Productive Government Spending and its Consequences for the Growth-Inequality Tradeoff*

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Abstract

This paper investigates the effects of productive government spending on the relationship between growth and inequality in an economy subject to idiosyncratic production shocks and heterogeneous endowments. Assuming lognormal distributions, we derive tractable closed form solutions describing the equilibrium dynamics. We show how the effect of government investment on the equilibrium dynamics of both inequality and growth depends crucially upon the elasticity of substitution between public and private capital in production. This has important consequences for the growth- and welfare-maximizing rates of government investment. Finally, we supplement our theoretical analysis with numerical simulations, calibrated to approximate the productive characteristics of a real world economy. With the empirical evidence strongly supporting the complementarity between public and private capital, our simulations suggest that conclusions based on the commonly employed Cobb-Douglas production function may be seriously misleading.

Key words: Government investment; Idiosyncratic shocks; Growth; Inequality
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1. Introduction

The role of public investment in infrastructure as a source of economic growth continues to be widely debated in both developing and advanced economies, with different economies adopting different policy options. Several emerging-market countries including India, China, and Brazil have undertaken extensive public investment, to which their high growth rates of recent years may at least in part be attributed. In contrast, several European countries have reduced public spending, as they have pursued austerity measures intended to deal with concerns related to their rising debt levels. Contemporaneously with these diverse policies toward public investment, we have witnessed increasing income inequality, both in emerging markets and in most OECD countries. This raises the important question that we address in this paper, namely the relationship between public investment directed toward growth enhancement and its consequences for income inequality.

Interest in the relationship between public investment, output, and growth has a long history, dating back to Arrow and Kurz (1970), who examined it in the context of a neoclassical economy. Beginning with Barro (1990), an extensive literature has evolved addressing the issue in an endogenous growth framework, with a general consensus that government spending on infrastructure can yield significant productivity gains and thus enhance growth. See e.g. Futagami et al. (1993), Glomm and Ravikumar (1994), Turnovsky (1997), and more recently Agénor (2011), who provides a detailed survey of the theoretical literature on this topic. The empirical literature is even more extensive. Most of it focuses on estimating the productive elasticity of government expenditure in producing output. The overwhelming consensus is that infrastructure contributes positively and significantly to output, though the productive elasticity is considerably smaller than Aschauer's (1989) original estimate of 0.39. Bom and Ligthart (2014) provide an exhaustive review of the literature and place the elasticity somewhere between 0.10 and 0.20, a range also shared by most other studies.[1]

By impacting factor productivity, and hence relative factor returns, public investment also plays a critical role in the evolution of wealth and income distributions as the economy grows over time. Indeed, infrastructure by virtue of its diverse nature is likely to have significant redistributive

[1] There is a much briefer literature analyzing the effect of infrastructure on the growth rate. While much of this is inconclusive, Sachez-Robles (1998) and Calderón and Servén (2004, 2010) obtain a substantial positive growth effect.

consequences, since depending on its type it will inevitably confer differential benefits across agents in the economy. Public investment in public transportation, directed toward the needs of the less affluent agents, is likely to reduce inequality, while public expenditure on enhanced communication, by favoring the wealthier owners of capital, will tend to exacerbate inequality. As our analysis highlights, the distributional consequences depend critically upon the substitutability-complementarity relationship between private and public capital in production.

In contrast to the public investment-growth relationship, empirical evidence on the relationship between infrastructure investment and inequality is less conclusive and more anecdotal. For example, Calderón and Chong (2004), Calderón and Servén (2004), Fan and Zhang (2004), Ferranti et al. (2004), and Lopez (2004) find that public investment has both promoted growth and helped mitigate inequality.[2] On the other hand, Brakman et al. (2002) find that government spending on infrastructure has increased regional disparities within Europe, while Artadi and Sala-i-Martin (2003) suggest excessive public investment has contributed to rising income inequality in Africa. In the case of India, Banerjee and Somanathan (2007) report that access to critical infrastructure services and public goods is in general positively correlated with social status, while a World Bank (2006) study also finds that the quality and performance of state-provided infrastructure services tend to be the worst in India's poorest states. The diversity of these empirical findings emphasizes the need for a well-specified analytical framework, within which the interaction between infrastructure spending, economic growth, and inequality can be systematically addressed.

This paper examines the distributional and growth effects of public investment within such a unified framework, where both growth and inequality are endogenously determined. In doing so, we address a key limitation of the existing literature. Most of the literature, both theoretical and empirical, assessing the productivity of public capital, employs a Cobb-Douglas production function. This is a serious shortcoming in light of the empirical evidence suggesting that the elasticity of substitution between public and private capital in aggregate output is most likely significantly less than unity, contrary to the Cobb-Douglas specification. Thus, since the elasticity of substitution turns out

[2] For example, using data for low and high income countries, between the period 1960 and 1997, Calderón and Chong (2004) find a negative and significant relationship between infrastructure measures (such as roads, railways, telecommunications, and energy) and income inequality.

to be a key mechanism whereby public investment influences the distribution of income and growth, it becomes even more important that the production function be generalized beyond the restrictive Cobb-Douglas form.

We develop an overlapping generations model in which individuals are subject to two sources of heterogeneity: (i) their initial endowments of private capital, and (ii) idiosyncratic productivity shocks. To examine the potential impact of the substitutability-complementarity relationship between public and private capital on the growth-inequality relationship, we assume that these two factors contribute to final output via a constant elasticity of substitution (CES) production function. By assuming that the technology has constant returns to scale in public and private capital, the macroeconomic equilibrium we generate is one of endogenous growth.

We assume that the two sources of heterogeneity can be represented by lognormal distributions. This assumption facilitates aggregation, enabling us to derive the joint distributional and aggregate dynamics of the CES economy in a very tractable closed form, thereby providing substantial insight into its evolution.[3] The equilibrium dynamics has a simple recursive structure. The dynamics of inequality drives the growth of the aggregate variables – private capital, public capital, and output – but not vice versa.

There are no credit or insurance markets, the absence of which is a key element generating inequality, as in Loury (1981), and Bénabou (2000, 2002). When individuals cannot fully insure themselves from future income uncertainty and are unable to borrow or lend unlimited amounts, inequality associated with the idiosyncratic productivity shocks persists.[4] This is because diminishing returns to investment imply that the poorly endowed can obtain a higher marginal return to their capital than do the wealthy. But, since they cannot borrow and invest efficiently due to credit constraints, Pareto efficiency cannot be achieved, as often is implicitly assumed in representative-agent models with complete markets. Consequently, inequality persists, leading to less efficient resource allocation and slower growth.

However, if redistributive public investment policies can mitigate the equity-efficiency trade-

[3] Some empirical support for the lognormal is that for several economies it describes quite well the distribution of income below the top 3%; see Clementi and Gallegati (2005).

[4] In contrast, the inequality originating with the initial endowments gradually disappears.

off they could potentially yield additional efficiency gains. To the extent that this is so depends upon the degree of substitutability between public and private capital. This is yet another reason why it is so important to generalize the production technology to the CES function, rather than restrict it to the Cobb-Douglas function as so much of the relevant growth literature does.

We show that an increase in public investment will decrease or increase inequality, both in the short run and over time, according to whether the elasticity of substitution between public and private capital is greater than, or less than, unity. The underlying intuition is straightforward. Public capital that is highly substitutable for private capital provides poor households with the opportunity to replace their restricted capacity to investment, due to their inability to borrow, by engaging in factor substitution. This leads to a more equitable distribution of wealth and a subsequent positive impact on productive efficiency. In contrast, public investment highly complementary to private capital will exacerbate inequality and have an adverse effect on productive efficiency. This is because public investment will now disproportionately favor the rich who, because they own much of the private capital in the economy, obtain lower marginal returns on their investments. The net efficiency gains obtained from enhancing the marginal productivity of aggregate private capital, while aggravating inequality in this latter case, nevertheless still most likely remain positive.

Thus, investment in infrastructure will impact the growth rate through two channels. The first is the familiar direct productivity effect, emphasized by Barro (1990). But, in addition, it will have an indirect impact through its effect on inequality. The presence of the second channel influences the choice of optimal (both growth-maximizing and welfare-maximizing) tax, which now also depends on the degree of wealth heterogeneity as well as the elasticity of substitution between the factors. As a result, the optimal tax rate is higher in societies with more inequality if the elasticity of substitution exceeds unity. This is because in these circumstances, in addition to enhancing the productivity of private capital, public investment also serves to offset the inefficiency arising from inequality.

This paper is related to several bodies of literature dealing with issues pertaining to public capital, economic growth, and income distribution. First, is the literature on public capital and endogenous growth, some of which is referred to earlier. Second, are papers studying distribution and growth with imperfect credit markets, (e.g., Loury, 1981, Galor and Zeira, 1993, Aghion and Bolton, 1997, Aghion, et al., 1999, Bénabou, 1996, 2000, 2002, and Seshadri and Yuki, 2004).

The paper is also related to the recent, but sparser, literature on the relationship between public expenditure and the distributions of wealth and income. In particular, Chatterjee and Turnovsky (2012) employ a CES production function to examine the impact of productivity-enhancing fiscal policies on inequality. This analysis, following Caselli and Ventura (2000), assumes the existence of perfect factor markets and unlimited borrowing, the effect of which is to fundamentally reverse the structural dynamics from that developed here. Specifically, under those assumptions, perfect aggregation across agents as in Gorman (1953) is straightforward, and the macroeconomic (aggregate) equilibrium is determined independently of any distribution characteristics. Instead, the factor returns generated by the aggregate equilibrium now determine the evolution of wealth and income inequality. In addition, these models treat the initial capital endowments as the primary source of inequality, and show further that, in conjunction with the evolution of factor returns, they remain a key determinant of long-run inequality. These two aspects – the dynamic structure and the long-run relevance of the heterogeneous endowments – are fundamentally different in these two classes of models. The contrasting conclusions derived from these alternative approaches underscore the fact that the growth-inequality relationship is a complex one, and confirms the need to study it from alternative viewpoints.

By focusing on uninsurable idiosyncratic shocks as the key underlying source of persistent inequality, the paper is also related to the approach pursued by Krusell and Smith (1998) and others.[5] However, the complexity of their framework precludes analytical solutions, and hence the analysis is entirely based on numerical computations. In addition, the specific nature of the stochastic structure and the focus of the analyses are very different, especially since that literature does not address issues pertaining to public policy. In this last respect the paper is related to Getachew (2010, 2012) who considers public investment and inequality in endogenous growth models and imperfect credit markets, although that work is highly restrictive. First, it does not consider idiosyncratic uninsurable risk, the key element generating the non-degenerate wealth distribution. Instead, the only source of heterogeneity it introduces is initial endowments, the effects of which on inequality are only transitory. Also, by assuming Cobb-Douglas production functions, it does not address the role of the

[5] See, for example, Castañeda, Díaz-Giménez, and Rios-Rull (1998), Díaz, Pijoan-Mas, and Rios-Rull (2003).

substitutability-complementarity relationship between public and private capital, which is a central element of the present analysis.[6]

Finally, one of the key contributions of the present paper is that to our knowledge it is the first study to derive a closed form solution for the equilibrium growth and distributional dynamics in a situation where heterogeneity is generated by idiosyncratic shocks and a more general production function, such as the CES, is employed. But despite being able to conduct most of our analysis formally, we also carry out numerical simulations, in part to substantiate its empirical relevance. To do so requires reliable estimates of the elasticity of substitution between private and public capital, on which empirical evidence is sparse. Thus, because this is such a crucial parameter we undertake a detailed calibration, by relating it to the widely available empirical estimates of the productive elasticity of public capital. We supplement this by also obtaining our own empirical estimates, based on a cross section of African countries. The upshot of this is that, at the aggregate level, the evidence suggests that public and private capital are complements, which we therefore take as the empirically more relevant case. As a consequence, our simulations extending over a wide range of elasticities of substitution suggest that computations based on the Cobb-Douglas production function may seriously understate the growth and welfare consequences of government investment policy.

