On the cubic law and variably saturated flow through discrete open rough-walled discontinuities: a review

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Highlights

- The role of the fractured intermediate vadose zone is becoming increasingly important.
- Discontinuity geometry affects flow, most notably due to changes in aperture, roughness and infill.
- The applicability of the cubic law is often queried.
- Better understanding will contribute to issues of slope and excavation stability and contaminant transport.
- Open questions requiring addressing concluded the paper.

Abstract: Fracture flow is fairly well documented with the widespread application of, for instance, the cubic law and assumed smooth parallel plate model. Geometrical intricacies such as aperture, roughness and infill do however significantly influence the validity of the cubic law with even its application to smooth parallel systems being contestable. Rock mechanical discontinuity surveys provide valuable information regarding the discontinuity geometry that can likely contribute to the evaluation of flow through individual fractures with variable properties. The hydraulic aperture is available for the transmission of flow, while normal and shear stresses alter discontinuity properties over time. In this, numerous advances have been made to better accommodate deviations of natural discontinuity geometry to that of smooth parallel plates and at partial saturation. The paper addresses these advances and details conditions under which the cubic law, even in local form, fails to adequately estimate the hydraulic properties. The role of roughness in open discontinuities is addressed in particular, as contact areas and high amplitude roughness cause most extensive deviation from the cubic law. Aperture of open fractures still governs hydraulic properties, but inertial forces control flow in very rough fractures, in which instances the applicability of the cubic law should be revisited. Open questions are finally posed,

assessment of which will contribute significantly to the understanding of flow through individual discontinuities as well as fracture networks.

Keywords: aperture; roughness; cubic law, parallel plate model; nonlinear flow

List of Symbols:

a – half mean fissure width ($2a$ = mean fissure width)

A_h – cross-sectional throughflow area

b – spacing between fractures

C – constant value

D_h – hydraulic diameter = F

e – aperture

e_h – hydraulic aperture

e_m – mechanical aperture

F – body force vector per unit mass

F – relative roughness = D_h

Fo – Forchheimer number

g – gravitational acceleration

i – roughness

i_1 – large-scale roughness

i_2 – small-scale asperities

Jc – critical hydraulic gradient

JRC – joint roughness coefficient

k – absolute wall roughness or asperity height

K – hydraulic conductivity

k – intrinsic permeability

K_f – fracture hydraulic conductivity

K_{fi} – fracture infill hydraulic conductivity

K_m – matrix hydraulic conductivity

l – characteristic length of fracture

P – pressure

Q – volumetric flow rate

Re – Reynolds number

Re_c – critical Reynolds number

REV – representative elementary volume

RQD – rock quality designation

T – transmissivity

$\mathbf{u}$ – velocity vector of flow field

v – flow velocity (lower case Roman letter 'vee')

w – width of fracture

α – non-Darcy effect factor

β – non-Darcy flow coefficient or Forchheimer coefficient

η – porosity

μ – dynamic viscosity of fluid

ν – kinematic viscosity (lower case Greek letter 'nu')

ρ – fluid density

σ – standard deviation

1. Introduction

A significant portion of the subsurface comprises unsaturated weathered and fractured rock. Given changes in stage of weathering and discontinuity development (which defines the primary and secondary porosity), geometry (related to continuity, roughness and pore size) and in-situ conditions (such as overburden stresses or induced changes), evaluation of flow through discontinuities at moisture contents below saturation poses certain challenges.

Water flow and contaminant transport are documented in significant detail. The well-known cubic law [1] estimates the viscous flow between smooth parallel plates. The model however loses applicability when the roughness and mean aperture are in the same order of magnitude, which has resulted in approaches to deal with these variations, such as the Reynold lubrication equation [2] or the Navier-Stokes equation.

It was as early as 1985 [3] and 1986 [4] that experimental work showed that the cubic law generally fails as the prior authors could only achieve a 30% contact area at effective stresses of 90 MPa in natural fractures. Flow through a fracture also decreases at a rate that exceeds the cube of the mean aperture, and a nonlinear relationship exists between mean aperture and normal stress [5][6].

Aydin [7] evaluated the applicability of the cubic law for low flow rates, as well as a wide range of friction and conductivity modification factors based on a number of flow domains.

Berkowitz [8] next identified key research issues, including scale and the correlation between geometry and hydraulic properties in unsaturated rock masses. Partially and variably saturated fractured media are also noted specifically. Neumann [9], on the other hand, highlights complex flow in fracture and matrix blocks, as well as their intersections. He continues to state that field tests likely provide more accurate findings than geometric correlations.

The implications of better understanding of partially saturated flow through rock mass fracture networks are numerous. Worldwide surface infrastructure is presently extending skywards as well as to deeper depths below surface. The increased need for natural resources also implies deeper mines, larger dewatering cones around mines, dewatering of rock aquifers, increased size and toxicity of waste disposal sites, deep nuclear waste repositories, both on land surface and in subsurface excavations. Prospects of deep hydraulic fracturing (fracking) for shale gas or coal-bed methane raise questions about upward migration of hydrocarbon liquids and gases, potentially rendering groundwater aquifers susceptible to contamination. These anthropogenic activities impact on the hydrosphere and lithosphere and need to be quantified to determine negative impacts and remedial measures.

The quantification of water movement through rock masses also has enormous economic implications. Water inflow into deep mines, building basements, civil engineering tunnels (water and transport), subsurface waste disposal sites (nuclear and other) and large rock caverns (hydro-electric and storage facilities) needs to be quantified to create safe operational conditions. Closer to surface, water seepage into road cuts and through rock foundations and rock slopes have serious stability consequences.

This paper aims to evaluate the influence of individual joint conditions on partially saturated flow, highlighting the constraints of the commonly assumed smooth parallel plate model. Orientation and fracture networks are not considered, as the focus is not on bulk flow, but rather on the properties of individual discreet fractures.

2. Discontinuities in Rock

Discontinuity and *fracture* are often used interchangeable, although the definition of the prior is mostly applied in a rock mechanical and the latter in a geological and hydrogeological sense. Given that flow occurs through fractures and that discontinuities determine the strength of rock masses, the application of the terms often overlap, notably when considering (as in this paper) the existing discontinuity-based terminology and classification to better understanding of flow through fractures. For clarification, some definitions are supplied below, although the terms are used synonymously in this paper.

A *discontinuity* refers to any plane of mechanical or sedimentary origin that serves to separate intact rock blocks within the rock mass, that has a tensile strength of zero or very close thereto, and in which the strength of the infill material or the shear strength of the discontinuity plane determines its mechanical behaviour [10].

A *fracture*, on the other hand, refers to any separation in a large body of solid earth materials, irrespective of the geological origin of the separation (i.e. joint, fault, bedding planes or shear zones), occurring within and interacting with intact rock material, the properties of the combined system referred to as *rock mass*. As fractures cannot be isolated from the intact rock, the flow behaviour within a rock mass is often considered to be a function of the intact rock and fracture properties [11].

Rock mass quality is to a large extent dependent on the density of discontinuities. Rock Quality Designation (RQD) is a measure of the degree of fracturing [12], is generally determined from borehole core, and is defined as the percentage of intact core pieces longer than 10 cm in the total length of core from boreholes. It is often the only method used for measuring the fracturing in a rock mass [13]. The RQD forms an important input into most engineering rock mass classifications.

