

# **PERSISTENCE, MEAN-REVERSION AND NON-LINEARITIES IN INFANT MORTALITY RATES\***

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## **Abstract**

This study examines the time series behavior of infant mortality rates within a long memory approach with non-linear trends using data for 37 countries. The main results show significant differences both in the degree of integration and non-linearities among the analyzed series. Furthermore, non-linearities in the time trends are found in most of cases, in contrast with the main assumption of linearity used in the literature. Finally, the results on the integration order of the series have important policy implications in many areas, such as on international convergence in mortality rates, on the income and infant mortality relationship, and, on whether health policy interventions will have transitory or permanent effects on infant mortality rates.

**JEL classification:** C22, C32, H51, I18

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## **1. Introduction**

Infant mortality rates can be considered an important indicator of a country's population health, and thus, of a country's welfare. In fact, one of the Millennium Development Goals of the United Nations Development Program was to reduce by two thirds, between 1990 and 2015, the under-five mortality rates (United Nations, 2000). However, although the global under-five mortality rate has declined by more than half, dropping from 90 to 43 deaths per 1000 live births between 1990 and 2015, this progress has not been enough to achieve the goal of a reduction of the infant mortality rates (IMR) by two thirds. Furthermore, when considering cross-country differences in IMR in 2014, the IMR in Afghanistan (117 per 1000 lives) is over seven times higher than that in the US (6 per 1000 live births) and over 50 times higher than those in Japan and Monaco (2 per 1000 live births), the countries with the lowest IMR. In addition, the rates of decrease in IMR have been very different across the different countries, according to their degree of development, to the efficiency of the different health and education policies carried out in each of the countries and to additional exogenous changes with an effect on infant mortality rates (wars, droughts, vaccination campaigns, etc.). The relevance of this variable as an indicator of the welfare of a country, its dynamic response to health or education policies and to other exogenous shocks, and the large cross-country differences in IMR justifies a long-run analysis of the dynamics of the infant mortality rates across different countries.

The time series properties of the IMR will shed some light on the degree of stationarity/nonstationarity of this variable in each of the countries and in different subsamples. For instance, if this variable is stationary, shocks will have transitory effects, unlike what happens if the IMR is nonstationary, where shocks will be permanent, and policy interventions will be required if the shock is negative and we

want to recover the underlying trend in the series. Furthermore, the order of integration of this variable will determine whether or not this variable is cointegrated with other variables, such as per capita GDP levels, which usually present non-stationary behavior. Finally, convergence among infant mortality rates among countries would be rejected if the series in each country presents different order of integration. Thus, the relevant policy implications of the time series properties of this variable explains the ample literature on the econometric modeling of this variable using alternative unit root tests (see, for example, Bishai, 1995; Dreger and Reimers, 2005; Erdogan et al., 2013, among others) or fractional integration techniques (Caporale and Gil-Alana, 2014).

The historical data on IMR reveals that these rates have declined a lot over the last decades. Furthermore, the rate of decline observed in these variables during these last decades has not been constant nor linear, as neither has been the evolution of its main drivers: economic growth or development, female education rates or the improvement of the national health systems. By contrast, several factors may have caused several disruptions, and thus, non-linearities in the temporal evolution of this variable. The nonlinear behavior of this variable has been mostly modeled in the literature by the inclusion of structural breaks (see, for example, Conley and Springer, 2001; Silva, 2007; or Siah and Lee, 2015, among others), but to the best of our knowledge this is the first paper which model infant mortality rates within the context of fractional integration by simultaneously allowing for non-linear deterministic trends. The analysis of non-linearities in the IMR will have important policy implications, since it will help us understand the heterogeneous historical evolution of each of the series, the differences among the analyzed countries and the potential effects of policy changes in those series.

The contribution of this paper is two-folded. First, we provide evidence of the long memory properties of IMR allowing for non-linear deterministic trends in the form of Chebyshev polynomials. Second, including in the analysis a sample of 37 countries, based on data availability, will allow us to compare time series properties of this variable among countries of different degrees of development. Note our sample of countries do not include Sub-Saharan Africa due to lack of continuous long-span time series data.