The rest of the paper is organized as follows. Section 2 develops the analytical framework. Section 3 examines the dynamics and equilibrium of the model. Section 4 analyzes the effects of government investment on the short-run and long-run evolution of growth and inequality. Section 5 describes the calibration employed to estimate the elasticity of substitution beween public and private capital as well as a brief summary of the new empirical estimates obtained. Section 6 complements the theoretical analysis by carrying out extensive numerical simulations. In conducting these, we pay particular attention to relating them to the relevant empirical evidence, and find that despite its parsimony the model performs well in this dimenstion. Section 7 concludes, while technical details are provided in the Appendix.

[6] There is also a vast literature that studies the relationship between public education and income distribution, to which this paper may also be related to by virtue of the common interest of examining the distributional effect of a productive public good in growth models, (e.g., Glomm and Ravikumar, 1992, 2003, Saint-Paul and Verdier, 1993, and Eckstein and Zilcha, 1994). Finally, it is also related to studies of the impact of the elasticity of substitution (between capital and labor) on growth; see e.g. Klump and de la Grandville (2000), Miyagiwa and Papageorgiou (2003).

2. The analytical framework

The analytical framework we employ is that of an overlapping generations economy populated by a continuum of heterogeneous households.

2.1 Preferences and technology

At any point in time, each household, indexed by $i \in [0,1]$, consists of a young – the offspring – and an adult – the parent. Population size thus remains constant over time. Each parent of the initial generation (at t=0) is endowed with private capital, k_0^i, a unit of time, and has access to public infrastructure, g_0, which is a pure public good equally available to all. The distribution of wealth assumes a known (given) initial probability distribution but evolves endogenously over time.

Individuals live for two periods. In the first period, when agents are young, they acquire capital from their parents, while their consumption is included in that of their parents. They do not take any economic decisions. Rather, all such decisions are made during the second period, when children have become adults. Parents earn income by supplying capital and labor to privately-held firms, which produce output, by combining these factors with publicly provided capital.[7] The government taxes income at a fixed flat rate in order to finance the public investment good that is used in final goods production. Agents allocate their after-tax income between consumption and saving, with the accumulated capital at the end of the second period being endowed to their offspring.

The ith household's utility function is specified by the logarithmic preference function,

$$W_t^i \equiv \ln c_t^i + \eta \ln\left(1 - l_t^i\right) + \beta \ln k_{t+1}^i \tag{1}$$

where c_t^i denotes its consumption during period t, l_t^i denotes its labor supply, and k_{t+1}^i is the amount of capital it leaves to its offspring for productive use in period (t+1). The coefficient β can be viewed as reflecting two elements: (i) the tradeoff in utility between its own consumption vs. bequests, reflecting the "joy of giving" and (ii) some intertemporal discounting due to the fact that the bequests

[7] This type of individual entrepreneurship is common in models with incomplete markets (see, for e.g., Loury, 1981, Bénabou, 2000, 2002 and Angeletos and Calvet, 2005, 2006).

are to be enjoyed by the next generation.[8]

Households choose c_t^i, s_t^i, and l_t^i to maximize the utility function, (1), subject to their budget constraint and capital accumulation relationship[9]

$$c_t^i + s_t^i = (1-\tau)y_t^i \tag{2a}$$

$$y_t^i = a\xi_t^i\left((1-\alpha)\left(k_t^i\right)^\rho + \alpha\left(g_t\right)^\rho\right)^{\frac{1-\theta}{\rho}}\left(l_t^i k_t\right)^\theta \qquad 0<\alpha<1,\ a>0 \tag{2b}$$

$$k_{t+1}^i = s_t^i \tag{2c}$$

where s_t^i denotes the individual household's saving, and y_t^i is its income, which is taxed at the flat rate, τ. Output is produced by the two-level production function specified in equation (2b). At the first level, the individual's private capital and public capital, g_t, are combined in accordance with the CES production function, with elasticity of substitution being $\varepsilon \equiv 1/(1-\rho)$. We assume that the two capital goods are cooperative in production, meaning that $\partial y_i/\partial k_t \partial g_t > 0$, which implies

$$\rho' \equiv \frac{\rho}{1-\theta} < 1 \tag{2d}$$

and imposes an upper bound on the degree of substitutability, $\varepsilon < 1/\theta$.[10] This intermediate output is then combined with labor in accordance with a Cobb-Douglas technology to produce final output. Labor productivity is augmented by the aggregate capital stock, k_t, in accordance with the Romer (1986) technology, thereby rendering (2b) constant returns to scale in k_t^i, k_t, g_t and generating an equilibrium of ongoing growth. The exponent θ reflects the productivity of labor in final output, while α reflects the importance of public capital in the intermediate output good.

Production by the *i*th agent (firm) is subject to an idiosyncratic productivity shock, ξ_t^i, assumed to be i.i.d. and lognormal with mean one, so that $\ln \xi_t^i \sim N(-\upsilon^2/2, \upsilon^2)$. The initial distribution of capital is also assumed to be lognormal, $\ln k_0^i \sim N(\mu_0, \sigma_0^2)$. Finally, for analytical

[8] As discussed at length by Galor and Zeira (1993), the general results and insights are robust with respect to variations in the specification of this class of utility function.
[9] The assumption of a unitary inter-temporal elasticity of substitution utility function of altruistic agents with a "joy of giving" motive is widely used in the literature of income distribution dynamics (see, for instance, Glomm and Ravikumar, 1992, Galor and Zeira, 1993, Saint-Paul and Verdier, 1993, and Bénabou, 2000).
[10] Setting $\theta \approx 2/3$ implies $\varepsilon < 1.5$, which the empirical evidence discussed in Section 5 suggests is not a severe restriction.

convenience equation (2c) asserts that private capital fully depreciates within the period, so that the capital stock accumulated by household *i* coincides with its savings.

There are no markets for insurance or credit. Though these are extreme forms of market incompleteness, adopted to maximize tractability and transparency, what matters is the existence of some forms of incompleteness; see, e.g., Aghion et al. (1999), Bénabou (2000 and 2002). In this respect we view our analysis as being particularly relevant for developing economies.[11]

The government provides inputs for goods production, and collects proportional taxes on marketed output. Its budget is always balanced. Public capital also completely depreciates within the period, so that its accumulation takes the form,

$$g_{t+1} = \tau \int_0^1 y_t^i di \equiv \tau y_t \tag{3}$$

where y_t is aggregate income. In treating public capital as a flow, we are following the original Barro (1990) approach, rather than some of the subsequent literature which treats it as a stock.[12]

Summing (2a) and (2c) over all the households, aggregate consumption and the private capital flow are given by, respectively

$$c_t = (1-\tau)y_t - s_t \tag{4}$$

and

$$k_{t+1} = s_t \tag{5}$$

where $c_t = \int_0^1 c_t^i di$, $s_t = \int_0^1 s_t^i di$, $k_t = \int_0^1 k_t^i di$.

2.2 Individual optimal capital accumulation

Performing the optimization set out in (1) and (2) yields household *i*'s first order conditions

$$s_t^i = \chi(1-\tau)y_t^i \tag{6a}$$

[11] There is mounting evidence that credit markets are imperfect in most of the developing countries (see, for e.g., Mel et al., 2008, McKenzie and Woodruff, 2006 and Banerjee and Duflo, 2004). But even in the developed world, the cost of administrating credit is substantial suggesting the presence of credit market frictions in these countries as well.

[12] See e.g. Futagami et al; (1993), Turnovsky (1997). The model can be extended to introduce public capital as a gradually accumulating stock, but little insight is lost by adopting the simpler flow specification.

$$l_t^i = \frac{\theta}{\eta(1-\chi)+\theta} \equiv l \tag{6b}$$

where $\chi \equiv \beta/(1+\beta)$. Eq. (6a) describes the *i*th household optimal saving allocated to capital accumulation, which for the logarithmic preference function simplifies to saving being a constant fraction of the agent's after tax income, and is independent of the rate of return to investment.[13] Equation (6b) determines the household's allocation of time to labor, which is similarly constant across agents and over time.[14] Substituting for s_t^i, y_t^i, and l_t^i from (2b), (2c), and (6b) yields the following relationship describing the dynamics of capital accumulation for the *i*th individual

$$k_{t+1}^i = (1-\tau)a'\chi\xi_t^i\left((1-\alpha)\left(k_t^i\right)^{\rho} + \alpha\left(g_t\right)^{\rho}\right)^{\frac{1-\theta}{\rho}}\left(k_t\right)^{\theta} \tag{7}$$

where $$a' \equiv a\left(\frac{\theta}{\eta(1-\chi)+\theta}\right)^{\theta}$$

Thus, capital accumulation of the *i*th offspring at time $t+1$ is a function of the parent's capital (k_t^i), the level of public capital in the previous period (g_t), the stock of aggregate private capital in the previous period (k_t), and the idiosyncratic productivity shock (ξ_t^i). With ξ_t^i being i.i.d., (7) implies that ξ_t^i and k_t^i are uncorrelated.

