Visualisation as a metacognitive strategy in learning multiplicative concepts: a design research intervention

by

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DECLARATION

I declare that the thesis, which I hereby submit for the degree Doctor of Philosophy at the University of Pretoria, is my own work and has not previously been submitted by me for a degree at this or any other tertiary institution.

MC du Ploooy Date

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The author whose name appears on the title page of this thesis, obtained the applicable research ethics approval for the research described in this work and declares that she observed the ethical standards required in terms of the University of Pretoria's *Code of ethics for researchers and the Policy guidelines for responsible research.*

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ABSTRACT

At the primary school level, the understanding of multiplication and division is critical, and division is a particularly challenging concept for most learners. I argue that the concept of division can be grasped more readily if learners learn to regulate their own mental processes intentionally; and that metacognitive strategies can be cultivated. Learners' cognitive development and their propensity for visual imagery at age 11 to 12 years provide an opportunity to mediate such a strategy while learning mathematics concepts. Visualisation gives learners access to what I call "the virtual space of the mind", where they can reconstruct an external life situation as an internal reality, upon which they can act mathematically. Based on a review of the relevant literature, the use of visualisation in problem solving (division in particular) at this developmental age was found to be under-explored, which gave rise to the need to develop a useful and feasible instructional design. Design Research was applied, as its iterative, cyclic nature allows the researcher to work through phases while developing various prototypes of the design. The fourth prototype of this design was tested with sixteen Grade 6 participants in the classroom setting of an English-medium primary school in Gauteng Province, South Africa, where the strategy was mediated for division in money-, area- and rate contexts. The research enabled the identification of specific design principles to underpin a final design for the mediation of visualisation as a metacognitive strategy in learning multiplicative concepts.

Keywords: conceptual understanding; Design Research ; mathematics education; metacognitive problem solving; metacognitive strategy; multiplicative concepts; Realistic Mathematics Education; self-regulating behaviour; visual imagery; visualisation.

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ABBREVIATIONS

ANA	Annual National Assessment
CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
IEA	International Association for the Evaluation of Educational Achievement
NCS	National Curriculum Statement
NCS-R	National Curriculum Statement: Revised
NRC	National Research Council
OBE	Outcomes Based Education
RME	Realistic Mathematics Education
SA	South Africa(n)
SoP	Sphere of practice
SSS	Specialised structural system
TIMSS	Trends in Mathematics and Science Study

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CHAPTER 1 – INTRODUCTION

From various perspectives, there is agreement about the educational quandary facing South Africa (SA), with the teaching of mathematics probably being the most critical challenge. The Minister of Basic Education, Mrs AM Motshekga, committed Government to strengthened efforts to improve mathematics education within the SA school system (Motshekga, 5 December 2013; Department: Basic Education South Africa [DBE], 4 December 2014).

The Department of Basic Education (DBE) is responsible for the school system in SA, which comprises a General Education and Training band for Grade R-9 and a Further Education and Training band for Grade 10-12. The General Education and Training consists of a Foundation Phase for Grade R-3, an Intermediate Phase for Grade 4-6, and a Senior Phase for Grade 7-9. The learning areas in Foundation Phase, numeracy and literacy, are extended into six subjects in the Intermediate Phase, namely Home Language, First Additional Language, Mathematics, Natural Sciences and Technology, and Life Skills.

This chapter starts with an outline of the background and context within which the study was undertaken. The situation that resulted in the identification of the research problem under investigation, is described, as well as the demarcation of the specific focus and aims of the study. The research questions that were formulated to guide the exploration of a response to the research problem are listed. These questions aimed to systematically address various identified facets of the problem, a process that is elaborated in the chapters following. Finally in this chapter, the methodology that was employed, and the approach towards data analysis are briefly outlined. The multi-faceted outcomes of the investigation need to be viewed from various perspectives and thus are not reflected here, rather they are discussed in Chapters 6 and 7.

Background and Context

A prominent goal of the newly-elected government, marking SA's transition to a democratic political dispensation in 1994, was to provide previously disadvantaged learners with equitable education opportunities. Whereas the Apartheid school education system was characterised by differences along racial divides with a subject-specific didactical emphasis, the new government adopted a learner-centred pedagogical approach towards education (DBE, 2012a; DBE, 2012c; Mouton, Louw & Strydom, 2012).

Accordingly, the principles of Outcomes Based Education (OBE) were phased into the school system through the policy statement labelled Curriculum 2005 in 1997. The assessment system changed from normative-based to criterion-referenced, and assignments replaced examinations. There was widespread controversy about this approach and difficulty was experienced with implementing the curriculum changes at the teacher level. From 1995 to 2011, learner performance at the Senior Phase was reported to have reached unacceptably low levels in international assessments like Trends in Mathematics and Science Study (TIMSS) (Howie, 1997; Human Sciences Research Council, 7 December 2012, p. 3). The outcomes of other international comparative studies, for examplethe South African Consortium for the Monitoring of Educational Quality (SACMEQ) in 2000 and Monitoring and Learning Achievement (MLA) supported the view that South Africa's mathematics education was in trouble and that policy interventions were inevitable (Howie, 2012).

The problems experienced with Curriculum 2005 prompted a review of the guiding principles of the education system (Chisholm, 2003), resulting in the revised National Curriculum Statement (NCS-R) of 2002. Ongoing perceived implementation challenges resulted in a further revision in 2009. The product of this revision was the National Curriculum Statement (NCS) as amended by the Curriculum and Assessment Policy Statement (CAPS) published in 2012 (DBE, 2012a). The purpose of the CAPS was to provide clearer specification of what must be taught and learned on a term-by-term basis (CAPS, DBE, 2012a). This CAPS document contains details per subject and per grade, in respect of its aims as well as the learner skills required to achieve those aims. The curriculum is accompanied by learner workbooks, aligned with the requirements of the CAPS. These books are distributed annually to schools across the country. It is clear from the spirit and the letter of all the recent developments in mathematics education in SA and from the curriculum policy documents, that the SA government assigns great importance to assessment. This focus is a shift away from the approach of what came to be known colloquially as OBE, towards alignment with international standards for mathematics (Chrisholm, 2003).

A countrywide systemic assessment, the Annual National Assessment (ANA), was implemented in 2012 for Grades 1-6 and Grade 9, externally verified for the exit levels of each school phase, that is, at Grades 3, 6 and 9. This assessment may be seen as one of the indicators of learner achievement and educational quality in the SA schooling system (Motshekga, 5 December 2013). From 2012, approximately seven million learners across 24 000 schools countrywide participated in the ANA and the number of learners increased to 7,3 million in 2014. A comparison of the results in mathematics at the three exit level grades of the Foundation Phase, the Intermediate Phase and the Senior Phase from 2012 to 2014, are presented in the table below:

Table 1

Mathematics Results: ANA 2012-2014: National Averages (DBE, 4 December 2014)

	2012	2013	2014
Grade 3	41%	53%	56%
Grade 6	27%	39%	43%
Grade 9	13%	14%	11%

Even though the validity and comparability across years of the ANA systemic assessments may have weaknesses, the information points to a substantial drop of scores from one phase to another, which remains reason for concern and therefore supports the need for intervention. At the exit level of the Senior Phase, concerns were expressed about the negative progression of Grade 9 learners and their inability to reach the desired targets (DBE, 4 December 2014).

Problem Statement

The CAPS (DBE, 2012a) aims for mathematical competency at the levels of factual knowledge, procedural efficiency and "deep conceptual understanding" (p. 6), yet the poor results from international- and national (systemic) assessments raise concerns that these goals have not yet been reached in many classrooms. Reflecting on the main findings of the International Association for the Evaluation of Educational

Achievement (IEA), Howie (2013) suggested that South Africa "has not been able to overcome its deprived legacy; that the new policies have not been implemented effectively or widely; and that the country has not yet seen the fruits of its more recent initiatives related to education quality" (Howie, 2013, p. 150).

Not only was the failure of South African learners to meet international benchmarks revealed in all international studies in which the country participated (Howie, 2013, p. 150), but also the results of the national systemic assessments, (see Table 1), raised serious concerns among the policy makers. The substantial deterioration in performance in the ANA from 2012 to 2014, especially from Grade 6 to Grade 9 (Motshekga, 5 December 2013; DBE, 4 December 2014), may be due to a number of factors. There is, for example, the possibility that the deterioration was an artefact of the test development, or that the standards and practices of the ANA as an assessment instrument had not been well established at the time of testing. Even so, the concerns about learners' level of performance, especially at the Grade 9 level, seem to be justified.

One critical factor accounting for the results, I hypothesise, is a shortcoming in deep conceptual understanding of mathematical ideas. Whatever mathematical understanding learners take with them from the exit level of the Intermediate Phase (Grade 6), is clearly not supporting optimal mathematical functioning at the Senior Phase. This assumption is reinforced from both the Intermediate Phase results, where the achievement of South African learners in the SACMEQ mathematics assessments at Grade 6 in 2000 and 2007 were below the SACMEQ average (Moloi & Chetty, 2010); and from the Senior Phase results, where national and international assessment results for Grade 8 and 9 provide evidence to this effect (Howie, 2013).

It can therefore be hypothesised that some existing conceptual deficiencies in the transition between the two phases may hinder the transition from the semi-abstract concepts at the end of the Intermediate Phase to the more abstract concepts of algebra, for example, at the senior level.

According to the national admission policy, learners should enter the Intermediate Phase (Grade 4) at the age of 9 years and therefore be 12 years by the end of Grade 6 (Moloi & Chetty, 2010). I focused on the teaching of division at the age group 11 to 12 years, for a number of reasons: Firstly, from a subject perspective, some mathematical concepts have been identified as troublesome specifically at the primary level. Within the content area “number”, the multiplicative conceptual field (Vergnaud, 1988) includes both challenging and crucial concepts. Division as a threshold concept within this field, is a particularly troublesome concept (Hart, 1989; Lamon, 2007; Long, 2011). A possible reason for the difficulty is that the set of integers is not closed under the operation of division and therefore division requires conceptual understanding of rational number. Furthermore, at the Grade 6 level, various applications of the division concept are incorporated in the curriculum, such as money- or currency contexts, area and rate. The challenges of learning these concepts are discussed in Chapter 2.

Secondly, from the learner perspective, this age is an opportune stage to learn cognitively challenging concepts. Learners are cognitively and metacognitively ready at this age to reason mathematically on the basis of imagined representations (Demetriou et al., 2011); and their cognitive structures are sufficiently developed to meet the intellectual demands of various crucial yet troublesome concepts. This perspective is elaborated in Chapter 3.

Thirdly, from a teaching perspective, I reasoned that visualisation as a metacognitive strategy may be mediated and employed intentionally to regulate problem solving requiring division in various realistic contexts. I have discovered in my own teaching, and later confirmed through the literature study, that the conscious regulating of one's own thought processes is a powerful aid in the conceptualisation of mathematical ideas. This type of self-regulation is seen as an essential metacognitive ability supporting concept formation (Efklides, 2011; Koriat, 2007; Panaoura, 2007; Pintrich, 2002; Veenman et al., 2005) – an ability that is functional in learners at the age of 11 to 12 years old . At this stage therefore, learners' initial- and ongoing understanding of mathematical concepts could benefit particularly from an improved ability to regulate their own mental processes.

However, from a review of the literature it emerged that , not many studies have been directed at the application of this type of strategy in comparable contexts. No study was encountered that incorporated these three perspectives for seeking an instructional solution to a prevailing educational problem. In fact, no study was found where the use of this strategy was explored in support of the mathematical concept "division" at the age of 11 to 12 years.

It can be concluded therefore, that the use of visual imagery as a self-regulating metacognitive strategy has been under-investigated to date in relation to its potential to assist multi-dimensional understanding of real life situations within the multiplicative conceptual field on all cognitive levels. This is the case, at least at the primary level in the developmental stage 9 to 12 years, and more specifically at age 11 to 12 years, the age of most sixth graders.

Aim of the Study

Consequently, this investigation aims to explore how the intentional regulation of mental processes can be employed through structured visual imagery as a metacognitive strategy; and how the use of this metacognitive strategy influences the understanding of some concepts within the multiplicative concepts. The study was conducted with a convenient sample of Grade 6 learners of age 11 to 12 years old. Through this exploration it was hoped that some relationship(s) between self-regulating behaviour and concept formation would be observed.

In developing an instructional design towards this aim, the research had to present a set of evaluated examples of didactical situations, as suggested by Prediger and Zwetzschler (2013). In this case learners were guided in the construction of representational mental models of real problem situations, the mental constructs with which they could engage mathematically. The outcomes of their mathematising would be assessed, not only in terms of their outcomes, but also in terms of three identified dimensions of understanding (Usiskin, 2012). Participants' subjective experiences when they used the metacognitive strategy, also had to be accounted for in relation to their demonstrated competency in a series of assessments.

Research Questions

The problem of a lack of depth in conceptual mathematics understanding and the potential of metacognition to assist concept formation, guided the formulation of the main research question: "How can a self-regulating metacognitive strategy comprising structured visual imagery be used at Grade 6 level for the understanding of multiplicative concepts as they arise in realistic situations?"

The specific research questions are:

1. Which mathematics education approach should be adopted in this research to meet the requirements for the teaching of multiplicative concepts at Grade 6?
2. How does the understanding of mathematics in general, and of the multiplicative concepts in particular, come about in learners?
3. How do the cognitive- and metacognitive functions of 11 to 12 year old learners support their understanding of mathematical concepts?
4. How can a metacognitive strategy comprising structured visual imagery be mediated for the understanding of division at Grade 6?

Research Methodology

Since studies of a similar nature had not been encountered in the literature review, guidelines were lacking for the instructional design that was to be used at the practical classroom level in response to the above questions. This gap necessitated a Design Research approach to developing a novel design for the purposes of the investigation. Van den Akker et al. (2006) summed up the motivation for choosing this method of research as follows: "By carefully studying progressive approximations of ideal interventions in their target settings, researchers and practitioners construct increasingly workable and effective interventions, with improved articulation of principles that underpin their impact (p. 2)."

Design Research

In order then to answer the specific research questions, three phases of the design development were identified, in accordance with the guidelines of Plomp

(2007), Plomp & Nieveen, (2013) and Van den Akker et al. (2006). Mainly, but not exclusively, the first three questions were investigated through a literature review during the Preliminary Phase. During the same phase, the first and second prototypes of the intervention design were developed on the basis of theory, thereby starting to address the fourth research question. The fourth question was mainly investigated however, during the Intervention Phase, through the refinement in further prototypes of an instructional design for use at the classroom level. The third phase of the Design Research, the Evaluation Phase, started at the end of the practical interventions and assessments at the classroom level and was concluded with writing up the findings, the analyses of the results (Chapter 6) and the conclusions (Chapter 7). On the basis of the principles acquired in the process, the final intervention was designed.

During the Design Research process, although iterative and cyclical in nature (Plomp, 2007; Plomp & Nieveen, 2013; Van den Akker et al., 2006), some markers of specific progress could be distinguished, as follows:

1. The review of existing research and theory enabled the demarcation of the problem space for the research and the postulation of a local theory upon which a solution for the problem could be proposed.
2. To embody this solution, Prototype I of the intervention was conceptualised and designed, aiming for both relevancy to the problem context and relevance within the existing theory.
3. Prototype II was subject to various forms of evaluation, including expert- and peer review, reflective journaling and further literature study, in the quest for content validity, and consistency, within the design itself.

4. Prototype III was informally tried out with a few individuals, which prompted multifaceted refinement of the design to improve its practicability.
5. Prototype IV was tested against a target group in a formal experimental setting. Over the period of six weeks of fieldwork, Prototype IV progressively improved with regard to its practicability within a classroom setting.
6. The outcomes of the fieldwork and the measure of its effectiveness within the confines of the experimental setting, could be analysed critically in relation to the various characteristics, mainly of Prototype IV.
7. The theoretical background, the practical experience and the analysis of outcomes enabled the identification of a set of design principles, embodied in Prototype V, the final intervention design (Appendix D).
8. Following a discussion of the design with seven practising teachers, they volunteered to test it out in their own classrooms. Here the design was applied in a natural setting, however, within the time constraints of the study, the outcomes of their interventions could not be followed up.

The two levels of research are now briefly introduced, as follows:

Theoretical Underpinnings of the Design: The Literature Review

Consistent with a Design Research approach, pragmatism as a methodological tradition that is able to accommodate both qualitative and quantitative data within a single study (Creswell, 1998), was adopted as the epistemology of this research. This approach allows for experiences to be reported from a first person point of view in search of their central underlying meaning, and emphasises “the intentionality of inward consciousness based on memory, image and meaning" (Creswell, 1998, p. 52). Furthermore, pragmatism supports a developmental approach towards educational design and enables the assessment of a theoretical assumption that is investigated for solving the research problem during its practical application.

Addressing the first research question, the mathematics education approach followed in this study corresponds with that of Realistic Mathematics Education (RME), an approach elaborated by Gravemeijer in collaboration with other specialists in the field (Gravemeijer, 1994; Gravemeijer & Cobb, 2006; Gravemeijer & Doorman, 1999) and professional experts in the field, like Van den Heuvel-Panhuizen (2003). In the design of this didactical intervention, RME theory, where a mathematical intervention is completely situated within a real life situation, proved ideal for the use of visual imagery. The learner transforms elements of the real life situation into a mental image, including the numerical values and other information attached to the situation. Through manipulation and control of the mentally-observed image (which includes numerical values), the learner becomes inclined towards the mathematical action that would be needed to reach the solution. This approach is elaborated in Chapter 3 and in further chapters, and is demonstrated in its practical application in the design put forward as the final product of this research (Appendix D).

The anchor theories that have been used with regard to the second research question about mathematical conceptualisation, are firstly, the theory of conceptual fields, with a specific focus on the multiplicative conceptual field (Vergnaud, 1988; 2009); secondly, the theory that mathematical understanding is multi-dimensional (Usiskin, 2012); and thirdly, the theory of cognitive levels of understanding (Anderson, 2002; Anderson & Krathwohl, 2001; Ferguson, 2002; Forehand, 2012; Krathwohl, 2002). These theories guided the selection of material for the interventions and assessments.

The third question prompted research into the cognitive factors that could contribute towards multi-dimensional understanding of mathematical concepts, and included firstly, the theory of cognitive development at the age of the Grade 6 learner, about 11 to 12 years, drawing on Demetriou et al. (2011); and secondly, theories pertaining to metacognition in general and concept formation through self-regulation in particular. This aspect of the literature was reviewed as broadly as was possible within the time constraints. The viewpoint presented in this study, was shaped by the research ideas and practices of Desoete and Ozsoy (2009), Efklides (2011), Flavell (1979), Kaniel (2000), Koriat (2007), Kramarski (2009), Nelson and Narens (1990), Panaoura (2007), Pintrich (2002) and Veenman et al. (2005). Within these metacognitive theories, the specific focus was on theories of mental visualisation in mathematics as was specifically addressed by Presmeg (2006) and Mason (2002).

This report varies in respect of the order and dynamics according to which the concepts of the above focal theories were integrated within the present design. The various theoretical arguments and research results encountered in the literature, culminated in the theoretical framework according to which the Design Research could be undertaken practically to investigate the fourth research problem.

Implementation of Theory on the Design Level: The Design Process

The various theoretical perspectives from the literature review provided the structural components of the Design Research, in five respects, firstly, the specific goal of the design itself could be refined and articulated; secondly, the anchor theories for each of the various aspects of the design could be selected and synthesised; thirdly, investigative tools, such as scaled assessment instruments, could be developed; fourthly, the protocol for mediating the metacognitive strategy could be adapted from existing models; and lastly, preliminary design principles, which were to be refined through the various cycles of the design development, could be identified. However, in the absence of interventions of a similar nature in the existing research, the intervention itself had to be created anew.

To this end, numerous design tasks were undertaken, manifesting in a number of developmental cycles, as the design progressed and took shape in different prototypes. In retrospect, the design activities were inherently systematic and could be ordered in a logical sequence, although they did not appear to happen in a neatly-defined chronological order during the design process. The developmental tasks that were undertaken, were as follows:

Keeping a design journal. A journal was kept for the period of developing the design. For the phases and cycles of the Design Research process, the reflective journal was used to capture and critically evaluate the products developed for the various prototypes of the design and the rationales accompanying their rejection, alteration or acceptance. As a subjective account, the journal includes experiences and self-reflection. From this point on, reference will be made to the reflective research journal as the "design journal".

In reviewing the reflective journal, both the preliminary and final products of the various prototypes were derived from the progressive development within the journal as it developed chronologically. The journal reflects the sequence of the design cycles and the progressive research work that went into the design, as well as the various stages of developing research- and assessment instruments, items and instructional models. The informal, reflective rationales within the original text of the journal, were formalised for use in the chapters of the main body of the thesis.

Defining the cognitive abilities of the average Grade 6 learner. The cognitive abilities range that can reasonably be expected from an 11 to 12 year old, the typical age of a Grade 6 learner, had to be identified. These abilities extend mathematical thinking to reasoning with ideas (Copeland, 1984) and to imagining the non-real. Flexible logical multiplication and proportional reasoning are within their cognitive reach and the logical validity of propositions can be proved; variables can be isolated and symbolic structures can be coordinated; and domains can be differentiated on the basis of their qualifying characteristics (Demetriou et al., 2011). These abilities of thinking- and reasoning demarcate the type of mathematical problems that can be set in teaching and learning mathematical concepts at this developmental stage.

Demarcating the conceptual field of the study. Given the previously mentioned identification of division as a particularly difficult concept, the choice for a mathematical domain was the multiplicative conceptual field (Vergnaud, 1988) with a specific, but not exclusive, focus on division. Mathematical work in the multiplicative conceptual field at the primary level in general and Grade 6 in particular, requires the acquisition and use of multiplicative concepts such as multiplication, division, ratio, rate, fractions and proportion – as all of these concepts are embedded in situations in the real world. The cognitive demands for concept formation within this field align well with the cognitive abilities that reasonably can be expected of learners 11 to 12 years of age.

Investigating the nature of the multiplicative conceptual field. Vergnaud (1988) proposes that there is a relationship between mathematical concepts and situations: a concept may be applied to many situations, likewise for any one situation, many concepts may be applied. It is necessary therefore to define a conceptual field.

In the case of this study, the concepts involve multiplication and its inverse, division and therefore is embedded in the multiplicative conceptual field. Situations that embody division were selected from the learners' experiential reality, as suggested in RME (Gravemeijer, 1994; Gravemeijer & Doorman, 1999), and were represented as mathematical problems. Three contexts within the multiplicative conceptual field were selected that require the understanding of division. These were a currency context (money), a rate context (speed) and a measurement context (area).

Standardising the mathematical problems for the intervention. The assessment items were analysed, using a matrix that captures the dimensions of understanding (Usiskin, 2012) and the levels of cognitive demand (Linn, 2002) or cognitive levels (Anderson & Krathwohl, 2001) required in each item. This matrix made it possible to evaluate pre- and post intervention assessment outcomes in a uniform manner, in addition to standard scoring assigned according to a memorandum.

Designing the intervention. Applications of the metacognitive strategy of visual mental imagery were subsequently prepared and a protocol was established according to which the strategy would be mediated within selected situations. The mediation steps were organised according to a model for metacognitive problem solving based on the work of Kaniel (2003). To evaluate the effect of the strategy on group- and individual understanding, a test-teach-test approach was followed, drawing on the work of Feuerstein and Rand (1974) and Tzuriel (2001). This approach included a baseline assessment, three interventions and a summative assessment.

Executing the intervention. With the sample group, the participants were guided to employ some of the available modes of representing a problem situation, namely verbal, visual and kinaesthetic modes as suggested by Kress (2009), internally within their mind space while building mental models (Ambrose et al., 2003). Irrespective of whether participants had pre-knowledge of the concepts used in the intervention, the instructional interventions included no mathematics teaching, only the mediation of the self-regulating metacognitive strategy within a given problem situation. This approach made the influence of the metacognitive strategy discoverable as it isolated, to an extent, the intervention effect of the metacognitive strategy as the sole influence on mathematising in the experimental situation.

Assessing the effect of the metacognitive strategy. In their written calculations, learners were free to follow their method of choice, be it formal algorithms, self-invented informal strategies (Greer, 1992) or other heuristic methods of problem solving. The effect of the metacognitive strategy on mathematics performance was assessed, before, during and after the interventions. The outcomes of the assessments were analysed and described, not only in terms of scores, but also according to the level of cognitive demand (Linn, 2002) displayed in participants' responses and their range of mathematical understanding(s) (Usiskin, 2012).

Research Data: Methods of Collection and Analysis

Research data included quantitative information and a minor qualitative component. The data was collected during the formally-arranged interventions at the research site in the experimental setting of a classroom, where sixteen Grade 6 learners from a public school participated once a week for six weeks. The data derived from these encounters made it possible to consolidate the findings about the effect of the interventions. The method of data collection in this phase was assessment of learner performance and the completion of a metacognitive questionnaire by participants. The relevant documents, as well as learners' work-in-progress and their reported metacognitive experiences according to a questionnaire, were collated in a learner portfolio per individual participant. The work was transcribed and assessed (Appendix C), and represented in the form of tables and graphics. Participants' subjective reporting about their metacognitive experiences, were added to the individual reports of performance and the responses were discussed in relation to their performances.

Metacognitive questionnaires. A minor contribution towards the data was of a qualitative nature and was obtained from questionnaires completed by participants about their experiences of using the metacognitive strategy. Metacognitive knowledge and experiences are highly subjective (Desoete & Ozsoy, 2009; Efklides, 2011; Flavell, 1979; Pintrich, 2002) and are not necessarily seen to reflect the factual situation. They did, however, serve to add a subjective dimension to the assessment results and, as mentioned, were treated in relation to the quantitative evidence of performance within the discussion of individual participant performance (Chapter 6).

Assessments. Quantitative data were collected from a baseline assessment, three intervention assessments and two post-intervention assessments. The reasons for repeating the post-intervention assessment are discussed in Chapter 5. Five sets of assessments containing fifteen situated problem items were therefore extended to six sets, containing a total of eighteen items from the selected mathematical contexts.

The assessments done by the participants in the experimental sample were (a) assessed according to a (traditional) memorandum and (b) plotted on a matrix of dimensions and levels of understanding. These two sub-sets of data were compared and allowed for findings about learner performance and progress.

The tabled scores of each participant were represented graphically and their metacognitive responses were transcribed, followed by a content analysis of the performance and progress per individual. The assessment outcomes were related to each individual's reported metacognitive experience. The group performance over the intervention period was reflected in the same way. The assessment items were also evaluated separately, per class of division problem.

Limitations and Suggestions for Further Research

The study was conducted on a small scale and it would be recommended that a larger-scale investigation be done to further investigate the effect of this type of metacognitive strategy on mathematical concept formation. Involvement and active participation of practitioners at various stages of the intervention did not materialise to the full as planned and it leaves the need for testing out the design in various classroom settings in further studies. These limitations are elaborated in Chapter 7.

Outline of Chapters

The study was arranged in the following chapters and appendices:

- In Chapter 1, a broad overview is provided to introduce the reader to the study.
- Chapter 2 contains the first two parts of the literature review according to the first two specific research questions, i.e. the teaching approach that should be followed in this study, and some perspectives on the dynamics of mathematics learning, with a specific focus on division at the primary level.
- Chapter 3 continues the literature review, addressing the third specific research question, i.e. the cognitive and metacognitive facilities available in learners of the age 11 to 12 years, to understand mathematics concepts. The chapter is concluded with a conjecture from the present situation towards the desired situation, my own local theory and the conceptual framework for the study.

- In Chapter 4, Design Research as the methodology of this study is explained, as well as the research planning according to which the study would progress.
- Chapter 5 reports the design process and the development and testing of the various prototypes of the instructional design.
- Chapter 6 contains the descriptive analysis of the research data obtained in the fieldwork from the group of participants, from the individual participants, and from the three assessment item types.
- In Chapter 7, a reflection of the study is offered, its findings are reported, and the conclusions and suggestions for further research are documented.
- Appendix A contains a journal that was kept to record the design process as it unfolded.
- Appendix B provides a record of the analyses of the assessment items used in the empirical phase, according to a standard memorandum and a matrix of understanding.
- Appendix C contains the verbatim responses of the participants, their scores and ratings of their understanding and some summative tables and graphic representations of the outcomes of assessments.
- Appendix D presents the final instructional design that is suggested as an example of the integration and mediation of visualising in a classroom lesson when division in a rate context is taught and learned at Grade 6.
- Appendix E contains the actual letters of consent and assent that were distributed to the prospective participants in the research.

CHAPTER 2 – LITERATURE REVIEW: MATHEMATICS TEACHING AND LEARNING

An instructional design is researched in this study, for the mediation of visual imagery as a self-regulating metacognitive strategy in the understanding of multiplicative concepts, as they occur in real-life situations. The study is positioned within the SA mathematics education context, with a focus on learners at the senior primary level, specifically at Grade 6, where learners are 11 to 12 years on average.

The specific questions that were employed to guide the selection of literature for this review, focus firstly on the mathematics education approach that is followed in the research to meet the curriculum requirements for the multiplicative concepts at Grade 6; secondly, on the process of understanding mathematics in general, and multiplicative concepts in particular; and thirdly, on the role of learners' cognitive- and metacognitive functions at age 11 to 12 years, in understanding mathematics concepts. Following this arrangement, the theoretical investigation of the study is organised according to these three focus areas, while the fourth specific question is used to direct the practical aspect of the research with regard to the way that a metacognitive strategy of visual imagery can be mediated for understanding of division at Grade 6.

Mathematics Education

In the first part of the literature review, some aspects of mathematics education are addressed. The South African policy context within which the specific focus of this study resides is now briefly discussed as a background to the further discussions.

South African Mathematics Education: Policy Context

The NCS as amended by the CAPS (DBE, 2012a) contains the SA mathematics curriculum. Two documents appeared consecutively, namely the final draft of CAPS (DBE, 2011) and the official CAPS policy (DBE, 2012a). The only difference between the two documents is that in the official document the weighting of the content area: numbers, operations and relationships for the Intermediate Phase is increased, following a heightened awareness of the importance of number sense (DBE, 2012a, p. 12). This increase is only in name, since no adaptation to the content areas was made, neither to the time allocation nor the depth within which the content area is addressed. In this thesis, reference is made to the official document (DBE, 2012a).

The CAPS implementation was followed up by two learner workbooks per grade (DBE, 2012c) based on the "concrete-representational-abstract" sequence of instruction. This approach corresponds broadly with the "enactive, iconic and symbolic modes of representation" that typified Bruner's (1977) theoretical stance. Success has been widely reported using this instructional approach with at-risk learners in remedial settings (Jordan et al.,1998).

The implementation of CAPS resulted in the publication of at least five series of CAPS-aligned textbooks, teacher guides and extra materials for school- or home-based use. Various e-sources, including a national education portal at the initiative of the DBE, provide similar support, both on- and off-line. Some schools print their own learner workbooks and teachers devise their own worksheets. School-based and private extra mathematics classes are available to assist in implementing the CAPS.

Structure of the mathematics curriculum. The instructional aims for mathematics in the CAPS (DBE, 2012a, p. 8) are firstly of an affective and attitudinal nature in keeping with its explicit goal to remediate the detrimental effects of the inequalities of education during the Apartheid period (Mouton et al., 2012). Secondly, the aims of the curriculum in the cognitive domain are to make sense of mathematical situations through deep conceptual understanding and through mathematical knowledge and skills that can be used to solve problems within and beyond the subject. The content knowledge and practical skills required for the Intermediate Phase are systematically set out across the content areas, showing a clear progression from Grade 4 to 6, with evidence of increasing integration of knowledge and skills into combined configurations and examples of problems requiring the application of skills.

Assessment requirements of the CAPS. The implementation of assessment prescribed by the CAPS (DBE, 2012a, pp. 294-296), as it relates to the promotion requirements (DBE, 2012b), is complex and therefore could prove problematic to implement in the classroom. Different types and forms of assessment are prescribed at different points in the year, and assessments span the content areas and cognitive levels in various ratios. The types of assessment are baseline, diagnostic, formative and summative assessments, in the form of tests, examinations, oral questions, teacher observation, learner interaction, workbook evaluation, projects, assignments and investigations. In a separate document (DBE 2012b), a scale of achievement is set out that contains a description of competence in seven levels of achievement, described in a range from “outstanding achievement” (7) to “not achieved” (1), in percentages from 100% to 0%. A moderate level of achievement (in the range 40 - 49%) is required for promotion in mathematics from Grade 4 to 6.

Some Theoretical Perspectives on Mathematics Education

The following general theoretical perspectives have informed the mathematics education approach that was adopted within the present design:

Positioning the learner, the teacher and the subject contents. Three dynamic elements of an educational situation are discerned in various instructional approaches, i.e. the learner, the teacher and the contents being taught and learned. In preparation for this Design Research, a brief look was taken at the difference between a pedagogical and a didactical approach, which is generally regarded as related to the distinction between a learner-centred approach and an approach where the teacher and the subject matter enjoy priority in education (Oerbaek, 8 June 2011; Fraser, 7 July 2000).

Lavigne (7 July 2000) distinguished the two positions as follows: “Pedagogy deals with the learning process, focused thus on the learner; didactics with the disciplinary content you want to teach, thus on a teaching standpoint.” He argued that even in a pedagogical approach, account has to be taken of the discipline specific knowledge that must be passed on to the learner. The inverse of this statement can be argued for including the learning process in a didactical encounter – which mitigates any notion of a stark contrast between the two approaches.

Oerbaek (8 June 2011) citing Hetmar (1996), argued that learners can become disciplinary participants in the didactical encounter. This notion resonates with my opinion that the learner as disciplinary participant, helps to position the subject knowledge in the centre of the educational encounter, that is, both the teacher’s existing knowledge and the learner’s emerging knowledge.

Hudson (2008; 2012) regarded the teacher as both a pedagogical leader and a didactical designer, in that “the central role for the teacher at the core of the teaching-studying-learning processes is seen in overall terms as the designer of teaching situations, pedagogical activities (studying) and learning environments” (2008, p. 4).

In conclusion, although the design in this study is termed a “didactical design”, it takes equal account of the interests of the teacher, the subject and the learner.

Realistic Mathematics Education. RME advocates mathematics instruction as a human activity based upon the modelling of contextual problems which are situated in the experiential reality of the learners (Gravemeijer, 1994; Gravemeijer & Doorman, 1999). This approach originated in the early seventies and has been researched and promoted in the Freudenthal Institute for Science and Mathematics Education at the University of Utrecht (Gravemeijer & Doorman, 1999). Freudenthal (1905-1990) followed a phenomenological approach towards learners’ experience in mathematics, not only because the contexts that give rise to mathematising are selected from their lived realities, but also because instruction facilitates the transcendence of this reality to a subjective experience. The following design principles in RME apply to the present study:

Realising situations promote concept- and model formation. An assumption in RME, considered to be a design principle, is that learners’ experience of phenomena in the real world will spontaneously result in concept- and model formation; and that these will enable further applications in mathematics. Prediger and Zwetzschler (2013) embarked on a study to concretise this general principle in a design for a specific mathematical topic. In this study, the formation of models of any kind did, however, seem not to follow naturally upon the teacher’s intervention or the learners’ experience of a real-world situation (2013). In-depth adaptations and various developmental cycles have to be undertaken to enable more effective model formation. In this way, instructional design in RME is constantly developed and refined through research, experimentation, analysis and reflection (Gravemeijer & Cobb, 2006; Gravemeijer & Doorman, 1999) to effect the shift away from traditional didactical approaches in mathematics education towards instruction based on the principles of RME.

Mathematics learning follows an informal-to-formal progression. In the traditional approach, instruction would typically progress from formal knowledge transfer through routine applications, to contextual applications. Within RME, however, an innovative space is created where learners are guided to start mathematising from real contexts through their own mathematical activity (Gravemeijer, 1994), before learning is formalised in mathematical language and algorithms. Following this instructional order, a second design principle in RME is, therefore, that informal problem solving serves as a foothold for more formal mathematics, eventually enabling generalisations to different contexts.

In 1978, Treffers elaborated Freudenthal's original distinction between formal and informal mathematising (Van den Heuvel-Panhuizen, 2003). He introduced the notion of (informal) "horizontal" mathematising where unmathematical matter is organised in a mathematical way and mathematical tools are used to solve realistic problems. He termed the formal way "vertical" mathematising, when some level of schematising is reached and insight into the general principles behind a problem enables systematic problem solving (Van den Heuvel-Panhuizen, 2003).

Evolving models enable the transition from informal- to formal mathematising. A third design principle in RME, related to the second, is that emergent self-developed models serve to bridge the gap from horizontal to vertical mathematising (Gravemeijer, 1994; Barnes, 2004). In RME, a wide range of representations of a problem situation can serve as models, such as physical materials, visual sketches, mental images, diagrams and symbols, to mention a few examples.

An important focus derived from the above design principles is that learners are accompanied in their upward journey from horizontal to vertical mathematising through "guided re-invention" (Gravemeijer, 1994; Doorman, 2001). In this journey, mathematical truths, even rules and theorems, proofs and properties are discovered and re-invented from their own mathematical experiences (Slavin & Lake, 2008).

Mediation as an instructional approach. Prominent in all of the above theoretical perspectives is the specific role of the teacher as guide of the learning process. I conceptualise guidance as mediation, as it is practised in the International Centre for the Enhancement of Learning Potential and is explained as being:

> "...the interactional process between the developing human organism (the learner) and an experienced, intentional adult (the mediator), who – by interposing him or herself between the learner and external sources of stimulation – mediates the experience by selecting, framing, focusing, intensifying, and feeding back environmental experiences... to produce appropriate learning sets and habits" (Feuerstein et al., 2006, pp. 14-15).

Mediated learning as it is described in the above quotation, is employed as a pedagogy (Tan, 2003) within the present design. The process of guided, or mediated logic in my final design (Appendix D), where learners adopt and use the proposed metacognitive strategy to experience situations mathematically, corresponds in all respects with the approach of mediation in this study.

Theoretical Perspectives on Multiplicative Concepts

The understanding of multiplicative concepts is required at the Intermediate Phase, for computational skills and proficiency in the content area, “Numbers, operations and relationships”, and for applications within the content areas, “Patterns, functions and algebra, Space and shape (geometry), Measurement, and Data handling (statistics)”.

CAPS requirements. At the Grade 6 level, 14 hours are allocated for the direct teaching of multiplication and 21 hours for division, which represents 20% of all available instruction time, that is excluding their applications within the other content areas. Grade 6 learners start by dividing 4 digits by 2 digits and continue to dividing 4 digits by 3 digits. Although various “methods” or algorithms for multiplication and division are introduced, learners are given the option to use the method they prefer.

The multiplicative conceptual field. Vergnaud (1988) demarcated conceptual fields according to clusters of related concepts. A conceptual field consists of a set of concepts and a set of situations that are aligned in some respects (Vergnaud, 1988; 2009; 2010). These structures do not exist in isolation, but have (conceptual and situational) links to other conceptual fields. The concept of multiplication and its inverse division, together with the situations requiring these operations, form the base of the conceptual field of multiplicative structures, and as such are observed in all the elements of the field. The multiplicative conceptual field, according to Vergnaud, takes anything from 7 to 18 years to develop (Greer, 1992, p. 282). Multiplicative concepts build on the additive conceptual field and provide links to elementary algebra (Long, 2011). Vergnaud (2009) summed up this theory as follows:

> "The theory of conceptual fields is a developmental theory. It has two aims: (1) to describe and analyse the progressive complexity, on a long- and medium-term basis, of the mathematical competences that students develop inside and outside school, and (2) to establish better connections between the operational form of knowledge, which consists in action in the physical and social world, and the predicative form of knowledge, which consists in the linguistic and symbolic expressions of this knowledge. As it deals with the progressive complexity of knowledge, the conceptual field framework is also useful to help teachers organize didactic situations and interventions, depending on both the epistemology of mathematics and a better understanding of the conceptualizing process of students" (Vergnaud, 2009, p. 83).

In the development of operational knowledge, the learner takes a series of action steps or follows a scheme, in which she uses an existing, initially intuitive and informal, concept to engage with a mathematical situation, termed a concept-in-action (Vergnaud, 2009). The implicit use of the structural properties during problem solving, is demonstrated in learners' explicit applications, which provides the teacher with clues of their reasoning and approach to the situational problems, a phenomenon which Vergnaud termed "theorem-in-action" (1988, p. 144).

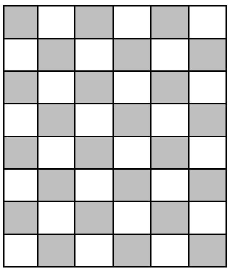
A modelling theory. Schwartz and Kaput (1988, cited in Greer, 1992, p. 284) viewed mathematics as a modelling activity where aspects of the world are counted or measured. They approach multiplication and division from an intensive and extensive point of view, where the quantity derived at, differs in kind from the two used to obtain that result, as follows: If we use 35 kWh electricity per week (extensive quantities), the electricity consumption per day is 5 kWh – an intensive quantity or the numerical expression of a constant multiplicative relationship (Greer, 1992, p. 283).

Classes of multiplicative concepts. Table 2 contains examples of situations set at a Grade 6 level, using Vergnaud's (1988; 2010) classification of problem situations within the multiplicative conceptual field. Pictorial (image) modelling and numbers are used to represent the elements and solutions to the problems, as follows:

Table 2

Classes in Multiplication and Division (Vergnaud, 1988; 2010)

<table>
<tr><td>Isomorphism of measures
(Vergnaud, 1988; 2010)
Corresponds with “mapping rule” (Nesher et al., 2003)</td><td>Product of measures
(Vergnaud, 1988)</td><td>Multiple proportions
(Vergnaud, 1988; 2010)
Corresponds with “multiplicative comparison” (Nesher et al., 2003)</td></tr>
<tr><td>Sets correspond in direct proportion to each other</td><td>Two measure spaces mapped onto a third (Greer, 1992)</td><td>Expressed as “n times as many as…” (Greer 1992, p. 277).</td></tr>
<tr><td>Ratio:
There are 8 boys and 12 girls in a dance group of 20 dancers. In the same ratio, how many boys will be in a dance group of 55?
b b b b b b b b b b b b b b b b b b b b bb
ggg ggg ggg ggg ggg ggg ggg ggg ggg ggg ggg
---- ---- (8)
----------- ----------- (22)
---- ---- ---- (12)
----------- ----------- ----------- (33)

Rate:
Thembi's car with a 35l petrol tank costs R437.50 to fill. Fred's car has a 45l tank. How much will it cost to fill his car's tank?
||||| ||||| ||||| ||||| ||||| ||||| ||||| = R437.50
| = R12.50 or ||||| = R62.50
||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| = R562.50</td><td>Area:
How many square tiles of 50cm length are needed to tile a floor that is 4m long and 3m wide?

Cartesian product:
With 5 T-shirts, two jeans, and one pair of shoes, how many different outfits do you have?
Ta Ta Tb Tb Tc Tc Td Td Te Te
Ja Jb Ja Jb Ja Jb Ja Jb Ja Jb
S S S S S S S S S S
1 2 3 4 5 6 7 8 9 10</td><td>A worm walks 4 times slower than an ant. The ant walks at 60mm/sec. If they both start walking on your 30cm ruler at the same time, how long will it take each of them to finish?

You are 12 years old and you are now 4 times as old as your sister. In how many years from now will she be half your age?
___|___|___|___ (12)
___ (3)
6 years from now...
___|___|___|___|___|___ (18)
___|___|___ (9)</td></tr>
</table>

Fischbein et al. (1985, cited in Greer, 1992) held that learners have primitive models of division, i.e. partitioning and quotition, and that they intuitively decide which operation to choose in a problem, using two pieces of numerical data. They asserted that partitioning is the original model (p. 187).

The role of division in the progression of the number concept. The integers are not closed under division. This property requires progression in learners' understanding of numbers to include rational numbers. Two instances of rational numbers that are encountered at Grade 6, are the infinite number of fraction representations (of whole numbers) and the concept of decimal numbers. The concept of rational numbers can be regarded as a threshold concept, or a gateway to the understanding of various other mathematical concepts (Long, 2011). Early misconceptions are common (Fischbein, 1985, cited in Greer, 1992, p. 287) and activities need to be carefully planned and thoughtfully presented to ensure conceptual understanding.

Young learners' strategies to deal with division computation mentally, play a role in the overall progression of their mathematics understanding. Computational strategies employed in this regard, have been researched, amongst others, by Ambrose et al. (2003), Verschaffel et al. (2000; 2007) and Voutsina (2011), as follows:

When dividing, young learners would use repeated subtraction (216 – 12, 204 – 12, 192 – 12...) or building up (12, 24, 36...), instead of counting down. What is used in multiplication as a partitioning number strategy, is also used for division, by expanding the number and creating sub-problems that are more easily dealt with (120 is 10 x12; 60 is 5x12; 36 is 3x12; so 216 is 18x12). They also use a compensating strategy for division, which is usually used for multiplication, or they fill up or round off a number to an easier dividend and then adapt the quotient accordingly, e.g. (240 = 20x12; 216 is 2x12 less; 20 – 2 =18). As a starting point, these learners used a direct modelling strategy, with a drawing or concrete manipulatives of the problem elements.

Language and terminology in mathematics education. Mathematics **t**erminology of which the meaning is unclear to the learner, poses a barrier to learning. In SA, numeracy at Grades R to 3 is taught in the eleven official languages to provide for education in the learners' first language. These languages are Afrikaans, English, isiNdebele, isiXhosa, isiZulu, Sepedi, Sesotho, Setswana, Siswati, Tshivenda and Xitsonga. From Grade 4 onwards, all non-language subjects, like mathematics, are taught either in English or in Afrikaans. The textbooks and workbooks based on CAPS are translated into the eleven official languages up to Grade 3, but the resource languages for education at the Intermediate Phase and further, are only English and Afrikaans. The challenges posed by this arrangement are particularly pertinent in subjects such as mathematics and science. Wababa (2010) found that many learners' command of English is not sufficiently well-developed to master mathematical concepts expressed in English and that there is often a lack of appropriate terms in their first language.

While words like "product" and "quotient" can be memorised to mean the answer of a multiplication or division sum, terminology like "rate", "average", "ratio", "per..." and "out of" require conceptual understanding and a functional command of the English language, to enable the distinguishing of intended nuances.

Nesher et al. (2003) placed special emphasis on the added complexity brought about by cultural differences in comprehension as a result of differences in language.

There is also a dynamic interaction between thought and language in mathematics learning (Greer, 1992, p. 286). Subtle distinctions between terms may improve the understanding of concepts. Confrey, for example (1994, cited by Ambrose et al., 2003, p. 66) obtained success in her research with replacing the word "dividing" by "splitting", which even resulted in easier conceptualisation of the division of fractions.

Synopsis of Literature Review: Mathematics Education

In this part of the literature review, the first research question is addressed, namely: Which mathematics education approach should be adopted in this research to meet the requirements for the teaching of multiplicative concepts at Grade 6?

In this part, the central position of the teacher within the teacher-subject-learner triad of education is highlighted, specifically the role as designer of the educational encounter. The focus is on the teaching of increasingly complex concepts and on the demand on teachers to balance the extrinsic and intrinsic subject requirements, through an approach conducive to learner receptiveness for meeting those requirements.

Following the review, the pivotal role of the teacher is seen as follows:

- The teacher takes a central position as the designer of interventions that meet both the subject demands and the learners' needs and abilities.
- The teacher designs interventions around mathematical concepts as they arise in realistic situations of the learners' lived reality.
- The teacher understands the subject-intrinsic structures underpinning the concepts, including situations and classes of the concepts; how they can be

modelled; multiplication and division methods; language and terminology; and the role of these concepts in the progression of the number concept.

- The teacher knows and complies with the extrinsic requirements of the curriculum for instruction and assessment in this mathematics content area, within the prescribed scope and at the prescribed level.
- The teacher adopts the role of mediator in the instructional situation, between the mathematical concepts and the developing learner understanding.

Mathematics Learning

In the second part of the literature review, the second specific research question is addressed, namely: How does the understanding of mathematics in general, and of the multiplicative concepts in particular come about in learners? The complex dynamics of learning and understanding mathematics require both openness to different viewpoints and cautious consideration of the merits of various mathematics learning theories. The norm applied in this review, is whether a learning theory views factual knowledge, computational skills and conceptual understanding of mathematics in a balanced way.

Aspects of Mathematics Knowledge- and Learning Theory

Many mathematics learning theories stress learning with understanding, as contrasted to rote learning and mechanical execution of procedures (Mayer, 2002). Some mathematics learning theories about learning goals, types of mathematical knowledge and the learning process are now briefly discussed.

Mathematics learning aimed at the construction of meaning. For Kilpatrick et al. (2005), the aim in mathematics education is the mediation of meaning. They regarded the classroom as a "Sphere of Practice" (SoP) where gaps in meaning are filled, where the evolution of meaning is stimulated and meaning is communicated. Mathematics has applications in many fields and therefore, Kilpatrick et al. (2005, p. 12) held that the meaning of a mathematical concept is discovered within the situation where a problem is solved, and should not be isolated as a classroom matter.

The concept division, for example, applies to meanings in other SoPs as well: $492 \div 6 = 82$ in the classroom may be linked to its meaning in the transport SoP, where 492km completed in 6 hours means an average speed of 82km/h. Teaching can also depart from the concept rate, as a situation where the division concept is used.

Mathematics knowledge components. Hiebert and Carpenter (1992) distinguished between factual, procedural and conceptual knowledge of mathematics. Whereas the notion of factual knowledge is almost self-evident, procedural knowledge is defined as "a sequence of actions… the manipulation of written symbols in a step-by-step sequence" (1992, p. 78). They argued for a relationship between procedural and conceptual understanding, rather than contrasting the two aspects. They reasoned that the external representation of mathematical quantities may start forming internal mathematical representations of quantities, which can be depicted as conceptual understanding. Ferguson (2002) concurred with this notion.

Progression in mathematics knowledge. With regard to the relation between procedural and conceptual knowledge, Pantziara and Philippou (2011) found that learners have a better command of fractions, if conceptual knowledge has been developed alongside procedural knowledge. Voutsina (2011) found supportive evidence for the "iterative model" in her study of young learners' conceptual mathematical development. Although changed problem-solving behaviour stems from the interaction between procedural and conceptual knowledge (Rittle-Johnson et al., 2001), the initial conception of a task was found to have a decisive influence on later representations (Voutsina, 2011). It seems entirely plausible therefore that failure to make the cognitive transition from knowledge and routine procedures to complex procedures and problem solving, may in part at least, account for deteriorating mathematical competence at the later grades.

Sfard (1991; 1998) described the long and difficult progression in the transition from computational operations to abstract objects. The starting point of this process is when an external or physical operation is interiorised as a mental representation. Following interiorisation, learners conceive a sequence of steps as belonging to a single action, from input though processing to output – a point in the transition which she termed condensation. Reification in Sfard's view, has popularly been described as the Aha! moment in conceptualisation, and happens when the condensed process is understood as an object in its own right, and which can be acted upon (Sfard, 1991).

In relation to the progression of mathematical knowledge, the idea of Treffers (1978, cited by Van den Heuvel-Panhuizen, 2003) can be mentioned, that learners start mathematising informally when they organise unmathematical matter mathematically and when they employ mathematical tools to solve problems. He called this stage "horizontal" mathematising, after which learners progress to formal- or "vertical" mathematising, that is they reach a level of schematising and they have insight into the principles behind problems, which then enables more advanced problem solving.

Mathematics Understanding

The relative simplicity of multiplying or dividing as mathematical computations can mask the psychological complexity of how they model situations

(Greer, 1992). Superficial success can be reached when numbers alone are manipulated without understanding their representations in real life situations (Greer, 1992, p. 286). The understanding of mathematics concepts in real life situations is now reported as they have been investigated from various theoretical perspectives.

Dimensions of understanding. Usiskin (2012) emphasised mathematical understanding from the learner's perspective as he introduced a perspective which he identified as positioned between Freudenthal's didactical phenomenology (discussed in Chapter 2), and Hiebert and Carpenter's (1992) approach involving networks of understanding.

One of the central theories of mathematical understanding employed in this study, is that of Usiskin (2012). The constituting elements and perspectives in his theory are listed below:

- Mathematical activity consists of concepts and problems.
- A concept can be termed a concept if it can be systematically analysed.
- Five dimensions, which all contribute to understanding, can be distinguished.
- No dimension is more crucial than another.
- No dimension implies a higher order of understanding than another.
- Dimensions of understanding do not necessarily develop in any set order.
- Each of the five dimensions describe some facets of the complex dynamics at work in mathematical problem solving.

Four of the five dimensions of understanding are discussed as follows:

The skill-algorithm dimension. Usiskin (2012) described this dimension of understanding as knowing how to find an answer. He defied the notion that skill is a lower order of thinking, but argued for a higher or lower level of understanding within the skill-algorithm dimension. An example in this regard is the standard algorithm of long division in a vertical column as a way of treating division. When understanding reaches a higher level, the learner becomes comfortable with various alternative methods too, like breaking up the dividend and the divisor.

The use-application dimension. Usiskin (2012) reasoned that the skill to perform algorithms does not necessarily constitute the knowledge of when, or under which circumstances, to apply the skill. The use-application dimension could assist the compilation of a series of logical actions to be taken, even if more than one operation is required in complex problems. The learner has to recognise within a problem or situation, where there are an infinite number of possible variations, which operation to use. This ability is pertinent to the logical processes involved in mathematical thinking; at the same time, it is often an illusive goal of instruction.

The representation-metaphor dimension. This dimension has bearing on the semiotics of mathematics, including the understanding of words, signs and symbols. Words like rectangular, wide, long, square and tessellate, are terms with specific meanings in mathematics and once misinterpreted, may result in faulty assumptions. Punctuation and abbreviations are equally important in the transition of the verbal problem to a solvable mathematical problem and representing the solution in a mathematically correct form. Mathematical notation, such as the decimal comma, as well as the operational signs need to be precisely understood as well. The formal teaching and learning of mathematical representations and metaphors may be a decisive factor in the success of solving the problem.

The property-proof dimension. According to Usiskin (2012), someone does not really understand mathematics unless they can identify the mathematical properties of a concept and also prove why their way of reaching the solution is valid. This dimension includes what can be called the "theory" of mathematics and contains not only knowledge, but also the understanding of that knowledge in applications.

Neither the property-proof dimension, nor the fifth dimension, the historical-cultural understanding of a mathematics concept is in the focus of my study. I was not sure whether learners age 11 to 12 were cognitively ready for this type of understanding. In this regard, a recommendation for further research is made at the end of Chapter 7.

Cognitive levels in understanding mathematics. Panaoura (2007) defined the term intelligence as "a hierarchical and multidimensional edifice", associated with individual differences in process appropriateness and/or working memory (2007, p. 31). I am not using the term "intelligence", rather I refer to cognition in the way it was characterised by Feuerstein and Rand (1974) and Feuerstein et al. (2006), namely as a developmental process which, over time, effects structural changes to the brain and enables applications of learnt experiences to new situations.

Theorists use different terms to describe various cognitive hierarchies or levels. The number of levels and the terminology used to describe those, also vary slightly. Linn (2002) used the term "level of cognitive demand" for what CAPS termed "cognitive levels" (DBE, 2012a, p. 296). Four cognitive levels at which assessment is to be conducted, are distinguished in CAPS, i.e. knowledge; routine procedures; complex procedures; and problem solving. This classification corresponds with the International Assessment of Educational Progress categories of "conceptual understanding, procedural knowledge and problem solving" as well as with the Subject Assessment Guidelines of the 1999 TIMSS Mathematics survey taxonomy of categories of mathematical demand (Stols, 2013, p. 13).

The Report of the National Research Council of the USA (NRC, 2001) described the progression in learning from a basic level of factual knowledge (retrievable through memory), to a level of "deep understanding", where concepts are reorganised for further application within the domain. Within a specific domain, "comprehension and thinking require a coherent understanding of the organising principles in any subject" (NRC, 2001, p. 238). Finally, through spending more time and practice, the expert level is reached, which is incremental in relation to understanding. At this level, the learner has a "detailed and organized understanding of the important facts within a specific domain" (NRC, 2001, p. 236). They also hold that mastery in one domain facilitates success in the next domain (p. 239).

The cognitive processes of educational activities across three learning domains are systematically classified in Bloom's taxonomy (1956). Three knowledge categories and six skills processes are distinguished within cognitive domains.

The knowledge categories (Anderson, 2002; Anderson & Krathwohl, 2001; Krathwohl, 2002) as explained by Forehand (2012), are (1) factual knowledge of subject specific terminology, i.e. details and elements; (2) conceptual knowledge of classifications and categories, principles, generalisations, theories, models and structures; and (3) procedural knowledge of subject-specific skills, algorithms, techniques and methods as well as knowing when to use appropriate procedures.

Skills within the cognitive domain range in a hierarchy from remembering, through understanding, applying, analysing and evaluating to creating new patterns or structures (Anderson & Krathwohl, 2001). This classification has been widely researched and applied to date in instructional designs, mainly for its use as an assessment tool of learners' abilities (Forehand, 2012).

In the revised taxonomy (Anderson, 2002; Anderson & Krathwohl, 2001; Krathwohl, 2002), the six knowledge categories are described in action words, rather than in noun form, and were rearranged slightly. For the purpose of easy plotting, the six cognitive processes were organised in a matrix with the three cognitive levels. A fourth knowledge category was added, namely metacognitive learning. The table below shows this matrix, followed by a graphic representation, reflecting the changes from the original, to the revised taxonomy, as follows:

Table 3

Matrix of Cognitive and Knowledge Dimensions (Anderson & Krathwohl, 2001)

	The Cognitive Dimension					
The Knowledge Dimension	*Remember*	*Understand*	*Apply*	*Analyze*	*Evaluate*	*Create*
Factual						
Conceptual						
Procedural						
Metacognitive						

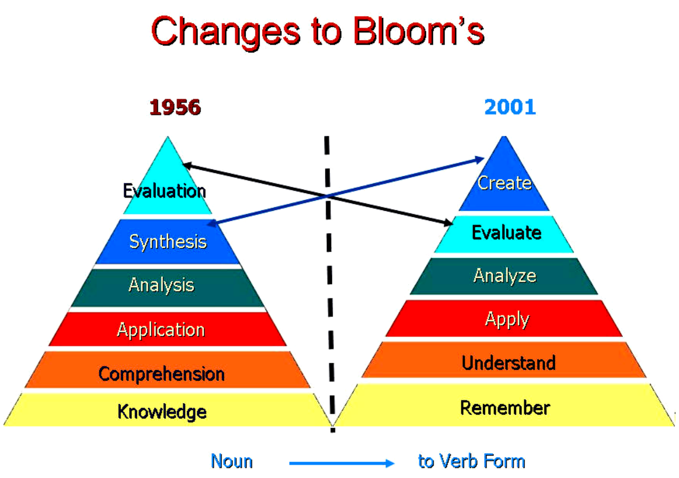


Figure 1. Bloom's revised taxonomy (Anderson & Krathwohl, 2001)

Internalised or interiorised representations of knowledge. From the learners' perspective, Hiebert and Carpenter (1992) explained the idea of "networks of understanding", where the object of understanding is a single mathematical aspect. Various single bits of knowledge form links with related single bits, loosely knitted together in a primitive network of understanding. At a more advanced level, networks become securely integrated, enabling their transfer to related mathematical situations.

The eventual goal of networks of understanding is remote mathematical applications, including wide-ranging contexts within and outside mathematics (Ambrose et al., 2003; Kilpatrick et al., 2005; Rittle-Johnson et al., 2001; Verschaffel et al., 2000; Voutsina, 2011). Key to network formation is the internalisation of representations (Hiebert & Carpenter, 1992). The representations are internally structured, as they connect with existing representations at points of similarity and consistency. New knowledge is not accommodated as stand-alone entities, but as making sense in terms of existing knowledge (Kilpatrick et al., 2005; Schoenfeld, 1992; 1994). The stronger this network becomes, the more fluently mathematical operations and problem solving can be carried out.

Threshold concepts in the understanding of mathematics. A threshold concept can be considered as "akin to a portal, opening up a new and previously inaccessible way of thinking about something" (Meyer & Land, 2003, p. 1). The learner who has not yet crossed the threshold, finds herself in a liminal space (Meyer & Land, 2003), which is a point where the concept can barely be perceived and below which it is not experienced. If the conceptual threshold stays uncrossed, progression towards further understanding is severely compromised.

Division is regarded as a threshold concept (Long, 2011), which often lies within the tension between learners' desired and actual performance. At the basis of division, there are another two major concepts, "number" and "place value". These concepts build up the threshold concept "division" (Long, 2011, p. 71).

Additionally, "the multiplicative *identity* of number" may be a concept required for building strategies in the application of division in modelling contexts like rate (Verschaffel et al., 2007). It needs to be understood that the whole number 1 (one) has a multiplicative identity for all rational numbers, as multiplier, as multiplicand and as divisor. "The multiplicative inverse property" may be found to be another threshold concept, required for division in some contexts, for factorising and for building up proofs of the correctness of products. This property is of special importance when taking into account that division extends the scope of mathematics from the whole numbers to the rational numbers.