Note that, we aim to provide a direct estimate of the degree of persistence in IMR, covering at times over one hundred and fifty years of data for some of the 37 countries considered (see Table 1). For our purpose, instead of relying on tests of unit roots, as commonly done in the literature (discussed below), we take a long memory approach. Unlike, standard unit root tests, which can only indicate whether a series is stationary or not by looking at 0 or 1 for the orders of integration, and have low power especially in cases where the series is characterized by a fractional process (Diebold and Rudebusch, 1991; Hassler and Wolters, 1994; Lee and Schmidt, 1996; and more recently, Ben Nasr *et al.*, 2014), the long memory approach provides us with an exact measure of the degree of persistence. This in turn, can provide us with a time span that it would take for the shock to die off, if at all.

However, long memory models are known to overestimate the degree of persistence of the series in the presence of structural breaks (Cheung, 1993; Diebold and Inoue, 2001; and more recently, Ben Nasr *et al.*, 2014), which are very likely in our case as it covers long samples for many countries in our sample. Given this, we supplement our long memory model to accommodate for non-linear (deterministic) trends as in Cuestas and Gil-Alana (2015), *i.e.*, through the use of Chebyshev polynomials, which, in turn, are cosine functions of time. This approach is preferred over the method

proposed by Gil-Alana (2008), whereby the number of breaks and the break dates in the series are determined endogenously, obtained by minimizing the residual sum of the squares at different break dates and different (possibly fractional) differencing parameters. The main reason is that, since we are using low-frequency data, structural breaks should ideally be modelled in a smooth rather than an abrupt fashion.

The remainder of the paper is structured as follows: Section 2 revises the literature on time series modeling of infant mortality rates. Section 3 describes the methodology and justifies its application in the context of infant mortality series. Section 4 presents the data and the main empirical results, while Section 5 contains some concluding comments and policy implications.

## **2. Literature review**

Modeling the dynamic behavior of infant mortality rates has become an important research issue based on the relevance of this variable as an indicator of a country's health, and thus, welfare. Thus, a growing literature has examined the stationarity of these series using different time series techniques. For example, Bishai (1995) applies Augmented Dickey Fuller (ADF, Dickey and Fuller, 1976) tests to infant time series mortality rates in Sweden (1800-1989), the UK (1839-1989) and the US (1915-1989) and he concludes that the null hypothesis of unit root cannot be rejected in any of the three cases. The non-stationary behavior of the series allows him to test for cointegration between infant mortality rates and per capita GNP levels, concluding that a bivariate long-run cointegrated relationship exists for the case of Sweden. Dregen and Reimers (2005) apply unit root tests to infant mortality rates to a sample of 21 OECD countries from 1975 to 2001, without obtaining conclusive results on the stationarity of the series.

Silva (2007) applies panel stationary tests that allow for structural breaks in infant mortality rates in Australia from 1911-2002, and finds that even allowing for structural breaks, the unit root null hypothesis cannot be rejected for all the states. On the contrary, Erdogan et al. (2013) test for panel unit root in IMR for a sample of 25 high-income OECD countries over the period 1970-2007 obtaining that these series are stationary. Caporale and Gil-Alana (2014) estimate the fractional order of integration of infant mortality rates in 24 countries and they obtain different results for each of the countries. However, in this paper the authors do not take into account the possibility of non-linear deterministic trends.

As far as the deterministic time trends of infant mortality rates are concerned, most of the studies assume an exponential or a linear time trend. Caporale and Gil-Alana (2014) allow for deterministic linear trends when they estimate the fractional integration order of the series. However, Bishai and Opuni (2009) analyze the time trends in infant mortality rates in 18 countries using data for the 20<sup>th</sup> century and conclude that the over time infant mortality decline is best modeled as exponential for the case of US, while for the rest of the 17 countries, the time trends can be modeled neither as exponential nor linear. Allowing in the present paper for non-linear deterministic trends in IMR series will help us test the adequacy of the assumption of linearity in each of the countries.

