# Explaining the harmonic sequence paradox*

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**Abstract**

According to the harmonic sequence paradox (Blavatskyy, 2006), an expected utility decision maker's willingness-to-pay for a gamble whose expected payoffs evolve according to the harmonic series is finite if and only if his marginal utility of additional income becomes zero for rather low payoff levels. Since the assumption of zero marginal utility is implausible for finite payoff levels, expected utility theory—as well as its standard generalizations such as cumulative prospect theory—are apparently unable to explain a finite willingness-to-pay. The present paper presents first an experimental study of the harmonic sequence paradox. Additionally, it demonstrates that the theoretical argument of the harmonic sequence paradox only applies to time-patient decision makers whereas the paradox is easily avoided if time-impatience is introduced.

*Keywords:* St. Petersburg Paradox, Expected Utility, Time-Preferences

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# 1 The harmonic sequence paradox

The so-called harmonic sequence paradox, due to Blavatskyy (2006), is based on the following hypothetical gamble $G$:

> "Consider an urn that initially contains one white and one black ball. An individual draws one ball from this urn and receives one dollar (nothing) should the ball be white (black). Whatever the drawn ball happens to be, it is subsequently put back into the urn. Additionally, one more black ball is added to the urn. The individual then draws one ball again and the cycle continues ad infinitum. At each iteration, the drawn white ball pays off one dollar and the number of black balls is increased by one." Blavatskyy (2006, p. 221)

Formally, consider the state space

$$X = \times_{t=1}^{\infty} \{0, 1\} \tag{1}$$

with generic element $x = (x_1, x_2, ...)$ and define the coordinate (random) variable $Y_t$, $t \in \mathbb{N}$, such that

$$Y_t (x_1, x_2, ...) = x_t.$$

Observe that the gamble $G$ is given as a sequence of independently distributed random variables $(Y_t)_{t \in \mathbb{N}}$ with distribution

$$P_t (Y_t = 1) = \frac{1}{t+1} \tag{2}$$

and define by

$$P_{t,...,t+k} = P_t \cdot ... \cdot P_{t+k} \text{ for } k \geq 0 \tag{3}$$

the so-called *finite-dimensional distributions* of the stochastic process $(Y_t)_{t \in \mathbb{N}}$ on $(X, \mathcal{F})$ whereby $\mathcal{F}$ denotes the $\sigma$-algebra generated by the following partition of $X$

$$\mathcal{X} = \{\{x\} \mid x \in X\}. \tag{4}$$

By Kolmogorov's existence theorem (cf. Billingsley, 1995, Theorem 36.1; Aliprantis & Border, 2006, Chapter 15.6), there exists a unique additive probability measure $P^*$ on $(X, \mathcal{F})$ such that $(Y_t)_{t \in \mathbb{N}}$ has (3) as finite dimensional distributions.

Given the measure space $(P^*, X, \mathcal{F})$ the monetary outcomes that can be earned by an individual who participates in $G$ for $T$ iterations are described by the $\mathcal{F}$-measurable random variable $Z_T = \sum_{t=1}^{T} Y_t$. Define now the $\mathcal{F}$-measurable (Billingsley, 1995, Theorem 13.4) random variable $Z_\infty = \sum_{t=1}^{\infty} Y_t$ with extended range space $\mathbb{R}_+ \cup \{+\infty\}$ and recall that Blavatskyy (2006, Proposition 1) reports the interesting result that

$$P^* (Z_\infty = +\infty) = 1. \tag{5}$$

That is, any individual participating forever in the gamble $G$ will earn with certainty an infinite amount of money.[1]

In spite of this extremely attractive feature of $G$ real-life people are only willing to pay (=WTP) up to some finite amount of money, say $M$, for a participation in $G$.[2] In order to describe such preferences by an expected utility functional we therefore have to assume that

$$E\left[U\left(Z_\infty\right), P^*\right] = \int_X U\left(Z_\infty(x)\right) dP^*(x) \tag{6}$$

$$= U(+\infty) \cdot 1 \tag{7}$$

$$= U(M) \tag{8}$$

whereby (weak) monotonicity of the von Neumann Morgenstern (=vNM) utility $U$ of money would imply that the individual is indifferent between all amounts of money greater or equal than $M$. This indifference, however, is quite implausible and apparently at odds with the fact that individuals are not willing to pay more than $M$ for a participation in the gamble $G$. In a nutshell, the harmonic sequence paradox therefore states:

> Suppose that an individual regards a participation in gamble $G$ as equivalent to the random variable $Z_\infty$. If this individual is an expected utility decision maker–or for that matter a cumulative prospect theory decision maker (Tversky & Kahneman, 1992)–her willingness to pay for $G$ is finite if and only if her marginal utility of additional monetary income becomes eventually zero.