Strategies for the Progression of Mathematics Understanding

Ambrose et al. (2003) asserted that learners employ three types of strategies to deal with division mentally, namely: direct modelling strategies; working with one group at a time; and using a partitioning number strategy. Fluency in these strategies depend on learners' prior understanding of place value, grouping and the properties of the four basic operations. A concept develops therefore not only through the knowledge about it, but also through the application of the operations to the concept, or procedures, including methods, algorithms and heuristics. In my view, this idea can be extended for the considered teaching of an algorithm or "method", as it is popularly known to learners. I reason that algorithms provide an opportunity for the systematic teaching and learning of the effect of operations on numbers when these are manipulated through addition, subtraction, multiplication and division. The effect on odd and even numbers, and in the case of division, on the switch to rational number, can be employed fruitfully and some properties of number − like the commutative and associative properties − can be introduced in the process of teaching algorithms for these operations.

Sense-making of the vertical computation algorithm that is regularly used for division, is challenging both on the operational and the conceptual level. Copeland (1982, p. 116) pointed out the differences that division shows as compared to the other operations: instead of working from units upwards, that is, from right to left, as in the other three operations, division works from left to right, that is, from the bigger place value down to the smaller. Answers are written at the top where learners are used to writing answers at the bottom of the calculation. Furthermore, answers often do not come out as whole numbers, but often manifest remainders or fractions as part of the answer and have to be reported in a comprehensible way.

Kilpatrick et al. (2005, p. 157) argued that traditional algorithms do not always contribute to the understanding of what learners are doing. The use of a single place value at a time with transitions to an adjacent position (trades, regrouping, “borrow,” or “carry”) presents a major conceptual challenge.

My view, as stated earlier, is that with skilful guidance, a traditional division algorithm offers an opportunity to enhance conceptualisation. The function of place value and the role of the divisor and the dividend, respectively, can become meaningful if carefully mediated throughout each step. The process can begin with a partitioning strategy that can then provide a bridge to the standard algorithm as a reliable computation method, lending itself to step-by-step control over the smaller calculations. In support of this view, Resnick (1987) suggested the use of errors in algorithmic computation to trace conceptual weaknesses. If the rationale behind a computational procedure is understood, fault detection may be improved and improved control of impulsivity may follow (Feuerstein et al., 2006).

Treffers et al. (1987), in their "guided reinvention" principle (cited by Van den Heuvel-Panhuizen, 2003), advocated for the use, firstly of mental calculation strategies and then, facilitating the formation of algorithmic procedures. They described the concepts and skills upon which the strategies depend that are used in multiplication and division (Treffers et al., 1978, cited in Verschaffel et al., 2007, p. 572). They found that in an instructional environment based on the principles of RME, elementary school learners gradually build up algorithms for column multiplication and division, starting from their invented informal strategies (Greer, 1992).

Representational modelling in division. Verschaffel et al. (2007) suggested that the commutative, associative and distributive laws of number underlie the mental capabilities used to build strategy, and that explicit teaching of these laws should be investigated as possible facilitators of more successful conceptualisation of multiplication and division.

Representational modelling strategies (Baek, 1998, cited in Ambrose et al., 2003; Verschaffel et al., 2007) like simple drawings, can assist to illustrate some basic aspects of the more complicated commutative property of number, in the transition from multiplication to division. Representational modelling can be applied to a simple mathematical situation of division, where the possibility is investigated of grouping the same number of items in different ways. The following example illustrates a few concepts that can be learned through representational modelling:

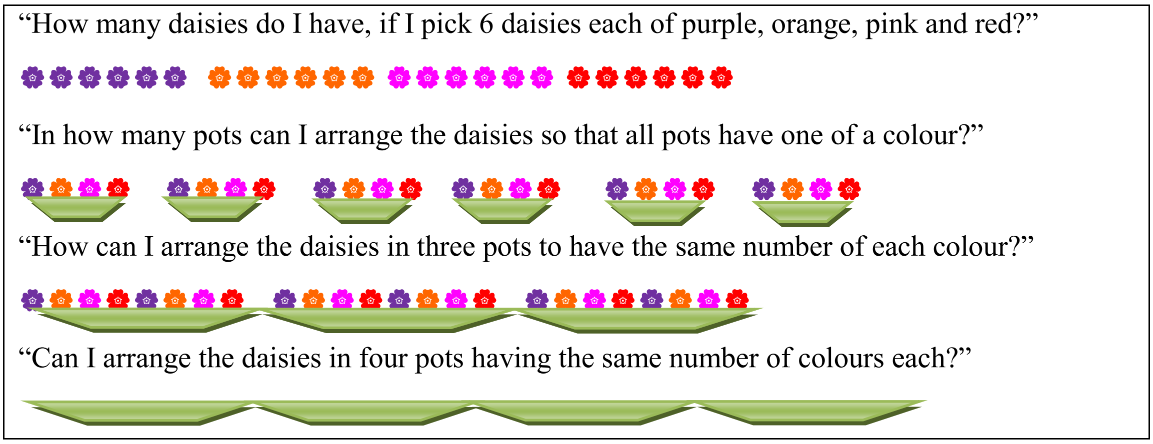


Figure 2. Representational modelling in division.

The basis for this is that 6 daisies in 4 colour groups, result in a total of 24 flowers (6x4=24), and 24 daisies grouped according to the even distribution of 4 colours, result in 6 groups (24÷4=6). Because the commutative property of number is used in the first case, a conceptual leap is needed from 6 daisies x 4 colours = 24 daisies to 24 daisies ÷ 4 colours = 6 groups, where a split between the four colours is required. Furthermore, because of the fact that 6 is a multiple of 2, there is the possibility of re-grouping the flowers in three groups only, by pairing the original groupings. However, when it comes to four similar groups, the grouping is prohibited by the provision that all pots have to contain the same number of each colour daisy. This type of reasoning is needed for conceptual progression in division.

Flexible methods in division. Flexible methods occur in many forms and make use of concepts like place value and number pairs that can combine with ease. The distributive property of multiplication and knowledge of how multiplication and division are related, are needed to invent flexible division strategies. The aim of promoting learners' invention of their own strategies is not to replace computation, but to aid the development of the number sense (Ambrose et al., 2003).

The emphasis in flexible methods is on the natural linking of strategy to existing knowledge and on understanding algorithms rather than performing them mechanically to obtain the correct answer. Although in the study of Ambrose et al. (2003), the more complicated applications of division such as rate, Cartesian products and rectangular area were not used, the findings provide insight into the conceptual development of learners in regard to division. Learners did not display particular difficulty to switch from multiplication to division and often used halving and multiple subtraction, although building up and doubling strategies were employed just as effectively. The acceptance of division as the inverse operation of multiplication proved to be more intuitive than would be expected (Ambrose et al., 2003).

Other strategies include decomposing the dividend, using tens, or dividing by 10, 5 or by 2 to solve partitive and quotitive problems in measurement contexts. Ambrose et al. (2003) further observed that learners do not necessarily break up numbers into manageable chunks to use their presumed well-known multiplication facts, as is done in standard algorithms. In fact, it appears possible that they could even construct strategies while still learning multiplication facts.

Ambrose et al. (2003) remarked that the concepts underlying the invented strategies are much closer to the surface than in the standard algorithm where the underlying concepts are obscured or masked by the procedure itself. Learners developed strategies using adaptive expertise, which entails that the knowledge they had, even though it would not traditionally be regarded as sufficient, was so well connected and their understanding of the properties of numbers so useful that they could use it in creative ways to arrive at solutions across operations.

Objectification as a goal of division strategies. The idea that knowledge can be objectively embodied, is a powerful notion in mathematics teaching. Learners can for example either objectify a division situation externally or internally, in their mind space. In my view, mathematical objects gain strength when they are created by learners in stead of provided to them. In her research, Sfard (1991; 1998) found that object formation is key to conceptual knowledge, in the transition of mathematical concepts from interiorisation through condensation to reification (Sfard, 1991;1998).

Objectification was systematically elaborated in the learning metaphors of Paavola and Hakkarainen (2005). The metaphor "knowledge acquisition" refers to teachers' "monological" transmission of logically organised concepts. "Dialogical" learning through interpersonal encounters in a social context is described as "knowledge participation" (2005). However, "knowledge creation" is "trialogical" since it comprises not only individual and collective learning, but also the "systematic development of shared objects of activity" (Paavola & Hakkarainen, 2005, p. 536). In this regard, learners' writing (external objectification) and visualisation (internal objectification) are seen as promising tools in knowledge building (p. 549).

The ideas of representational modelling, flexible methods and objectification all offer optimistic design alternatives for the present study, where a metacognitive strategy for mathematical transformation of situations in the multiplicative conceptual field is proposed. In this investigation, participants were encouraged to use flexible methods in calculating, and the process of objectification through visual imagery was supported by representational modelling. These ideas were employed both in the design prototype for the experimental situation during fieldwork (Prototype IV) and in the design (Appendix D), which is presented as the final product of this research.

Some Barriers to Understanding in Mathematics

Division is one of the more problematic mathematical areas, at least at the primary level, and various conceptualisations of the barriers obstructing the understanding of division provide theoretical perspectives worth considering.

Contra productive cognitive behaviours. Feuerstein (Feuerstein & Rand, 1974) identified some possible causes of mathematics problems, such as: not being able to control impulsivity; struggling with sequential thinking; not being acquainted with hypothetical reasoning; not comparing accurately; and/or failing to conserve constancies. Persistence of these cognitive behaviours makes it hard to cope with complex concepts like division in a reliable and systematic way.

Dysfunctional computation. Roberts (cited in Copeland, 1982, p. 133) found four main error categories in division problems, mainly focusing on computational skills, namely, the wrong choice of operation; computational errors involving number facts; defective algorithms; and random responses. A further finding was that learners who learn in a highly structured instructional situation, struggle to generalise.

Abstract introduction of division concepts. Copeland (1982) reckoned that because learners do not understand division easily on the abstract level, it should be introduced "in terms of object manipulation in the physical world" (p. 116). Since the wording used is decisive for understanding, he argues for a distinction between partitive and measurement division by saying: "Divide 18 into three groups" or "Divide 18 into groups of 3" (p. 117). Remedial procedures for division may include a place value board, verbalising each step and relating problems to the concrete level before they are written down (p. 132).

Conceptual difficulties intrinsic to division. Greer (1992) suggested a set of well-articulated problem areas, focusing on the conceptual aspects involved in division, including: difficulty to move from integers to fractions and decimals; bridging from number to text; moving from counting to measuring; failure to conserve constancies; and persistent difficulties in some contexts like volume. These factors bring to the fore the dynamic complexity of the processes at work when children grapple with division problems. Greer (1992, p. 293) also recommended a metacognitive approach, drawing on the work of Fischbein (1987, 1990), mainly as a measure of avoiding or countering misconceptions. Greer (1992, p. 293) proposed, amongst others, the synthesis of bodies of research on multiplication and division word problems, proportional reasoning and rational number concepts.

Duration of mathematical difficulties. Duval (2000) grouped mathematical difficulties in terms of their duration: Temporary difficulties typically occur with new or complex concepts, when earlier concepts have not been solidified. Recurrent difficulties often occur within changed contexts, e.g. from a familiar algorithm to a heuristic approach towards computation. Standing difficulties occur most frequently when the acquisition of knowledge inhibits further acquisitions, e.g. a division algorithm may inhibit the application of division to a problem in a realistic context.

Semantic factors in mathematics understanding. In an analysis of arithmetic word problems, Nesher et al. (2003) found that ineffective interaction between thought and language accounted for difficulty in mathematics cognition and learning . Gamaroff (2010) stated that where the medium of education is not the learner's first language, as is often the case in SA, the difference in the symbolic systems of the two languages can be a major contributor towards failure in learning.

In summary, the barriers in understanding division reside both within the learner and within the subject matter itself. Deficient cognitive processes and flawed mathematical behaviour are said to account for difficulties from a learner's perspective. From the subject perspective, intrinsic conceptual difficulties within division particularly challenge mastery of division type problems.

Assessment of Mathematical Understanding

A primary aim of assessment is to establish whether learners reached the "threshold of initial learning" (NRC, 2001, p. 235) and whether understanding of the subject matter has been developed in such a way that transfer to different situations can reliably take place. Assessment of knowledge recall alone reflects neither successful nor unsuccessful instruction. It is when transfer of knowledge to new contexts and later development stages is evaluated, that the success or failure of different instructional approaches becomes apparent (NRC, 2001).

Although international and systemic assessments are used for diagnostic and developmental purposes within the broad mathematics education arena, the majority of assessments take place in the mathematics classroom, as a teacher responsibility. This task poses great challenges, such as aligning learning expectations with assessment (Webb, 1997); detecting underlying reasoning, concepts and misconceptions through assessment (Black & Wiliam, 2001); transforming assessment outcomes into improved instruction (Bennett, 2011); employing dynamic ways of assessing mathematical understanding and processes, as opposed to static measures (Campione & Brown, 1987); and linking assessments to the real world in such a way that mathematical concept formation is facilitated (De Lange, 1999). Some salient aspects about the assessment of mathematical understanding are now addressed:

Progression in conceptual understanding. In this investigation, a test-teach-test method of assessment was adopted as one of the variations of dynamic assessment (Feuerstein & Rand, 1974; Kaniel, 2000; Tzuriel, 2001). This method of assessment can be employed in assessment-based development of didactic interventions. This concept aligns with the intention of the present study to effect and assess the progression of understanding. Subsequently, a baseline assessment was needed as a starting point, if the effect of instruction on learner understanding was to be investigated. Further interventions would then be based on the evidence of learning demonstrated in the baseline assessment. Within Design Research, the cycle of assessment, analysis and intervention would be repeated until the desired level of conceptual understanding was demonstrated in a summative type assessment.

Establishing the depth of mathematical knowledge. Webb (1997) argued that assessment of learner knowledge should be aligned with the depth of knowledge expected of learning. In such assessment, the following questions may be considered: Can the learner transfer knowledge to different contexts? Can the learner generalise the knowledge to new situations? Is the learner's knowledge integrated with other aspects within the conceptual field and also across fields? Does the learner reason logically? Does the learner demonstrate self-monitoring behaviour by fault-detection?

Demonstration of mathematical understanding. Steinbring (2005) argued that the teacher will most likely be the one to do the assessment based on the underlying principle of having a high degree of match between what learners are expected to know and what information is gathered about their knowledge (p. 31). Based on Duval's (2000) epistemology of mathematics knowledge, Steinbring remarked that learners' mental constructions are not readable in their descriptive statements and therefore an in-depth understanding of the nature of the mathematics classroom discourse is necessary to analyse a learning environment. Duval (2000) drew a distinction between assessing or analysing learners' conceptual understanding on the one hand and their underlying thought processes on the other. In my own view, traditional written assessments can display learners' understanding, but articulation from the learner's first person point of view is needed to enable inferences about their mathematical thinking.

Synopsis of Literature Review: Mathematics Learning

In the review of a sample of research studies on mathematics learning, a variety of approaches were found, accounting for the various emphases in theoretical perspectives. Consensus exists, however, on one matter, namely that mathematics learning aims to construct meaning through deep conceptual understanding (Hiebert & Carpenter, 1992; Kilpatrick, 2005). Recent mathematics learning research is characterised by an openness towards the processes that may be followed to reach conceptual understanding (Ferguson, 2002; NRC, 2001; Pantziara & Philippou, 2011; Rittle-Johnson et al., 2001; Sfard, 1991; 1998; Voutsina, 2011). Later theories steer away from prescriptive processes; they would rather advise process integration and freedom according to pragmatic considerations and available developmental amenities (Ambrose, 2003; Baek, 1998; Copeland, 1982; Greer, 1992; Kilpatrick, 2005; Long, 2011; Meyer & Land, 2003; Nesher et al., 2003; Paavola & Hakkarainen, 2005; Schoenfeld, 1992; 1994; Treffers, 1978; Verschaffel et al., 2007).

Extensive attention is given in the literature to the assessment and verification of mathematics learning. In the research, use is made of the demonstration of learning to extract indicators of, and enable inferences about, the underlying reasoning and thought processes involved in the acquisition of understanding and meeting the expectations of learning (Bennett, 2011; Black &Wiliam, 2001; Campione & Brown, 1987; Duval, 2000; Kaniel, 2000; Lange, 1999; Steinbring, 2005; Tzuriel, 2001; Webb, 1997). Assessment of mathematics learning aims to do justice to the exact nature and extent of learners' conceptualisation.

In conclusion, the mathematics learning process is reported to be a multi-faceted edifice at different levels (Anderson, 2002; Anderson & Krathwohl, 2001; Demetriou et al., 2011; Krathwohl, 2002; Linn, 2002; Panaoura, 2007) and dimensions (Usiskin, 2012). The transition from elementary to advanced mathematising hinges on a deep and full-rounded understanding of mathematical concepts in relation to factual knowledge and operational appropriateness. Both instruction and assessment should therefore have as their central focus, conceptual grasp within integrated networks of understanding. In Chapter 3, some internal processes in learning are highlighted.

CHAPTER 3 – LITERATURE REVIEW: COGNITION AND METACOGNITION IN MATHEMATICS

In Chapter 2, the first two of four specific research questions were addressed, as attention was given to matters concerning the teaching and learning of mathematics and specifically to the related aspects in the focus of the present study. In this part, the third specific research question comes into focus in a review of the literature on the cognitive and metacognitive amenities available within the learner, to respond to the requirements of mathematics learning at this developmental stage.

Mathematics Cognition and Metacognition

With regard to the facilities available within the learner to respond to the demands of learning, Nelson and Narens (1990) suggested two levels at which cognitive processes occur, i.e. the object level of cognition and the meta-level that falls mainly within the metacognitive abilities of the learner. This part of the review focuses on cognition and metacognition and concludes with a local theory that serves as the theoretical point of departure for the data collection phase, as well as the conceptual framework upon which this study is based.

Cognition

The following cognitive theories are used to a greater or lesser extent to guide the present Design Research:

Classic cognitive development theories. Piaget (1896-1980), as summarised in a lesson by McLeod (2009), discerned discreet developmental stages in his quest for insight into how knowledge grows throughout childhood and identified schemas and adaptation processes that characterise and facilitate the movement from one stage of development to the next. Many research projects have been undertaken and additional theories constructed based upon Piaget's idea of developmental progression from the sensory motor stage through the concrete operational to the formal operational stage. The contribution of Bruner (1977, discussed in McLeod, 2008) in this regard and the neo-Piagetian theory of Demetriou et al. (2011) are now briefly discussed.

Bruner (1977) suggested a more flexible view of development than Piaget, by proposing modes of representation, depicting the manner in which knowledge is encoded at different stages to process information, to remember and to learn. These stages would not coincide exactly with chronological age, but would rather convert naturally into the next stage, as follows:

1. The enactive stage is mainly ascribed − though not limited − to the first year of life, when the child mainly learns through motion and motor tasks.
2. The iconic stage spans the rest of the pre-school years, where information takes a visual form or a mental picture.
3. The symbolic representational stage covers the schooling years where the child makes use of symbols such as numbers and language to represent knowledge, and where symbols can be manipulated and adapted.

Subsequent to Bruner, some studies applied varied terminology for the same set of ideas. Some schools refer to the development stages as concrete-pictorial-abstract, while others prefer a "concrete, semi-concrete, semi-abstract, abstract" classification, all generally pointing to the same developmental sequence.

Whereas Piaget reasoned that thought precedes language, Bruner regards the importance of language as its ability to deal with abstract concepts. The role of internalised language (Vygotsky, 1962; as discussed by McLeod, 2007) is a precondition for understanding. A discrepancy between the use of one language of teaching (the social medium) and another language of learning (private speech) could be a potential barrier to learning (Vygotsky, 1978; 1987). This discussion is supported by the semantic analysis of Nesher et al. (2003), as mentioned earlier.

Vygotsky (1978), in his social development theory, stressed the socio-cultural influences on learning. In this regard, the core construct, "the zone of proximal development" (1978) pointed to the "distance" from what the learner can do, to what she cannot do without help, and for which mediation is needed.

Cognition in mathematics learning. In classrooms that are aligned with both the affective and cognitive goals of the CAPS, teachers have to strike the balance between building learners' subjective experiences of and motivation for learning mathematics (affection) and their demonstrated subject knowledge and competency (cognition). Mathematical competency maintains that learners reach mental control of concepts in relation to their factual mathematical knowledge and their procedural efficiency while solving problems.

A mathematical problem is an example of a situation where information processing takes place, in which case a series of cognitive operations would typically be required, leading to problem solving. At the cognitive- or object level (Nelson & Narens, 1990), during the input phase, the instruction is taken, interpreted and internalised (self-instructing) and the relevant information is collected; at the elaboration phase, knowledge is retrieved from the memory, clues and cues are linked with each other as well as with facts from the memory stores, inferences are made and a conclusion reached; and at the output phase, the solution is communicated and defended.

Some preconditions for understanding situated problems. The conditions to consider prior to learners solving mathematical problems, are as follows:

Command of the language. If we accept Duval's (2000) viewpoint that the only cognitive access we have to mathematics is through signs, symbols, words, expressions or drawings, then the understanding of mathematical problems and concepts is almost completely dependent on the command of language, both the language of education and the mathematical language.

In this regard, Steinbring (2005) asserted that apart from the situative and exemplary context conditions for analysis of learning, words and relations must be used that are already known and familiar to the learner. In the epistemological triangle below (2005, p. 22), he illustrated the relationship between three essential elements in mathematical learning, i.e. the context, the sign or symbol and the concept. Through interpretation, concepts are evoked by the contextual mathematical signs and symbols.

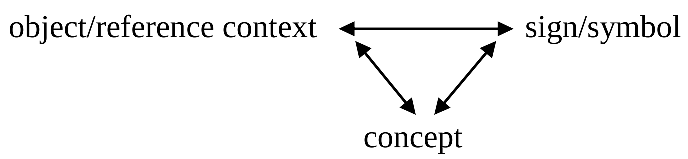


Figure 3. The epistemological triangle (Steinbring, 2005).

The introduction of a real world situation as a mathematical problem, presupposes a functional command of the language of teaching if the learner wants to form a mental image of the situation and its constituting elements. The everyday understanding of language is a function of the learner's literacy level, which, at a subject level, includes mathematical terminology and symbols.

Imagining a situation. Greer (1992) observed that the derived quantity of multiplication and division is more difficult to represent externally than sets of objects and measured quantities, even when a visual representation like a picture or a diagram is used. A picture can be limiting at higher levels; however, this study is optimistic that even static external representations can, up to a point, be metamorphosed to a mental representation which holds potential for manipulation.

The English adjective "realistic", in the language of origin of RME, namely Dutch (Van den Heuvel-Panhuizen, 2003), is understood to describe the action "zich realiseren", meaning to imagine something (pp. 9-10). Presented with a situation, learners would typically start by reading it, thinking it through and imagining the situation. Duval (2000, p. 6) considered the possibility of a discrepancy between the signifier, or the intention of the problem and the meaning of what is evoked in the mind. Even so, I reason that when the learner becomes aware of the mental image that has been evoked by a verbally-represented situation, such an image may be regulated at will. The information provided by the problem situation may be applied systematically to the mental image to aid the quest for a solution. In this way, visual mental imagery has the potential to transcend cognition to metacognition, falling within the category of metacognitive skills, actions and strategies (Desoete & Ozsoy, 2009; Flavell, 1979; Pintrich, 2002).

Supporting functions. As mentioned previously, various cognitive skills such as the control of impulsivity, hypothetical reasoning, accurate observation, comparison and conservation of constancies (Feuerstein et al., 2006) support problem solving.

Cognitive development at age 9 to12 years. Understanding a mathematical concept has to be viewed also from an educational psychological perspective. In this regard, the cognitive development theory of Piaget (cited in Copeland, 1982, 1984) and its extended novel interpretation by Demetriou et al. (2011) are considered.

Copeland (1984) explained the intellectual development of the learner aged 7 to 12 years as the logico-mathematical phase (p. 406). The learner’s mathematising is now “concrete operational”, entailing in part physical manipulation of objects and in part logical thinking. Invariance is a basic characteristic of this stage (Copeland, 1984, p. 407), comprising identity (something is what it is); conservation (something remains what it is); and reversibility (something returns to what it was). At this stage, learners also start grouping things on the basis of characteristics that can be ordered.

At some stage, the learner breaks away from concrete manipulation of objects to start generalising. The formal operation stage of reasoning with symbols or ideas rather than with physical objects, is however unlikely to occur before 11 to 12 years of age (Copeland, 1984). As soon as the learner is capable of combinations and permutations of the original, thought processes have gained strength, or in Piaget’s words, “extension of the power of thought” is reached (Copeland 1984, p. 410).

Piaget argued that the individual’s mental operations develop from simple to complex and from concrete to abstract, while mental schemata are modified through the expansion of own experience (Hergenhahn & Olson, 2001). This is the same type of progression that we see in the mathematics curriculum from the foundation years throughout the primary school years. The notion of progression phases in mathematics was recently explicated and refined by Demetriou et al. (2011).

Demetriou et al. (2011) held that the architecture of the human mind comprises specialised structural systems (SSSs) and processes, which change developmentally and should be accommodated within instructional contents and educational encounters. According to this theory (Demetriou et al., 2011), a set of mental processes interface with the environmental domain, necessitating representation and information processing (representational capacity system) in order to connect the information to the goal (general inferential system). The conscious system monitors, controls and regulates the processes at any given moment.

The specialised structural systems (SSSs) involve the logical processes of categorical-, quantitative-, causal- and spatial thinking, all of which pertain to mathematical thinking. Social thought is the fifth SSS, which is indirectly involved in the interpersonal encounter(s) within the mathematics learning environment. It is important though, to recognise mathematical thinking as a cognitively involved process and not singly as a numerical (or quantitative) process. This argument is strengthened if it is kept in mind that categorical thought deals with similarity-difference relations; causal thought deals with cause-effect relations; and spatial thought deals, amongst others, with “imaginal-iconic representation of the environment” (Demetriou et al., 2011, p. 604). The SSSs are organised at three levels, i.e. core processes, mental operations, knowledge and beliefs (Demetriou et al., 2011). This organisation is illustrated below:

Table 4

Examples of Mathematising at Different Levels of Thinking (Demetriou et al., 2011)

Level of thinking	**Mathematical example**
Causal thinking at the core process level	Observing that a vehicle in motion covers more and more distance as time passes
Causal thinking at the mental operation level	Forming a hypothesis that the faster the vehicle moves, the shorter the time to cover the same distance
Causal thinking at the knowledge level	Understanding that the time needed to cover a distance is dependent on the speed of the vehicle
Quantitative thinking at the core process level	Observing that 4 hours pass while 368km are covered
Quantitative thinking at the mental operation level	Calculating that a speed of 92km/h was maintained
Quantitative thinking at the knowledge level	Inferring the formula that states that speed is the quotient of distance and time

Likewise, this example applies in the spatial domain, where the image of the moving vehicle and the distance covered provide the basis for carrying out the mental operation. In the comparative domain, the example could serve to determine the difference between cars driving the same distance over various periods of time.

The characteristics of the specialised structural systems change with development. Different children reach specific cognitive milestones at different stages and circumstantial factors such as exposure to environmental stimuli impact on the tempo of development. Even so, the indicators below can be regarded as a well-researched general yardstick for pitching mathematical items and assessment expectations. Additionally, the cognitive level required in mathematics has to be raised progressively in tandem with the cognitive level reached per individual and at which the individual presently functions. The following modal characteristics of the specialised domains can be expected reasonably of learners ranging from 9-10 years at the lower end, to 11 to 12 years at the higher end (Demetriou et al., 2011, p. 606):

Table 5

Development of Specialised Structural Systems: 9 to 12 Years (Demetriou et al., 2011)

SSS class	**Range start at 9 to 10 years**	**To range end at 11 to 12 years**
Categorical thinking	Logical multiplication in an unfamiliar context	Flexible logical multiplication
Quantitative thinking	Construction of simple mathematical relations, e.g. $a + 5 = 8$	Proportional reasoning and coordination of symbolic structures
Causal thinking	Testable theories in action	Suppositions and isolation of variables
Spatial thinking	Representation of complex realities	Imagining the non-real
Inferential thinking	Logical necessity	Logical validity of propositions
Consciousness	Differentiation between cognitive functions	Differentiation between clearly different domains

.

Inferential thinking and consciousness are general cognitive functions overarching the specific structures, as they apply separately to each system. The meaning of the first four specific SSS categories, according to Demetriou et al. (2011), and some practical implications of the categorisation for teaching mathematics to a learner in the age group 9 to12 years, are interpreted as follows:

Categorical thinking. The ability to observe perceptual similarities and differences, is the cognitive skill involved in categorical thinking. Within the knowledge domain of mathematics, this skill enables a 9 to 10 year old learner to classify situations according to similar characteristics, while noting the differences between them, which enables similar computations in various contexts. At 11 to 12 years, this cognitive ability has developed to a level where the logic of multiplication for example, has become flexible for unfamiliar incidents, like the division of a large area (a m x b m) into small areas (c m x d m), for which various logical options could be considered, e.g. $ab\text{m}^2/cd\text{m}^2$ or (a m/c m) x (b m/d m). Additionally, the properties of real objects can be transformed into mental objects and conceptual systems can be constructed (Demetriou et al., 2011) as contrasted to tangible systems.

Quantitative thinking. Mathematical thinking in terms of quantitative variations and relations is at the core of this structural system. At the lower end of the age group 9 to 12, the relations still need to be simple, like counting, pointing, removing or sharing. At the higher end, more advanced quantitative transformations like proportional reasoning can be expected. Symbolic structures can be coordinated with understanding, for example the concept of average speed can be introduced as a calculated value typical of the relationship between the two variables, distance and time, based on a workable understanding of the quantitative aspects of the world.

Causal thinking. In the mathematics knowledge domain, this system would typically include the ability to reason hypothetically. At 9 to 10 years, a given theory in action can be verified by the learner's observation of the more overt causal relations, while at 11 to 12 years, the learner is able to construct own suppositions and isolate the (often covert) variables that would influence the outcome. A learner would for example observe: "The more times I cast the dice, the better chance each number has to come up equally frequently" – a supposition that can be tested statistically.

Spatial thinking. The spatial orientation enabling the representation of an intricate situation at age 9 to 10, develops into the unique cognitive ability of thinking up or reconstructing a real situation into an imaginal-iconic situation at age 11 to 12 years. "Mental maps of places or mental images of ... objects and operations on them, such as mental rotation, belong to this system" (Demetriou et al., 2011, p. 604). Direction tracking can therefore be done not only in the learner's external world, but also in the internal mental space. This cognitive ability is the point of departure and lies to the core of the metacognitive strategy, the mathematical application of which is investigated in this study.

In view of the above cognitive development characteristics, there is no time as appropriate to initiate the conceptualisation of intricate mathematical ideas, using the capacity for visual imagery, than at the age 11 to 12 years. Learners at this age are ready for visual imagery in mathematics, not only in terms of the cognitive functions that can accommodate mathematical ideas, but also in terms of the metacognitive capacity to consciously regulate, control and monitor the mental processes.

As far as the representational capacity system goes, Demetriou et al. (2011) argued that, to be grasped, a concept has a specific representational demand and this must reside within the representational and processing possibilities of the learner. A concept is defined on the basis of other concepts. The higher level concepts can be grasped when the integration of the lower level concepts are representationally possible, e.g. to teach speed, the lower level concepts (distance and time) should have a representational form already. A mental model of speed would then prepare the understanding of causal relations where the principles of covariance and correlation have to be discerned and considered. In building such a mental representation, the learner can be guided to observe that time is lapsing as the vehicle covers distance; the shorter the time to cover a given distance, the higher the speed; also, the lower the speed, the shorter the distance covered in a given period of time. The functional mental model representing a complex reality like speed, affords the learner the opportunity to validate the derived hypothesis or proposition logically. Also, pertinent to instruction within the numerical domain, the mental model of speed facilitates the construction of a simple mathematical relation such as speed = distance/time.

It is not easy to displace or rectify personal knowledge, which has been shaped by what previously have been perceived as salient properties of the phenomenon (Demetriou et al., 2011). However, to be functional within and across the operational domains, guided reconstruction of representations, especially integrated mental models, should not be compromised as an educational task.

In this study, I am not investigating the structural components of intellect, instead I observe the functional components of the cognition process through the signs of cognition that learners demonstrate in their numerical calculation of mathematical problems and in the verbal explanation of what happens in their minds while solving the problem. I am not following an approach where the correct answer in itself is the goal of mathematics; however, the successful outcome of the mathematical process is an important goal in modifying the thinking process and is one of the goals of an effective cognitive process. Because this goal cannot be compromised, I am not only investigating how the thinking, or in this case the meta-thinking takes place, but through intervention, I mediate specific strategies that have the potential to enhance the process, and subsequently the outcome of cognition.

Metacognition

Metacognition is the study of what lies behind thinking (“meta” as a prefix means “beyond” or “behind”). In various descriptions, the phenomenon of metacognition had long been acknowledged in the education- and psychology arenas. Dewey’s concept of reflection as a central part of learning, Piaget’s “consciousness of cognizance” and Vygotsky’s notion of the “inner voice” all contain elements of the greater phenomenon of metacognition (Darling-Hammond et al., 2003).

Flavell (1979) formally coined the term “metacognition”. He described metacognition as the “knowledge and cognition about cognitive phenomena” and concentrated on metacognitive monitoring in this early stage of the emerging concept as a category of cognitive science. Such monitoring would then broadly be exerted over “memory, comprehension and other cognitive enterprises” (1979, p. 906). Flavell (1979) set the tone by classifying metacognition into four sub-categories. These categories, as they are elaborated in more recent works, are now described as follows:

Metacognitive knowledge. The first category is metacognitive knowledge (Flavell, 1979), operating either through a conscious memory search or through unintentional, automatic retrieval of cues (Desoete & Ozsoy, 2009, p. 2). Three types of metacognitive knowledge are distinguished, as follows:

The first type, metacognitive knowledge about persons, refers to what learners know (or believe) about their own cognition, about another's cognition in relation to their own and about other people as cognitive processors (Desoete & Ozsoy, 2009). Statements like: "I am bad at mathematics. My teacher thinks I am dumb. Everybody else gets it easily, but not me", illustrate that metacognitive knowledge of persons is highly subjective is difficult to validate. The impact of this type of knowledge on self-concept and motivation may be significant if it becomes prevalent.

The second type of metacognitive knowledge pertains to the availability, quality and amount of information for a mathematical task (Flavell, 1979; Desoete & Ozsoy, 2009). This knowledge may still be subjectively coloured and detached from the factual reality. A typical remark about the task of constructing a square where one side is 8cm long, would be: "I cannot do this. There is not enough information to solve this problem". It is an explicit task of teaching to help the learner overcome resistance to new and more complex tasks to alter their beliefs towards greater realism.

Metacognitive knowledge of strategy lastly refers to what learners know or believe to be, effective strategies for reaching their goals (Flavell, 1979, p. 907). An example is, that a learner would say: "I can only do division if I have written down the multiples of the divisor from 1 to 9".

Metacognitive experiences. Following on metacognitive knowledge is that, as a result of what learners know (or believe) about persons, tasks and strategies, subjective experiences and feelings are evoked (Flavell, 1979), which in turn, may result in decisions about strategies that will be followed in future.

In later interpretations of the categories, it was found that metacognitive experience functions mainly by means of two feedback loops associated with negative affect (Desoete & Ozsoy, 2009; Efklides, 2011). These are firstly, error detection, or an estimate of discrepancy from the goal and secondly, a feeling of difficulty or a lack of process fluency. In the author's experience, if learners experience difficulty and/or know (or believe) that they do not have sufficient knowledge or skills to correct the error, they either quit trying, knowing that the error would cost them marks; habitually call for help and increasingly make themselves dependent upon external sources of skill belonging to those whom they believe have superior knowledge; or they keep on trying and in this way stand a chance of expanding their existing skills and correcting the error autonomously. In these reactions within the feedback loops, teacher guidance is pivotal in reaching not only the instructional goals, but also some positive attitudinal and affective goals of mathematics, as stated in CAPS (2012a, p. 6).

Metacognitive experience, according to Flavell (1979), also includes metacognitive judgements or estimates of own learning, amongst others in terms of the effort that was expended on the task. The judgement about how much time a task would need, or how much time was spent on a task, falls in this sub-category, as well as the estimate of the conclusion reached and/or the correctness of the solution. The judgement about the solution reached, is clearly a function of monitoring and operates through the feedback loops of error detection and feelings of processing difficulty.

In later developments concerning this sub-category, an important affective element of metacognitive experience is the judgement of a person's confidence (Desoete & Ozsoy, 2009; Efklides, 2011). The metacognitive perspectives that learners hold about their own abilities, and also about their abilities as compared to other's mathematical abilities (Desoete & Ozsoy, 2009; Flavell, 1979; Pintrich, 2002) tend to become predictors of their performance on a sub-conscious level. These estimates, feelings or judgements do not necessarily reflect the true situation. "I cannot, but Thembi can always"; "I can, if I spend more time"; "I have spent enough time trying"; "This is too complicated" and so on, are subjective judgements; however, this internal dialogue may have a profound influence on their proficiency.

Metacognitive goals, tasks, skills, actions and strategies. Flavell (1979) identified a third category, namely metacognitive goals or tasks, which refers to the objectives and directedness of a cognitive activity (p. 907), and immediately links to it the fourth category, metacognitive skills, actions or strategies, referring to the behaviours employed to achieve these goals or tasks. Metacognitive experiences would activate strategies aimed at either cognitive or metacognitive goals. Metacognitive strategies aimed at cognitive goals, would, for example, be the revision of a piece of work as a result of the awareness of insufficient knowledge. The example to illustrate metacognitive strategies aimed at metacognitive goals is the self-questioning technique to ensure that a piece of work is mastered. Flavell did not elaborate the demarcation between these two categories much further.

Desoete and Ozsoy (2009) did not refer to metacognitive goals as a separate category or class, but mentioned in a single sentence that "metacognitive skills refer to the voluntary control people have over their own cognitive processes".

Efklides (2011) stated that metacognitive strategy is a representation of cognition that informs the control function, the same phenomenon that Flavell (1979) described as metacognitive goals. Likewise, Darling-Hammond et al. (2003) portrayed metacognitive strategy as the conscious directing or regulating of one's thinking towards a desired goal. They argued that metacognitive strategies hinge on metacognitive knowledge (2003, p. 159) and mentioned planning and monitoring as tactics used in metacognitive strategy. Regulating of learning is equated to "executive control" (Darling-Hammond et al., 2003, p. 161).

In this regard, Nelson and Narens (1990) held that the activities and actions of the object level are overseen or monitored at the meta-level of cognition. Depending on the message that is sent back to the conscious mind about the object level, control is taken to actively regulate the activities on the object level. Such control could take many forms, e.g. the correction of a wrong calculation, the experimentation with a new strategy to address the problem, exploration of additional sources of information like references to the textbook and many more. The direction of self-regulation is from the meta-level towards the object level, and not vice versa (Koriat, 2007).

The consensus from the reviewed literature is that the construct metacognitive strategy refers to activities whose purpose is the regulating of learning. Following on the notion of mathematics learning as conceptualisation of mathematical constructs, I define metacognitive strategy, at least as it is used in my study, as the intentional regulating of mental activities to effect conceptualisation.

Further research on metacognition. The seminal work of Flavell (1979) is honoured as the initial demarcation of the field, and serves as the basis for later studies that shed more light on some specific facets and relations within metacognition.

In a concise conceptualisation, Panaoura (2007) defined metacognition as "the awareness and monitoring of one's own cognitive system and its functioning" (p. 32). This definition distinguishes two crucial elements of metacognition, namely declarative knowledge (or consciousness of cognition) and procedural knowledge (or control of cognition). Citing Brown's (1987) knowledge types, Panaoura (2007) explained this distinction as that between *what* we know (self-awareness) and *how* we go about knowing (self-regulation). To these two elements, she added conditional knowledge or knowing *when* and *why* (p. 33).

The above conceptualisation has some similarity with the four dimensions introduced by Usiskin (2012), i.e. the representational dimension (what?), the skills dimension (how?), the use and application dimension (when?) and the proofs and properties dimension of understanding (why?). At this point, conceptual clarity is needed about the difference between the "What?", "How?", "When?" and "Why?" of the metacognitive knowledge types (Brown, 1987, cited in Panaoura, 2007) and those of the dimensions of understanding (Usiskin, 2012), which I regard as cognitive knowledge types. In the second, activity occurs on the object level as a cognitive process and in the first, on the meta-level as oversight and control of the cognitive process (Nelson & Narens, 1990).

Pintrich (2002) made a major contribution in the application of metacognition in teaching, learning and assessment and metacognitive strategy takes a central position in his theoretical approach. He collapsed the strategy category into the knowledge category, described as "knowledge of general strategies that might be used for different tasks, knowledge of the conditions under which these strategies might be used, knowledge of the extent to which the strategies are effective, and knowledge of self" (p. 219).

Different classifications of metacognitive knowledge such as the above, would subsequently lead to different views of the same phenomenon. As an example, a statement like: "I am not good in doing calculations with fractions" would be classified by Flavell (1979) and Pintrich (2002) as metacognitive knowledge of person or self, whereas Brown (1987 cited in Panaoura, 2007) would classify the same statement as declarative knowledge. Concerning metacognitive knowledge, the position of the National Research Council of the USA (2001) is that learners develop knowledge of their own learning capacities very early, resulting in the ability to plan and monitor success and correct mistakes when necessary. However, these "natural capabilities require assistance for learning: Learners' early capacities are dependent on catalysts and mediation" (p. 234).

Pintrich (2002, p. 219) emphasised the ability to plan and control learning (metacognitive process), as well as the consciousness of own altered perspectives (metacognitive knowledge). Following this notion, it is the author's view that knowledge and strategy follow iterative metacognitive cycles as the knowledge and judgement about own strategies lead to the acquisition of more, improved or novel strategies, which in turn, create the consciousness of altered perspectives. The new strategies are once again tested out and their value for problem solving is judged.

Pintrich (2002, p. 220) distinguished three types of strategies that learners employ in learning: Rehearsal strategies reinforce the uptake of information into the memory stores; elaboration strategies deepen comprehension; and organisational strategies classify, outline, map and make connections between segments of learning.

Efklides (2011) defined the interaction of metacognition with motivation and affect by the construction of a metacognitive and affective model of self-regulated learning. Two levels of functioning in self-regulated behaviour are discerned: At the first level, self-regulative behaviour originates within the person, as a result of the self-concept where goal-directed self-regulation is set in motion in a top-down direction. At the second level, self-regulative behaviour originates from the task in a bottom-up direction, yet it is still personally connected. The self-regulative model (Efklides,

2011) depicts this relationship, which results from metacognitive experiences about the task, as *task* x *person*.

In the present study, the exploration of metacognition and consciousness in the research of Koriat (2007) is embraced, where he brought together in a coherent conceptualisation, the constructs of monitoring (knowledge) and regulating (control) in metacognition, as follows, (p. 2): "Metacognition research concerns the processes by which people reflect on their own cognitive and memory processes (monitoring), and how they put their meta-knowledge to good use in regulating their information processing and behaviour (control)." In this definition, the monitoring function entails the knowledge aspect and the control function entails the processing aspect of metacognition, but more than that, monitoring is the starting point of control.

Veenman et al. (2005) recommended further research into metacognitive learning to identify a strategy which can be developed under structured intervention conditions created for the acquisition of metacognition; and to establish the relation of metacognition with individual differences in mathematics performance. These recommendations are followed up to some extent within the present study.

Visual imagery as a metacognitive strategy. The human mind has multi-faceted capabilities: the abilities to experience sensory stimuli intra-personally, to control mind-processes intentionally, to be involved in motion in the mind, to capture information and access the appropriate memory stores, to switch subjective and objective roles in metacognitive activities such as self-check and to conduct internal dialogue and discourse. Tapping into these innate abilities, the metacognitive strategy of self-regulation presented in this study, is the intentional and guided creation of a virtual space in the mind where a situation can be engaged with mathematically.

During the early 1970s, Bishop (1980) initiated a series of research studies on the interface between mathematics and mental spatial abilities. Following Bishop, Presmeg (2006) simply defined a visual image as “a mental construct depicting visual or spatial information” (p. 3). The general impression is that the research regarding visual imagery and mathematics focuses mainly on spatial aspects of mathematics, like shape, geometry and trigonometry, aspects that are by their very nature visually represented in mathematics (Presmeg, 2006). In this study, however, the investigation centres around general problem solving arising from real life situations, irrespective of whether it falls within the spatial content area of mathematics or not.

Presmeg modified the various types of visual imagery as put forward by Dörfler (1991, cited in Presmeg, 2006, p. 5), as follows:

- Concrete imagery where a picture is created in the mind;
- Kinaesthetic imagery of the learner’s own physical movement;
- Dynamic imagery where the mental image itself is moved or changed;
- Memory images of formulae or methods of calculation; and
- Pattern imagery where pure relationships are stripped of concrete details.

In this study, use is made of the concrete and dynamic types of visual imagery, which would enable learners to “experience the mental world poised between the material world and the world of symbols” (Mason, 2002, p. 11).

Concrete imagery, where a picture is objectified in the mind, is employed in the present design in a partially structured and partially free manner. Structure is provided by requiring that the essential material elements of the problem situation, as well as the numerical values attached to these elements, must form part of the image. Learners are free with regard to the appearance and composition of these elements.

Kinaesthetic imagery of the learner's own physical movement has not been mediated in the experimental situation in this research, although kinaesthetically inclined individuals would probably have experienced themselves moving through the situation, or becoming part of the situation, for example walking there or coming closer to an object. Movement of self was not suggested to participants, taking into account that kinaesthetic imagery is a matter of personal preference.

Dynamic imagery, where the mental image itself is moved or changed, is seen as a crucial type of imagery, being the dynamic aspect of objectification of a situation. Dynamic imagery has, in my view and experience, the ability to prompt firstly, the application of an operation for solving the problem, secondly, the use (and order of use) of the numbers within the situation, and thirdly, the general rule that applies in solving similar problems. The manipulation of objects within the mind space is closely mediated in this design; however, the manipulation of the numbers is not mediated, rather this action is left to the learners' own devices when they start calculating, immediately after they had manipulated the objects mentally. In Appendix D, the way that concrete and dynamic visual imagery is employed, is illustrated in a practical application within an instructional design, where the real life idea of average speed is established as a mathematical concept through visual imagery.

The memory images of formulae and methods have been purposefully avoided, because the aim was to allow the visual images themselves, and particularly their manipulation, to prompt the appropriate operation that would assist the solution of the problem. It was believed, that through visual imagery, learners ideally would reach their own, albeit primitive formulae, which they could test in further applications. Learners were encouraged to use their own heuristics, the way they saw fit.

Not all learners (or teachers) use visual imagery spontaneously or with equal enthusiasm or ease (Dörfler cited in Presmeg, 2006, pp. 7-8). Although intuitive visual imagery is widely reported following research, learners' construction of mental images and the creative energy unleashed through visual imagery (Mason, 2002, pp. 11 to 12) can be assisted by "pictorial presentations of the teacher's own imagery as indicated by... spatial inscriptions" (Presmeg, 2006, p. 8).

Visual imagery is employed in the current research, irrespective of the content area where the problem is situated and not only in the content area labelled geometry, space and shape. Situations have been selected that would require applications within the additive- as well as the multiplicative conceptual field; however, the primary focus was on situations of division. As the design was evolving through its developmental cycles, the potential of visual imagery to support understanding in the use-application dimension at a conceptual level, emerged as a prominent benefit of the metacognitive strategy. The instructional ideal to assist understanding in the use-and-application dimension of a mathematical concept within a situation (Usiskin, 2012) at a conceptual cognitive level (Anderson & Krathwohl, 2001), therefore came within reach.

Operationalising a metacognitive strategy. In summary, the main theoretical construct upon which I base my own theory, is intentional self-regulation through a metacognitive strategy of structured visual imagery, called *the virtual space of the mind*, aimed to assist the mathematical understanding of real life situations. In building a model upon which the metacognitive strategy could be mapped for problem situations, I drew upon the "Model for decision-making as an optimal metacognitive process" by Kaniel (2003), which progresses through four distinctive phases, namely goal setting, planning, executing and feedback. This model is elaborated in Chapter 5.

A Local Theory of Mathematics Learning

The quest of this study is for improvement of mathematics education practice; however, this target is out of reach, given the constraints of a single study. It therefore aims to yield a set of design principles upon which a local instruction theory could be built that would ideally be tested in broader contexts. This theory may establish itself as a viable basis for improvement of future mathematics intervention designs.

The context within which my investigation takes place, is an educational encounter where Grade 6 learners must find a mathematical solution for a problem from the real world requiring conceptualisation of multiplication and division. Stemming from the foregoing literature review, the local theory upon which this research design is based, rests on two premises that I will now argue as follows:

Premise 1: Meaning is a meeting place. There is a paradox between the exact nature of mathematics as a subject and the difficulty for a learner to understand mathematics exactly. This statement is illustrated by an example in nature:

The physical features and behaviour of a local community of meerkats follow exact rules and patterns and can be understood by direct scientific research. If for some reason, a set of signs, symbols, formulae, graphics and theorems are set up in various operational systems and configurations to exactly represent the phenomenon, “meerkats”, and one is to study the representational system itself, it becomes a complex endeavour. However, if the understanding of the real little community of meerkats precedes the introduction of the elaborate system representing the phenomenon, chances are higher that the system would be understood, which could further facilitate the understanding of global communities of meerkats.

Following this analogy, mathematics represents concepts within real life situations within a structure of inter-related signs and symbols. Just as with the meerkats, the understanding of mathematical concepts has roots within understanding of phenomena in the real world. Mathematics is able to explain powerfully and organise complex realities in an agreed set of symbols and representations, rendering those real life situations (at least theoretically) manageable. Without the knowledge, observation and experience of the real life phenomenon or situation itself, however, the set of representations cannot be expected to appeal to the novice as meaningful. However, when it does become meaningful in relation with reality, mathematics becomes powerfully instrumental for further understanding of reality.

The first argument in my local theory is then that meaning in mathematics is situated, neither in the real life situation itself, nor in the mathematical representations thereof, but where reality meets mathematical representations. If mathematics teaching fails to arrange that meeting intentionally, meaning stays an illusive goal.

Premise 2: Reality enters the meeting place. To further my argument, I reason that it is an elementary school practice to arrange the meeting between real life phenomena or situations and mathematics by instructional designs which draw them together and cage them into the mathematics classroom. The situations are needed to introduce mathematical concepts: beads introduce the counting numbers and demonstrate addition and subtraction; learners arranged in groups of four introduce the dividend-divisor relation; the distance in paces to the school gate introduce units of measurement. At that point of interface between real life and representation, concrete manipulatives are used to aid the formation of meaning in mathematical concepts.

However, this practice is terminated at a certain point, and the weaning is quite abrupt when reality is pushed backward to make space for images, pictures, diagrams and tables that substitute for reality. This practice is potentially increasing the distance between reality and mathematical concepts, but is justified by the rationale that this move away from the concrete helps the learner to advance to abstract mathematising.

I reason that the concrete-representational modality of pictures and later the representational-abstract modality of diagrams are not the only or the most effective options we have towards conceptualisation. It is especially at the Intermediate Phase that real life should not be distanced further, fostering the perception that we have to get rid of reality when mathematising. On the contrary, real life should be hauled in closer, not as an external manipulative, but as an object within the mental space wherein mathematics may operate. In this way, the operational platform for mathematising is the interface between reality and mathematics in the mind space.

The second argument supporting my local theory is then that mental representations can substitute for real life situations themselves at a more advanced phase of mathematising, rendering them even more elegant and dexterous, but not less real, models to manipulate than concrete manipulatives. I argue that in this way, given enough practice, mathematics will eventually overlay reality in the mind to such an extent that mathematical models themselves are manipulated with understanding.

Consistent with the specific situation that I am investigating, I posit the local theory that, at the Intermediate Phase, the understanding of mathematics concepts within situations is still situated at the point of interface between real-life situations and mathematics, only now mental imagery makes it possible that the real situation is translated to a mathematical situation within the learner's internal mind space.

Implications of the Literature Review for this Design Research

As far as theoretical premises are concerned, the above literature review shaped the following outline for the didactical design researched in this study:

Mathematics takes the central position in the didactical encounter. The instructional goal is for learners to relate cognitively with mathematics and to become interested in the interaction between reality and mathematics. Learning takes place when the teacher's knowledge and design, and the learner's work and understanding of the subject meet each other.

For the instructional design, a central major concept that is embedded within a conceptual field, is chosen. Sub-concepts of the major concept are identified and situations representing the sub-concepts are selected, across the spectrum of classes within the field. Contextual problems are presented in simple written formulation, accompanied by as many modalities as feasible within the instructional setup. A self-regulating metacognitive strategy of visual imagery is mediated to guide learners to reciprocate by the creation of mental models of the situations.

A low degree of structure is provided after the mediation of strategy to open up the opportunity for learners to form and test their concepts in action, based on the mental model. The instructional situation allows for learners to create multi-modal signs of their understanding and present their theorems in action. The accommodating teacher mediates meaning within learners' discovery of mathematical concepts, truths, rules and properties derived from the mental models.

The didactical intervention starts at the level of existing cognitive possibilities and is further extended to unknown situations to enable the generalisation of concepts through flexible informal problem solving. Meaning is mediated before situations are escalated to more complex levels of cognitive demand which require self-discovery of concepts in both concrete and abstract contexts. As learners become autonomous in the self-regulation of mental processes, the teacher moderates her involvement.

Assessment items are set for whole concepts to represent the spectrum of real life situations within the conceptual field. The aim is to create a space for the demonstration of participants' knowledge, procedures and conceptualisation across the different dimensions of understanding and to allow for the analysis of the progression in the depth and breadth of understanding. Accommodating flexible methods and individual preferences of sign-making of individual participants, assessments are criterion referenced against a memorandum (score sheet) for performance rating as well as against a matrix for plotting the dimensions and levels of understanding.

Conceptual Framework

School mathematics demands that learner competencies meet the progressive complexity of the subject (Vergnaud, 2009, p. 83). Diagnoses of and remedial action to address the inability of mathematics education in SA to currently meet this demand, as noted by Motshekga (5 December 2013), may be suggested from various perspectives, including change in policy, review of the curriculum (DBE, 2012a), improvement of school facilities and learning environments, attention to subject intrinsic challenges (DBE, 4 December 2014), the redress of teaching practices (DBE, 2012c), as well as from the perspective of promoting learner proficiencies. This study focuses on the last, the individual learner, and specifically on the advancement of learners' metacognitive ability to regulate their own mental processes (Desoete & Ozsoy, 2009; Efklides, 2007; Kaniel, 2000; Koriat, 2007; Panaoura, 2007; Pintrich, 2002; Veenman et al., 2005) in support of mathematical concept formation.

The focus of this study is at the intersection between a critical phase in mathematics learning that occurs at the exit level of the Intermediate Phase, Grade 6; and a recurring troublesome mathematical concept within the multiplicative conceptual field, division (Lamon, 2007; Long, 2011).

Although division as a mathematical concept is built up from the early years of schooling, the conceptual understanding of division at Grade 6 becomes a requirement. Many real-world situations are associated with division, including incidents of equal sharing, grouping, area, ratio and rate, all within various contexts. Additionally, a conceptual leap is required in the application of division at this stage, namely the idea that the integers are not closed under division. Subsequently, the learner's knowledge of the number system is to be extended to include rational number.

Learners, in advancing to the Intermediate Phase, require a multi-dimensional understanding (Usiskin, 2005; 2012) of the concepts within the multiplicative conceptual field, as far as knowing when to use division, how to divide and how to represent the answer to the problem. Neither retrieval of what was previously stored about division through rote learning, nor operational efficiency in division calculations proves sufficient to sufficiently cover all cognitive levels (Anderson, 2002; Anderson & Krathwohl, 2001; Ferguson, 2002; Forehand, 2012; Krathwohl, 2002) and to meet the required level of conceptual demand (Linn, 2002).

Encouraging factors at this intersection point are firstly, that the cognitive abilities range that can reasonably be expected from an 11 to 12 year old, the typical age of a Grade 6 learner, extend mathematical thinking to reasoning with ideas and to imagining the intangible (Copeland, 1984; Demetriou et al., 2011). Two important quantitative cognitive abilities within their reach at this developmental stage, are flexible logical multiplication and proportional reasoning (Demetriou et al., 2011). Secondly, the latent capacity of the human mind to consciously regulate own thought processes in support of concept formation (Efklides, 2007; Koriat, 2007; Panaoura, 2007), has become an explicit metacognitive ability at 11 to 12 years. Self-regulation may be accomplished through various means, including visual, kinaesthetic and auditory mental imagery (Presmeg, 2005; Mason, 2002).

Drawing on the combined cognitive strategy of imagining mathematical ideas and the metacognitive ability of regulating own thought processes, the methodology of this research is a Design Research, where a didactical design (Van den Akker et al., 1999) is developed to explore the use of intentional visual imagery at Grade 6.

The aim of the designed intervention is to guide learners into discovering a mathematical concept within a realistic situation (Gravemeijer, 1994; Gravemeijer & Doorman, 1999; Van den Heuvel-Panhuizen, 2003) by means of visual imagery. The mathematical content chosen is division as it occurs in situations of equal sharing, area and rate. The teacher mediates the uninterrupted flow of learner consciousness from the perception of a situation through its objectification as a mental construct (Sfard, 1991; 1998), to the subsequent mathematical action. Based on the mental construct of real situations, learners regulate their own thought processes towards the subsequent mathematical action that would offer a solution to the problem situation. This strategy is used with a view to enhance multi-dimensional conceptual understanding (Usiskin, 2009) of a mathematical idea. The objective and subjective effects of the strategy are integratively analysed to enable conclusions about the proposed didactical design.

The conceptual framework for this study is conceived as follows below:

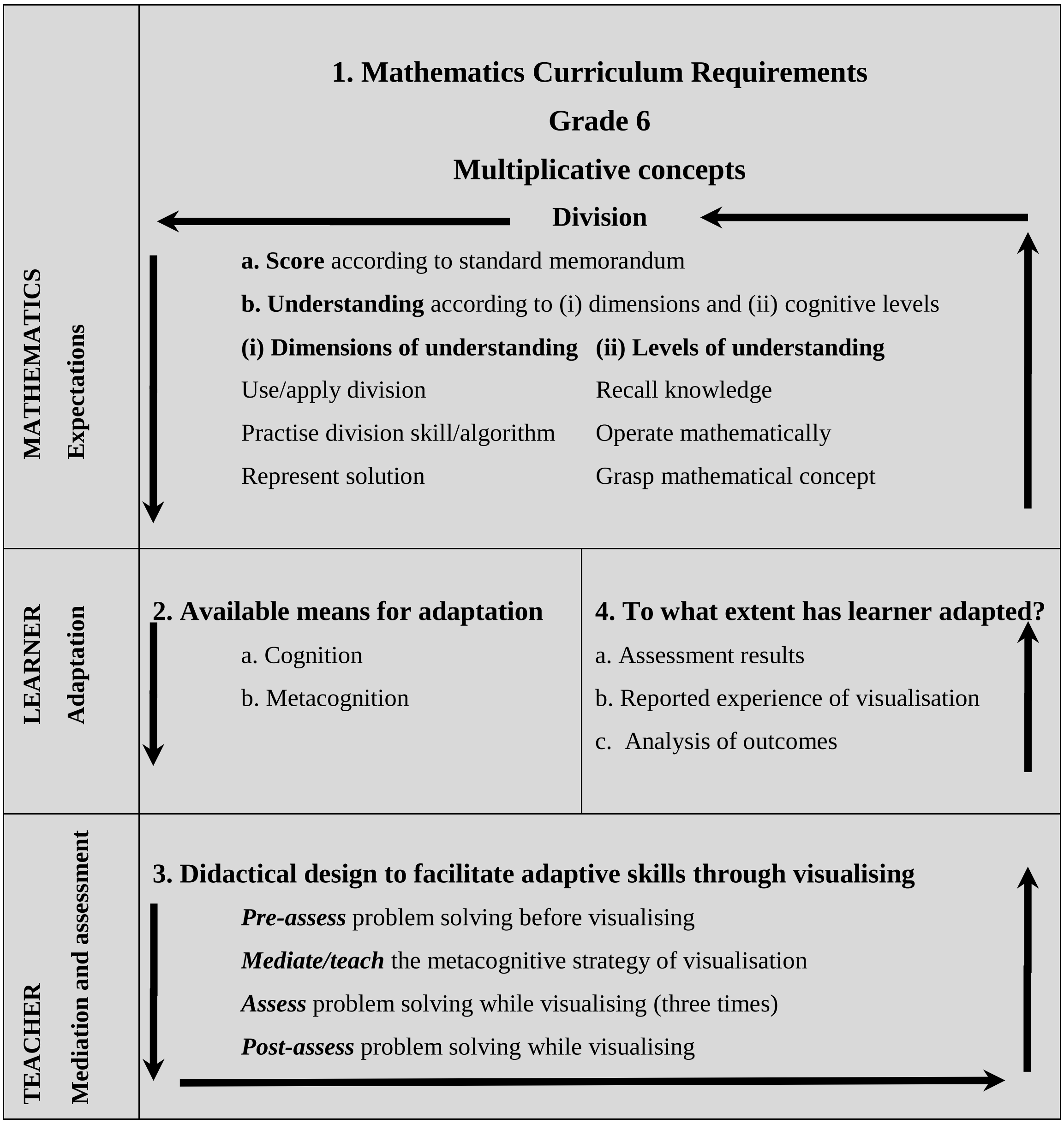


Figure 4. Conceptual framework for this study.

