1993; Vahid and Engle, 1993), and iii) Polynomial Serial Correlation Common Features (PSCCF) (Cubadda and Hecq, 2001, 2003). Panel A reports the full sample results while Panel B reports post-WW II result. In the full sample, no common cyclical features are found. In the post-WW II sample, WF and SSCF cases indicate a common cyclical feature. AIC, BIC and HQ information criteria all favor the SSCF specification.

Table 6 reports the forecast error variance decomposition for the full sample model for both the oil price and stock price series while the decomposition separates the variance into percentages accounted for by a shock in the common permanent component and a shock in the transitory (cyclical) component. The decomposition is based on the SSCF specification. This table is reported only for comparison purposes only, as we do not find a common cycle in the full sample (see Table 5 discussion). Therefore, the transitory shock should not be assumed as common. In the short-run 98% of the variance of the oil price is due to transitory shocks, declines to 82% in 60 months. Almost all of the variance (99% in 1 to 3 months and 100% afterwards) of the stock price is due to permanent shocks.

Table 7 reports the forecast error variance decomposition for the post-WW II model for the oil price and stock price series. The decomposition splits the variance into percentages accounted for by a shock in the common permanent component and a shock in the common

transitory (cyclical) component. The decomposition is based on the SCCF specification. Both the permanent and transitory shocks are common shocks. In the short-run 56% of the variance of the oil price is due to transitory shocks, this falls to 21% in 60 months. So, permanent shocks eventually dominate in the long-run and the impact of transitory shocks die off. In the short-run 44% of the variance of the stock price is due to transitory shocks, this falls to 12% in 60 months. So, permanent shocks eventually dominate in the long-run and the impact of transitory shocks die off. Transitory shocks account for almost half of the variance for both SP500 and WTI in the short-run, implying that common cyclical fluctuations play a significant role in the short-run but the common permanent shock dominate in the long-run.

#### **4. Conclusion**

The role played by the stock and oil markets in the US economy is an important area of inquiry. It is important for policy makers to determine what types of shocks drives these two markets. Ascertaining whether shocks are temporary (e.g. aggregate demand shocks) or permanent in nature (aggregate supply shocks), helps in better formulate policy. This paper investigates the role of permanent and transitory shocks, within the framework of common cycles and common trends, in explaining stock and oil prices. This is the first study to determine the nature of shocks that drives these two markets simultaneously. We perform a multivariate variance decomposition analysis of monthly data on the West Texas Intermediate (WTI) oil price and the S&P500. The dataset used in the study spans a long period of 150 years and therefore contains a rich history to examine both the short- and long-run comovement properties of the oil and stock prices. We examine the short- and long-run comovement of oil and stock prices.

This paper centers around two important themes: (a) existence of common stochastic trend (common long-run relation) (b) existence of a common features (i.e. comovement of short run, transitory components). We present decomposition of each series into the independent permanent and transitory components, impulse responses and forecast error variance decompositions in the presence of both common stochastic trends and common cycles. Stock prices and oil prices may commove in the long-run as well as in the short-run.

We employ the Vahid and Engle's (1993) approach to identify common trends and cycles in the context of the multivariate Beveridge and Nelson (1981) decomposition technique. If common-cycle restrictions are correctly imposed, estimates of the dynamic model (a vector autoregression model) are more precise, and leads to more accurate measurement of the relative importance of permanent and transitory shocks. Given that the oil and stock markets might comove both in the short- and long-run, it is of interest to see the relative impacts of transitory and permanent shocks on both variables. The analysis might reveal valuable information on what type of shocks drives these variables in the short- and long-run. The information might also help to disentangle the likely sources of transitory and permanent shocks, some of which are common to each variable. Our analysis considers both the common stochastic trends and common cycles in the bivariate model of oil and stock prices. Therefore, not only the common stochastic trend restrictions but also common cycle restrictions apply in our case. We present decomposition of each series into the independent permanent and transitory components, impulse responses, forecast error variance decomposition, in the presence of both common stochastic trends and common cycles. Our analysis reveals that there is a structural break in 1945 in the long-run linkage between log stock price and log oil prices.

For the post WW II period we find that the common trend component is driven primarily by the changes in the stock price. It is the oil price that deviates mostly from this long-run common trend. However, this deviation feeds back to the system as the cycle is common in the post-WW II model. We do not find a common cycle during the post-WW II period. The deviation of oil price should impact the stock market in the short-run as well. This impact is quite large in the short-run. The results for the full sample estimates are quite different from the post-WW II results. The results indicate that a common cycle is not found and most things are driven by permanent shocks. We find that the stock price is always close to its permanent component, while the oil price significantly deviates from its permanent component in several periods. So the common cycle is driven by the oil price deviating from the long-run trend. During the post-WW II sample, responses to permanent shocks are quite different for the stock market than in the full sample period. The response of oil price in the post-WW II period is negative and significant. The response of oil price in the full sample and post-WW II samples are similar. It is the response of stock market that is largely affected by the structural break. The responses of both oil and stock markets to transitory shocks in the post-WW II sample vary significantly compared to the full sample model. During the full sample period the response of oil price now becomes positive and exponentially decaying as well as the response of the stock market. In the post-WW II period we see more changes for oil price's response to transitory shocks. The stock market's response also becomes larger and significant in the post-WW II sample. In the post-WW II sample, the WF and SSCF cases indicate a common cyclical feature. AIC, BIC and HQ information criteria all favor the SSCF specification. The forecast error variance decomposition is based on the SCCF specification. For the full sample we find that in the short-run 98% of the variance of the oil price is due to transitory shocks, declines to 82% in 60 months. Almost all of the variance (99% in 1 to 3 months and 100% afterwards) of the stock price is due to permanent shocks.