While the famous St. Petersburg paradox (Bernoulli, 1954), respectively the super St. Petersburg paradox (Menger, 1934), can be resolved within the expected utility under risk framework by the assumption of a sufficiently concave vNM utility function, respectively by the assumption of a bounded vNM utility function, the above argument

[1] Observe that this feature makes $G$ much more attractive to risk-averse individuals than just an infinite expected value of $Z_\infty$.

[2] While Blavatskyy considers $M = \$10$ as plausible, we observe in our experimental study (cf. Section 2) a median WTP of $M = \$17.50$.

demonstrates that such a resolution is not at hand for the harmonic sequence paradox. Two alternative explanations for the harmonic sequence paradox come instead to mind: Either real-life people have a substantially biased (subjective) notion about the "objective" probability measure $P^*$ generated by $G$ or they do not regard the participation in $G$ as equivalent to the random variable $Z_\infty$. Both possibilities will be considered in the present paper.

In the next section we present an experimental investigation of the harmonic sequence paradox. We ask people for their willingness-to-pay (WTP) for participating in gamble $G$ and also for their beliefs about its expected monetary value. From responses to the latter question we can infer whether people have correct notions of the probability measure $P^*$ generated by $G$. As this paper's main contribution, we develop in Section 3 an explanation of the harmonic sequence paradox based on a possible difference between $G$ and the random variable $Z_\infty$. More precisely, we consider subjective expected utility decision makers whose subjective probability measures coincide with $P^*$ but who identify the possible outcomes of the gamble $G$ as infinite streams of monetary payoffs rather than as payoff-points in the range of $Z_\infty$. We further illustrate our formal argument in Section 4 by a simple example involving expected utility decision makers with additively separable time-preferences. Whenever these decision makers are sufficiently time-impatient, their WTP for a participation in $G$ is rather moderate. Since time-impatience appears as a rather natural assumption for an individual who is confronted with a gamble that goes on forever, our approach might contribute towards an explanation of why real-life people only offer a finite amount for participating in the gamble $G$. Moreover, the fact that an individual's WTP for participating in $G$ is rather low might then result from her correct understanding that the expected monetary payoff increases very slowly over time.

# 2 Experimental study

*(i) Introduction*

In order to obtain empirical evidence on real individuals' WTPs—compared to Blavatskyy's hypothetical WTPs—we conducted an experimental study. As one finding we obtain that real individuals have severe problems in understanding the payoff distribution of the harmonic sequence gamble. More importantly, however, in line with Blavatskyy's original argument we also find that better informed people still exhibit WTPs that cannot be explained by standard decision theoretic models for one-shot lotteries. This finding motivates our approach of introducing time-impatience as a possible explanation of the harmonic sequence paradox.

Blavatskyy describes the following hypothetical choice situation:

> "However, an individual MAY [highlighted by the authors] be informed that L is simulated on a computer with high processing speed for a finite (large) number of iterations (e.g. 10^15) within a short period of time (e.g. 5 minutes). Any observed discrepancy between actual WTP (e.g. <$10) and the theoretical prediction (e.g. $30 as demonstrated by Fig. 2) is then as damaging to the established decision theories as the infinite version of the paradox itself." Blavatskyy (2006, p. 224)

Of course, if real-life individuals were indeed to stick with a WTP of less than $10 after it was explained to them which lottery is played out on a computer within 5 minutes time, then this paper's explanation of the paradox through time-impatience would be no longer convincing: Only very few (risk-neutral) decision makers would be time-impatient enough to go for $10 immediately rather than waiting 5 more minutes for an expected value of $30. While we do not know the WTP of real individuals in the above hypothetical situation described by Blavatskyy (our WTPs would be roughly around $20), there are two alternative ways (plus their combination) of "resolving" the paradox. Firstly, real-life people may simply not understand the resulting payoff distributions of the harmonic sequence gamble since the according computations are too complex for them. In this case we can hardly speak of a "paradox" since standard decision theoretic models assume that people have preferences over well-understood objects. Secondly, as we have formally argued in the previous sections, the time-dimension of the harmonic sequence gamble may play a role to the effect that small WTPs result from time-discounting.