Fracture or discontinuity planes are not always smooth and parallel. Roughness of fractures comprises large-scale waviness (i_1), as well as small-scale asperities (i_2) on the fracture walls that influence strength and stress relationships (**Figure 1**) [14][15][16][17]. Field descriptions simplify roughness and asperities as shown in **Figure 2.**

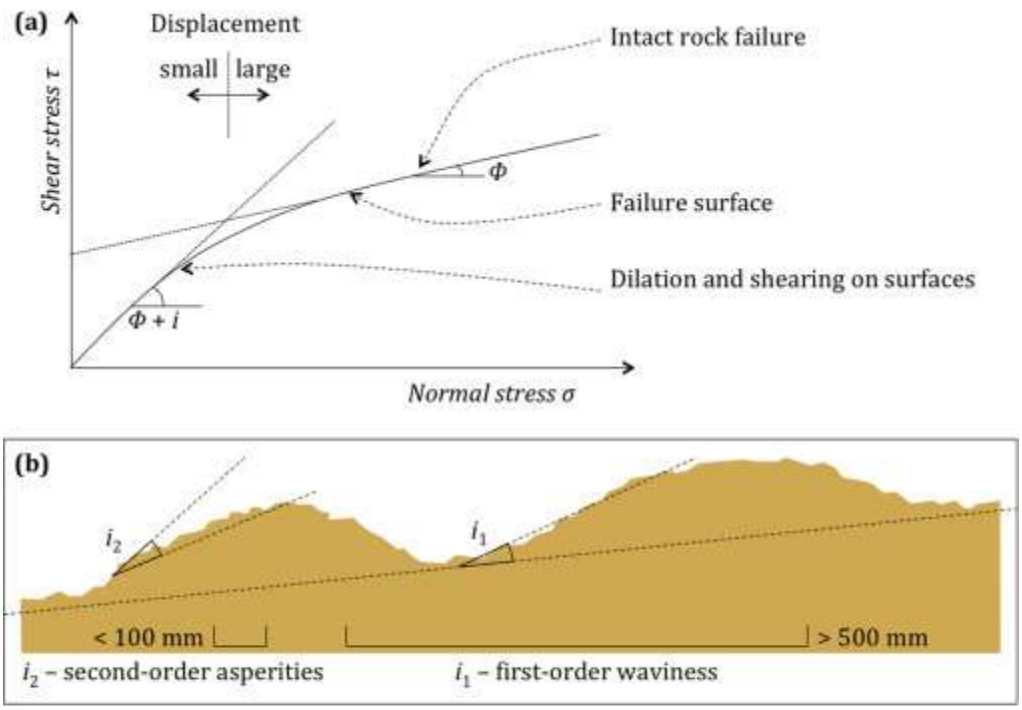

Figure 1. (a) Effects of surface roughness *i* on friction *Φ* and (b) different scales of roughness (e.g. [14][15][16]).

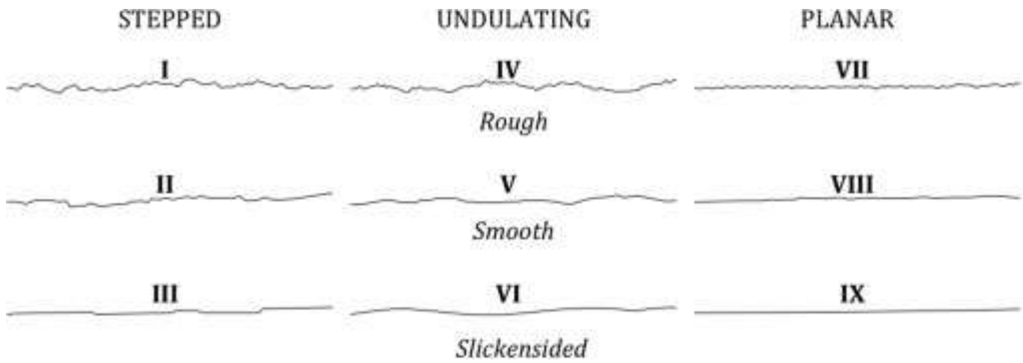

Figure 2. Field description of roughness (scale 10cm) (after [14][16][17].

The concept of *representative elementary volume* becomes important in identification of appropriate scales of consideration. Bear [1] addresses the concept in great detail, elaborating on finding the so-called REV where microscopic and macroscopic heterogeneities are addressed and the porosity of the investigation volume of medium accurately represents the behaviour of a material. Applied to fractures, very small-scale investigation will result in porosity η of either one or zero, depending on whether the point of investigation represents a solid mineral grain or an open fracture aperture. Excessively large scales of investigation are subject to geological variability such as geological contacts, shear zones and other regional influences. A certain REV is achieved intermediately where the microscale and macroscale influences no longer dominate.

Water movement through unsaturated fractured rock mass require understanding of the slow matrix flow and the fast fracture flow with the latter acting as preferential pathways. It has, for instance, been found that fracture permeability and water saturation can vary greatly on scales below one metre, and the REV needs to be increased for large-scale applications[18].

Typical discontinuity survey data collected in rock mechanical context are shown in **Figure 3**. One should, however, always be cognisant of the differences in data obtained from exposed rock (e.g. excavations, outcrops) and conditions at depths. For the sake of this paper, however, the focus is solely on individual joint geometry.

The shear strength of rock discontinuities, usually also being the shear strength of the rock mass, is either calculated from empirical equations using data collected during a discontinuity survey or determined in a laboratory on small rock samples incorporating a single discontinuity. The line survey data are also used in rock mass classifications with wide application in determining the stability and support needed in tunnels, slopes and other structures in rock.

1. TYPE OF STRUCTURE	
0	Fault zone
1	Fault
2	Joint
3	Cleavage
4	Schistosity
5	Shear
6	Fissure
7	Tension crack
8	Foliation
9	Bedding

2. APERTURE (mm)	
1	Wide (> 200)
2	Moderately wide (6 200)
3	Moderately narrow (20–60)
4	Narrow (6–20)
5	Very narrow (2–6)
6	Extremely narrow (< 2)
7	Tight

3. INFILLING – NATURE	
1	Clean
2	Surface staining
3	Non-cohesive
4	Inactive clay
5	Swelling clay
6	Cemented
7	Chlorite, talc, gypsum
8	Other – specify

4. INFILLING – COMPRESSIVE STRENGTH	
1	Very soft (< 40 kPa)
2	Soft (40–80)
3	Firm (80–150)
4	Stiff (150–300)
5	Very stiff (300–500)
6	Hard/ very weak (600–1250)
7	Weak (1.25–5 Mpa)
8	Mod. weak (5–12.5)
9	Mod. strong (12.5–50)
10	Strong (50–100)
11	Very strong (100–200)
12	Ext. strong (> 200)

6. WAVINESS	
1	Wavelength (m)
2	Amplitude (m)

5. FRACTURE ROUGHNESS	
1	Polished
2	Slickensided
3	Smooth
4	Rough
5	Defined ridges
6	Small steps
7	Very rough

7. WATER	
1	Dry
2	Damp
3	Seepage
	Flow
4	< 10 ml/s
5	10–100 ml/s
6	0.1–1 l/s
7	10–100 l/s
8	> 100 l/s

Figure 3. Descriptors for discontinuity surveys [17][20].

Discontinuity data collection generally depends on accessible, exposed rock outcrops or core from boreholes. The calculation of shear strength and determination of rock classes in rock mass classifications usually incorporate a parameter describing the water condition in the discontinuity [19][20][21]. These descriptions vary between dry, moist, wet, dripping and flowing.

Partial saturation is therefore accommodated in these descriptors although pore water pressure is generally calculated for saturated conditions. The normal effective stress component in both Mohr-Coulomb and Barton-Choubey equations allow for any value to be used from dry to saturated conditions. However, typically full saturation is assumed to calculate for the lowest effective stress.

Groundwater measurements are generally limited to visual assessment, water loss measurements in a single borehole or water pressure measurements [13].

3. Saturated Flow in Discrete Fractures

3.1. Saturated hydraulic conductivity of discontinuities

When considering rock mass, it is clear that the hydraulic conductivity of the fracture or discontinuity (K_f) and of the intact rock matrix (K_m) should vary considerably with fractures often perceived as pathways of low resistance through which most of the groundwater flow occurs [22]. The intact rock material poses a much greater resistance to flow as the water needs to pass through the small interstitial pore spaces. The conductivity of a rock mass can therefore be considered the sum of the matrix (primary) and fracture (secondary) conductivities as per Eq. 1 [15].

$$K = K_m + K_f \;\; and \;\; K_f \gg K_m \tag{1}$$

According to numerous authors, e.g. [15][22][23][24], flow through the intact rock matrix can be calculated according to Darcy's Law, assuming that slow groundwater flow is laminar. Exceptions exist where large cavities, large fractures or steep hydraulic gradients result in turbulent flow and Darcy's Law no longer applies.