3. Infrastructure, inequality and aggregate capital dynamics

In this section we characterize the aggregate dynamics and steady-state equilibrium. Of particular interest are the effects of infrastructure investment on the transitional dynamics of inequality, as measured by the variance of $\ln(k_t^i)$ and denoted by σ_t^2, and the growth rate, denoted by γ_t.[15] An appealing property of the lognormal distribution is that it facilitates aggregation, details of which are provided in the Appendix. There we show that the macroeconomic equilibrium is

[13] As is well known, when the inter-temporal elasticity of substitution is unity, the income effect exactly offsets the substitution effect.
[14] This contrasts sharply with the optimality conditions obtained in the heterogeneous agent models based on the infinitely-lived representative agent, where the differential allocation of time across agents is the crucial determinant of the equilibrium distribution of income.
[15] There are many measures of inequality having varying qualitative attributes; see Atkinson (1970). Our choice of σ_t^2 is essentially the squared coefficient of variation, and is convenient, given the assumed underlying lognormal distribution. Although it places more weight on extreme observations than does the coefficient of variation, σ_t, its qualitative implications are the same and it can be easily converted to the latter if one so chooses.

summarized by the following relationships, where suppression of the index *i*, identifies aggregates:[16]

(i) Dynamics of inequality:

$$\sigma_{t+1}^2 = \upsilon^2 + \frac{1}{\rho'^2}\ln\left[1+\left(\frac{z_t}{1+z_t}\right)^2\left(e^{\rho^2\sigma_t^2}-1\right)\right] \tag{8a}$$

where
$$z_t \equiv \frac{1-\alpha}{\alpha}\phi^{\rho}e^{\rho(\rho-1)\sigma_t^2/2} \tag{8b}$$

$$\phi \equiv \frac{k_t}{g_t} = \chi\left(\frac{1-\tau}{\tau}\right) \tag{8c}$$

(ii) Dynamics of aggregate private capital accumulation:

$$\begin{aligned}\ln k_{t+1} = {} & \ln(a'\chi^{1+\theta}\alpha^{1/\rho'}) + (1+\theta)\ln(1-\tau) - \theta\ln\tau + \ln g_t + \frac{1}{\rho'}\ln(1+z_t) \\ & + \frac{1-\rho'}{2\rho'^2}\ln\left[1+\left(\frac{z_t}{1+z_t}\right)^2\left(e^{\rho^2\sigma_t^2}-1\right)\right]\end{aligned} \tag{9}$$

(iii) Dynamics of public capital accumulation:

$$\begin{aligned}\ln g_{t+1} = {} & \ln(a'\chi^{\theta}\alpha^{1/\rho'}) + \theta\ln(1-\tau) + (1-\theta)\ln\tau + \ln g_t + \frac{1}{\rho'}\ln(1+z_t) \\ & + \frac{1-\rho'}{2\rho'^2}\ln\left[1+\left(\frac{z_t}{1+z_t}\right)^2\left(e^{\rho^2\sigma_t^2}-1\right)\right]\end{aligned} \tag{10}$$

(iv) Dynamics of aggregate output:

$$\begin{aligned}\ln y_t = {} & \ln(a'\chi^{\theta}\alpha^{1/\rho'}) + \theta\ln(1-\tau) - \theta\ln\tau + \ln g_t + \frac{1}{\rho'}\ln(1+z_t) \\ & + \frac{1-\rho'}{2\rho'^2}\ln\left[1+\left(\frac{z_t}{1+z_t}\right)^2\left(e^{\rho^2\sigma_t^2}-1\right)\right]\end{aligned} \tag{11}$$

[16] We should note that by virtue of (6a), measures of wealth inequality and income inequality coincide (though the latter lags by one period) and hence we just refer to "inequality".

3.1 Transitional dynamics

A number of observations follow from these equations. First, the constancy of the ratio of private to public capital, asserted in (8c), follows directly from equations (9) and (10). In addition, equations (10) and (11) reflect the constancy of the ratio of public capital to lagged output, as specified in (3). Thus (10) implies that during the period (*t*, *t*+1) private and public capital grow at the same rate

$$\begin{aligned}\gamma_{t+1}^{k} \equiv \ln g_{t+1} - \ln g_{t} = \ln(a'\chi^{\theta}\alpha^{1/\rho'}) + \theta\ln(1-\tau) + (1-\theta)\ln\tau + \frac{1}{\rho'}\ln(1+z_{t}) \\ + \frac{1-\rho'}{2\rho'^{2}}\ln\left[1+\left(\frac{z_{t}}{1+z_{t}}\right)^{2}\left(e^{\rho^{2}\sigma_{t}^{2}}-1\right)\right]\end{aligned} \tag{12a}$$

while during the same period output and consumption both grow at the common rate[17]

$$\begin{aligned}\gamma_{t+1}^{y} \equiv \ln y_{t+1} - \ln y_{t} = \ln(a'\chi^{\theta}\alpha^{1/\rho'}) + \theta\ln(1-\tau) + (1-\theta)\ln\tau + \frac{1}{\rho'}\ln(1+z_{t+1}) \\ + \frac{1-\rho'}{2\rho'^{2}}\ln\left[1+\left(\frac{z_{t+1}}{1+z_{t+1}}\right)^{2}\left(e^{\rho^{2}\sigma_{t+1}^{2}}-1\right)\right]\end{aligned} \tag{12b}$$

From (12a) and (12b) we see $\gamma_{t+1}^{k} = \gamma_{t}^{y}$; i.e. the growth of output leads that of capital by one period.[18]

From equations (8)-(12) we see that the dynamics of the economy are driven entirely by inequality, which operates through two channels. The first is through its impact on the average ratio of private to public capital employed in production, reflected in z_t, while the second is the direct systematic stochastic component of output as determined by $\text{var}(\ln\xi_t^i)$ in the aggregation of the production function, (2b). Starting from an initial inequality endowment σ_0^2, (8a)-(8c) generate dynamic time paths for σ_t^2 and z_t These solutions for σ_t^2 and z_t then feed into the remaining equations to generate the time paths for k_t, g_t, y_t and their growth rates, with k_t being subject to an initial condition, k_0, obtained by aggregating over k_0^i.

Comparing equations (8) and (12), we see that there is a contemporaneous relationship

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[17] To see that consumption and output grow at the same rate substitute (6a) into (2a) and aggregate, from which it immediately follows that $c_{t+1}/y_{t+1} = (1-\tau)/(1+\beta)$ implying a common growth rate.

[18] This is a consequence of the lag introduced in eq. (3); it is not particularly consequential.

between cross-sectional inequality and output growth, while inequality impacts the growth of capital similarly, but with a one period lag. The influence of inequality on growth reflects two factors – credit market imperfections and diminishing returns to individual investment. The inability to borrow implies that productive investment opportunities may be foregone. With diminishing returns to investment, the poor have a higher marginal product than do the rich. Therefore, greater inequality is associated with a loss of productive efficiency, leading to lower growth.

This causality contrasts sharply with that obtained in the class of inequality-growth models developed by Caselli and Ventura (2000), and Turnovsky and García-Peñalosa (2008), for example, where their underlying assumptions permit exact aggregation as pioneered by Gorman (1953).[19] Under those assumptions, the macroeconomic equilibrium is determined independently of the distribution across agents, while the distribution is then determined by returns to capital and labor generated by the aggregates, causing Caselli and Ventura to characterize this as a "representative agent theory of distribution".

3.2 Steady state

Assuming that the system is stable, the steady-state equilibrium wealth inequality, $\tilde{\sigma}^2$, and growth rate, $\tilde{\gamma}$, are determined jointly (together with $\tilde{z}$) by

$$\left(\frac{\tilde{z}}{1+\tilde{z}}\right)^2\left(e^{\rho^2\tilde{\sigma}^2}-1\right)=e^{\rho'^2(\tilde{\sigma}^2-\upsilon^2)}-1 \tag{13a}$$

$$\tilde{\gamma}=\ln(a'\chi^{\theta}\alpha^{1/\rho'})+\theta\ln(1-\tau)+(1-\theta)\ln\tau+\frac{1}{\rho'}\ln(1+\tilde{z})+\frac{1-\rho'}{2}\left(\tilde{\sigma}^2-\upsilon^2\right) \tag{13b}$$

$$\tilde{z}\equiv\frac{1-\alpha}{\alpha}\phi^{\rho}e^{\rho(\rho-1)\tilde{\sigma}^2/2} \tag{13c}$$

From these conditions inequality, $\tilde{\sigma}$, is seen to impact the long-run growth rate through two channels. The first channel is direct, reflecting the basic property of the lognormal distribution that expected value is an increasing function of its variance; see (13b). The second channel occurs via the

[19] The key assumptions are homogeneity of the underlying utility functions and perfect factor markets, in which all agents earn the same rates of return.

adjustment in the public to private capital ratio as reflected in $\tilde{z}$, and its direction depends upon whether the elasticity of substitution $\varepsilon \gtrless 1$.

If $\upsilon^2 = 0$, so that there are no idiosyncratic productivity shocks, (13a) implies $\tilde{\sigma}^2 = 0$, so that steady-state inequality is zero. In the absence of such shocks, the long-run impact of initial wealth inequality disappears, and the long-run distributions of wealth and income degenerate, as relatively resource poor individuals rapidly accumulate wealth due to their relatively high marginal productivity, which in turn is due to the presence of diminishing returns to investment. In contrast, if $\upsilon^2 > 0$, so that the economy is subject to idiosyncratic productivity shocks of constant variance, then the long run equilibrium is characterized by non-degenerate wealth and income inequality that exceeds υ^2. In the presence of such uninsurable idiosyncratic risks, individuals may be unable to overcome both their differences in luck as well as some of their initial differences in endowment. This in turn will impact the equilibrium growth rate. The fact that $\tilde{\sigma}^2 \geq \upsilon^2$ is immediately seen by writing (13a) in the form

$$\tilde{\sigma}^2 = \upsilon^2 + \frac{1}{\rho'^2}\ln\left[1+\left(\frac{\tilde{z}}{1+\tilde{z}}\right)^2\left(e^{\rho^2\tilde{\sigma}^2}-1\right)\right] \geq \upsilon^2 \qquad \text{(13a')}$$

To determine the consequences of the idiosyncratic productivity shocks for the long-run inequality-growth relationship we differentiate equations, (13a) - (13c), with respect to υ^2 to obtain:

$$\frac{d\tilde{\sigma}^2}{d\upsilon^2} = \frac{1}{1-D} \qquad \text{(14a)}$$

$$\frac{d\tilde{\gamma}}{d\upsilon^2} = \frac{1}{2(1-D)}\left[(1-\theta)(\rho-1)\left(\frac{\tilde{z}}{1+\tilde{z}}\right)+(1-\rho')D\right] \qquad \text{(14b)}$$

where $$D \equiv \frac{1}{\rho'^2}\left(\frac{\tilde{z}}{1+\tilde{z}}\right)^2\left[\frac{1}{1+\left(\tilde{z}/1+\tilde{z}\right)^2\left(e^{\rho^2\tilde{\sigma}^2}-1\right)}\right]\left[\frac{\rho(\rho-1)}{(1+\tilde{z})}\left(e^{\rho^2\tilde{\sigma}^2}-1\right)+\rho^2 e^{\rho^2\tilde{\sigma}^2}\right] > 0 \qquad \text{(14c)}$$

It is straightforward to show $D > 0$, in which case, as we will see in Section 3.3, $D < 1$ is a necessary and sufficient condition for the aggregate economy to be locally stable. In fact, for any plausible parameterization we obtain $0 < D < 1$, in which case $d\tilde{\sigma}_t^2 / d\upsilon^2 > 1$. This will be associated with a negative effect on growth if and only if this condition is strengthened to $D < \left[\tilde{z}/(1+\tilde{z})\right](1-\theta)$, a weak condition that we shall henceforth impose. Moreover, as shown in (15') below, D is closely approximated by $D \approx (1-\theta)^2 \left(\tilde{z}/(1+\tilde{z})\right)^2$, clearly meeting this constraint, and invoking this approximation leads to:

$$\frac{d\tilde{\sigma}^2}{d\upsilon^2} \approx \frac{(1+\tilde{z})^2}{[1+(2-\theta)\tilde{z}][1+\theta\tilde{z}]} > 1 \tag{14a'}$$

$$\frac{d\tilde{\gamma}}{d\upsilon^2} \approx -\frac{(1-\theta)\tilde{z}}{2(1+\tilde{z})^2}\left[1-\rho+\theta\tilde{z}\right]\frac{d\tilde{\sigma}^2}{d\upsilon^2} < 0 \tag{14b'}$$

Thus we may summarize these responses in the following:

> **Proposition 1: (i)** An increase in the variance of the idiosyncratic productivity shocks will increase inequality and will be associated with a lower growth rate, both in the short run and in steady state.
>
> **(ii)** An increase in the inequality in initial endowments will increase inequality and reduce the growth rate temporarily, declining over time, and vanishing in the long run.