For the post-WW II period we find both the permanent and transitory shocks are common shocks. In the short-run 56% of the variance of the oil price is due to transitory shocks, this falls to 21% in 60 months. Permanent shocks eventually dominate in the long-run and the impact of transitory shocks die out. In the short-run 44% of the variance of the stock price is due to transitory shocks, this falls to 12% in 60 months. Permanent shocks eventually dominate in the long-run and the impact of transitory shocks die out. Transitory shocks account for almost half of the variance for both (log) SP500 and (log) WTI in the short-run, implying that common cyclical fluctuations play a significant role in the short-run but the common permanent shock dominate in the long-run. The importance of both the permanent and transitory components in explaining the movements in both series (answered by the IRFs and FEVDs) show interestingly that in the short-run oil is driven mostly by cycles (transitory shocks) while the stock market is mostly driven by permanent shocks. But, permanent shocks dominate in the long-run.

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**Table 1: Descriptive statistics.**

	SP500	WTI
<b><i>Panel A: log levels</i></b>		
<b>Mean</b>	3.381	1.435
<b>SD</b>	1.947	1.343
<b>CV</b>	0.352	-2.303
<b>Min</b>	7.653	4.897
<b>Max</b>	0.732	0.707
<b>Skewness</b>	-0.726	-0.278
<b>Kurtosis</b>	208.434 <sup>***</sup>	162.122 <sup>***</sup>
<b>JB</b>	1865.708 <sup>***</sup>	1860.730 <sup>***</sup>
<b>Q(1)</b>	7417.626 <sup>***</sup>	7320.441 <sup>***</sup>
<b>Q(4)</b>	1866.888 <sup>***</sup>	1841.264 <sup>***</sup>
<b>ARCH(1)</b>	1863.956 <sup>***</sup>	1844.030 <sup>***</sup>
<b>ARCH(4)</b>		
<b><i>Panel B: log returns</i></b>		
<b>Mean</b>	0.0039	0.0005
<b>SD</b>	0.0476	0.0904
<b>CV</b>	-0.3563	-0.6931
<b>Min</b>	0.3524	0.7985
<b>Max</b>	-0.5286	-0.2482
<b>Skewness</b>	8.6242	13.024
<b>Kurtosis</b>	5899.1800 <sup>***</sup>	13270.6490 <sup>***</sup>
<b>JB</b>	25.1111 <sup>***</sup>	277.6604 <sup>***</sup>
<b>Q(1)</b>	34.0404 <sup>***</sup>	324.3194 <sup>***</sup>
<b>Q(4)</b>	93.6588 <sup>***</sup>	201.6846 <sup>***</sup>
<b>ARCH(1)</b>	230.4339 <sup>***</sup>	299.5312 <sup>***</sup>
<b>ARCH(4)</b>	0.0039	0.0005
<b>N</b>	1,871	1,871

**Note:** The table gives the descriptive statistics for Standard and Poor's S&P 500 Stock Market Index (SP500), and West Texas Intermediate spot crude oil price (WTI). All values are in natural logarithms in Panel A. Panel B gives the descriptive statistics for log returns. The sample period covers Sep. 1859-Jul. 2015 with  $n=1871$  observations. S.D. and C.V. denote standard deviation and coefficient of variation, respectively. In addition to the mean, standard deviation (S.D.), minimum (min), maximum (max), skewness, and kurtosis statistics, the table reports the Jarque-Berra normality test (JB), the Ljung-Box first [ $Q(1)$ ] and the fourth [ $Q(4)$ ] autocorrelation tests, and the first [ARCH(1)] and the fourth [ARCH(4)] order Lagrange multiplier (LM) tests for the autoregressive conditional heteroskedasticity (ARCH). <sup>\*\*\*</sup>, <sup>\*\*</sup> and <sup>\*</sup> represent significance at the 1%, 5%, and 10% levels, respectively.

**Table 2: Unit root tests.**

	LnSP500	LnWTI	LnSP500	LnWTI
	Full Sample: Sep. 1859 – Jul. 2015		Post-WW II Sample: Oct. 1945 – Jul. 2015	
<b>Panel A: Unit-root tests in levels</b>				
<b>ADF</b>	-1.3728 [21]	-2.1747 [25]	-2.5357 [6]	-2.2465 [21]
<b><math>Z_\alpha</math></b>	-5.1710 [1]	-4.4679 [1]	-9.9881 [1]	-12.1280 [1]
<b><math>MZ_\alpha</math></b>	-5.1669 [1]	-4.4660 [1]	-9.9389 [1]	12.1060 [1]
<b><math>MZ_t</math></b>	-1.4641 [1]	-2.1236 [1]	-2.2289 [1]	-2.4485 [1]
<b>DF-GLS</b>	-1.4620 [1]	-1.4326 [1]	-2.2335 [1]	-2.4313 [1]
<b>KPSS</b>	0.8678 <sup>***</sup> [1]	4.5657 <sup>***</sup> [1]	2.7545 <sup>***</sup> [1]	1.9687 <sup>***</sup> [1]
<b>Zivot-Andrews</b>	-5.4187 <sup>**</sup> [18]	-4.1562 [37]	-3.3566 [18]	-4.3794 [1]
<b>Panel B: Unit-root test in first differences</b>				
<b>ADF</b>	-10.1480 <sup>***</sup> [19]	-12.8360 <sup>***</sup> [23]	-27.6870 <sup>***</sup> [0]	-20.353 <sup>***</sup> [0]
<b><math>Z_\alpha</math></b>	-512.3800 <sup>***</sup> [9]	-2.5211E+05 <sup>***</sup> [14]	-485.6600 <sup>***</sup> [1]	-541.0000 <sup>***</sup> [1]
<b><math>MZ_\alpha</math></b>	-43.609 <sup>***</sup> [9]	-2.5176E+05 <sup>***</sup> [14]	-274.3300 <sup>***</sup> [1]	-356.6600 <sup>***</sup> [1]
<b><math>MZ_t</math></b>	-4.6549 <sup>***</sup> [9]	-354.7900 <sup>***</sup> [14]	-11.71100 <sup>***</sup> [1]	-13.2920 <sup>***</sup> [11]
<b>DF-GLS</b>	-6.9010 <sup>***</sup> [9]	-12.4930 <sup>***</sup> [14]	-14.5670 <sup>***</sup> [1]	-16.2530 <sup>***</sup> [11]
<b>KPSS</b>	0.9608 [9]	0.0951 [14]	0.0515 [1]	0.0697 [1]