3.2. Navier-Stokes and Reynolds equations

The Navier-Stokes equation governs fluid flow. This complex equation can be simplified to local scale in fractures if, for instance, viscous forces dominate inertial forces and the Reynolds number (calculated as per Eq. 2 as a function of the fluid density ρ, average fluid velocity along the fracture v, average aperture e, fluid viscosity μ and characteristic fracture length in the flow direction l) becomes very small [25][26][27].

$$Re = \frac{\rho v e^2}{\mu l} = \frac{\rho Q}{\mu w} \ll 1 \tag{2}$$

If the abovementioned conditions apply, and if the fracture aperture does not change too abruptly, the Navier-Stokes equation (Eq. 3) can be simplified to the Reynolds equation (Eq. 4) with derivation in [25][28]) which is dependent on fluid density (ρ), the body force vector per unit mass (**F**), the pressure (P), the fluid viscosity (μ) and the velocity vector (**u**).

$$\frac{\delta \mathbf{u}}{\delta t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = \mathbf{F} - \frac{1}{\rho}\nabla P + \frac{\mu}{\rho}\nabla^2 \mathbf{u} \tag{3}$$

$$\nabla \cdot (d^3 \nabla P) = 0 \tag{4}$$

3.3. Parallel plate model and cubic law

The *fracture aperture* (the separation between the opposing walls of a fracture) is considered to be most influential with respect to the fracture conductivity and the aperture can be seen as analogous to the pore geometry of the intact rock matrix. However, the intrinsic difficulty in determining an aperture for an entire fracture length resulted to a fundamental approach, viz. the parallel plate model. A natural fracture can be conceptualised as an opening with a fixed aperture between two parallel plates that represent the fracture walls as per **Figure 4**. Assuming laminar flow in this model, the

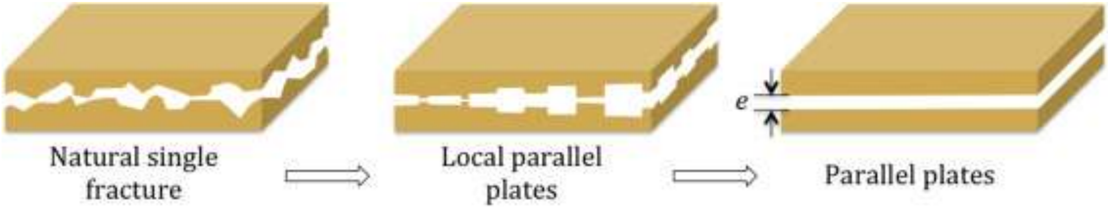

Figure 4. Conceptualisation of a rough natural fracture as two parallel plates separated by a constant aperture, e (after [32]).

hydraulic conductivity and transmissivity of such a fracture can be calculated according to the cubic law given in Eq. 5-7 as a function of gravitational acceleration g, fracture aperture e, characteristic fracture length l, water's kinematic viscosity v, and the spacing between fractures b (e.g. [1][8][9][14][15][22][29][30][31][32][33][34]. The cubic law represents a solution of the complex Navier-Stokes equation.

The derivation of the cubic law assumes a fracture represented by two smooth, parallel plates separated by aperture e and possessing a uniform pressure gradient within the plane of the fracture. Here the pressure gradient and the velocity field are expected to present components solely in the direction of flow. The velocity gradient only changes along aperture direction (closer or farther from the walls) and not along the flow direction, fulfilling the Navier-Stokes equation's no-slip boundary condition requiring the tangential velocity at the fracture walls (phase boundary) to be zero [28].

The cubic law is a function of the cube of the aperture and a constant term C as per Eq. 5 [34].

$$\frac{Q}{\Delta h} = C \cdot e^3 \ \ where \ \ C = \frac{w}{l} \cdot \frac{\rho g}{12\mu} \tag{5}$$

Incompressible Newtonian viscous fluid flow is then governed by Eq. 6 and Eq. 7 with the solution to achieve the cubic law detailed by [28] and, in terms of combining Darcy's and Poiseuille's Laws, as detailed by [35].

$$K_f = \frac{ge^3}{12vb} = \frac{\rho g e^3}{12\mu b} \tag{6}$$

$$T = \frac{gwe^3}{12vb} = \frac{\rho g w e^3}{12\mu b} \tag{7}$$

An alternative to the cubic law for a saturated individual fracture (i.e. where spacing of fractures is not know) also exists in the form of Eq. 8.

$$K_f = \frac{ge^2}{12v} \ \ or \ \ Q = \frac{ge^3}{12v} \cdot \Delta h \tag{8}$$

The highest hydraulic conductivity can be obtained through the cubic law (Eq. 2), although it applies solely to laminar flow through smooth parallel planar discontinuities.

The lowest equivalent hydraulic conductivity for a discontinuity system with infilling (with hydraulic conductivity K_{fi}), however, is given by Eq. 9 [14].

$$K = \frac{e}{b}K_{fi} + K_m \tag{9}$$

The influence of aperture and spacing of discontinuities on the hydraulic conductivity is shown in **Figure 5**.

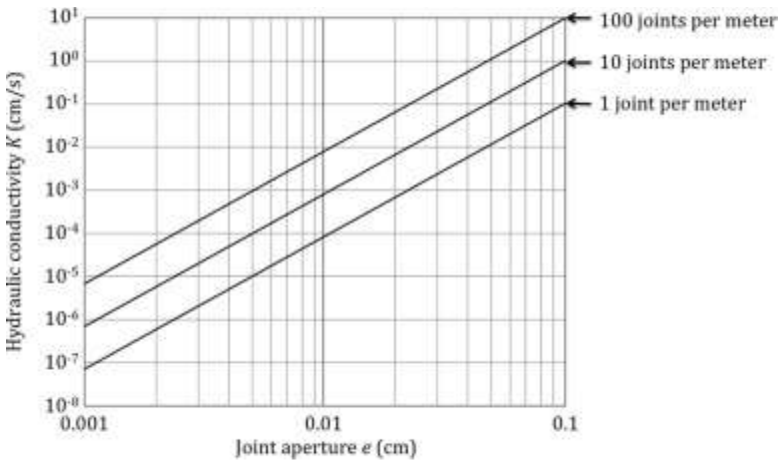

Figure 5. Influence of joint aperture e and spacing b on hydraulic conductivity K parallel to a set of smooth parallel joints (adapted from [14]).

3.4. Nonlinear flow

In many instances the inertial forces are not negligible when compared to the viscous forces. At higher flow rates, the linear relationship between flow rate and pressure drop (as per Darcy's Law and the Cubic Law) no longer applies and flow becomes nonlinear. The Forchheimer Law is mostly used to address this nonlinear flow as a function of two parameters, viz. *A*, the linear coefficient related to fluid properties, and *B*, the nonlinear coefficient related to geometries of the medium (Eq. 10 to Eq. 12) [26][27].

$$-\nabla P = AQ + BQ^2 \tag{10}$$

$$A = \frac{\mu}{kA_h} = \frac{12\mu}{we_H^3} \tag{11}$$

$$B = \frac{\beta\rho}{A_h^2} = \frac{\beta\rho}{w^2e_H^2} \tag{12}$$

The Forchheimer or non-Darcy flow coefficient, β, has dimension of [L^{-1}] and, when β=0, the Forchheimer's Law reduces to Darcy's Law. Together with the Reynolds number (which relates inertial to viscous forces), the Forchheimer number *Fo* relates nonlinear to linear pressure losses in the Forchheimer's Law (Eq. 13) [26][27].

$$Fo = \frac{BQ^2}{AQ} = \frac{BQ}{A} \tag{13}$$

A critical Reynolds number can be defined based on this, where the non-Darcy effect factor α relates to an α percentage contribution to the overall pressure drop at the critical Reynolds number (Eq. 14). Substituting this into the equations for the Reynolds number, the cubic law and the determination of coefficients A and B yields the determination of the critical Reynolds number (Eq. 15) [26][27].

$$\alpha = \frac{BQ^2}{AQ+BQ^2} \tag{14}$$

$$Re_c = \frac{A\rho\alpha}{B\mu w(1-\alpha)} = \frac{12\alpha}{1-\alpha}\frac{1}{\beta e_H} \tag{15}$$

4. Validity of the Cubic Law for Discrete Fractures

Parameters A and B of Forchheimer's Law addresses the properties of the fluid and the medium respectively. These are also the parameters significantly influencing the validity of the cubic law on rough open natural discontinuities.