It is instructive to relate the nature of the long-run distribution of wealth obtained here with that obtained using other approaches. First, Li and Sarte (2004) consider an economy in which agents have heterogeneous initial endowments of capital. They then show that if agents are subject to progressive taxes but have a common rate of time discount, the long-run wealth distribution will degenerate, precisely as (13a) implies in the absence of productivity shocks. But they also show that the long-run wealth distribution will not degenerate if agents have different rates of time preference.[20] That too turns out to be the case here, as one can easily show for the Cobb-Douglas technology, assumed by Li and Sarte. In effect, the assumption of incomplete financial markets being adopted here is playing the role of the progressive tax structure in their analysis. Another strand of literature, initially developed by Becker and Tomes (1979), Krusell and Smith (1998) and others, shows if agents are initially identical but are subject to idiosyncratic shocks, then this will lead to a long-run non-

[20] Following this general approach, Carroll and Young (2009) show that the assumptions of: (i) complete markets, (ii) progressive taxation, and (iii) heterogenous labor productivity can also produce a nondegernerate wealth distribution.

degenerate wealth distribution, just as we obtain here. A third body of literature, which assumes that the only source of heterogeneity is due to the initial endowments of capital, obtains non-degenerate long-run distributions of wealth and income that are directly tied to the initial distribution. This hysteresis arises because of the assumption of complete financial markets that these models also assume, making the distributional dynamics be path-dependent; see e.g. Caselli and Ventura (2000), and Atolia, Chatterjee, and Turnovsky (2012). These differences highlight how the assumptions pertaining to the presence or absence of financial markets play a crucial role in assessing the causality and nature of the growth-inequality tradeoff.

3.3 Local stability

To determine the local stability of the dynamics we evaluate $d\sigma_{t+1}^2 / d\sigma_t^2$ in the neighborhood of the steady state equilibrium, using (8a) and (8b), to obtain:

$$\frac{d\sigma_{t+1}^2}{d\sigma_t^2} = D \tag{15}$$

where D is defined in (14c) and for local stability $D < 1$. In the absence of production risk, when $\tilde{\sigma}^2 = 0$, $D = \left((1-\theta)\tilde{z}/(1+\tilde{z})\right)^2$ and (15) simplifies to

$$\frac{d\sigma_{t+1}^2}{d\sigma_t^2} = \left(\frac{(1-\theta)\tilde{z}}{1+\tilde{z}}\right)^2 \tag{15'}$$

and is clearly locally stable. Moreover, in the case that: (i) the production function is sufficiently close to Cobb-Douglas, and (ii) the idiosyncratic production risk is sufficiently small so that $e^{\rho^2\sigma_t^2} \approx 1 + \rho^2\sigma_t^2$, then (8a) can be approximated by[21]

$$\sigma_{t+1}^2 = \upsilon^2 + \left(\frac{(1-\theta)z_t}{1+z_t}\right)^2 \sigma_t^2 \tag{16}$$

and is also clearly stable. Equation (16) reflects the contrary effects of the two underlying sources of

[21] In deriving (16) we are also using the fact that if $e^{\rho^2\sigma_t^2} \approx 1 + \rho^2\sigma_t^2$, then $\ln(1+\lambda\rho^2\sigma_t^2) \approx \lambda\rho^2\sigma_t^2$ for $0 < \lambda < 1$. For the Cobb-Douglas function $z_t = (1-\alpha)/\alpha$, in which case (16) converges to $\sigma_{t+1}^2 = \upsilon^2 + \left((1-\theta)(1-\alpha)\right)^2 \sigma_t^2$.

heterogeneity – initial endowments and idiosyncratic productivity shocks. While the effects of the former decline over time, vanishing in the steady state, the effects of the latter grow over time. As a practical matter we view this approximation as being highly relevant, since in the most plausible instances $\rho^2\sigma_t^2$ is likely to be small.[22] But while our numerical simulations indicate that for all plausible parameters $D<1$, we cannot rule out $D>1$, and for (16) to diverge; this may occur if the variance υ^2 of the idiosyncratic production shocks is implausibly large and the elasticity of substitution, ε, is sufficiently small.

4. Effects of public investment on growth and inequality

We now employ the equilibrium relationships (8)-(13) to determine both the short-run and steady-state effects of public investment on inequality and growth.

4.1 Short-run effects

We shall assume that the economy is initially in steady state and in period 1 government investment increases by $d\tau$. From (8a) we see that with the one period lag, inequality remains unchanged until period 2, when using (8a) and (8b) we derive

$$\frac{\partial\sigma_2^2}{\partial\tau} = -\frac{(1-\theta)^2}{\rho}\frac{\left(e^{\rho^2\sigma_1^2}-1\right)}{\left[1+\left(z_1/(1+z_1)\right)^2\left(e^{\rho^2\sigma_1^2}-1\right)\right]}\frac{2(z_1)^2}{(1+z_1)^3}\frac{1}{\tau(1-\tau)} \tag{17a}$$

from which we infer that the short-run effect of an increase in investment in public goods on inequality depends upon the elasticity of substitution, ε; government investment will increase inequality if $\varepsilon<1$ and decrease it if $\varepsilon>1$. The underlying intuition is straightforward. The presence of diminishing returns to investment and credit market imperfections make different households face different problems with respect to their *productivity* and *investment* opportunities in the economy. The poor face investment constraints due to the lack of a credit market, while the rich obtain relatively lower marginal returns due to diminishing returns to investment. Which group benefits more from the

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[22] For our prefered calibration $\tilde{\sigma}^2=0.17$ and $\rho^2=0.133$, in which case $e^{\rho^2\sigma_t^2}=1.0228$, is closely approximated by $1+\rho^2\sigma_t^2=1.0226$.

provision of a specific form of public capital depends upon its substitutability or complementarity to private capital. Substitutable public investment disproportionately benefits the poor, as it provides the opportunity to circumvent their resource constraint by substituting public capital in production; inequality therefore declines. Complementary public investment, however, benefits the rich more, through enhancing the productivity of private capital, causing inequality to increase.

Setting $t=0$ in (12b) and combining with (8a) the growth rate of output in period 1 is

$$\gamma_1^y = \ln(a'\chi^{\theta}\alpha^{1/\rho'}) + \theta\ln(1-\tau) + (1-\theta)\ln\tau + \frac{1}{\rho'}\ln(1+z_1) + \frac{1-\rho'}{2}\left(\sigma_2^2 - \upsilon^2\right) \quad \text{(12b')}$$

from which we derive

$$\frac{\partial\gamma_1^y}{\partial\tau} = \frac{1}{\tau(1-\tau)}\left[\frac{(1-\theta)}{(1+z_t)} - \tau\right] + \frac{1}{2}\left(1 - \frac{\rho}{1-\theta}\right)\frac{\partial\sigma_2^2}{\partial\tau} \qquad \text{(17b)}$$

and we see that public investment impacts growth through two channels. The first is the net impact on the productive capacity of the economy, the effect of which depends upon τ relative to the productivity of public capital as reflected in z_t. This is the effect emphasized by Barro (1990), although in contrast to Barro, for the CES technology z_t depends upon the existing degree of inequality; see (8b).[23] But this response must be modified by its direct effect on inequality, expressed by the second term in (17b). If $\varepsilon < 1$, this term is positive. In this case the increase in inequality caused by increasing public investment increases the short-run growth rate. However, if $1 < \varepsilon < 1/\theta$, public investment decreases inequality, and this causes the short-run growth rate to decline. Thus, the overall impact of increasing public investment on the short-run growth rate will depend upon the current magnitude of τ and the degree of substitutability between the two productive inputs.

4.2 Steady-state effects

To derive the long-run effects of government investment on inequality and growth we return to (13a)-(13c), from which we show:

[23] For the Cobb-Douglas production function the term becomes $(\alpha(1-\theta)-\tau)/[\tau(1-\tau)]$, which reduces further to $(\alpha-\tau)/[\tau(1-\tau)]$ if, as Barro does, one abstracts from labor.

$$\frac{\partial\tilde{\sigma}^2}{\partial\tau}=-\frac{(1-\theta)^2}{\rho}\frac{\left(e^{\rho^2\tilde{\sigma}^2}-1\right)}{(1-D)\left[1+\left(\tilde{z}^2/(1+\tilde{z})^2\right)\left(e^{\rho^2\tilde{\sigma}^2}-1\right)\right]}\frac{2(\tilde{z})^2}{(1+\tilde{z})^3}\frac{1}{\tau(1-\tau)} \tag{18a}$$

$$\frac{\partial\tilde{\gamma}}{\partial\tau}=\frac{1}{\tau(1-\tau)}\left[\frac{(1-\theta)}{(1+\tilde{z})}-\tau\right]+\frac{1}{2}\left[\left(1-\frac{\rho}{1-\theta}\right)-\frac{(1-\theta)(1-\rho)\tilde{z}}{1+\tilde{z}}\right]\frac{\partial\tilde{\sigma}^2}{\partial\tau} \tag{18b}$$

The parallel between (18a) and (17a) is clear, with the qualitative impact of τ on inequality still depending upon $\operatorname{sgn}(\varepsilon-1)$, and the scale factor $(1-D)^{-1}$ reflecting the adjustment of inequality over time. Equation (18b) parallels (17b) in that it incorporates the two channels through which public investment impacts growth. But now account must be taken of the fact that in steady state, the choice of τ will also determine the degree of inequality reflected in $\tilde{z}$, reducing the coefficient of $\partial\tilde{\sigma}^2/\partial\tau$, and thus impacting the tradeoff between long-run growth and inequality. Strengthening (2d) to $\varepsilon<\theta^{-1}\left[1-\tilde{z}(1-\theta)^2(1+\tilde{z})^{-1}\right]$ ensures that the steady-state impact of inequality on growth remains positive, although with the modification being small it is only marginally weaker than the short-run effect. We may summarize these aggregate effects in:

Proposition 2: (i) An increase in the rate of public investment will increase (decrease) both short-run and long-run inequality according to whether the elasticity of substitution is less (greater) than one.

(ii) To the extent that more public investment increases inequality, this will tend to increase the growth rate both in the short run, and in the long run, although in all cases the net overall effect will also depend critically upon whether the current rate of expenditure $\tau \gtrless (1-\theta)(1+\tilde{z})^{-1}$.

4.3 Growth versus welfare maximization

The growth-inequality tradeoff is also manifested in the choice of long-run growth-maximizing rate of public investment. Setting $\partial\tilde{\gamma}/\partial\tau=0$, we see that the growth-maximizing rate of public investment, τ^*, and corresponding ratio of private to public capital, z^*, are related by

$$\frac{1}{\tau^*(1-\tau^*)}\left[\frac{(1-\theta)}{(1+z^*)}-\tau^*\right]+\frac{1}{2}\left[\left(1-\frac{\rho}{1-\theta}\right)-\frac{(1-\theta)(1-\rho)z^*}{1+z^*}\right]\frac{\partial\tilde{\sigma}^2}{\partial\tau}=0 \tag{19}$$

For expositional convenience we shall maintain the assumption $\varepsilon < \theta^{-1}\left[1-\tilde{z}(1-\theta)^2(1+\tilde{z})^{-1}\right]$, a condition that is almost certainly satisfied by the empirical data.[24]

> **Proposition 3**: To the extent that government investment in infrastructure increases (decreases) inequality, it will set the long-run growth-maximizing rate of public investment at a rate $\tau^* > (1-\theta)(1+z^*)^{-1}$, the optimality condition characterizing a riskless economy.

