Optimal metering plan for measurement and verification on a lighting case study

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Abstract

Measurement and Verification (M&V) has become an indispensable process in various incentive energy efficiency and demand side management (EEDSM) programmes to accurately and reliably measure and verify the project performance in terms of energy and/or cost savings. Due to the uncertain nature of the unmeasurable savings, there is an inherent trade-off between the M&V accuracy and M&V cost. In order to achieve the required M&V accuracy cost-effectively, we propose a combined spatial and longitudinal metering cost minimisation (MCM) model to assist the design of optimal M&V metering plans, which minimises the metering cost whilst satisfying the required measurement and sampling accuracy of M&V. The objective function of the proposed MCM model is the M&V metering cost that covers the procurement, installation and maintenance of the metering system whereas the M&V accuracy requirements are formulated as the constraints. Optimal solutions to the proposed MCM model offer useful information in designing the optimal M&V metering plan. The advantages of the proposed MCM model are demonstrated by a case study of an EE lighting retrofit project and the model is widely applicable to other M&V lighting projects with different population sizes and sampling accuracy requirements.

Keywords: Energy Efficiency, Lighting, M&V, Sampling.

Nomenclature

Variables

α	$\alpha = e^{-L}$, where L is the rated average life span of a certain type of lamp
$\bar{\chi}(K)$	the random variable denoting the cumulative sample mean across all lighting groups up to the Kth crediting year
$\bar{\chi}_i(K)$	the cumulative sample mean of the ith lighting group up to the Kth crediting year
$\bar{x}$	the sample mean
$\bar{X}(k)$	the random variable denoting sample mean of the daily energy consumption per lamp across all lighting groups in the kth year
$\bar{x}(k)$	the sample mean of the daily energy consumption per lamp across all lighting groups in the kth year
$\bar{X}_i(k)$	the random variable denoting the sample mean of the daily energy consumption per lamp of the ith lighting group in the kth year
$\bar{x}_i(k)$	the sample mean of the daily energy consumption per lamp of the ith lighting group in the kth year
β	the coefficient related to the slope of the lamp decay
δ	the δth year, $1 \le \delta \le K$
γ	the coefficient related to the initial percentage lamp survival at τ=0
$\Gamma(K)$	the random variable denoting the cumulative standard deviation across all lighting groups up to the Kth crediting year
$\Gamma_i(K)$	the random variable denoting the cumulative standard deviation of the ith lighting group up to the Kth crediting year
$\hat{\Psi}$	the set of post-implementation energy governing factors
λ	the design variable $\lambda = (\lambda(0), \ldots, \lambda(k), \ldots, \lambda(K))$, where $\lambda(k) = (z_1(k), \ldots, z_I(k), p_1(k), \ldots, p_I(k))$
λ^*	the optimal solution
λ_0	the search starting point to solve the optimisation model
μ	the true mean
$\mu(k)$	the true mean of the daily energy consumption per lamp across all lighting groups in the kth year

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$\mu_i(k)$	the true mean of the daily energy consumption per lamp of the ith lighting group in the kth year
Ψ	the set of baseline energy governing factors
σ	the standard deviation
$\sigma(k)$	the true standard deviation of the daily energy consumption per lamp across all lighting groups in the kth year
$\sigma_i(k)$	the true standard deviation of the daily energy consumption per lamp of the ith lighting group in the kth year, and $\sigma_i(k) = \overline{x}_i(k)CV_i(k)$
τ	the time after a lamp installation
$\theta(K)$	the random variable denoting the cumulative true mean across all lighting groups up to the Kth crediting year
$\theta_i(K)$	the random variable denoting the cumulative true mean of the ith lighting group up to the Kth crediting year
$\tilde{\beta}$	the coefficient in the discrete lamp population decay model
$\tilde{\gamma}$	the coefficient in the discrete lamp population decay model
a_i	the individual meter device cost in the ith lighting group
b_i	the installation cost per meter in the ith lighting group
$B_i(k)$	the backup meters of the ith lighting group in the kth year, $B_i(0)$=0
c_i	the monthly maintenance cost per meter in the ith lighting group
CV	the coefficient of variation
$CV_i(k)$	the estimated CV value of the ith lighting group in the kth year
$E_i(t)$	the daily energy consumption per lamp of the ith lighting group at time t
$F(\Psi)$	the baseline energy model
$G(\Psi)$	the post-implementation energy model
I	the total lighting groups
i	the counter of lighting groups
K	the total project crediting years
k	the counter of project crediting years, where k=0 denotes the baseline period
lb	the lower bound of the design variable
N	the lighting population
n	the sample size after population adjustment
$N(k)$	the total survived lamp population in the kth year
n_0	the initial sample size before population adjustment
$N_i(k)$	the lighting population of the ith lighting group in the kth year
$n_i(k)$	the sample size of the ith lighting group in the kth year
$N_i(t)$	the lamp population of the ith lighting group at time t
$O_i(t)$	the lamp daily burning hours of the ith lighting group at time t
p	the relative precision
$P(\delta)$	the cumulative precision level across all lighting groups up to the δth crediting year
$p(k)$	the combined relative precision across all lighting groups in the kth year
$P_i(\delta)$	the cumulative precision level of the ith lighting group up to the δth crediting year
$p_i(k)$	the relative precision of the ith lighting group in the kth year
$P_i(t)$	the lamp rated power of the ith lighting group at time t
$S(t_2)$	the reported energy savings
$s(\tau)$	the percentage of survived lamps at time τ
$S_i(k)$	the mathematical sign of $B_i(k)$ of the ith lighting group in the kth year
t	the project duration including both baseline and post-implementation periods
t_1	the project baseline period
t_2	the project post-implementation period
$TolCon$	the tolerance on the constraint
$TolFun$	the tolerance on the function value
$TolX$	the tolerance on the design variable
ub	the upper bound of the design variable
$X(k)$	the random variable denoting the daily energy consumption per lamp across all lighting groups in the kth year

$X_i(k)$	the random variable denoting the daily energy consumption per lamp of the ith lighting group in the kth year
z	the abscissas of the normal distribution curve that cut off an area at the tails to give desired confidence level, also known as z-score
$Z(\delta)$	the cumulative z-score across all lighting groups up to the δth crediting year
$z(k)$	the combined z-score across all lighting groups in the kth year
$Z_i(\delta)$	the cumulative z-score of the ith lighting group up to the δth crediting year
$z_i(k)$	the z-score of the ith lighting group in the kth year

Abbreviations

R^2	coefficient of determination
ASHRAE	American society of heating, refrigerating, and air-conditioning engineers
CDM	clean development mechanism
CFL	compact florescent lamp
CV	coefficient of variation
DLC	direct load control
ECM	energy conservation measure
EEDSM	energy efficiency and demand side management
EVO	efficiency valuation organization
HDL	halogen downlighter
HERO	home energy rebate offer
ICL	incandescent lamp
IPMVP	international performance measurement and verification protocol
kW h	kilowatt-hour
LED	light-emitting diode
M&V	measurement and verification
MCM	metering cost minimisation
n/a	not applicable
PD	project developer
R	South African currency Rand
RMSE	root mean squared error
TWC	tradable white certificate
UNFCCC	United Nations framework convention on climate change
USD	United States dollar
W	Watt

1. Introduction

Measurement and Verification (M&V) is the process of using measurement to accurately and reliably determine the savings delivered by an energy conservation measure (ECM) [11]. The M&V process is introduced in detail in various M&V guidelines, such as the IPMVP [11], the ASHRAE Guideline 14 [1], the California energy efficiency evaluation protocol [25], and the localised M&V guideline in South Africa [10]. The best practice and experience of M&V usually offer valuable feedbacks of the project performance, i.e., energy or cost savings to the project developers for energy efficiency (EE) technology deployment and project design. M&V has thus become an indispensable process in various incentive EE programmes such as clean development mechanism (CDM) [20], tradable white certificate (TWC) scheme [2], demand side management (DSM) programmes [10], and performance contracting [30]. According to [11], the most crucial part of the entire M&V process is the design of an M&V plan, in which baseline modelling and savings determination methodologies are proposed with a proper metering plan for the measurement of the relevant M&V data. The M&V savings are inherently uncertain as they are naturally inexist and not directly measurable [19]. As summarised in [11] and [1], the quantifiable savings uncertainties are comprised of the measurement uncertainty, sampling uncertainty, and modelling uncertainty. A number of existing M&V studies have proposed various baseline modelling techniques to deal with the modelling uncertainties that arise from the improper mathematical function form, inclusion of the irrelevant variables or exclusion of relevant variables. For example, [15] has proposed a normative energy model based on Bayesian calibration, which is able to model the energy consumption patterns in large sets of buildings efficiently with quantifiable uncertainties associated with model parameters. In [4], an M&V approach is proposed to compare actual energy performance of a building with its theoretical performance using calibrated thermal modelling. Different accuracy indicators such as the normalised root mean squared error (RMSE), relative bias, and median of the absolute relative total error are adopted in [13] to estimate the accuracy performance of five statistical baseline models for M&V applications. Regression models have been adopted in the following studies to develop baseline models for M&V purposes with detailed model identification and validation by the uncertainty indicators of coefficient of determination (R^2), and coefficient of variation of the RMSE (CVRMSE). Statistical criteria to assess goodness-of-fit of baseline models in terms of the R^2 and CVRMSE are discussed in [23]. And [16] develops a regression model to characterise the relationship between daily energy consumption and energy governing factors such as degree days, humidity, and fuel prices to assess the energy saving performance of the Louisiana Home Energy Rebate