4.1. Considerations with respect to Flow

Unsaturated seepage in soil is well documented [36]. Rock, however, poses different concerns as primary porosity, if present, essentially serves to store water while fractures serve to transmit water. Given the likely higher aperture of fractures compared to void diameters of interstitial porosity, adhesion (water–mineral attraction) becomes less dominant and cohesion (water–water attraction) predominates, resulting in less capillarity and suction, and more gravitational drainage.

Although many of the subsequent paragraphs relate to saturated systems as well, flow regimes, discontinuity geometry and influences of stresses likely have greater influence in partially and variably saturated systems.

4.1.1. Influence of flow mechanism and flow regime

Flow regimes in open fractures differ from those anticipated in soils. Depending on the aperture, continuity, roughness and other factors, air–water flow in discontinuities is governed by the wetting behaviour of water as well as the water saturation. Different classifications exist for such flow phases, although single vertical fractures with relatively low water saturation are mostly considered systems where water will flow as droplets or films (*flow mechanism*), either as laminar or turbulent flow (*flow regime*), on the discontinuity surfaces. Typical flow mechanisms are shown in **Figure 6** and are detailed extensively by e.g. [35], although the possibility of different mechanisms (e.g. drop flow on fracture walls of vertical fractures at low water saturation) and influences of discontinuity intersections (e.g. larger pore spaces) and orientations (e.g. vertical versus horizontal) should also be considered.

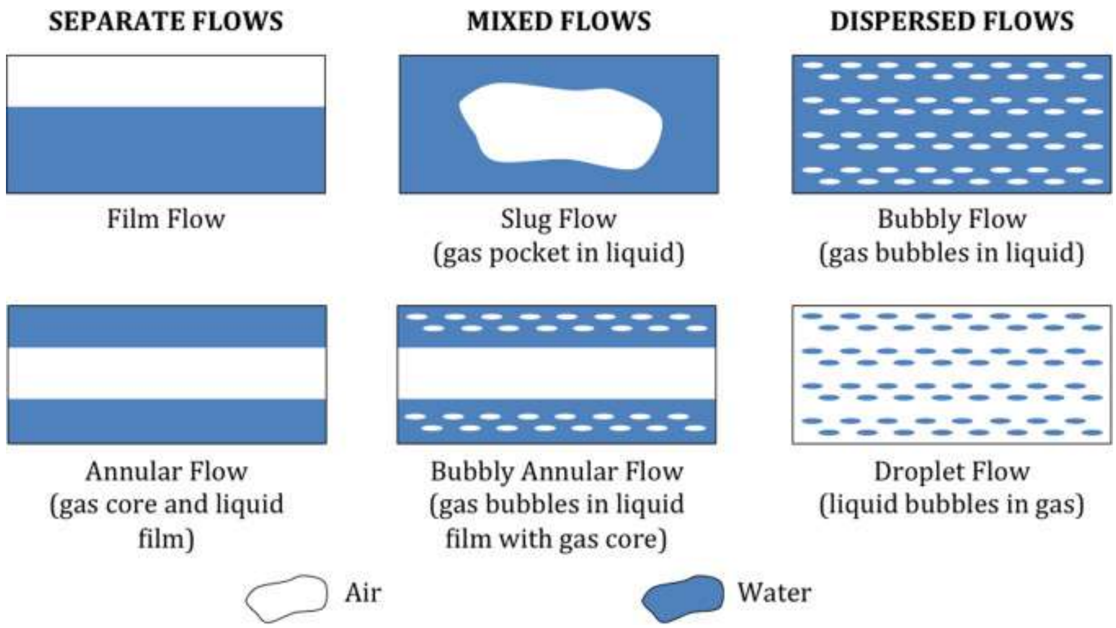

Figure 6. Flow mechanisms of water–air systems through single fractures or macropores (after [15][68]).

High viscosity, low density and small apertures have been shown to stabilize the flow field, rendering the cubic law more valid (e.g. [37] in [28]). Compensation for turbulence is also experienced earlier in rough-walled than smooth fractures through turbulence developing earlier, and transmissivities of rough fractures are lower than for smooth counterparts [38].

In certain non-Darcian conditions, the cubic law can still be applicable in situations in which the inertial component of flow is modest [39]. The Forchheimer Number (*Fo*) was used instead of the *Re* number to assess flow, and it was found that for a non-Darcian flow of *Fo* less than 1.30, the cubic law was able to produce meaningful results [40].

The dependence on the flow regime (laminar versus turbulent) has been addressed by other authors in recent research (e.g. [41]), as has the models of flow mechanisms such as film and droplet flow (e.g. [42]).

4.1.2. Influence of hydraulic gradient

The hydraulic gradient in fractures plays an important role in assessing the applicability of the cubic law. At too high hydraulic gradients flow is likely to be turbulent, rendering the cubic law inapplicable, as it relates back to the flow velocity and the Re number [43].

A critical hydraulic gradient value (*Jc*) exists which marks the boundary between linear and non-linear flow in fractures. Below this *Jc* value, the flow in the fractures is linear and the cubic law applies. Rougher surfaces, more fracture intersections and larger apertures, however, can cause flow to be non-linear even below Jc. Nonetheless, the application of the cubic law below this threshold still yielded sufficiently applicable results in most cases [44].

4.1.3. Influence of partial saturation

Probably the most notable implication of partial saturation of fracture networks is that gravity does not necessarily force water down near-vertical discontinuities. The likelihood of adhesion to discontinuity surface may result in significant lateral flow as water moves into near-horizontal discontinuities due to adhesive forces exceeding cohesive forces. In such instances, water molecules will be attracted to near-horizontal discontinuity roofs rather than to each other to induce early-time vertical drainage through fracture or discontinuity intersections. Mixed air-water flows therefore behave differently from water-saturated systems.

4.1.4. Influence of flow velocity

Laminar flow is required in order for the cubic law to hold. High velocities may, however, induce turbulent flow at Reynolds numbers exceeding 1150. Induced or forced flow induce large pressure gradients, and inertia due to tortuosity in rough-walled fractures may also result in deviations from the cubic law at low flow rates before turbulent flow is encountered [28][45]. Further to this, the linear relationship between flux and pressure gradient becomes non-linear at some critical high Reynolds number [46]. The flow trajectory changes with increasing flow velocity, resulting in changing flow trajectories in the active open flow zone and eddies forming in dead flow zones developing due to the high flow rates [47].

Originally the cubic law assumed that the velocity distribution is parabolic and symmetrical. Some amount of asymmetry does, however, still allow application of the cubic law [48]. Problems do however arise when the flow field is non-parabolic and non-zero in the aperture direction (e.g. [49]).

Flow in fractures is commonly faster when compared to porous flow due to the ease of water movement. High velocities do however result in flow possibly becoming non-Darcian, resulting in a non-linear relationship between permeability and velocity in which instance the cubic law fails [39][50][40]. These non-Darcian conditions have also been reported to be possible at low Re values due to the introduction of tortuosity through roughness [28].

The gradient of a graph of Re versus average flow velocity showed no effect due to changes in fracture roughness. The slope of the relation decreased as aperture decreases, showing in turn that flow is more stable at smaller apertures as Re is increased by larger changes in flow velocity [39].

4.2. Considerations with respect to Geometry

The validity of the cubic law is, however, often queried, given the apparent oversimplification of complex natural systems (**Figure 7**). Fractures resembling smooth parallel plates rarely exist in nature and a certain degree of deviation exists in natural systems, but is rather represented by highly variable aperture due to the irregular fracture surface and the mechanical behaviour of the fracture in a given stress regime. Surface roughness, fracture orientation and in situ stresses need to be incorporated (e.g. [8][15]). The use of a mean aperture is one possible solution, although the use of this *hydraulic aperture* may still depend on varying aperture and obstructed regions in the fracture [28].