The choice of τ^* implied by (19) thus depends upon the presence and degree of inequality in this economy. In comparing (19) to a riskless economy, we need to take account of the fact that the equilibrium ratio of private to public capital will also be affected. Optimal expenditure in the analogous riskless economy is characterized by $\bar{\tau} = (1-\theta)(1+\bar{z})^{-1}$ in which case (19) implies $\tau^*/\bar{\tau} > (1+\bar{z})(1+z^*)^{-1}$. The fact that government investment increases inequality and the growth rate causes them to reduce investment, thus increasing *z*, enabling them to reduce the tax rate. In our numerical simulations reported in the next section, we see that the growth-maximizing tax rate is relatively higher in an economy with no idiosyncratic risk, and therefore no inequality, if the elasticity of substitution between public and private capital is less than unity; the opposite applies if $\varepsilon > 1$.

To see the underlying intuition suppose that the government has set the appropriate growth-maximizing tax, given the degree of inequality. If the degree of inequality is now increased, this will decrease the growth rate; see (14b'). In order to offset this decrease, the government should adjust its investment, with the appropriate response depending upon the elasticity of substitution. If $\varepsilon < 1$, this is achieved by increasing τ, which will increase inequality, while simultaneously lowering $\tilde{z}$ and raising the long-run growth rate in accordance with (18b), and correspondingly reducing τ if $\varepsilon > 1$.

While growth and inequality are important, the key issue in assessing the consequences of structural changes and policy responses concerns their impact on social welfare. Defining such a measure as the discounted sum of the expected utility of all future generations, $W = E\sum_{t=0}^{\infty} W_t (1+R)^{-t}$, in Appendix A.2 we show that in steady state the implied social welfare function is

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[24] This condition is clearly met if $\varepsilon < 1$ and the two capital goods are complements. For $\varepsilon > 1$ and our benchmark parameterization it involves strengthening $\varepsilon < 1.5$ to around 1.4.

$$W=\frac{(1+R)(1+\beta)}{R}\left(C+\ln\left(1-\tau\right)+\frac{\tilde{\gamma}}{R}-\frac{1}{2}\tilde{\sigma}^{2}\right) \tag{20}$$

where the constant C is defined in equation (A.10). Thus, steady-state social welfare involves a tradeoff between the equilibrium growth rate, the degree of income inequality, and the loss in average income due to the tax rate used to finance government investment.

Maximizing (20), the welfare-maximizing rate of government investment satisfies

$$\frac{\partial\tilde{\gamma}}{\partial\tau}=R\left[\frac{1}{1-\tau}+\frac{1}{2}\frac{\partial\tilde{\sigma}^{2}}{\partial\tau}\right] \tag{21}$$

In the absence of idiosyncratic productivity shocks, (21) implies $\partial\tilde{\gamma}/\partial\tau=R(1-\tau)^{-1}>0$, so that maximizing social welfare is associated with a rate of government investment that falls short of the growth maximizing rate. In other words, maximizing the growth rate leads to too much government investment for intertemporal utility maximization across generations. Moreover, government investment resulting from growth maximization is even more excessive in the presence of idiosyncratic shocks, when for $\varepsilon<1$, government investment exacerbates income inequality.

5. Calibration

It is clear from the analysis of the previous sections that the magnitude of the elasticity of substitution between public and private capital is crucial in determining the effects of government investment in infrastructure on the dynamics of inequality, and growth. However, the availability of empirical estimates of this critical parameter is sparse. Most empirical studies examining the effect of public capital simply assume a Cobb-Douglas function ($\varepsilon=1$), while others adopt the more general translog function; see e.g. Berndt and Hansson (1991), Lynde and Richmond (1992), Nadiri and Mamuneas (1994). Although the generality of the latter function makes it difficult to map the estimates to a CES function, anecdotal evidence suggests that the complementarity-substitutability relationship between public and private capital can vary substantially, depending upon the specific types of capital. Thus, for example, while public and private transport may be highly substitutable, roads and cars are complementary.

In contrast to the lack of estimates of ε, abundant estimates of the productive elasticity of capital, $e \equiv d\ln y / d\ln g$ are available. In Section 5.1 below we show how using this information, together with the equilibrium conditions reported in Section 4, can yield plausible calibrations for the elasticity ε. But before doing so, we must parameterize the remainder of the model.

Table 1 parameterizes a benchmark economy. The parameters we employ are almost entirely conventional and reflective of plausible real economies.

Table 1: Calibration: Benchmark values

Preference and technology parameters:	$\beta = 0.30,\ \eta = 1.75,\ R = 0.30$
Production parameters:	$\alpha = 0.2 - 0.4,\quad e = 0.1 - 0.2,\quad \theta = 2/3$
Policy parameters	$\tau = 0.05$
Idiosyncratic productivity shocks	$\upsilon^2 = 0.16$

Assuming a period (generation) to be of the order of 30 years, at the end of which parents pass on their bequests, and assuming a psychological discount factor of 0.96, $\beta = 0.96^{30} \approx 0.30$; see e.g. de la Croix and Michel (2002, p.255). The weight $\eta = 1.75$ assigned to leisure in utility implies an equilibrium allocation of time to labor of around 0.33, consistent with the real business cycle literature. The discount factor $(1+R)^{-1}$ measures the relative weight assigned by the social planner (welfare maximizer) to the utility of successive generations and as is well known it has no relationship to β. Setting $R = 0.3$ implies a generational discount factor of 0.77, which while seemingly plausible, is purely illustrative. We should also add that the only role that R plays is in assessing the welfare associated with any given equilibrium.

The share of public capital in the first level of production ranges between $\alpha = 0.2$ and 0.4, which covers most of the plausible values parameterized by Eden and Kraay (2014), including their preferred estimate (0.4). As Bom and Ligthart (2014) document, empirical estimates on the productive elasticity of public capital, *e*, are far ranging. In their comprehensive study they summarize 578 estimates and find the average productive elasticity of public capital to be around 0.19, while their meta-regression analysis yields an estimate of around 0.10. Other authors, using different data sets and methods obtain similar estimates, so that our range $e = 0.10 - 0.20$ brackets the overwhelming bulk of the empirical evidence, obtained employing a variety of data sets and techniques.[25] The productive elasticity of labor, $\theta = 2/3$, is standard.

We set the benchmark government spending ratio, $g = \tau$ to be 5% of GDP, which is roughly consistent with evidence on the rate of public infrastructure spending for most OECD countries.[26] The final element is the specification of the idiosyncratic productivity shocks. Data on this are sparse, but Bartelsman, Haltiwanger, and Scarpetta (2013) provide estimates of the standard deviation of within industry log TFP shocks. For the seven countries they consider these average 0.4, with estimates for the US and UK being 0.39 and 0.42, respectively. On the basis of this we set $\upsilon^2 = 0.16$ ($\upsilon = 0.4$). Finally, *a* is chosen to match a long-run growth rate of 2%. To achieve this each parameterization of the CES production function requires a slight adjustment in *a*.[27]

5.1 Calibrating the elasticity of substitution

It is clear that ρ (or ε) is the critical parameter. Using the equilibrium conditions, in Appendix A.3 we establish the following relationship between the key parameters.

$$\rho \ln\left(\frac{\tau(1+\beta)}{(1-\tau)\beta}\right) = \ln\left[\frac{e(1-\alpha)}{\alpha(1-\theta-e)}\right] \tag{22}$$

Given assumptions on $\beta, \tau, \alpha, \theta,$ and estimates of the productive elasticity e, as reflected in Table 1, one can infer a value for ρ and hence an implied value for $\varepsilon = (1-\rho)^{-1}$. Table 2 reports the corresponding calibrated values for ε, for the grid of plausible parameter values spanning $\alpha = 0.20, 0.30, 0.40;\ e = 0.10, 0.15, 0.20$.

[25] For example, Henderson and Ullah (2005) obtain an estimate of 0.15. Using Belgian data, Everaert (2003) finds $e = 0.14$, while using Canadian provincial panel data, McDonald (2008) finds the elsticity to range between 0.10 and 0.15. Fosu et al. (2015), using data for Sub Saharan African countries, estimate e around 0.10. Finally, using a sample of data from developing countries Dessus and Herrer (2000) find *e* to range between 0.11. and 0.13.

[26] For developing countries it may be slightly higher. For instance, based on the World Bank (2012), Fosu et al. (2015) find the average public investment-GDP percentage for 42 Sub-Saharan African countries between 1960 and 2011 to be around 7.4%.

[27] It is worth noting that in order to target the same growth rate of 2%, the productivity parameter, *a*, must be reduced as the elasticity of substitution, ε, increases. This is a manifestation of the result due to Klump and de la Grandville (2000), and confirmed empirically by Yuhn (1991), that holding *a* constant, the growth rate increases with ε.

Table 2: Calibrated elasticity of substitution

	$e=0.1$	$e=0.15$	$e=0.2$
$\alpha=0.2$	$\varepsilon=0.733$	$\varepsilon=0.555$	$\varepsilon=0.452$
$\alpha=0.3$	$\varepsilon=1$	$\varepsilon=0.696$	$\varepsilon=0.541$
$\alpha=0.4$	$\varepsilon=1.426$	$\varepsilon=0.878$	$\varepsilon=0.646$

From Table 2, the following can be noted. The calibrated values of ε are sensitive to variations in both α and e. Over the plausible ranges of both these parameters we find that ε ranges between 0.452 (public and private capital strong complements) to 1.426 (strong substitutes). It is also compatible with the Cobb-Douglas production function for $\alpha=0.3, e=0.1$. While, the case of complementarity is clearly more prevalent, the possibility of substitutes cannot be ruled out, and indeed obtains if one combines the Bom-Ligthart (2014) meta-regression estimate of $e=0.10$ with the Eden-Kraay (2014) preferred calibration for $\alpha=0.40$.

Provided $\tau(1-\tau)^{-1}<\beta(1+\beta)^{-1}$, as our parameterization clearly implies, (22) yields a simple criterion for $\rho<0$ ($\varepsilon<1$) and for the two capital goods to be complements, namely

$$\varepsilon<1 \text{ if and only if } e>\alpha(1-\theta) \tag{23}$$

That is, public and private capital will be complements if and only if the productive elasticity of public capital, e, exceeds its share of total output, $\alpha(1-\theta)$.[28] Conversely, substitutability between private and public capital will be characterized by a public good that has a low productive elasticity but accounts for a high share of output.

Our finding that the elasticity of substitution is likely to be less than unity is generally characteristic of the empirical estimates. Eden and Kraay (2014), in focusing on the 'crowding in' of public investment, draw upon empirical estimates to calibrate the CES production function and find convincing evidence of complementarity.

[28] In the Cobb-Douglas case $e=\alpha(1-\theta)$ and $\varepsilon=1$.