This paper examines the long memory properties of the infant mortality rates of 37 countries allowing for non-linear deterministic trends in the form of Chebyshev polynomials in time. Our paper can hence, be considered as a robust extension of Caporale and Gil-Alana (2014), by incorporating nonlinear deterministic trends to capture possible structural breaks in IMR, which in turn, allows us to guard against possible evidence of long-memory. Also note, while Caporale and Gil-Alana (2014)

considered 24 countries till 2006, we analyze as many as 37 countries with updated time spans that includes more recent years, and thus have a more comprehensive data set. The different results obtained on the order of integration of the series for each of the countries and on the existence of significant non-linearities in each of the series will have different policy implications for each of the countries examined.

### 3. Methodology

The methodology used in this paper is based on the concept of fractional integration. An updated review of this issue can be found in Gil-Alana and Hualde (2009). We say a process  $\{x_t, t = 0, \pm 1, \dots\}$  is integrated of order 0 (and denoted as  $x_t \approx I(0)$ ) if it is a covariance stationary process with a spectral density function that is positive and finite at the zero frequency. Having said this, a process is integrated of order  $d$ , (and denoted as  $x_t \approx I(d)$ ) if it can be represented as

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, \dots, \quad (1)$$

with  $x_t = 0$  for  $t \leq 0$ , and  $d > 0$ , where  $L$  is the lag-operator ( $Lx_t = x_{t-1}$ ) and  $u_t$  is  $I(0)$ .

Thus, the parameter  $d$  refers to the number of differences required to render a series stationary  $I(0)$ . However, by allowing  $d$  to be fractional, we permit a much richer degree of flexibility in the dynamic specification of the series, not achieved when using the classical approaches based on integer differentiation. Processes with  $d > 0$  in (1) display the property of “*long memory*”, characterized because the spectral density function of the process is unbounded at the lowest (zero) frequency.

On the other hand, it is well known that fractional integration, non-linearities and structural breaks are issues which are intimately related (see, e.g., Cheung, 1993; Diebold and Inoue, 2001; Giraitis et al., 2001; Kapetanios and Shin, 2003; Mikosch and Starica, 2004; Granger and Hyung, 2004; etc.). In this context, some of these authors

argue that fractional integration can be an artificial artifact generated by the presence of breaks that are not taken into account. Further, changes can occur smoothly rather than suddenly as implied by structural breaks; Ouliaris et al. (1989), for example, proposed regular polynomials to approximate deterministic components in the data generation process (DGP). However, as later pointed out by Bierens (1997), Chebyshev polynomials might be a better mathematical approximation of the time functions, since they are bounded and orthogonal; being cosine functions of time, they are a very flexible tool to approximate deterministic trends in a non-linear way.

We then consider the following non-linear model:

$$y_t = \sum_{i=0}^m \theta_i P_{iT}(t) + x_t, \quad t = 1, 2, \dots, \quad (2)$$

with  $m$  indicating the order of the Chebyshev polynomial, and  $x_t$  following an I(d) process of the form as in (1). The Chebyshev polynomials  $P_{iT}(t)$  in equation (2) are defined as:

$$P_{0,T}(t) = 1, \\ P_{i,T}(t) = \sqrt{2} \cos(i\pi(t-0.5)/T), \quad t = 1, 2, \dots, T; \quad i = 1, 2, \dots \quad (3)$$

(see Hamming (1973) and Smyth (1998) for a detailed description of these polynomials). Bierens (1997) uses them in the context of unit root testing, i.e., imposing  $d = 1$  in (1). According to Bierens (1997) and Tomasevic and Stanivuk (2009), it is possible to approximate highly non-linear trends with rather low degree polynomials. If  $m = 0$  the model contains an intercept, if  $m = 1$  it also includes a linear trend, and if  $m > 1$  it becomes non-linear - the higher  $m$  is the less linear the approximated deterministic component becomes. The estimation and testing results displayed in the following section are based on Cuestas and Gil-Alana (2015). They proposed a joint specification of (1) and (2) such that using a Lagrange Multiplier (LM) test of the same form as in