**Note:** Panel A reports unit roots tests for the log levels of the series with a constant and a linear trend in the test equation. Panel B report unit root test for the first differences of the log series with only a constant in the test equation. ADF is the augmented Dickey-Fuller (Dickey and Fuller, 1979) test,  $Z_\alpha$  is the Phillips-Perron  $Z_\alpha$  unit root test (Phillips and Perron, 1988),  $MZ_\alpha$  and  $MZ_t$  are the modified Phillips-Perron tests of Perron and Ng (1996), DF-GLS is the augmented Dickey Fuller test of Elliot *et al.* (1996) with generalized least squares (GLS) detrending, KPSS is the Kwiatkowski *et al.* (1992) stationarity test, and Zivot-Andrews is the endogenous structural break unit root test of Zivot and Andrews (1992) with breaks in both the intercept and linear trend.  $Z_\alpha$ ,  $MZ_\alpha$  and  $MZ_t$  tests are based on GLS detrending. For the ADF unit root statistic the lag order is selected by sequentially testing the significance of the last lag at 10% significance level. The bandwidth or the lag order for the  $MZ_\alpha$ ,  $MZ_t$ , DF-GLS, and KPSS tests are select using the modified Bayesian Information Criterion (BIC)-based data dependent method of Ng and Perron (2001). <sup>\*\*\*</sup>, <sup>\*\*</sup> and <sup>\*</sup> represent significance at the 1%, 5%, and 10% levels, respectively.

**Table 3: Multivariate cointegration tests for the full sample: Sep.1859-Jul. 2015.**

**Panel A: VAR order-selection criteria**

Lag ( $p$ )	1	2	3	4	5	6	7	8
AIC	-5.2390	-5.4119	-5.4095	-5.4120	-5.4109	<b>-5.4150</b>	-5.4129	-5.4141
HQ	-5.2212	<b>-5.3822</b>	-5.3679	-5.3585	-5.3455	-5.3377	-5.3238	-5.3130
BIC	-5.2324	<b>-5.4010</b>	-5.3942	-5.3922	-5.3868	-5.3865	-5.3801	-5.3768

**Panel B: Johansen cointegration tests**

Eigenvalues	0.0175	0.0019	Critical values			Cointegration vector		
$H_0$	$\lambda_{\text{trace}}$	1%	5%	10%	LOIL	LSP	Trend	
$r \leq 1$	3.4905	16.5539	12.5180	10.6664	1.4989	-1.0396	0.0003	
$r = 0$	36.3757 <sup>***</sup>	31.1539	25.8721	23.3423	-0.1685	1.4667	-0.0056	

$H_0$	$\lambda_{\text{max}}$	1%	5%	10%	Loadings	
					LOIL	LSP
$r = 1$	3.4905	16.5539	12.5180	10.6664	-0.0108	0.0008
$r = 0$	32.8852 <sup>***</sup>	23.9753	19.3870	17.2341	-0.0007	-0.0020

**Panel C: Stock-Watson cointegration test**

$H_0: q(k,k-r)$	Statistic	Critical values for $q(2,1)$	
$q(2,0)$	-5.1557	1%	-38.539
$q(2,1)$	-48.6600 <sup>***</sup>	5%	-30.369
		10%	-26.501

**Note:** Table reports selection criteria and multivariate cointegration tests for the VAR( $p$ ) model of variables LSP, and LOIL. Panel A reports the AIC, BIC, and Hannan-Quinn (HQ) information criteria. The VAR order is selected based on minimum AIC and is 6. Panel B reports maximal eigenvalue ( $\lambda_{\text{max}}$ ) and trace ( $\lambda_{\text{trace}}$ ) cointegration order tests of Johansen (1988, 1991) with a restricted trend specification. Non-rejection of  $r=0$  for the Johansen tests implies no cointegration. Panel C reports the multivariate cointegration test of Stock and Watson (1988a). Under the null  $q(k,k-r)$  of Stock-Watson cointegration test,  $k$  common stochastic trend is tested against  $k-r$  common stochastic trend (or  $r$  cointegration relationship). Rejection of  $q(2,1)$  for the Stock-Watson test implies cointegration. Numbers in bold are the minimum information criterion values. \*\*\*, \*\* and \* represent significance at the 1%, 5%, and 10% levels, respectively.

**Table 4: Multivariate cointegration tests for the post-WW II sample: Oct. 1945-Jul. 2015.**

**Panel A: VAR order-selection criteria**

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Lag ( $p$ )	1	2	3	4	5	6	7	8
AIC	-6.2154	<b>-6.3274</b>	-6.3248	-6.3209	-6.3221	-6.3212	-6.3213	-6.3194
HQ	-6.1813	<b>-6.2704</b>	-6.2450	-6.2183	-6.1967	-6.1730	-6.1503	-6.1256
BIC	-6.2023	<b>-6.3056</b>	-6.2942	-6.2815	-6.2740	-6.2643	-6.2557	-6.2451

**Panel B: Johansen cointegration tests**

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Eigenvalues	0.0235	0.0042	Critical values			Cointegration vector		
$H_0$	$\lambda_{\text{trace}}$	1%	5%	10%	LOIL	LSP	Trend	
$r \leq 1$	3.5043	16.5539	12.5180	10.6664	-2.8381	-4.2907	0.0386	
$r = 0$	23.3639*	31.1539	25.8721	23.3423	-0.9193	2.3233	-0.0095	