5.2 Some new empirical evidence on the elasticity of substitution

In order to obtain additional information on the likely magnitude of the elasticity of substitution between public and private capital, particularly for developing countries, we have estimated an aggregate CES function of the form

$$y_t = \vartheta_t a\left((1-\alpha)k_t^{\rho} + \alpha g_t^{\rho}\right)^{\frac{1}{\rho}} \tag{24}$$

where y_t, k_t, and g_t are measured in per capita form, and ϑ_t, and a represent aggregate stochastic and non-stochastic TFP, respectively.[29] We adopt the approach originally employed by Arrow et al. (1961), and regress ln[y_t / g_t] on the log of the marginal product of g_t. This is done by taking the first derivative of (24) with respect to g_t, and rewriting the resulting expression in the form

$$\ln y_t^g = \lambda + \varepsilon \ln p + \nu_t \tag{25}$$

where $\lambda \equiv -\ln \alpha^{\varepsilon} a^{\rho\varepsilon}, \nu_t \equiv -\varepsilon\rho \ln \vartheta_t, y_t^g \equiv y_t / g_t, p \equiv dy_t / dg_t$. The estimation equation for (25) can then be easily specified in panel data form as follows:

$$\ln y_{it}^g = \varepsilon \ln p_{it} + \lambda_i + \nu_{it} \tag{26}$$

where λ_i and ν_{it} denote unobserved country-specific fixed effects and the error term, respectively; i refers to individual countries and t refers to particular time periods.

We estimate (26) using the fixed effects (FE) panel data estimation method.[30] The panel data we use cover 42 African countries, for the period 1960 to 2011, from World Bank (2012). Eleven African countries are excluded due to either lack of data, inconsistencies, or outlier effects.[31] In order to smooth out any potential business cycle effects we have estimated the equation using 5 year

[29]Note that with complete depreciation of capital k_t and g_t are flow variables.

[30]Some of the most widely employed estimation methods for panel data with large time are FE, Random Error (RE) or pooled OLS (see Wooldridge, 2010). The advantage of FE over the two is that it enables one to account for country-specific time-invariant unobserved factors.

[31]These are Somalia, Sao Tome and Pricipe, Nigeria, Seychelles, Angola, Algeria, Cape Verde, Madagascar, Libya, Cong, Dem. Rep, and Zambia. Data are not available for Somalia, Sao Tome and Pricipe Nigeria and Seychelles. Algeria, Cape Verde, Madagascar, Libya, Zambia, and Cong, Dem. Rep have too high average public or private investment or both over the sample period. For instance, private investment for Algeria, Cape Verde, Madagascar, Zambia and Cong, Dem. Rep are reported to be 109%, 71%, 159%, 79% and 71% of GDP, respectively. Public investment for Algeria, Cape Verde, Madagascar, Libya, and Zambia are shown to be 117%, 47%, 71%, 55% and 39% percents of GDP, respectively.

averages of the data. We have estimated equation (26) exactly as it appears, and also with a one period lag to control for possible problems of reverse causality.[32] In the first case we estimate $\varepsilon = 0.564$ (standard error, 0.038), in the latter $\varepsilon = 0.466$ (standard error 0.041). These estimates are in the range characterizing complementarity, summarized in Table 2.

6. Public investment, and the impact on inequality and growth

6.1 Welfare costs of idiosyncratic shocks

Table 3 reports the effect on the equilibrium growth rate and welfare obtained by eliminating the inequality arising from the presence of the idiosyncratic productivity shocks. Since the productivity elasticity, *e*, is typically determined endogenously, rather than specified as an exogenous parameter, the table is arranged relating the share of public capital, α, to the plausible range of the elasticity of substitution, ε, based on the calibration in Section 5. The implied productivity elasticities are consistent with the range of empirical estimates discussed by Bom and Ligthart (2014) and others, summarized in Section 5.

Thus, while the Cobb-Douglas function serves as the conventional benchmark, we view $\varepsilon < 1$ as the most likely scenario, and for expositional purposes will treat the parameterizaton $\alpha = 0.2, \varepsilon = 0.6$ as the base case. This position is consistent with the empirical analysis conducted by Lynde and Richmond (1992), who, using a more general translog approach, suggested that public and private capital tend to be complements rather than substitutes in production.[33] However, in light of our calibration results, we certainly cannot dismiss $\varepsilon > 1$ as a plausible possibility in some instances.

Table 3 indicates that the impact of eliminating entirely idiosyncratic shocks on growth and welfare is rather consistent across this range of plausible parameters. The long-run growth rate will increase by around 1.5 percentage points, while welfare will increase by around 7%. Both of these responses are consistent with empirical evidence. The magnitude of the growth effect is comparable

[32]As robustness checks we also estimated (26) using one year averages of the data. We also employed the approximation proposed by Kmenta (1967); see, Klump et al., 2012. In all cases we obtain estimates of the elasticity of substitution, well below unity.
[33] See also Baier and Glomm (2001). Since the public good is an aggregate, comprising a mix, some of which are complements and others substitutes to private capital, we restrict the range of variation of ε.

to the estimate of Alesina and Rodrik (1994), who find that a decrease in the Gini coefficient by 0.16 points raises the growth rate by around 0.8 percentage points.[34] In contrast to Lucas' (1987) finding that economy-wide supply shocks impose negligible welfare costs, Turnovsky and Bianconi (2005) found that idiosyncratic productivity shocks were many times more costly, yielding costs comparable to this, depending upon the assumed degree of risk aversion. In addition, Pallage and Robe (2003) provide evidence to suggest that the welfare costs of economic fluctuations in developing countries may be of the order of up to 25 times that of the United States!

6.2 Increase in rate of public investment

Although the equilibrium described in equations (8a) generates transitional dynamics (in contrast to the Romer-Barro model), the speed of the local dynamics, summarized by D, is exceedingly fast.[35] This is unsurprising, since both private and public capital depreciate fully each period and the only source of dynamics is the small fraction of current output that is saved and contributes to next period's capital.

Table 4 reports the long-run effects of raising the rate of public investment from its benchmark value of 5% by 1 percentage point to 6%. The steady-state growth rate is extremely sensitive to the productivity parameter, *a*, and in all cases, we have chosen it so as to normalize the initial steady-state growth rate at 2%.[36] The first line of the table shows the effects on both inequality and growth, corresponding to the calibrated value $\upsilon^2 = 0.16$, while the second line describes the case of zero productivity shocks and therefore no long-run inequality.

Taking the case $\varepsilon = 0.6, \alpha = 0.2$ as the benchmark, we see that the increase in government investment raises the growth rate by around 1.4 percentage points, which is consistent with the empirical evidence cited by Calderón and Servén (2014).[37] Looking across the table we see that the

[34] Reducing the Gini coefficient to zero would result in an increase in the growth rate of around 1.6%

[35] This is easily seen for the Cobb-Douglas case where for the benchmark parameterization $\alpha = 0.2, \theta = 2/3$ when $D \approx 0.071$. To obtain slower and more plausible transitional dynamics, capital must depreciate at a lower rate.

[36] The fact that we are changing a prevents comparisons of the magnitudes of the changes between the three cases.

[37] Calderón and Servén suggest that a 1% increase in the stock of infrastructure may raise the growth rate by between 1-2 percentage points. Assuming that public capital depreciates at around 5% per annum, in steady state the flow of government investment will be approximately 5% of the stock. Thus, a 1% increase in the stock is equivalent to around 20% increase in the net flow, which is comparable to an increase in τ from 5 % to 6% considered in the present analysis.

growth effect declines with the elasticity of substitution, but increases with the importance of public capital in production. In the presence of idiosyncratic productivity shocks ($\upsilon^2 = 0.16$) public investment increases or reduces inequality, depending upon whether $\varepsilon <$ or > 1, thus illustrating Proposition 1. As a result for $\varepsilon = 0.6$ the increase in inequality causes government investment to have a less positive effect on growth. Specifically, the increase in $\tilde{\gamma}$ is reduced to 1.38 percentage points, compared to 1.42 percentage points in the absence of idiosyncratic shocks. The opposite applies for $\varepsilon = 1.2$. Table 4 also clearly indicates how the prevalent Cobb-Douglas production function may seriously understate both the growth effects and welfare benefits of government investment if indeed public and private capital are strong complements as much empirical evidence suggests.

The final point to note is that for $\varepsilon = 1.2$ and $\alpha = 0.2$, increasing public investment from 5% to 6% actually reduces welfare. This is because with the two capital goods being highly substitutable and public capital being only mildly productive, increasing τ to 6% leads to over-investment in public capital beyond its socially optimal level (see Table 6).

A key issue concerns the impact of government investment or infrastructure on income inequality. In this regard, to the extent that $\varepsilon < 1$ our model suggests that public investment, as specified by an increase in τ from 0.05 to 0.06 will increase inequality. The empirical evidence on the relationship between infrastructure and income inequality is much less definitive. While much of the evidence obtains a negative correlation, the nature of this relationship has been subject to severe questioning; see Calderón and Servén (2010, 2014). First, there is the issue of causality as opposed to mere correlation. Second, several authors, including Calderón and Servén (2010), and Senevirane and Sun (2013) argue that data on public investment offer a poor proxy for infrastructure. One well documented reason for this is that infrastructure projects may involve a partnership between the public and private sector. In the present context it is plausible that the public investment may be devoted to improving the reliability of the existing infrastructure, resulting in a decline in the idiosyncratic productivity shocks. Thus, for example, assuming the benchmark technology, $\varepsilon = 0.6$, $\alpha = 0.2$, and if the increase in τ from 0.05 to 0.06 results in a decline in υ from 0.16 to 0.15, we find that the net effect is an *increase* in the growth rate of 1.49% together with a 2.98% *decline* in inequality.[38]

[38] We may note that for $\varepsilon = 1$ these responses are reduced to 0.32% and 0.70% respectively. Since richer countries tend to

6.3 Growth- and welfare-maximizingtax rates, inequality, and elasticity of substitution

Tables 5 and 6 summarize the growth- and welfare-maximizing rates of government investment, reporting both $\upsilon^2 = 0.16$ and $\upsilon = 0$, in order to see the role of the idiosyncratic shocks. For the widely employed Cobb-Douglas technology, the growth-maximizing rate of public investment is independent of the stochastic shocks, reducing to the well known condition in the Barro model, $\hat{\tau} = (1-\alpha)(1-\theta)$. But in contrast to the Barro model, this does not coincide with the welfare-maximizing condition, which is now $\hat{\tau} = (1-\alpha)(1-\theta) - R$.

Focusing on the benchmark technology, $\varepsilon = 0.6, \alpha = 0.2$, in the absence of productivity shocks, the growth-maximizing rate of government investment is 9.76% implying a corresponding optimal growth rate of 5.16%. With its adverse effect on inequality, the presence of productivity shocks $\upsilon^2 = 0.16$, reduces $\hat{\tau}$ to 9.67% and the corresponding growth rate to 5.03%. The welfare-maximizing rate of public investment is approximately 1.6 percentage points below the growth-maximizing rate, leading to an approximately 0.24 percentage point lower growth rate. This places it around 8.1%, slightly higher than the average rate of investment (7.4%) for Sub-Saharan countries estimated by Fosu et al. (2015). This suggests mild underinvestment in public capital and is consistent with the findings of Eden and Kraay (2014).

