TESTING OF THE FORCES IN CABLE OF SUSPENSION STRUCTURE AND BRIDGES

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ABSTRACT

This paper applies FEA theory on structural dynamic analysis. The relation between forces in cable and its dynamic character-natural frequency has set up by authors. Using the numerical solution, a more accurate measurement model of forces in cable is presented for those cable of suspension bridges in construction and maintenance.

Keywords: bridge testing forces in cable natural frequency

1. INTRODUCTION

With the development of road and transport, more bridges with longer spans need to build over wide rivers, valleys and bays. Their economic and reasonable structure types are very important. Among those bridges, suspension, cable-stayed bridges have the privilege for them. Due to tubular concrete technologies, arch bridge, the most ancient bridges have been largely applied. All those structures have common characteristics: suspension and cable systems.

In the construction of long span bridges, the self-frame system may not interrupt the navigation in order to save the cost. This method may result in complicate deformations. Their main deformations impact the changes of forces in cable. At mean time, adjusts of cables will influence the lay out and internal forces of neighboring cables. Therefore, it is necessary to adjust and measure the forces in cables in order to meet the needs of designs.

During the operation of bridges, the authority must inspect bridges and structures regularly. For the special bridges, it is very important to evaluate the changes of forces in cable. At present, testing methods for those bridges need improvements due to the accurate limitation. This paper applies numeral analysis method to establish the forces in cable and self vibration frequencies.

2. INTRODUCTION OF TESTING METHOD ON CABLES

The cable is one of important parts of suspension and cable-stayed bridges and structures. The magnitudes of the forces directly impact the status of forcing condition and deformations.

The current testing methods applied as follows:
1) Resisting gauge testing method
2) Cable extension testing method
3) Cable deformation relation method
4) Jacking testing method
5) Compressing sensor method
6) Vibrating testing method

The above 1) to 3) are not easily to operate. Normally, the above 4) and 5) can collect the extension of cable during construction. However they can not get information regarding the forces within cables. The vibrating methods may measure frequencies of the cable to calculate both relations.

The vibrating method is one popular method to test forces in cables. The principle is to make use of following equation:

$$\frac{W}{g}\frac{\partial^2 y}{\partial t^2} - T\frac{\partial^2 y}{\partial x^2} = 0 \tag{1}$$

Where Y-horizontal coordinate
X-vertical coordinate
W-unite weight along the cable
G-gravity accelerating speed
T-extending force of the cable
T-time

According to the assumption of two fixed ends, the equation (1) may compute the frequency of the cables as:

$$f_n = \frac{n}{2l}\sqrt{\frac{Tg}{W}} \tag{2}$$

$$T = \frac{4Wl^2}{g}\left(\frac{f_n}{n}\right)^2 \tag{3}$$

Where：

f_n - nth vibrating frequency of the cables
l - computing length of the cables
n- nth vibration

3. RELATION OF FORCES IN CABLE AND VIBRATING FREQUENCIES

3.1 Assumptions

Following assumptions are introduced:

- 1. The forces in cable are regard as external forces which can impact the stiffness of the cables
- 2. Damping of the cables is ignored

3.2 Impact of Damping

According to existing results from related researches, damping has the bigger impact for the vibrating magnitude and smaller for frequency. Before the model is established, damping within the cables is ignored. The following formula is adopted.

$$\omega_\tau = \omega\sqrt{1 - D^2}$$

Where:

ω_τ as frequency with damping， ω as frequency without damping, D as ratio of damping.

The ratio of damping of common materials is quite small, such as: steel D=0.03，concrete D=0.08.
For steel cables， $\omega_\tau = \omega\sqrt{1-D^2} = 0.9995\omega \approx \omega$

3.3 Computing Model

According to D'Alembert's principle, the free vibrating equation of the cables without damping as

$$M\ddot{V} + KV = 0 \tag{4}$$

The above: M-mass matrix of the cables；
K-stiffness matrix of the cables;
V-node deformation in the cables.

Ordering the solution of equation (4) as

$$V = \phi\sin(\omega t + \varepsilon) \tag{5}$$

Here ϕ as magnitude of vibration， ω as frequency， ε as phase

From (5)and（4）following equation may be got

$$(K - \omega^2 M)\phi\sin(\omega t + \varepsilon) = 0 \tag{6}$$

Because $\sin(\omega t + \varepsilon)$ is not always zero, the condition of the above equation is as

$$(K - \omega^2 M)\phi = 0 \tag{7}$$

when $\phi = 0$， the cable is static. The solutions of the equation do not exist.

The group of linear equation（7）must meet the following condition (8). Then the solution may be obtained.

$$\det(K - \omega^2 M) = 0 \tag{8}$$

The roots of the above matrix are the frequency of the cables.

The M matrix may be got from many methods. The matrix with equal value and combining mass have been considered. The stiffness matrix is focused as follows.

3.4 Stiffness Matrix of the Cables

Inclined cables have geometrical non-linear effectiveness including big deformations along the cable and vertical deformation.