$H_0$	$\lambda_{\text{max}}$	1%	5%	10%	Loadings	
					LOIL	LSP
$r = 1$	3.5043	16.5539	12.5180	10.6664	0.0069	0.0042
$r = 0$	19.8596**	23.9753	19.3870	17.2341	0.0024	-0.0021

**Panel C: Stock-Watson cointegration test**

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$H_0: q(k,k-r)$	Statistic	Critical values for $q(2,1)$	
$q(2,0)$	-7.2332	1%	-38.539
$q(2,1)$	-39.263***	5%	-30.369
		10%	-26.501

**Note:** Table reports selection criteria and multivariate cointegration tests for the VAR( $p$ ) model of variables LSP, and LOIL. Panel A reports the AIC, BIC, and Hannan-Quinn (HQ) information criteria. The VAR order is selected as 2 by all information criteria. Panel B reports maximal eigenvalue ( $\lambda_{\text{max}}$ ) and trace ( $\lambda_{\text{trace}}$ ) cointegration order tests of Johansen (1988, 1991) with a restricted trend specification. Non-rejection of  $r=0$  for the Johansen tests implies no cointegration. Panel C reports the multivariate cointegration test of Stock and Watson (1988a). Under the null  $q(k,k-r)$  of Stock-Watson cointegration test,  $k$  common stochastic trend is tested against  $k-r$  common stochastic trend (or  $r$  cointegration relationship). Rejection of  $q(2,1)$  for the Stock-Watson test implies cointegration. Numbers in bold are the minimum information criterion values. \*\*\*, \*\* and \* represent significance at the 1%, 5%, and 10% levels, respectively.

**Table 5: Common cyclical features tests using reduced rank regression.**

<b>Panel A: Full Sample, Sep. 1859 – Jul. 2015</b>								
Model	$H_0$	$\hat{\lambda}$	$\chi^2$	$\log L$	$p$ -value	AIC	HQ	BIC
(i) WF								
	$s \geq 1$	0.0269	50.8264 <sup>***</sup>	10337.60	> 0.0001	-11.072	-11.058	-11.033
	$s \geq 2$	0.1579	371.3850 <sup>***</sup>	10177.30	> 0.0001	-10.912	-10.910	-10.906
(ii) SCCF								
	$s \geq 1$	0.0273	51.7097 <sup>***</sup>	10337.20	> 0.0001	-11.073	-11.059	-11.037
	$s \geq 2$	0.1695	398.0150 <sup>***</sup>	10164.00	> 0.0001	-10.900	-10.900	-10.900
(iii) PSCCF								
	$s \geq 1$	0.0134	25.1295 <sup>***</sup>	10350.50	0.0015	<b>-11.085</b>	<b>-11.069</b>	<b>-11.043</b>
	$s \geq 2$	0.0242	70.8467 <sup>***</sup>	10327.60	> 0.0001	-11.071	-11.067	-11.059
<b>Panel B: Post-WW II Sample, Oct. 1945 – Jul. 2015</b>								
Model	$H_0$	$\hat{\lambda}$	$\chi^2$	$\log L$	$p$ -value	AIC	HQ	BIC
(i) WF								
	$s \geq 1$	0.0027	2.2253	5037.30	0.1358	-11.922	-11.918	-11.911
	$s \geq 2$	0.1164	105.7250 <sup>***</sup>	4985.55	> 0.0001	-12.039	-12.028	-12.011
(ii) SCCF								
	$s \geq 1$	0.0071	5.9435 <sup>*</sup>	5037.67	0.0512	<b>-12.042</b>	<b>-12.034</b>	<b>-12.020</b>
	$s \geq 2$	0.1304	122.7210 <sup>***</sup>	4979.28	> 0.0001	-11.912	-11.912	-11.912
(iii) PSCCF								
	$s \geq 1$	0.0033	2.7955	5034.29	0.2472	-12.039	-12.022	-11.994
	$s \geq 2$	0.0302	28.4194 <sup>***</sup>	5021.47	0.0001	-12.018	-12.009	-11.995

**Note:** Table reports the tests for  $s$  common cyclical features for three types of models (restrictions): (i) Weak Form (WF) reduced rank structure (Hecq et al. 2000, 2002, 2006), (ii) Serial Correlation Common Features (SCCF) (Engle and Kozicki, 1993; Vahid and Engle, 1993), and (iii) Polynomial Serial Correlation Common Features (PSCCF) (Cubadda and Hecq, 2001, 2003). All these restrictions lead to likelihood ratio tests, which distributed as  $\chi^2$  with degrees of freedom equal to number of restrictions. In the table  $\hat{\lambda}$  are the eigenvalues form canonical correlation problems relating to the three types of restrictions,  $\log L$  is the log likelihood of the model under the specification, and  $p$ -value is for the  $\chi^2$  test given in the fourth column. AIC, BIC, and HQ are the Akaika, Bayesian, and Hannan-Quinn information criterion, respectively. Numbers in bold are the minimum information criterion values. <sup>\*\*\*</sup>, <sup>\*\*</sup> and <sup>\*</sup> represent significance at the 1%, 5%, and 10% levels, respectively, and “>” means “less than” the following it.

**Table 6: Forecast Error Variance Decomposition for the full sample common features model: Sep. 1859-Jul. 2015.**

Variance Decomposition of LOIL			Variance Decomposition of LSP	
Step	Variance due to:			
	Permanent shock	Transitory shock	Permanent shock	Transitory shock
1	0.02	0.98	0.99	0.01
2	0.02	0.98	0.99	0.01
3	0.03	0.97	0.99	0.01
4	0.03	0.97	1.00	0.00
5	0.04	0.96	1.00	0.00
6	0.04	0.96	1.00	0.00
7	0.05	0.95	1.00	0.00
8	0.05	0.95	1.00	0.00
9	0.05	0.95	1.00	0.00
10	0.06	0.94	1.00	0.00
11	0.06	0.94	1.00	0.00
12	0.06	0.94	1.00	0.00
13	0.06	0.94	1.00	0.00
14	0.07	0.93	1.00	0.00
15	0.07	0.93	1.00	0.00
16	0.07	0.93	1.00	0.00
17	0.07	0.93	1.00	0.00
18	0.08	0.92	1.00	0.00
19	0.08	0.92	1.00	0.00
20	0.08	0.92	1.00	0.00
30	0.10	0.90	1.00	0.00
40	0.13	0.87	1.00	0.00
50	0.15	0.85	1.00	0.00
60	0.18	0.82	1.00	0.00