For $\varepsilon = 0.6$, both the growth-maximizing and welfare-maximizing rates of government investment decrease with υ^2, reflecting the fact that for $\varepsilon < 1$, public investment increases income inequality, while the reverse applies if $\varepsilon = 1.2$. Irrespective of the presence of idiosyncratic risk, an increase in the elasticity of substitution between the two capital goods reduces both the growth- and welfare-maximizing tax rates, while an increase in the productivity of public capital raises them. As a final observation, we see that if the government optimizes on the basis of the Cobb-Douglas production function, when in fact $\varepsilon \approx 0.6$, they may underinvest by over 2 percentage points, producing a substantially sub-optimally low growth rate, accumulating over time to yield significant welfare losses.

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have higher elasticities of subsitution (Duffy and Papageorgiou, 2000), this pattern is consistent with Calderón and Chong's (2004) finding that government investment tends to be more effective in developing countries.

7. Conclusion

It is widely accepted that public investment has important consequences for income distribution and for the nature of the growth-inequality tradeoff. Despite the importance of this issue, it has received little attention in the literature. This paper has addressed the question, focusing particularly on the role of the substitutability/complementarity relation between public and private capital in the production process. The source of heterogeneity is crucial and we have considered an overlapping generations economy in which both initial endowments and idiosyncratic productivity shocks are generated by lognormal distributions. Apart from its plausibility, lognormality offers the critical advantage of facilitating aggregation across the diverse agents and yielding a tractable closed form solution for the equilibrium dynamics. A key implication of the model is that over time the heterogeneity due to endowments vanishes, while that due to productivity shocks persists.

We also focus on an economy lacking financial markets, in which case, the diminishing marginal product of capital for the rich, coupled with the inability to borrow by the poor, introduces productive inefficiencies, reflected in inequality, and shown to be detrimental to growth. As a result of this, the degree of substitutability between public and private capital in production has important consequences for the growth-inequality relationship. In the case that the two productive factors are highly substitutable, public investment may improve the growth-inequality tradeoff. In effect, the flexibility in production it introduces substitutes for the lack of credit market, and serves as a means of relaxing the resource constraints that impede the investment opportunities for the poor. However, if the two factors are complementary in production, then by disproportionately benefiting the owners of capital – the rich – it may exacerbate any adverse growth-inequality tradeoff. This in turn has consequences for the optimal degree of public investment, as originally derived by Barro (1990).

The framework adopted, while sharing some aspects in common with Chatterjee and Turnovsky (2012) – in particular an endogenous growth framework – nevertheless generates a very different equilibrium structure yielding very different conclusions. Chatterjee and Turnovsky consider an economy in which the sole source of heterogeneity is the initial endowments of capital. By assuming that all agents have equal access to perfect financial markets, they show how the distributions of endowments are a key determinant of both wealth and income inequality, both in the short run and over time. Under their assumptions, they show how the economy-wide growth rate determines, but is not determined by, the equilibrium inequality, precisely the opposite to the causality emphasized here. Moreover, they find that public investment will increase wealth inequality over time, irrespective of how it financed, while the impact on income inequality is highly sensitive to the mode of public finance. On the other hand, the degree of factor substitutability, emphasized here, plays a lesser role. This is because in their analysis, public capital impacts directly on the productivity of labor.

The point of this comparison with the results of the present paper is to stress the complexity of the growth-inequality relationship, highlighting the need to consider it from alternative viewpoints. It emphasizes how the consequences of government policy are highly sensitive to the underlying source and nature of the heterogeneity, as well as the structural characteristics of the economy. These are important aspects to consider in exercising government investment policy and merit further careful investigation.

Table 3: Equilibrium

	$\varepsilon = 0.6$			$\varepsilon = 1$			$\varepsilon = 1.2$		
	$\tilde{\sigma}$	$d\tilde{\gamma}(0)$	$dW(0)$	$\tilde{\sigma}$	$d\tilde{\gamma}(0)$	$dW(0)$	$\tilde{\sigma}$	$d\tilde{\gamma}(0)$	$dW(0)$
$\alpha = 0.2$	(e=0.134)			(e=0.067)			(e=0.054)		
	0.4091	1.74%pt	7.43%	0.4150	1.68%pt	7.47%	0.4165	1.68%	7.47%
$\alpha = 0.3$	(e=0.178)			(e=0.100)			(e=0.084)		
	0.4056	1.50%pt	6.96%	0.4114	1.52%pt	7.09%	0.4130	1.47%pt	7.11%
$\alpha = 0.4$	(e=0.214)			(e=0.133)			(e=0.114)		
	0.4034	1.26%pt	6.50%	0.4083	1.34%pt	6.72%	0.4099	1.33%pt	6.75%

Table 4: Increase in τ from 0.05 to 0.06

	$\varepsilon = 0.6$			$\varepsilon = 1$			$\varepsilon = 1.2$		
	$d\tilde{\sigma}$	$d\tilde{\gamma}$	dW	$d\tilde{\sigma}$	$d\tilde{\gamma}$	dW	$d\tilde{\sigma}$	$d\tilde{\gamma}$	dW
$\alpha = 0.2$	0.244%	1.38%pt	1.84%	0%	0.22%pt	-0.16%	-0.048%	0.02%pt	-0.53%
	$\sigma \equiv 0$	1.42%pt	2.02%	$\sigma \equiv 0$	0.22%pt	-0.17%	$\sigma \equiv 0$	0.00%pt	-0.58%
$\alpha = 0.3$	0.173%	2.21%pt	3.30%	0%	0.88%pt	0.97%	-0.048%	0.59%pt	0.48%
	$\sigma \equiv 0$	2.27%pt	3.87%	$\sigma \equiv 0$	0.87%pt	1.00%	$\sigma \equiv 0$	0.58%pt	0.48%
$\alpha = 0.4$	0.124%	2.89%pt	4.51%	0%	1.52%pt	2.09%	-0.049%	1.18%pt	1.52%
	$\sigma \equiv 0$	2.97%pt	4.86%	$\sigma \equiv 0$	1.51%pt	2.19%	$\sigma \equiv 0$	1.17%pt	1.58%

Table 5: Growth maximizing government investment

	$\varepsilon = 0.6$			$\varepsilon = 1$			$\varepsilon = 1.2$		
	$\tilde{\sigma}$	$\tilde{\gamma}$	$\hat{\tau}$	$\tilde{\sigma}$	$\tilde{\gamma}$	$\hat{\tau}$	$\tilde{\sigma}$	$\tilde{\gamma}$	$\hat{\tau}$
$\alpha = 0.2$	0.4124	5.03%	9.67%	0.4150	2.26%	6.67%	0.4164	2.21%	5.58%
	$\sigma \equiv 0$	5.16%	9.76%	$\sigma \equiv 0$	2.26%	6.67%	$\sigma \equiv 0$	2.02%	5.53%
$\alpha = 0.3$	0.4099	8.55%	12.27%	0.4114	4.06%	10.0%	0.4123	3.23%	9.12%
	$\sigma \equiv 0$	8.81%	12.41%	$\sigma \equiv 0$	4.06%	10.0%	$\sigma \equiv 0$	3.19%	9.05%
$\alpha = 0.4$	0.4078	12.39%	14.66%	0.4083	7.12%	13.33%	0.4087	5.89%	12.80%
	$\sigma \equiv 0$	12.79%	14.85%	$\sigma \equiv 0$	7.12%	13.33%	$\sigma \equiv 0$	5.82%	12.73%

Table 6: Welfare maximizing government investment

	$\varepsilon = 0.6$			$\varepsilon = 1$			$\varepsilon = 1.2$		
	$\tilde{\sigma}$	$\tilde{\gamma}$	$\hat{\tau}$	$\tilde{\sigma}$	$\tilde{\gamma}$	$\hat{\tau}$	$\tilde{\sigma}$	$\tilde{\gamma}$	$\hat{\tau}$
$\alpha = 0.2$	0.4115	4.78%	8.09%	0.4150	2.04%	5.12%	0.4167	1.82%	4.11%
	$\sigma \equiv 0$	4.92%	8.20%	$\sigma \equiv 0$	2.04%	5.12%	$\sigma \equiv 0$	1.81%	4.07%
$\alpha = 0.3$	0.4089	8.22%	10.22%	0.4114	3.72%	7.69%	0.4126	2.88%	6.75%
	$\sigma \equiv 0$	8.49%	10.37%	$\sigma \equiv 0$	3.72%	7.69%	$\sigma \equiv 0$	2.84%	6.68%
$\alpha = 0.4$	0.4068	11.98%	12.14%	0.4083	6.64%	10.25%	0.4091	5.39%	9.53%
	$\sigma \equiv 0$	12.39%	12.34%	$\sigma \equiv 0$	6.64%	10.25%	$\sigma \equiv 0$	5.32%	9.45%

Appendix

A.1 Aggregation and distribution

This Appendix derives the basic dynamic equations generating the degree of inequality, (8), aggregate capital, (9), and output, (11), using key relations pertaining to the normal and lognormal distributions. These relationships imply that a lognormal random variable preserves its property under multiplication and addition by constant where the latter leads to a special case of a *shifted* lognormal distribution. Specifically, we use the following result stated by Kleiber and Kotz (2003, p.121): "If there exists $\lambda \in R$ such that $Z \sim \ln(X-\lambda)$ follows a normal distribution, then X is said to follow a three- parameter [shifted] lognormal distribution." Thus letting $Y \equiv \exp(Z)$, then $Y+\lambda = X$. This implies that if Y is lognormal then $Y+\lambda$ is a shifted lognormal. For this to be the case, the probability of X of taking any value below λ must be zero. But X could take any value greater than λ. This is further discussed by Aitchison and Brown (1957, p.14), who point out that as long as λ is given exogenously, the variate X has all the properties of the two parameter lognormal variate.

To utilize these standard results, it is convenient to rewrite (7) as

$$(k_{t+1}^i)^{\rho'} = b_t(\xi_t^i)^{\rho'}\left(1+z_t^i\right) \tag{A.1}$$

where

$$z_t^i \equiv \alpha^{-1}\left(1-\alpha\right)(\phi_t^i)^{\rho};\;\; \phi_t^i \equiv k_t^i / g_t;\;\; b_t \equiv \left(\left(1-\tau\right)a'\chi g_t^{1-\theta}k_t^{\theta}\right)^{\rho'}\alpha\,;\rho' \equiv \rho(1-\theta)^{-1}$$

Then, note that given the assumptions of lognormality, $\ln k_t^i \sim N(\mu_t, \sigma_t^2)$, $\ln \xi_t^i \sim N(-\upsilon^2/2, \upsilon^2)$ and the above discussion: (i) z_t^i is lognormal, (ii) If z_t^i is lognormal, then $1+z_t^i$ is (a shifted) lognormal[39] and (iii) as a product of two lognormal variables $(k_{t+1}^i)^{\rho'}$ is also a lognormal.

Also, from the basic properties of the lognormal distribution, we have:

$$\mathrm{E}\, k_t^i \equiv k_t = e^{\mu_t + 0.5\sigma_t^2}$$
$$\operatorname{var} k_t^i = \left(e^{\sigma_t^2}-1\right)e^{2\mu_t+\sigma_t^2} = k_t^2\left(e^{\sigma_t^2}-1\right)$$

[39] Note that $1+z_t^i$ can take any value exceeding unity but have a zero probablity of taking any value below unity. With $z_t^i > 0$, this condition clearly holds.