Offer (HERO) programme. In order to quantify the industrial energy savings, [17] uses multi-variable piece-wise regression models to develop energy baselines, which can be adjusted by weather and production data over the post-retrofit period. In [9], a primary multiple regression model is derived as a baseline model by incorporating three weather parameters, namely, outdoor temperature, relative humidity, and global solar radiation. Linear regression models are constructed in [8] for baseline calibration in order to quantify the energy and demand savings due to installation of motor sequencing controller on the conveyor belt. In addition, [31] introduces a cross-validation method to compute the baseline model uncertainty. Besides the modelling uncertainties of M&V, the measurement uncertainties usually come from inappropriate calibration of the metering equipment, inexact measurement procedure, or improper meter selection, installation or operation; and the sampling uncertainties result from inappropriate sampling approaches or insufficient sample sizes [1].

Although M&V metering plans can be designed to handle the measurement uncertainties by applying sophisticated measurement instruments while reducing the sampling uncertainties by taking sufficient sample sizes, M&V practitioners cannot enjoy such a luxury due to limited budgets for the projected savings verification, given that [11] clearly states that the annual M&V cost should be less than 10% of the annual savings realised by the EE projects. Hence M&V practitioners and project developers have great interest in designing the optimal M&V metering plan that helps to verify the savings accurately and cost-effectively. An M&V metering plan obtained by professional judgements of M&V practitioners may be far from optimal, especially when there are particular requirements on the M&V accuracy and M&V cost. In order to minimise the metering cost, and thus to maximise the project developers' profit, this study aims to design a cost-effective metering plan to satisfy M&V accuracy requirements.

An obvious observation is that the metering cost is lower whenever fewer samples are measured. However, the samples to be measured in some existing M&V case studies do not seem to have been determined optimally. In [18], instantaneous demand meters and run-time loggers are installed to monitor 10% of the lighting fixtures' energy consumption. Ref. [14] proposes to quantify the load reduction from a residential electric water heater load control programme by a "notch" test on substation level in order to reduce the metering and sampling cost of M&V. However, substations are not easily accessible for common M&V practice. In [22], a "deemed savings estimates" M&V approach is proposed by modelling historical data, which are sampled from 288 end users of the regional direct load control (DLC) programmes.

The general mathematical description of the optimal M&V metering plan problem has been proposed in [33]. However, with the guidance of [33], the optimal M&V metering plan for various M&V projects needs to be redeveloped with the consideration of project specific budget plans, technologies, measurement complexities, accuracy requirements, and population sizes. Among these projects, the optimal metering plan for lighting retrofit projects has attracted considerable research. The major reason is that sub-metering of the entire lighting population implies prohibitive measurement and sampling cost. The design of cost-effective metering plans to achieve the required sampling accuracy criterion with proper sample size becomes more difficult when the lamp population is large and decentralised. Ref. [35] has proposed a spatial metering cost minimisation (MCM) model to balance the sampling uncertainties across lighting groups. The idea in [35] is to minimise the sample sizes and metering cost for CDM lighting EE projects by assigning optimal confidence and precision levels to the lighting groups with different energy consumption uncertainties. The model in [35] is applicable and useful in optimising the M&V metering plan, but lacks of considerations on lighting population decay dynamics over the projects' life cycle. In practice, the lamp population will decay due to the lamp breakage, theft or other unpredicted damages. Sampling theory [26] indicates that the sample size can be reduced when the sampled population size becomes smaller. Several studies have proposed longitudinal MCM models to balance the sampling uncertainties across adjacent reporting years. The idea is to reduce the M&V metering cost for lighting EE projects by optimally deciding the required confidence and precision levels in different reporting years over the projects' crediting period. For instance, studies [34] and [36] present a longitudinal MCM model by incorporating a liner lamp population decay model that is widely used in CDM lighting projects. The longitudinal MCM model provided in [34] and [36] is further improved in [5], which provides more detailed discussions on the lamp population decay models, weighted measurement impacts, and price inflations of the metering devices. The longitudinal MCM models in [34], [36], and [5] are applicable to lighting projects with homogeneous lighting population that shares the same energy usage patterns and population decay dynamics.

On summary of existing M&V studies, the optimal M&V metering plans can be designed 1) without optimisation but by professional judgements; 2) by applying the spatial MCM model for project with no population decay; 3) by using the longitudinal MCM model to projects with homogeneous lighting population and similar population decay dynamics. However in practice, solely using the spatial or longitudinal MCM model is insufficient to accommodate lighting projects that have multiple homogeneous lighting groups but with different energy consumption patterns and population decay dynamics across groups. In this study, a combined spatial and longitudinal MCM model is proposed to further reduce the lighting project metering cost by balancing the sampling uncertainties both spatially across homogeneous lighting groups and longitudinally across adjacent reporting years. In this model, the design variables are the required annual confidence and precision levels for each lighting group. The objective function is a cost function that covers the procurement, installation and maintenance of the metering system for M&V. The sampling accuracy requirements are formulated as the constraints. In order to demonstrate the advantages of the proposed MCM model, an optimal metering plan is designed for a lighting retrofit project with two homogeneous lighting groups as a case study. Optimal solutions for the case study are obtained by the proposed combined spatial and longitudinal MCM model with the consideration of the project specific characteristics. The optimal solutions provide useful and sufficient M&V metering plan information such as the required lighting samples to be measured in each lighting groups, the achieved sampling accuracy in terms of confidence and precision levels as well as the annual and total M&V metering cost for the studied lighting project. In addition, the metering solutions obtained without optimization, with solely the spatial or the longitudinal MCM models are also calculated and compared. The comparisons among these solutions highlight the advantageous performance of the proposed spatial and longitudinal MCM model in designing cost-effective M&V metering plan whilst satisfying the M&V accuracy requirements. This combined optimisation model will be widely applicable to design the optimal metering plan for various M&V lighting projects with different population sizes and sampling accuracy requirements.

The rest of this paper is organised as follows: Section 2 provides the mathematical formulation of the optimal M&V metering plan problem. In Section 3, an optimal metering plan is designed for a hospital lighting retrofit project as a case study to demonstrate the advantages of the proposed model. The applicability of the proposed model is discussed in Section 4 while the conclusion comes in the last section.

2. Formulation of the optimal M&V metering plan problem

In this section, general metering plans for lighting retrofit projects are discussed. With the application of classic sampling approaches, sample size determination methodologies, and lamp population decay models, the optimal M&V metering plan problem is formulated as a combined spatial and longitudinal MCM model under necessary modelling assumptions.

2.1. The metering plan for lighting retrofit projects

Without loss of generality, the methodology to design the optimal M&V metering plans is discussed under the scope of lighting retrofit projects in this study. Given a lighting retrofit project with an initial lamp population of N, the lamp population can be classified into I homogeneous lighting groups when the same technical specifications, similar energy consumption uncertainties, and population decay dynamics of the lamps are identified in the ith lighting group, where i is the counter of the lighting groups. For lighting retrofit projects, various EE lighting technologies, i.e., compact florescent lamps (CFLs), light-emitting diodes (LEDs) or solar-powered lamps are employed to replace existing less EE lamps such as halogen downlighters (HDLs) and incandescent lamps (ICLs). The retrofit interventions do not change the existing lighting control configurations and illumination levels.