The four main elements of the above conceptual framework can be explained as follows:

1. The mathematics curriculum requires, amongst others, that certain applications of the concepts within the multiplicative structures, including division, are known and understood. The multiplicative conceptual field consists of situations and concepts and at each successive grade, learner

competencies must meet the progressive complexity of these concepts and their applications in different contexts. Division is a challenging concept at the senior primary level, as it extends the number concept from natural numbers to rational numbers. Also, the multiplicative concepts are becoming more abstract at the Grade 6 level, where understanding of division is required as it applies in money contexts, area and rate. The required proficiency levels are set out in the curriculum to indicate the level of attainment in terms of scores, however, it is also required that learners understand mathematics at a deep conceptual level. In this study, attention is given to measuring proficiency in terms of three dimensions of mathematical understanding and three cognitive levels of understanding in addition to measuring it in terms of scores according to the standard memorandum.

2. Learners of 11 to 12 years old have the available cognitive and metacognitive means to adapt to the above subject demands. Cognitively, they can reason with ideas as opposed to physical objects, they can imagine the abstract or non-real, they are capable of flexible logical multiplication and they are ready for proportional reasoning. Metacognitively, they are capable of self-regulation of their thoughts and visual mental imagery.
3. The didactical design that is developed in this study aims to facilitate the metacognitive aspect of the above adaptive skills in learners. The intervention is designed such that learners would discover within situations, mathematical concepts upon which one can act. Real situations are framed, which are incidents of division as equal sharing in money contexts, area and rate and learners are to discover the concepts through visual mental imagery. Before this metacognitive strategy is mediated to the participating learners, they are

assessed to establish their baseline proficiency in terms of scores and understanding. They undergo three consecutive sessions of exposure to the mediation and use of the strategy, each session including an assessment, before they are finally assessed while solving problems with the use of visualisation.

4. Finally, the participants' assessment outcomes are analysed, taking cognisance of their reported experience of the metacognitive strategy, to establish the extent to which they have adapted to the mathematical expectations as set out in the first paragraph, in comparison with their level of performance in the baseline assessment.

The methodology that was adopted to develop the design according to this conceptual framework, is discussed in Chapter 4.

CHAPTER 4 – RESEARCH METHODOLOGY

As stated previously, studies have not been encountered where visual imagery as a self-regulating metacognitive strategy has been investigated in relation to its potential to assist the understanding of multiplicative concepts within real-life situations. This omission is the case, at least at the primary level, specifically for the age group 11 to 12 years, the age of Grade 6 learners.

The investigation was conducted, informed by the main research question, namely, "How can structured visual imagery as a self-regulating metacognitive strategy be used at Grade 6 level for the understanding of multiplicative concepts as they arise in realistic situations?" In direct mapping with the first three specific research questions of this study, the aspects that were primarily investigated on an intellectual level through a literature review, were organised in three parts, as follows:

The first half of Chapter 2 contains a report of the literature reviewed in response to the first specific research question, namely, "Which mathematics education approach should be adopted in this research to meet the requirements for the teaching of multiplicative concepts at Grade 6?" The study was firstly positioned within the context of the present South African mathematics education policy. Classic and contemporary mathematics learning research and theories were reviewed, enabling the selection of anchor theories for the structural elements of the design. These theories pertain to the notions that the understanding of mathematical concepts can be reached as they are derived from situations of learners' lived reality; that concept formation can be mediated in an intentional manner; and that learners are co-constructing meaning of concepts as didactical partners of instruction.

The second part of the literature review in Chapter 2 responded to the specific research question: "How does the understanding of mathematics in general, and of the multiplicative concepts in particular, come about in learners?" Here, the dynamics of mathematics understanding was investigated, notably the understanding of concepts in the multiplicative conceptual field. The research studies and theories that have been reviewed here, have been used to guide the compilation of the mathematical contents of the design. It was understood in this part of the review, that the attainment of concepts at the primary level is a requirement for the transition from elementary to advanced mathematics; that modelling of situations mathematically is essential in conceptual development; and that indicators can be extracted in assessment of learners' underlying understanding of concepts in various dimensions and at various cognitive levels.

In Chapter 3 (the third part of the literature review), the third specific research question was addressed, asking, "How do the cognitive and metacognitive functions of 11 to 12 year old learners support their understanding of mathematical concepts? " The quest was for insights about the cognitive and metacognitive functions present within the minds of learners this age, that could be employed to improve mathematical understanding. These insights would enable the selection of theories to ground the strategies for reaching the instructional purposes of the design. Within this focus, it became clear that the cognitive engagement with concepts operates powerfully, at least at the primary level, within the interface of learners' lived reality and mathematics; that mental models of realistic situations can be created through intentional visual imagery; and that the cognitive and metacognitive abilities of learners this age, make it the opportune stage to mediate mathematical modelling through visual imagery.

The fourth specific question guiding the research on a practical level, is "How can a metacognitive strategy comprising structured visual imagery be mediated for the understanding of division at Grade 6?" Since educational designs addressing a problem of a similar nature have not been found in the literature, it was decided that the design function was best suited for such an investigation. Design Research as the preferred methodology for this study is discussed in this chapter, notably the process according to which the design principles for, and the characteristics of the instructional design were researched. This process included the development and design of various prototypes and their constituent elements, and their implementation and evaluation, in order to find the most effective instructional design.

The structure of this chapter includes four main sections, the first on the research design, the second providing an overview of the methods employed in the research, the third highlighting which, and to what extent, standard methodological norms for research were considered, and the last section reporting on the ethical considerations that were taken into account while conducting the study.

The Research Design

Epistemological Point of Departure

The aim of this study was to investigate and design an application that would contribute to addressing a particular problem in the primary school mathematics education domain in SA, for which there was no known solution.

According to Creswell (1994; 1998), pragmatism is the suitable philosophy to explore knowledge and knowledge acquisition in a problem-centred study within the real world practice. The characteristics of pragmatism are firstly, that it supports a developmental approach towards educational design; secondly that the theoretical assumption that is investigated for solving the research problem, can be assessed during its practical application (Creswell, 1998); and thirdly, that it has a capacity for logic in using combined data collection strategies to "best frame, address, and provide tentative answers to research question[s]" (Johnson et al., 2007, p. 125). Information can be derived from both "an external world, independent of the mind as well as that lodged in the mind" (Creswell, 2009, p. 11). Experiences that are reported from a first person point of view, enable the researcher to "search for the central underlying meaning of the experience and emphasize the intentionality of inward consciousness based on memory, image and meaning" (Creswell, 1998, p. 52). In such an investigative space, the insights provided by a narrative type of "mind data" are accommodated alongside empirical test results as both qualitative and quantitative data can be obtained and analysed within a single study.

Resonating with the above characteristics of a pragmatic paradigm, the research design of Design Research is also problem-centred and real-world orientated, a methodology which may use both qualitative and quantitative modes of enquiry. Johnson et al. (2007) therefore argued that pragmatism is best served by situating itself within a Design Research, which is the methodological approach that was adopted for the present investigation.

Design Research

Educational Design Research is an innovative approach, "appropriate to address open and complex problems in educational practice for which no clear guidelines for solutions are available" (Plomp, 2007, p. 9). Plomp (2007) described this approach as "the systematic study of designing, developing and evaluating educational interventions ... which also aims at advancing our knowledge about the...processes of designing and developing them" (p. 13). Just as important as the process(es), is the research requirement that the process should produce a set of design principles and the identification of the characteristics of such a design.

According to Plomp (2007), Plomp & Nieveen (2013) and Van den Akker et al. (2006), a Design Research approach follows an iterative and cyclical process, as it moves through three distinct phases, namely the Preliminary Phase, the Intervention Phase and the Evaluation Phase. Typically, the literature review of a Design Research would be conducted during the Preliminary Phase, as in the case of the present study, addressing the first three specific research questions. The development and testing of several design prototypes constitute the focal activity of the second phase, which is the Intervention Phase, which is in this study mainly to address the fourth specific question. The third phase of the Design Research, the Evaluation Phase, is designated for data analysis, the construction of the final design and reporting of the study. This phase provides a response to the main research question. The specific tasks of each phase are tabled in the research planning (Figure 5) and elaborated in Chapter 5.

The following characteristics of Design Research were derived from the ideas and theories of Barab and Squire (2004), Cobb et al. (2003), Kelly (2003), Nieveen (2007), Van den Akker (1999), Van den Akker et al. (2006) and Wademan (in Plomp & Nieveen, 2007), confirming the suitability of the approach for the present study:

- The interventionist characteristic allows for designing a novel intervention such as the mediation of visual imagery in mathematical problem solving.
- The iterative nature of Design Research involves repeated cycles of prototyping the design, product development, implementation, evaluation and revision, to refine its effectiveness and practicability in the classroom.

- The process-focused nature of Design Research seeks to understand both the processing of the mathematical problem in the mind of the learner and the effect that the intervention design has on that processing.
- The utility-oriented characteristic implies that the design aims to produce knowledge that can be used practically to explain how the intervention functions, in the context of Grade 6 learners, while they are coming to an understanding of division through visualising problem situations.
- Design Research as a theory-driven approach implies that the design is based in theoretical assumptions derived from the existing body of knowledge, which materialised within the present design.
- Involvement and active participation of practitioners at various stages of the intervention development were limited in this study, as a result of time constraints.

Design Research is promising in realising evidence informed educational reform (Nieveen, 2007; Plomp, 2011), which includes the knowledge about the interrelatedness of teaching, the subject matter and the learning and dynamics at work within the educational triad. This knowledge enables the postulation of a local instructional theory that sets in motion the quest for new evidence. A tentative instructional design is then created and practically tried out in several contexts. Each of the consecutive prototypes is critically analysed to ascertain why particular ideas worked and other ideas did not work. At the end of the cyclic refinement process, some design principles and a workable outline are formulated for a typical intervention aimed to respond to the initial instructional goal.

Quality Criteria for the Intervention Design

Nieveen (2007) proposed “four criteria for high quality interventions” (p. 26). The distinction between the quality criteria for the intervention and methodological norms for the research of the intervention, is subtle. In my view, the former refers to the criteria for the quality of the intervention as a product of the research, while the latter refers to the norms guiding the process of Design Research. To clarify this distinction, the two sets, the quality criteria aimed for, and the norms adhered to in meeting those criteria, are merged in terms of the activities undertaken during the various phases of the research (in Table 7). Before the overview of the design for the present study is presented, the quality criteria set for the intervention are explained briefly, as follows:

Relevance. As becomes apparent in the design planning (Figure 5), the confirmation of the need for the intervention was the main aim during the Preliminary Phase of the design. Mainly through the literature study, its scientific bases were established and it was ensured that the intervention would be designed using state-of-the-art knowledge, as suggested by Nieveen (Plomp & Nieveen, 2007, p. 26). As a methodological norm, relevance as referring to “content validity in context” is discussed later in this chapter.

Consistency. A second criterion is consistency. In the planning of this design in Figure 5, consistency refers to the principle that the intervention follows a process of logical design, where all components are consistently linked to each other (Plomp & Nieveen, 2007, p. 26).

Practicality. The quality criterion of practicality is met, if the intervention is usable in the setting for which it was intended and if it can be applied in a practical educational encounter, both by the researcher and educators in different settings. In this regard, a distinction needs to be drawn between expected and actual practicality (Nieveen, 2007, p. 94). Expected practicality refers to the expectation that the intervention would be usable in the settings for which it had been designed and developed. Actual practicality refers to the confirmation of the usability of the intervention in the settings for which it was intended. During the Intervention Phase, it was established that this study complies with the criterion of expected practicality. Actual practicality, however, remained to be proven by research following on the Intervention Phase, as it required a more substantial period of exposure to the practical situation than was possible within the time constraints of the second phase of the present study.

Effectiveness. The final test of the design is whether it actually reached the desired outcomes for which it had originally been designed, as compared to the expected effectiveness (Nieveen, 2007, p. 94). In the case of this design, if the design and the practical implementation of the intervention culminated in improvement in learner performance, it could be seen as effective – that is as far as the experiment goes. The actual effectiveness of the design in the everyday classroom practice should ideally be tested with larger cohorts by other teachers than the researcher.

Overview of the Design for This Study

The planning according to which the research was conducted, is reflected in Figure 5 below. In Figure 7 (Chapter 5) a summary is provided that was compiled during the Evaluation Phase, comparing the original planning and showing how this planning materialised in the actual Design Research process. The coherence between the final planning and the original planning confirms the suitability of the chosen approach towards the research. During the Intervention Phase the research process was envisaged as follows:

▫Preliminary Phase ⌖2012-2013	**▫Intervention Phase ⌖2014**			**▫Evaluation Phase ⌖2015**	
☆Literature study; ☆Curriculum and policy analysis; ☆Needs assessment	☆Design of intervention		☆Tryout with participants in classroom	☆Data analysis	☆Final intervention design ☆Write-up
⌖Conjecture; ⌖Local theory; ⌖Conceptual framework	⌖Prototype I	⌖Prototype II	⌖Prototype III	⌖Prototype IV	⌖Prototype V
	✪Expert advice	✪Teachers advice		✪Expert advice and review	
⋎Relevance	⋎Relevance	⋎Practicality		⋎Effectiveness	
	⋎Consistency		⋎Effective-ness		
⦿Questions 1-3	**⦿Questions 3-4**			**⦿Main research question**	

▫Phase and ⌖year of the design	☆Research tasks	⌖Products of the research process	✪Second party involvement	⋎Quality criteria for the intervention	**⦿Research questions**

Figure 5. Research planning schema.

In this figure it becomes apparent that the criteria for a high quality design as indicated by Nieveen (2007), were intentionally pursued from the start, while the research questions were systematically addressed within the phases; and that expert advisors were to be consulted at various points in the process. The Design Research approach makes it possible to conceptualise these elements of the research plan within a single framework. On the one hand a design plan serves to guide the process, on the other hand, it serves as a framework for systematic reporting in hindsight of the process. The planning for the various phases of the Design Research is briefly discussed here and the way that the plan materialised, is elaborated in Chapter 5. The research tasks are addressed as an aspect of the research methodology in this chapter and the prototypes of the design (as the products of the design process at various points) are discussed in Chapter 5.

The Preliminary Phase. In view of the importance of a thorough literature review for establishing the theoretical basis of the design, the greater proportion of the total research time was allocated to the Preliminary Phase. Through the literature study, a review of the curriculum and the relevant policies, the need for the study had to be established and its relevance for mathematics education in SA had to be confirmed. Subsequently, the goals in this phase were to position the study in the global context; to contextualise the research within the SA mathematics education arena; and to clarify the specific theoretical intent of the design – a critical component of Design Research, according to Gravemeijer and Cobb (2006, p. 48).

It was clear from the onset that the creation of an appropriate structure for organising the literature search would be subject to continuous refinement throughout the phase and beyond. Generally speaking, however, as has been discussed earlier, the first three research questions served to distinguish between relevant and irrelevant information, to select the main theoretical assumptions for the study, and to organise the review according to three themes corresponding to the specific research questions.

According to the theory of Design Research (Gravemeijer & Cobb, 2006; Plomp, 2007; Van den Akker et al., 2006), the Preliminary Phase is concluded with a conjecture from the present situation to the desired situation, the formulation of a local theory as the basis of the design and the articulation of a conceptual framework to guide the research process.

The Intervention Phase. As suggested by Gravemeijer and Cobb (2006), the aim of the practical phase was to bring the instructional starting point in closer proximity of the anticipated endpoint; and according to Plomp (2007), to gain insight into the mechanisms at work while improving the prototypes of the intervention. This process would progress in an iterative and cyclic manner, where a prototype or elements of a prototype would be subject to tightly integrated iterative reflection, analysis and further innovation. According to the planning scheme, it was expected that Prototype III of the intervention design would be suitable to be tested with participants in an experimental situation.

Two themes guiding the design in this phase were multi-dimensional mathematics understanding drawing on the work of Usiskin (2012) and the contribution of self-regulation, in particular the metacognitive strategy of visual imagery, towards conceptual understanding of situations requiring division.

All criteria for a quality intervention were to be met in this pivotal stage of the research with a special emphasis on practicality of the design. It was planned that academic, professional and practising experts would contribute to the research during the Intervention Phase.

The Evaluation Phase. Finally, in the Evaluation Phase, both the fourth and the final prototypes of the design were envisaged, research data were analysed and the outcomes of the study written up in a coherent form. This phase was to confirm the quality criterion of effectiveness of the design. As part of the evaluation of the design, two external evaluators reviewed the work prior to publication, resulting in intensive and meaningful alterations. The Evaluation Phase progressed through its own cyclic and iterative refinement processes, as will be discussed in Chapter 5.

The way that the process played out across the phases, is discussed in Chapter 5; the quantitative data derived from the fieldwork with participants is discussed in Chapter 6; the empirical outcomes are contained in Appendix C; and the final design is contained in Appendix D.

Overview of Methods

Given the iterative nature of the research, this section provides an overview of the research methods only, as the detailed methods are presented together with the findings in Chapters 5 and 6.

The present investigation was conducted as a small-scale research using convenience sampling. With the full knowledge of the disadvantages of such an approach where no claim could be made in terms of the representivity of the research outcomes (Cohen et al., 2007, p. 113) I had to consider the purpose of the investigation in justifying the scale of the research. The intention was not to generalise the findings beyond the sample in question, but to develop a design that was testable in an experimental situation in response to the main research question.

Research Site and Participants

Research site. A Government English-medium primary school in Centurion, Gauteng, was selected as the field research site. This school serves a low- to middle-income sector of the community according to the school zoning provision of the National Education Policy Act, 1996 (Act No. 27 of 1996), paragraphs 33 and 34 (Department of Education, 1996). Its racial composition is roughly representative of the broad population of SA and is based on fair admissions according to paragraph 8 of the Act (Act No 27 of 1996). The CAPS document (DBE, 2012a) is used to guide teaching and learning. Two aftercare facilities, one adjacent to the school, and another on the school grounds, accommodate learners from Grade R to Grade 7 after school.

Sample and participants. The principal was approached and permission was asked to conduct the investigation with Grade 6 learners who would be willing to participate on five consecutive Saturday mornings. He advised though that most learners were not able to make it to school over the weekend, mainly because of transport factors. Instead, he suggested that the research project would be conducted on five consecutive Friday afternoons at the aftercare facilities, because learners do not have homework sessions on a Friday afternoon. He requested the heads of the aftercare facilities to cooperate in the project, to allow the distribution of consent- and assent letters and to make a classroom available for the duration of the sessions.

With the cooperation of the heads of the aftercare facilities, the invitation to participate was extended to all Grade 6 learners (± 25) in the two aftercare facilities. They were called to a meeting to explain the purposes and expectations of the project as well as the principle of voluntary participation. Twenty-one learners opted to take part in the research study, of which the data of sixteen participants (four boys and twelve girls) could be used, as they attended at least four of the sessions and their parents had consented to their participation. Three learners did not have their parents' consent, but at their own request they were allowed to attend, although their information was not included in the research data set. Two boys chose to discontinue their participation after the first session. The age of the group was in the range 11 to 12 years. Six consecutive Friday afternoons were used from 14:00 for as long as was needed to finish the day's activities, normally about an hour.

The small sample of participants could be seen as a limitation of the study, however Nieveen (2007, p. 97) conceded that, when the assessments used in a research are of a formative nature, as was the case in this study, the sample size is less critical. It is however, she argued, a valid sampling approach in view of the study's main purpose "to locate shortcomings in the intervention and to generate suggestions for improvement" (Nieveen, 2007, p. 97). Similarly, Patton (1990) stated that "the key to purposeful sampling is to select cases for systematic study that are information rich" (p. 64). The purposes set by both authors were reached in this study, however it remains a future ideal to have the design tested with a large cohort.

Instruments and Data Collection Strategies

In the present study, these questions were best explored deriving information from both "an external world independent of the mind as well as that lodged in the mind" (Creswell, 2009, p. 11). Therefore, empirical test data as the outward demonstration of proficiency together with participants' subjectively reported inward experiences constituted the body of data that was gathered for this investigation. The use of more than one source of information allowed for the building of a complex picture of the dynamics at work when the proposed learning strategy was used (Costello, 2003) and added the feature of triangulation to the field research (Patton,1990).

Empirical data can be complemented by, understood in relation to, and analysed against the background of additional information derived from various other sources. The contribution of the literature review as a source of information in this study has already been clarified. As far as the design process is concerned, I have diarised the development of the design chronologically in a reflective design journal. If this progression had not been documented as the design unfolded, it would stay information "lodged in the mind" of the researcher (Creswell, 2009, p. 11).

In my quest for composite knowledge of the richness and complexities involved in mathematics learning (Cohen et al., 2007, p. 141), I had to synchronise the theoretical- or intellectual information, the chronological- or process information and the experimental quantitative- and subjective qualitative data that became available throughout all the phases and stages of the research. Table 6 below summarises the types of data and information generated by the research study, as the design was developed. The design journal and the assessment item analyses are regarded as sources of information, which primarily informed the design process, while the assessments and the metacognitive questionnaires (contained in the learner portfolios) are regarded as sources of the quantitative- and qualitative data, which informed the description of the statistics that were generated as a result of the fieldwork.

Table 6

Types of Information Derived and Data Generated by the Research Study

	Theoretical and intellectual information	**Qualitative subjective data and chronological information**	**Experimental quantitative data of assessments**
Type	Mathematics education and -learning, cognition and metacognition; SA mathematics curriculum (CAPS).	Documenting of, and reflection on design development process and work-in-progress; Reporting of metacognitive knowledge and experiences.	Pre-, during and post-intervention assessment; outcomes of participants' mathematics performance.
Instrument	Literature review.	Design journal; Metacognitive questionnaires.	Assessment item analysis; Participants' assessments.
Output	A conjecture; Conceptual framework; A local instruction theory.	Prototypes of intervention; Subjective metacognitive report; Design principles; Final intervention.	Test score to memorandum; Measure of understanding according to matrix of understanding.
Evaluation or analysis	Reflection, discussion, grouping and refinement of literature review in correspondence with the research questions.	Design development process description (Chapter 5); Metacognitive knowledge and experiences recorded, reported and discussed with test results per individual (Chapter 6).	Description of assessment data (Chapter 6) a. Counts b. Percentages c. Individual performance d. Group performance

Design journal. The development, reflection, planning and improvement of intervention prototypes were documented through journaling (Appendix A). This practice stemmed from the researcher's teaching profession, as a way of improving her own instruction, as related in Chapter 5.

Various situations were journaled: while prototypes of the intervention were in the planning, an entry would for example contain lesson expectations and objectives, the rationale for inclusion of elements in the intervention, reflection and critique of the preliminary products and design principles derived from the recent experience of design. In some entries the focus would be on assessment, the refinement of assessment items and measurement of assessment outcomes; some entries contained reflections of personal experiences and observations, subjective valuation of sessions, self-criticism and innovations for improvement of the existing design. In summary, the journal gives account of the design as a work-in-progress.

Evans (2002) proposed that empirical data as such is not necessarily a direct mirror of reality, but that reflection on the process of obtaining the data provides an additional source of interpretation and meaningfully adds to the quality of analysis. Reporting from a first person point of view allows comparison and merging of the subjective reality in relation to the objective reality. Additionally, a reflective approach enables the researcher to face contradictions between the researcher's expectations and the true outcomes of the experiment (Evans, 2002, p. 55).

The design journal assisted to shape the way in which the literature was viewed and the way in which theory was applied in practice, and together, the theoretical or intellectual contribution of the literature review and the empirical data, gave rise to the design principles upon which the design was founded.

Learner portfolios. Alabdelwahab (2002) highlighted the value of learner portfolios to showcase progress and accomplishments and to display self-evaluative insight into learning – an important metacognitive skill in self-regulating learning. Through learner portfolios teachers can also reflect on their own teaching strategies

(2002). The participant portfolios in this study contained only those materials that were relevant to the data gathering period, as follows:

- A protocol and planning for the intervention period;
- Three baseline assessment items;
- Nine intervention assessment items, three for each of three sessions;
- Six summative assessment items;
- The model for metacognitive problem solving;
- A metacognitive questionnaire with questions some of which were answered in written form and others in spoken form.

Assessments. The various assessments and assessment instruments that were developed for the fieldwork, underwent a number of developmental cycles, as part of the various prototypes of the design. Eighteen assessment items were eventually used in the fieldwork - six items were set in a money context, six in an area context and six in a rate context. Three items were used for the pre-assessment (baseline assessment), three items for each of the three intervention sessions and three items for the post-assessment (the summative assessment). The instruments according to which the participants' work were scored and their understanding was measured, the rationale behind these and their practical applications, are discussed in Chapters 5 and 6.

It was necessary to enter into the Intervention Phase with a clear picture of the participants' existing competency. The quest was for a practical and reliable way to assess progression of proficiency in realistic mathematical situations, covering three topics which were underpinned by the multiplicative structures. These topics were firstly, equal sharing and/or unit price in a money (currency) context; secondly, rectangular and square area using tessellation; and thirdly, speed as a concept of rate.

To assess participants' initial proficiency in these contexts, a baseline assessment had to be set, followed by assessments during the interventions and a summative assessment at the conclusion of the sessions. Within Prototype IV of the design, all interventions would include a set of three assessment items, dealing with one of the three contexts per session (See Figure 6), while the summative assessment would be similar to the baseline assessment, as illustrated in the figure below:

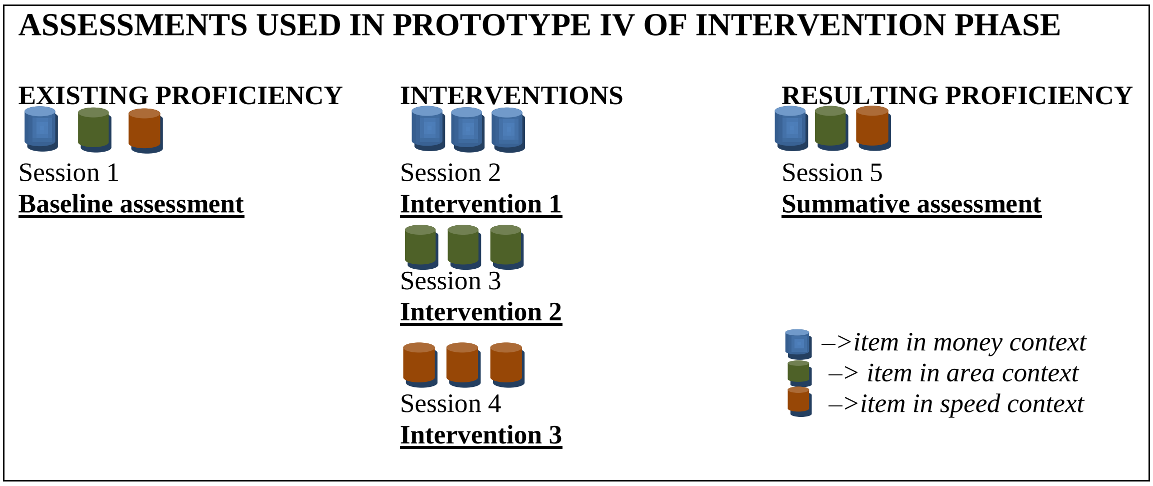


Figure 6. Assessments during the Intervention Phase.

The planning of five sessions changed to six sessions, as the post-assessment had to be repeated. The situation leading to this change is described in Chapter 5.

The items in a money context included the SA currency, rand and cent, and required calculations about cost and unit price. The items in a measurement context included concepts of rectangular area, length and width. In the rate context, concepts of speed, distance and time were used. Items were set in realistic contexts, following the prescribed mathematics contents for Grade 6 (CAPS) and representing different classes in the multiplicative structures. Scoring was done according to a conventional memorandum and a matrix of understanding (Chapter 6; Appendix A; Appendix B).

In Tzuriel's definition (2001, p. 238), a test-teach-test assessment is characterised as dynamic, namely "an assessment of thinking, perception, learning, and problem solving by an active teaching process aimed at modifying cognitive functioning." In much the same way, Embretson (1987, p. 143) described dynamic assessment as "testing that introduces material or instructions that can aid performance and in this way attempts to modify the performance level of an examinee." In her view, the assessment itself involves learning in a controlled situation.

Steinbring (2005, p. 31) stated with regard to assessment, that "the underlying principle is to have a high degree of match between what students are expected to know and what information is gathered on students' knowledge". For this research, assessments were therefore set according to what is required in terms of subject contents and cognitive levels of assessment in CAPS (DBE, 2012a, p. 296). These levels indicate depth of understanding and are closely related to the knowledge levels of Bloom's revised taxonomy for learning, teaching and assessing (Anderson, 2002; Anderson & Krathwohl, 2001; Raths, 2002). Additionally, mathematics proficiency should, in my view, also be articulated in terms of the width of the dimensions of understanding that learners attain while solving problems (Usiskin, 2012).

Metacognitive questionnaires. The second instrument for data collection was a metacognitive questionnaire, originally planned to be administered after each of the five sessions. The questionnaire was not administered after each session, because one of the objectives was to see how the participants viewed the change in their own metacognitive process of visualising, and the questionnaire did not make provision to track the process itself. It was therefore administered only after the last session.

The enquiry dealt with the following aspects of the participants' subjective experience and knowledge of visualising in solving mathematics problems:

- The judgement of the time spent, using the metacognitive strategy, as compared to the time spent in their usual way of problem solving;
- The judgement of the effort spent, using the metacognitive strategy to solve mathematics problems;
- The ability to recall the images that were created in visual imagery, including numerical data attached to the image;
- Knowledge about the goal that would be pursued, before visualising;
- Knowledge about the strategy that would be followed with the mathematics problem, after visualising;
- Preference for either an auditory or a visual modality of receiving information about the problem;
- Knowledge about their own process of visualising;
- Metacognitive experiences about aspects of ease and difficulty in visualising.

The subjective information gained from the questionnaires was not expected (and also not found) to be completely congruent with the assessment outcomes, however, they were informative and in some cases explained aspects of individual performance, difficulties or successes.

Data Analysis

The predominantly quantitative assessment data was the main basis for analysis; the questionnaires that were completed after the last session, delivered a minor, supplementary subset of qualitative data. The analysis of the assessment data was aimed at exploring any changes in learner proficiency when solving problems which required the understanding of multiplicative concepts, when they visualised situations mathematically. The questionnaires were expected to provide additional insight into participants' subjective knowledge and experience of visualisation.

Analysis of assessment data. In view of the small scale of the study, the sample size (16), the number of assessment items (18) and the number of questions in the questionnaire (9), each participant's verbal and written responses and all items were treated as meaningful. A measuring system was devised with many various indicators of learner proficiency and understanding, which enabled rich descriptors of each participant's progression.

In preparation for the data collection and analysis of this research, each assessment item was analysed (Appendix B) and four marks were allocated per item for indications of accurate problem solving, according to a standard memorandum. Additionally, individual items were analysed and described in terms of three dimensions and three levels of understanding, resulting in a total of 9 potential measures of the depth and width of the participants' understanding per item. The questionnaires contained six questions, each with multiple response options which had to be completed in writing as well as three questions which were asked to all participants verbally, while field notes were made.

The scales used to describe participants' marks are, according to Cohen et al. (2007, p. 503), regarded as a powerful level of data for the processing of information, also when the need exists to order or group selected aspects of the data sets. The values attached to the assessment responses of participants made it possible to analyse the main body of quantitative data generated in this study, by way of descriptive statistics (Durrheim, 1999), as follows:

After each assessment, all participants' work was marked on the two scales as mentioned before. Their responses per session were typed out verbatim. According to the memorandum (Appendix C), the parts of their work deserving a mark, were typed in green, those not deserving a mark in black and the actual number value of the mark that was allocated, was typed in red. The plotting according to the matrix of understanding, was recorded per session in a table form with nine empty blocks per learner for each of the three items of the session, and those aspects where participants had demonstrated understanding, were shaded in green (Appendix C). This coding provided for a visual image of a learner's depth and width of understanding, and for comparison of progress across the sessions. The shaded blocks per item, per learner, were counted to indicate the incidents of understanding demonstrated.

After marking the assessments, the first analysis task was to capture each individual's written responses and their unprocessed measurements electronically (Appendix C) in a table format that allowed for classification and ordering. Total scores, the range of scores and the mean group- and individual values were calculated (Appendix C). The ordered data was represented per group, per item type and per individual, each in a table form and also in a graphical format (Chapter 6).

In this way, the following variables were described:

- Scores according to the memorandum over six weeks;
- Counts on dimensions and levels of understanding over six weeks; and
- Performance per item type before, during and after interventions.

The marking of assessments against the memorandum was moderated to eliminate errors of judgement and mistakes in score calculations.

The marking against the matrix of understanding posed a greater challenge. In this regard the analysis of items before the onset of the assessments was helpful and responses were reflected against those descriptors for specific indicators of understanding. Additionally, the system and the researcher's judgement were reflected with an expert in the combined domains of mathematics, cognition and metacognition. Even so, I reconsidered my own judgement of each participant's responses at several more occasions to ensure that all responses were treated in a fair and comparable way.

By representing the variables numerically in tables and also in a graphic format and by ordering the measures, the relationship between group scores and counts of incidents of understanding (now referred to as "counts"), between measures at different intervals over time, between scores and counts per individual participant, between individual measures over time and between item types at different occasions each became evident. Mean group performance on the two scales was calculated and the range of scores and counts was determined for the group.

Although the sample size did not allow for inferences and generalisations, the degree of detail that could be obtained through the measurement instruments on two different scales (the memorandum and the matrix of understanding), proved valuable in the detection of characteristics within the sample that influenced the formulation of some findings. These findings were possible, to a lesser extent with regard to the group, since the range of measured performance was wide and did not permit many specific findings. Rather, individual patterns of performance were treated in detail.

Analysis of metacognitive questionnaires. The sub-set of qualitative data obtained from the metacognitive questionnaire, contained structured questions about metacognitive knowledge and experience and semi-structured questions about the use of the metacognitive strategy, according to guidelines derived from Costello (2003).

The purpose of the questionnaire was not to survey respondents' opinions, but to prompt their expression of inward experiences and knowledge of visualisation while solving problems. In fact, the questionnaires were useful to initiate interaction between the researcher and the participants, corresponding more with the characteristics of a structured interview than a typical questionnaire – resonating with the description by Cohen et al. (2007, p. 349), stating that "the interview is not simply concerned with collecting data... its human embeddedness is inescapable."

For practical reasons, the questions with set response options were answered in writing, while the open-ended questions were answered verbally. Both spontaneous participant contributions and verbal responses during the encounters were recorded in field notes and captured per participant (Appendix C). By comparing the assessment outcomes with metacognitive reporting, it was hoped, as Marrapodi (2006) suggested, that "metacognitive descriptions (would) provide insight into limitations and errors in learners' understanding (and) reveal their thinking".

Individual scores and understanding, the researcher's observations and participants' subjective knowledge and experiences of the metacognitive strategy were described in a collated form and discussed per individual (Chapter 6). This discussion was complemented by the background of events as recorded in the design journal.

Although it was clear that all participants had used visualisation as a newly acquired strategy in problem solving, their measured performance could not consistently be related to the participants' metacognitive experiences and knowledge. Some participants' metacognitive reporting aligned well with their demonstrated performance, while in other cases there was almost no link. This observation gave rise to some findings that in turn influenced the final design. In some instances though, the metacognitive reporting explained participants' demonstrated mathematical behaviour, which enabled a better understanding of the dynamics at work while solving problems.

Conclusive remarks about the analysis of data. The aim with the systematic exploration of the classroom outcomes of a series of structured interventions was to investigate any relationship between the proposed instructional design and progression in mathematics performance while using the metacognitive strategy. Although the sample of participants was small, events could be isolated that were incidents of such a relationship (Miles & Huberman, 1994, p. 7).

At the conclusion of the Preliminary Phase of the study, it was conjectured that Grade 6 learners' understanding at different cognitive levels, of the division concept situated in real life contexts, would improve, subject to a baseline assessment, from a presumed starting point of relatively narrow and shallow understanding towards a broader and deeper understanding, through the use of a metacognitive strategy of visual imagery.

The analysis of the quantitative data set, as supplemented by the minor qualitative sub-set, revealed that learners had moved from their instructional starting point towards an improved end point and that this change could be linked, with certain provisions, to the use of the metacognitive strategy. This observation, based in the analysis of data, is elaborated in Chapter 7.

Methodological Norms

The purpose of the investigation was to observe changes, if any, in mathematics performance and understanding when participants employed visualisation as a self-regulating metacognitive strategy while they were solving problems requiring division. The methods that were employed to reach this purpose, had to comply with the standard norms for research of this nature and due attention had to be given to those aspects of the research that would allow both critics and experts to verify the outcomes and findings of the investigation.

Robson (2002, p. 93) referred to the important concepts that are traditionally associated with research, namely validity, reliability and generalisability. Validity was simply seen as the requirement that a study would describe what it set out to describe and reliability as the requirement that the measures applied in a study could be repeated in another study with the same results. Generalisability or transferability is the requirement for applicability in further contexts.

Validity

Cohen et al. (2007) held a much broader view of validity though, as a quality of research that may be improved from many different perspectives and through the use of various strategies. At the same time, they state that perfect validity is an unattainable ideal of research. The best a researcher can do, is to “minimise invalidity and to maximise validity” (Cohen et al., 2007, p. 133).

I argue that certain forms of validity are only partially attainable: the norm of generalisability for one, cannot be applied to the data that was generated in this study, mainly in view of the sample size. Aspects of the study would be generalisable in other ways though, when the design principles, the measurement instruments and the design itself could be applied in contexts other than was the case during the research.

I further argue that the person of the researcher may impact generalisability. The presentation of the same design, if replicated by another teacher may, or may not have the same effect than was the case in the experimental situation. The teacher's unique style and the way that she relates with learners will differ from practitioner to practitioner, which may influence the effect of the strategy in other classrooms.

Furthermore, as far as the qualitative aspect of self-report in this study is concerned, I understand that validity is not about what the researcher regards as valid, but what is regarded by the respondent as valid. In the present study, some participants' knowledge and experience of visualisation were obviously off the mark or "wrong" in the eyes of the researcher as observer, because it showed no correlation with their mathematical performance. However, their honest responses are authentic and therefore "secure a sufficient level of validity and reliability" (Cohen et al., 2007, p. 135).

As a Design Research has an obvious concern with processes and not merely with the outcomes of an experiment, its descriptive validity is probably its strongest claim. Actual processes, the factual evolution of the various prototypes of the design and the true outcomes of the assessments and the questionnaire are directly reported in this thesis. Even so, the possibility that the narrative could be edited to distort the true situation, cannot be excluded completely, especially in terms of what was not reported.

The present study features strong theoretical validity and firm theoretical anchors, notably in the core theoretical assumption, the interface between the cognitive development of mathematical skills at age 11 to 12 years and learners' ability to model situations mathematically in their minds. This, in my view, is an important insight and integration of theories, which explains the potential of visualisation (at this age) to support conceptual understanding of troublesome mathematical ideas.

The matrix of understanding that was used to plot indicators of learner understanding, was a design element which could be regarded at face value as a strong feature of this design, but which has, in my opinion, a potential weakness in internal validity. Its weakness is not situated in the elements used to plot the indicators of understanding – those are based in strong and well researched existing theories. The possible weakness of internal validity is in the inferences made for the descriptors of categories. This aspect caused uncertainty and was therefore reflected with an expert after which it was revised. Plotting learner responses on this matrix, requires a level of judgement which could add an undesirable subjective element by the researcher.

On the other hand though, a remarkable alignment became apparent between participants' understanding according to the matrix and the scores that they obtained according to the memorandum. This alignment was encouraging and could be regarded as an indication of internal validity, despite my own insecurities.

This study cannot claim external validity, since it was not a research goal to generalise the results to a wider population in contexts other than the present study. Ideally, such generalisations may become possible through follow-up research in large cohorts and in different contexts.

Content validity was a further aspect of quality that was reasonably well attained within the limitations of the study. This is in application of Cohen et al.'s (2007, p. 237) requirement that the instrument "fairly and comprehensively covers the domain and the items that it purports to cover". The multiplicative concepts that were assessed, are representative of the levels and types of division required by the mathematics curriculum for the Intermediate Phase, and the items that are incidents of these types of division are representative of the lived reality of the participants. Also, the metacognitive questionnaires covered the categories of metacognition that have a direct bearing on the issues that were investigated.

Construct, in Cohen et al.'s view (2007), is an abstract: "In this type of validity agreement is sought on the 'operationalized' forms of a construct, clarifying what we mean when we use this construct" (p. 138). The main construct of interest in this Design Research is an innovative instructional design that was tested through investigation. The construct could not be reflected against similar designs, because, to the best of my knowledge, they have not been done. It can be confirmed with reasonable certainty that the study was researching what it originally set out to do in order to answer the research question (Robson, 2002), since the design was drafted and refined, developing rationally and logically (Plomp, 2007). In fact, reason and logic were the only amenities available to create the instructional design.

Evans (2002) stated that conceptual clarity "is an essential ingredient of methodological rigour and of the development of theory" (p. 72). Due attention was given to describe the exact dimensions of the key terms of the study for the sake of conceptual clarity and consensus between the reader and the researcher (Evans, 2002, pp. 52-57). In a few cases, I have explained terms on the basis of my own understanding as they apply in the context of this study.

Triangulation

Triangulation as a methodological norm (Patton, 1990; Cohen et al., 2007, p. 141) was employed through the use of two instruments for data collection; and through the cross-verification of assessment results by using two rating scales. In my view, the strongest claim for triangulation in this study is in the area of integrating multi-dimensional theories into a single, coherent intervention design.

Reliability

The quest for reliability posed a major challenge, especially in view of the small scale of the study and the fact that it produced a novel design, the application of which had not been tested on a wide scale, and will now be discussed.

Cohen (2007, p. 146) expressed the sentiment that reliability is concerned with precision and accuracy. In his view of reliability as stability, this study is complying to the norm, in that it has delivered similar data from the same respondents at regular intervals over time. The intervals between interventions and from pre-test to post-test were, in my view, long enough to exclude the immediate effect of the previous intervention and short enough to prevent external influences of teaching and learning to impact significantly on the differential effect of the strategy on participants' performance. The remaining ideal in this regard, is that it would deliver similar data from similar respondents in large cohorts over longer periods of time.

A strong feature of this design was its compliance with reliability as equivalence (Cohen, 2007, p. 147). Not only have equivalent forms of testing been used throughout as has been explained earlier, but also the rating of responses was consistent (Robson, 2002) as they were all undertaken by myself as the researcher. Bias in this regard was diminished by the influence of an expert advisor who moderated my judgement. Relevance or content validity, requiring that the design is based upon modern, acknowledged, researched, proven and accredited knowledge in the field, is, in my opinion, also a strong feature of this research.

I argue that the two concepts, methodological norms and the criteria for high quality interventions, can be seen in a cause-effect relationship to each other: if the norms are complied with, the result will meet the criteria. Table 7 merges the two concepts with the systematic actions taken in addressing each research question:

Table 7

Methodological Norms, Quality Criteria and Research Procedures, Applied to Research Questions

Research Question and Phase of the Design Research	**Methodological Norms**	**Research Procedures**	**Quality Criteria**
Preliminary Phase **Question 1.** Which mathematics education approach should be adopted in this research to meet the requirements for the teaching of the multiplicative concepts at Grade 6?	***-Theoretical validity*** ***-Content validity*** ***-Construct validity*** ***-Conceptual validity***	-Review literature on SA mathematics education situation and curriculum. -Review literature about mathematics education approaches.	***-Content validity or relevance***
Preliminary Phase **Question 2.** How does the understanding of mathematics in general, and of the multiplicative concepts in particular come about in learners?	***-Construct validity*** ***-Internal validity*** ***-Triangulation***	-Review literature about the mathematics focus of study. -Identify theories for an intervention prototype. -Apply theories to assessments.	***-Content validity or relevance*** ***-Practicality*** ***-Construct validity or consistency*** ***-Effectiveness***
Preliminary Phase **Intervention Phase** **Question 3.** How do the cognitive- and metacognitive functions of children 11 to 12 years old, support their understanding of mathematical concepts?	***-Theoretical validity*** ***-Triangulation*** ***-Reliability (precision and accuracy)*** ***-Content validity*** ***-Construct validity***	-Review literature about cognition and meta-cognition in mathematics. -Apply visual imagery in an intervention for use in experiment with participants.	***-Relevance*** ***-Practicality*** ***-Construct validity or consistency*** ***-Effectiveness***
Intervention Phase **Evaluation Phase** **Question 4.** How can a metacognitive strategy comprising structured visual imagery be mediated for the understanding of division at Grade 6?	***-Generalisability*** ***-Replicability*** ***-Authentic validity*** ***-Descriptive validity*** ***-Triangulation***	-Ensure optimal intervention results by continuous refining of prototypes. -Apply the best prototype in the field experiment with participants. -Assess effects of learning in relation to the metacognitive strategy that was mediated.	***-Effectiveness*** ***-Practicality*** ***-Consistency***

Research Ethics

The ethical requirements for research in which children are involved, compel special attention (Louw & Edwards, 1997) to avoid any harm to learners, including infringement on their dignity, forced participation, participation without consent, violation of privacy and unacceptable levels of disclosure of information. Before the fieldwork for the research could be undertaken, formal ethical clearance from the University of Pretoria, consent from the principal of the school, informed consent from participants' parents and assent from participants themselves had to be obtained.

Ethical Clearance From the University of Pretoria

The application at the Research Ethics Committee of the University of Pretoria was duly processed and prior to commencement of the fieldwork on 16 May 2014, approval was received to continue with the fieldwork. The Integrated Declarations form (D08) was completed and an ethical clearance certificate was obtained from the Faculty of Education Ethics Committee (University of Pretoria). This certificate is attached in Appendix E.

Permission From the Principal of the School

A school was selected as the research site where the researcher had not previously taught and did not know any of the participants. Permission to conduct the research had to be obtained from the principal of this school. I contacted him and discussed my request and the intended procedures for the fieldwork with him after which his written consent had been gained. The letter of consent is attached in Appendix E. Thereafter, I approached the heads of the aftercare facilities, discussed the matter with them and arranged for a convenient time slot that would suit their programmes and those of the Grade 6 learners involved.

Informed Consent and Voluntary Participation

All Grade 6 learners who made use of the aftercare facilities, and their respective parents, had to be informed about the research in writing, in order to obtain their respective consent and assent. Examples of these letters are attached in Appendix E. The purpose of the investigation was explained and the relevant information that would have an influence on their decision to take part in the research, was clearly communicated. In the letters of invitation it was also explained how the results would be used. Having read and understood the purpose of the study, and what would be expected of them, learners could sign a letter of assent and their parents could sign a letter of consent for their participation in the study. In the letters and during sessions, it was made clear that learners participated voluntarily, that they were free to withdraw at any time and that withdrawal would not affect them in any way.

From the 26 potential participants, 21 wished to participate with due consent. During the intervention, two learners wished to withdraw and they were released. Three learners chose to participate, although they could not produce the letters of signed consent from their parents. They were allowed to join in, although their results were not used, since parental consent could not be proven.

The language of the letters, the interventions and assessments was English, which is the language used for education at the participating school. No participants had therefore been excluded from understanding the conditions of the invitation.

Learners were aware that their written and spoken responses would be documented and that they had the right to access their transcribed verbal responses and assessment results if they so wished. No marks or scores were announced in class, since the outcomes were kept confidential from the other participants.

Safety in Participation

No potential risk or harm to learners had been anticipated in this study, and none occurred. The sessions focussed on success and not on failure, input built onto existing skills and aimed at improvement of performance. Exposure to other learners in the classroom setup was nothing other than what they were used to during the course of a normal school day.

Privacy, Confidentiality and Anonymity

Anonymity was guaranteed and data was not linked to a particular name, but to a code allocated to each participant. The letters of invitation to participate were returned to the researcher and not to the teacher or principal, therefore the principal was not aware which learners had participated. Only the researcher had the final list of names of participants and this information was not disseminated to any other person.

The learner portfolios, with their contents of tests and question items, bore the code number and letter allocated to each particular learner. All participant work and the class discourse were recorded by the researcher and kept for control purposes. They were duly classified and access to them were not given to any outsider. All parties involved knew that the research results would be published anonymously in the thesis following the research, in articles and in presentations at seminars and conferences.

CHAPTER 5 – DEVELOPMENT OF A MODEL FOR MEDIATING VISUALISATION IN DIVISION LEARNING

The investigation in this study followed a Design Research approach, as a pragmatic way of developing an instructional design. As was proposed in the literature about Design Research drawing on the work of Gravemeijer and Cobb (2006), Plomp (2007) and Van den Akker et al. (2006), the research progressed through three phases. The literature review was the main focus of the Preliminary Phase. The resulting theoretical-, conceptual- and methodological frameworks for the study have been documented in Chapters 2, 3 and 4. The development of the design prototypes and the try-out of the design were the main focus of the Intervention Phase. The design process was recorded in a design journal (Appendix A), and is documented in Chapter 5. In the Evaluation Phase, the data collected during the try-out was captured and analysed (Appendix C; Chapter 6), the final design was composed (Appendix D) and conclusions were drawn in the reflection on the study (Chapter 7).

Background: Journaling the Design Process

Before the commencement of this study, I had entered into an informal process of improving my own mathematics teaching, which I later found, was a practice advocated by researchers such as Miller (2007), Evans (2002) and Parsons and Brown (2002). Byrd (2002) advocated teacher reflection, posing the question: "About what should teachers reflect?" and answering: "Teachers should reflect on the fundamental questions that have plagued them for some time" (2002, p. 247).

Costello (2003) also appraised the teacher as researcher when they engaged in their own practice and developed their own theories deriving from that practice. As a routine task of my everyday teaching, the learning process was continually observed and data collected to improve not only my instructional practices, but also my mathematics understanding and learning; and to find solutions to problems.

I used to invite learner reporting about their subjective learning experiences, an approach which I learned later, corresponded with a phenomenological view of knowledge acquisition. In the systematic journaling of the learner reports and my own experiences, use was made of what Evans (2002) termed “reflective practice”. The teacher journal provided a framework for understanding why, when, and how learning improved or did not improve within the everyday classroom teaching and learning.

The offshoots of journaling were two-fold: firstly, a recurring factor in learner success came into focus, namely the use of metacognitive strategies for mathematical problem solving; and secondly, journaling as an instrument in improving instructional design. As a result, a metacognitive strategy of visual imagery was identified as the focus of this study and journaling became instrumental in the development of the design. As the teacher became a researcher, so did the teacher journal become a Design Research journal, now regarded as the main design instrument in this research.

The literature is explicit about the advantages of journaling for the research process: Ortlipp (2008) appraised reflective journaling in research as formative, allowing visibility of the “constructed nature” (p. 695) of the research process; and how the construction was built on choices and decisions. Additionally, the researcher is “engaging with the notion of creating transparency in the research process, and explore(s) the impact of critical self-reflection on research design” (2008, p. 695).

Evans (2002) stated that reflective practice strives to concretely improve research within the teaching reality as subjective self-criticism and objective perspectives are merged to dynamically reform the design. This journal contains subjective verbal reports laden with perceptions (Yin, 1994), as well as arguments, motivations for change, planning for interventions and assessments, rationale and self-criticism. Each of the design prototypes developed in a cyclical way and delivered a useful element, instrument or idea that could be retained and carried forward, albeit in a refined form, for the eventual design.

The Design Research Planning

The research process had been envisaged as depicted in Figure 5 (originally presented in Chapter 4, p. 95). For convenience of comparison with the actualised plan, the original plan is replicated below. This plan materialised slightly differently from the planning, and the actualised process is presented in Figure 7 (p. 124).

▫Preliminary Phase ⌖2012-2013	**▫Intervention Phase ⌖2014**			**▫Evaluation Phase ⌖2015**	
☆Literature study; ☆Curriculum and policy analysis; ☆Needs assessment	☆Design of intervention		☆Try-out with participants in classroom	☆Data analysis	☆Final intervention design ☆Write-up
⊕Conjecture; ⊕Local theory; ⊕Conceptual framework	⊕Prototype I	⊕Prototype II	⊕Prototype III	⊕Prototype IV	⊕Prototype V
	✪Expert advice	✪Teacher's advice		✪Expert advice and review	
⋎Relevance	⋎Relevance	⋎Practicality		⋎Effectiveness	
	⋎Consistency		⋎Effective-ness		
⦿Questions 1-3	**⦿Question 4**			**⦿Main research question**	

▫Phase and ⌖year of the design	☆Research tasks	⊕Products of the research process	✪Second party involvement	⋎Quality criteria for the intervention	**⦿Research questions**

Replica: *Figure 5*. Research planning schema.

Visualisation as a metacognitive strategy in learning multiplicative concepts: a Design Research intervention

Preliminary Phase 2012-2013	Intervention Phase 2014a	Evaluation Phase 2014b-2015
Objectives: **To define frameworks for the study: - theoretical - conceptual - methodological**	**Objectives:** **- To develop the elements for instructional design prototypes - To evaluate and improve the prototypes during each design cycle - To test the semi-final design prototype with participants - To collect and evaluate empirical data in a formal research setting**	**Objectives:** **- To analyse data - To construct a final design - To write up the study**
Research tasks 1. Review relevant literature of Theories Research CAPS 2. Construct A conjecture Local theory Conceptual framework 3. Design Prototype I	**Research tasks** 3. Develop Instructional models Assessment items Assessment instruments Design Prototype II Design Prototype III 4. Evaluate and improve prototypes **Design instrument** Design journal 5. Compose Design Prototype IV Intervention Protocol 6. Try out Prototype IV The intervention The effect on learning Participant experiences **Data collection instruments** a. Mathematics assessments b. Metacognitive questionnaire	**Research tasks** 7. Analyse empirical data 8. Evaluate design process 9. Extract design principles 10. Construct final design Prototype V 11. Write up the study
Documented in Chapter 2 (Literature review: mathematics teaching, learning) Chapter 3 (Literature review: cognition and metacognition) Chapter 4 (Research methodology)	**Documented in** Chapter 5 (Development of the model for intervention) Appendix A (Design journal) Appendix B (Analysis of assessment items) Appendix C (Assessment outcomes) Chapter 6 (Results of assessment) Appendix E (Letters to participants)	**Documented in** Chapter 1 (Overview of the study) Chapter 7 (Reflections, conclusions, design principles, recommendations) Appendix D (Final design)
Quality criterion Relevance	**Quality criteria** Relevance; consistency Practicality; effectiveness	**Quality criterion** Effectiveness; practicality; consistency
Secondary research questions addressed 1. Mathematics education approach in this study 2. Mathematics learning 3. Cognition and metacognition in mathematics learning	**Secondary research questions addressed** 3. Cognition and metacognition in mathematics, specifically at 11 to 12 years 4. Characteristics of an intervention to mediate structured visual imagery for understanding division at Grade 6.	**Main research question addressed** "How can structured visual imagery as a self-regulating metacognitive strategy be used at Grade 6 level for the understanding of multiplicative concepts as they arise in realistic situations?"

Figure 7. Summary of the actual Design Research process and reporting.

The research for the design of the intervention proceeded through three phases, as illustrated in Figure 7 and elaborated in this chapter, as follows:

The Preliminary Phase

The objectives set for this phase, pertained to the essential frameworks of the study, namely to establish a theoretical framework for the research, a methodological framework for the design process and a conceptual framework for the design itself.

The theoretical framework for the study was established through a review of the literature containing theories and research within the global mathematics education arena; and also through a review of the South African mathematics curriculum to contextualise the study within the local mathematics education arena. The need for the present study was eminent from a research point of view as the specific focus of the research problem had not been encountered in a comparable study in the literature. This need was found to be especially pertinent within the SA context. In this way, the specific theoretical intent of the design was clarified, as is advised by Gravemeijer and Cobb (2006, p. 48) for this phase of a Design Research.

The creation of an appropriate structure for organising the literature search, was subject to continuous refinement, even beyond the Preliminary Phase. The review was eventually structured according to the themes of research questions 1-3, namely:

- The mathematics education approach for the instructional design;
- Mathematics learning, specifically of multiplicative concepts; and
- The cognitive- and metacognitive functions of learners 11 to 12 years old, that support their mathematics understanding.

Whereas these three themes were mainly investigated on a theoretical level, they were applied practically and tested within the Intervention Phase, while the fourth research question, the question pertaining to the characteristics of a design for the mediation of visual imagery, was addressed.

The Preliminary Phase delivered four distinct research products: a conjecture for the research; a local theory that formed the basis of the research process; the conceptual framework of the research; and Prototype I of the design.

A Conjecture

Towards the end of the Preliminary Phase, an ambitious conjecture was envisaged for this study to bridge the gap from an assumed starting point of "relative absence", to an end point of a "proven relative presence of conceptual grasp of applied division in realistic contexts".

At the first design attempt, it was, however, realised that both points of the conjecture were based mainly on academic and theoretical assumptions: "relative absence of conceptual grasp" could firstly not be assumed as the starting point for all participants, and secondly, could not be assumed without having proven it; and "relative presence" provided too vague a qualifier to be either pursued as a goal or proven as an outcome. The assumptions were all too general and unstructured and they had to be subject, according to Gravemeijer and Cobb (2006) and Plomp (2007), to the escalating and refining proofs of the Intervention Phase. Following expert advice, it was realised that a baseline assessment would be suitable to demarcate a realistic starting point for the conjecture. In a rephrased starting point, reference would therefore be made to "the level of performance as established through a baseline assessment", rather than to "relative absence of conceptual grasp."

Similarly, since the conjectured end point should remain a constant goal according to Gravemeijer and Cobb (2006, p. 49), the existing end point was found vague and inadequate to direct the design. The theoretical end point was subsequently changed into a realistic research goal of "improvement in conceptualisation of division" rather than "relative presence of conceptual grasp."

The second and third versions of the conjecture only evolved at the onset of the Intervention Phase. As a final phrasing, it was conjectured that Grade 6 learners' understanding at different cognitive levels, of the division concept situated in real life contexts, would improve, subject to a baseline assessment, from a presumed starting point of relatively narrow and shallow understanding towards a broader and deeper understanding, through the use of a metacognitive strategy of visual imagery.

A Local Theory

Secondly, in this phase, as suggested by Gravemeijer and Cobb (2006, p. 51), an empirically founded local theory has to be formulated, which can be elaborated and refined while practically conducting the experiment (Thijs & Van den Akker, 2009). As discussed in Chapter 3, the local theory centred around the use of a metacognitive strategy in exploring and mastering sustainable and valid mathematical concepts as they apply in everyday situations. The theoretical assumption in the local theory is that, in the transition from concrete learning, through representational learning, learners may create a workable mental model of a realistic situation upon which they can act mathematically, in preparation for more abstract mathematising.

Also during the Preliminary Phase, the first prototype of the instructional design was developed. This prototype was not practicable and it fell short of construct validity. However, a few design principles could be extracted from it for later use.

Prototype I

As related in the design journal, the first efforts towards an instructional design were made during the Preliminary Phase (2012b), as the need for the study became clear and some theories were derived from the literature review. Its two constituting elements were: a set of required and demonstrated proficiencies; and a test-teach-test type of intervention design (contained in the design journal, Appendix A, Entries 5-8).

Required and demonstrated proficiencies. A set of cognitive and metacognitive proficiencies that would be required for mathematics problem solving, were listed as the point of departure for Prototype I (Appendix A: Design Journal: Table 32, Entry 5). The reason for developing such a list, was the quest for clarity about what would be assessed and how it could be assessed in a reasonable manner. The list had formative value as a background for later assessment developments; however, it was not retained in later prototypes.

Test-teach-test intervention design. For this prototype, a test-teach-test sequence within a single lesson was followed (Tables 33, 34 and 35 of Entry 6 in the Design Journal, Appendix A). Learners would do a pre-intervention assessment, using their existing skills and then complete a questionnaire to establish which metacognitive strategies and processes they had followed. A post-assessment intervention of the metacognitive strategy of visual imagery would then be mediated, based on the same problem. After this, a post-intervention assessment would be conducted on a comparable item, followed by the same metacognitive questionnaire. The process would be repeated for five sessions to establish change in mathematics performance.

Reflection on Prototype I. Reflecting on Prototype I, I decided to retain the notion of a test-teach-test approach with inclusion of a pre-intervention assessment, followed by the intervention itself and concluded with a post-intervention assessment. It also seemed useful to intervene on the basis of the pre-assessed item. Using real life situations as problems, became a constant design feature as it contributed towards making the design relevant. Another useful idea was obtaining subjective feedback about the use of metacognitive strategies.

Weaknesses in Prototype I. The following list of weaknesses disabled the application of the prototype as an intervention for a practical research situation:

- In Table 32, cognitive and metacognitive requirements were not clearly distinguished and the list was not easy to understand.
- Items of the pre- and post-intervention assessments (Tables 35 and 37) were not comparable. The first item was a complex situation in a measurement context, requiring conversions and an understanding of the concept of rate, whereas the second item was a less complex, multi-step number application in a currency context.
- The textbook style intervention was so prescriptive that it would not leave educators any freedom of application.
- The metacognitive questionnaire (Tables 35 and 37) overshadowed the mathematical activity.
- Questions in the questionnaire were random and enquired into metacognitive categories that were neither taught nor investigated.
- The self-administered questionnaire required too much reading and was too complicated and too finely nuanced.