Robinson (1994), i.e., testing  $H_0: d = d_0$ , for a given real value  $d_0$ , the null model becomes linear with respect to the (non-linear) parameters,<sup>1</sup>

#### 4. Data and empirical results

The infant mortality rates data employed in this paper corresponds to the number of infants dying before reaching one year of age, per 1000 live births in a given year, and have been obtained from the Human Mortality Database at the University of California, Berkeley ([www.mortality.org](http://www.mortality.org)). The number of countries included in the analysis is 37, and the time period for each of the country depends on the data availability. Table 1 reports the time period and the number of observations for each of the countries. As shown in the table, the largest time series correspond to Sweden (261 observations, from 1751-2011), to France (198 observations, from 1816 to 2013), to Denmark (177 observations, from 1835 to 2010), to Island (176 observations, from 1838 to 2013) and the UK (176 observations, from 1841 to 2011). In contrast, due to the small number of observations in Chile (14 observations), Israel (27 observations) and Slovenia (27 observations), empirical results are not displayed for these countries due to its lack of reliability.

**(Insert Table 1 around here)**

Table 2 displays the estimates of the fractional differencing parameter  $d$  along with the  $\theta$ -coefficients in a model given by (1) and (2) under the assumption that the error term ( $u_t$  in (1)) is white noise. In other words, the estimated model is

$$y_t = \sum_{i=0}^m \theta_i P_{iT}(t) + x_t, \quad (1-L)^d u_t = \varepsilon_t, \quad t = 1, 2, \dots,$$

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<sup>1</sup> See Cuestas and Gil-Alana (2015).

And we use for this purpose a Whittle function in the frequency domain (See, Dahlhaus, 1989). The confidence intervals correspond to the 95% non-rejection values of  $d_0$ , when testing  $H_0: d = d_0$ , for  $d_0 = 0, 0.01, \dots, 2$ .

**(Insert Table 2 around here)**

Firstly, we focus on the deterministic coefficients, and the first thing that we observe is the presence of significant non-linear coefficients (i.e.,  $\theta_2$  and  $\theta_3$ ) in the majority of the series. In fact, only 9 out of the 37 series present evidence of linearity. These series correspond to Austria, Belgium, Canada, Czech Republic, West Germany, Ireland, Russia, Ukraine and USA. (See also Table 3). With respect to the non-linear series, strong evidence of it (in the sense that both  $\theta_2$  and  $\theta_3$  coefficients are statistically significant) is obtained in 18 of the countries: Australia, Bulgaria, Switzerland, Denmark, Estonia, France (both civilian and total population), UK (Scotland), Island, Italy, Japan, Norway, New Zealand (Maori, non-Maori and total population), Portugal, Sweden and Taiwan. (See again Table 3), while the remaining countries (Belarus, East Germany, Spain, Finland, UK (North Ireland), UK and UK (England and Wales civilian population)) present some degree of non-linear behaviour, with at least one of the two coefficients significant.

**(Insert Tables 3 and 4 around here)**

Finally, if we focus on the degree of integration of the series, evidence of  $d > 1$  is obtained for the cases of Czech Republic and Russia and there are another fourteen series (Austria, Belgium, Bulgaria, Belarus, Germany (East and West), Hungary, Japan, Lithuania, Poland, Slovakia, Taiwan, Ukraine and USA) where the unit root null hypothesis (i.e.,  $d = 1$ ) cannot be rejected. (See Table 4). For the remaining 26 countries, the series are found to be mean reverting implying transitory shocks and disappearing in the long run. Thus, in the event of a negative shock, increasing sharply

the IMR special attention should be taken in the countries belonging to the first two groups (those where the hypotheses  $d = 1$  and  $d > 1$  cannot be rejected.)