Let a lighting retrofit project have a three-months' baseline measurement period and K years of the project crediting period with its savings performance being measured, verified and reported; $k = 1, 2, \ldots, K$ denotes the counter of the crediting years and $k = 0$ denotes the baseline year. $F(\Psi)$ and $G(\hat{\Psi})$ denote the energy models, where notations Ψ and $\hat{\Psi}$ represent a set of energy governing factors that determines the lighting energy consumption in the baseline and post-retrofit periods, respectively. For the lighting technology, $F(\Psi)$ and $G(\hat{\Psi})$ should at least include the following energy governing variables, i.e., the lamp population $N_i(t)$, rated power $P_i(t)$, and daily operating hours $O_i(t)$. In order to simplify the measurement and sampling uncertainty analysis, $P_i(t)$ and $O_i(t)$ can be determined in combination as the daily energy consumption $E_i(t)$. Then the project baseline can be denoted by $F(N_i(t_1), E_i(t_1))$ and similarly the post-retrofit is denoted by $G(N_i(t_2), E_i(t_2)$, where t_1 refers to the baseline period, t_2 refers to the post-retrofit period, and t refers to both periods. To ensure a fair comparison, the projected energy savings $S(t_2)$ under the post-retrofit condition are calculated by Eq. (1)

$$S(t_2) = \tilde{F}(N_i(t_2), E_i(t_1)) - G(N_i(t_2), E_i(t_2)), \tag{1}$$

where $\tilde{F}(\cdot)$ is the adjusted baseline when the lamp population decays in the post-retrofit period.

In order to accurately report the savings $S(t_2)$ in Eq. (1), the modelling uncertainties, measurement uncertainties, and sampling uncertainties must be handled properly. The modelling uncertainty is not applicable to lighting retrofit projects when the lighting energy usage is directly measured in isolation. The measurement uncertainties are usually negligible when suitable and high accuracy metering equipment is applied for measurement. For lighting projects with large and decentralised population, sampling uncertainty is the major contributor to the savings uncertainty. The sampling uncertainties can be reduced by taking sufficient sample sizes with suitably selected sampling techniques such as simple random sampling, stratified sampling, systematic sampling, cluster sampling, and multi-stage sampling [7]. In order to report the M&V savings accurately in this study, the sample sizes for the lighting projects are optimally decided to satisfy the 90/10 criterion. [1]

According to the previous discussions, the metering plans for the EE lighting projects can be summarised as follows:

1) The two energy governing variables namely the survived lamp population $N_i(t)$ and daily energy consumption per lamp $E_i(t)$ in the ith lighting group need to be continuously sampled and metered. More precisely, $N_i(t)$ needs to be sampled regularly and $E_i(t)$ will be monitored by long-term metering over the projects' baseline and crediting period. Each monitored and sampled variable must satisfy the 90/10 criterion.

2) The meters will be purchased and installed during the baseline period. The baseline lighting system will be measured for 3 calendar months.

3) The decay dynamics of $N_i(t)$ will be discussed in Subsection 2.3. The required sample sizes for metering $E_i(t)$ will be decided by the proposed combined spatial and longitudinal MCM model.

4) Meters will be installed to monitor the sampled lamp appliance individually. Meters with different functionalities and

[1] For the 90/10 criterion, precision is an assessment of the error margin of the final estimate and confidence is the likelihood that the sampling result of an estimate lies within a certain range of the true values. Following the notation of the 90/10 criterion, x/y denotes x% confidence and y% precision in this study.

prices will be applied in different lighting groups. Calibration and maintenance of the metering systems will be performed regularly.

2.2. The sampling approach and sample size determination

According to [7], the simple random sampling approach is applicable when the sampled units are homogeneous. However, the stratified random sampling is most applicable for a lighting retrofit project with multiple lighting groups, when characteristics of the lighting units are more similar within groups than across groups. In this study, the lighting population are firstly stratified into I homogenous strata and then the simple random sampling is performed within each stratum where each lighting unit has the same probability of being sampled and metered. The sampling uncertainties in different lighting groups are characterised by coefficient of variation (CV), which is defined as the standard deviation of the metering records divided by the mean. CV is a positive value and a greater CV value corresponds to a higher sampling uncertainty.

As provided in standard statistics text books [26], the initial sample size n_0 to achieve certain confidence and precision level of homogeneous population is calculated by

$$n_0 = \frac{z^2 CV^2}{p^2}, \tag{2}$$

where z denotes the abscissas of the normal distribution curve that cut off an area at the tails to give desired confidence level, also known as the z-score, and p is the relative precision. For the 90/10 criterion, z=1.645 for 90% confidence and p=10% as the allowed margin of error. The values of z at various confidence levels are tabulated in many statistics books [7]. z can be calculated by the Z-transformation formula

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}, \tag{3}$$

where $\bar{x}$ is the sample mean; μ is the true mean while σ denotes the true standard deviation of the sampled population.

CV can be estimated from spot measurements or derived from previous metering experience. In some cases, it may be desirable to initially conduct a small sample for the sole purpose of estimating a CV value to assist in planning the sampling design. If CV is unknown, 0.5 is historically recommended by [28] as the initial CV since numerous projects have shown this to be reasonable guess for most applications. After the first year of monitoring, the CV can be projected from the results of the metering in the previous year, which can be used as an updated initial CV value for the sample size determination of the coming year. Usually more samples are required to achieve a higher confidence level and a better precision level for a given CV value. The initial sample size n_0 can be adjusted by Eq. (4) [26] when the population N is a finite number. As can be observed in Eq. (4)

$$n = \frac{n_0 N}{n_0 + N} = \frac{CV^2 z^2 N}{CV^2 z^2 + N p^2}, \tag{4}$$

when N reduces from $+\infty$ to 0, the sample size will become smaller.

2.3. Lamp population decay modelling

As discussed in Eqs. (1) and (4), the survived lamp population is crucial for the M&V baseline adjustment, savings calculation, and sample size determination. Without an accurate model to characterise the lamp population decay dynamics, the survived lamp population needs to be identified by conducting samples of questionnaires, telephone interviews, and onsite surveys for various energy efficient lighting retrofit projects. The inspections on the lamp population at different time intervals over the projects' crediting period are helpful for the project performance evaluation, M&V metering plan design, and necessary maintenance planning. But the regular inspection approach is usually very costly and time-consuming as such inspections have to be conducted repeatedly for various lighting projects with different characteristics. In order to alleviate the lamp population inspection burdens, the lamp population decay dynamics are characterised by various models that have been established from biological population dynamics study or from reliability engineering experiments. For instance, previous study [6] has performed an informative review on the existing lamp population decay dynamics. In addition, [5] has proposed a reliable lamp population decay model that is improved from existing models as given in both the Poland efficient lighting programme evaluation report [21] and the technical report of South African national CFL mass roll out programme [3]. The general form of the model is provided in Eq. (5)

$$s(\tau) = \frac{1}{\gamma + \alpha e^{\beta \tau}}, \tag{5}$$

where $s(\tau)$ is the percentage of survived devices at time τ for a lighting project, τ is counted from the beginning of lamp installations. $\alpha = e^{-L}$ and L is the rated average life span of a certain type of lamps. Following CDM guidelines [27], the rated average life span is declared by the manufacturer or responsible vendor as being the expected time at which 50% of any large number of EE devices reach the end of their individual lives. β is the slope of decay, and γ is initial percentage lamp survival at $\tau = 0$. Thus, values for β and γ can be obtained by solving the following system of equations:

$$\begin{cases} s(0) = 1, \\ s(L) = 0.5. \end{cases} \tag{6}$$

The discrete and dynamical form of model (5) is also given in [6] and [5] as follows

$$s(k+1) = \tilde{\beta}\tilde{\gamma} s(k)^2 - \tilde{\beta} s(k) + s(k), \tag{7}$$

where $s(k)$ is the survived percentage of the lighting project population at the kth sampling interval. Note that for different lighting groups, the parameters $\tilde{\beta}$ and $\tilde{\gamma}$ are different and they can be obtained by the system identification approach proposed in [5]. Eq. (7) is further applied in the design of optimal maintenance plans for lighting retrofit project population by a control system approach in [37].

2.4. Modelling and assumptions

According to the proposed metering plan, $E_i(t)$ needs to be monitored by long-term measurement over the baseline and post-retrofit periods. The measurement uncertainties and sampling uncertainties must be properly handled to ensure the satisfaction of the required 90/10 criterion. In order to reduce the measurement uncertainties, the metering devices need to be carefully selected with full consideration of their accuracy levels and cost implications. According to [29], the key components of the metering cost include meters procurement, installation and maintenance cost. In order to design a cost-effective metering plan, it is suggested to use different metering devices with different procurement prices, memory capacities, data transmission functions and accuracy levels for lighting groups with different sampling uncertainties. Hence the meters will be selected according to the estimated CV values in various lighting groups in this study. Particularly, if $CV < 0.25$, then less expensive meters with acceptable accuracy will be chosen. Otherwise if $CV \geq 0.25$, then expensive and sophisticated meters will be applied. In general, measurement uncertainties are ignorable when the accuracy levels of the selected M&V meters are much better than the 90/10 criterion.