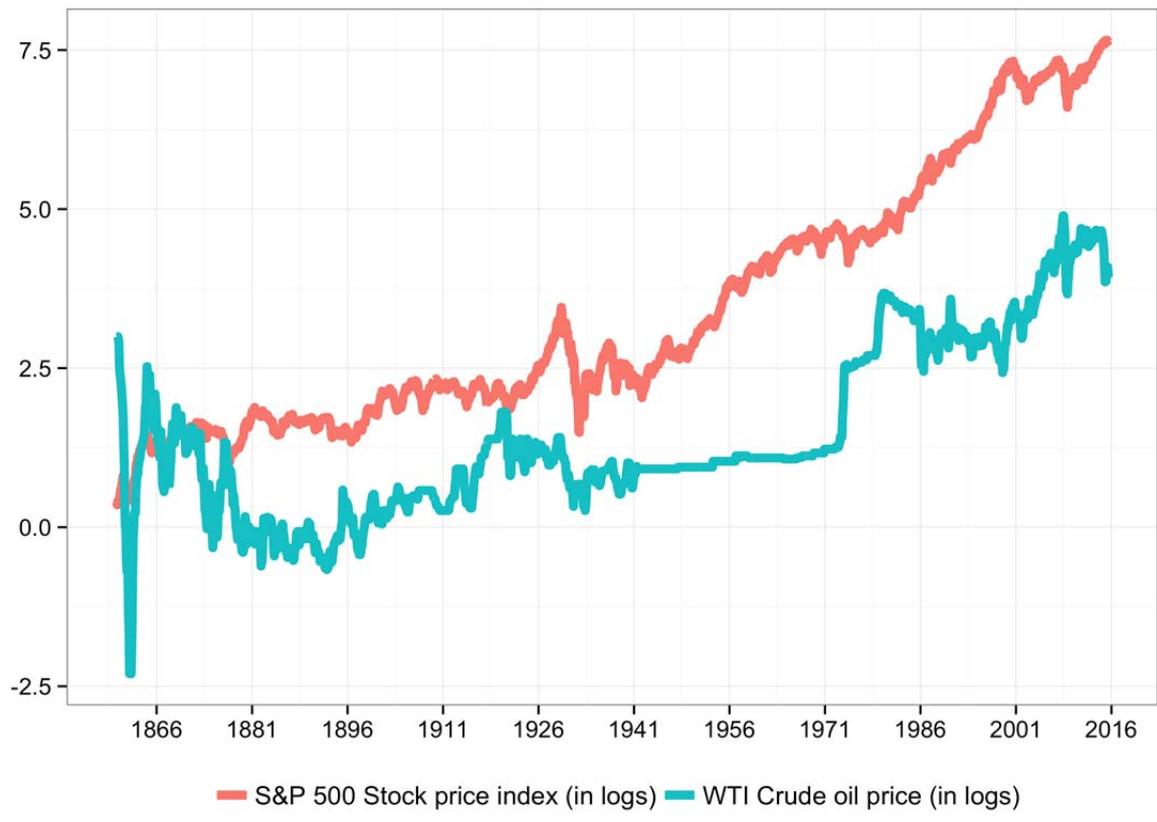
**Note:** The decomposition is based on the SSCF model, which selected jointly by all information criteria.