Since $(k_t^i)^{\rho'}$, $(k_{t+1}^i)^{\rho'}$ and $(\xi_t^i)^{\rho'}$ are all log-normal, it immediately follows that:

$$\mathrm{E}(k_t^i)^{\rho'} = k_t^{\rho'} e^{0.5\rho'(\rho'-1)\sigma_t^2} \qquad \text{for all } t \tag{A.2a}$$

$$\mathrm{E}(\xi_t^i)^{\rho'} = e^{0.5\rho'(\rho'-1)\upsilon^2} \tag{A.2b}$$

$$\mathrm{var}(k_t^i)^{\rho'} = k_t^{2\rho'} e^{\rho'(\rho'-1)\sigma_t^2}\left(e^{\rho'^2\sigma_t^2}-1\right) \tag{A.2c}$$

$$\mathrm{var}(\xi_t^i)^{\rho'} = e^{\rho'(\rho'-1)\upsilon^2}\left(e^{\rho'^2\upsilon^2}-1\right)=\left(e^{\rho'^2\upsilon^2}-1\right)\left(E(\xi_t^i)^{\rho'}\right)^2 \tag{A.2d}$$

Using (A.2a) and (A.2c), and recalling the definition of ϕ from (8c) we compute the mean and variance of z_t^i:

$$\mathrm{E}\, z_t^i \equiv z_t = \alpha^{-1}\left(1-\alpha\right) g_t^{-\rho}\, \mathrm{E}(k_t^i)^{\rho} = \alpha^{-1}\left(1-\alpha\right)\phi_t^{\rho} e^{0.5\rho(\rho-1)\sigma_t^2} \tag{A.3a}$$

$$\mathrm{var}\, z_t^i = \alpha^{-2}\left(1-\alpha\right)^2 g_t^{-2\rho}\, \mathrm{var}(k_t^i)^{\rho} = \alpha^{-2}\left(1-\alpha\right)^2 \phi_t^{2\rho} e^{\rho(\rho-1)\sigma_t^2}\left(e^{\rho^2\sigma_t^2}-1\right)= z_t^2\left(e^{\rho^2\sigma_t^2}-1\right) \tag{A.3b}$$

Using these results we can easily aggregate (A.1), first aggregating both sides and then using (A.2a) at time $t+1$, (A.2b), and (A.3a), and using the fact that with ξ_t^i being i.i.d., ξ_t^i and z_t are independent to obtain:

$$\mathrm{E}(k_{t+1}^i)^{\rho'} = b_t\, \mathrm{E}(\xi_t^i)^{\rho'}\left(1+\mathrm{E}\, z_{it}\right)= b_t\, \mathrm{E}(\xi_t^i)^{\rho'}\left(1+z_t\right)$$

so that

$$k_{t+1}^{\rho'} e^{0.5\rho'(\rho'-1)\sigma_{t+1}^2} = b_t e^{0.5\rho'(\rho'-1)\upsilon^2}\left(1+z_t\right) \tag{A.4}$$

Next, taking the variance of both sides of (A.1) yields:[40]

$$\begin{aligned}\mathrm{var}(k_{t+1}^i)^{\rho'} &= b_t^2\, \mathrm{var}\left[(\xi_t^i)^{\rho'}\left(1+z_t^i\right)\right] \\ &= b_t^2\left(\left(\mathrm{E}(\xi_t^i)^{\rho'}\right)^2 \mathrm{var}\left(1+z_t^i\right)+\left(\mathrm{E}\left(1+z_t^i\right)\right)^2 \mathrm{var}(\xi_t^i)^{\rho'}+\mathrm{var}\left(1+z_t^i\right)\mathrm{var}(\xi_t^i)^{\rho}\right)\end{aligned} \tag{A.5}$$

Using (A.2d) enables us to simplify this expression to:

$$\begin{aligned}\mathrm{var}(k_{t+1}^i)^{\rho'} &= b_t^2\left(\left(\mathrm{E}(\xi_t^i)^{\rho'}\right)^2 \mathrm{var}\, z_t^i+\left(\mathrm{E}(\xi_t^i)^{\rho'}\right)^2\left(e^{\rho'^2\upsilon^2}-1\right)\left(\left(1+z_t\right)^2+\mathrm{var}\, z_t^i\right)\right) \\ &= b_t^2\left(\mathrm{E}(\xi_t^i)^{\rho'}\right)^2\left(\left(e^{\rho'^2\upsilon^2}-1\right)\left(1+z_t\right)^2+e^{\rho'^2\upsilon^2}\,\mathrm{var}\, z_t^i\right)\end{aligned}$$

[40] In deriving (A.5) we are using the result that the variance of the product of two independent variables, x, y is: $\mathrm{var}(xy)=\left(Ex\right)^2 \mathrm{var}\, y+\left(Ey\right)^2 \mathrm{var}\, x+(\mathrm{var}\, y)(\mathrm{var}\, x)$

Substituting (A.2b), (A.2c), (A.3b) and (A.4) into the above equation and simplifying this reduces to

$$e^{\rho'^2\sigma_{t+1}^2} = e^{\rho'^2\upsilon^2}\left(1+\frac{z_t^2}{\left(1+z_t\right)^2}\left(e^{\rho^2\sigma_t^2}-1\right)\right) \tag{A.6}$$

which after taking logarithms is reported as eq. (8a) in the text.

Writing (A.4) in the form

$$k_{t+1}e^{0.5(\rho'-1)\sigma_{t+1}^2} = b_t^{1/\rho'}e^{0.5(\rho'-1)\upsilon^2}\left(1+z_t\right)^{1/\rho'}$$

and taking logarithms of this equation yields

$$\ln k_{t+1} = \frac{1}{\rho'}\ln b_t + \frac{1}{\rho'}\ln\left(1+z_t\right) + \frac{1-\rho'}{2}\left(\sigma_{t+1}^2 - \upsilon^2\right)$$

Substituting (8a) into this equation and using the definition of b_t and ϕ we obtain

$$\begin{aligned}\ln k_{t+1} = {} & \ln(a'\chi^{1+\theta}\alpha^{1/\rho'}) + (1+\theta)\ln(1-\tau) - \theta\ln\tau + \ln g_t + \frac{1}{\rho'}\ln(1+z_t) \\ & + \frac{1-\rho'}{2\rho'^2}\ln\left[1+\left(\frac{z_t}{1+z_t}\right)^2\left(e^{\rho^2\sigma_t^2}-1\right)\right]\end{aligned} \tag{A.7}$$

which corresponds to (9) in the text. To derive (11), substitute (6a) in (4), after aggregating to get,

$$\ln y_t = \ln k_{t+1} - \ln\left(\left(1-\tau\right)\chi\right)$$

and substitute (A.7) into the above. Finally, combining (3) and (11) leads to (10).

A.2 Steady-State Welfare

To derive a measure of steady-state economy-wide welfare, we begin by taking expected values of (1) across agents, to yield the average welfare of agents born at time *t*:

$$W_t = \mathrm{E}\left[\ln c_t^i + \eta\ln(1-l_t^i) + \beta\ln k_{t+1}^i\right] \tag{A.8}$$

Substituting (6a), (6b) and (2c) yields

$$W_t = \ln\left(1-\chi\right) + \beta \ln \chi + \eta \ln(1-l) + \left(1+\beta\right)\ln\left(1-\tau\right) + \left(1+\beta\right)\mathrm{E}\ln y_t^i$$

But since

$$\mathrm{E}\ln y_t^i = \ln \mathrm{E}\, y_t^i - \frac{1}{2}\mathrm{var}\ln y_t^i = \ln y_t - \frac{1}{2}\sigma_{t+1}^2$$

and $\sigma_{t+1}^2 = \mathrm{var}\ln k_{t+1}^i = \mathrm{var}\ln y_t^i$ we obtain

$$W_t = \ln\left(1-\chi\right) + \beta \ln \chi + \eta \ln(1-l) + \left(1+\beta\right)\ln\left(1-\tau\right) + \left(1+\beta\right)\ln y_t - \frac{1}{2}\left(1+\beta\right)\sigma_{t+1}^2 \quad \text{(A.9)}$$

where σ_{t+1}^2 is given by (8). Along a balanced growth path, y_t grows at a constant rate, $\tilde{\gamma}$ so that starting from some initial predetermined income level given at time 0, we have $\ln y_{t+1} = \ln y_t + \tilde{\gamma}$ and $\sigma_{t+1}^2 = \tilde{\sigma}^2$, implying

$$W_t = (1+\beta)\left[C + \ln\left(1-\tau\right) + t\cdot\tilde{\gamma} - \frac{1}{2}\tilde{\sigma}^2\right] \quad \text{(A.10)}$$

where $C \equiv (1+\beta)^{-1}\left[\ln\left(1-\chi\right) + \beta\ln\chi + \eta\ln(1-l) + \left(1+\beta\right)\ln y_0\right]$.

To obtain the economy-wide welfare, *W*, we aggregate welfare over all generations, (A.10), discounted at rate R, which may or may not coincide with β. Thus

$$W = \sum_{t=0}^{\infty} W_t \left(1+R\right)^{-t}$$

and substituting (A.10) yields

$$W = \frac{(1+R)(1+\beta)}{R}\left(C + \ln\left(1-\tau\right) + \frac{\tilde{\gamma}}{R} - \frac{1}{2}\tilde{\sigma}^2\right) \quad \text{(A.11)}$$

A.3 Calibration of elasticity of substitution

We begin by taking logarithms of the production function (2b) to obtain

$$\ln y_t^i = \ln a + \frac{1-\theta}{\rho}\ln\left(\left(1-\alpha\right)\left(k_t^i\right)^\rho + \alpha\left(g_t\right)^\rho\right) + \theta\ln\left(l_t^i k_t\right)$$

implying the productive elasticity of public investment,

$$e_t^i \equiv \frac{dy_t^i / y_t^i}{dg_t / g_t} = \frac{\alpha(1-\theta)\left(g_t\right)^\rho}{\left(1-\alpha\right)\left(k_t^i\right)^\rho + \alpha\left(g_t\right)^\rho} \quad \text{(A.12)}$$

Since we are calibrating from estimates based on aggregate data, we abstract from the heterogeneity of the individual firms. Hence we assume $e_t^i = e$, $k_t^i = k_t$ so that (A.12) implies

$$\left(\frac{g_t}{k_t}\right)^{\rho} = \frac{e(1-\alpha)}{\alpha(1-\theta-e)}$$

and taking logarithms:

$$\rho \ln\left(\frac{g_t}{k_t}\right) = \ln\left[\frac{e(1-\alpha)}{\alpha(1-\theta-e)}\right] \tag{A.13}$$

Combining (A.13) with the equilibrium condition (8c) and recalling the definition of χ we obtain

$$\rho \ln\left(\frac{\tau(1+\beta)}{(1-\tau)\beta}\right) = \ln\left[\frac{e(1-\alpha)}{\alpha(1-\theta-e)}\right] \tag{A.14}$$

Given assumptions on $\beta, \tau, \alpha, \theta,$ and estimates of the productive elasticity e, one can infer the value of ρ and hence obtain ε.

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