*(ii) Method*

In order to go beyond a mere hypothetical choice situation and to get some insights into people's actual evaluation of the gambles underlying the harmonic sequence paradox, we ran an experimental study at the University of Kiel, Germany. Our subject pool consists of 48 advanced students enrolled in economics, psychology, or business administration who participated in a elective interdisciplinary course on behavioural economics. All subjects have passed introductory courses in mathematics and statistics at a rather high level compared to other German universities. Our design does not directly address the issue of time-impatience; that is, we do not mention any time-frame for running the gamble. The main focus is the question how people evaluate the harmonic sequence and whether their evaluations are biased by insufficient knowledge about their distributions.

After the gamble has been carefully described to them, subjects had to respond to five hypothetical questions:[3]

1. What is the maximal amount you would be willing to pay for participation in the gamble with an infinite number of rounds?

2. What do you think is the expected value of the payoff in the gamble with an infinite number of rounds?

3. What is the maximal amount you would be willing to pay for participation in the gamble with 10ˆ15 rounds?

4. What do you think is the expected value of the payoff in the gamble with 10ˆ15 rounds?

5. Note that the expected value of the gamble with an infinite number of rounds is infinity and the expected value of the gamble with 10ˆ15 rounds is 34.43. What is the maximal amount you would be willing to pay for participation in the gamble with 10ˆ15 rounds?

Questions 1-4 should give us just some insights about the evaluation of gambles and subjects' knowledge about their expected values. Responses to Question 5 allow us to judge whether evaluations are biased by insufficient knowledge about the gamble with 10ˆ15 rounds. If subjects understand the property of the gamble perfectly, there should be no difference between responses to Questions 3 and 5.

*(iii) Results*

Table 1 presents the results of our experiment. The single columns give mean and median responses, standard deviation (*SD*), and median absolute deviation from the median (*MAD*) for the five questions above, presented in the same order. We do not report mean and *SD* in the second and third columns as at least one subject responded with infinity in both questions. The high mean and *SD* in the fifth column is caused by three outliers. Figures 1 and 2 additionally give an overview of the distribution of responses to Questions 1 and 2, respectively to Questions 3, 4, and 5 by reporting relative frequencies of responses in the intervals $[0, 1[$, $[1, 5[$, $[5, 10[$, $[10, 20[$, $[20, 50[$, $[50, 100[$, $[100, 200[$, $[200, \infty[$, and responses of $\infty$. Despite the huge difference of true values (infinity versus 34.43) the median responses to the questions concerning the expected value of the gamble with an infinite number of rounds $E(\infty)$ and the gamble with 10ˆ15 rounds $E(10\hat{}15)$ are identical (i.e., 5). This result shows that subjects have severe problems in understanding the

[3] As reward for participation 8 randomly selected subjects received a payment of 20 Euros.

properties of both gambles. The medians of maximal amounts people would be willing to pay in order to participate in the gamble with an infinite number of rounds $WTP(\infty)$ and in the gamble with 10ˆ15 rounds $WTP(10\hat{}15)$ do not differ substantially from responses to expected values which gives no indication of paradoxical behavior. While responses to Question 5 still suggest that people have problems in evaluating the gambles properly, the mean and median of $WTP^{*}(10\hat{}15)$ incorporate in a very sensible way the distributional information we have provided. After learning the expected values, the median WTP for the gamble with 10ˆ15 rounds becomes five times higher (17.50 versus 3.5). According to a Wilcoxon test this difference is significant ($N = 48, z = -3.53, p < 0.01$).

**Insert Table 1 here**

Figures 1 and 2 show that a high fraction of responses to Questions 1-4 is within the interval $[1, 5[$. This can explain the low values of *MAD* for these questions compared to Question 5 for which the highest fraction of responses is within the interval $[20, 50[$, see the light grey bars in Figure 2. Interestingly, *SD* of responses to Question 5 is lower than that for Questions 3 and 4. One reason for this low *SD* is the absence of very high responses: the highest response to Question 5 is given by 37 whereas four subjects stated a WTP of 48 or higher for Question 3. Therefore, additional information about the distributional properties of the gamble increases the average of stated WTP (both in terms of mean and median) and decreases the variation, at least if measured in terms of *SD*.