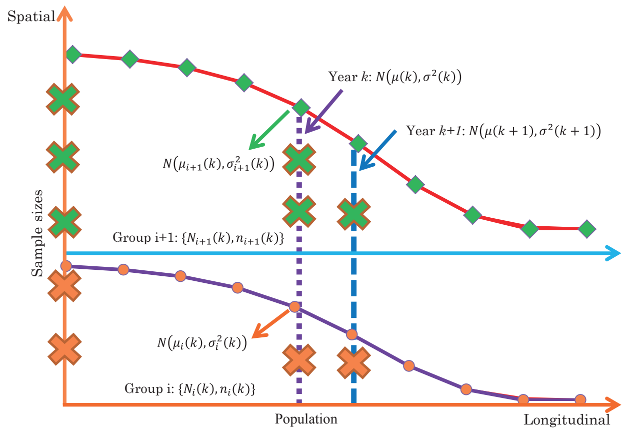

Figure 1: Illustrations for the metering cost minimisation modelling.

The sampling uncertainties are analysed and handled by a combined spatial and longitudinal MCM model in this study. The optimisation ideas of the modelling are illustrated by Figure 1. In Figure 1, the curve (in red) with squared-markers (in green) and the curve (in purple) with circled-markers (in orange) denote the lamps with different population decay dynamics over time. On the spatial domain at the kth year, the lighting project population is classified into I homogeneous strata according to different sampling uncertainty levels of the daily energy consumption of an individual lamp. Let $z_i(k)$ and $p_i(k)$ denote the z score and the precision levels in the ith group, $z(k)$ and $p(k)$ denote the combined z score and precision levels across all subgroups, respectively; $N_i(k)$ and $n_i(k)$ denote the survived lamp population and the required sample size of the ith lighting group in the kth year, respectively; $N(k)$ denote the total survived lamp population in the kth year. The spatial sampling uncertainties across all lighting groups in the kth year can be analyzed as follows. Let $X_i(k)$ be the random variable that denotes the daily energy consumption of an individual lamp of the ith lighting group in the kth year. From the well-known Central Limit Theorem [12], it is assumed that $X_i(k)$ follows normal distribution $X_i(k) \sim \mathcal{N}(\mu_i(k), \sigma_i(k)^2)$ given the large lamp population in the ith lighting group, where $\mu_i(k)$ is the true mean value, and $\sigma_i(k)$ is the true standard deviation of the ith group in the kth year. If any $n_i(k)$ samples are drawn from the ith lighting group, the sample mean distribution satisfies a normal distribution $\overline{X}_i(k) \sim \mathcal{N}(\mu_i(k), \sigma_i(k)^2/n_i(k))$ [32]. Assume the $\overline{X}_i(k)$'s are independent and the combined distribution for the $\overline{X}_i(k)$'s in all lighting groups in the kth year is denoted by $X(k) \sim \mathcal{N}(\mu(k), \sigma(k)^2)$, where the combined sample mean value $\overline{x}(k)$ for the total lighting population in the kth year is calculated by

$$\overline{x}(k) = \frac{\sum_{i=1}^{I} N_i(k)\overline{x}_i(k)}{N(k)}, \tag{8}$$

the true mean value $\mu(k)$ for the total lighting population in the kth year is calculated by

$$\mu(k) = \frac{\sum_{i=1}^{I} N_i(k)\mu_i(k)}{N(k)}, \tag{9}$$

and the true standard deviation $\sigma(k)$ for the total lighting population in the kth year is calculated by

$$\sigma(k)^2 = \sum_{i=1}^{I} \frac{(\sigma_i(k)N_i(k))^2}{n_i(k)N(k)^2}. \tag{10}$$

According to Eq. (3), the z transformation function in the ith lighting group of the kth year is given by

$$\overline{x}_i(k) - \mu_i(k) = z_i(k) \cdot \frac{\sigma_i(k)}{\sqrt{n_i(k)}}, \tag{11}$$

where $\overline{x}_i(k)$ is the sample mean of the ith lighting group in the kth year and $\sigma_i(k) = \overline{x}_i(k)CV_i(k)$. Assume that the estimated daily energy consumptions and CV values of the ith lighting group will not change over the credit period, then the standard deviation $\sigma_i(k)$ of the ith lighting group in the kth year will also remain unchanged.

The combined annual z score $z(k)$ and relative precision level $p(k)$ are calculated by

$$z(k) = \frac{\overline{x}(k) - \mu(k)}{\sigma(k)}, \tag{12}$$

and

$$p(k) = \frac{\overline{x}(k) - \mu(k)}{\overline{x}(k)}. \tag{13}$$

On the longitudinal domain over the crediting period, the project performance may need to be reported regularly at fixed reporting intervals, i.e., in the years of $\delta = \{2, 4, ..., K\}$, to the project developers and relative stakeholders by M&V practitioners. For both the baseline year and the reporting years δ, the sampled parameters are required to satisfy a required accuracy level, i.e., the 90/10 criterion. It is clear that fewer samples are required to achieve the 90/10 criterion when the project population decreases. For a performance report covers the years k and $(k+1)$, the possible sample sizes over the two years might be 30 and 10, respectively to achieve the 90/10 criterion. Then the initial investment must be made available for 30 meters in the kth year while the surplus 20 meters become unnecessary in the $(k+1)$th year. An optimal metering plan may be designed to install 20 meters for both the years k and $(k+1)$, such that a lower accuracy level, i.e., 85/15 is reached in the year k but a higher accuracy level, i.e., 95/5 is obtained in the year $(k+1)$, while the combined accuracy level across the years k and $(k+1)$ satisfies the 90/10 criterion.

In order to quantify the sampling uncertainties on the longitudinal domain, assume the installed metering system will not be relocated over the K years and the same sampled lighting units will be continuously measured. And the sampled lamps need to be monitored to ensure immediate replacement on occurrence of a lamp failure. Thus the metered data from the Years 1 to $(k-1)$ will also be analyzed together with the metered data in the kth year. Further assume that $\bar{X}(k)$'s are independent, then the combined distribution for the $\bar{X}(k)$'s over the K years will follow a normal distribution $\bar{\chi}(k) \sim \mathcal{N}(\theta(k), \Gamma(k)^2)$, where

$$\bar{\chi}(K) = \frac{\sum_{k=1}^{K} N(k)\bar{x}(k)}{\sum_{k=1}^{K} N(k)}, \tag{14}$$

$$\theta(K) = \frac{\sum_{k=1}^{K} N(k)\mu(k)}{\sum_{k=1}^{K} N(k)}, \tag{15}$$

$$\Gamma(K)^2 = \sum_{k=1}^{K} \left(\frac{\sigma(k)N(k)}{\sum_{k=1}^{K} N(k)} \right)^2. \tag{16}$$

Let $Z(\delta)$ and $P(\delta)$ denote cumulative z score and cumulative precision levels by end of the δth year, respectively, then

$$Z(\delta) = \frac{\bar{\chi}(\delta) - \theta(\delta)}{\Gamma(\delta)}, \tag{17}$$

$$P(\delta) = \frac{\bar{\chi}(\delta) - \theta(\delta)}{\bar{\chi}(\delta)}, \tag{18}$$

where $\bar{\chi}(\delta)$, $\theta(\delta)$, and $\Gamma(\delta)$ are calculated by Eqs. (14)-(16), respectively by substituting δ for the value K.

As the lamp population decays, the number of required meters may also decease. If fewer meters are required in the kth year than the available meters installed in the $(k-1)$th year, then the surplus meters remain onsite for backup use. Let a_i, b_i and c_i denote the meter procurement, installation and monthly maintenance cost for each metering device of the ith lighting group, respectively. Then the combined spatial and longitudinal MCM model is formulated under follow assumptions. **1)** The lighting population will not decay during the baseline period. The time for the project implementation can be ignored. **2)** During the credit period, maintenance will only be performed to the active meters. **3)** The inflation/deflation of the metering cost will not be considered. **4)** The uncertainty of the lamp population decay model is neglectable.

Let the design variable be $\lambda = (\lambda(0), \ldots, \lambda(k), \ldots, \lambda(K))$, where $\lambda(k) = (z_1(k), \ldots, z_I(k), p_1(k), \ldots, p_I(k))$. The objective function is denoted by

$$\begin{aligned} f(\lambda) = &\textstyle\sum_{i=1}^{I} (a_i + b_i + 3c_i) n_i(0) \\ &+ \textstyle\sum_{k=1}^{K} \sum_{i=1}^{I} [12 c_i n_i(k) + B_i(k) S_i(k)(a_i + b_i)], \end{aligned} \tag{19}$$

where $n_i(k)$ is calculated by Eq. (4); and $B_i(k)$ denotes the surplus meters in the kth year, which is calculated by

$$B_i(k) = max(B_i(k-1), 0) + n_i(k-1) - n_i(k),$$

where $B_i(0) = 0$, and $S_i(k)$ is the mathematical sign of $B_i(k)$, which is defined as

$$S_i(k) = \text{sgn}(B_i(k)) = \begin{cases} 0, & \text{if} \quad B_i(k) > 0, \\ -\frac{1}{2}, & \text{if} \quad B_i(k) = 0, \\ -1, & \text{if} \quad B_i(k) < 0, \end{cases}$$

where $sgn(\cdot)$ is the sign function. The constraints are summarised as

$$\begin{cases} z(0) \geq 1.645, \\ p(0) \leq 10\%, \\ Z(\delta) \geq 1.645, \\ P(\delta) \leq 10\%, \end{cases} \tag{20}$$

where $\delta = \{2, 4, ..., K\}$; [2] $z(0)$ and $p(0)$ are the combined z score and relative precision across all lighting groups during the baseline period, while $Z(\delta)$ and $P(\delta)$ are the cumulative z score and relative precision up to the δth year in the post-implementation period. The combined spatial and longitudinal MCM model is denoted by $C((19), (20))$.