## **5. Concluding comments**

In this paper we have examined the time series behaviour of the infant mortality rates (IMR) within a long memory approach, including the possibility of a non-linear deterministic trend to accommodate for smooth regime switching due to structural breaks (which could lead to spurious evidence in favour of long-memory), and using data for a sample of 37 countries. The main results obtained in the paper, together with the main policy implications, are the following. First, the results suggest that in most of the cases (28 out of 37), the assumption of a linear time trend in infant mortality rates can be rejected, as suggested in Bishai (2009). That is, the rate of decrease observed in infant mortality rates over the analysed period is not constant but non-linear, a result which is against one of the most common assumption used in the literature on mortality rates. Second, we find significant differences in both the fractional order of integration of the series and in the non-linear behaviour of the series. Based on this result, it is difficult to assume international convergence among infant mortality rates. Furthermore, while health or education policies will have a transitory effect on the infant mortality rates in some of the countries, they will only have transitory effects in other countries. Third, we find evidence of fractional integration orders below 1 ( $d < 1$ ) for many of the infant mortality series (19 out of 37 countries) and in only two countries (Island and New Zealand (total population)) the hypothesis of stationarity ( $d < 0.5$ ) cannot be rejected. These characteristics of the infant mortality rates, both the non-linearities and the nonstationarity of the series should be taken into account when analysing the main determinants of infant mortality rates.

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**Table 1: Time series data**

Initial	Country	Dates	N. of obs.
AUS	Australia	1921 - 2011	91
AUT	Austria	1947 - 2010	64
BEL	Belgium	1919 - 2010	94
BGR	Bulgaria	1947 - 2010	64
BLR	Belarus	1959 - 2010	55
CAN	Canada	1921 - 2010	91
CHE	Switzerland	1876 - 2010	136
CHL	Chile	1992 - 2010	14
CZE	Czech Republic	1950 - 2010	62
DEUTE	East Germany	1956 - 2010	56
DEUTNP	Germany, Total population	1990 - 2010	22
DEUTW	West Germany	1956 - 2010	56
DNK	Denmark	1835 - 2010	177
ESP	Spain	1908 - 2010	105
EST	Estonia	1959 - 2011	53
FIN	Finland	1878 - 2012	135
FRACNP	France, Civilian population	1816 - 2013	198
FRATNP	France, Total population	1816 - 2013	198
GBR_NIR	UK, Northern Ireland	1922 - 2011	90
GBR_NP	UK,	1922 - 2011	90
GBR_SCO	UK, Scotland	1855 - 2011	157
GBRCENW	UK, England and Wales civilian population	1841 - 2011	171
GBRTENW	UK, England and Wales total population	1841 - 2011	171
HUN	Hungary	1950 - 2009	60
IRL	Ireland	1950 - 2009	60
ISL	Island	1838 - 2013	176
ISR	Israel	1983 - 2009	27
ITA	Italy	1872 - 2009	138
JPN	Japan	1947 - 2012	66
LTU	Lituania	1959 - 2011	53
LUX	Luxembourg	1960 - 2009	50

(cont.)

**Table 1: Time series data (cont.)**

Initial	Country	Dates	N. of obs.
LVA	Latvia	1959 - 2011	53
NLD	Netherlands	1850 - 2012	163
NOR	Norway	1846 - 2009	164
NZL_MA	New Zealand Maori	1948 - 2008	61
NZL_NM	New Zealand Non Maori	1901 - 2008	108
NZL_NP	New Zealand, Total population	1948 - 2008	61
POL	Poland	1958 - 2009	52
PRT	Portugal	1940 - 2012	73
RUS	Russia	1959 - 2010	52
SVK	Slovakia	1950 - 2009	60
SVN	Slovenia	1983 - 2009	27
SWE	Sweden	1751 - 2011	261
TWN	Taiwan	1970 - 2010	41
UKR	Ukraine	1959 - 2009	51
USA	United States	1933 - 2010	78