**Insert Figures 1 and 2 here**

*(iv) Discussion*

On the one hand, the median (informed) WTP of 17.50 clearly exceeds the low hypothesized WTP of Blavatskyy ($< 10$). As a consequence, peoples' improved understanding of the distributional properties of the harmonic sequence gamble results in a less "paradoxical" behaviour than hypothesized by Blavatskyy. On the other hand, however, this WTP is still less than the WTP implied by expected utility or cumulative prospect theory ($> 30$, cf. Figure 2 in Blavatskyy, 2006) whenever 10ˆ15 rounds of the harmonic sequence gamble are described as a one-shot lottery for which time-preferences do not matter. While there is evidence that people still have problems in understanding the payoff-distribution of the harmonic sequence gamble after learning the correct expected value, we would conjecture that at least some of the remaining discrepancy between reported WTPs and theoretical predictions for the evaluation of one-shot lotteries could be due to time-preferences.

Since we did not mention in the experimental set-up that the 10ˆ15 rounds are to be played on a fast computer within a few minutes, it seems plausible that the participants of the experiment did not evaluate the gamble just as an one-shot lottery but rather as sequence of payoffs whereby theses payoffs may take a long time to occur. Based on this insight, we develop in the remainder of this paper a formal argument according to which the harmonic sequence paradox can be explained through the introduction of time-impatience. It would be a desirable avenue for future research to design and run an experiment that may directly check for time-preferences within the context of the harmonic sequence gamble.

# 3 The general argument

The original statement of the harmonic sequence paradox considers a situation of decision making under risk where the individual has preferences over lotteries with given objective probability distributions. For our purpose it is convenient to slightly change the perspective and consider an individual who is a subjective expected utility decision maker in the sense of Savage (1954). That is, we consider a probability space $(P^s, \Omega, \Sigma)$ where $P^s$ is a subjective probability measure derived from the individual's preferences over Savage-acts which are mappings from the state space $\Omega$ into some set of consequences $C$.[4] Under Savage's axioms such preferences are representable by an expected utility functional, i.e., for any two Savage-acts $f, g$,

$$f \succeq g \text{ if and only if } E\left[U\left(f\right), P^s\right] \geq E\left[U\left(g\right), P^s\right]$$

for a unique $P^s$ and a unique (up to positive affine transformations) vNM utility function $U : C \rightarrow \mathbb{R}$.

In order to recast the harmonic sequence paradox within this Savage framework, we assume that the set of consequences is given as the possible payoff-sequences of $G$, i.e.,

$$C \equiv X = \times_{t=1}^{\infty} \{0, 1\} . \tag{9}$$

[4]The Savage framework is convenient for two reasons. Firstly, since Savage acts can map into arbitrary consequence spaces, our definition of consequences as (infinite) payoff-streams makes it very obvious from the outset that we are not treating the HSP as a problem of preferences over lotteries with monetary payoffs. Secondly, since our formal results are derived for arbitrary additive probability measures, they also hold for subjective probabilities that do not coincide with the "objective" probabilities generated by the gamble $G$.

Furthermore, we suppose that the Savage state- and event space are sufficiently rich in the sense that they can reflect the previous section's definitions of $X$ and $\mathcal{F}$. To this end we assume that

$$\Omega = X \times U, \tag{10}$$

with generic element $\omega = ((x_1, x_2, ...), u)$, for some space $U$ whereby the event space

$$\Sigma = \mathcal{F} \otimes \mathcal{U} \tag{11}$$

is given as the product $\sigma$-algebra of $\mathcal{F}$ and some $\sigma$-algebra $\mathcal{U}$ on $U$. Observe that within this Savage framework a participation in the gamble $G$ can be defined as the Savage act $f^G : \Omega \to C$ such that

$$f^G((x_1, x_2, ...), u) = (x_1, x_2, ...) \tag{12}$$

whereby the consequence $(x_1, x_2, ...)$ is interpreted as one possible outcome of the stochastic process $(Y_t)_{t\in\mathbb{N}}$ as introduced in the previous section. The following assumption finally connects our subjective model to the harmonic sequence paradox by stating that the individual's subjective probability measure $P^s$ coincides in an appropriate way with the "objective" probability measure $P^*$.

**Assumption 1.** *For all $A \in \mathcal{F}$,*

$$P^s(A \times U) = P^*(A).$$

Given this re-interpretation of the harmonic sequence gamble within a Savage framework, the next proposition follows easily.