- The interpretation of responses of the metacognitive questionnaire could not be used readily by educators other than the researcher.

Intended improvements to Prototype I. Based on the above shortcomings, the following improvements were intended for Prototype II:

- Only the essential metacognitive aspects that were used in the intervention had to be enquired into in a short and easy to administer questionnaire.
- Since the intervention aimed to cultivate a specific metacognitive strategy, the enquiry should focus on learners' experience and knowledge of that strategy.
- Items that would be used during a single session, had to be set at the same level of complexity, and from the same category within the multiplicative structures.
- The intervention guidelines should be set up with the next user in mind, that is, it should be revised for practicality and usability.

The Intervention Phase

The process of designing the elements of an instructional intervention follows as many cycles as are feasible within practical- and time constraints (Plomp, 2007, p. 13). The primary purpose of the second phase during which the design experiment was to be conducted is, according to Gravemeijer and Cobb (2006), was to bring the instructional start point closer to the anticipated end point and to gain insight into the mechanisms at work while improving the intervention prototypes (Plomp, 2007).

The main design quest in this phase was for the characteristics of a design where self-regulation, in particular the metacognitive strategy, would be mediated for understanding situations requiring division. This investigation required a cyclic and iterative process of design, reflection, analysis, improvement and further innovation.

Prototype II

Prototype II was developed at the beginning of the Intervention Phase, after further reading about the multiplicative concepts, a factor that had a major impact on the assessment design in this prototype. Two developments of Prototype II were:

Baseline assessment development. Impressed by the reading about the multiplicative conceptual field and the theoretical underpinnings of multiplication and division, the main emphasis was on representing all classes of division within the pre-assessment (Appendix A: Design Journal: Entry 9). The baseline assessment was set for application in a group setup. It deviated from the previous idea of assessment and was never applied in practice. After expert review, it was altered completely.

The assessment contained a set of 25 questions, representing all classes of problems within the multiplicative structures at Grade 6 level, in a multiple-choice answer format, including a set of options for various operations and a set of options for various solutions, which, in retrospect, was complicated and potentially confusing to interpret. Furthermore, the answer options probably did not cover all possibilities and would restrict responses to the options provided, not allowing for learners' own innovations. The illustrations provided in the assessment could be more inhibiting to the imagination than assisting it. An extensive rationale for the assessment design followed on the test (Appendix A: Design Journal: Entry 10), explaining in detail the multiplicative concepts, with examples of their application in situations.

The inherent weaknesses of the assessment were evident, however, the main reason for the rejection of the question set for the purpose of the research, was that it would not be possible to cover all the classes of division in the interventions during the practical experiment. It would therefore be aimless to pre-assess them all.

An important spin-off of the question set was that a need for scaling or plotting assessment items developed as a requirement for setting the test. The idea of a matrix for plotting test items started here and was to be developed in Prototype III.

An intervention for the mediation of visual imagery. The second development in Prototype II was the re-think of the extensive emphasis placed in Prototype I on enquiring about participants' use of the metacognitive strategy, while the strategy had not been properly mediated to them. In Entry 11 of the design journal, this emphasis was reconsidered and was shifted towards the mediation of the metacognitive strategy, rather than assuming that all learners would be using the strategy spontaneously. The idea of creating what was called "the virtual space of the mind", was a notion that was retained throughout the design, and was also tested with participants. The intervention planning (Appendix A: Design Journal: Entry 12) contained the characteristics of mediation and step-by-step guidance for learners to create such a space in their minds. The main ideas within this design prototype were retained for the duration of the design, but had to be modified to be less rigid and prescriptive.

Prototype III

A major improvement to Prototypes I and II was the logical structure added to the intervention in Prototype III, in the form of a model for metacognitive problem solving (Appendix A: Design Journal: Entry 13).

A model for metacognitive problem solving. This model, upon which the metacognitive strategy could be mapped, is based on a "model for decision-making as an optimal metacognitive process" (Kaniel, 2003), which progresses through four distinctive phases, i.e. goal, planning, executing and feedback.

Development of the model (Appendix A: Design Journal: Entry 16). Some adaptations were made to the original model by Kaniel (2003). Two changes to the terminology are the use of "goal setting" instead of "goal", and "execution" instead of "executing" (2003, p. 214). The "supportive systems" in Kaniel's model are not used in my model in a supportive role, but as the actual actions by which the strategy comes into operation, including mental imagery, verbalisation, time regulation, resource management, precision and justification. In my model, a cycle is discerned, comprising the four sequential phases and secondly, actions, indicating the primary activities of each phase to solve the problem. Having used the model in the lesson (Entry 13), it was later formalised in a visual representation, as follows in Figure 8:

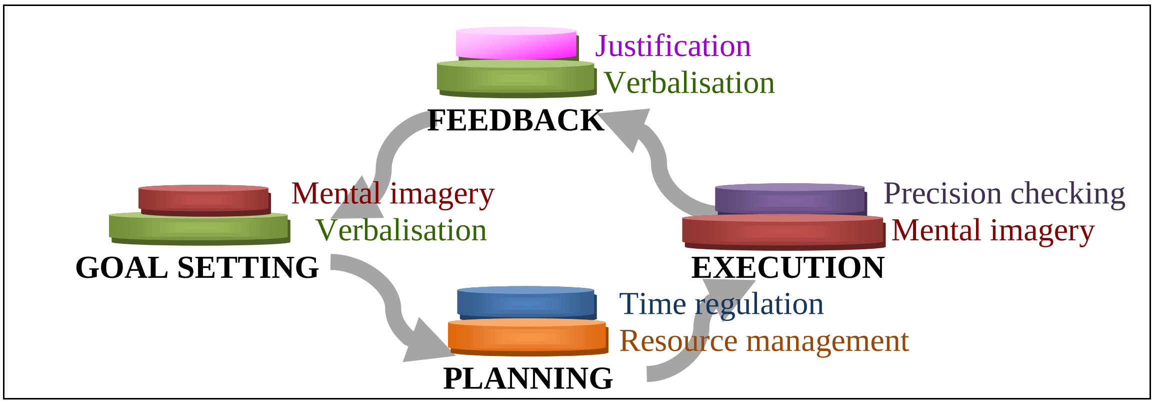


Figure 8. A model for metacognitive problem solving, based on "An Optimal Model for Decision-making by Individuals" (Kaniel, 2003).

Rationale for the model adapted for the present study. The ideal of mediating problem solving systematically, using visualisation in a structured way, is key to the suggested response to the main research question, and forms a key element of the conceptual framework for this study. Problem solving starts with goal setting, which according to Kaniel (2003, p. 216), entails unintentional, spontaneous data collection from interaction with other people and the environment. In the case of a structured intervention, problem solving is a planned activity, initiated by the educator. The problem is posed directly and the necessary elements for solving the problem have already been extracted so that the problem solver needs to set the goal only in terms of problem identification and a description of "the results of the goal in clear and observable terms" (Kaniel, 2003, p. 216).

In goal-setting, it is suggested that participants create a visual image of the problem situation to enable entry into "the virtual space of the mind". They create the image in the mental space according to the problem specifications, but also according to their own unique set of references. The mental image makes swift processing of the problem possible, as contrasted to the presumably restrictive external representations of, for example, a textbook picture or a blackboard drawing.

In the planning stage, the two main actions are time regulation and resource management (Kaniel, 2003, pp. 214, 216). Time regulation refers to the determining of a chronological sequence from the known to the unknown. The available resources are the information provided by the problem statement itself, which needs logical organisation and the learner's available knowledge and skills to solve the problem.

Execution is the determining factor in problem solving through visual imagery. Still in the mind space, the situation is systematically played out from the present state towards the desired state. The change is informed by the available information and guided by the verbalised problem statement. At this point, the individual, I surmise, has the urge to spontaneously leave the virtual space of the mind to start the actual calculation. A central action of the calculation is monitoring by precision checking, to ensure that the correct information is used for calculation and that the calculation was executed correctly. The solution is now ready to be presented.

During feedback, the individual verbalises the solution. Feedback serves the control purpose of judging the reasonableness of the answer in relation to the question. Judgement is not merely an estimation of correctness, but justification of the solution, which can be defended by proving how and why the answer is correct. Justification serves both a monitoring and control function, to either support the answer, or to revisit the problem solving process. Note that "precision checking" is used both as an action and as a monitoring tool in problem solving. "Justification of solution" is used as a control action after problem solving. I considered these monitoring and control functions (Entry 15 of the Design Journal, Appendix A) and I realised that they are iterative and interdependent throughout the process, as summed up in Figure 9 below:

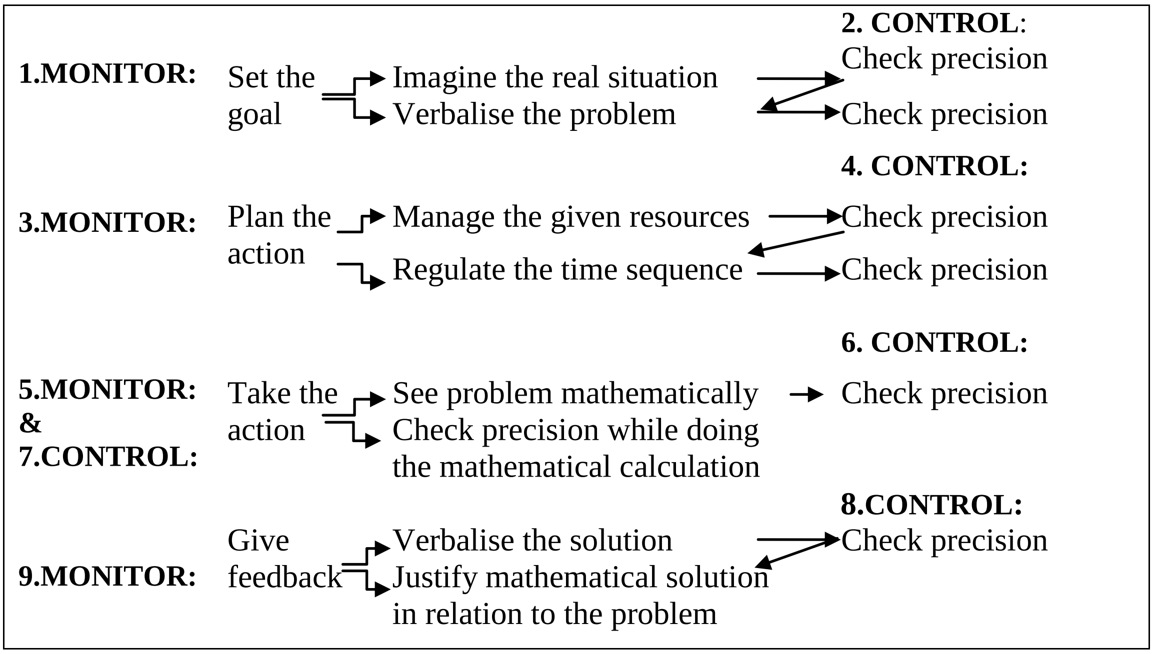


Figure 9. The cyclic and iterative nature of metacognitive control and monitoring.

Implications of the model for the design development process. The model was applied to an instruction/assessment item (Appendix A: Design Journal: Entry 13) and it appeared promising in that the problem solving steps followed logically and could be applied with ease by teachers and learners alike. Its generalisability allowed for the creation of a contents free generic template to use with various items to follow (Appendix A: Design Journal: Table 37, Entry 17). The template was applied to a test item (Appendix A: Design Journal: Table 38, Entry 17), to envisage learners' reactions while the instruction was in process, in preparation for the actual interventions. This application was for the sake of the researcher's own preparation, as it never featured in the intervention.

A further implication, mainly as a result of the improved structure of the design, was that the mediation used in the intervention was less complex and more concise. The intervention was specifically focused on the metacognitive strategy.

The model based intervention of Prototype III. Following an untested Prototype II, Prototype III was still theory based. Since, however, Gravemeijer and Cobb (2006) reminded the researcher that the design is meant for the practice of teaching in a classroom, Mafumiko's proposal (according to his model of 2006, cited in Plomp & Nieveen, 2007, p. 30) of expert appraisal was followed at this point in the design process. A mathematics education specialist was consulted, who appraised the legitimacy of the domain-specific elements of the intervention while an educational psychologist advised on the metacognitive elements that were employed.

An oversight on my part was that I did not know at the time that consultations for expert advice should be included as evidence into the Design Research report. Although their advice was incorporated in my work and I reported the encounters with names and dates in my regular student progress reports for purposes of supervision, I cannot deliver formal proof of their contributions. By the time I realised my omission, it was too late to expect of them to remember their advice and write such reports.

The sequence of problem solving suggested in the model for metacognitive problem solving was informally tried out with a few individuals, to test for ease of understanding on the participant side and for ease of mediating on the teacher side. The perceived weaknesses of Prototype III led to several revisions, which were recorded in the reflective research Design Journal. At this stage, it was necessary to envisage how learners would respond to the interventions and to anticipate and prepare for possible deviations from the expected. The try-out with individuals was valuable to provisionally test for practical relevancy and practicability.

Reflection on the intervention of Prototype III. The most useful aspect of the intervention part of Prototype III was the coherent structure according to which future versions of the intervention design could be developed, which was mainly provided by the model for metacognitive problem solving. An aspect to be improved was the high level of teacher control of the intervention, which still was found to be restricting the metacognitive activity of the participants.

Matrix for assessment of understanding. Apart from the model for metacognitive problem solving, also useful in Prototype III was the development of a matrix by which problem items could be gauged according to their level of cognitive demand and their dimensions of mathematical understanding. This development has been recorded in Entry 19 of the Design Journal (Appendix A). At different points along its own cycles of development, this matrix was introduced to colleagues and submitted for expert scrutiny for reliability of its theoretical premises. Table 8 reflects the first version of this matrix and Table 39 reflects the second version. It was then used to set comparable questions for all assessments during the Intervention Phase and it was used as a reference for establishing the width and depth of learner understanding in the assessments during the encounters with participants. In the beginning of the research on site, it was reflected upon for validity of scoring with an expert advisor, which resulted in the final version of the matrix (Table 43, Appendix B).

Rationale for the matrix of understanding. As argued in Chapter 2, the transition from "horizontal" to "vertical mathematising" (Treffers, 1987, cited by Van den Heuvel-Panhuizen, 2003) does not automatically result from the direct recall of mathematical facts or computational efficiency in mathematical procedures, but rather from conceptual mathematical understanding. Steinbring (2005) argued that for the assessment of the expected and the attained depth and breadth of understanding, "the underlying principle is to have a high degree of match between what learners are expected to know and what information is gathered on learners' knowledge" (2005, p. 31). To enable such assessments, the matrix of understanding was developed.

The initial matrix (Table 8) provided for the breadth of mathematical understanding across four dimensions, namely the skills-algorithm, use-application, representation-metaphor and property-proof dimensions (Usiskin, 2012). The depth levels of understanding were termed factual recall, operational efficiency and conceptual grasp, a concept derived from Anderson and Krathwohl's knowledge levels (2001). Because I see understanding as a more active concept than knowledge, the term "knowledge levels" (Anderson & Krathwohl, 2001) was changed to "levels of understanding". The metacognitive level of understanding was omitted since it is a primary focus of this study and would be treated separately. The dimensions were arranged in the order, factual, procedural, conceptual, the latter being, in my view, the deepest form of understanding. The principal gain from this development in Prototype III was that it enabled me to gauge both the expectations for, and the attainment of understanding, the two ends of assessment as emphasised by Steinbring (2005). This could now be done according to a single measure.

Versions of the matrix. As is apparent in Table 8 below, the form of the initial matrix of understanding (Version 1) was cryptic, containing only the basic ideas that gave rise to it. The categories had not been described yet, as the need for description only arose at its first exposure to a group of teacher colleagues, after which a second version was constructed (Table 39). At the start of the interventions with participants, the model was finalised, following expert advice (Appendix B: Tables 43-60).

Table 8

Matrix of Mathematical Understanding: Version 1

	Dimensions of understanding (Usiskin, 2012)			
Levels of understanding (Adapted from Anderson & Krathwohl, 2001)	**Use / application**	**Skill / algorithm**	**Representation / metaphor**	**Property / proof**
	Factual recall	Factual recall	Factual recall	Factual recall
	Operational efficiency	Operational efficiency	Operational efficiency	Operational efficiency
	Conceptual grasp	Conceptual grasp	Conceptual grasp	Conceptual grasp

Levels of understanding. The "factual recall" level is self-descriptive, while "procedural efficiency" is seen as "a sequence of actions…the manipulation of written symbols in a step-by-step sequence" (Hiebert & Carpenter, 1992, p. 78). The idea of "grasp" as a noun is a construct adopted from the idea of "episodic grasp of reality" (Feuerstein & Rand, 1974), referring to the incidental and isolated experience of reality, unrelated in time and space to other experiences. I use this idea "conceptual grasp" to describe the interrelatedness of an idea with other ideas in a coherent mental unit, describable in a single term, and encompassing various sub-concepts. In so doing, the term "conceptual grasp" differs greatly from the term used by Feuerstein.

Improvement of the matrix within Prototype III. The matrix of understanding was improved substantially as a result of reflection with colleagues. While plotting some problem items onto the matrix, the categories had to be described for general use. This improvement was helpful in the analysis of the items which were used for the instruction of the metacognitive strategy and for the assessments.

Through the affordances of the matrix, alignment of, and coherency between instruction and assessment were possible. Following the ideas of Airasian and Miranda (2002), this notion is regarded as a basic design principle. Following expert advice, "efficiency" was changed to "appropriateness". The term "efficiency" would, in my understanding, imply a pre-judgement of correctness of the solution and therefore be a predictor of competence (in terms of score). However, "appropriateness" expresses the true intention with this dimension and level of understanding, meaning that irrespective of the score obtained, the learner knew and understood the appropriate operation for the specific problem situation.

Reflection on Prototype III. Prototype III was by far the most extensive design. The matrix of understanding contributed towards standardising the assessment items and the metacognitive model was a valuable addition. Through an informal try-out prior to the actual sessions with participants in the classroom setting, it was evident that the intervention was still too directing and left little space for learner initiatives. The mediator input needed to be restricted, since imposing own ideas of what should happen in the mind space, could constrain learners' own creative mind processes. There was also a need to allow for differences in individual processing speed.

Intended improvements. Having reflected on Prototype III, the following improvements were regarded as necessary:

- Mediation guidelines must allow for individual visualising style and tempo.
- Educator communication should be restricted to the minimum to allow for uninterrupted visualising and information processing.
- For assessments and interventions alike, a single class of problem within the multiplicative structures should be selected as the focus of instruction.
- All items have to be at the same level of complexity.
- The matrix of understanding needed fine-tuning and better descriptors for easier application and improved general use.
- To gain insight into learners' use of the metacognitive strategy that is taught, an easily self-administered, short and essential questionnaire had to be set up.

Design principles. A few preliminary design principles had been accumulated up to this point, which could be used for Prototype IV:

- The design makes use of dynamic assessment of the test-teach-test type to include a pre-intervention assessment, a post-assessment intervention and a post-intervention assessment.
- The intervention is based on mathematical problems in real-life contexts.
- The problem items are assessed according to the matrix of understanding to determine the width and depth of mathematical understanding that they require.
- A structured and logical metacognitive processing sequence is mediated to learners according to the model for metacognitive problem solving.
- The design allows for individual differences in processing of mental images, as well as differences in tempo of processing.
- A questionnaire follows intervention clusters, focused on learners' judgement, knowledge and experience of the metacognitive strategy of visual imagery.

Prototype IV

Reflecting on the process up to this point, the academic, contextual and theoretical relevancy of the study had been secured during the Preliminary Phase, while addressing the first three specific research questions. In my view, Prototypes II and III started to satisfy the requirement for construct validity, a quality criterion described by Nieveen (2007, p. 94) meaning that the intervention had to be designed in a way that the goal (as set in the fourth research question) would be reached logically.

Prototypes I, II and III started to embody the local theory and set the scene for realising the conjecture set up at the Preliminary Phase of the design. Prototype IV was designated to be piloted for practicality within the true experimental situation. In its original form, Prototype IV was used for the baseline assessment and for the initial introduction to the metacognitive strategy in the first session with the participants at the research site. For the subsequent sessions, only the protocol of events was changed slightly, while all other elements were retained.

After the proceedings and outcomes of each session had been reflected upon critically, minor practical adaptations were effected. The useful elements were retained and the hindering elements were discarded, although the basic structure of interventions for Sessions 2 to 4 was kept constant to ensure comparability of the assessment outcomes across the sessions.

Prototype IV aimed for practicality and effectiveness, qualities that had to be tested in the experimental situation with the participants. The ideal to seek input from other teachers during and after the classroom try-out, had not materialised as desired.

The composition of Prototype IV. The design elements of Prototype IV, as they were used directly in the fieldwork try-out with participants, or instrumentally (marked with *) to enable the setting of assessment items, marking and scoring of assessments, and the sequence of events during the intervention, were as follows:

1. Assessment items (Appendix B)

    a. Fifteen assessment items set for five sessions (Appendix B)

    b. Three assessment items for the second summative assessment (Appendix B)

    c. Standard memoranda for scoring each item (Appendix B)

    d. Item analysis in terms of width and depth of understanding (Appendix B)

    e. The matrix of understanding (Version 3, Tables 43 to 60)*

2. An intervention plan for the mediation of visual imagery (Table 37)

    a. Two consecutive protocols of events (Appendix A: Entries 22 and 27)

    b. A generic template for visual imagery for learner use (Table 42)

    c. A teacher plan for mediating visual imagery (Table 43)

    d. The model for metacognitive problem solving (Entry 16, Figure 8)*

3. A metacognitive questionnaire for participants (Appendix A: Entry 23)

4. Visual prompts

    a. Visual prompt for the area items (Figure 10)

    b. Visual prompt for the speed items (Figure 11)

Before the onset of the practical phase of the research, fifteen new assessment items had been set and reflected against the revised matrix of understanding (Version 2 of the matrix, Table 39). Table 9 shows how the item types were organised per session:

Table 9

Assessment Items for the Interventions

Session 1 Baseline assessment	Session 2 First intervention	Session 3 Second intervention	Session 4 Third intervention	Session 5 Summative assessment
Money problem	Money problem	Area problem	Speed problem	Money problem
Area problem	Money problem	Area problem	Speed problem	Area problem
Speed problem	Money problem	Area problem	Speed problem	Speed problem

As mentioned before, the scoring of each item was provided for in two ways, namely a standard memorandum and the plotting of responses according to the matrix of understanding. The analysis of each item according to these scales, was revisited, reconsidered and refined on several occasions. The final product, according to which the marking of participants' work was done, is documented in Appendix B.

When setting the questions, it became clear that they did not make provision for the property-proof dimension of understanding, and a rider was added: "Prove that your answer is correct" after each item (Design Journal: Table 40, Entry 20). This was later found to have been a misguided step, and had to be rectified, as will be discussed further down in this chapter.

Last preparations before the encounters with participants included a protocol of events for each session (Design Journal: Entry 22) and a revised metacognitive questionnaire (Design Journal: Entry 23) for participants to report their experience with using visual imagery. The learner portfolios were compiled, each with a copy of the protocol of events, the assessments for five sessions, blank paper to work on, the generic template for metacognitive problem solving (Table 37) and a metacognitive questionnaire. The first encounter proceeded as follows:

Try-out of Prototype IV: Session 1. On 16 May 2014, eleven girls and five boys from the Grade 6 group attended the first session and received their individual portfolios. As with each new group setting, participants positioned themselves, especially in relation to me, since they were used to each other already. They were representative of at least three different Grade 6 class groups in the same school. The protocol of events for the research period, which contained the planning for the series of encounters at the research site, was explained to them. The idea was that the first item per intervention session would be used as a mini-baseline assessment, the second item to mediate the metacognitive strategy and the last item as a mini-summative assessment per session.

Baseline assessment. After explaining the protocol of events, participants received the baseline assessment items in writing and I read the items out to them. They had to solve the three problems in the way they saw fit, at their own tempo. The group had numerous questions about my expectations for them. They wanted to know where to write (although blank sheets were provided in their files), which method I wanted them to follow when solving the problems, and whether I would not penalise them for using the method of their own choice. They called me to ask whether the way they were solving the problem, was acceptable and whether the answers were correct. I reassured them that they were free to solve the problems in their own way, but they remained doubtful. It was clear that the mindset of the group was completely score oriented and "right and wrong" was in the forefront of their minds.

Introduction of visual imagery. After the baseline assessment, I introduced the participants (in my own way) to conscious visual imagery, to prepare them for the intervention sessions. I asked them to close their eyes and create something in their minds like a computer screen. The screen would form the base on which to set up the elements of the situation, in the colour of their choice. They had to imagine a farm dam and set it on the base, with a stream flowing from it. Five ducklings stood on the edge of the dam, ready to race. They started swimming downstream from the dam. Duckling number one was followed by number three, but overtaken by number four. Ducklings number two and five swam together behind number three. After this, I asked them to open their eyes and tell which duckling ended up first, second and last.

I have not read in the literature how to introduce learners to conscious visualisation, therefore, I invented this little exercise for the purpose. The exercise was not about the correct answer, although all participants agreed that duckling number one had won the race. The aim was to observe how they took on to visual imagery, how they organised the elements of the image, how they effected movement and change, and how they came up with an appropriate answer. Following the exercise then, I requested individual feedback from the participants.

All participants reported that they could set up a base, that they managed to set up the elements of a dam, a stream, a farm and five ducklings. Their bases and ducklings had various colours. They could make them move and swim, and they could see who was first and who was last, however, the tempo at which they processed the situation and managed to manipulate the images, varied.

One boy was troubled by a purple banana that jumped into the screen and wanted to play tennis. Interestingly, this was the boy who showed the greatest improvement in using the metacognitive strategy of visual imagery. His concern gave me the opportunity to explain "bugs" such as those that appear in computerised games, as well as the need to control the elements of the visual image, if one wants to work purposefully with those. By the end of the first session, I had the sense that learners realised that this was not going to be a traditional mathematics class setup, but that other mental facilities would be used, which they found entertaining.

Scoring the baseline assessment. The scoring of the baseline assessment went according to plan; however, it was apparent that the inclusion of the property-proof dimension of understanding did not serve a purpose, as learner responses showed that they understood proof as checking whether the calculation had been done correctly.

I experienced a measure of uncertainty about the plotting of items according to the matrix when they were assessed. I was uncertain of the objectivity and validity of my own judgement in scoring and I repeated the scoring a few times, realising I needed expert advice to ensure standardisation of scoring. The reflection of scoring with an expert in order to standardise the outcomes of the assessments, was therefore done after Session 1. This meeting also resulted in the final adaptations to the matrix of understanding (Appendix B: Version 3 of the matrix, as used in Tables 43 to 60).

Reflection on Prototype IV after Session 1. With the experience of Session 1, Prototype IV was evaluated in the Design Journal from the researcher's first person point of view. The following reflection was extracted from Entry 26 of the journal:

- *The baseline assessment gave me the opportunity to demonstrate to participants that they were free to solve the problems in their own individual ways and to assure them that their own methods would have no repercussions.*
- *The introduction to visual imagery served as an ice-breaker and introduced something that they had not associated with mathematics up to then.*
- *It also gave me the opportunity to ensure, by the questions I asked them, that all in the group were capable of visual imagery.*

- *The informal nature of the duckling scenario was non-threatening and had a relaxing effect on participants.*
- *The protocol of events provided internal structure and coherency to the design.*
- *The assessment items with memoranda for scoring and a refined version of the matrix to evaluate participants' levels and dimensions of understanding contributed towards standardising the assessment of participants' responses.*
- *The idea that three items of the same class would be used in different ways and for different purposes within a single intervention session, was fragmentary. The high level of structure that it would provide to the sessions, was potentially inhibiting visual imagery. Participants had to be properly exposed to the metacognitive strategy before the post-assessment in Session 5.*
- *Adding a rider "Prove that your answer is correct", reflected poor planning and was artificial and futile. Since the intention was not to teach mathematics during the sessions, there would be no opportunity for teaching how to prove a solution. The "property-proof dimension of understanding" (Usiskin, 2012) will be omitted for future items and the matrix adapted accordingly.*

Try-out of Prototype IV: Session 2. Five participants who had not attended the first session, were present for the second session, two who had attended the first session, were absent and never attended again, exerting their right to withdraw from participation in the research at any stage. This meant that nineteen participants took part in the second session, three of whose results could not be used, since they had not obtained parental consent to take part in the experiment. They wished to participate informally, however, because they believed there would be gain in attending.

The revised protocol of events applied for Session 2. One of the three baseline assessment items would be used to mediate visual imagery for the three intervention sessions respectively. For Session 2 then, the focus was on division in a money (currency) context. Visual imagery would be mediated for three items in a money context and these items would be assessed while the strategy was being applied.

Participants wanted to see the marks of their baseline assessment, but I had intended to show no sign of marking or marks in their portfolios. According to the template for metacognitive problem solving in their portfolios, they followed me step by step as I explained how a problem is approached through visual imagery. Having made sure they understood what was expected of them, they focused on the set of problem items in a currency context.

I read out the first question and they followed in their own files. Then I requested them to close their eyes and see the situation in the virtual space of the mind. I talked them through the elements of the problem and asked them to see these elements and attach number values to them. Then I asked them to open their eyes and calculate the solution according to what they had seen. Participants' tempos of calculation varied widely and those who had finished early, disturbed those who were still busy. We went on to problems two and three, and each time I asked them to close their eyes and imagine the situation as it was read out to them, to organise the elements, attach number values, and then open their eyes to do the calculations.

Reflection on Prototype IV after Session 2. A useful feature of Prototype IV was its coherent structure; however, having reflected on the session, some points of criticism could be brought in mainly on instructional style and methodology, which necessitated reconsideration and improvement for Session 3. The following reflection on Session 2 was extracted from the Design Journal (Entry 30):

- *It is inappropriate to illustrate visual imagery using the auditory mode only;*
- *Items should be paced, rather than to read out all items at the beginning of the session. Learners would probably retain attention better if an item was to be finished before a further item was brought forward;*
- *Teacher talking had to be restricted to a minimum, since it distracted from the participants' inner processes when they were interrupted by talking; and*
- *Participants had to get used to the freedom that they were allowed to use any "method" as long as they reached an appropriate solution.*

The main improvement from Session 2 to Session 3 was the preparation of a visual prompt on the area concept (Figure 10), which was the focus of Session 3.

Try-out of Prototype IV: Session 3. Seventeen participants attended the second intervention, Session 3. The inclusion of a visual prompt in the focus area of the session, a measurement context (area), was useful in illustrating how a situation could be set up in what by then we called "the virtual space of the mind", as follows:

I stood with my back to the class and read out a problem of an area, 80m long and 40m wide, which had to be divided in $5m^2$ blocks. The bigger area was shown as a blank board, without any writing on it. I said that that was the base in my mind space. Without talking, I wrote the dimensions mentioned in the problem situation onto the base, illustrating by this action how the mind image could be labelled with numbers. Still without talking, I marked a separate small red square "5 x 5". The board and the small squares had both been fitted with magnets. I started clicking one small square after another onto the larger area, without a word. I refrained from doing any counting or calculating of the number of small squares that would fit onto the large area, as I wanted to demonstrate visual imagery only, not to teach mathematics.

I then removed the visual prompt and read out the three items of the day with them, after which I kept quiet, to allow them to enter into their own mind spaces. Each participant worked at their own tempo from start to finish. I had to remind them to enter into the mind space each time before starting with a new problem, because they were not yet in a habit of visualising before they approached a mathematics problem. Below is the visual prompt that was used to demonstrate an area problem:

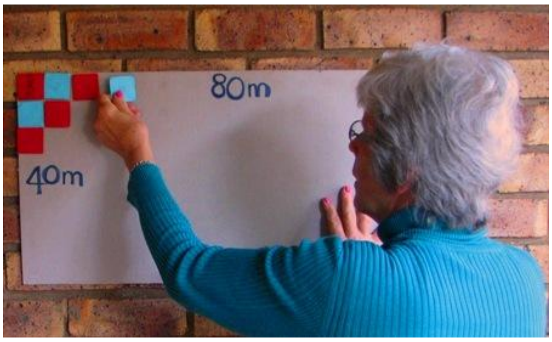


Figure 10. Demonstrating visual mental imagery of an area problem before Session 3.

Reflecting on Session 3, I realised that, although there was an improvement in learner focus during this session, my talking had to be restricted even more to ensure participant focus. A disturbing aspect during this session was that the terminology used in one of the items (Item 8: tiles were 30cm square, instead of 30cm^2) had created confusion and therefore slightly distorted the scores. Expert advice was obtained in this regard and the area item for post-assessment was adjusted to avoid a repetition of this mistake. They were also unfamiliar with the meaning of cm^2.

Try-out of Prototype IV: Session 4. Fifteen participants attended the third intervention, Session 4. The structure of the intervention remained constant to ensure comparability with the previous session(s).

From the outcomes of the speed item in the baseline assessment, it was evident that the participants had no idea of speed (rate). On enquiry, the group confirmed that they had not done speed in their mathematics classrooms up to then, which explained why the group had a performance of 0% during the baseline assessment. This was regarded as an opportunity to solely rely on the affordances of the metacognitive strategy and further refrain from teaching any mathematics during Session 4.

A visual prompt was prepared (Figure 11) for speed, as the focus of the third intervention. Having read out the problem, two houses were pasted onto the base, and then a blank road connecting them. Two number tags were written down, “90km” and “2½h”. Upon the road, an overlay tape was stretched, marked at two fifths and at four fifths to indicate at what distance an hour would have lapsed (I said, “one hour”), then another hour (I said, “two hours”) and then half that length (I said, “half an hour”). The demonstration of how distance is covered over time, was enough to excite participants. They eagerly started with their assessment items.

Figure 11. Visual prompt on distance, time and speed before the onset of Session 4.

Try-out of Prototype IV: Session 5a. The events of this session are related in Entry 33 of the Design Journal (Appendix A). In summary, the post-intervention assessment planned for this session, was conducted under the cloud of a detention session which most of the participants had to attend. A damper was put on the usual relaxed atmosphere and the majority of participants were under stress. The situation was such, that we could not sing and dance and laugh the way we used to do during the previous sessions. Accordingly, the assessment outcomes of this session reflected the negative influence of the day's events on the learners (Chapter 6; Appendix C) and the session had to be repeated the next week, on advice of the study supervisor. Three new assessment items were set, analysed and prepared with memoranda (Items 16-18, Appendix B). The participant portfolios were amended with the new items.

Try-out of Prototype IV: Session 5b. The second summative assessment proceeded in the same relaxed atmosphere as that which was maintained during the first four encounters. After the assessment, participants completed Questions a (i) and (ii); b (iv) and (v); and c (i) and (ii); of the metacognitive questionnaires in writing (Entry 23 of the Design Journal, Appendix A). Questions b (i), (ii) and (iii) were asked to each participant orally and field notes were taken of their responses.

Metacognitive questionnaires. Although the metacognitive questionnaire had been filed in the learner portfolios from the start, I had them complete it only after the last session, reflecting on the complete experience. The questionnaires enquired not only about participants' metacognitive experiences and knowledge, but also about the changes in their experience of visual imagery from the beginning to the end. During Session 5a, some of the participants completed the questionnaire, but I realised that the situation was not conducive for feedback. There was also no time to speak to each participant about their metacognitive experiences, because they had to rush back to detention, therefore participant feedback was better suited at the end of Session 5b. For this reason, some portfolios contain two completed questionnaires; however, for analysis purposes the last questionnaire was regarded as the valid response.

Reflection on Prototype IV. An improvement in Prototype IV was the revision of terminology in the items and ensuring that items were precisely focused on what they were intended to measure.

The revised protocol of events was conducive for repeated mediation of the use of visual imagery in mathematics problem solving, at the following occasions during the six encounters with participants at the research site:

1. Visual imagery was introduced and explained at the end of Session 1, when participants were exposed to the strategy, using a simple situation from nature.
2. At the onset of Session 2, visualisation was explained orally, according to the model for metacognitive problem solving of which there was a copy in each portfolio, based on the money context problem of the baseline assessment.
3. Also in Session 2, having read out one of the day's problems to the group, participants were reminded before they started with the first money item, to close their eyes and enter into the virtual space of their minds, to see the scenario and then to open their eyes to do the calculations. This procedure was repeated for all of the three money items assessed during Session 2.
4. In the first part of Session 3, visual imagery was explained verbally again, using the model for metacognitive problem solving, this time based on the area context problem of the baseline assessment. In this session, the explanation of visual imagery was aided with a visual prompt as explained before.
5. Also in Session 3, having read out all the problems to the group, participants were reminded to close their eyes and enter into the virtual space of their minds, before they started with each of three area items, to see the scenario and then to open their eyes and do the calculations.

6. In the first part of Session 4, participants were reminded of the sequence of metacognitive problem solving according to their templates, and visual imagery was demonstrated with a visual prompt, based on the rate context problem of the baseline assessment, as explained earlier.
7. In Session 4, having read out all of the rate problems to the group, participants were reminded to close their eyes each time before they started with a new problem and to enter into the virtual space of their minds, to see the scenario and then to open their eyes and do the calculations.
8. For the summative assessment sessions, participants were merely reminded to approach all the problems by entering into the virtual space of their minds. No mediation was done during these sessions.

An unexpected additional benefit was that speed had not been covered with this group at school at the time of the research, which offered me a rare research opportunity to observe and isolate the effect of an intervention without interference of any pre-conceptualisations, in this case of visual imagery on mathematics concepts.

The practical experience with presenting Prototype IV of the design, was a valuable resource in the construction of the final design, Prototype V (Appendix D). The set of design principles for the final design (Chapter 7) gradually took shape throughout the entire design process. The reflective practice of journaling the development process, proved valuable in the build-up towards the final design.

The Evaluation Phase

There was a high correlation between the theory and ideal progression of a Design Research as a research methodology, and my own findings concerning the actual unfolding of the research process.

As stated previously, a Design Research follows an iterative and cyclical process, as it moves through three typical phases, namely the Preliminary Phase, the Intervention Phase and the Evaluation Phase. Typically, the literature review of a Design Research would be conducted during the Preliminary Phase, in the case of the present study addressing the specific research questions 1-3. The development and testing of several design prototypes constitute the focal activity of the second phase in this study, which is the Intervention Phase, whose purpose was mainly to address the fourth specific question. The third phase of the Design Research, the Evaluation Phase, is designated for data analysis, the construction of the final design and writing up the study. This phase provides a conclusion of the response to the main research question.

The purpose of the Evaluation Phase is neatly described above, as containing a few clearly demarcated research tasks in rounding off and wrapping up the study. Additionally however, I regard the Evaluation Phase as an opportunity for reflection on the complete design process in all its phases, not the least to assist other students that would choose to use this method of research. In my experience, the intensity of the reality and the depth of demand in Design Research escalated almost exponentially as the study progressed towards the last phase. The reflections on the Preliminary Phase and the Intervention Phase had been recorded originally in the design journal, but were better suited in Chapter 7, as they also contain conclusions and recommendations.

The Evaluation Phase previously brought to mind the metaphor of funnelling a liquid, where the completed work of the previous phases was to be condensed into clear conclusions and where writing up would flow smoothly and continuously. This idea was replaced however, by the bewildering thought of having to bake, decorate and package a cake according to the standards of the industry, with the unmeasured raw ingredients available, but no recipe. The literature review and the fieldwork delivered the raw material that would be used to compile the design principles, to construct the final design and to write up the study, and in the Evaluation Phase it had to be processed. The main tasks in this phase were as follows:

Research Tasks of the Evaluation Phase

The research tasks of the Evaluation Phase also progressed through a cyclic development and refinement process before they could be regarded as completed.

Data analysis and findings. The attention during the Intervention Phase was focused on gathering data. The preparation for the assessments (analysing each item beforehand in terms of a standard memorandum and the matrix of understanding) was helpful, however what would be done with the captured, marked and scored responses, was not anticipated.

The initial idea was to collate all responses into a group performance report, represent them in a table and a graph, and then make inferences about the progress made in the group. It was expected that the participants' metacognitive knowledge and experiences would support these inferences. Additionally, the group response to the three types of assessment items would be statistically and graphically represented. Having attempted this, the following became evident:

- The graphical and tabular representation of the group performance on its own offered very little opportunity for a detailed description of the outcomes of the interventions.

- A numerical and/or graphic description of the group data would not be sufficient to explain the dynamics of events that took place within each individual member of the group.
- A statistical and graphical group performance report per item would not sufficiently enable a discussion and critique of the assessment instrument, its strengths and weaknesses.

Another approach was then adopted, which resulted in various descriptive statistics based on a content analysis of the assessment responses, which in turn were instrumental in answering the following questions:

- What was the influence of visualising on individual participants' progression?
- Was the trialled intervention prototype ideal? And was the way in which the interventions were presented, ideal?
- Were the assessment items comparable, and did they enable reliable and valid inferences? If not, what were the weaknesses?
- Were participants' reports about their metacognitive experiences and knowledge of the mediated strategy in accordance with their performance?
- Is there a relation between learner scores and their understanding of the problems over various dimensions and at various levels?

It was therefore decided to take the following steps:

- The group performance was arranged in ascending order of progress.
- Individual performances across the sessions were represented per individual, also in ascending order of progress, both graphically and in a table format.

- The course of each individual's performance was discussed and their own metacognitive reporting was linked with this discussion, as well as the observations made by the researcher during the sessions.
- The item types were grouped, compared for level of complexity, critiqued, and suggestions were made about adaptations that would reflect the influence of the metacognitive strategy in a more valid way.

Description and evaluation of the design process. The design process was reported in this chapter, essentially guided by the Design Journal (Appendix A).

Extraction of instructional design principles. An accumulated set of design principles were formulated and documented in Chapter 7. Extracting design principles, was experienced as the fruition of the study in all its facets, both in terms of the process and the products of the research. The experience corresponded with what I regard as the essence of learning: the knowledge and understanding that I acquired both before and during the study, were in the end hardly discernable in terms of their original sources only. They had been moulded, shaped and transformed into a unique set of principles, suitable for the specific application they were meant to guide and support. I found myself unable to cite sources precisely while compiling the design principles, because I was their original author. Writing down the principles, was completely unforced.

Final prototype of the instructional intervention. A coherent design was finalised which integrated the information obtained from various sources during the previous two phases (Appendix D). As was the case with the extraction of design principles, the design of the final prototype was a spontaneous process. The following characteristics of the design are noteworthy:

- The lesson was constructed according to an accepted form for a lesson design (Wiggins & McTighe, 2011) to provide structure and logical sequence.
- The aspect or concept addressed in the design was positioned within the context of the curriculum.
- The teacher guidelines were separated from the learner worksheets.
- The mediation of the concept included formative assessment.
- The memorandum for scoring the formative assessment formed part of the teacher guidelines.
- The focus was on concept formation and therefore intentional visualisation was used instrumentally, along with other strategies, to promote understanding.
- Visualisation as a regulating metacognitive strategy was applied within the lesson to demonstrate its place and use in an integrated way.
- New terminology was introduced by linking it to known terminology.

Writing up the study. The Evaluation Phase was concluded by writing up the study. The writing up was a complicated task and needed various reviews. The main aims with writing up the study, were as follows:

- To communicate the essence of what was researched in a way that would leave the reader with an answer to the main research question;
- To organise the theoretical, empirical and creational elements of the report logically, in such a way that they would no longer stand as individual components of the study, but as a coherent unity;

- To provide a work through which further research may be stimulated, to improve the design and the assessments, to prove or disprove assumptions, to expand theoretical bases, to contradict or confirm findings, and to apply and try out some of the suggestions for intervention in the mathematics classroom.

The reflections upon the Evaluation Phase with its demanding challenges, and the main findings from this phase are elaborated in Chapter 7. At this point in the study, the researcher is looking to formalise some research findings concerning the following matters:

- What was the ratio of research time spent on the various phases of the design? and can these proportions be justified in retrospect of the total study?
- How clearly did I express my own understanding and the line of argument for the benefit of the reader, and for the motivation of further research in the area?
- Was I able to extract clearly defined design principles as a basis for similar designs in future?
- Is the final design the optimal culmination of what was learned, researched and created during the study? and does is clearly make a novel contribution to the pool of knowledge in the field?
- Was it possible to identify areas calling for further research, not as a matter of a standard duty at the end of a thesis, but as a matter of need to compensate for the limitations or incompleteness of my own research?
- Could I make worthwhile recommendations for improved practice in my field of research?

CHAPTER 6 – OUTCOMES OF INTERVENTION ASSESSMENTS

The practical investigation was conducted over a period of six weeks, and the sessions were organised as explained in Chapters 4 and 5. Session 5 was repeated as explained earlier, and therefore, Sessions 5a and 5b are described separately.

Clarification of Aspects Pertaining to the Assessments

The participants' responses to the assessment items were captured verbatim (Appendix C) and marked (Appendix B), firstly according to a standard memorandum with marks allocated to parts of the calculation. The count of marks was expressed as a percentage, now referred to as "scores". The responses were secondly reviewed according to the matrix of understanding, which served as a rubric for the plotting and counting of incidents of understanding, now referred to as "understanding". For a general overview of performance, both the score per item and the incidents of understanding were recorded in table form. These two measures served as a comparison across all the participants' work. For reporting comparisons in scores, I have used percentage point differences, for example, if a participant attained 29% for the baseline assessment and 43% for the final assessment, the progress is reported as 14%. (No marks were shared with the participants).

Assessment Items

The assessment items are listed according to types of division and session:

Assessment items in a money context. The following items were set:

Item 1 (Session 1). Mom kept all her grocery receipts over a period of three months. She found that the total amount that she spent on milk was R1 365 and the amount spent on bread was R1 073.70. What was the average amount per month that she spent on milk and bread together?

Item 4 (Session 2). For the upcoming soccer tour, coach John received a donation of R25 000 for the school's first team. 15 players were selected for the team. How much money is available to buy gear for each member of the team if coach holds back R2 000 for the transport costs and R50 to tip the bus driver?

Item 5 (Session 2). The principal of the school had the team logo printed on a towel for each of the 15 members who will be going on the tour. The price of an unprinted towel was R95 each. The total amount for the towels plus the printing was R1 725. What did the printing cost per towel?

Item 6 (Session 2). A parent sponsored R800 for caps for each member of the team of 15 plus the two coaches. What was the price per cap if there was R43.50 left from the donation after paying the caps?

Item 13 (Session 5a). Vali bought 25 pockets of oranges and each pocket had 20 oranges. If he paid a total amount of R350.00, how much did he pay per orange?

Item 16 (Session 5b). You received R800 for your birthday. There is a special sale of PC games for R95 each. You decide to buy as many games as you can from your money. How many games can you buy and how much change will you get?

Assessment Items in an Area Context. The following items were set:

Item 2 (Session 1). For the school entrepreneurs' day, the rugby field of 110m x 80m will be divided up into equal blocks of 5m x 5m to set up the stalls. How many stalls will there be on the rugby field?

Item 7 (Session 3). Our teacher gave us a sheet of block paper with 308 blocks of 1cm^2 each. The sheet is 14cm wide. How long is it?

Item 8 (Session 3). Grandmother requested Squarit to tile her dining room floor. The floor is a rectangular shape of 390cm x 450cm. If a tile is 30cm square, how many tiles will be needed to tessellate the floor?

Item 9 (Session 3). The government of the Northern Cape bought a rectangular area of 204km^2 for nature conservation. The short side of the area is 12km long. How long will the fence be that camps the area in?

Item 14 (Session 5a). From a piece of fabric 150cm wide and 400cm long, the Grade 6 teacher cut up square scarves of 50cm long and 50cm wide. How many scarves could she cut from the piece of fabric? Prove that your answer is correct.

Item 17 (Session 5b). For bandana day, one Grade 6 learner makes yellow bandanas to sell. Her piece of fabric is 320cm long and 240cm wide. She wants to cut square bandanas of 40cm long and 40cm wide. How many bandanas will she be able to cut from her piece of fabric?

Assessment Items in a Speed Context. The following items were set:

Item 3 (Session 1). It took the school bus 5 hours from school to the Grade 6 camping site which is 351km away. On our way, we stopped for 30 minutes at a shop. Taking only the time that the bus was moving, at what average speed did the bus go?

Item 10 (Session 4). Dan left Bela-Bela at 04:00 to drive to Mangaung. He arrived in Mangaung at 11:00. If Bela-Bela is 686km from Mangaung, what was the average speed per hour that he drove?

Item 11 (Session 4). Thembi stays in Nelspruit and wants to visit in Soweto. She takes the 08:00 bus in Nelspruit. The bus goes at an average speed of 75km/h. If Soweto is 450km from Nelspruit, at what time will Thembi arrive in Soweto?

Item 12 (Session 4). Petros flies 3 hours from Nelspruit to Cape Town. If the plane flies at an average speed of 616km/h, how far is Cape Town from Nelspruit?

Item 15 (Session 5a). When my uncle drives to work, it takes him 1½ hours to get there. If he drives at 50km/h, how far is his work place from home?

Item 18 (Session 5b). We are going to Grandmother for the holidays. It takes us 2½ hours to get there. If we drive at 90km/h, how far is Grandmother from us?

Abbreviations

In Table 10 the abbreviations for the types of understanding are described.

Table 10

Meanings of Abbreviations in Tables and Figures in Chapter 6

Abbreviation	**Dimension of understanding**	**Level of understanding**
Uf	Use/application	Factual recall
Uo	Use/application	Operational appropriateness
Uc	Use/application	Conceptual grasp
Sf	Skill/algorithm	Factual recall
So	Skill/algorithm	Operational appropriateness
Sc	Skill/algorithm	Conceptual grasp
Rf	Representation/metaphor	Factual recall
Ro	Representation/metaphor	Operational appropriateness
Rc	Representation/metaphor	Conceptual grasp

Score to Memorandum

The following is an example (for Item 2) of the memorandum according to which participants' work was scored (for all memoranda, see Appendix B):

"For entrepreneurs' day stalls, the rugby field of 110m long and 80m wide is divided into blocks of 5m long and 5m wide. How many stalls can fit on the field?"

110m x 80m = 8 800m^2 (1)

5m x 5m = 25m^2 (1)

8 800m^2 ÷ 25m^2 = 352 blocks (2) - 1 mark for answer, 1 for representation

OR 110m ÷ 5m = 22 (1)

80m ÷ 5m = 16 (1)

22 x 16 = 352 blocks (2) -1 mark for answer, 1 for representation

OR own method of choice **Total [4]**

The learners' work was marked with a view to seeing whether computations had the potential of steering in the direction of the correct solution. If the provisional work showed, for example "110 x 80 = 880", a mark would be given for the fact that the mathematical reasoning behind the idea of obtaining the area through multiplying the length and the width of the field, would lead to the correct answer, even if the calculation was incorrect. As shown in the memorandum, as well as in the transcripts of participants' work in Appendix C, provision was made for heuristic methods and flexible strategies, which were all considered on their own merits.

Given the above leniency in marking, it could be reasoned that a higher score is reported than would be the case in a real classroom, where marking is often about the correct answers. However, the point of departure here is not marking work down for mistakes, but recognising any indication of knowledge and skill. Also, the same style was maintained throughout, which makes the score sets consistent and comparable over the assessment period. The scoring is documented in Appendix C, where marks allocated are indicated within the transcriptions of the individual participants' work. Each item was marked out of 4, which came to a total of 12 marks for the three items per day, and a total of 60 marks for the five sessions of the intervention period.

Seen in retrospect, the allocation of four marks for all items was forced somewhat artificial in an attempt to compare items. All items could not possibly fit the same mould – some problems warranted more, and some less marks, depending on the complexity and number of logical steps that had to be followed towards solving the problem. The scores are, therefore, to be seen at most as an indication of a trend, or a yardstick of the general executive performance levels of participants.

Understanding to Matrix

Prior to marking according to the matrix, each assessment item had also been analysed according to the understanding(s) required for its effective solving (Appendix B). The characteristics that were looked for while counting participants' responses according to the matrix of understanding, are illustrated with the example as for scoring (Item 2). The questions guiding the judgement of whether understanding had been attained, were as follows:

"For entrepreneurs' day stalls, the rugby field of 110m long and 80m wide is divided into blocks of 5m long and 5m wide. How many stalls can fit on the field?"

1. Did she know that the bigger area must be divided by the smaller area? (Use-application understanding on the level of factual recall).
2. Did she understand which elements/dimensions to use for the calculation(s)? (Use-application understanding on the level of operational appropriateness).
3. Could she arrange the elements in an order that would result in an appropriate solution? (Use-application understanding on the level of conceptual grasp).
4. Did she know how to multiply and divide? (Skill-algorithm understanding on the level of factual recall).
5. Did she understand how to multiply and divide correctly? (Skill-algorithm understanding on a level of operational appropriateness).

6. Did she demonstrate an understanding of the aspects involved in the computation, like place value, or dividing/multiplying by multiples of 10? (Skill-algorithm understanding on a level of conceptual grasp).
7. Did she know the notation for area and dimensions? (Representation-metaphor understanding on the level of factual recall).
8. Did she represent/formulate the answers in metres (m) and square metres (m^2) ? (Representational understanding on the level of operational appropriateness).
9. Did she demonstrate the understanding that a large area divided up into smaller areas, amount to a number of stalls or blocks and not to a measurement unit? (Representation-metaphor understanding on the level of conceptual grasp)

A maximum of 9 counts could be obtained per item, which came to a total of 27 for the three items assessed per day, and a total of 135 counts for five sessions. The plotting of nine incidents of understanding per item provided for a definition of the extent and nature of understanding. The item analysis according to the dimensions of understanding was natural and it was not necessary to force a fit. Scores were found to be slightly higher than understanding, however, generally they aligned well. I think that the "understanding" perspective of learner performance provided better insight into learner performance than "score", and that this perspective therefore holds the potential to enhance the planning of interventions and the design of individual remedial action.

Group Performance

Those participants who had missed one session were still accepted in the research study, since the ideal condition of having all participants at all sessions, had not materialised. The group was small and with two participants absent, it would distort the group score. In my view, the participants' individual performances have greater interpretative value and meaning than the group scores and I therefore discuss the individual outcomes in detail. For a general overview of the group results, and for later reference, their results are reflected below in the form of three tables and a graph. Some more perspectives are represented in tables and figures in Appendix C.

Table 11

Summary of Group Results: Scores and Incidents of Understanding

Session	**Score**	**Understanding**
Baseline assessment (13 participants present, 3 absent)	8% (13/156)	33 incidents / potential 351
Intervention 1 (16 participants present, 0 absent)	18% (35/192)	73 incidents / potential 432
Intervention 2 (14 participants present, 2 absent)	33% (56/168)	108 incidents / potential 378
Intervention 3 (14 participants present, 2 absent)	36% (61/168)	132 incidents / potential 378
Summative assessment a (15 participants present, 1 absent)	21% (38/180)	70 incidents / potential 405
Summative assessment b (16 participants present, 0 absent)	38% (73/192)	157 incidents / potential 432

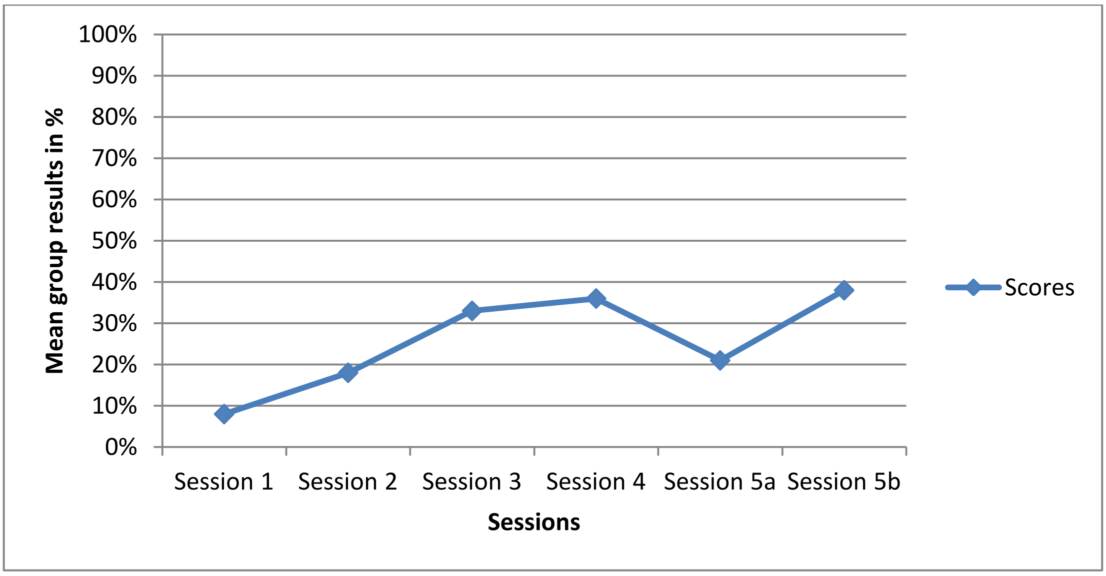


Figure 12. Group scores.

In Table 12, the group's individual results are listed in ascending order, both their scores and the number of incidents of understanding. The order was calculated taking into account both the scores and the progression in understanding, as follows:

Table 12

Summary of Individual Assessment Outcomes in Ascending Order of Progression

Name		Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Participant n	Score	17%	42%	17%	17%	8%	8%
	Understanding	3	5	4	4	2	2
Participant t	Score	8%	0%	0%	Absent	8%	8%
	Understanding	2	0	0	Absent	2	0
Participant f	Score	8%	8%	50%	8%	33%	8%
	Understanding	1	2	6	7	7	2
Participant o	Score	0%	0%	0%	33%	8%	8%
	Understanding	1	0	1	7	2	3
Participant m	Score	8%	0%	0%	35%	17%	33%
	Understanding	2	0	0	5	3	7
Participant g	Score	0%	0%	Absent	33%	8%	25%
	Understanding	0	0	Absent	8	2	6
Participant b	Score	Absent	17%	25%	17%	50%	42%
	Understanding	Absent	3	4	4	9	11
Participant k	Score	Absent	8%	50%	0%	0%	50%
	Understanding	Absent	2	8	0	0	9
Participant d	Score	8%	0%	17%	58%	50%	33%
	Understanding	2	0	2	13	12	9
Participant l	Score	25%	58%	50%	83%	Absent	50%
	Understanding	6	15	15	23	Absent	14
Participant c	Score	Absent	8%	50%	50%	33%	67%
	Understanding	Absent	2	12	13	6	17
Participant p	Score	17%	17%	58%	58%	17%	42%
	Understanding	4	5	17	15	2	16
Participant q	Score	0%	0%	25%	0%	0%	42%
	Understanding	1	0	6	1	2	9
Participant e	Score	0%	8%	75%	33%	0%	50%
	Understanding	1	2	16	9	0	11
Participant v	Score	8%	92%	58%	Absent	8%	67%
	Understanding	8	26	16	Absent	2	20
Participant w	Score	8%	33%	Absent	100%	75%	75%
	Understanding	1	12	Absent	26	21	20

The difference from Session 1 to Session 5b in each individual's scores in percentage points is presented in the table below in ascending order of overall progression – from -9% to +67%. (* indicates who was absent at Session 1, and whose intervention results were used to compare their results):

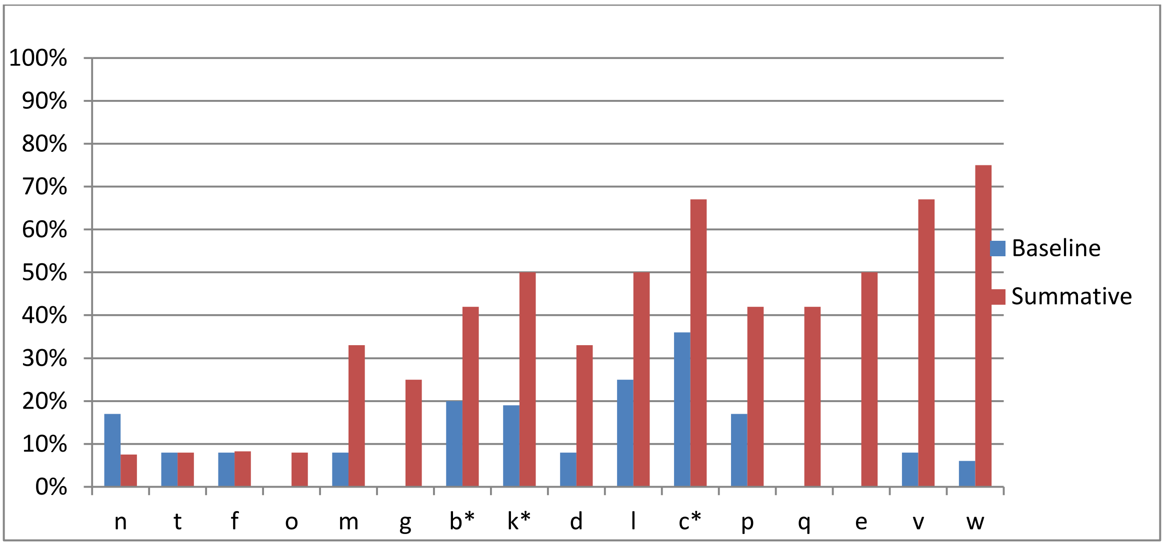


Figure 13. Individual performance: Baseline- to summative (Session 5b) assessments.