**Table 2: Estimates of d and the Chebyshev polynomials in time**

Country	d	$\theta_0$	$\theta_1$	$\theta_2$	$\theta_3$
AUS	0.63 (0.43, 0.86)	<b>-4.0456 (-57.36)</b>	<b>0.8413 (20.94)</b>	<b>-0.0509 (-1.73)</b>	<b>0.1146 (4.96)</b>
AUT	0.94(0.78, 1.14)	<b>-3.8548 (-17.26)</b>	<b>0.9672 (7.46)</b>	0.0007 (0.10)	0.0072 (1.51)
BEL	0.86 (0.69, 1.09)	<b>-3.6922 (-18.68)</b>	<b>1.1872 (10.44)</b>	-0.0655 (-0.98)	0.0451 (0.95)
BGR	0.84(0.68,1.03)	<b>-3.4965 (-17.55)</b>	<b>0.7210 (6.37)</b>	<b>0.2093 (3.09)</b>	<b>0.1579 (3.26)</b>
BLR	1.03 (0.82, 1.25)	<b>-3.8837 (-10.81)</b>	<b>0.5233 (2.46)</b>	-0.0891 (-0.86)	<b>0.1890 (2.79)</b>
CAN	0.72(0.59, 0.91)	<b>-3.7542 (-48.34)</b>	<b>1.0944 (24.92)</b>	0.0297 (1.00)	0.0309 (1.38)
CHE	0.62(0.48, 0.77)	<b>-3.3330 (-43.89)</b>	<b>1.3148 (30.20)</b>	<b>-0.0899 (-2.80)</b>	<b>0.1217 (4.79)</b>
CHL	---	---	---	---	---
CZE	1.21(1.07, 1.38)	<b>-3.3626 (-5.23)</b>	<b>0.6052 (1.51)</b>	-0.2028 (-1.28)	0.1109 (1.14)
DEUTE	0.77(0.58, 1.00)	<b>-4.4986 (-39.14)</b>	<b>0.8586 (13.32)</b>	0.0286 (0.69)	<b>0.1192 (3.89)</b>
DEUTNP	---	---	---	---	---
DEUTW	1.10(0.91, 1.33)	<b>-4.3232 (-20.64)</b>	<b>0.7802 (6.16)</b>	0.0309 (0.54)	-0.0075 (-0.20)
DNK	0.56 (0.46, 0.69)	<b>-2.9715 (-35.01)</b>	<b>1.2668 (25.57)</b>	<b>-0.4128 (-10.81)</b>	<b>0.1219 (3.95)</b>
ESP	0.64(0.49, 0.84)	<b>-3.3695 (-34.10)</b>	<b>1.3915 (24.72)</b>	<b>-0.2508 (-6.15)</b>	0.0246 (0.77)
EST	0.51(0.06, 0.95)	<b>-4.3344 (-42.36)</b>	<b>0.5430 (8.95)</b>	<b>-0.1652 (-3.43)</b>	<b>0.2057 (5.14)</b>
FIN	0.57(0.45, 0.73)	<b>-3.3706 (-38.56)</b>	<b>1.4375 (28.32)</b>	<b>-0.3028 (-7.80)</b>	0.0448 (1.42)
FRACNP	0.54(0.46, 0.65)	<b>-2.7415 (-32.65)</b>	<b>1.2081 (24.45)</b>	<b>-0.5864 (-15.19)</b>	<b>0.1964 (6.20)</b>
FRACTNP	0.54(0.46, 0.65)	<b>-2.7415 (-32.65)</b>	<b>1.2081 (24.45)</b>	<b>-0.5864 (-15.19)</b>	<b>0.1964 (6.20)</b>
GBR_NIR	0.53(0.43, 0.67)	<b>-3.8157 (-46.25)</b>	<b>0.9909 (20.41)</b>	<b>-0.0642 (-1.68)</b>	0.0456 (1.45)
GBR_NP	0.73(0.58, 0.93)	<b>-3.9551 (-42.61)</b>	<b>0.9337 (17.79)</b>	-0.0059 (-0.17)	<b>0.0882 (3.34)</b>
GBR_SCO	0.52 (0.44, 0.63)	<b>-3.0314 (-50.11)</b>	<b>1.1118 (30.34)</b>	<b>-0.4357 (-15.03)</b>	<b>0.0721 (3.00)</b>
GBRCENW	0.63 (0.56, 0.72)	<b>-3.0314 (-34.00)</b>	<b>1.1871 (23.21)</b>	<b>-0.3696 (-9.87)</b>	0.0330 (1.12)
GBRTENW	0.63 (0.56, 0.72)	<b>-3.6445 (-34.06)</b>	<b>1.1870 (23.25)</b>	<b>-0.3696 (-9.89)</b>	0.0333 (1.13)
HUN	0.84 (0.55, 1.12)	<b>-3.6445 (-22.24)</b>	<b>0.8099 (8.71)</b>	-0.0569 (-1.02)	<b>0.1129 (2.83)</b>
IRL	0.73 (0.51, 0.97)	<b>-4.3212 (-35.62)</b>	<b>0.7917 (11.63)</b>	0.0186 (0.41)	0.0524 (1.53)
ISL	0.29 (0.20, 0.42)	<b>-3.1312 (-40.36)</b>	<b>1.6228 (29.46)</b>	<b>-0.2654 (-5.43)</b>	<b>0.1902 (4.29)</b>
ISR	---	---	---	---	---
ITA	0.70 (0.59, 0.83)	<b>-2.8249 (-25.16)</b>	<b>1.2913 (20.21)</b>	<b>-0.4182 (-9.50)</b>	<b>0.1882 (5.61)</b>
JPN	0.89 (0.19, 1.17)	<b>-4.3709 (-25.22)</b>	<b>1.0706 (10.76)</b>	<b>0.1871 (3.31)</b>	<b>0.1192 (3.02)</b>