**Proposition 1.**

*Consider a (weakly) increasing vNM utility function, i.e., $U(c) \geq U(c')$ whenever $c \geq c'$ in the standard vector order.*

*Then the individual's subjective expected utility of participating in the gamble $G$ is finite if $U$ is bounded from above, i.e., $\sup_C U(c) < +\infty$, whereby*

$$E\left[U\left(f^G\right), P^s\right] \leq U(1, 1, ..).$$

**Proof.** Observe that

$$\begin{aligned}
E\left[U\left(f^{G}\right), P^{s}\right] &= \int_{\Omega} U\left(f^{G}(\omega)\right) dP^{s}(\omega) \\
&= \int_{X\times U} U\left(f^{G}\left(\left(x_1, x_2, ...\right), u\right)\right) dP^{s}\left(\left(x_1, x_2, ...\right), u\right) \\
&= \int_{X} U\left(x_1, x_2, ...\right) dP^{s}\left(\left(x_1, x_2, ...\right) \times U\right) \text{ by (12)} \\
&= \int_{X} U\left(x_1, x_2, ...\right) dP^{*}\left(x_1, x_2, ...\right) \text{ by assumption 1} \\
&\leq U(1, 1, ..) \cdot 1 \text{ by (weak) monotonicity} \\
&< +\infty.
\end{aligned}$$

The final line thereby follows from

$$U(1, 1, ..) = \sup_{C} U(c),$$

which is finite by assumption.□

By Proposition 1, the paradox is trivially avoided as long as an individual evaluates an infinite stream of one dollar payoffs by some finite vNM utility. It is obvious from the formal proof that this result holds for any subjective probability measure and therefore as well for the harmonic sequence probability distribution $P^*$ according to which some consequence $c = (x_1, x_2, ...)$ such that $\sum_{t=1}^{\infty} x_t = +\infty$ occurs with probability one.

# 4 An illustrative example

As an illustrative example we consider in this section the special case of a vNM utility function that results from additively time-separable preferences whereby we are in line with the standard assumption of a time-impatient individual.[5]

**Assumption 2.** *The individual's preferences*[6] *over infinite payoff-sequences $x \in X$ are additively time-separable with discount factor $\beta \in (0,1)$, i.e., for all $c = (x_1, x_2, ...)$,*

$$U(c) = \sum_{t=1}^{\infty} \beta^{t-1} u(x_t) \tag{13}$$

*for some strictly increasing function $u : \{0,1\} \to \mathbb{R}$.*

[5] The reader can easily verify that the assumption of a time-*patient* individual, i.e., $\beta = 1$, in our model would bring us immediately back to the original formulation of the harmonic sequence paradox.

[6] Recall that in a Savage-framework preferences over consequences are interpreted as preferences over constant Savage-acts, i.e., acts that give in every state of the world the same consequence.

Observe that we have, by Assumption 2,

$$
\begin{aligned}
U(1,1,..) &= \sum_{t=1}^{\infty} \beta^{t-1} u(1) \\
&= \frac{u(1)}{1-\beta},
\end{aligned}
$$

so that Proposition 1 immediately implies the following result.

**Corollary.** *If an individual is time-impatient with additively time-separable preferences, her willingness to pay for participating in the harmonic sequence gamble $G$ will never exceed the amount*

$$\frac{u(1)}{1-\beta}.$$

Recall that a risk-averse (risk-loving) expected utility decision maker is modelled through a concave (convex) vNM utility function $U$ whereby concavity (convexity) of $U$ results for the additive separable case from concavity (convexity) of $u$. Whenever we speak of a risk-neutral individual, i.e., linear $U$, we normalize (without loss of generality) $u(0) = 0$ and $u(1) = 1$. For example, in case of a risk-neutral individual the corollary determines $\frac{1}{1-\beta}$ as an upper bound for the individual's willingness to pay to participate in $G$. While the Corollary characterizes only an upper-bound for an individual's evaluation of the gamble $G$, the following proposition gives the exact expected utility.