**Table 7: Forecast Error Variance Decomposition for the post-WW II common features model: Oct. 1945-Jul. 2015.**

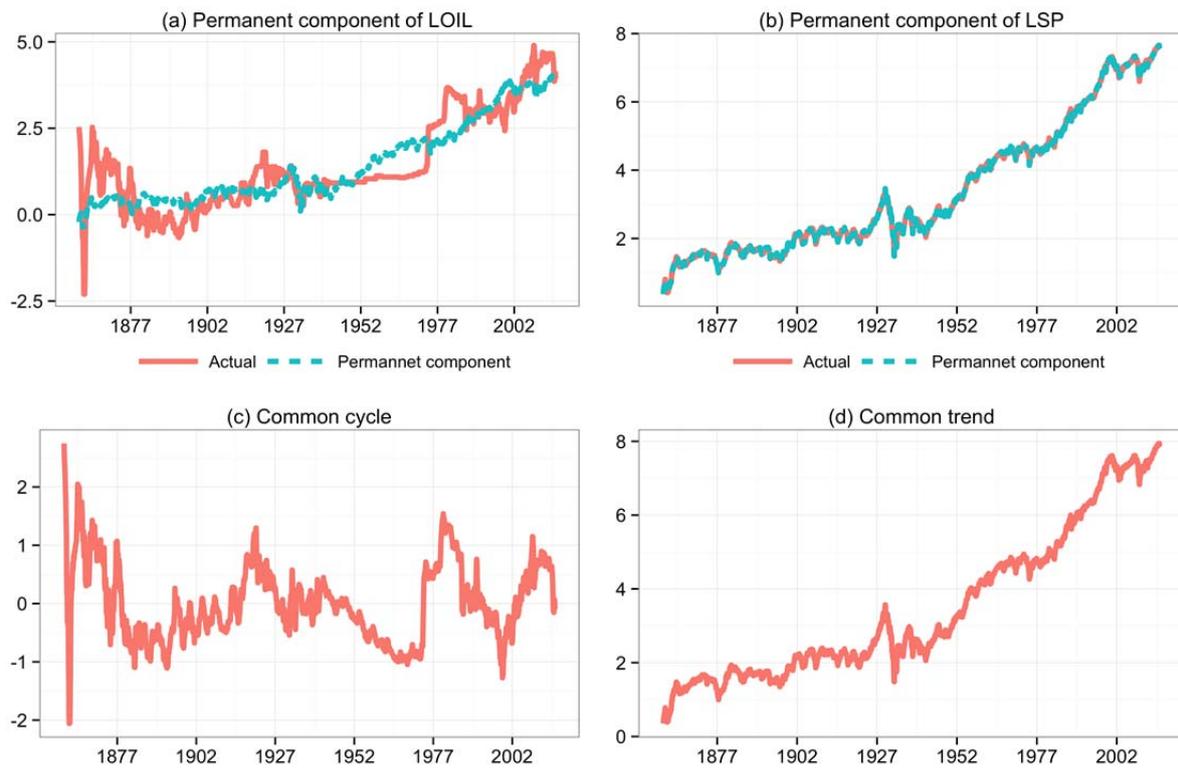
Variance Decomposition of LOIL		Variance Decomposition of LSP		
Step	Variance due to:			
	Permanent shock	Transitory shock	Permanent shock	Transitory shock
1	0.44	0.56	0.56	0.44
2	0.45	0.55	0.59	0.41
3	0.46	0.54	0.59	0.41
4	0.47	0.53	0.59	0.41
5	0.48	0.52	0.59	0.41
6	0.49	0.51	0.60	0.40
7	0.50	0.50	0.60	0.40
8	0.51	0.49	0.61	0.39
9	0.52	0.48	0.62	0.38
10	0.53	0.47	0.63	0.37
11	0.54	0.46	0.64	0.36
12	0.54	0.46	0.65	0.35
13	0.55	0.45	0.66	0.34
14	0.56	0.44	0.67	0.33
15	0.57	0.43	0.68	0.32
16	0.58	0.42	0.69	0.31
17	0.59	0.41	0.69	0.31
18	0.60	0.40	0.70	0.30
19	0.60	0.40	0.71	0.29
20	0.61	0.39	0.72	0.28
30	0.68	0.32	0.78	0.22
40	0.73	0.27	0.83	0.17
50	0.76	0.24	0.86	0.14
60	0.79	0.21	0.88	0.12