(cont.)

**Table 2: Estimates of d and the Chebyshev polynomials in time**

Country	d	$\theta_0$	$\theta_1$	$\theta_2$	$\theta_3$
LTU	0.85(0.45, 1.17)	<b>-3.9869 (-14.18)</b>	<b>0.5515 (3.46)</b>	0.0102 (0.10)	<b>0.1660 (2.47)</b>
LUX	0.42(0.06, 0.83)	<b>-4.6613 (-44.06)</b>	<b>0.7309 (10.96)</b>	-0.0557 (-1.00)	<b>0.1480 (3.08)</b>
LVA	0.73(0.51, 0.98)	<b>-4.1634 (-21.71)</b>	<b>0.34565 (3.23)</b>	-0.0334 (-0.46)	<b>0.1413 (2.61)</b>
NLD	0.61(0.50, 0.74)	<b>-3.1544 (-26.19)</b>	<b>1.4314 (20.65)</b>	<b>-0.2159 (-4.19)</b>	-0.0118 (-0.28)
NOR	0.78(0.68, 0.90)	<b>-3.4137 (-17.88)</b>	<b>1.1138 (10.17)</b>	<b>-0.3127 (-4.48)</b>	<b>0.1059 (2.06)</b>
NZL_MA	0.17(-0.16, 0.57)	<b>-3.8574 (-151.9)</b>	<b>0.7374 (35.70)</b>	<b>0.0990 (5.15)</b>	<b>0.1569 (8.63)</b>
NZL_NM	0.50(0.39, 0.65)	<b>-3.8803 (-60.85)</b>	<b>0.8262 (21.66)</b>	<b>-0.0743 (-2.43)</b>	<b>0.1744 (6.84)</b>
NZL_NP	0.15(-0.09, 0.46)	<b>-4.3654 (-348.7)</b>	<b>0.5658 (54.27)</b>	<b>-0.0546 (-5.58)</b>	<b>0.0606 (6.50)</b>
POL	0.80(0.45, 1.13)	<b>-3.8364 (-31.20)</b>	<b>0.6967 (10.09)</b>	-0.0159 (-0.37)	<b>0.1782 (5.76)</b>
PRT	0.40(0.25, 0.59)	<b>-3.5984 (-87.36)</b>	<b>1.2962 (49.12)</b>	<b>-0.1855 (-8.34)</b>	<b>0.0409 (2.11)</b>
RUS	1.48(1.29, 1.65)	<b>-2.9382 (-4.31)</b>	-0.1541(-0.34)	-0.0031 (-0.02)	0.0603 (0.78)
SVK	1.01(0.85, 1.19)	<b>-3.2821 (-7.26)</b>	<b>0.6669 (2.49)</b>	-0.0012 (-0.09)	<b>0.1522 (1.73)</b>
SVN	---	---	---	---	---
SWE	0.66(0.59, 0.73)	<b>-2.7949 (-20.97)</b>	<b>1.2725 (16.63)</b>	<b>-0.5598 (-10.24)</b>	<b>0.2564 (6.04)</b>
TWN	0.69(0.37, 1.09)	<b>-4.9229 (-49.97)</b>	<b>0.3171 (5.78)</b>	<b>0.1423 (3.74)</b>	<b>0.1124 (3.85)</b>
UKR	1.25(0.94, 1.44)	<b>-3.3990 (-6.16)</b>	0.1060 (0.30)	-0.0102 (-0.07)	0.0420 (0.53)
USA	1.05(0.93, 1.23)	<b>-3.8985 (-22.82)</b>	<b>0.7120 (6.94)</b>	0.0242 (0.50)	0.0490 (1.55)