**Proposition 2.**

*If an individual is time-impatient with additively time-separable preferences, she evaluates a participation in the harmonic sequence gamble $G$ by the following expected utility*

$$E\left[U\left(f^G\right), P^s\right] = \left(\sum_{t=1}^{\infty} \beta^{t-1} \left(\frac{1}{t+1} u(1) + \frac{t}{t+1} u(0)\right)\right).$$

*If, in addition, the individual is risk-neutral, her expected utility coincides with the expected value of the discounted random payment-stream, i.e.,*

$$E\left[U\left(f^G\right), P^s\right] = \sum_{t=1}^{\infty} \frac{\beta^{t-1}}{t+1}.$$

**Proof.** Observe that

$$\begin{aligned} E\left[U\left(f^G\right), P^s\right] &= \int_X U\left(x_1, x_2, ...\right) dP^*\left(x_1, x_2, ...\right) \\ &= \int_X \left(\sum_{t=1}^{\infty} \beta^{t-1} u\left(x_t\right)\right) dP^*\left(x_1, x_2, ...\right) \\ &= \sum_{t=1}^{\infty} \beta^{t-1} \int_X u\left(x_t\right) dP^*\left(x_1, x_2, ...\right) \\ &= \sum_{t=1}^{\infty} \beta^{t-1} E\left[u\left(Y_t\right), P^*\right] \\ &= \sum_{t=1}^{\infty} \beta^{t-1} \left(\frac{1}{t+1} u\left(1\right) + \frac{t}{t+1} u\left(0\right)\right) \end{aligned}$$

The first line follows from the proof of Proposition 1, the third line results from the linearity of the expectations operator, and the final line uses the probability distribution of the harmonic sequence gamble.□

Suppose that an individual's WTP for participating in the harmonic sequence gamble $G$ coincides with the rather fast converging series of Proposition 2. In that case we can easily come up with good approximations for the amount a risk-neutral individual of our model would be willing to pay in order to participate a reasonable number of rounds, say 1000, in the harmonic sequence gamble. For example, for $\beta = 0.99$ we obtain as WTP (in dollars)

$$\sum_{t=1}^{1000} \frac{\beta^{t-1}}{t+1} \simeq 3.69$$

(which is also a good approximation for the infinite case) and for $\beta = 0.999$ we have

$$\sum_{t=1}^{1000} \frac{\beta^{t-1}}{t+1} \simeq 5.70.$$

To sum: For already small degrees of time-impatience a risk-neutral individual of our model is willing to pay only moderate amounts of dollars in order to participate in the harmonic sequence gamble (even considerably less than the ten dollars mentioned by Blavatskyy). Obviously, this effect would be even more pronounced for risk-averse individuals.

# 5 Conclusion

Our formal analysis demonstrates that the theoretical argument behind the harmonic sequence paradox does not apply to time-impatient decision makers who are offered a random sequence of payments rather than a one-shot lottery. That is, a simple reinterpretation of the random object in question that takes into account the time dimension of payoffs might explain the decision makers' choice behavior in accordance with the harmonic sequence paradox.

Our proposal of including time-impatience in the description of consequences is a straightforward approach for "solving" the harmonic sequence paradox through existing decision theoretic models under the assumption that the decision maker correctly understands the payoff distribution of the harmonic sequence gamble. Of course, if decision makers do not understand the harmonic sequence gamble we cannot speak of a "paradox". On the one hand, experimental evidence suggests that real people have severe problems in understanding the distributional properties of the harmonic sequence gamble. On the other hand, however, this evidence also demonstrates that if people receive further information about distributional parameters, their WTPs are still below the theoretical predictions from expected utility and cumulative prospect theory for one-shot lotteries. In contrast to the assumption of an one-shot lottery, the assumption of time preferences may here easily account for the observed—otherwise "paradoxical"—WTPs.

In our opinion the harmonic sequence paradox does not reveal the inadequacy of existing decision theoretic models (including expected utility theory) but it rather cautions us in the way we identify the random objects people supposedly have preferences about. Just because we can mathematically define a random variable through the sum of infinitely many (different) random variables, this does not mean that real-life decision makers actually evaluate this infinite sequence of random payoffs as a lottery over monetary payoffs within the static von Neumann Morgenstern framework. In contrast, the Savage framework—with its arbitrary space of deterministic consequences—might provide us with the flexibility needed for describing choice situations over sequences of random payoffs in a more realistic way.

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Table 1

*Median, Mean, SD, and MAD for Responses to the Five Questions of the Experimental Study*

| | $WTP\,(\infty)$ | $E\,(\infty)$ | $WTP(10\hat{}15)$ | $E(10\hat{}15)$ | $WTP^{*}(10\hat{}15)$ |
|---|---|---|---|---|---|
| Median | 5 | 5 | 3.50 | 5 | 17.50 |
| Mean | – | – | 8.80 | 366686.17 | 16.83 |
| *SD* | – | – | 14.15 | 1714183.51 | 10.78 |
| *MAD* | 3 | 3.25 | 1.50 | 3.25 | 8.50 |

## List of figure captions

*Figure 1.* Distribution of responses to WTP and expected value for the gamble with an infinite number of rounds.