**Note:** The decomposition is based on the SSCF model, which selected jointly by all information criteria.

**Figure 1: Natural Logarithms of SP500 and WTI: Sept.1859-Jul. 2015**

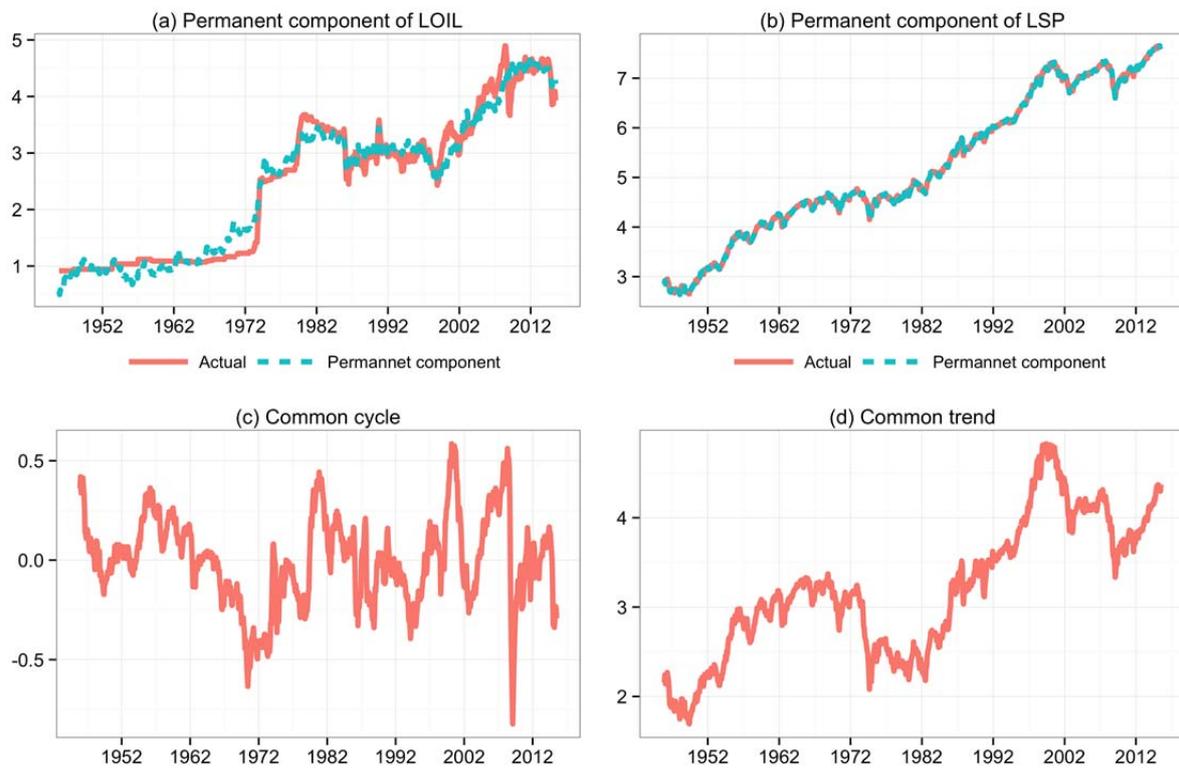


**Figure 2: Estimates of the common features for the full sample: Sep. 1859-Jul. 2015.**



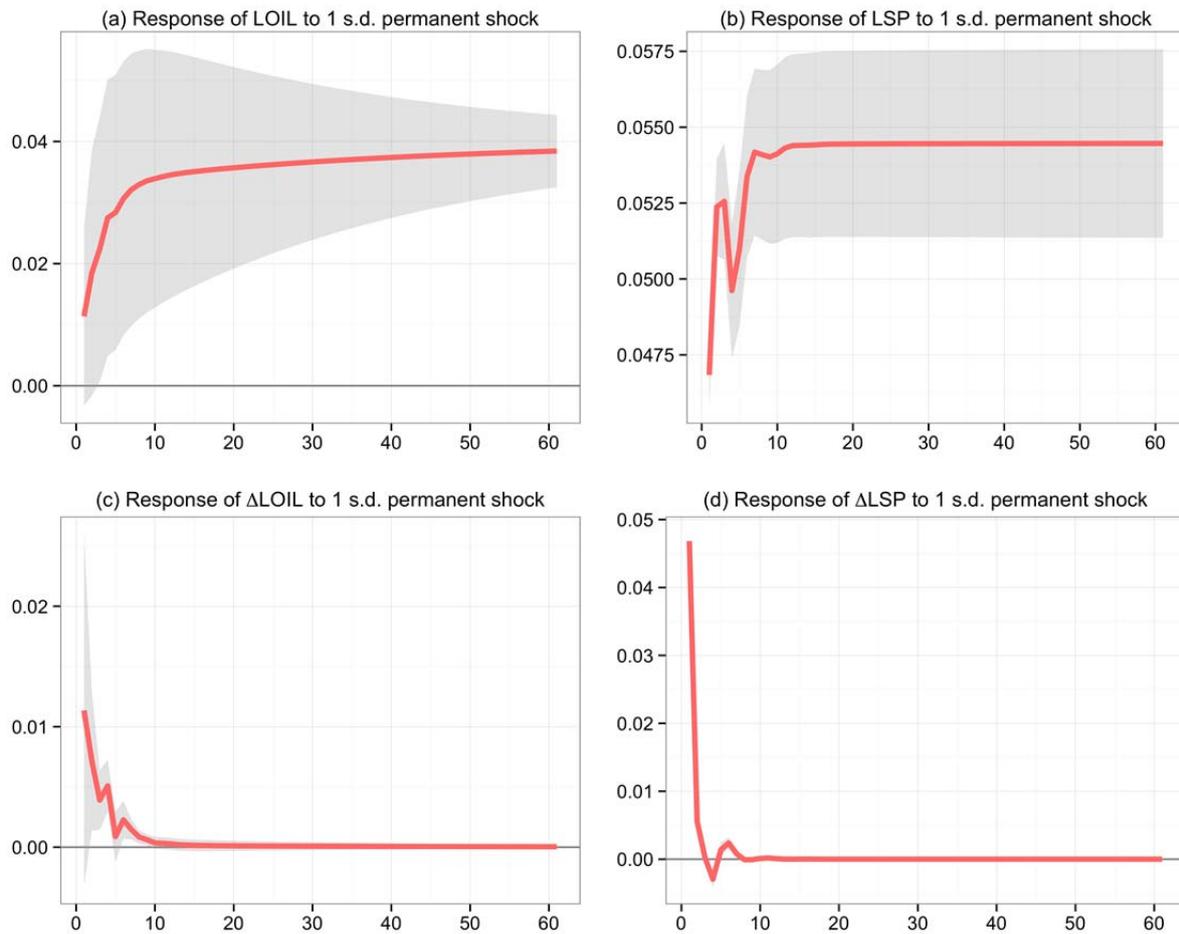
**Note:** The decomposition is based on the SSCF model, which selected jointly by all information criteria.

**Figure 3: Estimates of the common features for the post-WW II sample: Oct. 1945-Jul. 2015.**



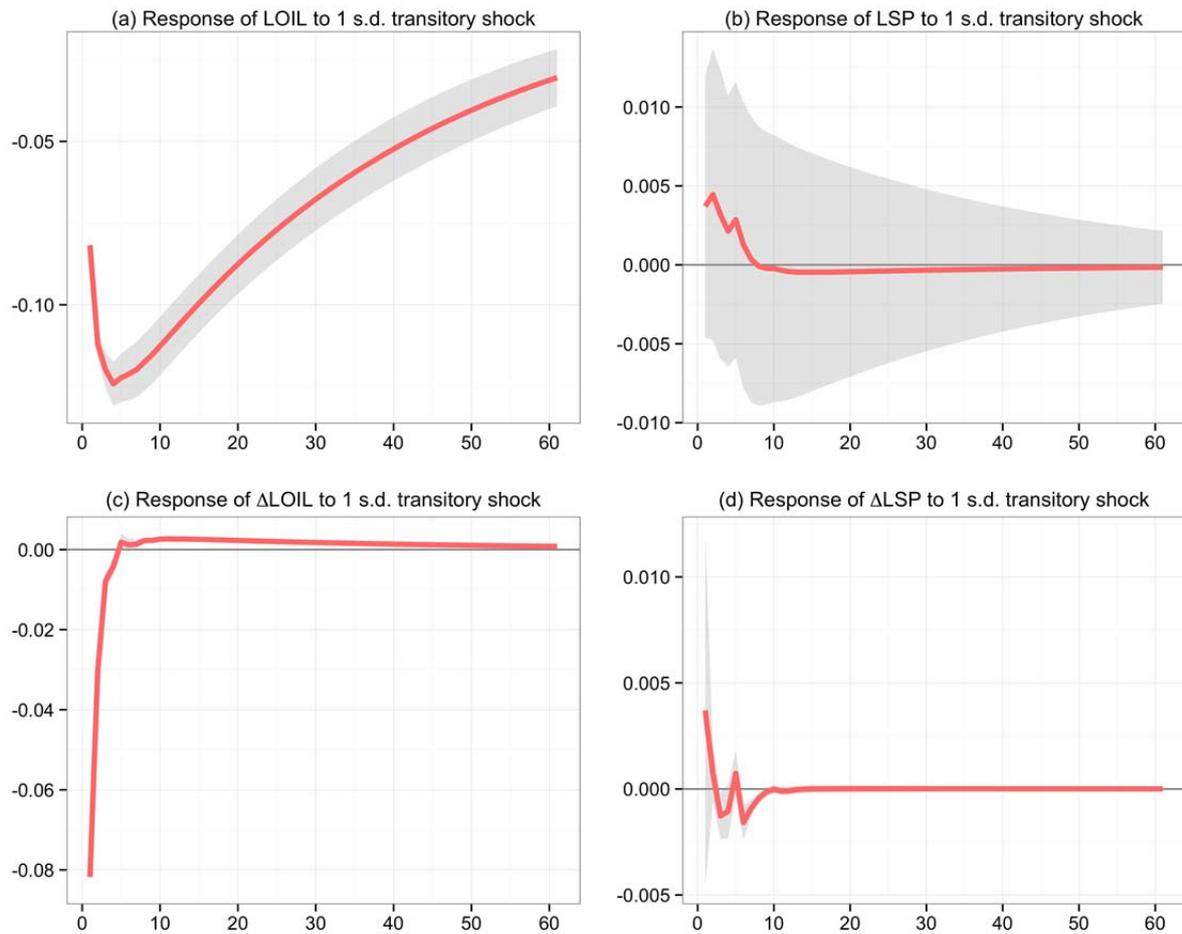
**Note:** The decomposition is based on the SSCF model, which selected jointly by all information criteria.