3. Case study

In this section, an optimal M&V metering plan is designed for a lighting retrofit project as a case study to illustrate the advantages of the proposed combined spatial and longitudinal MCM model.

3.1. Background of the lighting project

A lighting retrofit project is going to be implemented to reduce the lighting load in 45 provincial hospitals in South Africa. It is planned to install 263 519 CFLs to replace existing inefficient ICLs. In addition, 140 777 units of LEDs will be installed to replace the less energy efficient HDLs. The 12 Watt (W) CFLs and 6 W LEDs will be adopted to replace the 60 W ICLs and 50 W HDLs, respectively. The ICLs are mainly installed in office rooms and burning during 8:00-16:00 everyday. The HDLs are installed in the corridors and hallways where motion sensors are currently in use to control the HDL lighting systems. The CFLs and LEDs will be directly installed to replace the ICLs and HDLs without changing the existing lighting control systems. The EE lamps have equivalent lumen to

[2] Obviously, one can also let $\delta = \{1, 4, 7, ..., K\}$ when other reporting intervals are agreed by the project stakeholders.

the replaced old lamps. The CFLs have a rated life of 3 years while the LEDs have a rated life of 6 years. According to the agreements between the project sponsors and project developers (PDs), the energy saving performance of this project must be verified and reported in every 2 years' interval over the 10 years' crediting period. PDs are responsible for the M&V cost that at least covers the metering system procurement, installation and maintenance. The energy consumption of the lighting system will be sampled and measured over the 3 months' baseline period and the entire crediting period.

The involved lamps are naturally classified into two subgroups according to their different daily energy consumption uncertainties. Group I is the 263 519 ICLs and Group II is the 140 777 HDLs. The lighting classification remains unchanged after project implementation. The energy consumption uncertainties can be estimated by spot measurement during the on site project survey. For instance, the estimated daily energy consumption per lamp in Group I is 0.48 ± 0.09 kWh in the baseline period and 0.096 ± 0.018 kWh in the crediting period. CV value of the daily energy consumption per lamp in Group I is around 0.19. The energy consumption uncertainties in Group II are greater than those in Group I as the lamps are controlled by the motion sensors. In this case, a CV value as high as 0.5 is recommended by [28] for Group II over both the baseline and crediting periods. The estimated daily energy consumption per lamp in Group II is 0.20 ± 0.10 kWh in the baseline period and 0.024 ± 0.012 kWh in the crediting period based on an assumption that on average the lamps are burning 4 hours per day with low confidence. Since the energy consumption behaviours in Group II change more frequently than those in Group I, the metering devices to be installed in Group II should be more advanced, i.e., with more intelligent control units, faster sampling frequency, and larger memory capacity. The Group II meters are capable of capturing the real time energy consumption in both lighting groups but Group I meters are not applicable for the measurements in Group II. More detailed project information is summarised in Table 1 from the on site project survey.

Table 1: Lighting project details.

Parameters	Group I	Group II
Meter unit price	a_1=R 876	a_2=R 3 146
Installation per meter	b_1=R 195	b_2=R 320
Monthly maintenance	c_1=R 45	c_2=R 98
CV values	$CV_1(k) = 0.19$	$CV_2(k) = 0.50$
Baseline estimates	$\bar{x}_1(0) = 0.48$ kWh	$\bar{x}_2(0) = 0.20$ kWh
Post-retrofit estimates	$\bar{x}_1(k) = 0.096$ kWh	$\bar{x}_2(k) = 0.024$ kWh
Coefficient $\tilde{\beta}$ in Eq. (7)	$\tilde{\beta}_1 = 1.1438$	$\tilde{\beta}_2 = 1.0297$
Coefficient $\tilde{\gamma}$ in Eq. (7)	$\tilde{\gamma}_1 = 0.8553$	$\tilde{\gamma}_2 = 0.9201$

Once the coefficients $\tilde{\beta}$ and $\tilde{\gamma}$ in Eq. (7) are identified with the given lamp life span, the lamp population decay dynamics are determined for the studied project. In Figure 2, the horizontal axis denotes the count of years where Year k corresponds to the duration [k, k+1). For instance, Year 0 corresponds to the duration [0,1), denoting the baseline period and Years 1-10 correspond to the duration [1,11), denoting the crediting period. The vertical axis denotes the survived lamp population. It shows that Group I has a greater initial lamp population. However, as

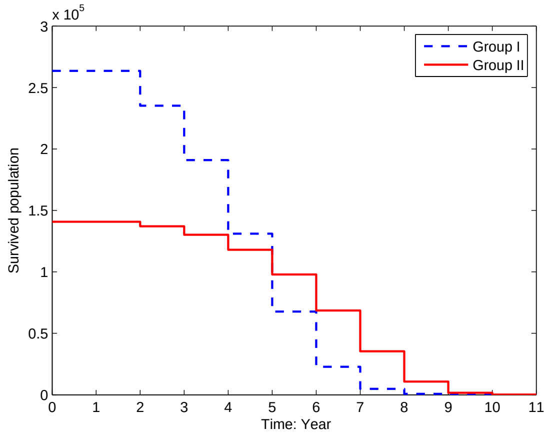

Figure 2: Survived lamp populations.

the CFLs have shorter life spans than the LEDs, the lamp population in Group II becomes greater than that in Group I from Years 5-10 of the lighting project.

The optimal metering plan can be obtained by solving the model $C((19), (20))$ with the application of the project specific information as given in Table 1. Solutions are calculated using the software program [24]. In particular, the optimal solutions are computed by the "fmincon" code of the Matlab Optimisation Toolbox. The optimisation settings of the "fmincon" function are shown in Table 2, where a search starting point λ_0 and the boundaries of the design variable are also assigned. From a mathematical perspective, the sample sizes, which are

Table 2: Optimisation settings.

Categories	Options
Algorithm	interior-point
TolFun	10^{-45}
TolCon	10^{-45}
TolX	10^{-45}
Hessian	'lbfgs', 20
lb: $(z_i(k), p_i(k))$	(0, 0)
ub: $(z_i(k), p_i(k))$	$(+\infty, 1)$
λ_0: $(z_i(k), p_i(k))$	(0.2, 0.2)

integer numbers, must be solved through integer programming algorithms. Since this study arises from the practical issues of minimising the metering cost, real-valued sample sizes are used during the optimisation. After the optimal solution λ^* is found, the *ceil* function is applied to obtain the integer sample sizes. Mathematically, the rounded sample sizes by the *ceil* function are only sub-optimal solutions. Henceforth, the terms "optimal/optimise" and "minimal/minimise" refer to the rounded sub-optimal solutions.

The studied hospital lighting retrofit project includes different lighting groups with different daily energy consumption uncertainties. In addition, these lighting groups exhibit different lamp life spans and population decay dynamics. The project characteristics strongly indicate the applicability of the combined spatial and longitudinal MCM model as discussed in Section 2. In order to fully reveal the superiority of the proposed model $C((19),(20))$, the optimal solutions obtained solely by the spatial MCM model in [35] and the longitudinal MCM model in [36] are also given in the following subsections for comparison purposes.

3.2. Benchmark and optimal solutions

In order to maximise the PDs' profits, the proposed MCM model $C((19),(20))$ will be applied to find the most suitable M&V metering plan for the hospital lighting retrofit project. As to demonstrate the advantages of the proposed combined spatial and longitudinal MCM model, the metering plan without optimisation is calculated as a benchmark for comparison purpose.

For the hospital lighting retrofit project, a possible solution without optimisation might be that the 90/10 criterion is applied to the sampling target in both lighting Groups I and II, where $\lambda_i(k) = (1.645_1(k), 1.645_2(k), 0.1_1(k), 0.1_2(k))$. The corresponding z scores, precisions, sample sizes and metering costs are calculated as shown in Table 3. It shows that the precision levels are better than 10% while the lowest z score is greater than 1.645 for each monitoring report. The total metering cost over the baseline and crediting period is R 1 115 732. In this scenario, the expected sampling accuracy is better than the required 90/10 criterion, which is not necessary.