3.4.1 Vertical Effectiveness of Inclined Cables

The movements of two ends of cable units undergo three factors' impacts:

- Elastic strain and modules after forcing in cable
- Vertical deformation has nothing to do with materials and stress. They are related to geometrical deformation under extensions in cables, length of the cable and gravity. The extensive stiffness varies with axial forces.

- Under loading actions, every single strain of the cables moves relatively. The cross sections become more close with new arrangements. Most cables deform permanently. However they may be removed during manufacture. Some non-permanent deformation may deducted with effective modules.

Germany scholar Ernst raised the effective module in non-linear stage. It includes material elastic deformation, structural extension and vertical changes. The formula expresses as follow:

$$E_i = \frac{E}{1 + \frac{W^2 L_x{}^2 A}{12T^3} E} \tag{9}$$

L_x - Horizontal length of the cables;
A- Area of cross section of the cables;
E- Effective elastic modules of the cables.

For the vertical cable, using $L_x = 0$ then $E_i = E$. The above (9) is a comprehensive formula for the inclined and straight cables.

The effective elastic modules may replace the modules in elastic matrix. It is called as namely elastic stiffness matrix K_E. It contains some non-linear factors, which is regard as namely stiffness matrix.

3.4.2 Large Deformation Effect
Under loading actions, geometric changes of cable are obvious. From the FEM, the coordinate of the nodes vary with their increment. The length of unites, inclined angles have bigger changes as well. Because the stiffness deformation is a function of geometrical deformation, the equation $\{F\} = [K]\{\delta\}$ is no longer as linear relation. The assumption on overlapping with small deformations is not reasonable at this case.

In order to calculate the geometrical effect in cables, the initial stress stiffness matrix may be applied. It is called as geometrical stiffness matrix K_G.

The namely stiffness matrix of the cables consists two parts as follows:

$$K = K_E + K_G \tag{10}$$

Using the above analyses, the forces in cables and frequencies may be obtained from (8).

4. EXAMPLES

Authors calculated the different lengths of cables, such as 0.5m、1m、1.5m、2m、3m、4m、5m 和 10m. Under the same extension, the results with stiffness and without stiffness are shown in Table 1 and Fig. 1.

Table 1. Ratio of the 1st frequency with and without bending stiffness.

Length of Cable(m)	0.5	1	1.5	2	3	4	5	10
Ratio of length/diameter	33	65	97	130	195	260	325	609
Frequency (f2/f1)	2.77	1.64	1.32	1.19	1.09	1.05	1.03	1.01

NB: f1 as the 1st frequency with bending stiffness
f2 the 1st frequency as without bending stiffness

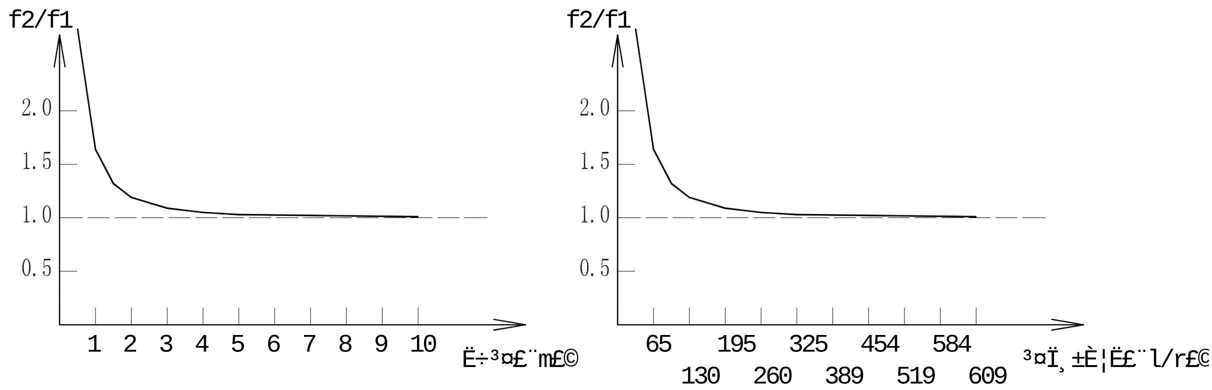

Figure 1. Ratio of the first frequency and length and length/Diameter of the cables.

In the case with the length of steel cable as 2m and length/diameter as 130, ratio of the 1st frequency with and without bending stiffness is 19% from Fig. 1. In the case with the length of steel cable as 1m and length/diameter as 65, ratio of the 1st frequency with and without bending stiffness is 64%. Therefore, the forces in cable and frequency have big interactions. It is necessary to consider them thoroughly, especially for short cables.

5. CONCLUSIONS

1. When the ratio of the length/diameter of cables is relatively bigger, their forces in cable are smaller difference under the conditions of with and without bending stiffness. When the ratio is bigger than 130, the error is within 5%.
2. For the smaller ratio of the length/diameter of cables, the deviation is bigger. If it is less 130, the deviation is over 20%. Especially for tubular concrete cables, this error will increase obviously.

It seems not reasonable for the short cables to used the chord differential equation on fixed frequencies. The numerical analysis method has better accuracy to meet the needs of engineering requirements.

6. REFERENCES

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