**Figure 4: Impulse responses to a permanent shock for the full sample: Sep. 1859-Jul. 2015.**



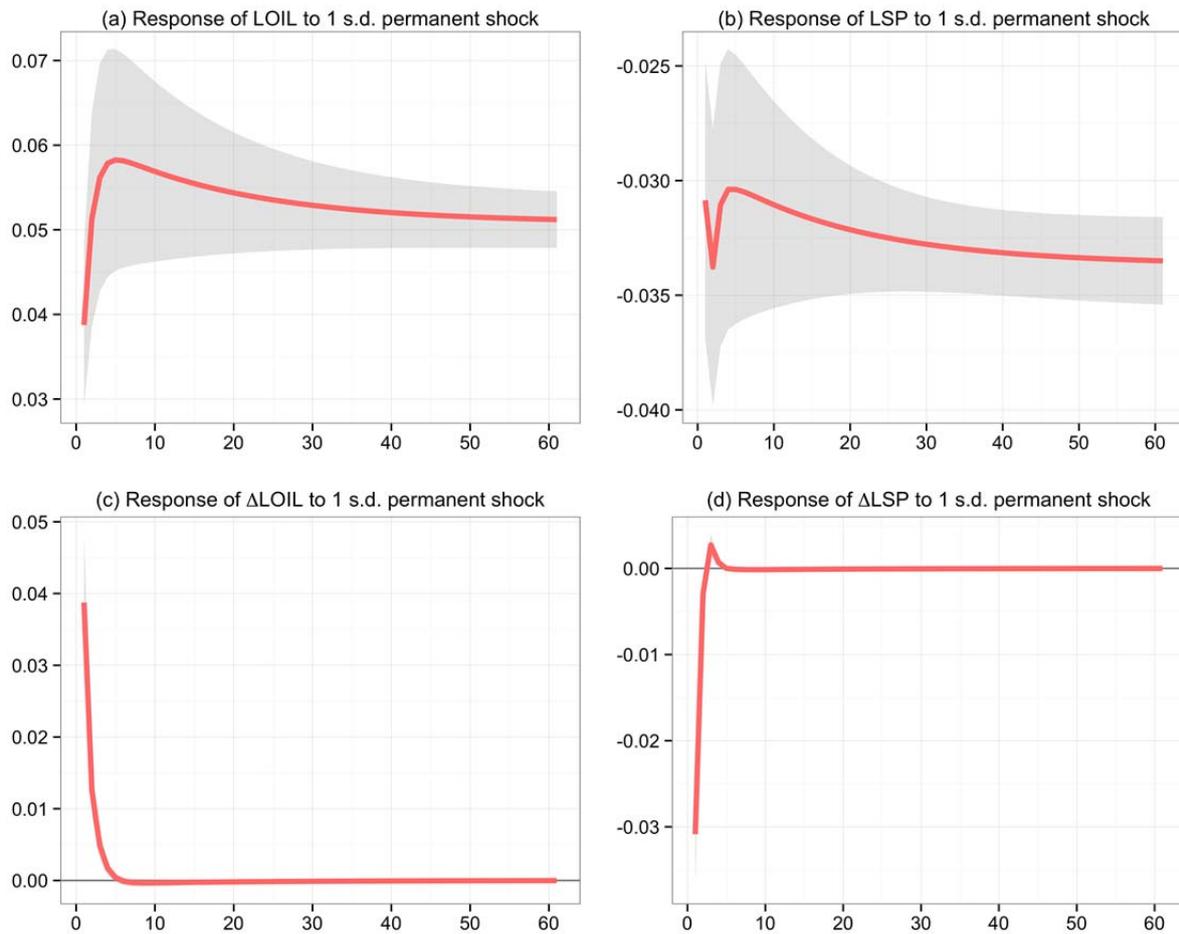
**Note:** The figure gives the impulse responses to a 1 standard deviation shock in the permanent component. The impulse responses are obtained the SSCF model, which selected jointly by all information criteria. The horizontal axis represents the steps in months. The solid line denotes the impulse response, while shaded region around the impulse response line represent the 68% ( $\pm 1$  standard deviation) confidence interval.

**Figure 5: Impulse responses to a transitory shock for the full sample: Sep. 1859-Jul. 2015.**



**Note:** The figure gives the impulse responses to a 1 standard deviation shock in the transitory component. The impulse responses are obtained the SSCF model, which selected jointly by all information criteria. The horizontal axis represents the steps in months. The solid line denotes the impulse response, while shaded region around the impulse response line represent the 68% ( $\pm 1$  standard deviation) confidence interval.

**Figure 6: Impulse responses to a permanent shock for the post-WW II: Oct. 1945-Jul. 2015.**



**Note:** See Note to Figure 4.