Table 3: Metering cost without optimisation.

Year	$Z(k)$	$C(k)$	$P(k)$	$n_1(k)$	$n_2(k)$	Cost (R)
0	**1.9665**	**95.06%**	**9.90%**	**10**	**68**	**R 267 740**
1	1.8500	93.57%	9.89%	10	68	R 85 368
2	**2.6234**	**99.13%**	**9.89%**	**10**	**68**	**R 85 368**
3	3.2122	99.87%	9.90%	10	68	R 85 368
4	**3.6682**	**99.98%**	**9.90%**	**10**	**68**	**R 85 368**
5	3.9677	99.99%	9.90%	10	68	R 85 368
6	**4.1110**	**100%**	**9.90%**	**10**	**68**	**R 85 368**
7	4.1617	100%	9.90%	10	68	R 85 368
8	**4.1747**	**100%**	**9.90%**	**10**	**68**	**R 85 368**
9	4.1767	100%	9.90%	9	68	R 85 368
10	**4.1769**	**100%**	**9.90%**	**7**	**65**	**R 85 368**
Total	**n/a**	**n/a**	**n/a**	**10**	**68**	**R 1 115 732**

3.2.1. Spatial optimisation

The spatial MCM model in [35] aims to balance the sampling uncertainties across lighting groups by assigning optimal confidence and precision levels to the lighting groups with different energy consumption uncertainties. For this case study, the spatial optimisation model is formulated as follows. The design variable is $\lambda = (\lambda(0), \dots, \lambda(k), \dots, \lambda(K))$, where $\lambda(k) = (z_1(k), z_2(k), p_1(k), p_2(k))$, $k = 0, 1, \dots, K$. The objective function is given in Eq. (19), which is subject to the constraints

$$\begin{cases} z(k) \geq 1.645, \\ p(k) \leq 10\%. \end{cases} \tag{21}$$

The spatial optimisation model is denoted by $S((19),(21))$. The model $S((19),(21))$ is solved with the initial values given in Table 1 and the optimisation settings in Table 2. The obtained confidence levels, precision levels, and optimal sample sizes are shown in Figures 3-4. In addition, the numerical optimal solutions and metering cost are summarised in Table 4.

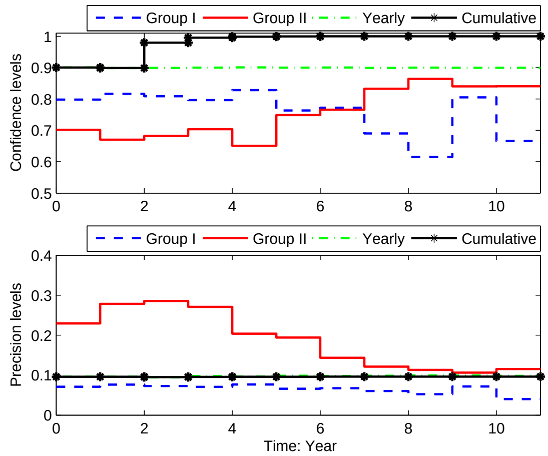

Figure 3: Confidence levels (spatial optimal only).

In Figures 3-4, the horizontal axes denote the counter of years. In Figure 3, optimal confidence and precision levels are presented, where the dashed line (in blue) and the solid line (in red) denote the confidence and precision levels for Groups I and II, respectively; the dash-dotted line (in green) denotes the combined confidence and precision levels across lighting groups over the kth year, while the starred line (in black) denotes the cumulative confidence and precision levels up to the kth year. As shown by the dash-dotted lines (in green) in both sub-figures of Figure 3, the constraints in Eq. (21) are satisfied.

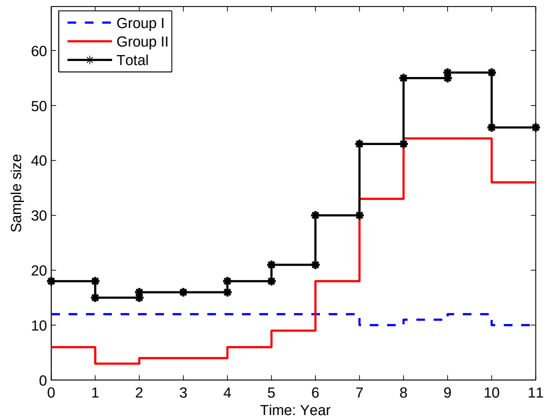

Figure 4: Sample sizes (spatial optimal only).

In Figure 4, the sample sizes in Group I and Group II are denoted by the dashed line (in blue) and the solid line (in red), respectively. The total sample sizes are denoted by the starred solid line (in black). It is observed that the sample sizes in Group I is greater than those in Group II during the years [0, 5) but becomes smaller than those in Group II during the years [6, 11). As discussed in [35], the sample sizes change when population changes. To achieve a certain level of sampling accuracy, greater number of samples are usually required for a bigger sampling population. However, it is worthy mentioning that in Year 5, $N_1(5) < N_2(5)$ but $n_1(5) > n_2(5)$ as shown in Figure 4. The reason is that the model $S((19),(21))$ attempts to use as many as less expensive meters in Group I in order to minimise the total metering cost.

In Table 4, $Z(k)$ is translated into the confidence levels $C(k)$. One may be surprised to see that more samples are required when population decays in Group II. This is because that Group II has a relatively greater population and a higher CV than those in Group I in the years [6, 10), which results in requiring greater sample sizes to satisfy the desired sampling accuracy.

Table 4: Metering cost with spatial optimisation.

Year	$Z(k)$	$C(k)$	$P(k)$	$n_1(k)$	$n_2(k)$	Cost (R)
0	**1.6452**	**90.01%**	**9.58%**	**12**	**6**	**R 37 032**
1	1.6448	90.00%	9.73%	12	3	R 10 008
2	**2.3185**	**97.96%**	**9.59%**	**12**	**4**	**R 11 184**
3	2.8278	99.53%	9.64%	12	4	R 11 184
4	**3.2124**	**99.87%**	**9.64%**	**12**	**6**	**R 13 536**
5	3.4659	99.95%	9.65%	12	9	R 27 462
6	**3.5883**	**99.97%**	**9.66%**	**12**	**18**	**R 58 842**
7	3.6331	99.97%	9.66%	10	33	R 96 198
8	**3.6447**	**99.97%**	**9.66%**	**11**	**44**	**R 95 810**
9	3.6464	99.97%	9.66%	12	44	R 58 224
10	**3.6466**	**99.97%**	**9.66%**	**10**	**36**	**R 47 736**
Total	**n/a**	**n/a**	**n/a**	**12**	**44**	**R 467 216**

3.2.2. Longitudinal optimisation

The longitudinal MCM model proposed in [36], with its improvements provided in [5], aims to balance the sampling uncertainties across adjacent reporting years by designing optimal confidence and precision levels in each reporting years. For this case study, the longitudinal optimisation model is formulated as follows. The design variable is $\lambda = (\lambda(0), \ldots, \lambda(k), \ldots, \lambda(K))$, where $\lambda(k) = (z_1(k), z_2(k), p_1(k), p_2(k))$, $k = 1, \ldots, K$. The objective function is given in Eq. (19) that is subject to the constraints

$$\begin{cases} z_i(0) \geq 1.645, \\ p_i(0) \leq 10\%, \\ Z_i(\delta) \geq 1.645, \\ P_i(\delta) \leq 10\%. \end{cases} \tag{22}$$

Assume $\overline{X}_i(k)$'s are independent over the years [0, K), then the $\overline{X}_i(k)$'s over the K years of the ith lighting group will follow a normal distribution $\bar{\chi}_i(k) \sim \mathcal{N}(\theta_i(k), \Gamma_i(k)^2)$, where

$$\bar{\chi}_i(K) = \frac{\sum_{k=1}^{K} N_i(k)\bar{x}_i(k)}{\sum_{k=1}^{K} N_i(k)}, \tag{23}$$

$$\theta_i(K) = \frac{\sum_{k=1}^{K} N_i(k)\mu_i(k)}{\sum_{k=1}^{K} N_i(k)}, \tag{24}$$

$$\Gamma_i(K)^2 = \sum_{k=1}^{K} \left(\frac{\sigma_i(k)N_i(k)}{\sum_{k=1}^{K} N_i(k)} \right)^2. \tag{25}$$

Let $Z_i(\delta)$ and $P_i(\delta)$ denote cumulative z score and the cumulative precision levels by end of the δth year of the ith lighting group, respectively, then

$$Z_i(\delta) = \frac{\bar{\chi}_i(\delta) - \theta_i(\delta)}{\Gamma_i(\delta)},$$

and

$$P_i(\delta) = \frac{\bar{\chi}_i(\delta) - \theta_i(\delta)}{\bar{\chi}_i(\delta)},$$

where $\bar{\chi}_i(\delta)$, $\theta_i(\delta)$, and $\Gamma_i(\delta)$ are calculated by Eqs. (23)-(25), respectively by substituting δ for the value K. The longitudinal optimisation model is denoted by $L((19),(22))$. The model $L((19),(22))$ is solved with the initial values given in Table 1 and the optimisation settings in Table 2. The obtained confidence levels, precision levels and optimal sample sizes are shown in Figures 5-6. In addition, the numerical optimal solutions and metering cost are summarised in Table 5.