Individual Performance

Participant n

Participant n was generally low functioning, but functioned marginally better during the interventions than during the formal assessment sessions. Her scores gave a different impression initially, however, her understanding stayed at a low level and did not seem to improve as a result of the metacognitive strategy. The few successful responses were mainly in the skill/algorithm dimension of understanding, on a factual recall cognitive level. Her scores and understanding aligned well from Session 4 on. She reported that visualising was a positive experience to her, but that she could not attach names and numbers to the objects in the virtual space of the mind (VSM). As a result, it seems as if she could not act mathematically on the image in her mind space.

In Session 1, her score was 17%, from Session 2 to 4 it was 25% and the score for both Sessions 5a and 5b was 8%. She demonstrated a decline of 9% in her score and a decline in the observed incidents of understanding too.

Table 13

Overview of Specific Outcomes of Assessment: Participant n

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	1/4 Sf So	2/4 Sf So	0/4	2/4 Uf Uo Sf Rf	0/4	1/4 Uf Sf
Score Understanding	1/4 Sf	3/4 Uf Sf So	1/4 Uf Sf	0/4	1/4 Uf Sf	0/4
Score Understanding	0/4	0/4	1/4 Uf Sf	0/4	0/4	0/4
Score Understanding	**17% 3**	**42% 5**	**17% 4**	**17% 4**	**8% 2**	**8%**

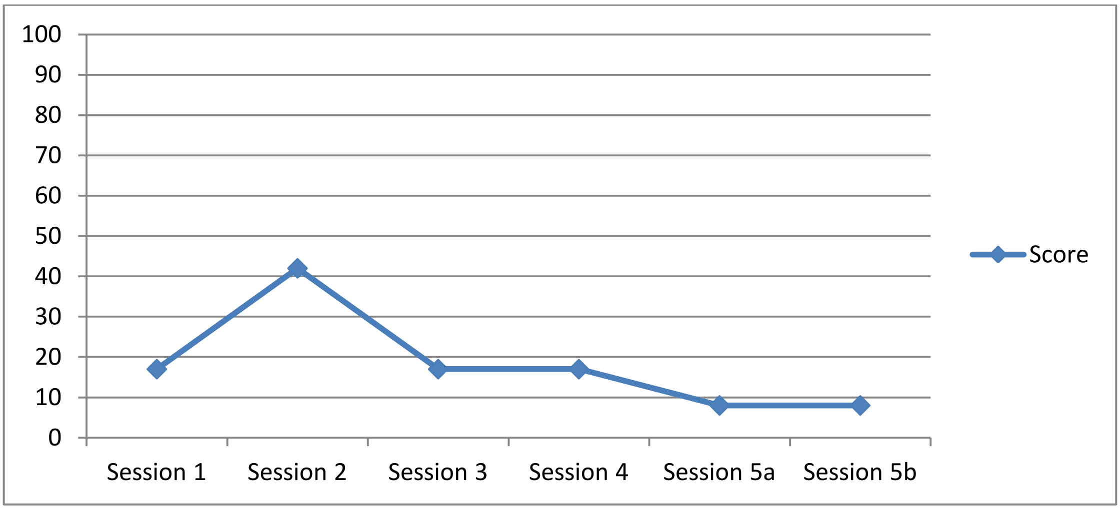


Figure 14. Graphic image of progression: Participant n.

Of the few successful responses, 80% were on a factual recall level and 20% were on the level of operational appropriateness. For 60% of successful responses, her understanding was in the skill/algorithm dimension, 35% in the use/application dimension and 5% in the representation dimension. Her initial score was comparatively higher than her understanding, but the score later tapered off to tally with the understanding.

About her metacognitive experience and knowledge, she reported that having read and listened to the problem, she had an understanding of the situation before she closed her eyes. She found it easy to enter into the VSM and to see the situation as a picture, but it was hard to attach numbers and words to the picture. She reported that she knew what to do when she opened her eyes and it was easy to remember all the numbers once she started to do her calculations. These subjective experiences and knowledge are neither consistent nor helpful to explain all aspects of her performance.

Participant t

Participant t was absent for Session 4. She was low functioning throughout, but was not able to function mathematically at all during Sessions 2 and 3. Her scores and understanding both stayed on a low level and she did not benefit in any way from the metacognitive strategy. Both measures aligned well and displayed the same pattern.

Table 14

Overview of Specific Outcomes of Assessment: Participant t

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	1/4 Sf So	0/4	0/4	Absent	0/4	0/4
Score Understanding	0/4	0/4	0/4	Absent	1/4 Uf Sf	0/4
Score Understanding	0/4	0/4	0/4	Absent	0/4	1/4
Score **Understanding**	**8%** **2**	**0%** **0**	**0%** **0**	**Absent**	**8%** **2**	**8%** **0**

In the baseline assessment, she had a score of 8%, no points during the interventions and 8% in both Sessions 5a and 5b, showing no nett progression. Three of the four successful responses were on the factual recall level and one on the level of operational appropriateness, three were in the skill/algorithm dimension and one in the use/application dimension of understanding. No indication of understanding on any level or in any dimension was found in her work of Sessions 2, 3 and 5b.

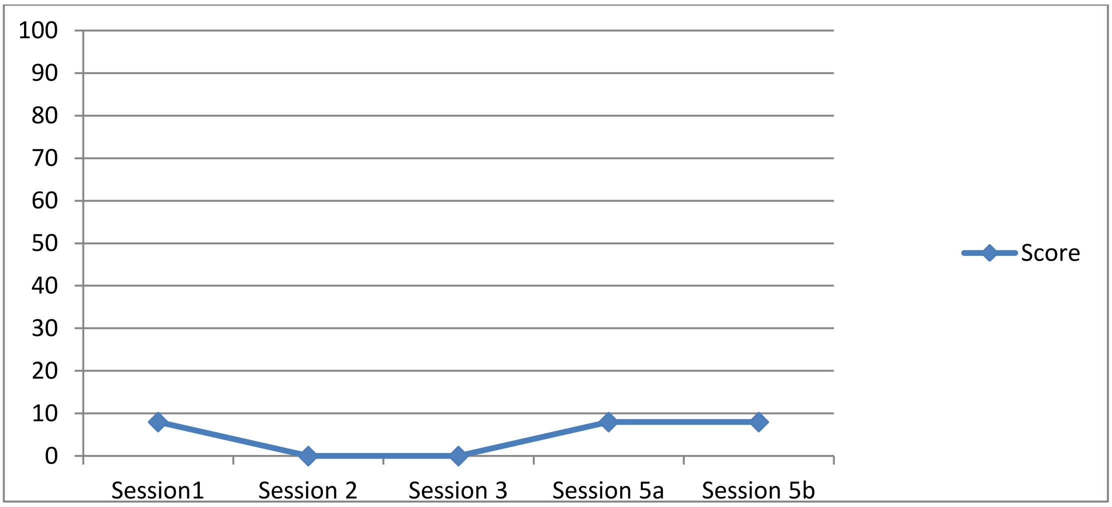


Figure 15. Graphic image of progression: Participant t.

Her metacognitive report reflected her insecurity about applying the strategy. She was well in touch with the problems that she experienced regarding her mental image and reported that she thought that she understood the problem before she closed her eyes, but she could not see a mental picture. She could not make things move or change, or remember the numbers attached to the problem situation. She knew what to do to solve the problem from her initial understanding of it, and not from the VSM.

In the duckling experiment, however, she could visualise the situation and act upon it, yet she reported that she could not manipulate the objects in her images. One possibility is that she had an image, but it was not functional: that the static picture of the situation in her mind, did not help her problem solving, because she could not change or move the objects. She had to revert to her original understanding, or lack of understanding, of the problem, which clearly did not support her performance either. It would be interesting to observe how, given more time to mediate her engagement with the strategy, performance could be influenced as visual imagery capabilities were improved. The period of contact was too short to engage in any such mediation.

Participant f

Participant f was present for all sessions. She displayed a steady progression in understanding, which took an unexpected plunge in Session 5b. Also, her scores fluctuated randomly as she displayed a sharp peak in Session 3. In this assessment she had correct answers, but did not provide enough evidence to substantiate an inference about understanding that would reflect the same pattern as in her score (see Appendix C, Session 3). Her scores and understanding consequently did not align throughout. There is not enough information to make inferences about the fluctuation of scores and the big difference between scores and understanding; however, it must be noticed that her understanding followed a more steady pattern than the scores. This measure probably reflects her actual performance more realistically than the scores do. It is possible that the term "test", used for Sessions 1, 5a and 5b, could have influenced her, if she had a negative attitude to test taking. Also, her correct responses corresponded with another participant's, but after I had separated friends, her score dropped.

In Session 1 she had a score of 8%, during the interventions, 22% and in Session 5a and 5b she had 33% and 8%, respectively. She demonstrated no nett progression on score, but some improvement in understanding.

Table 15

Overview of Specific Outcomes of Assessment: Participant f

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	0/4	1/4 Uf Sf	2/4 Uf Uo	0/4 Uf Ro	0/4	1/4 Uf Sf
Score Understanding	1/4 Sf	0/4	3/4 Uf Uo	1/4 Uf Uo Sf Ro	1/4 Sf So	0/4
Score Understanding	0/4	0/4	1/4 Uf Uo	0/4 Ro	3/4 Uf Uo Sf So Sc	0/4
Score **Understanding**	**8%** **1**	**8%** **2**	**50%** **6**	**8%** **7**	**33%** **7**	**8%** **2**

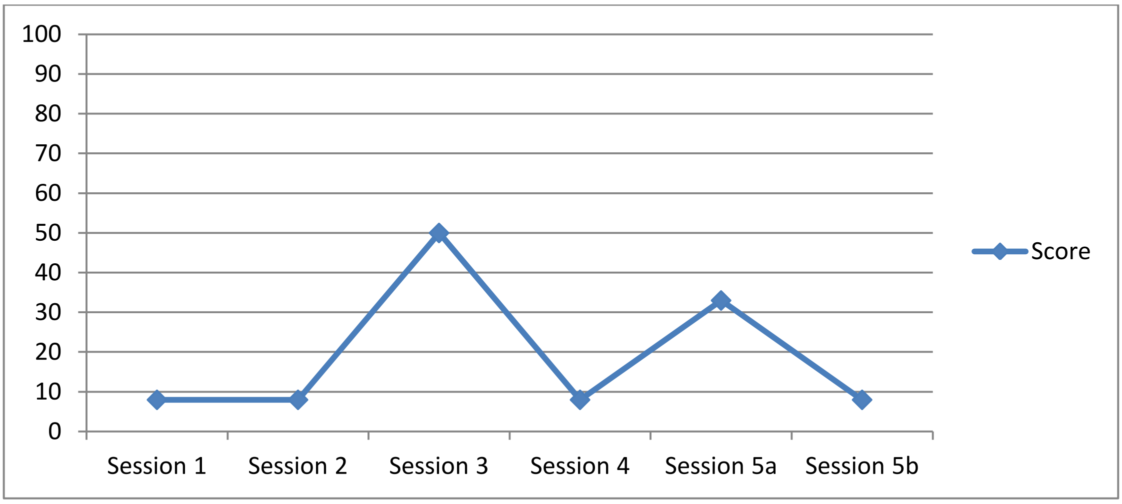


Figure 16. Graphic image of progression: Participant f.

Of the successful responses, 56% were on the factual recall level, 40% were on the level of operational appropriateness and 4% on the conceptual level, whereas 36% were in the skill/algorithm dimension, 52% in the use/application dimension and 12% in the representation/metaphor dimension of understanding.

In terms of her subjective experience, she reports on her own insecurity about most aspects of the process. She did not understand the problem clearly before closing her eyes, even though she had read it herself and in addition it had been read out by the teacher. According to her report, she seemed to struggle with goal setting when the problem was posed. She found it quick and easy to enter into the VSM, but it was hard to manipulate things in the VSM. She could not remember the numbers and did not know what to do when she opened her eyes. She was not sure what to write down, what to calculate or how to solve the problem.

Participant f was well in touch with her own lack of mathematical direction. In the light of the steady incline in understanding that she demonstrated during the interventions, I maintain that, given more time and exposure, her confidence could be built and she could have benefitted from this strategy.

Participant o

Participant o was present for all sessions. She was low functioning for the duration of the research period, except for a peak in a specific item (item 12) in Session 4. I would not regard the peak item as a flash in the pan, as speed was not a theme that the group had done formally in school before this research study and the only tool available to the group to manage the problem, was guided visual imagery. This incident can be interpreted as an indication of her potential to manage problems by way of guided visual imagery and also as the potential of the strategy to assist conceptualisation in a novel theme. Her scores and understanding aligned well throughout. Her metacognitive reporting reflects her experience of neither being able to apply the strategy on her own without guidance from the teacher, nor to attach the mathematical labels to objects in the mind space. It is also clear from her performance that the image she had built in her mind, did not serve her mathematical functioning.

In the baseline assessment, she had no score, during the interventions, 11%, and in the summative assessments, 8% and 8%, respectively. She demonstrated an overall progression of 8% on score and some improvement in understanding too.

Table 16

Overview of Specific Outcomes of Assessment: Participant o

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	0/4 Sf	0/4	0/4	0/4	0/4	0/4 Uf
Score Understanding	0/4	0/4	0/4 Uf	0/4	1/4 Uf Sf	0/4
Score Understanding	0/4	0/4	0/4	4/4 Uf Uo Uc Sf So Sc Rf	0/4	1/4 Uf Sf
Score **Understanding**	**0%** **1**	**0%** **0**	**0%** **1**	**33%** **7**	**8%** **2**	**8%** **3**

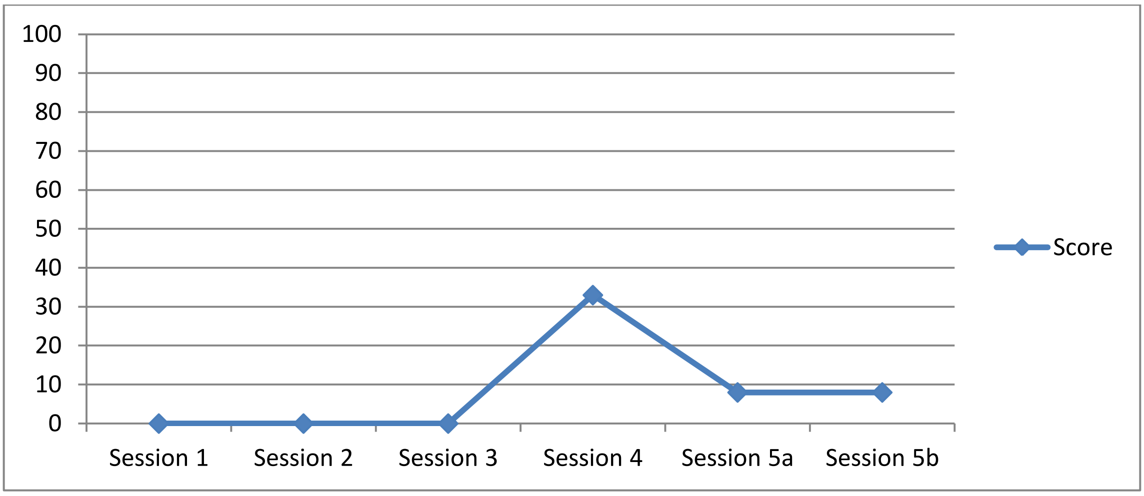


Figure 17. Graphic image of progression: Participant o.

Of the successful responses, 72% were on the factual recall level, 14% on the level of operational appropriateness and 14% showed signs of conceptual grasp. From the few successful responses, 43% were in the skill/algorithm dimension, 50% in the use/application dimension and 7% in the dimension of representation/metaphor.

In terms of metacognitive experience, she reported that before closing her eyes, she had no understanding of the problem from hearing or reading it. She had to depend on the teacher to create the VSM by providing clues and examples, as she could not do it on her own. She found it hard to write down any information in that space. She thought though, that she knew what to do when she opened her eyes and it was not difficult for her to remember all the numbers that she had to work with. It is clear that her subjective report of her metacognitive experience did not correspond with the real situation. In general, she did not benefit substantially from the strategy.

Participant m

Participant m was present at all the sessions. She was low functioning during the first half of the encounters, but steadily improved during the second half. Her scores and understanding aligned well throughout. The interruption of the incline in Session 5a could be due to the disruption caused by the unfortunate incident of detention on that specific afternoon. Her eventual improvement leaves the impression that, with further exposure, she would probably have shown even better results.

Table 17

Overview of Specific Outcomes of Assessment: Participant m

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	1/4 Sf So	0/4	0/4	3/4 Uf Uo Sf So Sc	0/4	2/4 Uf Uo Sf
Score Understanding	0/4	0/4	0/4	0/4	2/4 Uf Sf So	2/4 Uf Uo Sf So
Score Understanding	0/4	0/4	0/4	0/4	0/4	0/4
Score Understanding	**8% 2**	**0% 0**	**0% 0**	**25% 5**	**17% 3**	**33% 7**

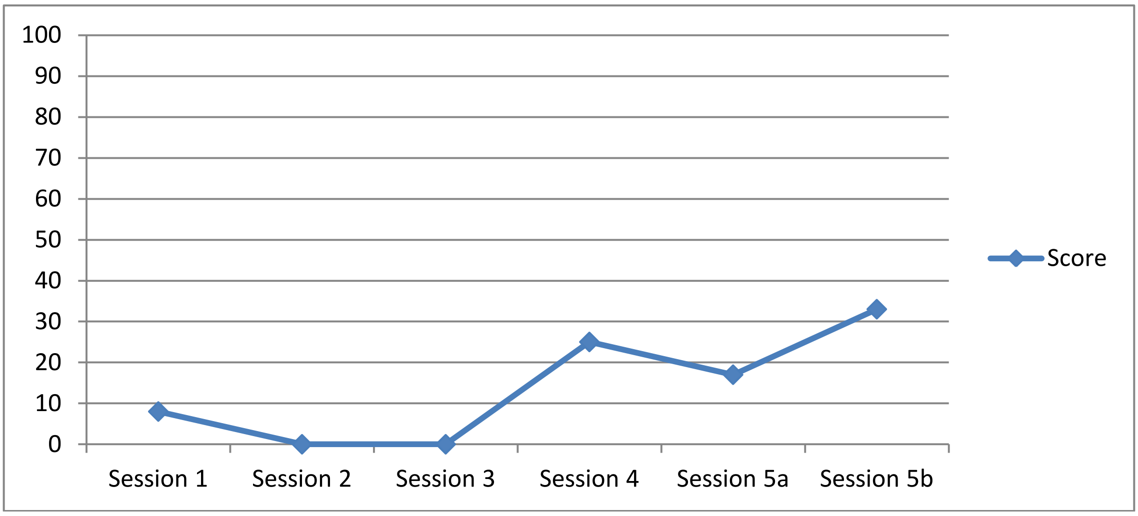


Figure 18. Graphic image of progression: Participant m.

In Session 1, she had a score of 8%, in Sessions 2 to 4 she had 12% and in Sessions 5a and 5b, 17% and 33%, respectively. She demonstrated an overall progression of 25% on score and also an increase in understanding.

From the successful responses, 53% were on the factual recall level, 41% on the operational appropriateness level and 6% on the conceptual level, while 59% were in the skill/algorithm dimension and 41% in the use/application dimension of understanding.

In terms of her metacognitive experience and knowledge, she reported that, from reading and hearing the question, she had understood the problem before entering into the VSM. She found it a slower way of dealing with the problem, yet it was easy for her to form a picture and to manipulate things in the VSM, an indication that she accepted the strategy and managed to see the mental image as a mathematical situation. She was not completely sure what to do when she opened her eyes, thus reflecting a measure of insecurity in the dimension of use/application understanding. She could however clearly remember the numbers that she saw in the VSM, so that she could work with them. That she could reportedly effect change and movement in the VSM is important in view of the nature of the items in her better intervention sessions, namely subdivision of area, and speed, both types of problem situations that need to undergo change or movement in the mind space in order to solve the problem.

Participant m's subjective reporting of her metacognitive experience and knowledge confirm the observations of her demonstrated mathematical performance.

Participant g

Participant g was absent for Session 3. She was low functioning during the first intervention, absent during the second and then showed a steep improvement during the third intervention session, which was fairly well maintained at the last assessment. Her scores and understanding aligned well throughout. The decline in the first summative assessment session can reasonably be ascribed to anxiety about the detention that afternoon. Her profile raises the expectation that, with further exposure, she could show even better results.

Table 18

Overview of Specific Outcomes of Assessment: Participant g

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	0/4	0/4	Absent	0/4	1/4 Uf Sf	2/4 Uf Uo Sf So
Score Understanding	0/4	0/4	Absent	0/4	0/4	0/4
Score Understanding	0/4	0/4	Absent	4/4 Uf Uo Uc Sf So Sc Rf Ro	0/4	1/4 Uf Sf
Score Understanding	**0% 0**	**0% 0**	**Absent**	**30% 8**	**8% 2**	**25% 6**

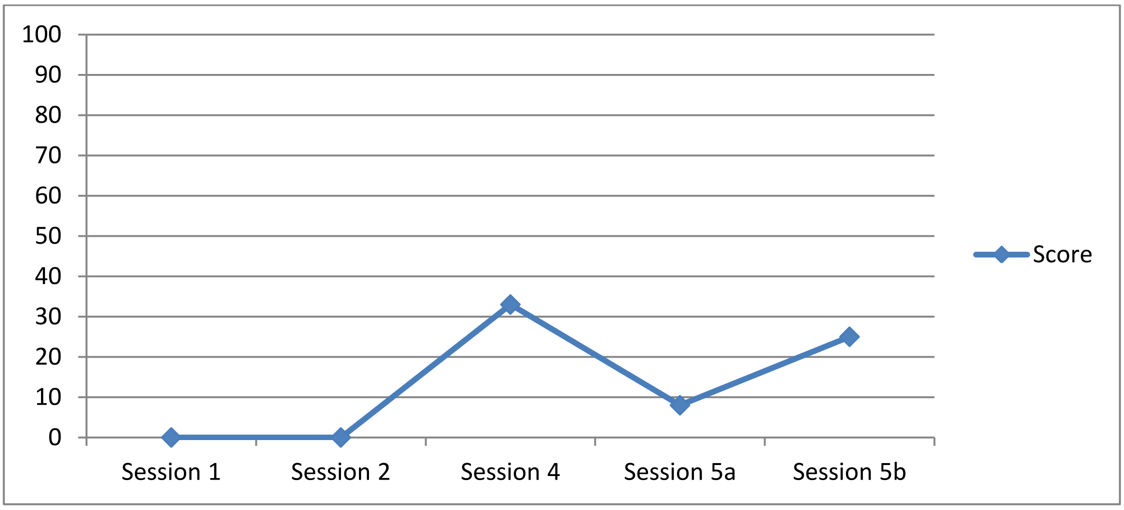


Figure 19. Graphic image of progression: Participant g.

In the baseline assessment, Participant g had 0%, during the interventions, 16,5% and in the summative assessments, 8% and 25% respectively. She demonstrated an overall progression of 25% on score and an improvement in understanding, roughly in line with her improved score. Of the successful responses, 56% were at the factual recall level, 31% at the operational appropriateness level and 13% on a conceptual level; 44% were in the skill/algorithm dimension, 44% were in the use/application dimension and 12% were in the representation/metaphor dimension of understanding.

In terms of her metacognitive experience and knowledge, she reported that she had already understood the question from reading and hearing it, before entering into the VSM. She found that it became easier and quicker to apply the method as time passed. She opened her eyes with a clear picture of what she was about to do, yet was not sure about the numbers that she had seen in the VSM. Her subjective reporting of the metacognitive experience reflects a positive experience of the strategy which, according to her own report, allowed her to see and act upon the mental image mathematically. In this case, it is not unreasonable to relate the participant's improvement in score and understanding, with the use of visual imagery.

Participant b

Participant b was absent at the baseline assessment and his performance could therefore not be compared to an initial performance. He cooperated well, and despite having missed the introduction to the strategy, he picked it up soon and generally showed an improvement towards the last assessment. Comparatively, his scores were markedly higher than his demonstrated understanding. He reported a positive experience of the strategy, which enabled him to see the situation in mathematical terms and to act upon the mental image mathematically. Of importance is his reported specific focus on the numbers which were associated with the situation, an indication that the situation was transformed mathematically to him in the VSM.

Table 19

Overview of Specific Outcomes of Assessment: Participant b

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	Absent	1/4 Sf	1/4 Uf	2/4 Uf Uo Sf So	3/4 Uf Uo Sf Rf Ro	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc
Score Understanding	Absent	1/4 Sf So	2/4 Uf Uo Sf	0/4	2/4 Uf Sf	1/4 Uf Sf
Score Understanding	Absent	0/4	0/4	0/4	1/4 Uf Sf	0/4
Score **Understanding**	Absent	**17%** **3**	**25%** **4**	**17%** **4**	**50%** **9**	**42%** **11**

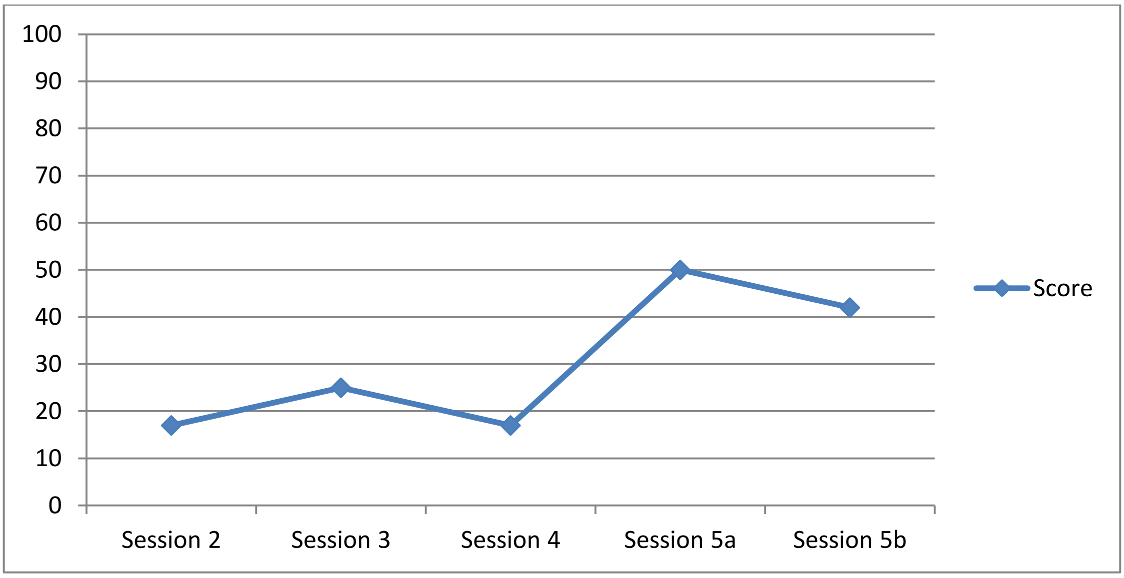


Figure 20. Graphic image of progression: Participant b.

During the interventions, he had a mean score of 19,6% and in the summative assessments he had 50% and 42%, respectively. In total, he showed a progression of 22,4% from the interventions to the final summative assessment. A prominent 61% of successful responses were on the factual recall level, 29% on the level of operational appropriateness and 10% on the conceptual level. From the successful response, 42% were in the skill/algorithm dimension, 42% in the use/application dimension and 16% in the representation/metaphor dimension of understanding. His understanding demonstrated a balance between dimensions. If that was not the case, the high success on the factual recall level would have raised concerns about the sustainability of his performance.

About his metacognitive experience and knowledge, he reported that he had already understood the question before entering into the VSM. He had no difficulty to see the picture of the problem situation, and neither to add numbers to the mental picture. After opening his eyes, those numbers were still usable to work with. He found the process hard in the beginning, but it became easier with time.

Participant k

Participant k did not attend the first session and therefore no baseline assessment was available, and he also had no initial orientation and detailed guidance about the strategy. He seems to have picked up on the metacognitive strategy fairly easily though, despite the less detailed guidance during the subsequent interventions.

During the interventions, he had an average of 19,3% and in the summative assessments he had 0% and 50%, respectively. In total, he showed a progression of 30,7% from the average of the three interventions to the final assessment. The decline in Session 5a could be ascribed to the fact that the group was disturbed and anxious as a result of the detention session that afternoon. The dramatic drop in performance for the third intervention may be explained by the fact that participant had no formal exposure to speed calculations prior to these sessions. Where performance peaked, it is remarkable that her understanding seems to be significantly lower than her score. The higher score was not substantiated by her understanding and thus, her understanding showed less fluctuation than her score. There is no certain indication that the gain is pointing towards a growth tendency as a result of the application of the metacognitive strategy, as the fluctuation in performance could well be ascribed to other, unknown factors.

Table 20

Overview of Specific Outcomes of Assessment: Participant k

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	Absent	1/4 Sf So	2/4 Uf Uo	0/4	0/4	2/4 Uf So
Score Understanding	Absent	0/4	2/4 Uf Uo	0/4	0/4	2/4 Uf Sf Ro
Score Understanding	Absent	0/4	2/4 Uf Uo Sf So	0/4	0/4	2/4 Uf Uo Sf Ro
Score **Understanding**	Absent	**8%** **2**	**50%** **8**	**0%** **0**	**0%** **0**	**50%** **9**

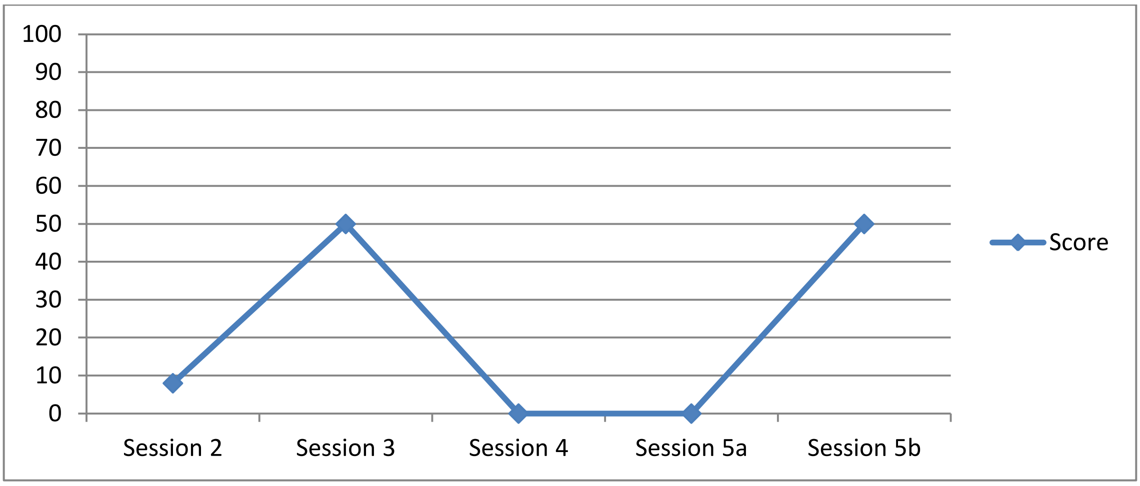


Figure 21. Graphic image of progression: Participant k.

From the successful responses, 57% were on the factual recall level and 43% on the level of operational appropriateness, 38% were in the skill/algorithm dimension, 52% in the use/application dimension and 10% in the representation/metaphor dimension of understanding. It is encouraging that the majority of successful responses were in the use/application dimension of understanding, as this dimension is regarded here as crucial for further progression to more advanced problems.

His subjective reporting reflects confidence in the strategy, as well as in his own ability to apply it. He reported that he had understood the question before entering into the VSM. He found it a slower way of dealing with the problem; however, it became easier with more practice. When he opened his eyes, he had a clear memory of the numbers and knew what to do to solve the problem.

Participant d

Participant d was present at all sessions. Her progress was slow initially and she gradually accepted the metacognitive strategy up to Session 4, where she showed a remarkably improved response. During this session, "speed" was the topic, a theme that had not been formally covered in school and for which the participants had only the strategy available to manage the problem mathematically. There were two girls who were not given detention and therefore I assume that she was not disturbed by the occurrence. Her performance improved in sync with her reported estimation of the value and ease of the strategy. That the majority of successful responses were in the use/application dimension of understanding, is encouraging and is supported by her reported focus on what to do when faced with a mathematical problem situation.

In the baseline assessment, she had 8%, during the interventions 25% on average and in the summative assessments she had 50% and 33%, respectively. She showed an overall progression of 25% on score and a comparable improvement in understanding. The majority of her successful responses (47%) were on the factual recall level, 37% were on the level of operational appropriateness and 16% on the conceptual level, while 37% were in the skill/algorithm dimension, 42% in the use/application dimension and 21% in the representation/metaphor dimension of understanding.

Table 21

Overview of Specific Outcomes of Assessment: Participant d

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	1/4 Sf So	0/4	0/4	1/4 Sf	2/4 Uf Uo Uc Sf Rf Ro	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc
Score Understanding	0/4	0/4	2/4 Uf Uo	2/4 Uf Uo Sf So	2/4 Uf Sf Ro	0/4
Score Understanding	0/4	0/4	0/4	4/4 Uf Uo Uc Sf So Sc Rf Ro	2/4 Uf Uo Sf	0/4
Score **Understanding**	**8%** **2**	**0%** **0**	**17%** **2**	**58%** **13**	**50%** **12**	**33%** **9**

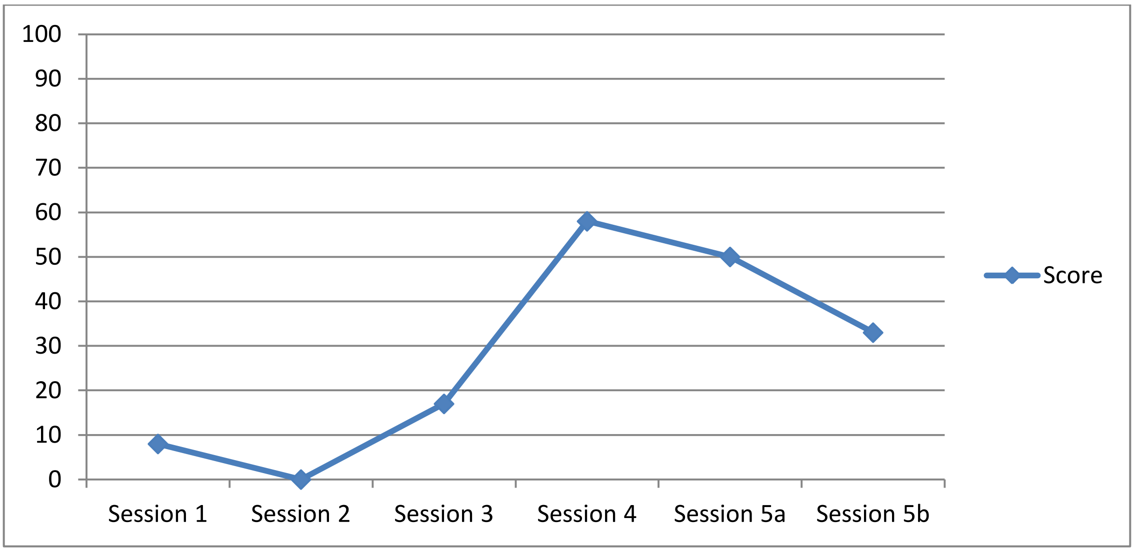


Figure 22. Graphic image of progression: Participant d.

In terms of her metacognitive experience and knowledge, she reported that from reading and hearing the question she had understood the requirements before entering into the VSM. She found it a quicker way of dealing with the problem than her usual way and it became easier as time passed and as she worked with the method more. She opened her eyes with a clear picture of what she was about to do, yet she was not completely sure about the numbers that she saw in the VSM. In her case, it would probably serve a good purpose to give her insight into her own profile.

On review of the process, this participant would be the ideal candidate for more mediation. She was not only responsive to the suggestion of using visual imagery, but also showed mathematical potential that had to date not been realised.

Participant l

Participant l was absent for Session 5a. He resisted using the method, yet attended all the intervention sessions. Despite his resistance, he demonstrated a substantial improvement from the baseline assessment, the highest being in “speed” (from 0% to 83%), of which he had no previous experience in school and for which he had only the benefits of guided visual imagery. His scores and understanding aligned well throughout all sessions. His strength was in the use/application dimension of understanding, confirming his own emphasis on “what to do” with a problem.

In the baseline assessment he had 25%, in Sessions 2 to 4 he had 63,6% and in Session 5b he had 50%. He demonstrated a progression of 25% in his score and even more, comparatively, on understanding. From the successful responses, 41% were on the factual recall level, 36% on the level of operational appropriateness and 23% on a conceptual level, while 29% of successful responses were in the skill/algorithm dimension, 38% in the use/application dimension and 33% in the representation/metaphor dimension of understanding.

The question arises, whether this participant’s improvement was in any way linked to his exposure to visual imagery on a sub-conscious level, and if not, what could be the reason for his substantial improvement? Considering that many other factors could be at play here, one possibility is that he accepted the structured metacognitive problem solving approach and that this influenced his performance.

Table 22

Overview of Specific Outcomes of Assessment: Participant l

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	0/4	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc	Absent	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc
Score Understanding	3/4 Uf Uo Uc Sf Rf Ro	3/4 Uf Uo Sf So Rf Ro	2/4 Uf Uo Uc Sf Rf Ro	2/4 Uf Uo Uc Sf Rf Ro	Absent	0/4
Score Understanding	0/4	0/4	0/4	4/4 Uf Uo Uc Sf So Sc Rf Ro	Absent	2/4 Uf Uo Sf Rf Ro
Score Understanding	**25% 6**	**58% 15**	**50% 15**	**83% 23**	**Absent**	**50% 14**

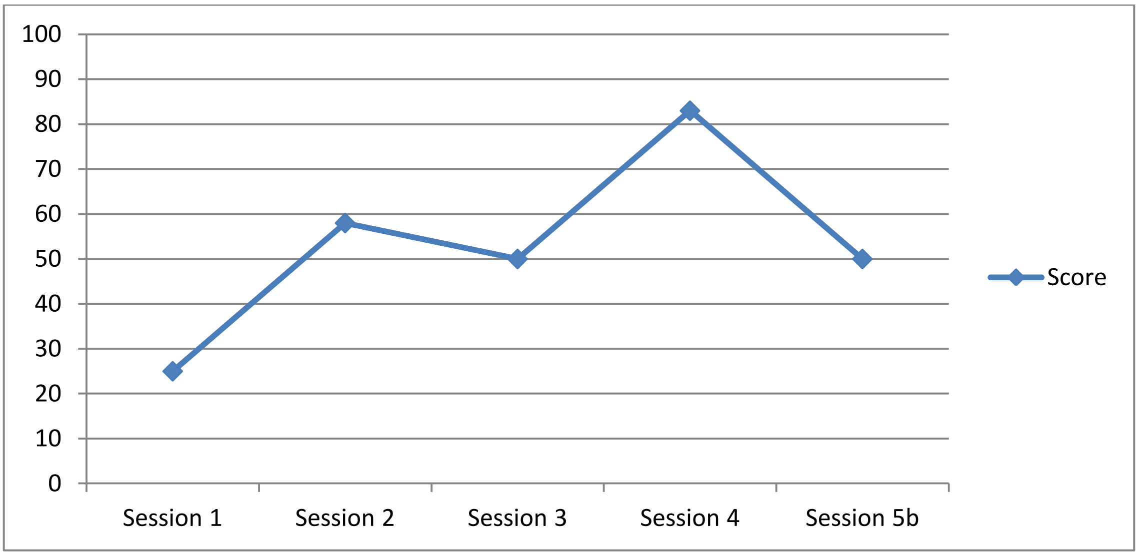


Figure 23. Graphic image of progression: Participant l.

In terms of his metacognitive experience and knowledge, he reported that he had understood the question from the start, even without going into the VSM. He could immediately see the problem in terms of numbers and did not feel the need to make use of the VSM to be able to solve the problem, since the situation automatically turned to a sum in his mind. He reported not being able to see pictures while working mathematically, only numbers. He found this strategy a slower method than his usual way of doing mathematics and had difficulty making things move or change.

Participant c

Participant c was absent during Session 1. She showed a steady improvement in performance, except for the drop in Session 5a, where it is possible that she was disturbed by the detention session. Her scores and understanding aligned throughout. Her metacognitive experience and appreciation of the method contains elements that call for further exposure to the strategy. Her uncertainty about "what to do" is confirmed by her lesser success in the "use/application" dimension of understanding.

Despite her metacognitive report of an interruption of goal setting before she entered into the VSM, she showed steady and substantial improvement towards the end.

During the interventions, she had an average of 36% and in Sessions 5a and 5b she had 33% and 67%, respectively. In total, she showed a progression of 31% from Session 2 to 5b. From the successful responses, 44% were on the factual recall level, 34% on the level of operational appropriateness and 22% on the conceptual level, while 40% were in the skill/algorithm dimension, 38% in the use/application dimension and 22% in the representation/metaphor dimension of understanding.

Table 23

Overview of Specific Outcomes of Assessment: Participant c

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	Absent	0/4	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc	2/4 Uf Sf So	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc
Score Understanding	Absent	0/4	2/4 Uf Uo Sf	2/4 Uf Uo Sf So	0/4	0/4
Score Understanding	Absent	1/4 Uf Sf	0/4	0/4	2/4 Uf Sf So	4/4 Uf Uo Uc Sf So Sc Rf Ro
Score Understanding	Absent	**8% 2**	**50% 12**	**50% 13**	**33% 6**	**67% 17**

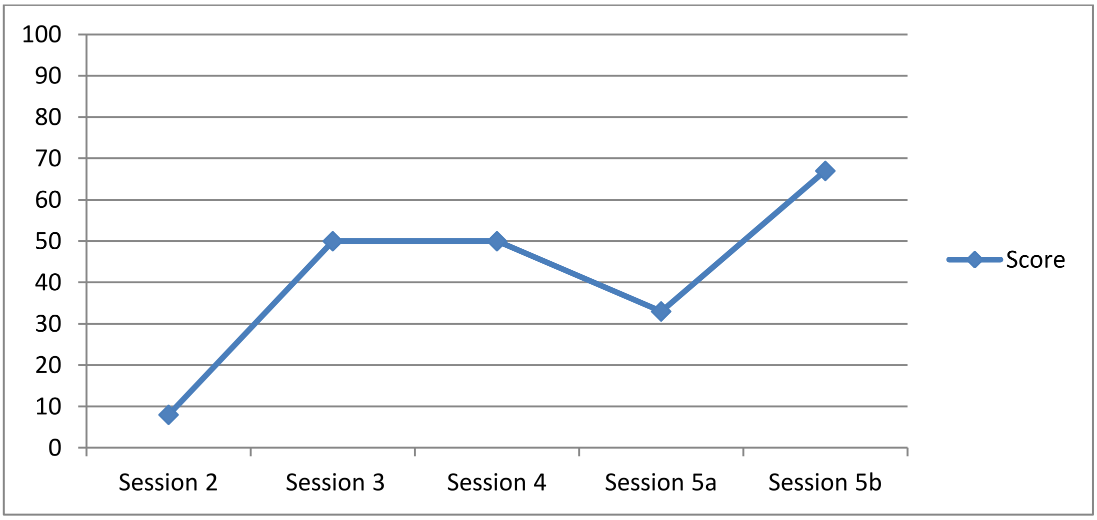


Figure 24. Graphic image of progression: Participant c.

In terms of her metacognitive experience and knowledge, she reported that she had understood the question from reading and hearing it, before entering into the VSM. She found this a quicker way of dealing with the problem than her usual way and found no difficulty to construct the picture. To move or change things was hard. Also, when she opened her eyes, the picture had “disappeared” and she was uncertain about what she had to do and what she saw there.

It is considered that this participant is an ideal candidate for further mediation of the strategy, in the light of her insight into her own mental processes and her mathematical response to visual imagery.

Participant p

Participant p was present for all sessions. She demonstrated a steep improvement in performance, except for the drop during Session 5a, where it can be accepted that she was emotionally disturbed by the detention session. Her scores and understanding aligned well throughout. She is in touch with her inner experiences and metacognitive knowledge, and expressed confidence and appreciation of the method, which is substantiated by her improved performance over the period of encounters.

Her baseline assessment and first intervention session may be seen to reflect the possibility that she was still holding on to her usual way of handling the problem situation mathematically. She adhered to the instruction in school that she had to write the problem out in an algebraic number sentence before commencing the calculations. Her focus on the “use/application” dimension of understanding is encouraging.

In Session 1, her score was 17%, in Sessions 2 to 4 her average was 44,3% and in Sessions 5a and 5b, 17% and 42% respectively. She showed an overall progression of 25% in score and almost double that, comparatively speaking, in her understanding.

Of the successful responses, 46% were on the factual recall level, 32% on the level of operational appropriateness and 22% on the conceptual level, whereas 35% of successful responses were in the skill/algorithm dimension, 41% in the use/application dimension and 24% in the representation/metaphor dimension of understanding. Her understanding as measured by the researcher's observation exceeded her scores from Session 2 onwards, when she started applying visual imagery in problem solving.

Table 24

Overview of Specific Outcomes of Assessment: Participant p

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	1/4 Sf So	2/4 Uf Sf So	1/4 Uf Uo Uc	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc	1/4 Uf	1/4 Uf Uo Uc Sf Rf Ro Rc
Score Understanding	1/4 Sf	0/4 Uf Sf	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc	3/4 Uf Uo Sf So Rf Ro	0/4	0/4
Score Understanding	0/4 Uf	0/4	2/4 Uf Uo Sf So Sc	0/4	1/4 Uf	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc
Score Understanding	**17% 4**	**17% 5**	**58% 17**	**58% 15**	**17% 2**	**42% 16**

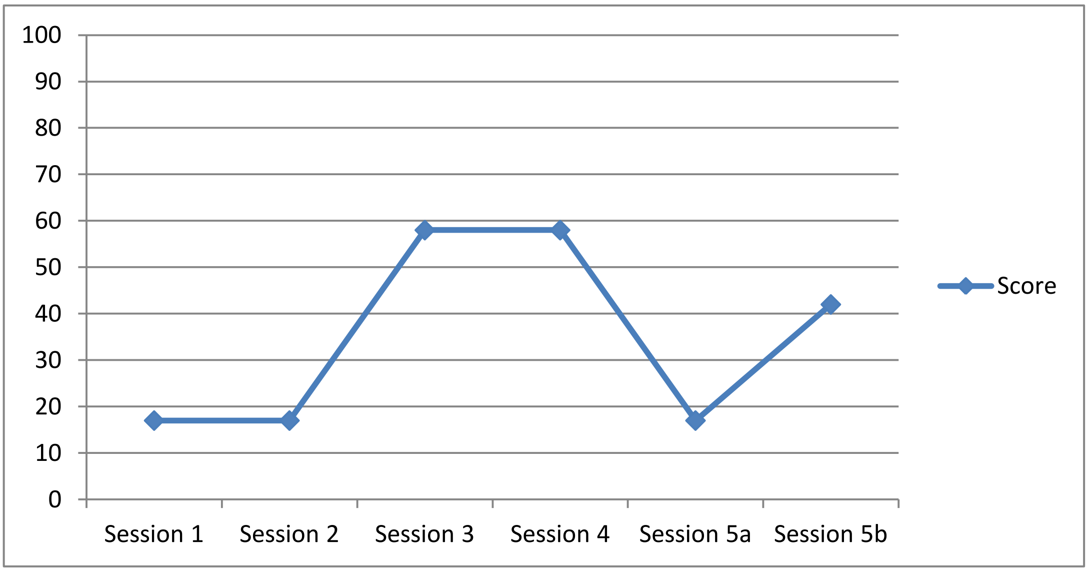


Figure 25. Graphic image of progression: Participant p.

In terms of her subjective reporting, she seemed to be in touch with her inner experiences and she reported confidently that she understood the problem from the start and could easily access and easily manage the problem situation in the VSM. Then she knew and remembered the numbers with certainty and knew what to write down to solve the problem. Nothing of the VSM was difficult for her to deal with.

Participant q

Participant q was present at all sessions. She demonstrated an eventual steep incline in performance. She was low functioning during the first half of the research study, though there was a moderate peak in the "area" intervention. Her scores and understanding aligned well throughout. Her subjective reporting reflected a low self-concept with regard to her mathematical ability, which is quite understandable in the light of the initial performance(s). I have reason to believe though that she would benefit from feedback, though feedback was not part of the protocol and no marks were disseminated to participants during or after the interventions.

The relaxed atmosphere of the second assessment (Session 5b) in comparison with the first stressful summative assessment (Session 5a), is in my opinion, an indication that learners like this participant respond better to a positive and open learning environment. Her eventual improvement leaves the impression of unrealised potential which, with further patient exposure, could result in even better results.

In Session 1, she had 0%, in Sessions 2-4, an average of 8,3% and in Sessions 5a and 5b, 0% and 42%, respectively. She showed a progression of 42% in her score and also some improvement in understanding, however lower, comparatively speaking. Of the successful responses, 63% were on the factual recall level, 26% on the level of operational appropriateness and 11% on a conceptual level; 58% of successful responses were in the skill/algorithm dimension, 37% in the use/application dimension and 5% in the representation/metaphor dimension of understanding.

Table 25

Overview of Specific Outcomes of Assessment: Participant q

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	0/4 Sf	0/4	3/4 Uf Uo Uc Sf So Sc	0/4 Uf	0/4 Sf	2/4 Uf Sf So
Score Understanding	0/4	0/4	0/4	0/4	0/4 Sf	2/4 Uf Sf So
Score Understanding	0/4	0/4	0/4	0/4	0/4	1/4 Uf Sf Ro
Score **Understanding**	**0%** **1**	**0%** **0**	**25%** **6**	**0%** **1**	**0%** **2**	**42%** **9**

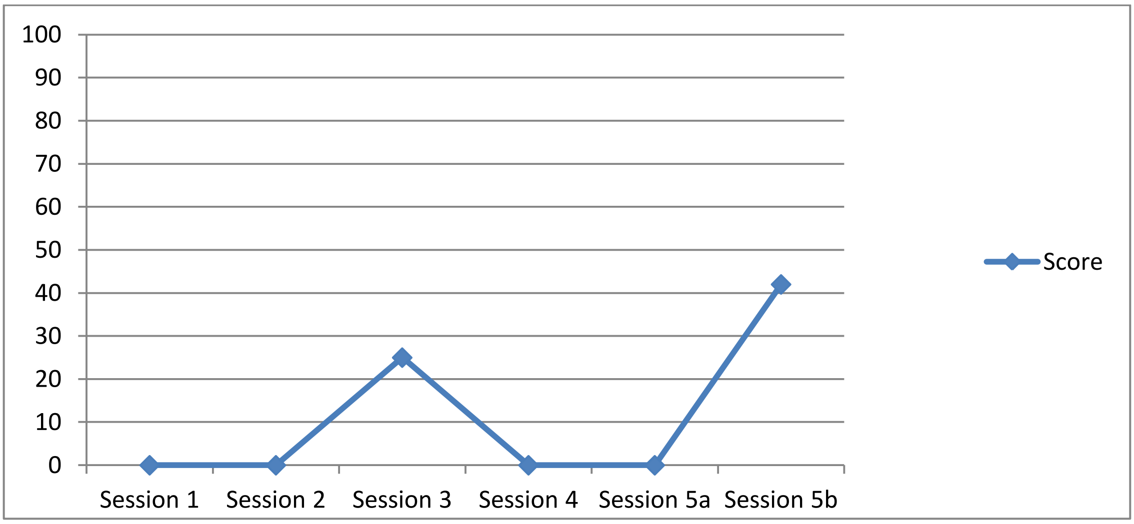


Figure 26. Graphic image of progression: Participant q.

In terms of her metacognitive experience and knowledge, she reported that, although she found it progressively easier to enter and manage the VSM, she was still uncertain whether she had been successful in doing so. She was pessimistic about how she understood the problem and she doubted whether she could remember the numbers and whether her strategy of doing the sums was good enough. She showed potential to perform better; however, she would need encouragement to accomplish that.

Participant e

Participant e was present at all sessions. She demonstrated a very steep incline in performance from a low starting point to the final summative assessment. She was still low functioning during the first intervention session, though from there on she demonstrated a peak performance in the “area” intervention and a moderate performance in the then unfamiliar “speed” intervention. She reacted very badly upon the unfortunate situation of Session 5a and could not perform at all. The majority of her successful responses were in the skill/algorithm dimension of understanding, on a factual recall cognitive level. Her scores and understanding aligned well throughout.

This participant is a diligent and respectful learner, whose behaviour is not problematic. She had to attend the detention on the date of Session 5a and was probably one of the two learners that were most disturbed by the event.

In Session 1 she had 0%, in Sessions 2-4 she had 38,6%, and in Sessions 5a and 5b she had 0% and 50%, respectively. She showed an overall progression of 50% on score and an improvement in understanding, a little lower comparatively. Of all successful responses, 48% were on factual recall, 40% on operational appropriateness and 12% on a conceptual level, 43% in the skill/algorithm-, 40% in the use/application dimension and 17% in the representation/metaphor dimension of understanding.

Her metacognitive reporting reflects a positive take on the utility of the strategy. Remarkable in terms of goal setting, is that she was uncertain about the problem before she entered into the VSM, but having gone through the visual imagery mode, she was able to formulate clear mathematical goals. This observation, together with her demonstrated improvement, vouches for the value of the strategy in enhancing her general mathematical problem solving strategies.

Table 26

Overview of Specific Outcomes of Assessment: Participant e

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	0/4 Sf So	1/4 Uf Sf	4/4 Uf Uo Uc Sf So Sc Rf Ro	0/4 Uf	0/4	3/4 Uf Uo Sf So Sc
Score Understanding	0/4	0/4	2/4 Uf Uo Sf Ro	0/4	0/4	0/4
Score Understanding	0/4	0/4	2/4 Uf Uo Sf So	4/4 Uf Uo Uc Sf So Sc Rf Ro	0/4	3/4 Uf Uo Sf So Rf Ro
Score Understanding	**0% 1**	**8% 2**	**75% 16**	**33% 9**	**0% 0**	**50% 11**

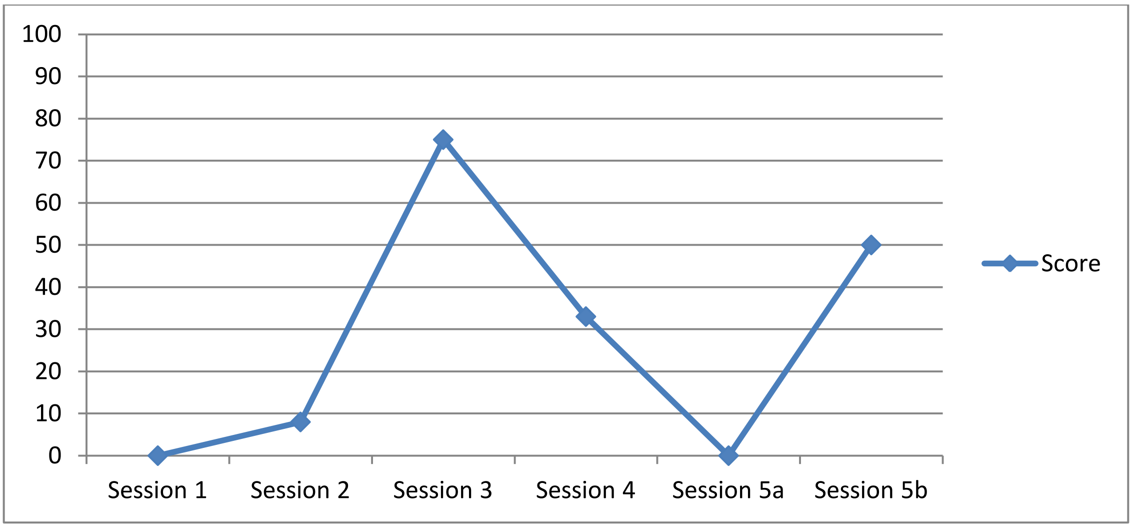


Figure 27. Graphic image of progression: Participant e.

In terms of her metacognitive experience and knowledge, she reported that although she was not certain about what the problem required before entering into the VSM, she found that the process became progressively easier as time passed. She had a clear idea of the numbers and of what to do after opening her eyes.

As was the case with the previous participant, her eventual improvement leaves the impression of a positive reaction to visual imagery which, with further exposure, would probably result in even better results.

Participant v

Participant v was absent for Session 4. A remarkable feature of her performance, is that although her scores and understanding as observed by the researcher aligned fairly well, she was the only participant whose understanding exceeded her scores throughout. Reviewing her responses, this observation is due to the fact that she lost marks in the skill/algorithm-operational appropriateness area, because she tended not to be accurate in calculating. She demonstrated a higher understanding in the use/application than in the skill/algorithm dimension, which was uncommon in the group. This feature was most prominent in the baseline assessment.

Another interesting feature is her complete lack of confidence in her own ability to set mathematical goals, to create and work upon mental images and to retain data from the mental images. She was as much distraught in the first summative assessment as the previous participant, because she is generally an exemplary student and detention was a traumatic experience to her. Both of the last observations, together with her remarkable improvement, point to a sensitive learner who would thrive on long-term investment of positive feedback and reinforcement of success.

In the baseline assessment, she had a score of 8%, during the interventions, 79%, and in summative assessments, 8% and 67%, respectively, with an overall progression of 59% in score and a remarkable improvement in understanding. Of the successful responses, 30% were on the factual recall level, 30% on the operational appropriateness level and 28% on the conceptual level, while 31% were in the skill/algorithm dimension, 43% in the use/application dimension and 26% in the representation/metaphor dimension of understanding.

Table 27

Overview of Specific Outcomes of Assessment: Participant v

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understanding	1/4 Uf Uo Uc Sf Rf Rc	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc	3/4 Uf Uo Uc Sf So Sc	Absent	1/4 Uf Sf	3/4 Uf Uo Uc Sf Rf Ro
Score Understanding	0/4	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc	2/4 Uf Uo Uc Sf	Absent	0/4	2/4 Uf Uo Sf Rf Ro
Score Understanding	0/4 Uf Ro	3/4 Uf Uo Uc Sf Sc Rf Ro Rc	2/4 Uf Uo Uc Sf So Sc	Absent	0/4	3/4 Uf Uo Uc Sf So Sc Rf Ro Rc
Score Understanding	**8% 8**	**92% 26**	**58% 16**	**Absent**	**8% 2**	**67% 20**

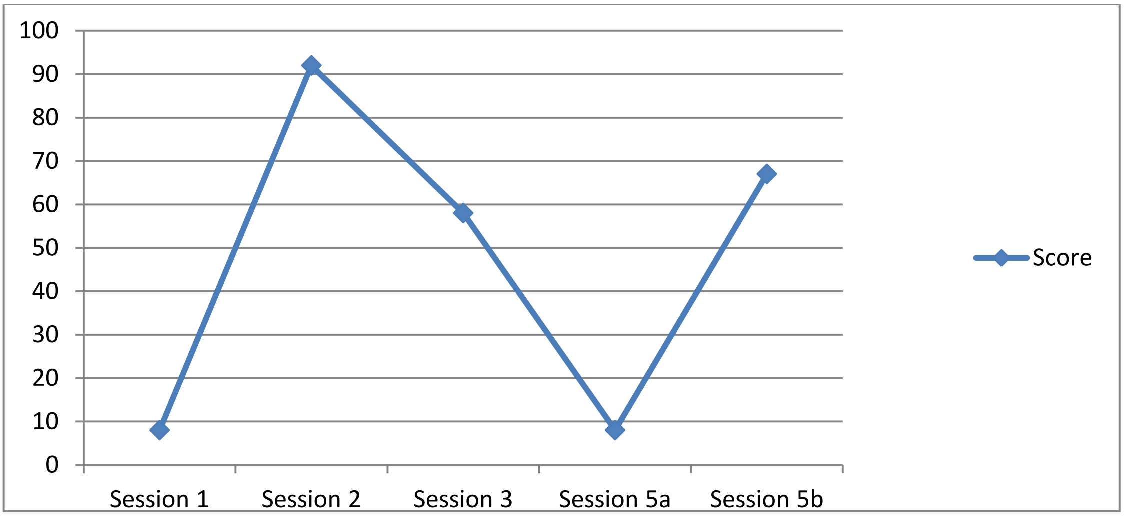


Figure 28. Graphic image of progression: Participant v.

She reported in a pessimistic way about her metacognitive experience and knowledge and that she had not understood the problem before she entered the VSM. She found it hard to imagine the problem situation as a picture and could not see a complete picture. Also, the picture "vanished" when she opened her eyes and she could not work on it. She found it hard to make things move or change and could not remember the numbers once she opened her eyes. Her metacognitive report aligns neither with her proven results, nor with her rate of improvement.

Participant w

Participant w was absent for Session 3. He started off at a low performance point from the baseline assessment to the final summative assessment. He was extremely sceptical about the strategy and had very low motivation in applying the process. He sought clear guidance as to the method they had to use to solve the problem, because he was scared that he would lose marks if he used his own preferred method, as had happened to him in the past. Once he had caught onto the strategy however, he seemed to thrive and became very competitive to be the best performer in this research study. He would not give up, although he reported in the metacognitive questionnaire that he struggled to stay focused.