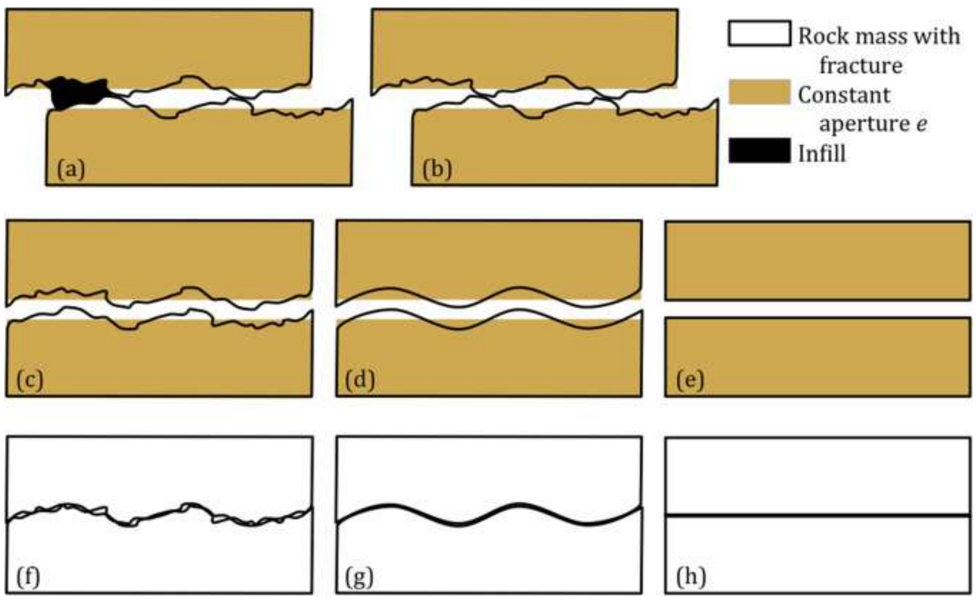

Figure 7. (a) A natural open fracture with aperture e can be progressively simplified through (b) removal of infill, (c) readjustment of alignment, (d) removal of asperities and finally (e) removal of roughness. For closed fractures such as (f), simplification (shown by (g) and (h)) will have less influence on the hydraulic properties as e → 0.

4.2.1. Aperture

Aperture is best defined as "… the perpendicular distance between the adjacent rock surfaces of the discontinuity…", and will be (i) constant in the instance of parallel planar adjacent surfaces, (ii) linearly variable for non-parallel planar adjacent surfaces, and (iii) totally variable for rough adjacent surfaces [51]. The influence of infill material obviously affects the aperture available for the transmission of water. The term *hydraulic aperture* therefore refers to the open aperture which can be related to a localised smooth parallel plate model for which the local cubic law (LCL) holds, whereas *infill aperture* refers to the aperture with infill material of given hydraulic conductivity, which is different from that of the rock itself (e.g. [52]). This is shown schematically in **Figure 8.**

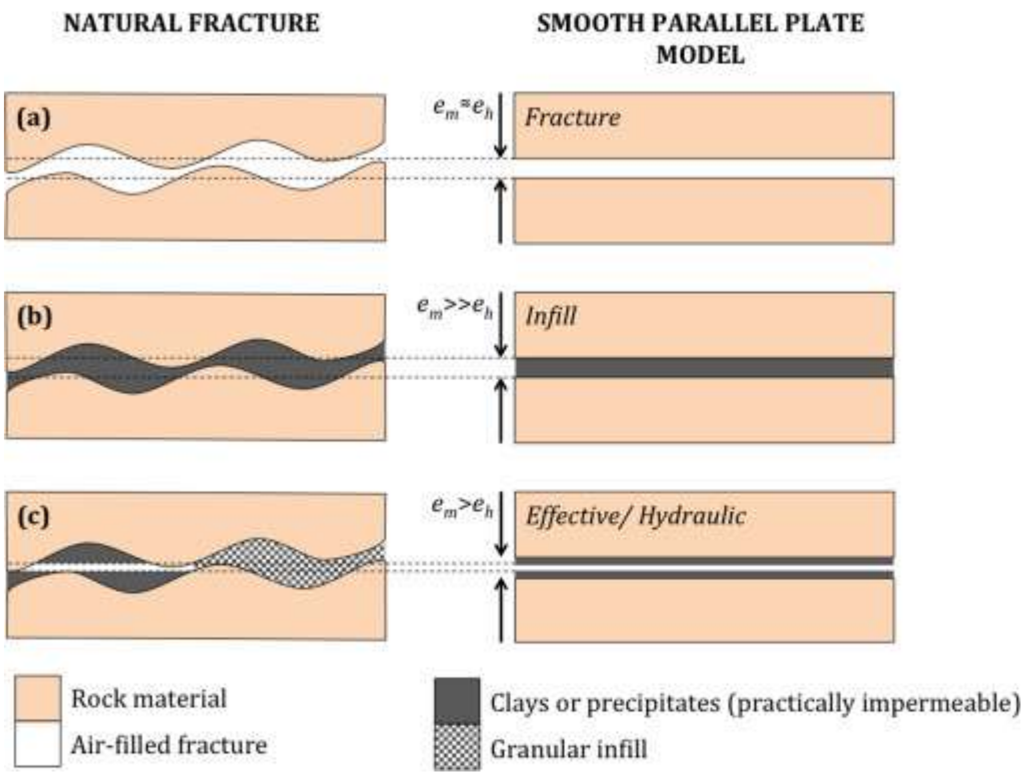

Figure 8. Relationships between mechanical (e_m) and hydraulic (e_h) apertures due to roughness and infill.

The cubic law is applicable to open and closed fractures with aperture being the governing geometrical property. Good agreement with the cubic law has been reported for apertures of 4-250 µm [34] although smaller apertures, higher fluid viscosity and lower fluid density do stabilize flow, causing it to become more linear [28]).

Aperture measurement is often contested due to the strong influence of this measurement on the results of the cubic law [8][53]. The vertical aperture (**Figure** 9) has been shown to correlate poorly with the cubic law, but remain recommended by some authors [53], whereas average segment apertures are preferred by others, cautioning against measurement on a point-to-point basis [54]. Normal to flow direction apertures and ball apertures have also been reported to pose problems with changing resolution given their sensitivities to drastic changes in apertures [8].

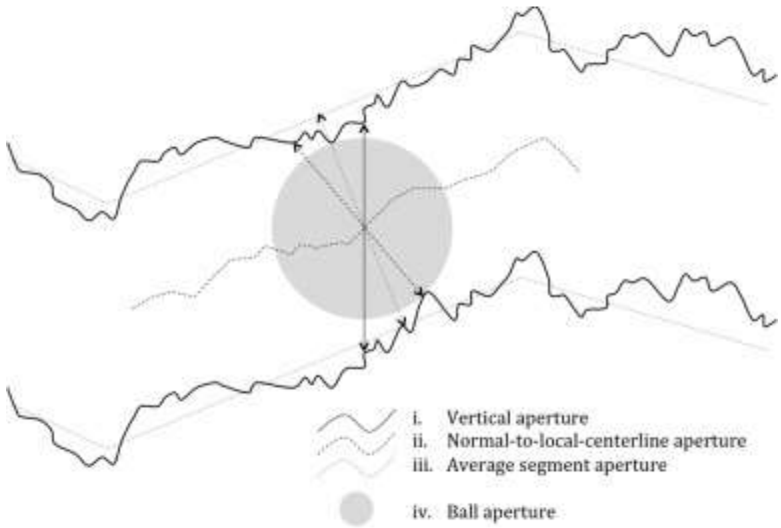

Figure 9. Measurement of aperture in a rough fracture: (i) the vertical aperture, perpendicular to the discontinuity, (ii) normal-to-local-centerline aperture, (iii) ball aperture, or (iv) average segment aperture [54]

Hydraulic apertures, on the other hand, can possibly be determined through knowledge of the means and standard deviations of fracture apertures [28]. Hydraulic aperture decreases as *Re* increases, and it increases with increase in roughness and decrease in mechanical aperture, further emphasising the sensitivity of the cubic law [29].