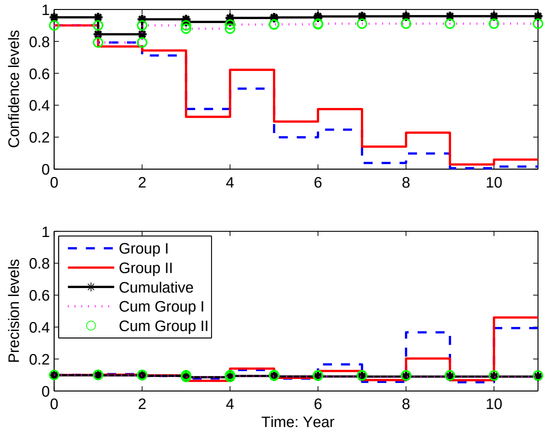

Figure 5: Confidence levels (longitudinal optimal only).

Figures 5-6 share the same presentation style as Figures 3-4 in terms of the horizontal and vertical axes. In Figure 5, optimal confidence and precision levels are presented, where the dashed line (in blue) and the solid line (in red) denote the confidence and precision levels for Groups I and II, respectively; the starred line (in black) denotes the cumulative confidence and precision levels up to the kth year; in addition, the dotted line (in purple) and the circle line (in green) denote the cumulative confidence and precision levels up to the kth year in the ith lighting group. As shown by the dotted lines (in purple) and the circle lines (in green) in both sub-figures of Figure 5, the constraints in Eq. (22) are satisfied.

In Figure 6, the sample sizes in Group I and Group II are denoted by the dashed line (in blue) and the solid line (in red), respectively. The total sample sizes are denoted by the starred solid line (in black). In Figure 6, the samples required during the baseline period are determined without optimisation. During the reporting period, the required sample sizes within every two years' reporting interval are very close. For instance, 35 and 34 samples are required in Years 1-2 while 11 and 10 samples are required in Years 3-4 in Group II. Similar sample size commitment pattern is also observed in Group I. The samples are optimally decided over the reporting period within lighting groups with the application of the model $L((19),(22))$. However, it is expected that the metering cost can be further minimised when spatial optimisation ideas can also be incorporated during both the baseline and reporting periods.

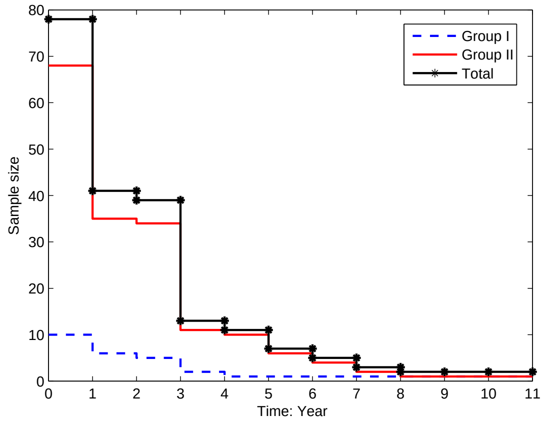

Figure 6: Samples sizes (longitudinal optimal only).

Table 5: Metering cost with longitudinal optimisation.

Year	$Z(k)$	$C(k)$	$P(k)$	$n_1(k)$	$n_2(k)$	Cost (R)
0	**1.9655**	**95.06%**	**9.90%**	**10**	**68**	**R 267 740**
1	1.4179	84.38%	9.80%	6	35	R 44 400
2	**1.8639**	**93.77%**	**9.48%**	**5**	**34**	**R 42 684**
3	1.7629	92.21%	8.67%	2	11	R 14 016
4	**1.9278**	**94.61%**	**9.41%**	**1**	**10**	**R 12 300**
5	1.9577	94.97%	9.09%	1	6	R 7 596
6	**2.0113**	**95.57%**	**9.06%**	**1**	**4**	**R 5 244**
7	2.0192	95.65%	9.00%	1	2	R 2 892
8	**2.0267**	**95.73%**	**9.01%**	**1**	**1**	**R 1 716**
9	2.0268	95.73%	9.01%	1	1	R 1 716
10	**2.0268**	**95.73%**	**9.01%**	**1**	**1**	**R 1 716**
Total	**n/a**	**n/a**	**n/a**	**10**	**68**	**R 402 020**

3.2.3. *Combined spatial and longitudinal optimisation*

In this subsection, the combined spatial and longitudinal MCM model $C((19), (20))$ is solved with the initial values given in Table 1 and the optimisation settings in Table 2. The obtained confidence levels, precision levels and optimal sample sizes are shown in Figures 7-8. In addition, the numerical optimal solutions and metering cost are summarised in Table 6.

Table 6: Metering cost with combined spatial and longitudinal optimisation.

Year	$Z(k)$	$C(k)$	$P(k)$	$n_1(k)$	$n_2(k)$	Cost (R)
0	**1.6585**	**90.28%**	**9.65%**	**14**	**5**	**R 35 684**
1	1.2148	77.55%	9.21%	7	2	R 6 132
2	**1.6727**	**90.56%**	**9.28%**	**6**	**2**	**R 5 592**
3	1.6680	90.47%	9.22%	2	1	R 2 256
4	**1.7252**	**91.55%**	**8.96%**	**2**	**1**	**R 2 256**
5	1.7276	91.59%	8.69%	1	1	R 1 716
6	**1.7540**	**92.06%**	**8.64%**	**1**	**1**	**R 1 716**
7	1.7529	92.04%	8.55%	1	1	R 1 716
8	**1.7549**	**92.07%**	**8.54%**	**1**	**1**	**R 1 716**
9	1.7550	92.07%	8.54%	1	1	R 1 716
10	**1.7550**	**92.07%**	**8.54%**	**1**	**1**	**R 1 716**
Total	**n/a**	**n/a**	**n/a**	**14**	**5**	**R 62 216**

In Figure 7, optimal confidence and precision levels are presented, where the dashed line (in blue) and the solid line (in red) denote the confidence and precision levels for Groups I and II, respectively; the starred line (in black) denotes the cumulative confidence and precision levels up to the kth year. As shown by the starred lines (in black) in both sub-figures of Figure 7, the constraints in Eq. (20) are satisfied.

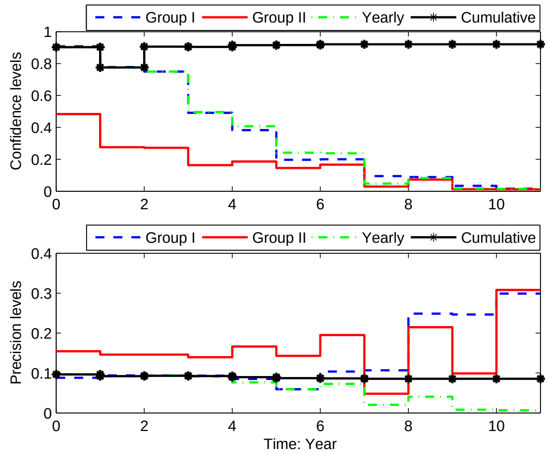

Figure 7: Confidence and precision levels (combined optimisation).

In Figure 8, the sample sizes in Group I and Group II are denoted by the dashed line (in blue) and the solid line (in red), respectively. The total sample sizes are denoted by the starred solid line (in black). As can be seen in Figure 8, the samples required during the baseline period are determined solely by spatial optimisation. In addition, as both the spatial and longitudinal MCM ideas are applied during the reporting period, the required sample sizes are optimised whereas the metering cost is significantly reduced.