*Figure 2.* Distribution of responses to WTP—uninformed versus informed—and expected value for the gamble with 10ˆ15 rounds.

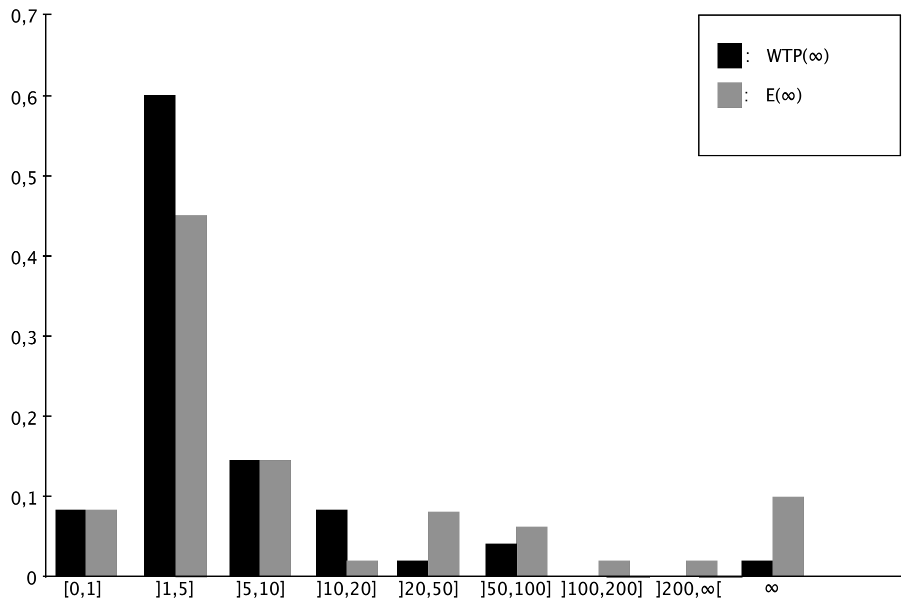


*Figure 1.* Distribution of responses to WTP and expected value for the gamble with an infinite number of rounds.

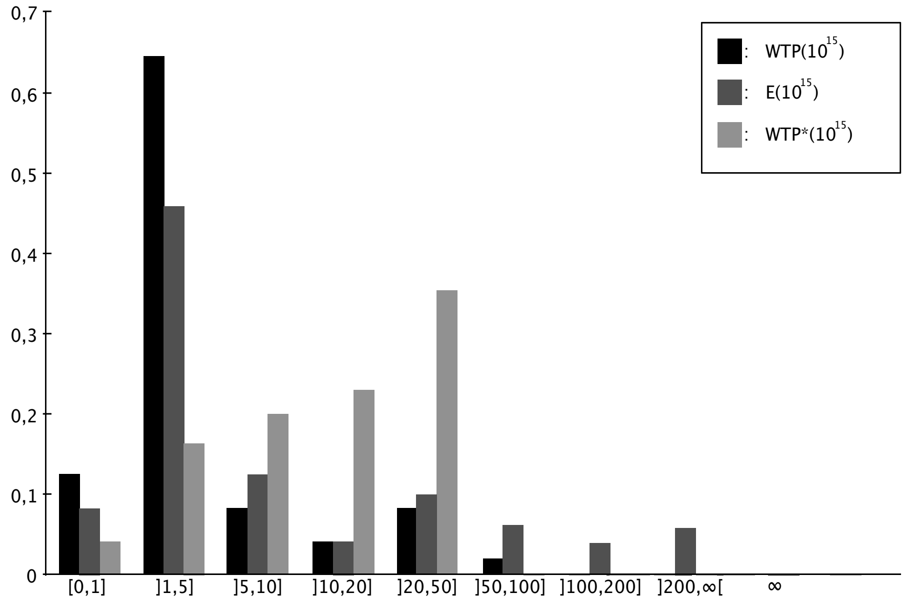


*Figure 2.* Distribution of responses to WTP—uninformed versus informed—and expected value for the gamble with 10ˆ15 rounds.