4.2.2. Roughness and contact obstacles

The *roughness* of a fracture has a direct effect on the behaviour of groundwater in a fracture [15]. Depending on the orientation of the roughness relative to the hydraulic gradient, roughness may either inhibit or enhance the flow through a fracture. Contact points or obstacles further result in bridging, selectively obstructing flow. Together with roughness, however, contact obstacles serve to keep discontinuities open, even in the absence of infill, except where joint wall matching is perfect.

The cubic law was devised under a situation in which the fracture walls were represented as smooth, and thus any roughness present in a fracture hinders the applicability of the cubic law [50][55]. In order to address this, the use of the cubic law locally at each void space in a fracture (hence sometimes called LCL) is often assumed to be applicable [43].

Special considerations for fracture flow are therefore fundamentally important, and include, for instance, the following:

- Roughness in general tends to increase inertial forces (e.g. [53][54]).
- Large-scale roughness (waviness) may result in offsets in the wall matching, which may lead to the formation of a stepped fracture surface. In this instance K_f is likely substantially increased due to the large apertures formed by the steps. Offsets as small as 0.5 mm have been shown to significantly affect hydraulic conductivity and deviations up to five orders of magnitude have been reported [8].
- If the mean aperture and roughness are in the same order of magnitude, the cubic law generally fails application (e.g. [5]) and sinusoidally varying aperture with shorter wavelengths and larger amplitudes deviate more from the cubic law [29].
- *K*-values deviate from those predicted if the wavelength of the dominant roughness component becomes smaller than the order of magnitude of the amplitude of that roughness, i.e. when roughness forms very steep sinusoidal curves (e.g. [25]).
- Special adaptations for cylindrical roughness and the influence of contact area are also documented (e.g. [56][57]).
- Further investigation on contact obstacles and variable aperture based on numerical flow simulations, resulting in corrections terms to better predict fracture flow [58].
- Flow paths due to contact points in rough fractures, under steady flow conditions, are 3-9% longer than the straight-line paths, resulting in a decrease in hydraulic gradient [43]. At the same hydraulic gradient, the flow velocity increases as the aperture increases and the roughness remains unchanged, but the flow velocity decreases as roughness increases and the aperture remains unchanged [39].

The *relative roughness* (*F*, often labelled D_h for hydraulic diameter) relates the difference between the maximum and minimum *asperity height* or *absolute wall roughness* (*k*) to the *fracture aperture* (*e*) or *mean fissure width* (2*a*) as per Eq. 16 and **Figure 10**. Louis (1969 in [15][59]) found that parallel (irrotational) flow would occur in a fracture where $F < 0.033$ and non-parallel (rotational) flow when $F > 0.033$. Associated with this, fracture flow may be laminar (Reynolds number $Re < 2000$) or turbulent (Re > 2000), depending on the fluid viscosity and the fracture aperture. This was evaluated in great further detail (e.g. [7]).

$$F = \frac{k}{2e} \quad (16)$$

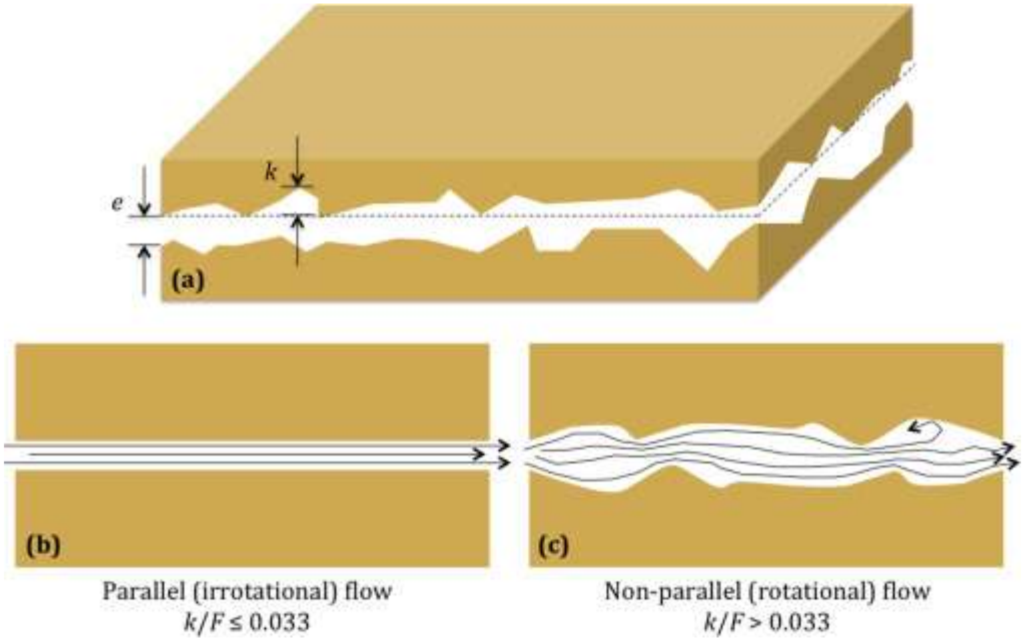

Figure 10. Variables related to roughness (a) and types of flow (b,c) (adapted from [59]).

4.2.3. Hydraulic aperture

A simplification to estimate *hydraulic aperture* is detailed by Barton et al. [60], relating the aperture to the joint roughness coefficient (JRC) as per Eq. 17.

$$e_h = \frac{e^2}{JRC^{2.5}} \quad (17)$$

Two-dimensional computational fluid dynamics (CFD) simulations were used to calculate the influence of roughness through the joint roughness coefficient (JRC) on pressure and hydraulic aperture [61]. With increasing JRC, the normalised pressure drop shows an increase while the normalised hydraulic aperture decreases (**Figure 11**).

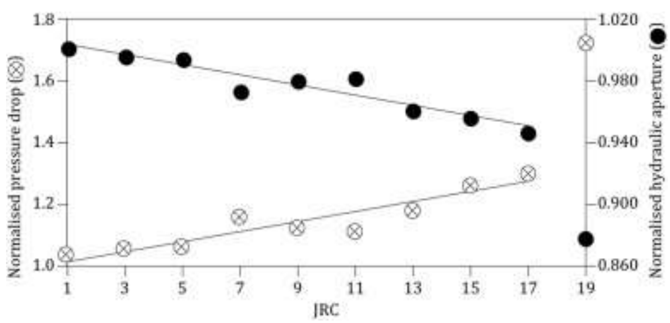

Figure 11. Pressure drops and hydraulic aperture for 10 JRC flow channels ([61]).

Modelling by means of the Navier-Stokes equation and other computational fluid dynamics (CFD) codes showed that hydraulic aperture (e_h) exceeds mean mechanical aperture as per Eq. 18 (e_m) [62] or as per Eq. 19 [63], where σ refers to the aperture's standard deviation. Roughness was also found to generally cause mechanical aperture to exceed hydraulic aperture, with the latter decreasing significantly with increase in roughness or decrease in mechanical aperture [29].

$$\frac{e_h}{e_m} \approx 0.94 - \left(\frac{5.0\sigma^2}{e_m^2}\right) \tag{18}$$

$$\frac{e_h}{e_m} \approx exp\left(-\frac{\sigma^2}{2}\right) \tag{19}$$

More recently, surface morphology has been scanned to determine the Gaussian aperture distribution of fractures. Localised apertures were used with infinitesimal parallel plates to evaluate saturation and capillary pressure in unsaturated rough-walled fractures [64].

4.3. Considerations with respect to normal stress, shear stress and fluid pressure

The validity and applicability of the cubic law based on differences in geometry for specific discontinuity surfaces are shown in **Figure 12**. A rough discontinuity surface with large aperture, for example, can allow for a valid application of cubic law but once normal stress increases, contact points increase and turbulent flow is induced [15].