4. Model performance comparison and discussion

In Section 3, four different M&V metering plans have been obtained for the same lighting retrofit project by the no optimisation approach, the spatial MCM approach, longitudinal MCM approach, and the combined spatial and longitudinal MCM approach, respectively. Detailed numeric solutions obtained from the four approaches are provided in Tables 3-6. In order to compare the performance among the four approaches, key information in terms of the sample sizes and the M&V metering cost are also presented graphically as shown in Figures 9-11. In Figures 9-10, the horizontal axis denotes the counter of years where Year 0 denotes the baseline period and Years 1-10 denote the crediting period. The vertical axis denotes the sample sizes. And in Figures 9-10, legend “Benchmark” (in red) denotes the sample sizes obtained without optimisation; legend “Spatial” (in green) denotes the sample sizes obtained by the spatial MCM model; legend “Longitudinal” (in purple) denotes the sample sizes obtained by the longitudinal MCM model; and legend “Combined” (in blue) denotes the sample sizes obtained by the combined spatial and longitudinal MCM model. Data labels are given in Figures 9-10 to denote the benchmark sample sizes. The M&V metering cost obtained by the four approaches are shown in Figure 11. Figure 11 has two vertical axes, in

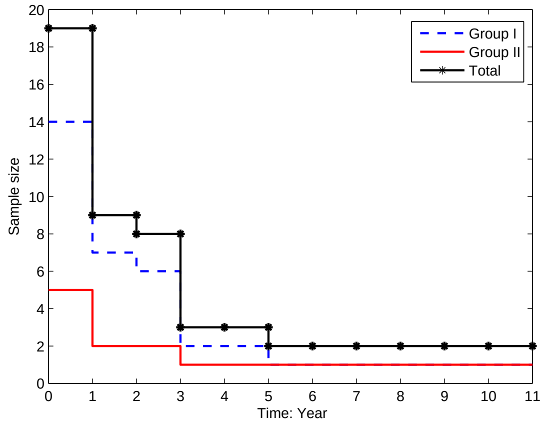

Figure 8: Sample sizes (combined optimisation).

which the primary axis denotes the M&V metering cost (in Rand), and the secondary axis denotes the percentage of cost savings against the benchmark, which is calculated as

$$Cost\ saving\ (\%) = \frac{Benchmark\ cost - Optimised\ cost}{Benchmark\ cost}.$$

More precisely, comparing to the benchmark, the spatial MCM model saves 58%, the longitudinal MCM model saves 64%, and combined spatial and longitudinal MCM model saves 94% of the metering cost that would have been spent without optimisation. Thus the model $C((19),(20))$ offers a minimal metering cost in terms of total metering cost of the hospital lighting project without violating the sampling accuracy requirements.

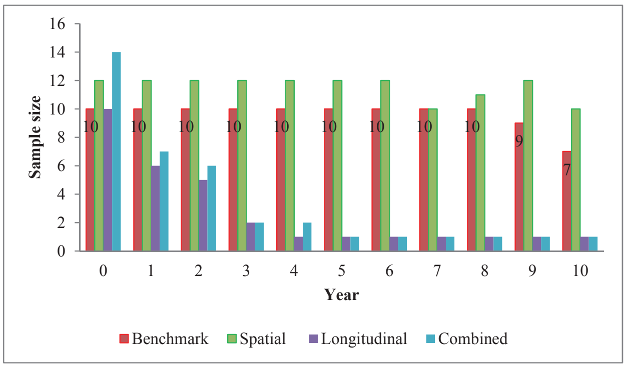

Figure 9: Sample size comparison in Group I.

The presented case study suggests that three MCM models $C((19),(20))$, $S((19),(21))$, and $L((19),(22))$ are all useful in designing the optimal M&V metering plans for lighting retrofit projects. When lighting projects have multiple homogeneous lighting groups with different sampling uncertainties, the spatial MCM model $S((19),(21))$ is most applicable when the lighting population is properly maintained to avoid lamp population decay. If no lighting maintenance activities are carried out, then the lamp population will decay as time goes by. In such a case, the longitudinal MCM model $L((19),(22))$ is most applicable to optimise the sample sizes within reporting intervals for each homogeneous lighting groups. Also learnt from the case study, the model $C((19),(20))$ exhibits the best performance in terms of metering cost minimisation whilst satisfying the required 90/10 criterion for each reporting interval.

In order to apply the model $C((19),(20))$ more flexibly, the lamp population decay dynamics for different homogeneous lighting groups need to be specifically identified by addressing the lamps' life spans, usage patterns, and technologies. In addition, if the lighting retrofit projects are sponsored under different EEDSM programmes, then performance reporting schedule δ in the model $C((19),(20))$ may be altered, which will result in different optimal sample size regimes. Moreover, it is likely that the M&V practitioners may need to design optimal M&V metering plans under different sampling accuracy requirements other than the 90/10 criterion. The optimal sample size regimes and relative metering cost are also calculated and provided in Table 7. It is obvious that better sampling accuracy requirement implies higher M&V metering cost over the project crediting period. For instance, requiring the 99/1 criterion for sampling implies higher metering cost than the 90/10, 85/5, and 95/5 criteria.

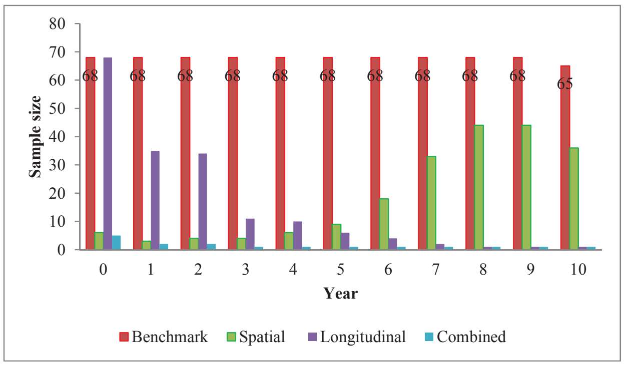

Figure 10: Sample size comparison in Group II.

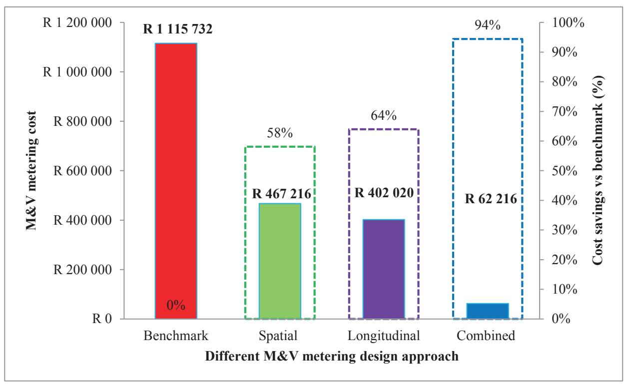

Figure 11: Metering cost comparison.

Table 7: Metering costs for different accuracy criteria.

Criteria	**85/5**			**95/5**			**99/1**		
Year	$n_1(k)$	$n_2(k)$	Cost (R)	$n_1(k)$	$n_2(k)$	Cost (R)	$n_1(k)$	$n_2(k)$	Cost (R)
0	41	14	R 102 086	76	26	R 189 416	3 268	1 080	R 8 002 008
1	19	5	R 16 140	35	9	R 29 484	1 488	362	R 1 229 232
2	17	5	R 15 060	31	9	R 27 324	1 348	349	R 1 138 344
3	7	2	R 6 132	11	4	R 10 644	454	138	R 407 448
4	5	2	R 5 052	8	3	R 7 848	313	125	R 316 020
5	2	2	R 3 432	4	3	R 5 688	160	106	R 211 056
6	1	1	R 1 716	2	2	R 3 432	55	73	R 115 548
7	1	1	R 1 716	1	1	R 1 716	14	33	R 46 368
8	1	1	R 1 716	1	1	R 1 716	1	12	R 14 652
9	1	1	R 1 716	1	1	R 1 716	1	5	R 6 420
10	1	1	R 1 716	1	1	R 1 716	1	2	R 2 892
Total	**41**	**14**	**R 156 482**	**76**	**26**	**R 280 700**	**3 268**	**1 080**	**R 11 489 988**

5. Conclusion

In this study, a combined spatial and longitudinal MCM model is proposed to assist the optimal M&V metering plan designs of the EE lighting retrofit projects. The proposed model is capable of designing optimal M&V metering plan for lighting projects that have multiple homogeneous lighting groups but with different lamp population decay dynamics across lighting groups. With the application of this model, the M&V metering cost is minimised by optimising the confidence and precision levels in different lighting groups over the projects' crediting period. As illustrated by the case study, the combined spatial and longitudinal MCM model is able to reduce 94% of the M&V metering cost that would have been spent under the no optimisation scenario, which exhibits better performance in terms of minimising the M&V metering cost under specific sampling accuracy requirements than both the spatial MCM and longitudinal MCM models. The proposed combined spatial and longitudinal MCM model can be flexibly applied to other similar lighting retrofit projects with different technologies, different project population variations, different reporting intervals, and different sampling accuracy requirements.

However, besides the advantages of the combined spatial and longitudinal MCM model, the proposed model has the following limitations: 1) this model focuses on handling sampling uncertainties cost-effectively in the M&V process, but pays less attention to the modelling and measurement uncertainties; 2) extra efforts are required to characterise the population decay dynamics when applying this model to design optimal M&V metering plans for other technologies.

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