A remarkable feature of his performance, was that he demonstrated the most balanced distribution of dimensions of understanding as well as levels of understanding, of all participants. His scores and understanding aligned well, sometimes understanding exceeded scores and sometimes scores exceeded understanding. He was so focused on improving his performance, that he was not at all distracted by the situation of detention in the first summative assessment.

In Session 1, he had a score of 8%, during the interventions, 66,5% and in the summative assessments both scores were 75%. He showed an overall progression of 67% on his score and a comparable improvement in understanding. Of the successful responses, 40% were on the factual recall level, 32,5% on the level of operational appropriateness and 27,5% on the conceptual level, while 36% were in the skill/algorithm dimension, 35% in the use/application dimension and 29% in the representation/metaphor dimension of understanding.

Table 28

Overview of Specific Outcomes of Assessment: Participant w

	Session 1	Session 2	Session 3	Session 4	Session 5a	Session 5b
Score Understan-ding	1/4 So	0/4 Sf	Absent	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc	4/4 Uf Uo UcSf So Sc Rf Ro Rc	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc
Score Understan-ding	0/4	4/4 Uf Uo Uc Sf So Sc Rf Ro	Absent	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc	1/4 Uf Sf Rf	1/4 Uf Sf
Score Understan-ding	0/4	0/4 Uf Uo Sf	Absent	4/4 Uf Uo Uc Sf So Sc Rf Ro	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc	4/4 Uf Uo Uc Sf So Sc Rf Ro Rc
Score Understanding	**8% 1**	**33% 12**	**Absent**	**100% 26**	**75% 21**	**75% 20**

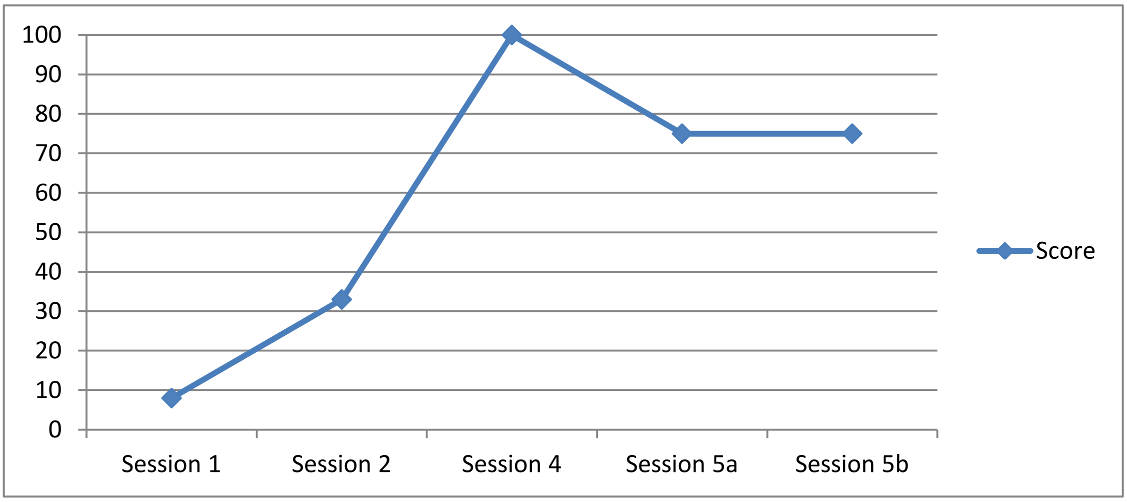


Figure 29. Graphic image of progression: Participant w.

In terms of his metacognitive experience and knowledge, he reported that he had an understanding of the problem before he closed his eyes. He found it easy to enter into the VSM and to see the situation as a picture and easy to see it as a sum, but it was hard not to be distracted by unwanted elements inside the VSM and also by people and things around him. He was uncertain about the numbers when opening his eyes. Although he thinks it is a slow method, the process became easier.

Some inferences, conclusions and recommendations that are based upon the individual assessments, reporting and observations, are made in Chapter 7.

Group Performance per Item Type

In terms of the three types of items, the following findings were made for the baseline assessment, the intervention that was focused on the specific item type and the summative assessment (Session 5b). Various factors influenced the outcomes of the respective types of problems, as well as the assessment outcomes as a whole.

Money Items

Assessment outcomes of money items. The money items have been listed in the first part of this chapter in the order that they were used. The table and graphic image below reflect the group scores and understanding for the money items:

Table 29

Group Performance: Money Items

Assessment		Score	Understanding
Session 1 (baseline assessment):	Item 1	13%	20 incidents / potential 117
Session 2 (first intervention):	Item 4	25%	31 incidents / potential 144
	Item 5	23%	30 incidents / potential 144
	Item 6	6%	11 incidents / potential 144
Session 5a (summative assessment)	Item 13	23%	29 incidents / potential 135
Session 5b (summative assessment)	Item 16	58%	80 incidents / potential 144

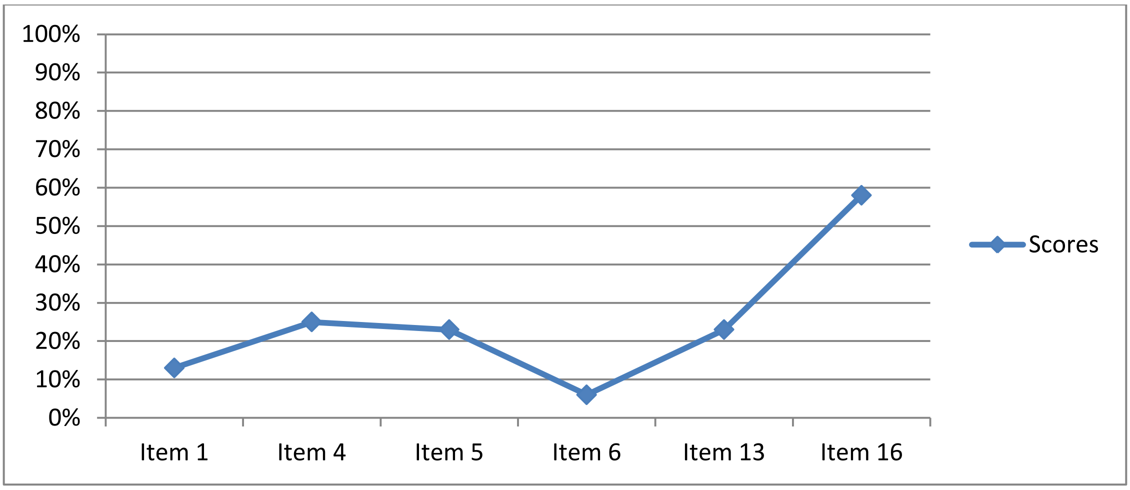


Figure 30. Group performance: Money items.

Discussion of money items. The language and words of items were comprehensible at a Grade 6 level and it is assumed that this factor did not hinder the understanding of problems. To further assist understanding, the items were also read out aloud before participants entered the VSM. The complexity levels of items 1, 4, 5 and 13 seem comparable, where the dividend amount had to be calculated initially before solving the problem. This was not the case for item 16, which made the problem less complicated, at least as far as multi-step complexity is concerned. Equal sharing was required in all items, but in the case of item 6, where rate, an alternative way of conceptualising equal sharing, was required. This mental leap complicated the item. Although item 13 was well comparable with the general complexity level of the money items, the group performance could be expected to be higher in view of the restraining circumstantial and emotional factors that had influenced Session 5a.

Area Items

In the introduction to this chapter, the items in an area context have been listed.

Assessment outcomes of area items. The assessment outcomes of the second item type are not at all comparable to those of the money items. A few factors influenced the two profiles, as will become clear in the brief discussion. The table and graphic image below reflect the group scores and understanding for the area items:

Table 30

Group Performance: Area Items

Assessment		Score	Understanding
Session 1 (baseline assessment):	Item 2	12%	9 incidents / potential 117
Session 3 (second intervention):	Item 7	40%	47 incidents / potential 126
	Item 8	39%	38 incidents / potential 126
	Item 9	18%	23 incidents / potential 126
Session 5a (summative assessment)	Item 14	18%	20 incidents / potential 135
Session 5b (summative assessment)	Item 17	16%	19 incidents / potential 144

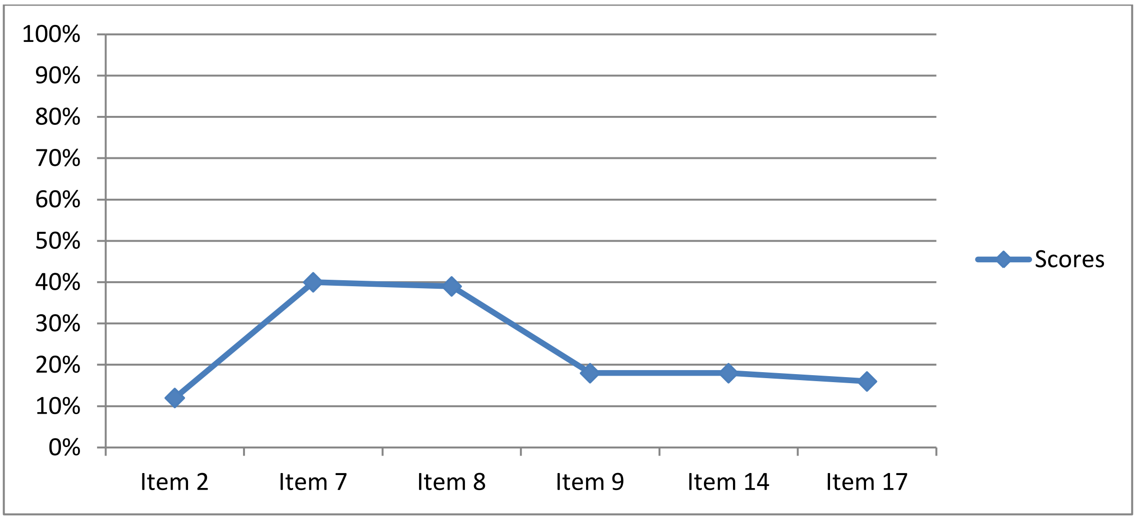


Figure 31. Group performance: Area items.

Discussion of area items. Although the language of the items was presumed easy to understand and clear at a Grade 6 level, it was soon apparent that the specific group was not familiar with the representational form, “110m x 80m” and “5m x 5m” (item 2), and “390cm x 450cm” (item 8). They interpreted this representational form of area as a prompt to multiply the dimensions, which gave a false impression of understanding the concept. When this tendency was picked up in scoring, marks were withheld from any such calculations of which true understanding was not substantiated by further insight(s) into the concept of area. Before starting with the assessments where these types of problems were to be presented, items 13 and 17 were corrected to avoid the same mistake. The representational forms “cm^2” and “km^2” were explained to the group on request, as the units that we use to describe a square block of a cm x cm or a km x km.

The complexity levels of items 2, 8, 14 and 17 were fairly comparable, the requirement in all these items being to divide a bigger area up so as to fit smaller areas into the larger area. In items 7 and 9, the complexity levels were lower and higher, respectively, accounting for the corresponding scores and understanding. Although the group had no exposure to area at the Grade 6 level up to the time of the research encounters, the “1cm^2” in item 7 was a concept with which they were familiar from counting square blocks of an area in Grade 5. On the other hand, item 9 required a counter-intuitive mental leap with clear insight into the theory of surface area, which raised its complexity level. Although item 14 (used in Session 5a) was comparable with the general complexity level of the other area items, the group performance could be expected to drop during that session.

The fact that performance on the comparable items 8 and 17 differ vastly, cannot be explained other than referring to the success of the related visual prompt used during the intervention (see Figure 10 in this chapter), which mediated to the group the entry into the VSM when an area problem is approached. The fact that this mediation did not have a lasting effect, reinforces firstly the notion that more exposure to the method would assist solidifying the concepts, and secondly, that the applicable visual prompt is an equaliser in class for those learners who find it hard to visualise.

Speed Items

Assessment outcomes of speed items. The table and graphic image below reflect the group scores and understanding for the speed items (listed in the beginning of this chapter):

Table 31

Group Performance: Speed Items

Assessment		Score	Understanding
Session 1 (baseline assessment):	Item 3	0%	3 incidents / potential 117
Session 4 (third intervention):	Item 10 Item 11 Item 12	43% 25% 41%	54 incidents / potential 126 33 incidents / potential 126 48 incidents / potential 126
Session 5a (summative assessment)	Item 15	22%	23 incidents / potential 135
Session 5b (summative assessment)	Item 18	40%	57 incidents / potential 144

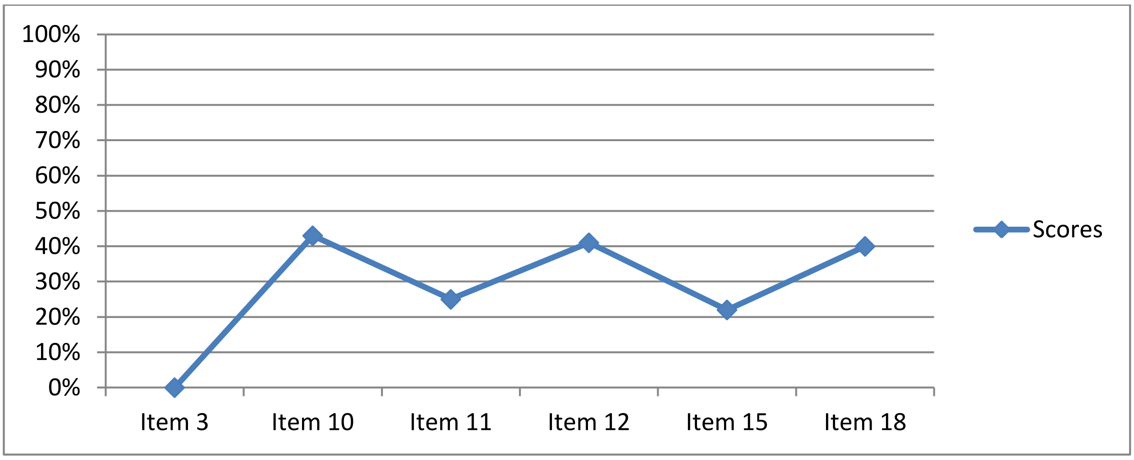


Figure 32. Group performance: Speed items.

Discussion of speed items. The words and language of the items were everyday knowledge for the participants; however, the baseline assessment outcome raised the suspicion that speed as a form of the rate concept had not been covered in their mathematics classes to date. This suspicion was confirmed. Since I had not taught any mathematics to the group on the previous themes, I was left with no option, but to rely completely on the use of the metacognitive strategy to create "speed" in their minds as a mathematical construct. What seemed to be a restrictive factor, turned out to be an opportunity to isolate the effect of the strategy as an important influence in creating a mental construct upon which participants could act mathematically. I thought it justified though, to clarify the representational form "km/h" to the group.

It is hard to judge the complexity level of the above items, since such a judgement is not clear-cut. In ascending order, the complexity levels of the items seemed to range from item 12 as the least complex, followed by items 3, 10, 15 and 18, and leaving item 11 as the most complex item. The use of fractions of hours could be seen as a complicating factor in items 3, 15 and 18.

A visual prompt was used (Figure 11) in the mediation of the visual imagery during the intervention session, also using a fraction of an hour. Although the group had no formal (school) exposure to the concept of speed up to the time of the research encounters, the level of conceptualising that took place, was substantial. The level of performance on item 15, which is seen as moderately complex, is once again seen as caused by the circumstances on the day, which called for a repetition to do justice to the process. In this research, the speed items are seen as the purest illustration of the distinctive effect of visual imagery on mathematising.

CHAPTER 7 – REFLECTION AND SUMMARY OF FINDINGS

The final chapter of my thesis begins with a summary of the research and an abbreviated version of the conceptual framework. The first part of the reflection is on my experience of the Design Research process. In the second part I reflect on my findings with regard to each of the research questions, starting from the secondary questions to the main research question, of which the design principles that have been extracted throughout the design process, represent the main body. Some conclusions are drawn and a few final recommendations are made.

Summary of the Research

I researched a didactical design for the mediation of visualisation, currently an under-explored aspect of metacognitive regulation in solving mathematical problems at the primary level. I based my investigation on three related premises: learners of 11 to 12 years old are cognitively ready to reason mathematically on the basis of imagined representations; their cognitive structures are sufficiently developed to meet the cognitive demands of the troublesome concept of division; and visualisation may be mediated and employed to regulate division problem solving in realistic contexts.

A Design Research approach was followed, which allowed for the cyclic and iterative development of the intervention. The fourth prototype of the design was applied during six encounters with a sample of sixteen Grade 6 learners in an English medium primary school in the Gauteng Province. No mathematics was taught; a visualisation strategy was mediated for the solving of division problems in three different contexts. During each intervention participants' performance was assessed and evaluated on two scales, one for standard scoring and one for plotting their understanding in three mathematical dimensions and on three cognitive levels.

Irrespective of how the participants reported their metacognitive experience, the assessment results indicated that fourteen participants' scores and understanding improved when solving division problems, while they were employing the strategy. One learner's performance remained the same and one was lower. This chapter is a reflection on the study, its findings and conclusions.

The study was conducted according to the conceptual framework (Chapter 4), here presented in an abbreviated form. I have omitted the references cited in the comprehensive framework for the sake of ease of reading.

Abbreviated Conceptual Framework

The move from the Intermediate Phase to the Senior Phase is one of the critical school transitions for learners, who are about 11 to 12 years old. At this stage, the fundamental mathematical concepts required for understanding the advanced concepts of the new phase, should have been firmly established. The learner, who remembers how to do a calculation and even does it correctly, is not ready for its advanced applications, unless they also understand when and why the operation is used.

Within the multiplicative conceptual field, division is perceived as a difficult concept to teach and to learn, and it also requires multi-dimensional understanding to manage its applications in various realistic situations. At Grade 6, semi-abstract ideas within this conceptual field, like speed, area and average, are learned – ideas that require understanding of rate and relationships between quantities and measure spaces.

Despite this challenge, the cognitive and metacognitive functions in the narrow 11 to 12 year developmental band offer an opportunity for effective teaching and learning of such troublesome concepts. At this age, learners' cognitive structures have developed sufficiently to allow for the logical flow of thought, which is involved in proportional reasoning, the coordination of symbolic structures and the isolation of variables. It is, however, the metacognitive ability of intentional and conscious mental imagery at this age, which was demarcated as the research space for this study.

The study brings together the concept of division, and learners' ability to objectify and operate on a realistic situation as a mental model through visual imagery. The instructional intervention proceeded according to a metacognitive model for problem solving, where visualisation in the learning of division was mediated.

Conclusions are drawn at the end of this chapter about the feasibility and potential of such an approach in relation to its capacity to enhance conceptual understanding of troublesome mathematical concepts such as division.

Reflections on the Design Research Process

Guidance from an experienced supervisor was probably a decisive factor in the decision about a suitable research approach that would best serve the purposes of the study and that would open up opportunities for answering the research questions. Many and confusing possibilities seemed appealing at the onset. The investigation was undertaken amidst a lack of existing research of interventions in my area of interest. I was advised to follow a Design Research approach, of which the first task was to adopt an epistemological position as a philosophical point of departure.

Pragmatism was an appropriate epistemology for Design Research. If two ideas can be singled out as those that I struggled most to conceptualise, they would be a research epistemology and a conceptual framework. In retrospect, I realised that both ideas contained the intangible elements of perspective and essence – two elements that were falling in place at an advanced stage of the study. Whereas the conceptual framework could await its opportune stage when it would be formalised, the researcher's philosophical stance needed to be established at a fairly early stage.

In line with the characteristics of a Design Research approach to instructional design development, I was advised, after consideration of many other options, to adopt pragmatism to support the investigation of my theoretical assumption, which aimed at solving the research problem during the practical application thereof. The pragmatic approach indeed proved appropriate within a study informed by quantitative- as well as qualitative data (Creswell, 1998), as reported in Chapter 6.

Both the objective data from the assessments and the subjective reports of the participants and the researcher assisted the search for the central underlying meaning of the research process. The participants' reporting of their experiences from a first person point of view was in the form of a questionnaire, which reflected their "inward consciousness based on memory, image and meaning" (Costello, 1998, p. 52), in relation to their demonstrated mathematical performance. On my part, I constantly journaled the research process in a reflective design journal (Appendix A).

Design Research requires patience and offers few certainties. The nature of Design Research made it impossible to fast track the investigation, since the design had to "grow" naturally through its own stages, which were iterative and cyclical, and of which the first, the literature review, was the most extensive and time consuming.

The large proportion of attention spent in reviewing the literature for a Design Research was inevitable and was not compromised: the theoretical elements constituting the foundations of a newly created design had to be carefully identified and understood to such a depth that they could be applied and adapted if necessary in a novel formation. Further than that, the commonalities and divergences between theories have to be recognised and opportunities for innovation have to be discerned.

Since the dynamics at work within the creation, growth and maturation of the instructional design could not be foreseen from the start, I had to (temporarily at least) forfeit predictive certainties and control. The Design Researcher may experience confusion during the initial, somewhat chaotic exploration, for which very little structure is available. Even though the planning of the research may be set out in a neat table for the purpose of defending the research proposal, it does not do justice to the ongoing cyclical trial-and-improvement process required by the reality of a Design Research. A serious mistake in this regard is to desire structure, certainty and finality too early in the process and then revert to existing designs, instead of seeing the innovative cycles through to conclusion and delivering an authentic design.

Another possible mistake is the forceful accommodation of theories that impress on the researcher with strong appeal. The reluctance to part with theoretical views that were worthwhile in their own right, but which did not contribute to the coherency and quality of the design, had unduly delayed the progress of my study. The initial acceptance of theoretical assumptions is of a provisional nature, pending its practical testing. Therefore, the decision about core theoretical assumptions should be continuously informed by the developing prototypes and should be regarded as provisional, pending their practical application. In my view, researchers with tendencies towards exact planning, control, order and clear-cut product output, would find the provisional and uncertain nature of Design Research exasperating.

The research questions provided a flexible design framework. A didactical design can be compared to a building, and the research questions to the structural framework according to which the building is erected. The research questions should not be seen as rigid and untouchable – they constitute a flexible framework for the design, and are subject to moderation. Although the main research question remained constant throughout this research, to define the eventual aim of the design, the framework according to which the design would materialise to comply with that aim, had to be continuously challenged. Any effort towards early finalisation of the research questions proved premature and not productive.

The research questions that would serve as a design framework had to be aligned with the greater function they were to fulfil as a collective set. The decision for or against a specific anchor theory, was taken in accordance with its capacity to contribute towards an appropriate response to one or more research questions, and in that, towards complying with the ultimate intended functional purpose of the design. In turn, the theoretical perspectives assisted the formulation of the research questions.

A Design Research delivers a researchable design. This Design Research resulted at least in a suggestion for a workable design that had been brought about according to sound research practices, and at most, in a design that invites further research. The eventual design is therefore neither provisional nor conclusive.

The validity of the claim of the design to be more than provisional, can be judged against its compliance with the principles for high quality educational interventions (Nieveen, 2007), as is elaborated in Chapter 3.

The non-conclusive character of the design can best be understood when the limitations of this study are considered, in particular the sample size used in the Intervention Phase. Although no claim can be made to the statistical representivity of my sample, I have analysed each individual's assessment results against the theoretical framework that I had set up in structuring the assessments and the questionnaire. This, according to Yin (1994) can also be seen as a way of generalising the outcomes of a study and contributing the theory – to my own, and possibly to other theories as well.

My understanding of the phenomenon that I have been investigating is restricted to an extent though, by the number of participants that took part in the investigation. Although the metacognitive strategy seems promising from what was found and reported in this study, it is entirely plausible that I have accounted partially or simplistically about the effect of the strategy, for lack of a broader scope and insight that would be attainable within a large cohort. It is also possible that the outcomes could be either exaggerated or understated, not intentionally, but because of structural shortcomings in the design. All of these possibilities may be researched further to improve the effectiveness of the design.

The Preliminary Phase requires discipline and routine. For me, the Preliminary Phase was characterised by the wonder and confusion of the knowledge that existed out there, and of which I was oblivious. For the first year, my rule was to read one article or one chapter per day, and write one paragraph in summary of what I read. These paragraphs contained the author's views, findings and theories, and my own insights and understanding of those writings. Many months into the process, I could start to group the paragraphs roughly by setting up three "bins" into which I could "deposit" concepts, ideas, statements, theories and my own insights and interpretations. These bins were re-arranged later to correspond with the first three research questions, in turn shaping the phrasing of the questions and the different parts of the literature review.

The discipline of daily reading and the time spent in gaining insight, was fruitful; however, being deeply involved with the core question of the research, three aspects of my research showed up as under-emphasised, under-explored, in fact, neglected. The oversight was costly in terms of time, as it caused a major hold-up in the Evaluation Phase and forced me to extend the literature review, to change the format of the thesis and to re-arrange the writing up of the study at an advanced stage. The shortfall of knowledge and understanding were concerning the following matters:

- The dynamics of the research methodology that I would follow;
- The nature of data that would be created through the fieldwork, and the method that would be employed to analyse data;
- The format into which the study had to be written up, the writing conventions and the requirements of style.

Having taken time in rectifying my oversight, I realised that I could set up more “bins”, and spend more time from the start on reading about what could be regarded as peripheral matters. I would divide my reading time in a ratio of core matter – 3: methodology – 1: data and analysis – 1: writing form and style – 1.

The Intervention Phase breeds research integrity. I had to face the dualism that characterised this phase in more than one way:

- I experienced excitement and eager expectation, also uncertainty and doubt.
- The tasks in this phase were both theoretical and practical: as much as the intervention needed to be grounded academically, the organisational and institutional requirements for arranging the fieldwork had to be met on a practical level too. Gaining consent(s), organising the research site, recruiting

participants, and managing the logistical arrangements, were indispensable tasks for ensuring the smooth (and legal) flow of the encounters.

- Discrepancies were likely to be discovered during this phase, between what had been accepted theoretically, and what was found in practice, and these had to be accepted, processed and managed.
- Along the same lines, one should be prepared to face, process and report any contradictions as far as the intended and the actual outcomes of the experiment go. For me, this was probably the most demanding challenge.

Researchers may find themselves at odds with the last possibility. In this regard, the greatest (and most timeous) lesson I have learned, was offered to me by my second expert advisor – an educational psychologist – with whom I consulted to reflect my educational concepts. She sent me off with a question, which she had adopted as a guideline for her own academic integrity, from Professor Jonathan Jansen during his office at the University of Pretoria: “How have you protected yourself today from being right?” In preparation for the fieldwork, I could only prepare myself mentally to face the possibility of not being right, but the challenge was, dealing with the outcomes of the fieldwork, to face contradictory findings; to critique my own efforts and propositions; and to be fiercely honest when describing the events. Her advice lingered and guided me for the duration of the second, decisive phase of the study.

The Evaluation Phase embodies the essence of Design Research. As has been illustrated in Chapter 5, the Evaluation Phase can be compared metaphorically to the task of baking a cake, for which only the raw ingredients are available (the literature review, the semi-final design prototype and the data sets) – without a recipe. Not that the raw ingredients were ready and provided – they had to be grown and cultivated too. However, I could draw from the existing theories and research studies as the sources. The Evaluation Phase is the period where the design is logically argued, motivated, described, finalised and presented – all the time taking account of the requirements of the standards, packaging and presentation of the (academic) industry. In this phase I made two important observations:

With respect to time, this study required an estimated ratio of 4:1:5 across the phases. The first phase was lingering, and took up about 40% of the dedicated study hours to review the literature. The Intervention Phase was short, intense and productive, and required about 10% of the time; and the last, highly demanding Evaluation Phase needed 50% of all available time, for the tasks of writing the study up, developing the final design and rounding off the thesis.

Following regular supervision at frequent intervals, I learned that I was the producer of the design, knowing my own intentions very well; however, the reader as the consumer of my work, needed to understand clearly what these intentions were, and I was responsible for enabling such understanding. I had to switch roles constantly from writer to reader, and still, I could not alienate myself completely from myself as the primary creator of the work. The role of the study supervisors and of the proof-reader is indispensable in this regard.

Findings According to the Research Questions

The development of the design was guided by the main research question: "How can structured visual imagery as a self-regulating metacognitive strategy be used at Grade 6 level for the understanding of multiplicative concepts as they arise in realistic situations?" In response to this question, the research was conducted according to four specific research questions, as follows:

1. Which mathematics education approach should be adopted in this research to meet the requirements for the teaching of multiplicative concepts at Grade 6?
2. How does the understanding of mathematics in general, and of the multiplicative concepts in particular, come about in learners?
3. How do the cognitive- and metacognitive functions of 11 to 12 year old learners support their understanding of mathematical concepts?
4. How can a metacognitive strategy comprising structured visual imagery be mediated for the understanding of division at a Grade 6 level?

Findings About Questions 1-3

According to plan (Chapters 4 and 5, Figures 5 and 7), the research questions were addressed systematically within the three phases of the Design Research, each phase delivering their own distinct findings. The first three research questions were addressed during the Preliminary Phase through a literature review and by an initial design prototype. The academic findings of this phase contributed both to the theoretical grounding of the formative elements of the design prototypes and to the formation of the design principles, an important aim of Design Research (Plomp, 2007; Plomp & Nieveen, 2013; Van den Akker et al., 2006).

Even though the literature review was reported in Chapters 2 and 3 directly according to these three questions, the findings of this phase do not necessarily correspond one-on-one with the specific questions. Rather, the integrated insights obtained from the review enabled me to identify some general conditions, which may optimise mathematics learning in my specific focus area. In an effort to separate the research findings to correspond directly with a specific research question, I realised that the following findings each contains a part-response to all the research questions. I found that the SA curriculum at the Intermediate Phase (CAPS, 2012a) offers a substantial basis for the teaching and assessment of mathematics, however, the realising of its aims may be more successful if instruction takes cognisance of research findings. Through the theories and research reports reviewed in this study, three findings for optimising interventions have emerged, as follows:

Subject demands should be correlated with learner abilities. The curriculum- and subject intrinsic requirements of mathematics (Hart, 1989; Lamon, 2007; Long, 2011, Vergnaud, 1988; 2009; 2010), even those of high demand (Linn, 2002; Meyer & Land, 2003; 2005; 2006), should be seen in relation to learners' available developmental abilities to respond to those demands (Copeland, 1984; Demetriou et al., 2011). If account is taken of learners' cognitive and metacognitive abilities at this age, instruction and assessment of mathematical concepts can be adapted accordingly (Demetriou et al., 2011), in aid of functional concept formation (Efklides, 2007; Koriat, 2007; Panaoura, 2007). I have adopted this finding as a core assumption for didactical design (Appendix D), an assumption that can be generalised across developmental stages as an instructional design principle.

Visualising enables mental objectification of mathematical situations. This finding is based on the assumption that concept formation and the progression to advanced mathematising hinge on the ability of learners to model mathematical situations mentally (Ambrose et al., 2003; Sfard, 1991; 1998). Therefore, it is advisable that metacognitive strategies (Desoete & Ozsoy, 2009; Efklides, 2007; Koriat, 2007; Panaoura, 2007; Pintrich, 2002; Veenman et al., 2005) such as visual imagery (Presmeg, 2005; Mason, 2002), will be employed to build mental models for the understanding of concepts as they arise from realistic situations (Gravemeijer, 1994; Gravemeijer & Doorman, 1999; Van den Heuvel-Panhuizen, 2003). These strategies can be mediated intentionally (Feuerstein & Rand, 1974; Feuerstein et al., 2006; Kaniel, 2000; 2003) and systematically, as logical guidance is given towards the understanding of situated mathematical concepts. Mental modelling extends the available range of modalities employed to teach and learn mathematics (Kress, 2009). The above finding became a fundamental aim of my instructional design and was tested with the participants during the fieldwork (Chapter 5 and 6; Appendix A and D).

The dimensions and levels of understanding should be assessed. The third finding is that the indicators of conceptual progression are not to be derived at only in terms of the (static) scores obtained for correct- and incorrect problem solving. Rather, that indicators of effective teaching and learning of mathematical concepts be tested in a dynamic way (Tzuriel, 2001) and sought in learners' demonstration of their underlying conceptual understanding in various dimensions (Usiskin, 2012) and at various cognitive levels (Anderson, 2002; Anderson & Krathwohl, 2001; Ferguson, 2002; Forehand, 2012; Krathwohl, 2002). I have applied this finding in my own design in an extensive way (Chapter 6 and Appendices A, B and D).

Findings About Question 4

The fourth specific research question was addressed mainly during the Intervention Phase of the study. It guided the research on a practical level, asking: "How can a metacognitive strategy comprising structured visual imagery be mediated for the understanding of division at a Grade 6 level?" A distinction is made between "How wás..." it done in this study for research purposes (reported in Chapter 5), and "How cán..." it be done in practical teaching for classroom applications (exemplified in the lesson plan in Appendix D). The findings below are mainly derived from the data obtained in testing Prototype IV in an experimental situation with the participants.

Participants were able to objectify situations through visual imagery. All participants in the experimental group reported that they could visualise situations and all of them could intentionally effect a flow of their consciousness for the duration of the situation (Chapter 6). This is a local finding and can be generalised only within a representative cohort of learners this age, where gender, race and cultural background, amongst other factors, are investigated as variables of this metacognitive ability.

Reported metacognitive knowledge and experience are coloured subjectively. The short metacognitive questionnaire (6 written questions and 3 questions answered orally) was complemented by a discussion with individual participants, of which field notes were taken (Chapter 6, Appendices A and C). My naïve aim with the participants' reports of their metacognitive knowledge and experience when using visualising to solve division problems, was to help explain the assessment outcomes. However, as should have been expected, many of the obviously subjective responses did not support or correspond with the objective indicators of their performance as reflected in their assessment scores and marks (Chapter 6).

The metacognitive reports were regarded as authentic, although they did not correspond consistently with assessment outcomes, as have been reported alongside the discussion of individual assessment outcomes in Chapter 6. However, the initial objective was not reached. Further research may establish whether this is a general trend; how such reports may be interpreted; and whether metacognitive knowledge and experiences can be mediated towards narrowing the "subjective-objective gap".

Visual imagery enhanced understanding of situated division concepts. The main limitations of the experiment are seen to be the small sample size and the weaknesses in some of the assessment items, as will be discussed later in this chapter. On the other hand though, it could be argued that if participants had more exposure to mental visual imagery in problem solving than was feasible within the empirical research period, a more substantial improvement could be expected. Furthermore, they had to close their eyes while visualising, but it was observed that, probably because they were not in the habit of visualising yet, some participants remembered to visualise directly after they had been reminded to do so with the first of three assessment items, but by the third item, they approached the problem without intentionally visualising – which could also have influenced the effect of the strategy.

Notwithstanding these limitations, and not accounting for any possible further improvement with more exposure or habit formation, it can be stated that participants' understanding of division improved, within the sample, with the use of visualisation. In this regard, the group improvement in scores was not regarded as a specific enough descriptor of the change that happened; rather both measures of the assessment outcomes according to the two scales, were reflected in a table per individual, thus providing an image of the improvement or decline in the measured performance.

Main Findings

Technically, it would be correct to state that the main aim of the study was to develop an instructional design towards a specific purpose; in fact, this primary goal was pursued for the duration of the study, up to the stage of writing it up. The general perspectives in response to the main research question turned out to typify a model for the generic development of designs with a similar purpose, namely:

- Design principles for similar educational intervention designs.
- Findings about teaching, aimed to improve the everyday classroom practice.
- An example, illustrating how these ideas and principles may be embodied within a didactical intervention that would serve the instructional purpose. This example is the final prototype of the design (Prototype V), and is included in the form of a lesson plan in Appendix D.

Design Principles

For the duration of the research process, various principles had crystallised for the design of the ideal didactical intervention as a response to the main research question. These principles have been collated and refined and are presented here as essential design principles, upon which the final design is built, as follows:

Principle 1: Major concepts consist of minor concepts. Long's statement (2011, p. 268) that there are "threshold concepts on the mathematical development path to proficiency... that provide the conceptual gateway to higher levels of mathematics", is understood to imply that a threshold concept does not necessarily have to qualify as a major mathematical concept to either obstruct or open the progressive pathway. The absence or misunderstanding of a minor, or base concept, can equally obstruct progression of understanding in higher levels of mathematics. This conception of threshold concepts qualifies a wide variety of minor, sub-concepts of a major concept also as threshold concepts.

The mathematics teacher needs to have a view of the course of a conceptual pathway before, during and after the particular phase that is taught. Knowledge of the concepts that are in place already, and of the concepts towards which she is working, provides the starting point and the end-point of the planning of the present work. The example of a didactical intervention shown in Appendix D, had been designed for the Intermediate Phase, which is preceded by the Foundation Phase and followed by the Senior Phase and finally leads to Further Education and Training.

The assumption in this intervention is that the separate base concepts of time and distance that have been established in the Foundation Phase, are related to each other in the concept of speed at the Intermediate Phase. The concept of speed is then broadened at the Senior Phase as an incident of the encompassing major concept of rate, and deepened in its application towards the concept of velocity at the following level. Keeping in mind the eventual conceptual requirements of Further Education and Training in this regard, it was noted in the practice of mathematics teaching at the Intermediate Phase, that an accompanying basic concept of speed at the Intermediate Phase, is “average”. If the essential characteristics and understanding of the concept of average have been missed, learners struggle to progress towards a concept of rate, and will, therefore, also be hindered from understanding the some calculus concepts. In this way, “average” has qualified itself, so to speak, as a threshold concept. In summary, a major concept embedded within a conceptual field is chosen for a set of instructional designs. Sub-concepts of the major concept at the particular level taught are then identified and situations representing the sub-concepts are selected as a basis for the instructional design, to ensure situated conceptual understanding.

Principle 2: Mathematical concepts are introduced through situations. A mathematics curriculum selects and organises sub-concepts within a mathematical theme, in the combination and order that has the best probability of building up towards the understanding of the major concept(s) in the specific field. Vergnaud (1988; 2009) stated that, within a conceptual field, a set of concepts are interrelated with a set of situations. Resonating with both the theory and practice of conceptual fields and those of RME (Gravemeijer, 1994; Gravemeijer & Doorman, 1999), the ideal would then be to introduce a mathematical concept by way of selecting and observing situations within the everyday experiences of the learner.

Principle 3: Concepts guide the selection of situations. For arguing the above design principle, a concept is pragmatically defined as: "A mathematical idea, the grasping of which enables progression of mathematising, from a problem situation towards solving the problem". To illustrate this definition, the major- and sub-concepts involved in the problem situation: "Sello goes to town", are illustrated below. This problem is also used for the final prototype of the intervention (Appendix D). The major- and sub-concepts involved in the problem are set out in the figure below: "Sello rides to Zap Store with his bicycle every day, which is exactly 2,4km from his house. Let us find out at what average speed he rides (in metres per second), if it takes him 10 minutes to reach the store".

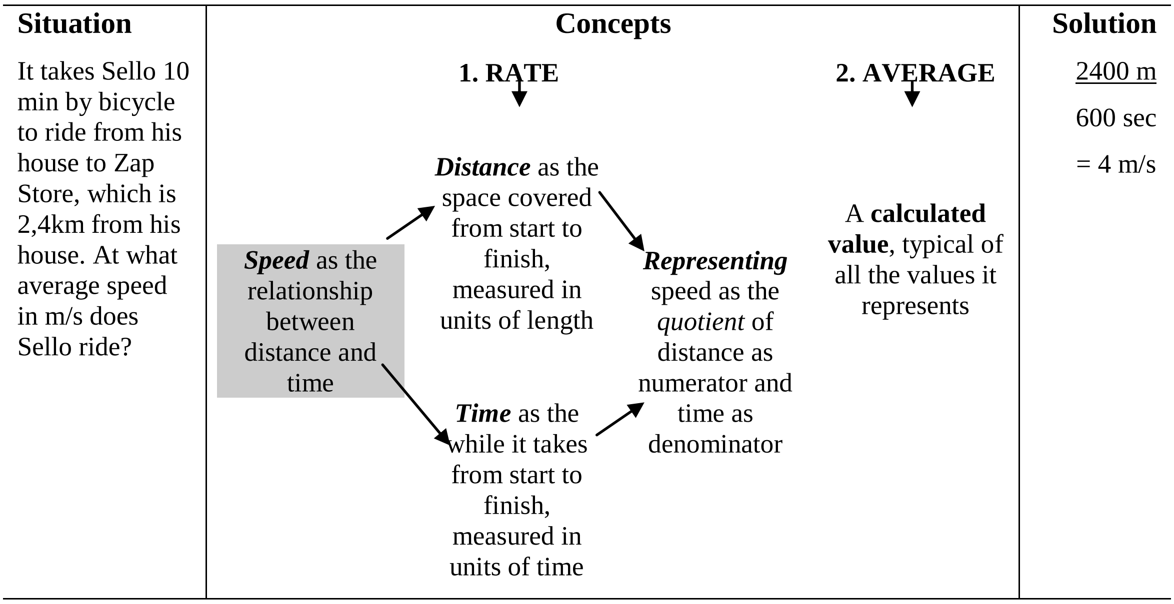


Figure 33. Major- and sub-concepts in solving a speed problem.

The central consideration in the identification, compilation and framing of a situation, is the target concept that is about to be established, confirmed, expanded or generalised, irrespective of the computations that would be required to work towards a solution for the problem. This argument is made in reaction to the practice of compiling situations solely to drill computational skills in isolation, where the target operation is the leading consideration for the selection.

From a subject point of view, the notion that the mathematical concept determines the situation that is framed is in essence the mathematical requirement for selecting the situation. From the learner's point of view, as counterpart for the mathematical requirement for selecting a situation, some human and circumstantial requirements are now considered.

Principle 4: Situations must have a natural fit to concepts. The sequence in which learners encounter everyday situations, does not necessarily correspond with the sequence planned out in a structured mathematics curriculum. These situations do not always neatly fit into the well-structured and advisable sequence of initially introducing complex and potentially troublesome mathematical concepts like those found in the later years of schooling, and for which the basic sub-concepts must have been prepared in primary school.

Real world incidents that may be used regularly to introduce the said elementary concepts could be less familiar to the experiential reality of the learner than those associated with the more complex ideas. For example, incidents of constant acceleration like an object in free-fall, or the uniform velocity reached by a projectile, are not always in close proximity of the living world of a primary school learner. Furthermore, seeing the rapidity of change taking place in the speed and the high velocity of falling or projecting, direct observation of these changes is hardly feasible. Conceptualisation of the elementary mathematical underpinnings of such phenomena is therefore a remote goal, at least at the primary level.

Non-constant acceleration, as a much more complex mathematical concept, is possibly a more frequently encountered and directly observable phenomenon in the living reality of a learner than constant or uniform acceleration. In many situations in the real world, neither acceleration nor deceleration is constant. On the sports field, the sprinting athlete accelerates rapidly from the start point and keeps (almost) the same velocity towards the finish line; when riding a bicycle, learners know very well from their own experience that, apart from the initial acceleration, their speed is set to change as a result of interruptions like stop signs, speed humps and variations in pedalling speed. Likewise, the speed of a car, a bus, a train, an aeroplane or a boat does not follow a constant acceleration pattern. Even the pace at which one walks across the school terrain varies all the time. A meat-eating bird may get airborne slowly, but dash down swiftly at its prey. An ant may hurry from one point to another, slow down at the sign of food and scurry off at a different pace again.

I argue that the human cognitive structures are capable of conceptualising even deeply-involved ideas like the above if at least three salient conditions are met: observability, connectedness and guided logic. The identification of these conditions culminated from both my theoretical and practical research experience.

Principle 4 (a): Observability. The ideal is that the concept that must be learned would be directly observable through the human senses to make the transition to mathematical processing easier. However, if that is not feasible, it can be observed secondarily in the form of an acute representation, which should enable learners to understand the phenomenon mathematically. The most common representation of situations in a visual pictorial form has specific limitations, especially in terms of scale and movement and is only partially helpful for conceptualisation. The representation of situations in graphic form requires a mental leap for understanding.

I argue that the situation may also be relocated to the mind space, where it is made observable through the mind's eye in the virtual space of the mind. The transition is swift and individualised and images can be intentionally manipulated. This practice is particularly useful within the confines of a classroom group setting and has dynamic affordances, which often cannot be reached with static representations like pictures. Additionally, visual imagery is equally available to all learners since it makes use of an innate human cognitive and metacognitive capacity.

Where this capacity is either not well developed, or under-utilised, it can be mediated fruitfully and supported with visual prompts to set visualisation in motion, as it is demonstrated through the foregoing report of the Design Research in Chapter 4.

Mental imagery and visualisation can, for example, accommodate change, a prominent requirement in the conceptualisation of complex ideas in the everyday lived experience of learners. However, a situation hardly can be imagined or mentally characterised if it is isolated or significantly disconnected from the experiential reality of the learner. It follows that a further precondition for observability is the connectedness of the situation to the learner's lived reality.

Principle 4 (b): Connectedness. I further argue that a precondition for mathematical conceptualisation of situations is connectedness, which refers to the links that situations have with what the learners have already experienced first-hand. At face value, the foregoing statement appears to be self-evident; however, if there is not a high degree of certainty that the situation has firm experiential links with the lived reality of each learner, a situation is perpetuated where certain learners are excluded from deep conceptual understanding of ideas.

To ascertain that situations have functional links with what learners already know, insight is required into the contexts within which learners live in a wide variety of settings. A child, for example, in a household mainly driven by high technology devices and self-sufficient electronic entertainment, may not be familiar with the lever force needed to roll a log of wood home for firewood. Likewise, an underprivileged child in a remote rural area may not be familiar with the power distribution in a four-wheel-driven quad bike over rough terrain. These situations would, therefore, be unsuitable for fostering mathematical concepts in the typical South African classroom.

In a vastly diverse country like South Africa, the selection of connected situations with which learners can mathematise, poses a challenge. However, if it can be assured that learners can relate equally to the situation, the skilful selection of situations could have a powerful influence on equity for all children in mathematical conceptualisation. My recommendation is that situations that are in close proximity of each learner's reality may be used, such as the situation in the design in Appendix D.

Principle 4 (c): Guided logic. As a last, but critical requirement, I argue that the mediation of sequential thinking, or guided logic, is a prerequisite for conceptualisation.

Guided logic, in my view, refers to the methodology of bridging, firstly from the generally connected (familiar) situation to the less familiar, and secondly, from the observable to the less observable. Visualisation is a potentially powerful strategy in rendering situations observable in the learners' mind space. The instructional intervention (Appendix D) practically exemplifies this process.

Not only do children in the age group 9 to 12 years have the need for logic, but they also have the ability for logical validity of propositions (Demetriou et al., 2011). Bridging the space between the familiar and the less known through logic, is therefore reasoned to be acceptable, feasible as well as advisable at this developmental stage.

The bridging from the familiar to the less familiar may extend the experiential reality of the learner to become an understood reality, even in the absence of personal or direct exposure to the situation. This notion is supported by the cognitive ability of a learner in the age group 9 to 12 years to imagine the non-real. What is experienced as real, may serve as a gateway for imagining what is not yet real. A learner who has no experience of riding a bicycle for example, has inevitably witnessed bikers before and can comfortably be included in the discourse without any adverse outcome.

Furthermore, the equalising influence of guided logic comes into effect when an equally unknown situation is accessed by all learners, from the observable to the less- or not observable. In the case of the cyclist, observable may be the time that it takes to cover a certain distance and the irregularity of acceleration and deceleration. The less observable would be the changes in acceleration and the virtual impossibility of accounting for the infinite number of changes in velocity.

An illustrative example of guided logic from the familiar to the less familiar and from the observable to the less observable is provided in the intervention design: "Sello goes to town" (Appendix D). Within a single intervention, all learners are guided equally through a slow and logical process into the conceptualisation of actual speed, average speed, (non-constant) acceleration and deceleration.

Principle 5: Didactical interventions should be designed according to age-appropriate cognitive development expectations. As has now become clear, a design needs to be cognitively age appropriate. Having stated that, although well founded in research and theory, it is acknowledged that the range of cognitive expectations within an age group can be used as a general guideline for setting the design. Individual variations within a group may require individual adaptations, both in terms of the amount, proximity and duration of mediation and in terms of the time allocated to the learning experience.

Additionally, it is advisable that a design has many checkpoints where the teacher may bring the learner and herself into contact with previously missed- or neglected concepts that would stand in the way of the establishment and confirmation of a new concept. This practice supports the assessment of learning in the everyday classroom setting, as exemplified in the final prototype of this design (Appendix D). In this way, interventions gain a formative character for both teachers and learners.

Findings About Classroom Practice

A few findings about the improvement of classroom practice are listed and briefly discussed below, as the majority of them have been explained in previous chapters. At the same time, I offer the following findings as recommendations to teachers wishing to improve their professional proficiency and classroom practice:

Journaling supports improved design development. As was the case with the design journal kept in developing the design for this study, the value of this practice of teacher journaling in enhancing everyday intervention design for classroom use is presented. The main benefits are:

- Journaling is a systematic and constructive way of keeping track of the development of a didactical design.
- The process encourages self-evaluation of failures and successes alike.
- It inspires creativity in the quest for solutions to problems.
- Journaling begets insight into learner thinking if it includes reflection on incidents demonstrating learners’ reasoning patterns.

Assessment of understanding helps teaching and learning. One of the single most useful ideas resulting from this research, is the notion that one can − with relative ease − assess learner understanding, both in a continuous and in a summative manner. One can allocate a number value to various dimensions, forms and depths of understanding. This may happen comfortably in tandem with the traditional marking according to a memorandum, as argued in Chapter 2, as practised in the Intervention Phase and as illustrated in the design in Appendix D.

The advantages of assessing understanding alongside scoring of learner responses according to a standard memorandum are, in my view:

- Learner progress is monitored, not only according to a proficiency standard after instruction has taken place, but on cognitive considerations during the process of concept building, inevitably improving their eventual proficiency.
- Assessment of understanding enables richer insights into the underlying factors in learner responses, which enables the reinforcement of understanding and mediation for the identified factors that lead to misunderstanding.
- Assessment of understanding provides a group profile of generally misunderstood concepts, which encourages the search for innovative solutions and planning of strategies to improve learners’ understanding.

- Teacher knowledge of learner understanding significantly improves lesson planning, as she becomes aware of the loopholes where misunderstanding may occur, pays due attention to explaining and contextualising terminology and builds in several checkpoints to monitor the progression of understanding.

Multi-modal teaching and assessment enables differentiation. Learner preferences for a specific mode of teaching and learning vary significantly; they are reflected in their learning strategies and in the varying measures of success, while they are employed in different modes of representation in learning.

Wary of my own preference to lean towards a specific mode, I incorporated the visual, auditory, written, kinaesthetic and numerical modalities of learning within the presentation of Prototype IV of this design, in the following ways:

- The majority of participants reported better understanding when assessment items were read out while they followed the written instruction (Appendix C).
- Participants found it helpful when I demonstrated with a visual aid, how a picture of a problem situation is translated into a mind picture (Chapter 5).
- Dancing and moving along on the rhythm of a little song about visualisation ("In the virtual space of my mind, ke na le plane" – Appendix A; Chapter 5), accommodated the kinaesthetically- and auditory-inclined participants – and created a relaxed and free atmosphere for learning.
- Whispering in the ears of learners, remarkably improved participants' attention during the intervention sessions and promoted order in the classroom.
- Participants were invited to draw pictures while solving problems, but very few made any use of the opportunity, probably because they were not in the habit of doing so in their everyday problem solving experience in the classroom.

Intervention planning promotes structured problem solving. In this study, lessons have been structured according to a model for metacognitive problem solving. Likewise, participants learned how to approach each mathematical problem according to the model (Chapter 5 and Appendix A).

- From my perspective, I anticipated the outcome from the beginning, and the structured approach of the metacognitive problem solving sequence assisted me to mediate the strategy in a systematic way. It was also helpful to control impulsivity amongst those participants who were inclined to rush to solutions.
- The problem solving sequence of goal setting, planning, execution and feedback can be repeated a few times in the course of a single lesson. It also provides an opportunity to assess learning and monitor understanding at various intervals within the lesson. The lesson plan in Appendix D is an example of mini-cycles building up towards the concept of "average speed".

Learning is promoted in a free instructional environment. As mentioned in Chapter 5 and Appendix A, I encountered an unexpected bonus insight during the Intervention Phase of my research: I have seen reflected in the scores and understanding of the majority of participants, the detrimental effects of a stressful situation as compared to the performance and progress they had demonstrated within a free and friendly instructional environment.

My assertion is that, irrespective of the constituents of a teacher's educational environment, it remains the task of teaching to arrange the instructional encounter in a way that all learners have the full advantage of free learning. The mood that prevails in the classroom is either evoking or suppressing learners' will to learn and their ability to respond to teaching in an appropriate manner.

Conclusions and Recommendations for Further Research

This study is concluded by presenting my considered judgements, as follows:

Final Conclusions

A pre-scientific observation in my everyday classroom practice was that visualisation was a generally available metacognitive facility in learners of about ten or eleven years old; that they were able to manage their images towards mathematical problem solving; and that teaching and learning division could be improved through visual imaging. This idea was investigated and confirmed in part, through the reading of theories and academic research reports, which were employed as a basic premise for my research experiment, and were reinforced by the findings of the research. My main research question was concerned, therefore, with the characteristics of an intervention where visualisation was mediated in the learning of multiplicative concepts at a Grade 6 level, upon which the following conclusions were drawn:

Visualisation is a human cognitive ability, which does not qualify by design as a metacognitive ability. From the literature review I gathered that learners would spontaneously, almost inescapably, visualise situations because of a generally available human cognitive function which reaches a certain developmental level of maturity at the age of 11 to 12 years. I state that this cognitive function does not necessarily serve monitoring and regulating functions in problem solving – functions associated with metacognition – unless it is consciously directed and intentionally employed to do so. The conscious use, intentionality and directedness of visualisation make it eligible as a metacognitive strategy and render it useful in problem solving.

The effect of visualisation on problem solving varies. Developmental theories and research suggest that learners in this age group are capable of functional management of their own visualisation, a notion that pertains to metacognitive directedness or purpose with regard to the imagined situation. I have found though, that participants employed the strategy with varying levels of ease and with vastly different effects on their problem solving (see Chapter 5 and Appendix C). A low functioning participant, for example, showed a decline of 6,5% in performance during the experiment, while a learner who had departed from the same low start level, demonstrated a sharp incline of 67,5% in performance and understanding.

The functional management of a visual image does not translate automatically into improved performance. This conclusion was reached, based on the assessment outcomes of the experiment (Chapter 6; Appendix C). The dynamics involved in the phenomenon as it was observed in this study, could not be investigated fully, given the time constraint and the sample size of this study, a limitation that did not allow for generalisations across the given population. However, it is highly possible that other factors influence and even override the functionality of the visual image, factors such as remembering the image, the availability of the various elements of the problem for mathematical use, and deficient calculation skills.

Reported metacognitive experiences and knowledge may deviate from the demonstrated performance, and therefore may not be a factor in performance. As mentioned before, participants' reported knowledge, experiences and valuation of managing and using mental images, were inevitably subjective and did not always have a direct correlation with their mathematical progression while using the metacognitive strategy. The trend goes both ways: participants who demonstrated a steep incline in performance and understanding while using the strategy, reported their inability to utilise the visual images in their minds, while some who showed very little or no progress at all, were optimistic about the affordances of their newly discovered ability to visualise while solving mathematics problems (see Chapter 5).

A possibility in this regard is that participants were not in the habit of judging their metacognitive knowledge, experiences and feelings. It is believed that with more exposure and feedback regarding their progress, the distance between their experiences and their performances may become smaller. No feedback was given to participants in this study regarding their progression; therefore, there was little or no external influence on their reported experiences.

Visual imagery is an available, flexible and dynamic mode of learning. Visualisation is probably in closer reach of all individuals, and more readily accessible than any other visual representation in a picture-, a chart-, electronic form or any other forms. The visual image can be created autonomously, or it can be prompted in the learner's mind by the teacher, through announcing its constituting elements within the description of the problem situation. The colour, size, shape, direction, means of tagging and manipulation of these elements are the learners' choice and are under their control – confirming the propensity of mental imagery for individuation. Visual imagery may (and preferably should) be supplemented by auditory elements where learners speak out words to their minds' ears and listen to their own in-talking, answer their own questions and run a discourse in their minds. The possibility of adding kinaesthetic elements to the image adds a dynamic feature to the strategy. Visual imagery is therefore an equaliser in terms of language of thought and availability of visual aids in learning, even in a resource-deprived learning situation.

Metacognitive strategies are embedded in metacognitive problem solving. This conclusion is a key response to the main research question and emerged mainly from prototyping the various versions of the instructional design (Chapter 5). I assert that metacognitive problem solving encompasses the range of goal setting, planning, execution and feedback; and that whatever metacognitive strategy is mediated, it is instrumental to self-monitoring and self-regulating learning. Any metacognitive strategy should, therefore, not be mediated in isolation from problem solving, but positioned strategically and functionally within all the phases of problem solving. Key to problem solving is its intentionality and directedness, which render it a conscious process with all the educational benefits of monitored- and self-regulated learning.

Metacognitive objectification adds a dynamic dimension to mathematising. The final, most significant conclusion is based on the argument in my local theory that meaning is derived when concepts meet situations in the internal space of the mind. I reason that the understanding on a conceptual level, of the use-and-application of a specific operation within a problem situation, benefits the most from metacognitive objectification. "Objectification" is used here to express the action of transforming a situation such that the person can view, control and manipulate it objectively for mathematical purposes. This assertion was reached during the Evaluation Phase when the outcomes of the assessments of the field-work were analysed. I conclude that the objectification of realistic situations by visualisation in the mind space facilitates the judgement of the mathematical actions that need to be taken to solve the problem.

Measuring on more scales broadens insight into learner performance. In this study, scoring work according to a standard memorandum and also measuring understanding according to a matrix, were complementary in describing learner performance and in shedding light on the dynamics involved in learners' mathematical processing. Prototype V of the design suggests this approach for everyday teaching.

Recommendations for Further Research

It is recommended that the following matters be investigated through research:

- What is the appropriate age at which the cognitive structures of the developing mind are ready to deal with the underlying properties and proofs involved in mathematical concepts? What is the possibility that properties and proofs of mathematical concepts may be learned from a young age at various levels of complexity?
- Can we accept that the ability to visualise and mentally objectify situations is equally present in individuals across gender, race and culture? Is the ability to manipulate mental images generally present? What are the variations in the phenomenon?
- How can we interpret subjective metacognitive reports and how do they contribute to explain objective realities? Can metacognitive knowledge and experiences be modified to narrow the "subjective-objective gap"?
- Can visualisation reliably enhance mathematical problem solving? Could the strategy be detrimental to some learners' mathematical understanding? Can self-regulation be extended to the learning of abstract mathematical concepts?
- What are the dynamics that are involved in the translation of a visual image to a situation that can be engaged with mathematically? What are the conditions under which such translations may take place? Which factors play a role in this transition, such as remembering the image, the availability of the various elements of the problem for mathematical use, and deficient calculation skills?

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APPENDIX A

Journaling my Journey: A Reflective Design Journal (Edited from the Original)

Note 1: *Explanatory notes* (in Italics font) were added in editing the journal.

Note 2: *Table- and figure numberings* were added during the writing up of the thesis.

Note 3: The classification of design products according to their relevant *research phases and design prototypes*, was done while editing the journal.

Entry 1: How I Started Journaling

As a teacher, I sat down at night, reflecting on the day's events in class. I wanted to learn from what happened in class. It took me hours to record events, to make sense of failures and successes and to plan lessons, learning materials and ongoing assessments that would improve learner performance. This made me believe in the value of reflective journaling. The main headings of a journal entry would be:

- *What happened in the Mathematics class? – reflecting on the day's lesson.*
- *How did it go? How did the learners do? – a record of learner responses.*
- *What was observed following this exercise? – analysis of responses.*
- *How can sense be made of the outcomes of the exercise? – a reflection of the possible reasons underlying the patterns.*
- *What was done? – a reflection of how I have handled the didactical encounter.*
- *What can possibly work to help learners overcome this difficulty? – a creative session, tapping resources and planning new ways to mediate understanding.*
- *What next? What must happen now? – a plan for the next step, which could improve on the previous performance (mine and that of the learners).*
- *What lessons were learned?*

Entry 2: Expectations for my Study: 15 November 2011

During the course of this study (2012-2015), I expect insights and solutions to develop and mature. I am prepared to walk many miles in pursuit of the existing arsenal of wisdom. I am a slate where some notes have already been scribbled, ready to be erased for correction or amended to fullness.

Entry 3: Vision for My Journey: 3 July 2012

It's not about the mundane;

It's not about solving the ills of the world;

It's not about developing programmes;

It's not about fads...

    It's about pushing the boundaries;

It's about knocking at the doors of people

    that have not been visited or asked for their opinion;

It's about sitting quietly in the market place

    and smelling their spices and perfumes;

It's about plodding the planet in search of the spikenard

    in altitudes that take my breath away;

It's about capturing the precious drops in the alabaster flask.

Most of all,

It's about anointing those

    who never knew they were meant to rule.

Entry 2: My Learning and Teaching Approach: 30 August 2012.