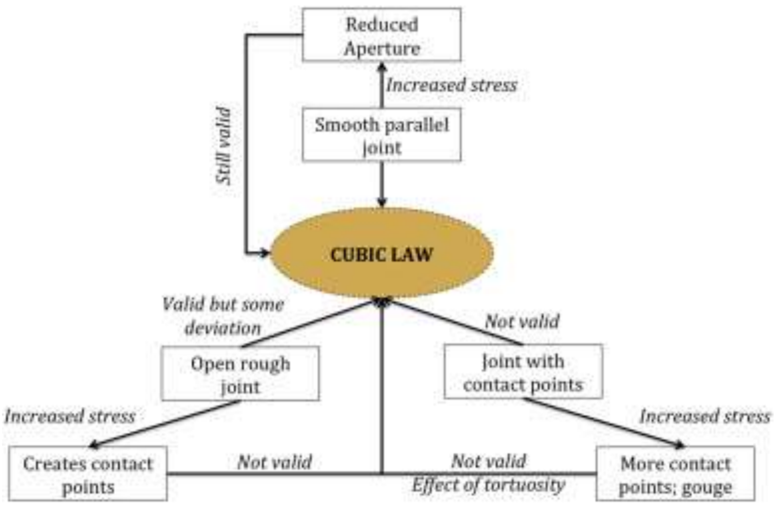

Figure 12. Validity of the cubic law for different fractures (after [15]).

The effective stress acting on a fracture is a function of the in situ stresses surrounding the fracture, being a combination of the normal and shear stresses experienced by the fracture and the fluid pressure within the fracture. Depending on the magnitude of the applied stresses and the orientation of the fracture, they suggest that the mechanical behaviour of the fracture will vary [15].

The relationship between cohesion, friction, normal stress and shear stress is shown in **Figure 1**(a). Based on the application of an axial load (normal stress), [15] found that the change in K_f is likely to occur in three stages, viz. constant conductivity, followed by decreasing conductivity, and finally increasing conductivity. If the fracture is oriented parallel to the direction of the axial load, the hydraulic conductivity will remain fairly constant, whereas the aperture and K_f will decrease if the fracture is oriented perpendicular to the axial load.

In the case of a rough discontinuity, K_f is reduced at a slower rate when compared to a smooth fracture. The roughness (asperity) of a discontinuity results in contact points between the rock walls when a load is applied. As a result, these contact points provide more resistance to the closure of the discontinuity and the aperture is reduced at a slower rate. Subsequently, rough discontinuities have a residual aperture when an axial load is applied (**Figure 13**) whereas smooth discontinuities will close completely and not have a residual aperture. An increase in K_f in the third stage is expected when the applied load results in the formation of new micro-cracks and the dilation of existing discontinuities. New discontinuities form and the apertures of existing discontinuities increase, resulting in increased possibility to transport groundwater [11][15]. The applied load can exceed the strength of the fracture sidewalls, in which instance failure will occur and rock fragments can potentially clog the fracture. This results in a reduction of K_f. with increasing normal stress as contact area and flow path tortuosity also increase [11].

Residual aperture results as a fracture is subjected to normal stress, resulting in a reduction of the aperture beyond the influences of joint-water pressure dissipation and consolidation. Whereas normal tensile stress increases both the aperture and permeability, normal compressive stress intrinsically reduces both these parameters [59].

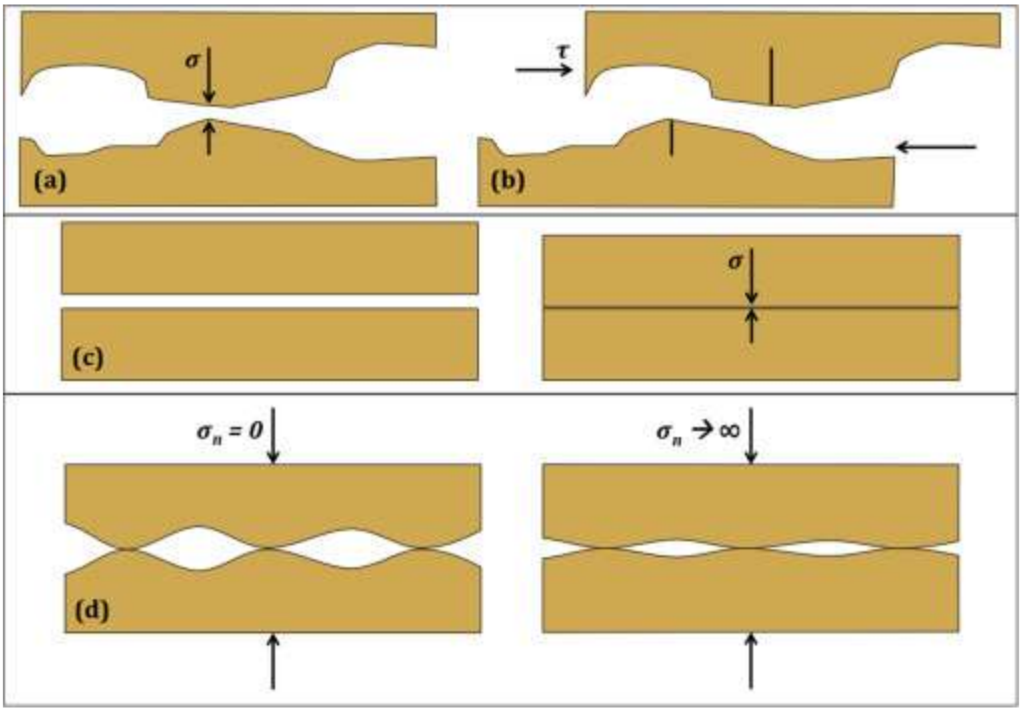

Figure 13. Influence of (a) normal stress and (b) shear stress on the aperture of a rough fracture (after Indraratna and Ranjith 2001); the reduction in aperture due to normal stress applied on a (c) smooth fracture and (d) a rough fracture ([59]).

4.3.1. Influence of normal stress

The cubic law could be applied to rough open fractures under low normal stress and discontinuities with negligible roughness (**Table 1**). Nevertheless, if a rough discontinuity is exposed to a high normal stress, as is often the case in a natural fracture, the cubic law then becomes invalid. Although imperfect, the parallel plate model and the cubic law are yet to be replaced by alternative and more reliable approaches, and are subsequently used to conceptualise flow through natural discontinuities. The assumptions of the cubic law furthermore remain convenient for practical use in the field, as it is impossible to incorporate the roughness and aperture of each measured fracture into a rock mass model [8][15].

At greater depths where compressional stresses are greater, the apertures are smaller near asperities and larger where there are openings between the discontinuity planes. In environments where the latter is prevalent, discontinuities are partly saturated and form pathways for water to flow between rock blocks [22].

Not only do effective stresses have a significant effect on fluid flow in fractures but also on the mechanical behaviour. In general, discontinuities which are parallel to the maximum stress tend to be open compared to those which are perpendicular tending to be closed (e.g. [22]).

Table 1. Validity of the cubic law to different types of fractures under varying stress conditions (after [15]).

Fracture Description	***Normal Conditions***	***Increased Stress***
Smooth parallel joint	Valid	Reduced aperture; still valid
Open rough joint	Valid with some deviation	Creates contact points; invalid
Joint with contact points	Invalid	More contact points or gouge deposition; invalid

4.3.2. Influence of shear stress

Shear stress applied parallel to the fracture orientation, on the other hand, is dependent on the geometry of the surface roughness (**Figure 14**). The result is an offset of the surface roughness, which may either increase or decrease K_f depending on geometry of the roughness of each individual fracture wall and the magnitude of the applied shear stress. If points of asperities are in contact, shear may move the points apart and result in increasing aperture. However, if the fracture walls are interlocking and if the shear stress exceeds the rock strength, roughness may be sheared off, causing the fracture to smooth out and the aperture to decrease [15]. It was, for instance, found that shearing takes place through dilation at low normal stresses and discontinuities are smoothed during failure [65].