“ --- “ means that convergence is not achieved. This is related with the small number of observations involved in these series.

**Table 3: Summary of the results in terms of non-linearities**

Evidence of NON-LINEARITIES	Some evidence of NON-LINEARITIES	No evidence of NON-LINEARITIES
AUS (0.63)	BLR (1.03)	AUT (0.94)
BGR (0.84)	DEUTE (0.77)	BEL (0.86)
CHE (0.62)	ESP (0.64)	CAN (0.72)
DNK (0.56)	FIN (0.57)	CZE (1.21)
EST (0.51)	GBR_NIR (0.53)	DEUTEW (1.10)
FRACNP (0.54)	GBR_NP (0.73)	IRL (0.73)
FRATNP (0.54)	GBRCENW (0.63)	RUS (1.48)
GBR_SCO (0.52)	GBRTENW (0.63)	UKR (1.25)
ISL (0.29)	HUN (0.84)	USA (1.05)
ITA (0.70)	LTU (0.85)	
JPN (0.89)	LUX (0.42)	
NOR (0.78)	LVA (0.73)	
NZL_MA (0.17)	NLD (0.61)	
NZL_NM (0.50)	POL (0.80)	
NZL_NP (0.15)	SVK (1.01)	
PRT (0.40)		
SWE (0.66)		
TWN (0.69)		

**Table 4: Summary of the results in terms of the value of d**

Evidence of $d < 1$	Evidence of $d = 1$	Evidence of $d > 1$
AUS (0.63)	AUT (0.94)	CZE (1.21)
CAN (0.72)	BEL (0.86)	RUS (1.48)
CHE (0.62)	BGR (0.84)	
DNK (0.56)	BLR (1.03)	
ESP (0.64)	DEUTE (0.77)	
EST (0.51)	DEUTEW (1.10)	
FIN (0.57)	HUN (0.84)	
FRACNP (0.54)	JPN (0.89)	
FRATNP (0.54)	LTU (0.85)	
GBR_NIR (0.53)	POL (0.80)	
GBR_NP (0.73)	SVK (1.01)	
GBR_SCO (0.52)	TWN (0.69)	
GBRCENW (0.63)	UKR (1.25)	
GBRTENW (0.63)	USA (1.05)	
IRL (0.73)		
ISL (0.29)		
ITA (0.70)		
LUX (0.42)		
LVA (0.73)		
NLD (0.61)		
NOR (0.78)		
NZL_MA (0.17)		
NZL_NM (0.50)		
NZL_NP (0.15)		
PRT (0.40)		
SWE (0.66)		