I know what helped me to understand, which were the factors contributing to breaking through ignorance to understanding. When I encounter concepts that are not firmly rooted, I study them, I chase their meaning, I hunt them down. I sleep, eat and drink ways to make learners understand concepts so well that they are applying them comfortably on a daily basis. In this way, I have found ways of helping them break through to true understanding of what mathematical concepts are about.

I reckon that there are many teachers in SA who feel ashamed of what they understand, but especially of what they do not understand about that which they are supposed to teach to the children. I am optimistic about their prospects of becoming successful teachers – because I have travelled that road myself. What I know, is founded both in hard learning about the way children think and learn, and in my personal journey with children, who started understanding and mastering mathematics.

Entry 5: Seeking Initial Direction: (*Preliminary Phase: Prototype I)*

I know that I need to make visible the metacognition that goes on in the learners' minds when they are busy solving mathematics problems. I know that metacognition must be linked with some requirement of the mathematics learners are doing. My thinking about this is not clear yet. I have to link required proficiencies with demonstrated proficiencies.

From previous studies, experience and observation, I started a list of requirements, modified it many times. I cannot tell how many times I have changed my first basic list of requirements. If anything, this list helped me to shape my own reasoning. Below is the list that eventually made it to the design journal:

Table 32

Required and Demonstrated Cognitive and Metacognitive Proficiencies for Assessment (Preliminary Phase: Prototype 1) Adapted from Feuerstein and Rand, 1974; Feuerstein et al., 2006

	Required proficiencies	**Demonstrated proficiencies**
a.	**Symbols, words and semantics** Know meaning of symbols, phrases and words Use appropriate words, symbols and signs Access appropriate memory stores Relate symbols and signs to what they represent	Explain how instruction is understood Tell how instruction is internalised Draw/write/tell content of mind glossary Draw/write/tell meaning of mind glossary
b.	**Decision making** Understand the situations that require division	Tell the rationale for choosing division
c.	**Self-regulation** Control impulsivity to act hastily/thoughtlessly Pace work according to time allocation	Verbalise self-instruction (input phase) Complete work in due time
d.	**Planning** Foresee the logical procedure to solution Obtain optimum speed-precision ratio	Map the process to be followed Allocate time per sum
e.	**Execution** Know different options of calculation Apply rules of operations Know effect of manipulating symbols Know how entities are derived from others	Follow a finite number of steps Draw/write/tell rule from mind glossary Predict an estimated answer Explain reason for calculation effects
f.	**Precision** Work accurately and check accuracy	Show and explain self-correction
g.	**Pattern formation** Compare for similarities, constancies, changes	Explain pattern formations
h.	**Output appropriation** Think hypothetically	Express and explain cause and effect
i.	**Generalisation** Transfer present knowledge to other contexts	Present situation in multiplicative structures

Entry 6. I reason, there must be a way for learners to demonstrate outwardly the metacognition going on in their minds. I am looking to externalise their internal metacognitive process through the assessment of a mathematics problem.

Table 33

Pre-Intervention Assessment (Preliminary Phase: Prototype I)

Instruction: Do the sum below, showing all your work. Every day, S'bu rides to school, which is 6000m from his house. In how many weeks does he ride 300km?		
Instruction: After you have completed the sum, highlight one answer that you pick from each row of questions below, like this:		
Example: (a) This is my best kind of sum	(b) This is just like any other sum for me	(c) This is my worst kind of sum
1 (a) I know some people hate word sums, but I love them mck> mce	1 (b) I know some people hate word sums, but I do not mind doing them mck> mce	1(c) I know some people hate word sums, and so do I mck> mce
2 (a) When I read the sum, it clearly brings the situation to my mind in a picture form mcas	2 (b) When I read the sum, there are only numbers and sums in my mind, no pictures, sound or movement mcas	2 (c) When I read the sum, I see words and numbers on the paper in front of me, but nothing comes into my mind about it mcas
3 (a) I knew what I had to find out mck > mcgt	3 (b) I think I knew what I had to find out mck >mcgt	3 (c) I did not know what I had to find out mck> mcgt
4 (a) I immediately knew that there were a few steps to take to get the answer mck>mcas	4 (b) At first I thought it was simple, then I saw there were more steps to take mck>mcas	4 (c) When I saw that there were more steps to take, I got confused mck>mcas
5 (a) I knew exactly what to do to get the answer mck>mcas	5 (b) I knew how to start to get to the answer mck >mcas	5 (c) I did not even know where to start mck >mcas
6 (a) I know my answer is right mck>mce	6 (b) I do not know if my answer is right or wrong mck>mce	6 (c) I know my answer is wrong mck> mce
7 (a) I do not worry that I will get it wrong, because I know I am right mce	7 (b) It does not worry me if it is right or wrong. To me is all the same mce	7 (c) I worry that I will get it wrong mce
8 (a) If I found a mistake when I checked, I corrected it, now I am sure everything is right mce> mcgt	8 (b) I could not see if I made mistakes, even when checking mce> mcgt	8 (c) If I found mistakes when checking, I would not know how to correct them anyway mce> mcgt
9 (a) I quickly got it right mce	9 (b) I had to take a long time before I got it right mce	9 (c) No matter how long I took, I never got it right mce
10 (a) Look at my work, it shows how I thought everything out mcas	10 (b) Look at my work, it shows how hard I tried to find the right answer mcas	10 (c) Look at my work above, it shows how confused I was mcas
Metacognitive knowledge mck Metacognitive goals and tasks mcgt	Metacognitive experience mce\ Metacognitive actions and strategies mcas	

Entry 7. This is how I will do a lesson based on a mathematics problem:

Table 34

Post-Assessment Intervention (Preliminary Phase: Prototype I)

Problem statement: Every day, S'bu rides to school, which is 6000m from his house. In how many weeks does he ride 300km?		
Rationale for the intervention	**Intervention**	**Learner response**
The instruction is conveyed into the mind space, using the auditory facility.	Read the word sum out silently so that you can hear the words in your head. Do not speak the words out aloud.	
The situation is constructed in the mind space, making use of the visual and kinaesthetic facilities. This may result in the awareness that the ride is double the distance, because he has to go back home.	Close your eyes and think about S'bu and how he rides from his house to school in the morning. Think where his bicycle is while he is in school. Think how he rides back home after school.	
The facts that inform the task are consciously extracted. This may give rise to awareness of incongruent units of distance and time.	Read the sum slowly again, stressing all numbers and units of measurement like metres, days and so on. Keep a pencil handy and, underline numbers and the units as you read.	Every <u>day,</u> S'bu rides to school, which is <u>6000m</u> from his house. In how many <u>weeks</u> does he ride <u>300km</u>?
The format of the information about the distance must be changed to make both units of measurement similar. (First person plural is used when clues or direct guidance come from the teacher)	We want to see how we can make the units of measurement of distance the same. Shall we convert them all to metre or to kilometre?	
The format of the time information has to be addressed, but must be postponed for later attention, at the appropriate time during the process. This action has to control the impulse to do everything in one go and has to put an action aside in the planning for later attention.	You saw that the time is also in different units, he rides every <u>day</u>, but the question is, "In how many <u>weeks</u> does he ride 300km?" We can work in days for now and calculate the weeks later, before we give the answer.	
Having established the situation and its information with their conversions, the scene is set to make a decision about the operation that will be used. Recapping the instruction within	Read the sum, using the new bits of information instead of the old.	Every <u>day,</u> S'bu rides <u>6km</u> to school and <u>6km</u> back from school (<u>12km</u> altogether). In how many <u>days</u> he ride

the new and ordered mind space may assist the decision.		300km?
Operational decision-making seems to be largely intuitive. It will be guided only if the decision proves to be contra-productive. Multiple subtraction, multiple addition, trial-and-error multiplication and division can all be productive methods, although not all of the same degree of ease.	You can decide now what to do to find the number of days it will take S'bu to ride 300km.	
An opportunity for intermediate checking is created, with an appeal to reason or reasonableness. This may provide the opportunity for learners to correct the calculation or revise the decision in favour of a specific operation.	Now you may check if this makes sense: read it back, not as a question, but as an answer with the number of days that you found. Does this make any sense to you?	S'bu rides 300km to and back from school in 25 days.
Returning to the original question may assist directedness of action or goal orientation. An appeal to logic is once again made.	Now you know that S'bu rides 300km to and back from school in 25 days. So far, so good. But what does the question ask? What is the important word in this question?	In how many <u>weeks</u> does he ride 300km?
The postponed action is retrieved to the appropriate answer to the question.	How are you going to answer the question? (If necessary, prompts may be provided: How many days are in a school week? OR How many days per week will S'bu ride to school?)	
Opportunity for a final check is created, bringing together the question and the answer. An appeal is made on both a cognitive and an affective level.	You can do a final check: read the question first. And now read out your answer. Do you think this answer is a good one to the question? Does it make sense? Won't anybody laugh at you when you give this answer?	Every day, S'bu rides to school, 6000m from home. In how many weeks does he ride 300km? <u>He rides 300km to school in 5 weeks.</u>
The confidence- and logic-strengthening effect that defending one's own right answer or decision has, is facilitated by creating the opportunity to explain the reasons for what was done and decided.	You can explain why you did not use 6000m as it was, or why you calculated how many days he took, when the question was how many weeks he took. You can explain why he rode 12km per day and why you used 5 days per week, not 7 days.	

Entry 8. I can also apply the same metacognitive questions to another type of problem. I can actually use it for all lessons, to see how learners think about their thinking. If I have them complete this questionnaire after every problem they have done, I will be able to monitor and observe progress in their metacognitive processes.

Table 35

Post-Intervention Assessment (Preliminary Phase: Prototype I)

<u>Instruction</u>: Do the sum below, showing all your work. A box with 24 packets of sour balls costs R96. If every packet has 8 sour balls, what is the price of one sour ball?		
<u>Instruction</u>: After you have completed the sum, highlight one answer that you pick from each row of questions below, like this:		
<u>Example</u>: (a) This is my best kind of sum	(b) This is just like any other sum for me	(c) This is my worst kind of sum
1 (a) I know some people hate word sums, but I love them [mck> mce]	1 (b) I know some people hate word sums, but I do not mind doing them [mck> mce]	1(c) I know some people hate word sums, and so do I [mck> mce]
2 (a) When I read the sum, it clearly brings the situation to my mind in a picture form [mcas]	2 (b) When I read the sum, there are only numbers and sums in my mind, no pictures, sound or movement [mcas]	2 (c) When I read the sum, I see words and numbers on the paper in front of me, but nothing comes into my mind about it [mcas]
3 (a) I knew what I had to find out [mck > mcgt]	3 (b) I think I knew what I had to find out [mck >mcgt]	3 (c) I did not know what I had to find out [mck> mcgt]
4 (a) I immediately knew that there were a few steps to take to get the answer [mck>mcas]	4 (b) At first I thought it was simple, then I saw there were more steps to take [mck>mcas]	4 (c) When I saw that there were more steps to take, I got confused [mck>mcas]
5 (a) I knew exactly what to do to get the answer [mck>mcas]	5 (b) I knew how to start to get to the answer [mck >mcas]	5 (c) I did not even know where to start [mck >mcas]
6 (a) I know my answer is right [mck>mce]	6 (b) I do not know if my answer is right or wrong [mck>mce]	6 (c) I know my answer is wrong [mck> mce]
7 (a) I do not worry that I will get it wrong, because I know I am right [mce]	7 (b) It does not worry me if it is right or wrong. To me is all the same [mce]	7 (c) I worry that I will get it wrong [mce]
8 (a) If I found a mistake when I checked, I corrected it, now I am sure everything is right [mce> mcgt]	8 (b) I could not see if I made mistakes, even when checking [mce> mcgt]	8 (c) If I found mistakes when checking, I would not know how to correct them anyway [mce> mcgt]
9 (a) I quickly got it right [mce]	9 (b) I had to take a long time before I got it right [mce]	9 (c) No matter how long I took, I never got it right [mce]
10 (a) Look at my work, it shows how I thought everything out [mcas]	10 (b) Look at my work, it shows how hard I tried to find the right answer [mcas]	10 (c) Look at my work above, it shows how confused I was [mcas]

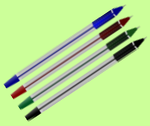
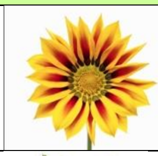
Entry 9: Planning the Intervention: ***(Intervention Phase: Prototype II)***

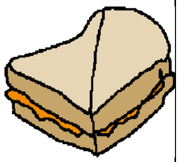
In my practical experiment, I must first see where learners are at, before I can intervene. I must conduct a baseline assessment. I will compile a set of questions that can be used for establishing a baseline in assessment. I will explain the reasons for including them in a rationale afterwards.

Table 36

Pre-Intervention Baseline Assessment Items (Intervention Phase: Prototype II)

CODE NAME ______________________________ **DATE** _____________
Read carefully through each word problem.
Picture the problem for yourself.
Decide which of the three number sentences make the most sense and **tick** them.
Tick the answer that belongs to that number sentence.
Write the full answer to the problem in the last column.

Word problem	**Picture it**	**Number sentence**	**Number answer**	**Full answer**
Example: How many girls will get pens from a box of 200 pens if Tim gives 8 pens to each girl?		a. 8 ÷ 200 b. 200 ÷ 8 ✓ c. 200 x 8	25 ✓ 40 1600	**25 girls will get pens**
1. All sunflowers have 34 petals each. How many sunflowers have 510 petals altogether?		a. 34 x 510 b. 510 ÷ 34 c. 510 x 34	17 340 17340 15	
2. If each bag contains 36 potatoes and sells for R40, how many potatoes are needed to make R1000?		a. 1000 ÷ 36 x 40 b. 1000 ÷ 40 x 36 c. 1000 x 36 ÷ 40	900 1111 900	
3. A bag of medium sized oranges contains 25 oranges. How many oranges are in 5 such bags altogether?		a. 5 x 25 b. 25 – 5 c. 25 ÷ 5	5 125 20	
4. Susan found a total of 120 pips from 15 apples. What is the average number of pips in each apple?		a. 120 x 15 b. 15 ÷ 120 c. 120 ÷ 15	1 800 8 8	

5. A twelfth of 180 Grade 6 learners failed last year. How many passed?		a. 180 ÷ 12 b. 180 ÷ 12 x 11 c. 12 ÷ 180	165 15 90	
6. How many days does S'bu take to ride 156km rides to school and back every day if he lives 6km from school?		a. 6 x 156 b. 156 ÷ 12 c. 156 ÷ 6	13 936 26	
7. Bongi uses 12,5 g butter on her sandwich every day. How many days would she be able to spread her bread from a 500 g block of butter?		a. 12,5 x 500 b. 500 ÷ 12,5 c. 500 + 12,5	625 40 512,5	
8. Bontle pours cold drink from a 3 l bottle into 150 ml cups. How many cups can she pour?		a. 3000 ÷ 150 b. 3 x 150 c. 150 ÷ 3	450 50 20	
9. Vali sells bananas and apples in a ratio of 3:1. If he sold 324 fruits in total, how many were bananas?		324 ÷ 4 x 3 324 ÷ 3 324 ÷ 4	108 243 81	
10. Andrew's Dad is 3 times as old as Andrew is. If Andrew is 12 years old, how old is his Dad?		a. 3 x 12 b. 12 + 12 + 12 c. 12 ÷ 3	36 4 36	
11. For every R5 that Tim saves, his Dad saves twice as much for him. How much money will Tim have altogether once he himself has saved R40?		a. R40 ÷ 5 x 2 b. (2 x R40) + R40 c. 3 x R40	R120 R80 R16	
12. Bibi's Mom is 35 years old; Bibi is 5 times younger than her Mom. How old is Bibi?		a. 35 – 5 – 5– 5– 5– 5– 5– 5 b. 5 x 35 c. 35 ÷ 5	175 7 x 5 7	
13. Lu read 172 pages of a book; Guy read 25 pages of a book. About how many more pages did Lu read than Guy?		a. 172 ÷ 25 b. 172 – 25 c. 175 ÷ 25	6 rem 22 147 7	
14. Five SA tennis players have to play a match each against 6 Namibian players. How many matches will be played altogether?		a. 5 + 6 b. 6 x 1 c. 5 x 6	30 6 11	

15. A rectangle of $28cm^2$ has a length of 7cm. What is the breadth of the rectangle?		a. 28 - 7 b. 28 ÷ 7 c. 7 x 28	196cm $4cm^2$ 21cm	
16. What is the area of a triangle with a base of 16cm and a height of 15cm?		a. 8 x 15 b. 16 x 15 c. (16 x 15) ÷ 2	$120cm^2$ $240cm^2$	
17. Thabo runs 21km in 2 hours and 6 minutes. What is his running speed in meters per minute?		a. 21 x 2 + 6 b. 126 ÷ 21 c. 2100 ÷ 126	≈ 167 m/min 2h 6 min 48m/min	
18. I drive for 5 hours from Soweto to Mangaung at an average speed of 90km/h. What is the distance from Soweto to Mangaung?		a. 90 ÷ 5 b. 5 x 90 c. 90 x 60	450km 5400km 18km	
19. A box of 24 packets of sour-balls costs R96. If there are eight sour-balls in each packet, what is the price of one sour-ball?		a. 24 x 8 ÷ R96 b. R96 ÷ 24 x 8 c. 9600c ÷ 192	R2 R32 50c	
20. I use 35 kWh electricity per week. What is my average daily electricity use?		a. 35 ÷ 7 b. 35 x 7 c. 7 ÷ 35	0,2 kWh 245 kWh 5 kWh	
21. 82 children must stand in groups of 12. How many groups can they form?		a. 12 x 7 b. 82 – 12 c. 82 ÷ 12	≈7 (6 r 10) 84 70	
22. An ant walks a total distance of 12,5m around an anthill. If the ant walks 10 times, what is the perimeter of the anthill?		a. 12,5 ÷ 10 b. 12500 ÷ 10 c. 12,5 + 12,5 + ... add up to 10 times	01,25 1250 125	
23. 9 brownies fit on a polystyrene tray. On how many trays do I pack 144 brownies?		a. 144 ÷ 9 b. 144 x 9 c. 144 – 9	135 16 1296	
24. I have to share 135 balloons between 5 classes. How many balloons does each class get?		a. 5 ÷ 135 b. 135 x 5 c. 135 ÷ 5	27 675 27	
25. I must join a hosepipe of 20m long. Hosepipes are packaged in lengths of 6 m. How many packets must I buy to be able to make the 20m long hosepipe?		a. 20 x 6 b. 20 ÷ 6 c. 24 ÷ 6	4 3 rem 2 120	

Entry 10: Rationale for the baseline/initial test

Type and level of assessment. For this test, I used the type of assessment called baseline testing in CAPS (2011, p. 293), with a view to apply the outcomes in a diagnostic way. Although the test is formally written, it requires mental calculations and therefore falls within the mental mathematics part of what is required at the Grade 6 level. This also explains why the numbers are not at the most advanced level for Grade 6. I used (at most) 3 digit numbers divided by simple 2 digit numbers with or without remainders, and did not go up to 4 digit numbers divided by 3 digit numbers as required by the CAPS at the Grade 6 level (2011, p 212).

Purpose of the baseline assessment. The assessment serves diagnostic purposes. The instrument should enable me to establish whether learners have an understanding on the level of conceptual grasp (Anderson & Krathwohl, 2001), of the use and application (Usiskin, 2012) of the multiplicative structures (Vergnaud, 2009) in mathematising real life situations towards a solution (Van den Heuvel-Panhuizen, 2003). Using the assessment outcomes, I would subsequently establish if and how learners mentally reconstructed the real life situation towards a mathematical representation thereof, both in case of correct and incorrect solutions.

Number sentences. According to the CAPS requirements (2011), learners have to write number sentences to describe problem situations, not only to develop the concept of equivalence, but also to apply various number concepts. In some cases, the number sentence serves to discover or express a rule, for example the area of a triangle (item number 16); the order of operations within the multiplicative structures (items 2 and 5); distribution according to a ratio (item number 9); and the commutative property of number in multiplication (item number 7). The commutative property of number in multiplication is also used to identify equivalent statements, for example in item 1, whereas the inapplicability of this property in division is demonstrated in items 4, 5 and 20. CAPS emphasises the desirability of treating number sentences in an integrated way with daily problems, and not only in an isolated manner. I therefore used examples of situations from the everyday life world known to the typical Grade 6 learner in South Africa.

Multiple choice format. Practice in the multiple choice question-and-answer format is required in CAPS (2011); especially − according to CAPS − because external systemic assessments frequently use this format. In the above assessment, I used a self-invented extended form of the multiple choice format that allows for two steps, a choice of three number sentences and a choice of three possible answers, followed by a consolidated answer that the learners have to create themselves, linking the problem to the numeric solution. The purpose of writing out the full answer in this last column is to provide an opportunity to the learners to judge the reasonableness of their answers.

Multiplicative structures. Division is used as a way of solving many problems at the Grade 6 level. As far as the types of division problems within the multiplicative structures are concerned, learners are required, according to CAPS (2011), to solve problems where they compare two or more quantities of the same kind (ratio) as well as two quantities of different kinds (rate). They must also be able to group or share equally, with or without remainders. Not only simple procedures are to be mastered, but also some complicated procedures, requiring either sequential calculations or combined operations. I included examples of the four different types of division problems in the Grade 6 curriculum, as explained below (Greer, 1992):

Equal groups. In the equal groups type of division, a distinction is drawn between partitive division, where the total is divided by the number of groups, to find the number in each group, also called equal sharing; and quotitive division, where the total is divided by the number in each group to find the number of groups, also called measurement multiplicative structures. Greer (1992) regarded rate as an alternative way of conceptualising equal groups (p. 277).

Multiplicative comparison may be expressed as *"n* times as many as…" (Greer 1992). The multiplicative factor may be regarded as the multiplier, but it can also be conceived of as a many-to-one (or ratio) correspondence (p. 277). In division application of multiplicative comparison is also found in "*n* times less than" (item 12).

A "conceptual switch" is needed for ***Cartesian products*** (Greer, 1992, p. 277), where the members of two different sets have to be grouped in distinctly ordered groups by multiplying set one (*m*) with set two (*n*). In the inverse operation of division, it would not make much sense to alternate the two possibilities. In fact, a division problem would rarely sprout from Cartesian products because of the symmetry between the roles of the two numbers (See question 14 as an example).

In ***rectangular area,*** the sides are integral and the resulting rectangle can be partitioned in squares (Greer, 1992, p. 277). The number of squares then can be physically counted in the rectangular array of *m* rows and *n* columns. Multiplication as a binary operation (including the commutative property of numbers) with its inverse operation − division − becomes highly conceivable when described in this way. With some basic understanding of the properties of a triangle; subsequently, this type of problem can be managed with ease at the Grade 6 level (see items 15 and 16).

Context of problems. I reflected in my assessment instrument on the emphasis of CAPS on financial and measurement contexts, including volume, mass, length, area and distance. Although the formulae involved in area or perimeter of shapes fall out of the scope of the Intermediate Phase, I included the terms in a few items like 15, 16 and 22, aspects which by now should have been mastered conceptually.

Various aspects of the multiplicative structures mentioned in the CAPS. The properties of whole numbers and the number range for multiples and factors are not focus points, although they may feature in some items. The test includes a single example (item 2) of treating a group as a unit. In two cases, rounding off is provided as an option (items 17 and 21), as suggested in CAPS. Division and multiplication by multiples of 10 are included in items 2 and 22. Brackets are used in items 11 and 16 for multiple operations within the same number sentence. The associative property of number is also illustrated in item 16. Calculation techniques are not explored. The test provides an opportunity for the learner to consider the reasonableness of solutions as explained above, by writing down the solution in a sentence. For this assessment calculators will not be used because the calculations should be managed mentally. Even then the learner's computing skills are only a secondary focus point.

The rationale for the test and the test items was probably the best formative experience of the design journal. It helped me to bridge from the literature and policy to practical applications. However, I had to criticise the items and the assessment as a whole and the assessment gave rise to something completely different.

Entry 11. Before this test, my greatest emphasis was on how learners metacognitise. Now, after I have gone through the design of the assessment instrument and the reasoning behind that, I realise it cannot work.

My main interest in this design must be on the mathematics learning. Only in service of mathematics learning, visual imagery must be used to help learners to form division concepts. I want learners to see the situation in their minds and then work on that in their calculations. This test and the metacognitive questionnaire are not going to serve the purpose. There is merit in the lesson planning *(of Prototype I)* but the assessment is not right. Just think about it:

There are too many questions. Yes, it gives an idea of where learners are at, but only that. I will not be able to follow up on each item. I must reduce my items drastically, so that each question becomes a base for concept formation.

The pictures in the questionnaire are contra-productive. If I put a picture there, I restrict their own imagination to form a mental picture.

The options for answers are confusing and are restricting their own initiatives to find answers. I know learners, they will look at the answers first thing and not try to calculate or find their way on their own.

I am too controlling in this test. While I am advocating freedom of imagination in their minds, at the same time I want to control what is going on there. This is not right. I must find another way, completely different to this, and with different emphasis. I have to start with the mathematics problem.

This is how I will teach them to visualise and use a metacognitive strategy to solve mathematics problems:

Entry 12. Mediation of the intervention: "Ke na le plane" (Ek sien 'n plan; I see a way). ***(Intervention Phase: Prototype II)***

Song and dance: "In the virtual space of the mind, *ke na le plane (3x)*."

1. *Start with the problem situation that is described in words.*

Read the problem. I must find out ____

2. *Take this problem situation into the virtual space of your mind.*

Close your eyes. Quick-play the event from start to finish. Keep your eyes closed.

Create a fixed base for the situation.

> Fix everything that will remain the same throughout the event, onto the base. If a name was given in the problem to any of the fixed objects, label them likewise. If a number value was given in the problem to any of the fixed objects, label them likewise. If no number value was given to a fixed object in the problem, label it with a letter of the alphabet, like x, y or z.

Select those objects that will move or change during the event.

> Put the objects that will move or change, in one corner, ready to be fetched. If any of these objects were given a start and/or a finish number value in the problem, label them likewise. If any of these objects were not given a start and/or a finish number value in the problem, label them with a letter of the alphabet like p, q or r.

All the bits of information given in the problem, are now in one screen. Still with your eyes closed, slow-play the event from start to finish. Keep your eyes closed.

> Watch the changes that happen as you slow-play the event. You know some things, but you do not know the answer to the question in the problem. While you slow-play the event, keep on asking that question, until you see a plan to the solution.

3. *Return this problem as a mathematical calculation*

Open your eyes and write down in numbers what you have seen in the virtual space of your mind, checking with the problem that you are using the numbers accurately.

Do your calculations and arrive at an answer, checking for precision. Repeat the question and give your answer, checking if it is a reasonable answer.

Entry 13. Thinking on paper now: In my research proposal, I developed an adapted form of "A metacognitive model for dynamic assessment and intervention" (Kaniel, 2000) to use in the interventions of my research experiment. The interventions take the form of a dynamic assessment event of the type: test-teach-test.

I think, during each intervention there must be three comparable assessment items based on a specific mathematical concept for the day, by which testing and teaching take place iteratively, in three mini-cycles of dynamic assessment. I can only compare the items if I have a set of criteria against which I can reflect all items.

I started with setting a single new item (the "Bela-Bela" item). This gave rise to Prototype III, which I set up in the form of Question and Answer, following the metacognitive model.

Metacognitive Model Applied to Intervention Item (Kaniel, 2000): *(Intervention Phase: Prototype III)*

Dan left Bela-Bela at 04:00 to drive to Mangaung. His business in Mangaung took him 3 hours and he was back home at 21:00. If Bela-Bela is 686km from Mangaung, what was the average speed per hour that he drove?

1. Goal setting <u>Verbalisation</u>

Q: What must I know in the end?

A: At what average speed he was driving.

<u>Mental imagery (general)</u>

Q: What do I see in my mind?

A: Dan leaving very early from home to Mangaung, doing business for 3 hours, driving back and arriving late after dark.

2. Planning <u>Resource management</u>

Q: a. What do I know?

b. What do I need to find out before I can get to the solution?

A: a. Dan left at 04:00, arrived back at 21:00, spent 3 hours in Mangaung. Bela-Bela is 686km from Mangaung.

b. How long did he drive in total?

What is the total distance that he drove?.

Time regulation

Q: What do I have to do?

A: a. Add the *to distance* to the *from distance* to calculate total distance; OR take the distance one-way.

b. Minus the meeting hours from the total hours; this is the total driving time; OR subtract the meeting hours from the total hours; halve the time for one-way.

3. Execution Mental imagery (detail)

Q: What do I do?

A: a. 686km + 686km = 1372km OR use just 686km

b. Count from 04:00 – 21:00 = 17 hours

Driving hours 17 – 3 = 14/ or half of 14 for one-way

c. Divide 1372km by 14 hours/ or 686km by 7 hours

Precision

Q: Check all calculations

a. Calculations accurate

b. Used consistent measures, either two way or one-way

4. Feedback Verbalisation

Q: What is my answer?

A: Dan drove at an average speed of 98km/h..

Justification

Q: Does my solution answer the question sensibly?

A: The question was: At what average speed did Dan drive? My answer is: Dan drove at an average speed of 98km/h. That sounds possible and sensible.

Following this application, I wanted to test this item for the levels of cognitive demand and the dimensions of understanding that it would require, and that gave rise to the design of a matrix of understanding, following the item below:

Entry 14. *(Table 8 was used in Chapter 3 and the same numbering was retained)*

Table 8

Matrix of Mathematical Understanding: Version 1(Intervention Phase: Prototype III)

	Dimensions of understanding (Usiskin, 2012)			
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	**Use / application**	**Skill / algorithm**	**Representation / metaphor**	**Property / proof**
	Factual recall	Factual recall	Factual recall	Factual recall
	Operational efficiency	Operational efficiency	Operational efficiency	Operational efficiency
	Conceptual grasp	Conceptual grasp	Conceptual grasp	Conceptual grasp

Reflecting my previous assessment items against the matrix for plotting an item in terms of the ideal of conceptual understanding, I spotted a serious shortcoming in the items. I have to adapt once again, going back to my design to alter it.

In the spirit of seeking optimistic alternatives, my interventions should bear the logo "Ke na le plane" ("There is a plan" or "I have a plan"; in Afrikaans: "Ek sien 'n plan"). The Sesotho phrase is used where people are faced with a problem and one would come up with an optimistic idea or way of addressing or solving the problem.

Entry 15. I envisage a group of 15 Grade 6 learners in a classroom on a Friday afternoon, 14:00. It is weekend, no homework, no pressure for school work. I am facing a real challenge. We had our first session the previous week, where I explained how we were going to go about the sessions. On that occasion, I did the baseline assessment. Therefore, at a cognitive level I can approximately gauge their present (demonstrated) competency in the multiplicative structures. What I do not know, is how, how much and how effectively they metacognitise.

During this session, I am going to firstly give them a problem, which they have to solve without any interference from my side. I will see their solutions one by one. After that, I will guide them through a similar problem, using metacognition. During this intervention, I want to expose them to self-regulating behaviour including both metacognitive control and metacognitive monitoring, alternating in that order. Metacognitive control is mediated firstly with regard to time regulation and resource management for goal setting and execution of the task and secondly with regard to a metacognitive strategy including mental imagery, verbalisation and justification of the solution. Each control action is monitored for precision, and if the action passes the test of precision, the next action step can be taken. In this view, control and monitoring oscillate within self-regulating behaviour. For the sake of clarity of what I aim for in this intervention session, I have to put it in a figure: *(Figure 9 was used in Chapter 3 and the same numbering has been retained whilst editing the journal)*

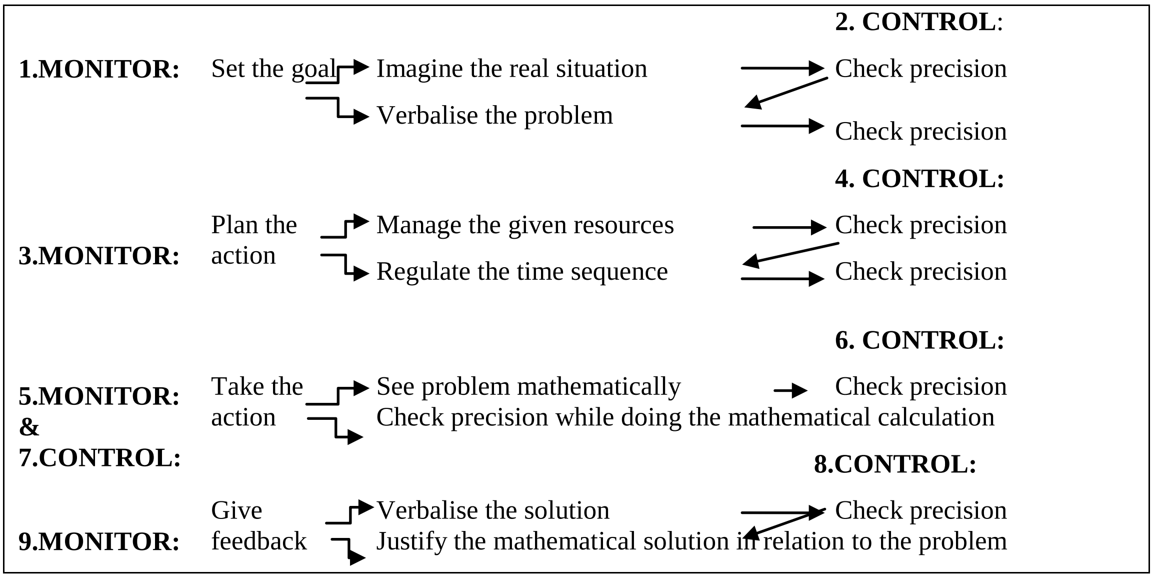


Figure 9. The cyclic and iterative nature of metacognitive control and monitoring. *(Intervention Phase: Prototype III)*

Working with a real life problem is organic. Although it is systematic, it is not rigid. It is not a manufacturing process following exact steps and isolated or distinct phases. There may be repetitions and omissions of certain aspects, as the problem requires. As a general guideline, however, the process would develop along a sequence of goal setting, planning, execution and monitoring. However, this sequence happens in micro cycles, rather than in a single macro cycle. Not only for the bigger problem, but also for every step taken towards solving the problem, one may need goal setting, planning, execution and monitoring. I will dare to jump, and see how it works.

Entry 16. I must operationalise my theory. I realise everything is not so clear cut. I have to reshuffle, cut superfluous and repetitive parts, and most of all, adapt my habitual picture-drawing to make space for "the virtual space of the mind". I realise that, if I start drawing pictures of the situation, I sabotage my own construct. Graphic images must originate in the mind and stay there to be manipulated, before they can be entered on a piece of paper and translated into mathematical terms. I can use the model. *(This figure was used in Chapter 4 and the numbering has been retained)*

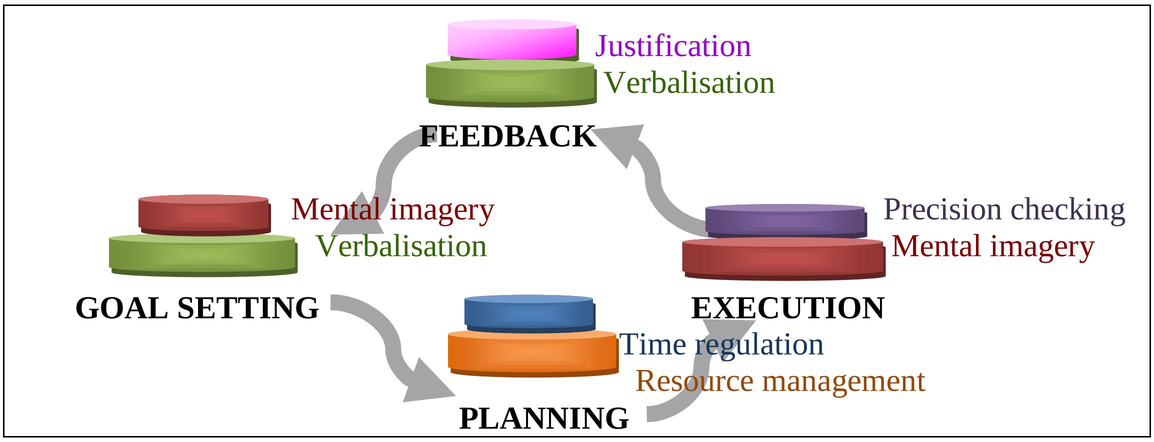


Figure 8. A model for metacognitive problem solving. Based on "An Optimal Model for Decision-making by Individuals" (Kaniel, 2003). *(Intervention Phase: Prototype III)*

Entry 17. I have now tested this model informally on a learner. I was not prepared to guide him properly for what I expected him to do. I do not feel in control. I must set out the process according to the model for problem solving and then add a set of possible responses. I will be prepared for any responses, using this template:

Table 37

A Generic Template for Learner Use with Different Mathematical Items (Intervention Phase: Prototype III)

1	**See the goal**	Read the problem	Tell yourself what you must know in the end.
		Close your eyes. ★	In your mind, fast-play the event from start to finish.
2	**Make the plan**	The base ★	Zoom out and create a base for the event/situation.
			Fix the objects that do not move/change, onto the base.
			Label the fixed objects with names or number values.
		The action objects ★	Put the objects that are going to move/change, in one corner, ready to be fetched later.
			Decide which moving/changing objects have number values.
			For each of those objects that are moving/changing by a number value, set a separate start- and finish number.
3	**Start the action**	The action ★	Fetch your moving/changing objects and place them each in their start positions with their start numbers.
			Start slow-play. Move the object from start to finish.
			Watch if anything else is changing too, while the moving object is in motion.
			Let the moving and the changing objects keep in pace with each other, so that they can finish together.
		The calculations	Open your eyes and write down the numbers you have seen while your eyes were closed.
			Write down all mathematics actions that the events in your mind space cause you to do, subtract, multiply, divide or add.
			Check each calculation for accuracy. If incorrect, re-do.
4.	**Close the problem**	The solution	Read the question in the problem again.
			Read out your solution as if you answer this question.
		Judge your solution	Decide whether this is a reasonable answer to the question.
			If it is reasonable, write it down. If not, check for a mistake in your reasoning or calculation.

At the ★ I explain to them how to use the facility of "the virtual space of the mind".

Table 38

Learner Template Applied to Item and Anticipated Learner Responses (Intervention Phase: Prototype III)

	Stage	Action	Instruction with anticipated learner response
1	**See the goal**	Read the problem	Tell yourself what you must know in the end. *"The average speed at which Dan drove."*
		Close eyes.	In your mind, fast play the event from start to finish.
2	**Make the plan**	The base	Zoom out and create a base for the event/situation. *"A 2-D land area."*
			Fix the objects that do not move/change, onto the base. *"A road and two towns at both ends*
			Label the fixed objects with names or number values. *"Road: <u>686km</u>; town: <u>Bela-Bela</u>; town: <u>Mangaung</u>."*
		The action objects	Put the objects that are going to move/change, in one corner, ready to be fetched later. *"Car and watch."*
			Decide which moving/changing objects have number values. *"Watch."*
			For each of those objects that are moving/changing by a number value, set a separate start- and finish number. *"04:00 and 21:00."*
3	**Start the action**	The action	Fetch your moving/changing objects, place them in their start positions with their start numbers. *"Car at Mangaung; watch at 04:00."*
			Start slow play. Move the object from start to finish.
			Watch if anything else is changing too, while the moving object is in motion. *"Time changes as car moves."*
			Let the moving and the changing objects keep in pace with each other, so that they can finish together. *"Time on the watch changes as car moves. When car stops, time moves on for 3 hours. Then car moves and time changes again."*
		The calculations	Open your eyes and write down the numbers you have seen while your eyes were closed. Check with your written problem that you have them right. *"04:00; 21:00; 3 hours; 686km."*
			Write down every mathematics action that the events in your mind space caused you to do, subtract, multiply, divide or add. *"From 4:00 to 21:00 = 17 hours; 17 hours – 3 hours = 14 hours; 686km + 686km = 1372km; 1372km ÷ 14 hours = 98km/h."*
			Check each calculation for accuracy. If incorrect, re-do.
4	**Close the problem**	The solution	Read the question in the problem again.
			Read the solution as to answer the question. *"Dan drove at 98km/h."*
		Judge your solution	Decide whether this is a reasonable answer to the question. *"That Dan drove at an average speed of 98km/h is reasonable."* If it is reasonable, write it down. If not, check for a mistake and correct it.

And so I have to go on, doing the same thing for more assessment items, like:

a. Dad drives 440km at 80km/h. How long will it take him?

But now, I have to give specific attention to ½ of, ¼ of and so on. In this example, when the learners have reached 400km (80, 160, 240, 320, 400), they will realise that they are left with 40km. Will it be spontaneous for them, as it is for me, to see that 40 is ½ of 80? And will it come spontaneously for them to realise that if 80km is covered in one hour, then 40km is covered in half an hour?

b. It takes us 3¼ hours to drive to our Grandmother in Soweto, which is 325km from our house. At what speed are we driving?

But now, how do they understand the concept of ¼? Can I accept that they take 15 minutes as ¼ of an hour? And then, how do they work out that the ¼ is referring to the 25km? Shall I leave them and see how they do it, then ask those who had it right, what they did, or shall I give them strategy or hints? No, let those who have their own plan, do it and if it seems workable, let me not tamper with it.

Entry 18: Nearing the Intervention: *(Intervention Phase: Prototype III)*

Walk-through: March 2014: Today I am going to test it on two people. The first person has a positive feel for mathematics and is confident. The second is a person with a very bad mathematics history up to now. For her it has been an emotional, turbulent, negative ride.

I went through the Dan/Bela-Bela item with both of them. I was so clumsy, but they helped me make sense from the experience. I was completely surprised to realise how their virtual space differed from mine. How much time they needed, how I created misconceptions by poor communication, and many more.

No, I cannot assume that everybody sees things the way I do. I must be more generic, more open, more free, more flexible. It is about flexible methods after all, not about impressing my ideas onto their minds. Their mind space is not my mind space. Their virtual reality goes in different directions and colours and at different tempos.

The rigid, prescriptive way of envisaging the application of the model with anticipated learner responses *(Table 38*: *Learner Template Applied to an Item and Anticipated Learner Responses)* has to be changed. I must give them more flexible, non-prescriptive guidelines.

Entry 19. Now I have to reflect upon the problem items that I will be using and make sure they are representing the dimensions of understanding and the levels of understanding in my matrix of understanding. Let me be clear about what I mean by every part of the matrix, so that I can judge my items fairly.

Table 39

Matrix of Understanding: Version 2 (Intervention Phase: Prototype III)

	Dimensions of understanding (Usiskin, 2012)			
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	**Use / application**	**Skill / algorithm**	**Representation / metaphor**	**Property / proof**
	Factual recall (e.g. a **rule** that applies in using division) **Uf**	Factual recall (e.g. **steps** in doing a long multiplication calculation) **Sf**	Factual recall (e.g. the **meaning** of mathematical symbols/signs) **Rf**	Factual recall (e.g. **attributes** of the area of a square) **Pf**
	Conceptual grasp (e.g. the **situations** that require division) **Uc**	Conceptual grasp (e.g. the **rationale** for each step in an algorithm) **Sc**	Conceptual grasp (e.g. what the **proportions** in a pie chart refer to) **Rc**	Conceptual grasp (e.g. the square **root** of a square number) **Pc**
	Operational efficiency (e.g. **applying** a specific operation to solve a problem) **Uo**	Operational efficiency (e.g. **calculating** towards a correct answer) **So**	Operational efficiency (e.g. **formulating** an answer) **Ro**	Operational efficiency (e.g. **proving** why a non square number has no square root) **Po**

The analyses of all items (eighteen in total) that had been set for use during the Intervention Phase, including the memoranda for marking and scoring the participant responses, are contained in Appendix B: Analysis of Assessment Items.

Entry 20. I have gone through the first three items, marking the appropriate blocks like this, as I judge what dimension and level of understanding they would need to solve the problem, for example, what is required for solving the following problem?

Item 1: Dan left Bela-Bela at 04:00 to drive to Mangaung. His business in Mangaung took him 3 hours and he was back home at 21:00. If Bela-Bela is 686km from Mangaung, what was the average speed per hour that he drove?

Table 40

Provisional Plotting of Assessment Items on Matrix of Understanding (Intervention Phase: Prototype III)

L	DIMENSION			
E	U/A	S/A	R/M	P/P
V	Fr	Fr	Fr	Fr
E	Oe	Oe	Oe	Oe
L	Cg	Cg	Cg	Cg

When I started with the third example, I realised two things: I should not trust only my own judgement and insight to plot the item. I must reflect this with an expert. But also, why is the property/proof dimension not covered? There is something lacking in the example, if this dimension is not covered. Then I amended the questions: “Explain why your answer is correct”.

The date for interventions is set. I have to decide on a protocol according to which I will do the intervention itself at the research site.

Entry 21. Interruptions: April 2014. While I was writing up the justification and rationale for my model, I had to wonder how I could test learners' real connection with a problem-situation-in-reality-as-a-mathematical-problem. As teachers we sit and think out realistic problems. Are they realistic in the child's world? I want to assume so, but how am I going to make out if my assumptions are correct? "How do I protect myself against being right?"

I have to read further. I come across something I have not been looking for, and it distracts me. While I am reading Darling-Hammond's Session 9 (2003), I react the way I always do when I encounter something of real worth: feel small, insignificant, unworthy to deal with a great theme like metacognition. Those who went before me, had it so clear, they could even explain it to me. Until... I get all fired up, inspired and ready to improve my own work. So, Journal, wait for me. I was interrupted. I used Darling-Hamilton's Session 9 in my writing about metacognition.

Then came the next interruption, this time, Gunther Kress (2009) with "Assessment in the perspective of a social semiotic theory of multimodal teaching and learning". I realised that by suggesting the virtual space of the mind as a metacognitive strategy, I am trying, amongst other things, to facilitate multimodality in meaning-making.

Entry 22: The Intervention: *(Intervention Phase: Prototype IV)*

Intervention protocol: April 2014. At the same time, I was working on my protocol for interventions that had to be submitted to the Ethics committee. I realised I had to adapt the feedback after the dynamic assessment during interventions. I had to make provision for feedback in a multimodal way. What shall I do with the pre-test (baseline assessment)? Keep it? In the first encounter, I will include the dynamic assessment of 3 items, as explained in the Protocol. I will test the questions and then adapt items, approach, questions as needed. For now, this is how I think about it.

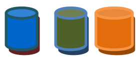
Protocol for the Intervention Phase of the Design Research. *(Intervention Phase: Prototype IVa)*

1. Session 1*

1.1 Introduction of the researcher to the participants and vice versa.

1.2 Coming to an agreement of mutual understanding:

a. Researcher expectations
b. Participant expectations
c. Clarification of "voluntary participation"
d. Explanation of conditions to receive an incentive
e. Explanation of "learner portfolio"

1.3 Baseline assessment of three items, one each of a specific type of problem making use of division: a pen-and-paper test. 

1.4 Explanation of the "virtual space of the mind". Questions and answers.

2. Sessions 2, 3*** and 4******

2.1 Introduction to the sequence of events for the session.

2.2 Formative dynamic assessment of three items per session, each session of a specific type of problem, making use of division. Researcher reads out the question to the group and they make notes. The question is then provided in a written form.

a. First assessment item: Participants do the item without guidance
b. Instructional item: Participants do the item with step-by-step mediation
c. Second assessment item: Participants do the item without guidance

Session 2: 

Session 3: 

Session 4: 

2.3 Audio recording each participant's answers to three sets of questions.

3. Session 5***

3.1 Final summative assessment of three items, a pen-and-paper test.

3.2 Discussion of the provisional findings with the participants.

3.3 Audio recording each participant's conclusive comments.

3.4 Handing out of certificate of attendance to the participants.

Entry 23. I will follow this up in each session with the following questionnaire to have an indication of how they experienced the metacognitive strategy: (*I have not followed up each session, only after completion of the full period, because I realised that participants are still building their experience of the metacognitive strategy.)*

Metacognitive questionnaire. ***(Intervention Phase: Prototype IVa)***

Your code name ________________ Date __________ Session no ________

WRITE DOWN THE ANSWERS TO THE BLUE QUESTIONS
ANSWER THE GREEN QUESTIONS WITH VOICE RECORDING

a. (i) Before you closed your eyes, did you understand the word sum completely?

1. Yes 2. No, because

(ii) With eyes open still, what helped you to understand the problem better, reading it yourself, or listening to it while it was read to you?

1. Reading it 2. Listening 3. Both together

b. (i) Tell me what happened in your mind while your eyes were closed.

(ii) What was easy for you while your eyes were closed?

(iii) What was difficult for you while your eyes were closed?

(iv) Is this a slower or a quicker way for you than drawing a picture?

1. Slower 2. Quicker 3. It became quicker

(v) Was it easy or difficult to let things move or change?

1. Easy 2. Difficult 3. It became easier

c. (i) When you opened your eyes, did you have a plan of what you would write down?

1. Yes 2. No 3. Not completely

(ii) Could you clearly remember the numbers that you had fixed in the virtual space of your mind?

1. Yes 2. No, not at all 3. I was not sure

Entry 24. Finalising the items: May 2014. With all the ideas about the items, I started compiling the items for the baseline assessment, for the three intervention sessions and for the summative assessment. This I did for 15 items. I reflected the items with my supervisor, printed them out and compiled the learner portfolios. *I had all assessment items in this journal as they developed, but as mentioned before, I have extracted them to form an appendix on their own (Appendix B), together with their memoranda for scoring and their analysis according to the matrix of understanding.*

Entry 25. All the practical and administrative arrangements in place, I was eagerly and a bit anxiously looking forward to the real thing... the intervention. What would I find? Am I not chasing a myth? Brace yourself. Just do it. Five weeks in a row... what is that in the bigger scheme of things? Give it your best shot, I encouraged myself. And I declared myself unavailable for anything else until this was over. I would focus completely on the intervention period, not work on chapters for now, making sure that all is ready for the next encounter. Above all, I had to identify what worked, what did not work, what I could do better. After all, this is a Design Research.

Entry 26. The first encounter with participants for baseline assessment: 16 May 2014. I had a group of Grade 6 learners in a classroom on a Friday afternoon, 14:15. The group consisted of 16 participants, 4 boys and 12 girls. It was a challenge, though not as much as I expected. We had our first session. We handled administrative matters and we introduced ourselves. They had questions of clarity, which I answered. I explained how we were going to go about the sessions.

We started with the baseline assessment. I read out the three items and they also received it in hard copy to read it themselves. Very uncertain about the unknown format, they started and wanted to know what method I expected them to do it. I tried to reassure them that it does not matter really how they went about it, they were free to tackle the problem in the way they deemed best. They kept asking me whether they were supposed to +, –, ÷ or x. I encouraged them to follow their own thinking, but I did not interfere or direct.

After they had handed in, I explained to them what "the virtual space of the mind" is as I see it. I explained how the creators of games started by creating a base upon which they fixed objects and icons, which would be movable. They had to close their eyes and create a base with a pond, a river and a farm. These things would stay the same throughout. Five ducklings stood on the edge of the dam, ready to race. They started swimming downstream from the dam. Duckling number one was followed by number three, but overtaken by number four. Number two and five swam together behind number three. (I made a mistake by saying they should put the ducklings in the right hand corner – that was too prescriptive). They all reached the farm. Who reached the farm first? "Number one" was the answer they all gave me.

I asked some general questions about their experience of creating such a scenario in the virtual space of their mind. I also asked a specific question about an aspect of the scenario to each individual. They clearly enjoyed telling me, except for one boy who was hiding his face from me, sitting behind another learner, and said that he could not imagine anything. But maybe he was just trying me, testing the boundaries – he told me what he saw soon after that. I asked:

- Did you see anything while your eyes were closed? (All but this boy said they could, later he joined in)
- What colour was your base? (Mostly green)

- Which side of the river was the pond and which side was the farm? (Some, left to right, others, right to left, no one top to bottom or bottom to top or diagonal)
- What colour were your ducks? (Brown, white, yellow)
- Which duckling was fastest? (No 1, response was immediate, no hesitation)

One boy stayed behind and reported that a purple banana kept on entering the virtual space of his mind, wanting to play tennis. I explained to him how bugs in games do the same and that it was in our own power to chase bugs out of the space, because they were hindering and not helping. He had to take mind control of the purple banana.

I must remember to say that to the rest of the group also. I should have asked them which duckling was the slowest – and see if they dared to say: uncertain. All-in-all, I can say with certainty that all of them, without exception, were able to visualise, to manipulate objects in their mind space and to "see" how things move and change.

I explained to them that we were going to start in the following sessions to use the virtual space of the mind to help us coping with word sums. Only then I did not know what I know now, how problematic their coping with word sums was really. I was encouraged by their skills to imagine a virtual space and to manipulate objects within that space, but I became doubtful when I saw their work. Would I ever be able to bring these two together? They handed in their scripts. At home, I captured all they had written, for marking, scoring and evaluating their understanding. *All of this, together with those of the next sessions, are contained in Appendix C.*

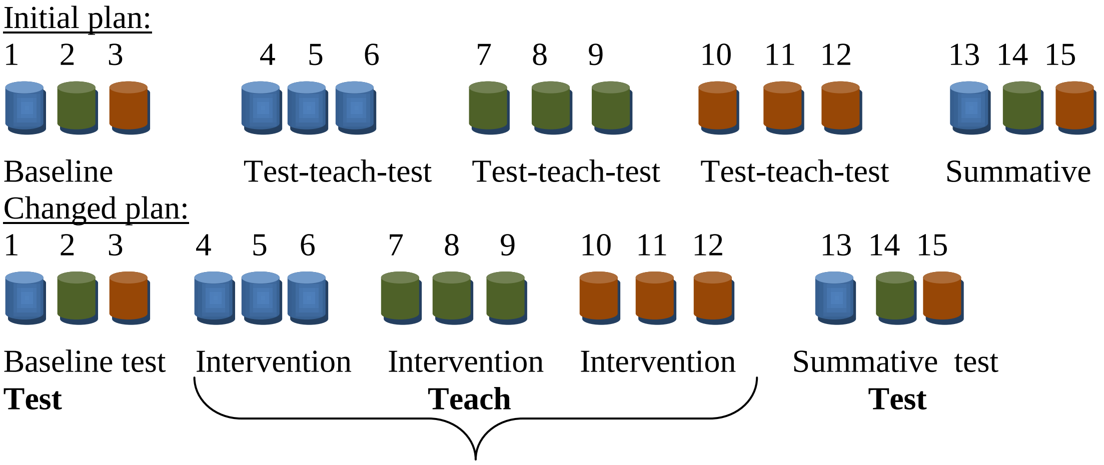
Entry 27. Intervention plan change: 19 May 2014. After this first session, when I reflected upon the little swimming duckling exercise, I thought I should change my planned protocol. I wanted to do my sessions like this:

The changed protocol for the interventions. ***(Intervention Phase: Prototype IVb)***

SESSION 2 (first intervention session)

Following the baseline assessment (Session 1) the concept was changed slightly:

According to the evaluation of Prototype III, the session planning changed as follows:



My reasons are: The metacognitive strategy of imposing mathematical ideas and processes onto and into the virtual space of the mind would take some time to practice in. Being faced by the three types of division sums, money (with decimals) area and speed, together in the baseline assessment was clearly overwhelming. I shall work out item 1, 4, 5 and 6 for the first session with the metacognitive strategy.

This is my plan: Item 1 (which they have done during the first session) with close mediation, slowly, step by step applying the strategy. Item 2 of the new money items, with close mediation. Item 3 just starting them off. Item 4 leaving them on their own. I must give them a generic plan for the strategy, which they can keep handy and use for further problems. The first session needs closer mediation than the rest.

Entry 28. The second encounter and first intervention: 23 May 2014. Session 2 went according to plan, up to a point. The participants were restless, as it certainly could be expected on a Friday afternoon just after two o'clock. When I had their attention, I went through the blue item of the baseline test, the bread-and-milk item. I slowly guided them through a process of seeing the problem in the virtual space of their minds, according to the model for metacognitive problem solving, as I had planned. When they opened their eyes and started working on "what they saw" when their eyes were closed, I just did not feel like interrupting them again, because they had come to rest.

Then we moved on to the three blue (money problem) items of Session 2. I did the same, and they did their writing work after they had opened their eyes. But I think I made a mistake – at least according to my own planning, maybe it was not really a mistake: I did not go through any of the instructions that I had planned for AFTER the experience inside the virtual space of the mind, that is after they had opened their eyes. I felt that I was going to interrupt them while they were working.

Firstly, in my planning for the next session, I will go all the way. I shall work out the items now, one by one and make sure that I make the last part possible, not to be seen as interruption of thinking, but as support for thinking and reasoning.

Secondly, I am going to plan the session such that I demonstrate the technique with item 2 (green, area problem) of the baseline assessment, then go on to walk closely with them through items number 7, 8 and 9, all green (area) items.

Thirdly, I am going to introduce them to the steps of goal setting, planning, executing and reflecting. When I am demonstrating, I have to point out that there is an order and method in what we are doing.

Lastly, I will see my expert confidante this week to reflect upon my judgement of Session 1 and 2 results, the way I judged them against the matrix of understanding. Then I will be able to journal my impression of what is happening with the results. For now, I am going to work out, step by step, how I will take them through the next three green (area) problems, items 7, 8 and 9, starting with a demonstration of item 2. I must prepare a visual prompt for the area items.

I went on, capturing and marking the work. I was pleasantly surprised and cautiously optimistic about what I observed. There was a slight improvement in scores as well as in understanding. Having said this, I was uncertain about my own judgement about understanding. I had to get advice before I could go on.

Entry 29. Ke na le plane: 24 May 2014. I travelled to my expert confidante. With wisdom, experience and insight he gave me advice to describe the categories so that I could discern demonstrated understanding as belonging to a specific category. I revised the matrix as well as my previous scoring according to these changes.

Table 41

Matrix of Understanding: Version 3 (Intervention Phase: Prototype IV)

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering which operation to use.	Factual recall Remembering what to do in the algorithm or method.	Factual recall Remembering the correct notation/representation.
	Operational appropriateness Knowing what info to use for the calculation.	Operational appropriateness Knowing how to do the calculation towards the right answer.	Operational appropriateness Formulating answers appropriately in relation to the question.
	Conceptual grasp Understanding how to arrange the relevant elements & operations in the right order to reach the solution.	Conceptual grasp Understanding the reasoning behind the calculation, demonstrated e.g. by fault detection.	Conceptual grasp Understanding how the elements of the problem situation result in the specific type of formulation, ratio, price, number, etc.

Entry 30. I have now formalised the steps as I had planned, of metacognition and added it to each participant's portfolio to refer to each time that we started with a problem. I reckoned, this is shorter, less prescriptive and it allows for freedom of thinking. It is simply a reminder of what they needed to do to get into the habit of working according to the metacognitive model for problem solving.

Table 42

Learner Template: In the Virtual Space of My Mind... "Ke na le Plane" (Intervention Phase: Prototype IV)

1	**See the goal**	Read the problem	What is the question?
		Close your eyes	In your mind, say what you are looking for.
2	**Make the plan**	The base	Create a base.
			Fix all the information onto the base.
3	**Start the action**	The action	Start doing.
			Go on doing.
			Finish doing.
		The calculations	Open your eyes. Write down the numbers you have seen while your eyes were closed.
			Write down every mathematics action that the events in your mind space caused you to do, subtract, multiply, divide or add.
			Check each calculation for accuracy. If incorrect, re-do.
4.	**Close the problem**	The solution	Read the question in the problem again, imagine you are asking it.
			Read out your solution as if it is an answer to your question.
		Judge your solution	Does that make sense when you say that out? Decide whether this is a reasonable answer to the question.
			If its reasonable, write it down. If not, check for mistakes.

Entry 31. The third encounter with participants and second intervention: 30 May 2014. We did area today. I refuse to teach any mathematics. I am only giving metacognitive strategy. And so I did. I said to them that I would illustrate to them how a virtual space is built up in my own mind. I read out the problem situation to them, much like the one they had to do in the baseline assessment, item 2. I read the problem out to them: “A soccer field that is 80m long and 40m wide, is divided in square blocks of 5m long and 5m wide. How many blocks can be metered out?”

Table 43

Application to Item: In the Virtual Space of My Mind “Ke na le Plane” (Intervention Phase: Prototype IV)

<table>
<tr><td colspan="4">For entrepreneurs day, the soccer field, 80m long and 40m wide, has to be divided in square blocks of 5m long and 5m wide for the stalls. How many blocks can fit into the soccer field?</td></tr>
<tr><td rowspan="2">1</td><td rowspan="2">See the goal</td><td>Read the problem</td><td>What is the question? How many blocks can be metered out on the soccer field?</td></tr>
<tr><td>Close eyes</td><td>In your mind, say what you need to know. Number of blocks on the field.</td></tr>
<tr><td rowspan="2">2</td><td rowspan="2">Make the plan</td><td rowspan="2">The base</td><td>Create a base. I held up the plain grey hardboard as my base.</td></tr>
<tr><td>Fix all the information onto the base. I wrote 80m on the long side and 40m on the short side. I fixed one small block onto the base, writing 5m on one side and 5m on the other side.</td></tr>
<tr><td rowspan="3">3</td><td rowspan="3">Start the action</td><td rowspan="3">The action</td><td>Start doing. I started clicking small squares onto the larger area, on the long side, until there were 16 fitting into the 80 m. All along I kept counting: 5, 10, 15, 20, 25...</td></tr>
<tr><td>Go on doing. I started clicking the second row, but argued that it would be the same as the first. So how many such lines will I be filling downwards? Counting: 5, 10, 15, 20.</td></tr>
<tr><td>Finish doing. If I went on, I would have filled the space with 16 blocks to the side, 8 times over. But I did not. I only showed how it would fill up.</td></tr>
<tr><td></td><td></td><td>The calculation</td><td>Open your eyes. Write down the numbers you have seen while your eyes were closed. Then I left them. I did not write down anything.</td></tr>
</table>

I started with a plain grey hardboard as a base in the true proportions of a problem situation, which I had prepared with small magnets at the back. I also came ready with squares in another colour, in the true proportions. These small squares had magnets fixed to their backs as well. I went through the steps slowly.