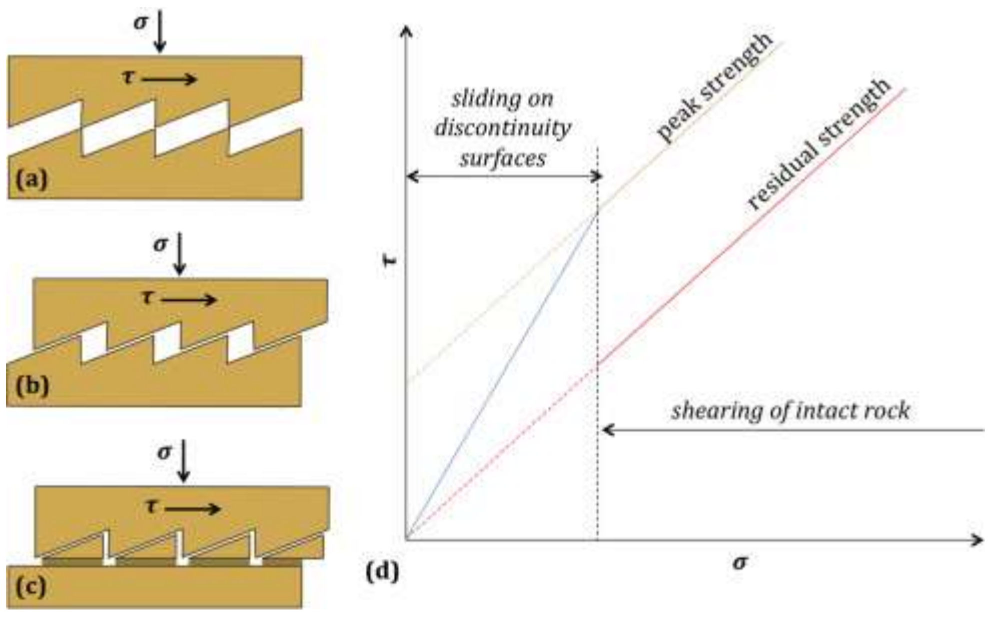

Figure 14. Modes of shear failure of a discontinuity: (a)-(b) shearing on discontinuity surfaces and (c) shearing of intact rock (adapted from [59]).

As discussed by [8], minor amounts of shear (for instance resulting in 0.5 mm displacement of fracture walls relative to each other) can change K_f by five orders of magnitude. Shear stress inducing offset of roughness also increases heterogeneity and anisotropy, and has been shown to increase permeability in the x-direction corresponding to roughness ridges rather than in the y-direction parallel to the shear displacement [66]. Further to this, shearing results in dilation of the discontinuity and gouge production as asperities degrade [67].

4.3.3. Influence of fluid pressure

The final component of the in-situ stresses – the fluid pressure within the fracture – depends primarily on the fluid density and the surrounding stress conditions. If the fluid pressure in the fracture is in balance with the surrounding stresses, the fracture will remain open. However, if the fluid pressure were to decrease (i.e. draining of a discontinuity), the aperture will be reduced and the discontinuity will close. The opposite would occur if the fluid pressure was to exceed the surrounding stresses, in which instance the aperture will increase, discontinuities might link up through hydrofracturing, and both K_f and flow volume through the discontinuities will increase [30].

4.4. Modifications to the Cubic Law

Some recent modifications to the cubic law have been recommended. These include, for instance, the following:

- Geometrical-related modifications account for roughness through treating the rough walls as sinusoidally varying along the length of the fracture, even more recently incorporating roughness profiles not in phase [29].
- The use of roughness factors have been suggested with varying levels of agreement with the cubic law [53].
- Flow assessment modifications include, for instance, employing the *Fo* number conjunctively or preferentially to the *Re* number [40].
- Adaptions to the Navier-Stokes equations that incorporate for inertia, pressure and shear stress have been documented.[43].
- Numerous authors have highlighted the possible from the cubic nature of the law through, for instance, recommended e^{6-Dt} where Dt is a number between one and two related to flow tortuosity [44], e^5 for the quantic law, or as high as e^{13} [50].

5. Final Comments

5.1. Fracture or Discontinuity?

For all practical purposes, it is accepted that *fracture* and *discontinuity* are essentially used synonymously by different disciplines. Based on available literature, it is clear that engineering geologists, hydrogeologists and rock engineers all consider distinctly different aspects with respect to variably saturated flow through discontinuities. The confusion between terminology related to joints, discontinuities and fractures contribute to the lack of cross-field interpretation of the implications of flow through fractured or discontinuous rock mass and the strength of the rock mass as a result thereof.

5.2. The Issue of Hydraulic Aperture

The convention in rock mechanics is to measure the discontinuity aperture as the perpendicular distance between the rock walls defining the discontinuity. This space is further defined during a joint line survey by adding the roughness profile as well as the infill with specific reference to the infill grain size. Both roughness and infill will influence the discontinuity shear strength. The aperture, roughness and infill will obviously also have influences on the discontinuity permeability. It is therefore clear that aperture for some implies the opening between the walls of the discontinuity, whereas, for others, the infill is seen as an intrinsic reduction. It should be clarified that (*mechanical*) *aperture* in itself applies to the prior, and *aperture with infill* to the latter. Measurements taken in the field are the true aperture, and additional input is needed with respect to the infill.

Hydraulic aperture, on the other hand, refers to that aperture within context of the local cubic law (LCL), which relates to the same flow as the natural discontinuity.

5.3. Constraints on the Cubic Law

The cubic law applies only in cases of laminar-parallel flow. Whereas the aperture in the parallel plate model is constant, the variable aperture in natural systems makes the applicability of the cubic law problematic. In addressing the issue of variable aperture, Berkowitz [8] suggests taking the aperture as the average over a specific length rather than measurements at discrete positions. Note should however be taken that fractures are generally measured on surface exposures or small diameter borehole core, and that subsurface tests such as packer and tracer tests only address the rock mass response due to the presence of the fractures (e.g. [8]).

Based on a comparison between the aperture and flow rate predicted by the cubic law and measurements from naturally fractured specimens, Raven and Gale [11] were able to illustrate the invalidity of the cubic law. The cubic law prediction varied significantly from the data that were measured from a rough natural discontinuity subjected to high normal stresses, and is attributed to the many random contact points along the fracture due to the roughness of the surface. As a result, Indraratna and Ranjith [15] suggest that the cubic law tends to overestimate the flow rate for a rough discontinuity subjected to high normal stresses, but shows little deviation in the flow rates for open (low normal stress) rough fractures and fractures with a relatively constant aperture.

Given that fracture geometry and flow regime rarely are constant in natural systems, even the Local Cubic Law (LCL) – which sees discrete sections of a localised parallel plates – does not always hold as it overestimates the flow [45].

As detailed at the hand of literature, the following conditions generally cause the cubic law to fail for discrete fractures:

- Roughness prolongs the travel path of water and has an onset of turbulent flow at very early stages.
- When roughness exceeds mean mechanical aperture, laminar flow is not achieved.
- High-amplitude roughness (i.e. roughness amplitude exceeds roughness wavelength) causes changes in the velocity profiles and dead spaces where eddies can form.
- Highly variable roughness profiles (i.e. highly variable changes in roughness profile) makes even the localised smooth parallel plate model inadequate.
- Contact areas and bridging induce turbulence at earlier onset.
- High flow velocities results in high Reynolds numbers.
- Discontinuity wall offset causes irregular roughness and induces turbulence.
- Flow regimes and flow mechanisms are associated with highly variable moisture content and degrees of water saturation of the fracture.

5.4. Further Research Needs

The following statements require further validation:

i. How we describe roughness, aperture and other geometrical properties of natural discontinuities may be insufficient for anticipating flow behaviour.

ii. The validity of the cubic law and its many adaptations applied to non-smooth fractures at saturated conditions is still not adequately addressed in international literature, and further laboratory scale and field scale experimental work is required.
iii. Any potential influence on the linear flow path induces some degree of turbulence and affects the flow velocity profile.
iv. Highly variable and changing saturation behaves in a hysteretic manner where wetting and drainage continuously affect the flow mechanism.

Finally, beyond the issue of single discontinuities, another question exists in how one upscales from a single discontinuity not in full compliance with the cubic law, to a fracture network without simplification to an equivalent porous medium or bulk flow. The implications are numerous as increased velocity increases contaminant travel times and exerts greater pore water pressures at more localised positions.

6. Acknowledgements and Declaration

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