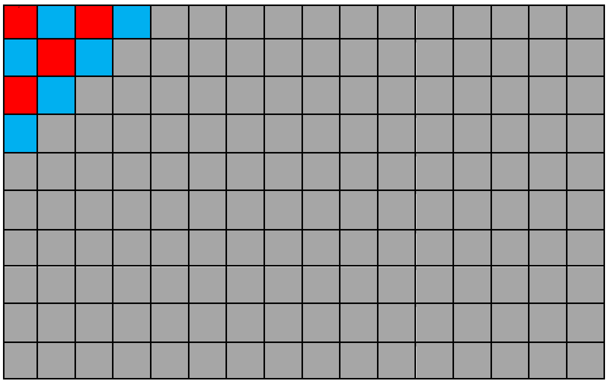


Figure 34. Visual prompt planned for mediation of visual imagery in area items. (Intervention Phase, Prototype IV)

Should I have written down something? Would they understand better if I wrote out how I would calculate the sum? But I did not. No, I have intended not to teach mathematics, but to leave it to them to do things their own way. I am not turning back. I want to see the effect of visualising on their concept formation, not the effect of my teaching mathematics. It is fine like this, I will continue to do so.

I am making this comment in retrospect, but I fit it under this heading, because it belongs here: There is a sharp decrease in scores and understanding with an item containing a similar problem that I set for the summative assessment. It was not that I haven't been spending time to clarify how one would go about such a problem in the virtual space of the mind. I consulted my supervisor about it. She reckoned that items 2 and 17 are counter-intuitive. Items 2, 8, 14 and 17 are similar items in my view. Do I think that something made sense to them on the day, but was too fluid to last? Is it the item's problem or the problem of fragile insight that has not yet crystallised? If they could master the concept then, even if it was temporarily, I would not regard it as a flash in the pan, anyway. I learned this from Professor Feuerstein. He said if children manage something at any point, that should be regarded as potential.

Entry 32. The fourth encounter with participants and the third intervention: 4 June 2014. I worked so much on the marking, scoring and capturing that I could not journal as much as I would want to do. However, there is much to be reported and to be reflected upon. Firstly, I want to write down all the lessons I have learned in terms of mistakes made, and secondly, in terms of what I see as successes. And finally, I would want to set up the ideal intervention that I will use during the last session.

Lessons learned: mistakes

1. I have not scrutinised my items enough.

- Item 3 was too complicated. I have to adapt the speed items to Grade 6 level. I get the impression they have not done speed up to now.
- I have overlooked the misprint 10mm^2 in item 7 that was meant to be 100mm^2 or 1cm^2. I had to correct that in class and some participants did not pay attention to correct it. I ensured in marking and scoring that it did not have a negative impact on their scores and my interpretation of their understanding.
- I created confusion in item 8, by stating that the tiles are 30cm square, whereas I meant they are 30cm x 30cm. I only realised this mistake once I was marking and had reflected the correct expression with my supervisor. The result was that I had to accommodate 30cm^2 as well as 900cm^2 as elements of the calculations.

2. I forced the "Property-Proof" dimension of understanding by artificially adding "Prove that your answer is correct". This I have removed from my memorandum.

3. I allowed them to have all items at once, which resulted in them not doing the metacognitive drill for the subsequent items, but just carry on the way they normally

do it. Most of them did it for the first item only. I will let them have item by item, to get them into the habit of doing the metacognitive drill before they calculate.

Lessons learned: successes

1. During the previous session, I have explained to them the metacognitive cycle: goal setting, planning, action and reflection. That was good.

2. The demonstration of the area item worked well. They saw what I meant by the virtual space of the mind.

3. It was good to demonstrate and teach on one of the baseline assessment items each time and then let them do the new items. The new items were then all "uncontaminated" by my instruction.

4. It was good to reflect my judgement about the indicators of the participants' understanding with an expert before I went too far with it. I gained valuable insight.

The ideal intervention. For the fourth intervention, I will do the ideal intervention. We are going to do speed items and this is what I plan to do:

I will illustrate how I use the virtual space of the mind to help me to do an item similar to item 3 of the baseline assessment, which was a speed problem. At the same time, point out when the goal is set, how the plan is made, where the action takes place (in the mind and on paper) and that the answer is evaluated or checked.

It took the school bus 4½ hours from school to the Grade 6 camping site which is 328km away. On our way, we stopped for 30 minutes at a Quick Shop. Taking only the time that the bus was really driving, at what average speed did the bus go?

Memorandum for marking Speed: 328km (1) ÷ 4h (1) = 82km/h (2) **[4]**

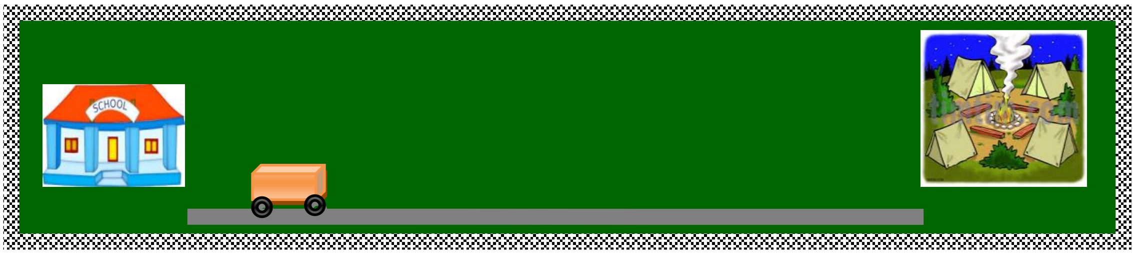


Figure 35. Visual prompt planned for mediation of visual imagery in speed items. (Intervention Phase, Prototype IV)

I will provide item 10 on a lined page, read it out with the group and allow time for them to do it. The same procedure is followed for items 11 and 12. Items 10, 11 and 12 must be reworked, because I vaguely realise they have not done speed yet and I cannot complicate the items. We have to stick to the basics of speed. The fourth session on 6 June went well, I marked their work and I was delighted by the results.

Entry 33. The fifth encounter for summative assessment: 15 June. Then came 13 June. I prepared a little party to celebrate the conclusion of the sessions and participants' willingness to assist in the research, went to school, all fired up for the final encounter, when disaster struck. Afterwards, I wrote an e-mail to my supervisor:

"14 June 2014

Dear .... (Supervisor)

I have officially completed my fieldwork, except for some wrap-up next week.

I have gone through the post-assessment very carefully. I was wary of the possibility that I wanted to see improvement and that I would therefore be biased in marking and assessing understanding. What also bothered me, was whether I have marked consistently, and whether I have judged their understanding the same way as I have come to do it now. So, I went back, a bit scared, to make sure, and it proved worthwhile. I had to adapt initial scores and rating of understanding slightly and I have a comparable set of outcomes now.

Yesterday was not a good day for the participants. I went there at 14:00 as usual, enthusiastic and excited, just to find 13 of them sitting detention on the bare cold floor of the school hall under threats and scolding. They had a book each in which they had to complete work as punishment. It looked like they had huge parts of their books to copy before 15:00 otherwise they could not be released.

The sweet art teacher in whose classroom we meet, came up to me when she saw me at the door and undertook to talk to the "big" teacher in charge of this exercise. By grace alone she granted me an audience and said she would send the participants. After a long time, they arrived in the classroom. They rushed their test, and grabbed the punishment work to complete it. They did not go through the drills as we normally do, the metacognitising and imagining. They were so stressed up. She patrolled at the windows of the classroom.

Needless to say, their tests do, in my view, not reflect the natural outcome of what we have accomplished in this period. There is a little difference, upwards from the baseline assessment to this disastrous summative assessment. When I have all the data ready, you can tell me whether it is significant or not."

My supervisor phoned me, upset. She reckoned I should not settle for this outcome, but re-do the summative assessment. I started preparing new items, although anxious and discouraged. But I have learned many years ago as a nurse, there is only one time to act when a patient is bleeding internally – now.

Entry 34. Sixth encounter with participants and the re-do session: 20 June 2014.

Returning to the school, I was uncertain of what I would find. The aftercare teacher was extremely helpful and organised the participants to come, happily so and not even grumpy that they had to sit for another session.

Back home, I marked, was overjoyed, worked fervently on the final statistics and arranged a meeting with my supervisor to reflect the outcomes. I had a data set! A dream come true. Am I really on this side of the study?

Entry 35: The ideal intervention. (*Evaluation Phase: Prototype V)*

All along, throughout the development of the design, I have consistently written down which aspects were valuable, which products were promising and what I needed to improve. These reflections filtered out to be design principles upon which the ideal design would be based. *The combined lists of principles are mainly incorporated in Chapter 5 and applied in Appendix D: The Didactical Design*.

APPENDIX B

Analysis of Assessment Items

Session 1: Baseline Assessment

Item 1

Mom kept all her grocery receipts over a period of three months. She found that the total amount that she spent on milk was R1 365 and the amount spent on bread was R1 073.70. What was the average amount per month that she spent on milk and bread together?

Memorandum for scoring **[4]**

R1 365 ÷ 3 = R455 (1)

R1 073.70 ÷ 3 = R357.90 (1)

R 455 + R 357.90 = R812.90 per month (2)

OR R1 365 + R1 073.70 = R2 438.70 (2)

R2 438.70÷ 3 = R812.90 per month (2)

OR **own method of choice**

Dimensions and levels of understanding **[9]**

Table 44

Analysis Item 1: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	**Use / application**	**Skill / algorithm**	**Representation / metaphor**
	Factual recall Remembering how to find total and average. **(1)**	Factual recall Remembering how to add and divide. **(1)**	Factual recall Remembering the notation for money and average. **(1)**
	Operational appropriateness Deciding which amounts to add and to divide. **(1)**	Operational appropriateness Adding and dividing correctly. **(1)**	Operational appropriateness Formulating answer in Rand/cent per month. **(1)**
	Conceptual grasp Understanding the order in which to arrange elements and order of operations to reach the solution. **(1)**	Conceptual grasp Understanding reasoning for calculations, e.g. by the right place value for the decimal number. **(1)**	Conceptual grasp* Understanding that answer is rate as quotient of the dividend/divisor, expressed as cost/time relation **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Item 2

For the school entrepreneurs' day, the rugby field of 110m x 80m will be divided up into equal blocks of 5m x 5m to set up the stalls. How many stalls will there be on the rugby field?

Memorandum for scoring **[4]**

110m x 80m = 8 800m^2 (1)

5m x 5m = 25m^2 (1)

8 800m^2 ÷ 25m^2 = 352 blocks (2)

OR 110m ÷ 5m = 22 (1)

80m ÷ 5m = 16 (1)

22 x 16 = 352 blocks (2)

OR **own method of choice**

Dimensions and levels of understanding **[9]**

Table 45

Analysis Item 2: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering to divide the smaller area into the bigger area. **(1)**	Factual recall Remembering how to multiply and divide. **(1)**	Factual recall Remembering the notation for area and dimensions. **(1)**
	Operational appropriateness Deciding on the dimensions to use to enable calculation(s). **(1)**	Operational appropriateness Understanding how to multiply and divide correctly. **(1)**	Operational appropriateness Formulating the calculations in m and m^2 and the answer correctly as blocks or stalls. **(1)**
	Conceptual grasp Understanding the alternative ways of arranging these elements in an order that would result in the appropriate solution. **(1)**	Conceptual grasp Understanding e.g. the role of the divisor and the dividend, or dividing and multiplying by multiples of 10. **(1)**	Conceptual grasp* Understanding that bigger area divided up into smaller areas, amount to the number of blocks. **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Item 3

It took the school bus 5 hours from school to the Grade 6 camping site which is 351km away. On our way we stopped for 30 minutes at a Quick Shop. Taking only the time that the bus was really driving, at what average speed did the bus go?

Memorandum for scoring **[4]**

Time: 5h - ½h = 4½h (2)

Speed: 351h ÷ 4½h = 78km/h (2)

OR own method of choice

Dimensions and levels of understanding **[9]**

Table 46

Analysis Item 3: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering which operation to use to calculate speed. **(1)**	Factual recall Remembering how to subtract and divide. **(1)**	Factual recall Remembering the notation for time, distance and speed. **(1)**
	Operational appropriateness Deciding which elements to select to divide. **(1)**	Operational appropriateness Subtracting and dividing correctly. **(1)**	Operational appropriateness Formulating the answer correctly in km/h. **(1)**
	Conceptual grasp Understanding how to arrange the relevant elements and operations in the right order to reach the solution. **(1)**	Conceptual grasp Understanding the reasoning behind the calculations, demonstrated e.g. by calculating the driving time. **(1)**	Conceptual grasp* Understanding rate as a relation between distance and time and the role of time (denominator) and distance (numerator) in speed. **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant’s demonstrated understanding in the full context of the item.

Session 2: First Intervention

Item 4

For the upcoming soccer tour, coach John received a donation of R25 000 for the school's first team. 15 players were selected for the team. How much money is available to buy gear for each member of the team if coach holds back R2 000 for the transport costs and R50 to tip the bus driver?

Memorandum for scoring **[4]**

R25 000 – R2 050 = R22 950 (2)

R22 950 ÷ 15 = R1 530 (2)

OR **own method of choice**

Dimensions and levels of understanding **[9]**

Table 47

Analysis Item 4: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering how to find difference and average. **(1)**	Factual recall Remembering how to subtract and divide. **(1)**	Factual recall Remembering the notation for money and average. **(1)**
	Operational appropriateness Deciding which amounts to subtract and divide. **(1)**	Operational appropriateness Subtracting and dividing correctly. **(1)**	Operational appropriateness Formulating answer in Rand/cent per player. **(1)**
	Conceptual grasp Understanding the order in which to arrange elements and order of operations to reach the solution. **(1)**	Conceptual grasp Understanding reasoning for calculations, e.g. by the right place values when dividing. **(1)**	Conceptual grasp* Understanding that answer is rate as quotient of the dividend/divisor, expressed as cost/player relation **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Item 5

The principal of the school had the team logo printed on a towel for each of the 15 members who will be going on the tour. The price of an unprinted towel was R95 each. The total amount for the towels plus the printing was R1 725. What did the printing cost per towel?

Memorandum for scoring **[4]**

R15 x R95 = R1 425 (2)

R1 725 – R1 425 = R300 (1)

R300 ÷ 15 = R20 (1)

OR **own method of choice**

Dimensions and levels of understanding **[9]**

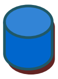
Table 48

Analysis Item 5: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering how to find difference, total and average. **(1)**	Factual recall Remembering how to add, subtract and divide. **(1)**	Factual recall Remembering the notation for money and average. **(1)**
	Operational appropriateness Deciding which amounts to add, subtract and divide. **(1)**	Operational appropriateness Adding, subtracting and dividing correctly. **(1)**	Operational appropriateness Formulating answer in Rand/cent per towel. **(1)**
	Conceptual grasp Understanding the order in which to arrange elements and order of operations to reach the solution. **(1)**	Conceptual grasp Understanding reasoning for calculations, e.g. by the right place values when dividing. **(1)**	Conceptual grasp* Understanding that answer is rate as quotient of the dividend/divisor, expressed as cost/towel relation **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Item 6

 A parent sponsored R800 for caps for each member of the team of 15 plus the two coaches. What was the price per cap if there was R43.50 left from the donation after paying the caps?

Memorandum for scoring **[4]**

R800 – R43.50 = R756.50 (2)

R756.50 ÷ 17 = R44.50 (2)

OR **own method of choice**

Dimensions and levels of understanding **[9]**

Table 49

Analysis Item 6: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering how to find difference, total and average. **(1)**	Factual recall Remembering how to add, subtract and divide. **(1)**	Factual recall Remembering the notation for money and average. **(1)**
	Operational appropriateness Deciding which amounts to add, subtract and divide. **(1)**	Operational appropriateness Adding, subtracting and dividing correctly. **(1)**	Operational appropriateness Formulating answer in Rand/cent per cap. **(1)**
	Conceptual grasp Understanding the order in which to arrange elements and order of operations to reach the solution. **(1)**	Conceptual grasp Understanding reasoning for calculations, e.g. by the right decimal place values when dividing. **(1)**	Conceptual grasp* Understanding that answer is rate as quotient of the dividend/divisor, expressed as price/cap relation **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Session 3: Second Intervention

Item 7.

Our teacher gave us a sheet of block paper with 308 blocks of 1cm^2 each. The sheet is 14cm wide. How long is it?

Memorandum for scoring **[4]**

308cm^2 (1) ÷ 14cm (1) = 22 (1)cm (1)

OR **own method of choice**

Dimensions and levels of understanding **[9]**

Table 50

Analysis Item 7: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering to divide the area by the dimension of width. **(1)**	Factual recall Remembering how to divide. **(1)**	Factual recall Remembering the notation for area and dimensions. **(1)**
	Operational appropriateness Deciding on the dimensions to use to enable calculation(s). **(1)**	Operational appropriateness Understanding how to divide correctly. **(1)**	Operational appropriateness Formulating the calculations in cm^2 and cm and the answer in cm. **(1)**
	Conceptual grasp Understanding that the single small square provides the unit for calculating towards the solution. **(1)**	Conceptual grasp Understanding e.g. the place values in dividing. **(1)**	Conceptual grasp* Understanding that the bigger area divided by one dimension, results in the other dimension. **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Item 8

Grandmother requested Squarit to tile her dining room floor. The floor is a rectangular shape of 390cm x 450cm. If a tile is 30cm square, how many tiles will be needed to tessellate the floor?

Memorandum for scoring **[4]**

390cm x 450cm (1) = 175 500cm^2 (1)

175 500cm^2 ÷ 900cm^2 (1) = 195 tiles (1)

OR 390cm ÷ 30cm = 13 (1)

450cm ÷ 30cm = 15(1)

15 x 13 = 195 tiles (2)

OR **own method of choice**

Dimensions and levels of understanding **[9]**

Table 51

Analysis Item 8: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering to divide the smaller area into the bigger area. **(1)**	Factual recall Remembering how to multiply and divide. **(1)**	Factual recall Remembering the notation for area and dimensions. **(1)**
	Operational appropriateness Deciding on the dimensions to use to enable calculation(s). **(1)**	Operational appropriateness Understanding how to multiply and divide correctly. **(1)**	Operational appropriateness Formulating the calculations in cm and cm^2 and the answer correctly as tiles. **(1)**
	Conceptual grasp Understanding the alternative ways of arranging these elements in an order that would result in the appropriate solution. **(1)**	Conceptual grasp Understanding e.g. the role of the divisor and the dividend, or dividing and multiplying by multiples of 10. **(1)**	Conceptual grasp* Understanding that bigger area divided up into smaller areas, amount to the number of tiles. **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant’s demonstrated understanding in the full context of the item.

Item 9

The government of the Northern Cape bought a rectangular area of 204km^2 for nature conservation. The short side of the area is 12km long. How long will the fence be that camps the area in?

Memorandum for scoring **[4]**

204km^2 ÷ 12km = 17km (2)

12km + 12km + 17km + 17km = 58km (2)

OR **own method of choice**

Dimensions and levels of understanding **[9]**

Table 52

Analysis Item 9: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering to divide the area by the dimension of width and add dimensions for perimeter. **(1)**	Factual recall Remembering how to divide and add. **(1)**	Factual recall Remembering the notation for area, perimeter and dimensions. **(1)**
	Operational appropriateness Deciding on the dimensions to use to enable calculation(s). **(1)**	Operational appropriateness Understanding how to divide and add correctly. **(1)**	Operational appropriateness Formulating the calculations in km^2 and km and the answer in km. **(1)**
	Conceptual grasp Understanding which dimensions enable calculation of perimeter. **(1)**	Conceptual grasp Understanding e.g. the place values in dividing. **(1)**	Conceptual grasp* Understanding that the bigger area divided by one dimension, results in the other dimension. **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Session 4: Third Intervention

Item 10

Dan left Bela-Bela at 04:00 to drive to Mangaung. He arrived in Mangaung at 11:00. If Bela-Bela is 686km from Mangaung, what was the average speed per hour that he drove?

Memorandum for scoring **[4]**

11h – 4h = 7h (1)

686km ÷ (1) 7h = <u>98km/h</u> (2)

OR own method of choice

Dimensions and levels of understanding **[9]**

Table 53

Analysis Item 10: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	<u>Factual recall</u> Remembering which operation to use to calculate speed. **(1)**	<u>Factual recall</u> Remembering how to subtract time and how to divide. **(1)**	<u>Factual recall</u> Remembering the notation for time, distance and speed. **(1)**
	<u>Operational appropriateness</u> Deciding which elements to select to divide. **(1)**	<u>Operational appropriateness</u> Subtracting and dividing correctly. **(1)**	<u>Operational appropriateness</u> Formulating the answer correctly in km/h. **(1)**
	<u>Conceptual grasp</u> Understanding how to arrange the relevant elements and operations in the right order to reach the solution. **(1)**	<u>Conceptual grasp</u> Understanding the reasoning behind the calculations, demonstrated e.g. by calculating the driving time. **(1)**	<u>Conceptual grasp*</u> Understanding rate as a relation between distance and time and the role of time (denominator) and distance (numerator) in speed. **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Item 11

Thembi stays in Nelspruit and wants to visit her aunt in Soweto. She takes the 08:00 bus in Nelspruit. The bus goes at an average speed of 75km/h. If Soweto is 450km from Nelspruit, at what time will Thembi arrive in Soweto?

Memorandum for scoring **[4]**

450km ÷ (1) 75km/h (1) = 6h (1)

08:00 + 6h = 14h (1)

OR **own method of choice**

Dimensions and levels of understanding **[9]**

Table 54

Analysis Item 11: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering which operation to use to calculate driving time. **(1)**	Factual recall Remembering how to divide and add time. **(1)**	Factual recall Remembering the notation for time, distance and speed. **(1)**
	Operational appropriateness Deciding which elements to select to divide. **(1)**	Operational appropriateness Dividing and adding time correctly. **(1)**	Operational appropriateness Formulating the answer correctly in hours. **(1)**
	Conceptual grasp Understanding how to arrange the relevant elements and operations in the right order to reach the solution. **(1)**	Conceptual grasp Understanding the reasoning behind the calculations, demonstrated e.g. by adding the driving time. **(1)**	Conceptual grasp* Understanding rate as a relation between distance and time and the role of time (denominator) and distance (numerator) in speed. **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Item 12

Petros takes 3 hours to fly from Johannesburg to Cape Town. If the plane flies at an average speed of 616km/h, how far is Cape Town from Johannesburg?

Memorandum for scoring **[4]**

616km/h (1) x 3h (1) = 1 848 (1)km (1)

OR **own method of choice**

Dimensions and levels of understanding **[9]**

Table 55

Analysis Item 12: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering which operation to use to calculate distance. **(1)**	Factual recall Remembering how to multiply. **(1)**	Factual recall Remembering the notation for time, distance and speed. **(1)**
	Operational appropriateness Deciding which elements to select to divide. **(1)**	Operational appropriateness Multiplying correctly. **(1)**	Operational appropriateness Formulating the answer correctly in km. **(1)**
	Conceptual grasp Understanding how to arrange the relevant elements and operations in the right order to reach the solution. **(1)**	Conceptual grasp Understanding the reasoning behind the calculations, demonstrated e.g. by place values. **(1)**	Conceptual grasp* Understanding rate as a relation between distance and time and the role of time (denominator) and distance (numerator) in speed. **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Session 5a: Summative Assessment

Item 13

Vali bought 25 pockets of oranges at the market and each pocket contained 20 oranges. If he paid a total amount of R350.00, how much did he pay per orange?

Memorandum for scoring **[4]**

25 x 20 = 500 (1)

R350 = 35 000c (1)

35 000 ÷500 = 70c (2)

OR R350 ÷ 25 = R14 (2)

R14.00 ÷ 20 = 70c (2)

OR **own method of choice**

Dimensions and levels of understanding **[9]**

Table 56

Analysis Item 13: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering how to find product and average. **(1)**	Factual recall Remembering how to multiply and divide. **(1)**	Factual recall Remembering the notation for money and average. **(1)**
	Operational appropriateness Deciding which amounts to multiply and divide. **(1)**	Operational appropriateness multiplying and dividing correctly. **(1)**	Operational appropriateness Formulating answer in Rand/cent per orange. **(1)**
	Conceptual grasp Understanding the order in which to arrange elements and order of operations to reach the solution. **(1)**	Conceptual grasp Understanding reasoning for calculations, e.g. by the right decimal place values when dividing. **(1)**	Conceptual grasp* Understanding that answer is rate as quotient of the dividend/divisor, expressed as proce/orange relation **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Item 14

From a piece of fabric 150cm wide and 400cm long, the Grade 6 teacher cut up square scarves of 50cm long and 50cm wide. How many scarves could she cut from the piece of fabric? Prove that your answer is correct.

Memorandum for scoring **[4]**

150cm x 400cm = 60 000cm^2 (1)

50cm x 50cm =2 500cm^2 (1)

60 000cm^2 ÷ 2 500cm^2 = 24 scarves(2)

OR 150cm ÷50cm = 3 (1)

400cm ÷ 50cm = 8 (1)

8 x 3 = 24 scarves (2)

OR own method of choice

Dimensions and levels of understanding **[9]**

Table 57

Analysis Item 14: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering to divide the smaller area into the bigger area. **(1)**	Factual recall Remembering how to multiply and divide. **(1)**	Factual recall Remembering the notation for area and dimensions. **(1)**
	Operational appropriateness Deciding on the dimensions to use to enable calculation(s). **(1)**	Operational appropriateness Understanding how to multiply and divide correctly. **(1)**	Operational appropriateness Formulating the calculations in cm and cm^2 and the answer correctly as scarves. **(1)**
	Conceptual grasp Understanding the alternative ways of arranging these elements in an order that would result in the appropriate solution. **(1)**	Conceptual grasp Understanding e.g. the role of the divisor and the dividend, or dividing and multiplying by multiples of 10. **(1)**	Conceptual grasp* Understanding that bigger area divided up into smaller areas, amount to the number of scarves. **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Item 15

When my uncle drives to work, it takes him 1½ hours to get there. If he drives at 50km/h, how far is his work place from home?

Memorandum for scoring **[4]**

1h x 50km/h = 50km (1)

½h x 50km/h = 25km (2)

50km + 25km = 75<u>km</u> (1)

OR **own method of choice**

Dimensions and levels of understanding **[9]**

Table 58

Analysis Item 15: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	<u>Factual recall</u> Remembering which operation to use to calculate distance. **(1)**	<u>Factual recall</u> Remembering how to multiply by a fraction. **(1)**	<u>Factual recall</u> Remembering the notation for time, distance and speed. **(1)**
	<u>Operational appropriateness</u> Deciding which elements to select to divide. **(1)**	<u>Operational appropriateness</u> Multiplying correctly. **(1)**	<u>Operational appropriateness</u> Formulating the answer correctly in km. **(1)**
	<u>Conceptual grasp</u> Understanding how to arrange the relevant elements and operations in the right order to reach the solution. **(1)**	<u>Conceptual grasp</u> Understanding the reasoning behind the calculations, demonstrated e.g. by fraction multiplication. **(1)**	<u>Conceptual grasp*</u> Understanding rate as a relation between distance and time and the role of time (denominator) and distance (numerator) in speed. **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Session 5b: Summative Assessment: Re-do

Item 16

You received R800 for your birthday. There is a special sale of PC games for R95 each. You decide to buy as many games as you can from your money. How many games can you buy and how much change will you get at the shop?

Memorandum for scoring **[4]**

R95 + R95 + R95 + R95 + R95 + R95 + R95 + R95 = R760 (2)

OR R95 x 8 = R760 (2)

R800 – R760 = R40 (1)

8 games and R40 change (1)

OR R800 ÷ R95 = 8 games (2)

and R40 change (2) **OR own method of choice**

Dimensions and levels of understanding **[9]**

Table 59

Analysis Item 16: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering how to find the number of items, given the price. **(1)**	Factual recall Remembering how to divide and subtract. **(1)**	Factual recall Remembering the notation for cost, item price and change. **(1)**
	Operational appropriateness Deciding which amounts to divide and subtract. **(1)**	Operational appropriateness Dividing and subtracting correctly. **(1)**	Operational appropriateness Formulating answer in number of items and money units as remainder. **(1)**
	Conceptual grasp Understanding the order in which to arrange elements and order of operations to reach the solution. **(1)**	Conceptual grasp Understanding reasoning for calculations, e.g. by the right place values when dividing. **(1)**	Conceptual grasp* Understanding answer as quotient of dividend/divisor, expressed as number of items and money remaining. **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Item 17

For bandana day, one of the Grade 6 learners makes yellow bandanas to sell. Her piece of fabric is 320cm long and 240cm wide. She wants to cut square bandanas of 40cm long and 40cm wide. How many bandanas will she be able to cut from her piece of fabric?

Memorandum for scoring **[4]**

320cm x 240cm = 76 800cm^2 (1)

40cm x 40cm =1 600cm^2 (1)

76 800cm^2 ÷ 1 600cm^2 = 48 bandanas(2)

OR 320cm ÷40cm = 8 (1)

240cm ÷ 40cm = 6 (1)

8 x 6 = 48 bandanas (2) **OR own method of choice**

Dimensions and levels of understanding **[9]**

Table 60

Analysis Item 17: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering to divide the smaller area into the bigger area. **(1)**	Factual recall Remembering how to multiply and divide. **(1)**	Factual recall Remembering the notation for area and dimensions. **(1)**
	Operational appropriateness Deciding on the dimensions to use to enable calculation(s). **(1)**	Operational appropriateness Understanding how to multiply and divide correctly. **(1)**	Operational appropriateness Formulating the calculations in cm and cm^2 and the answer correctly as bandanas. **(1)**
	Conceptual grasp Understanding the alternative ways of arranging these elements in an order that would result in the appropriate solution. **(1)**	Conceptual grasp Understanding e.g. the role of the divisor and the dividend, or dividing and multiplying by multiples of 10. **(1)**	Conceptual grasp* Understanding that bigger area divided up into smaller areas, amount to the number of bandanas. **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

Item 18

We will be driving to our grandmother during the holiday. It will take us 2½ hours to get there. If we drive at 90km/h, how far is grandmother's place from us?

Memorandum for scoring **[4]**

2h x 90km/h = 180km (1)

½h x 90km/h = 45km (2)

180km + 45km = 225km (1)

OR own method of choice

Dimensions and levels of understanding **[9]**

Table 61

Analysis Item 18: Dimensions and Levels of Understanding

	Dimensions of understanding (Usiskin, 2012)		
	Use / application	**Skill / algorithm**	**Representation / metaphor**
Levels of under-standing (Adapted from Anderson & Krathwohl, 2001)	Factual recall Remembering which operation to use to calculate distance. **(1)**	Factual recall Remembering how to multiply by a fraction. **(1)**	Factual recall Remembering the notation for time, distance and speed. **(1)**
	Operational appropriateness Deciding which elements to select to divide. **(1)**	Operational appropriateness Multiplying correctly. **(1)**	Operational appropriateness Formulating the answer correctly in km. **(1)**
	Conceptual grasp Understanding how to arrange the relevant elements and operations in the right order to reach the solution. **(1)**	Conceptual grasp Understanding the reasoning behind the calculations, demonstrated e.g. by fraction multiplication. **(1)**	Conceptual grasp* Understanding rate as a relation between distance and time and the role of time (denominator) and distance (numerator) in speed. **(1)**

***** If this level of understanding cannot be proven, an inference must be made, taking into account the participant's demonstrated understanding in the full context of the item.

APPENDIX C

Participant Responses

Table 62

Session 1: Baseline Assessment: Participant Responses and Scores to Memorandum

CODE	Item 1	Item 2	Item 3
Participant b	Absent		
Participant c	Absent		
Participant d	**R1365 + R1073.70 = R2438.70** She spent R2438.70 on milk and bread **(1)**	(110 x 5) + (80 x 5) = 550÷ 400 = (550 ÷100) x 4 = 23 There will be 23 stalls on the field	351 ÷ 5 = 7 rem 1 The average speed was 7 rem 1km
Participant e	R1365 + R1073.70 = R1074 365 She spent R1074365 on milk and bread	110m x 80m + 5m x 5m (110 x 5) + (5 x 80) = 550 ÷ 400 = (550 ÷100) ÷ 4 = 55 ÷ 4 = 14 rem 9 There will be 14 rem 9 stalls left	351 x 30 = 10530 The speed was 10 530
Participant f	R1073.70 x R1365 = R312 751 950	**110 x 80 = 8800m** **5 x 5 = 25 (1)** 8800 + 25 = 8825	(6x5) + (□ x 351) 6 x 5 = 30 351 x 30 = 10 530
Participant g	R1 073.70 x R1365 = R312 751 950	110m x 80m = 8800	6 x 5(5x30) = 351 x 30 10 531 The speed was 10 530
Participant k	Absent		
Participant l	R1 365 - R1 073 = R292 Proof: R1 073 + 292 = R1 365 The average amount spent is R292	**(110 x 80) ÷ (5 x 5)** **= 8800 ÷ 25** = 320 There will be 320 stalls **(3)**	351km ÷ (5 x 30) = 351km ÷ 150 = 2 rem 51
Participant m	**R1365 + R1073.70 = R2438.70** Spent R2 438.70 on milk and bread **(1)**	110m x 80m + 5m x 5m = 880 +25 = 905m	351 + 30 = 381
Participant n	**R1365 + R1073.70 = R2438.70** She spent R2 438.70 on milk and bread **(1)**	**110 x 80 = 8800m** **5 x 5 = 25 (1)** 8800 + 25 = 8825m	60 x 5 = 300 300 + 351 + 30 = 681 It took the bus 681 minutes
Participant o	R1 365 + R1 07370 = 11 735 She had to add her money to get the answer	110 x 80 = 8800m She had to times the 110 and 80 she had to add her answer	351 ÷ 30 =11 You had two divide multiply subtract
Participant p	**R1365 + R1073.70 = R2438.70** **(1)** The total amount is R2 438.70	**110 x 80 = 8800m** **5 x 5 = 25 (1)**8800 x 25 = 180000	351 ÷ 5 = 70 rem 1= 71km/h Checking: 70 x 5 = 350 + 1 = 351
Participant q	R1073.70 + R1365 =R1087.35 Checking 1087.35 + R1073.70 =R1365	110m x 80m = 880m 5m x 5m = 25m = 880m +25m = 905m 905m – 880m = 25m	351 ÷ 5 = 75
Participant t	**R1365 + R1073.70 = R2 438.70** R2 438.70 that is what the answer is**(1)**	110 x 80 = 8800m 8800 + 25 = 8825 The field is 8825m long	351 x 30 = 10530 10530 +6 =10536 The speed was 105.36km per hour
Participant v	**1364 ÷3 = 455 (1)** 107370 ÷ 3 =3579 3597 + 455 =4034 Average amount spent R403.4	110 x 80 = 8800m 8800 ÷ 5 =1760 There will be 1760 stalls	351 ÷ 5 = 70 rem 1 70km/h
Participant w	**R1365 + R1073.70 = R2438.70** 2438.70 – 1365= 1073.70 **(1)**	22 x 2=110 110 ÷ 5 = 22	70 ÷ 2= 35 351 ÷ 5=71
Sub	**7/52 = 13%**	**6/52 = 12%**	**0/52**
Total			**13/156** **8%**

Table 63

Session 1: Baseline Assessment: Participant Understanding According to Matrix

Abbreviations
Uf: Use/Application: factual recall; Uo: Use/Application: Operational efficiency; Uc: Use/Application: Conceptual grasp
Sf: Skill/Algorithm: factual recall; So: Skill/Algorithm: Operational efficiency; Sc: Skill/Algorithm: Conceptual grasp
Rf: Representation: factual recall; Ro: Representation: Operational efficiency; Rc: Representation: Conceptual grasp

CODE	**Item 1**				**Item 2**				**Item 3**			
Participant b	Absent											
Participant c	Absent											
Participant d	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant e	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant f	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant g	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant k	Absent											
Participant l	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant m	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant n	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant o	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant p	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant q	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant t	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant v	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant w	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Total	**Uf 1/13**	**Sf 9/13**	**Rf 1/13**	**11/39**	**Uf 1/13**	**Sf 4/13**	**Rf 1/13**	**6/39**	**Uf 2/13**	**Sf 0/13**	**Rf 0/13**	**2/39**
	Uo 1/13	**So 6/13**	**Ro 0/13**	**7/39**	**Uo 1/13**	**So 0/13**	**Ro 1/13**	**2/39**	**Uo 0/13**	**So 0/13**	**Ro 1/13**	**1/39**
	Uc 1/13	**Sc 0/13**	**Rc 1/13**	**2/39**	**Uc 1/13**	**Sc 0/13**	**Rc 0/13**	**1/39**	**Uc 0/13**	**Sc 0/13**	**Rc 0/13**	**0/39**
	3/39	**15/39**	**2/39**		**3/39**	**4/39**	**2/39**		**2/39**	**0/39**	**1/39**	
											33/351	

Table 64

Session 2: First Intervention: Money: Participant Responses and Scores to Memorandum

CODE	Item 4	Item 5	Item 6
Participant b	**25 000 – 2 000 = 23 000 (1)** 50 – 23 000 = 12 000	**90 x 15** = 1 350 **(1)**	800 x 17
Participant c	R25 000 ÷ R50 = R500 R500 x 50 = 25 000 R500 + 15 = R515	R1 725 x R95 = R8625	**R43.50 x 17 (1)** = R482.50
Participant d	R25 000 ÷ 15 = R166.10	1 725 ÷ 95 = 119.60	800 ÷ 17 = 47.1
Participant e	**R25 000 – R50** = R24 050 **(1)** R24 050–**R2 000**=R22 050 R22 050 -15= 22 036 He spent R22 036 on the team	R1 725 – 95 = R16 150 The printing cost R1 615	R43,50 – 800 = R35,50 The caps were R35,50
Participant f	**R25 000 – R2 000 – R50** = R23 015 **(1)** Coach spent R23 015	R1 725 x 95 = R170 840 The principal will have to pay R170 840	R43.50 +17 = R43.67 R800 – R43.47 = R370.67 R800 – R370.67 = R43.67 Coach paid them R800 and R370 was amount of each cap and yes R43 is change but R67 must be added.
Participant g	R2 500 – R50 = R2 450 R2 500 ÷ 50 = 50	2 000÷ 15 = 135 rem 5 5 x 5 = 25 95 x 25 = 2375	800 ÷ 17 = 47 rem 1
Participant k	**R25 000 – 2 000 = 23 000 (1)** R25 000+R2 000+R50=R27 050 R25 000 + R 50 = R25 050	R1725 + (5 = 1820 1725 ÷ 95 = 90	800 + 15 – 43 = 772
Participant l	**R25 000 – R2 050 ÷ 15 = x** **R25 000 – R2 050 = R 22 950** **R22 950 ÷ 15 = R1 530 (4)** for soccer gear	**R1725 – (R95 x 15) = x** **95 x 15 = R1 425** **R1725 – R1 425 = R300 (3)** The printing for each towel cost R300	R800 – R739.50 = x R800 – R739.50 = R60,50 Each cap costs R60.50
Participant m	R25 000 ÷ 15		
Participant n	**R25 000 – R2 050 = R23 000** **R23 000 – R50 = R 22 950 (2)**	**95 x 15 = R1 425** **R1 725 – R1 425 = R300 (3)**	R800 ÷ 43 = 11
Participant o	R25 000 + 2 000 = 27 000	R1 725 + 95 = 1 820	R800 + R4 350 = R5 150
Participant p	**R25 000 – R2 000 – R50 = R22 950 (2)** R22 950 is still available to buy soccer gear	**1 725 ÷ 15 = x** 2 000 ÷ 15 =R133.50 It was R133,50 per towel	R43,50 ÷ 17 = R2,50 Caps was each R2,50
Participant q	R25 000 ÷ 15 = 1666	R1 725 – R95 = R1 630	(R800 + 17) + R43.50 = R51.67
Participant t	R25 000 + R2 000 = R27 000 – R50 = R26 935	95 – 15 = 80 R1 725 – 80 = R1 630	3 850 – 17 = 783 R43.50 – 783 = R670
Participant v	**R25 000 – R2 050 ÷ 15 = x** **R25 000 – R2 050 = R 22 950** **R22 950 ÷ 15 = R1 530 (4)** **Each child gets R1 530 for soccer gear**	**R1 725 ÷ 15 = x – R95** **1725 ÷ 15 = R115** **R115 – R95 = R20 (4)** **The printing cost R20**	**R800 – R43.50 = x ÷ 17** **R800 – R43 = 756.50 (3)** 756 ÷ 17 = 38 **Each cap cost** R38
Participant w	R25 000 – R2 550 = R2 245	**1 725 ÷ 15 = 115** **115 – 95 = 20** **R20 (4)**	791 – 43 = 757 757 ÷ 17 = 41
Sub	**16/64** **25%**	**15/64** **23%**	**4/64** **6%**
Total			**35/192** **18%**

Table 65

Session 2: First Intervention: Money: Participant Understanding According to Matrix

CODE	Item 4				Item 5				Item 6			
Participant b	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant c	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant d	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant e	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant f	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant g	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant k	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant l	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant m	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant n	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant o	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant p	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant q	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant t	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant v	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant w	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Total	**Uf 5/16**	**Sf 9/16**	**Rf 2/16**	**16/48**	**Uf 5/16**	**Sf 6/16**	**Rf 3/16**	**14/48**	**Uf 3/16**	**Sf 3/16**	**Rf 1/16**	**7/48**
	Uo 2/16	**So 5/16**	**Ro 2/16**	**9/48**	**Uo 3/16**	**So 5/16**	**Ro 3/16**	**11/48**	**Uo 2/16**	**So 0/16**	**Ro 1/16**	**3/48**
	Uc 2/16	**Sc 2/16**	**Rc 2/16**	**6/48**	**Uc 2/16**	**Sc 2/16**	**Rc 1/16**	**5/48**	**Uc 1/16**	**Sc 0/16**	**Rc 1/16**	**2/48**
	9/48	**16/ 48**	**6/48**		**10/ 48**	**13/48**	**7/48**		**6/48**	**3/48**	**3/48**	
												73/432

Table 66

Session 3: Second Intervention: Area: Participant Responses and Scores to Memorandum

CODE	Item 7	Item 8	Item 9
Participant b	**308 ÷ (1)** 24 =100cm	**390 x 450** = 1 650 **(1)** 1 650 ÷ **30 (1)** = 510	76 ÷ 24 = 3
Participant c	**308 (1) ÷ 14 (1) = 22cm (2)** It was 22cm long	**450 x 390 (1)** = 165 000 165 000 ÷ **30 (1)** = 5 500 There will be 5 500 tiles	$204km^2$ x 12 = 2 $448km^2$ The fence is 2 $448km^2$ long in total
Participant d	308 + 14 ÷ 10 = x 308 + 14 = 422 322 ÷ 10 = 32 rem 2 The sheet is 32^2cm long	**390 x 450** = 1650 **(1)** 1 650 ÷ **30 (1)**= 55 55 tiles will be needed	204 x 12 = 2 448 The fence is 2 448^2km long in total
Participant e	**308 (1) ÷ 14 (1) = 22 (2)** It was 22cm long	**390 ÷ 30 (1)** = 130 **450 ÷ 30 (1)** = 150 150 + 130 = 280 She will need 280 tiles	**204 ÷ 12 = 17 (2)** 204 + 12 = 216 It is 17 in total. It is 216 in total.
Participant f	**308 (1) ÷ 14 (1)** = 217 14 x 10 = 140	**390 ÷ 30 (1) = 13 (1)** **450 ÷ 30 (1)** = 9	**204 ÷ 12 (1)** = 19 Its 19cm
Participant g	Absent		
Participant k	**308 (1) ÷ 14 (1)** = 23	**390 x 450** = 1 669 500 **390 ÷ 30 (1) = 13 (1)**	**204 ÷ 12 = 17 (2)**
Participant l	**308 x 1(1) = 308 (1)** **308 ÷ 14 (1) = 22cm (1)**	**390 x 450 (1)** = 275 500 275 500 ÷ **900 (1)** = 306 rem 100	204 ÷ 12 + 12 = x 204 ÷ 24 = 14 rem 1
Participant m	308 ÷ 10 = 30 rem 8 12 ÷ 10 = 1 rem 2 14 ÷ 10 = 1 rem 4	12 x 15 = 390	12 + 12 = 24 110 + 70 = 180 24 + 180 = 204
Participant n	306 x 14 = 4312	**390 x 450** (1) = 3600	**204 ÷ 12 (1)** = 18 rem 8 The other side of the fence is 8cm
Participant o	308 + 10 = 318 318 + 14 = 332 332 + 10 = 342	390 x 450 = 1680	204 + 12 = 216km
Participant p	**308 ÷ 14 (1)** = 112 This is 112 long	**390 x 450 (1) = 175 500 (1)** **175 500 ÷ 30 = 5 850 (2)** 5 850 is the number of tiles needed	**$204km^2$ ÷ 12 = 17 (2)**km^2 The other side of the fence is $17km^2$ long
Participant q	**308 ÷(1) 14 (1) = 22cm (1)**	390 x 30 = 11700	304 x 8 = 2448
Participant t	308 – 10 = 298 298 x 14 = 4182	390 + 450 = 840 840 – 30 = 810	204 x 12 = 2448
Participant v	**308 ÷(1) 14 (1) = 22 (1)** 14 by 22	**390 x 450** = 19 500 **(1)** 19 500 ÷ **30 (1)** = 550 She will need 550	**204 ÷ 12 = 17 (2)** 12 by 17
Participant w	Absent		
Sub	**24/56** **40%**	**22/56** **39%**	**10/56** **18%**
Total			**56/168** **33%**

Table 67

Session 3: Second Intervention: Area: Participant Understanding According to Matrix

CODE	**Item 7**				**Item 8**				**Item 9**			
Participant b	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant c	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant d	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant e	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant f	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant g	Absent											
Participant k	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant l	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant m	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant n	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant o	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant p	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant q	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant t	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant v	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant w	Absent											
Total	**Uf 9/14**	**Sf 5/14**	**Rf 3/14**	**17/42**	**Uf 12/14**	**Sf 7/14**	**Rf 2/14**	**21/42**	**Uf 6/14**	**Sf 5/14**	**Rf 0/14**	**11/42**
	Uo 8/14	**So 5/14**	**Ro 3/14**	**16/42**	**Uo 9/14**	**So 1/14**	**Ro 3/14**	**13/42**	**Uo 5/14**	**So 4/14**	**Ro 0/14**	**9/42**
	Uc 6/14	**Sc 5/14**	**Rc 2/14**	**13/42**	**Uc 3/14**	**Sc 1/14**	**Rc 1/14**	**5/42**	**Uc 1/14**	**Sc 2/14**	**Rc 0/14**	**3/42**
	23/42	**15/ 42**	**8/42**	**46/ 126**	**24/42**	**9/42**	**6/42**	**39/ 126**	**13/42**	**11/42**	**0/42**	**23/ 126**
											108/378	

Table 68

Session 4: Third Intervention: Speed: Participant Responses and Scores to Memorandum

CODE	Item 10	Item 11	Item 12
Participant b	**686 ÷ 7** = 70 **(2)**	75 + 450 = 525	616 ÷ 3 = 202
Participant c	**11:00 – 04:00 = 7:00** **686 ÷ 7 = 98km/h (4)** He drove 98km/h or 100km/h rounded off	**450 ÷ 75 = 6 (2)** 450 + 6 = 16:00 They arrived at 16:00 in Soweto	161 ÷ 3 = 53 53 x 424km Cape Town is 424km from Johannesburg
Participant d	**11:00 – 04:00** x 686 686 x 7 = 102km/h **(1)** He drove 102km/h	**450 ÷ 75 = 6 (2)** She will arrive at 6pm	**616 x 3 = 1848 (4)** Cape Town is 1 848km far.
Participant e	11:00 + 04:00 = 15:00 686 ÷ 15 = 450 r 11 The speed was 450 r 11km	450 x 75 = 327.50 She got there at 24:00	**616 x 3 = 1 848 (4)** Cape Town is 1848km from Johannesburg
Participant f	686 ÷ 4 = 171 rem 3 171 x 4 = 784 784 + 3 = 787 The average speed is 787km/h	**450 ÷ 75** = 5 r 70 **(1)** 08+ 5 = 13:00 They will arrive at 13:00	616 ÷ 3 = 235 rem 1 Johannesburg is 235km away from Cape Town
Participant g	4 x 11 = 44 686 ÷ 44 = 15 r 26 15 x 7 = 105 105 + 26 = 131 He drove at a speed of 131 per hour	75 x 8 = 600 600 – 450 = 150 She got there at 15:00 pm at Soweto	**616 x 3 = 1 848 (4)** 1 848km long from Johannesburg to Cape Town and back is double.
Participant k	0400 + 686 = 10.86 86 ÷ 60 = 1 r 26 My answer is 10.26 686 ÷ 0400 = 1 r 286 My answer is 1 r 286	450 x 75 = 33750 450 + 75 = 525 My answer is 525 o'clock	616 ÷ 3 = 235 It is 205km away from Joburg
Participant l	**11:00 – 04:00 = x** **11:00 – 04:00 = 7:00 hrs** **686 ÷ 7 = x** **686 ÷ 7 = 98km/hr (4)** He travelled at 98km/h	**450 ÷ 75 = x** **450 ÷ 75** = 5 r 75 **08:00** + 6h 15 min = 14:15 **(2)** They will arrive at 14:15	**616km/h x3 = x** **616km/h x3 = 1 848km (4)** Cape Town is km away from Johannesburg.
Participant m	**686 ÷ 7 = 98 (3)**	75 x 8 + 4 + 560 = 6:00	616 ÷ 3 = 205 205 x 3 + 1 = 616
Participant n	**11:00 – 04:00 = 07:00** **686 ÷ 7** = 63 **(2)** Dan left at 04:00 and got back at 11:00 wich is 7 hours	250 ÷ 75 = 3750 75 x 50 = 3750 It will take Thembi 50km	616 ÷ 3 = 1 848 616 x 3 = 1 848 It will take Petros 213 kl
Participant o	04:00 + 11:00 + 686 = 21:86	08:00 + 75km + 450km = 13:35km	616 x 3 = 1 848km **(4)**
Participant p	**11:00 – 04:00 = 7:00** **686 ÷ 7 = 98km/hr (4)** He drove 98km/h or 100km/h rounded off	**450 ÷ 75 = 6 (3)** They arrive at 18:00	616 ÷ 3 = 205 rem 1 205 + 1 = 206 It is 206km
Participant q	686 ÷ 4 = 171 rem 2 The average is 171km/2h	450km – 75km/h = x 450km – 75km = 375km 15:75 + 60 = 16:35 She will arrive at 16:35	616 ÷ 3 = 202 It will be 202km/h
Participant t	Absent		
Participant v	Absent		
Participant w	**R686 ÷ 7 = 98km/h (4)**	**450 ÷ 75 = 6** **6 + 8 = 14 (4)** They will arrive at 14:00/2 o'clock	**616 x3 = 1 848km (4)** It is 1 848km away.
Sub	**24/56 = 43%**	**14/56 = 25%**	**23/56 = 41%**
Total			**61/168** **36%**

Table 69

Session 4: Third Intervention: Speed: Participant Understanding According to Matrix

CODE	Item 10				Item 11				Item 12			
Participant b	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant c	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant d	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant e	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant f	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant g	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant k	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant l	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant m	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant n	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant o	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant p	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant q	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant t	Absent											
Participant v	Absent											
Participant w	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Total	**Uf 10/14**	**Sf 8/14**	**Rf 5/14**	**23/42**	**Uf 6/14**	**Sf 6/14**	**Rf 1/14**	**13/42**	**Uf 6/14**	**Sf 6/14**	**Rf 5/14**	**17/42**
	Uo 7/14	**So 6/14**	**Ro 5/14**	**18/42**	**Uo 6/14**	**So 4/14**	**Ro 4/14**	**14/42**	**Uo 6/14**	**So 6/14**	**Ro 6/14**	**18/42**
	Uc 4/14	**Sc 5/14**	**Rc 4/14**	**13/42**	**Uc 2/14**	**Sc 1/14**	**Rc 1/14**	**4/42**	**Uc 6/14**	**Sc 6/14**	**Rc 0/14**	**12/42**
	21/42	**19/42**	**14/42**	**54/126**	**14/42**	**11/42**	**8/42**	**31/126**	**18/42**	**18/42**	**11/42**	**47/126**
											132/378	

Table 70

Session 5a: Summative Assessment: Participant Responses and Scores to Memorandum

CODE	Item 13	Item 14	Item 15
Participant b	**350 ÷ 20 ÷ 25 (3)** = 11 One orange is R11	**400 ÷ 50 (1)** = 40 **150 ÷ 50 (1) = 3** 40 scarves	90 **x 50 (1)** = 450 Is 450km
Participant c	**350 ÷ 25 = R14 (2)** He paid R14 per bag	400 + 150 = 650 650 ÷ 50 = 13 She could cut 13 scarves	**50 ÷ 2 = 25 (2)** His work place is 25km away.
Participant d	25 x 20 ÷ 350 = x **25 x 20** = 50 **(1)** **R350 ÷ (1)** 50 = R7.00 He will pay R7 for 1 orange	**400 ÷ 50 (1)**= 80 **150 ÷ 50 (1)**= 30 60 + 50 = 110 She could cut out 110 pieces	**1½ of 50 = 25 x 3 (2)**= 65 It takes him 1h 5 min
Participant e	350 ÷ 20 = 17.10 It cost R17.10 per orange	400 -150 = 250 They cut 250 scarves	50 ÷ 5 = 1 50 -1 = 49 It will take him 49 minutes to get to work
Participant f	R350.00 x 45 = R18 750.00 18750 ÷ 20 = 931	400 ÷ 150 = 2cm **50 x 50 = 2 500cm (1)** = 25cm	**1½ of 50 = 3/2 x 50** **= 150/2 = 75 (3)**
Participant g	**25 x 20 = 500 (1)** 500 x 350 = R152 500 152 500 ÷ 500 = 3 rem 25c He paid 25c per orange	150 + 400 = 550 550 ÷ 100 = 5 rem 50 She could make 550 scarves	90 x 50 = 450 He would get there in four hours 50 min
Participant k	350 ÷ 50 = 70 70 + 25 = 95 My answer is 95	400 ÷ 150 = 2 2 + 50 52 My answer is 52	1500 ÷ 50 = 30 My answer is 50
Participant l	Absent		
Participant m	20 x 1½ = 25 1½	**150 ÷ 50m = 3** **400 ÷ 50 = 8 (2)** 3 + 8 = 11	300 ÷ 90 = 3 rem 30 30 minutes
Participant n	350 ÷ 20 = 17 rem 10 17 x 25 = 425 r 10 He paid R1 for each orange	**400 x 150 (1)** = 24 000 24000 – 50 = 23 950 23950 – 50 = 23 900 She used 23 900 pieces of fabric	90 ÷ 5 = 18 It took the uncle 90 minutes to get to his work
Participant o	25 + 25 + 25 ....= 350 Each orange was R50	**50 + 50 + 50......= 400 (1)** 8 pieces	60 +30 = 90 50 + 50 + 50 = 150 150km
Participant p	**R350 ÷ (1)** (20+25) = x = 777 r 35 7,77 + 35 = R8,12 It is R8,12 for one orange	(15x2) + (400x2) – (50x 2) –(50x2) (300 + 800) – (100 -100) = 1 100 -0 = 1 100 She can cut 1 100 scarves	**1h = 50km (1)** + ½ of 60 minutes 50km + 30 min 80km It is 80km from home
Participant q	350 ÷ 20 = 17 rem 10 Each orange cost R17	150cm + 400cm = 550cm 550 ÷ 50 = 11	650 ÷ 50 = 13 It is 13km long
Participant t	350 -20 = 325 He paid R325 per orange	**400 x 150 (1)** = 24 000 24 000 -100 = 23 891 23 891 scarves for the Grade 6s	150 – 50 = 100 His work is 100km
Participant v	**25 x 20 = 500 (1)** 500 ÷ 350 = 1 rem 150 She paid 150	150 + 400 = 550 550 ÷ 50 = 12 She can make 12	50 min ÷ 50 = 1 rem 40 He lives 40km away from home
Participant w	**25 x 20 = 500** **350 ÷ 25 = 14** **The answer is 70c an orange (4)**	**50 x 3 = 150 (1)** She can only make 3 scarves	**50 + 25 = 75** **His work is 75km away (4)**
Sub	**14/60 = 23%**	**11/60 = 18%**	**13/60 = 22%**
Total			**38/180** **21%**

Table 71

Session 5a: Summative Assessment: Participant Understanding According to Matrix

CODE	**Item 13**				**Item 14**				**Item 15**			
Participant b	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant c	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant d	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant e	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant f	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant g	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant k	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant l	Absent											
Participant m	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant n	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant o	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant p	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant q	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant t	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant v	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant w	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Total	**Uf 6/15**	**Sf 6/15**	**Rf 3/15**	**15/45**	**Uf 7/15**	**Sf 9/15**	**Rf 1/15**	**17/45**	**Uf 6/15**	**Sf 5/15**	**Rf 1/15**	**12/45**
	Uo 3/15	**So 2/15**	**Ro 3/15**	**8/45**	**Uo 0/15**	**So 2/15**	**Ro 1/15**	**3/45**	**Uo 3/15**	**So 3/15**	**Ro 1/15**	**7/45**
	Uc 2/15	**Sc 1/15**	**Rc 1/15**	**4/45**	**Uc 0/15**	**Sc 0/15**	**Rc 0/15**	**0/45**	**Uc 1/15**	**Sc 2/15**	**Rc 1/15**	**4/45**
	11/45	9/45	7/45	27/135	7/45	11/45	2/45	20/135	10/45	10/45	3/45	23/135
											70/405	

Table 72

Session 5b: Summative Assessment: Participant Responses and Scores to Memorandum

CODE	Item 16	Item 17	Item 18
Participant b	**You get 8 games and R40 change (4)**	**320 x 240 (1)** = 560 560 ÷ 40 = 11	120 x 2 = 240km
Participant c	**800 ÷ 95 = 8 r 40 (2)** **He could buy 8 games and his change was R40 (2)**	320 – 240 = 80 She could cut 80 bandanas out of the piece of fabric	90 of 2½ **90 ÷ 2 x 5 = 225 (2)** km/h **Grandmother's place is 225km away (2)**
Participant d	**800 ÷ 95 = 8 r 40 (2)** **I buy 8 games and R40 change (2)**	320 + 240 = 560 560 ÷ 40 = 14 She can cut 14 bandanas	250 ÷ 90 = 270 We will drive 270km to our grandmother's house
Participant e	**800 ÷ 95 = 8 r 40 (2)** **You can buy 8 games and (1)** and 40c change	320 x 6 = 1 920 320 x 40 = 12 860 320 + 40 = 360 320 + 240 = 560 She can cut 1 920; 360; 560	**90 x 2 = 180 (1)** **90 ÷ 2 = 45 (2)** It is 45km away
Participant f	**800 ÷ 95 (1)** = 7 r 65 7 games and your change will be R65	320 – 240 = 80 She can make 80 fabric bandana's	90 x 3/2 = 270/2 13 ½ Granny's house is 13 ½km/h
Participant g	**800 ÷ 95 (1) = 8 rem 40 (1)** I would get R8.40 change	320 + 240 = 560 560 ÷ 40 = 14 She can cut 14 bandanas from the fabric	**90 + 90 = 180 (1)** 180 ÷ 120 = 1 rem 60 It is 1hour and 60 min away from home
Participant k	**800 ÷ 95 (1)** = 7 My answer is R7 **8 x (1)** (games)	**6 (1) x (1)** 7 = 42 She can cut out 42	**90 + 90 (1) + (1)** 50 = 240km The answer is 240km
Participant l	**800 ÷ 95 (1) = 8 rem 40 (1)** **I'll buy 8 games and I'll get R40 change (2)**	(320 + 240) ÷ 40 = x 560 ÷ 40 = 14 She can make 14 bandanas	**90 x 2 (1) = 180 + (1)** 50 = 230km Grannie's house is 230km away
Participant m	**800 ÷ 95 (1) = 8 (1)** rem 80	**320cm ÷ 40cm = 8 (1)** **240cm ÷ 40cm = 6 (1)**	1h = 60min 2h= 120min ½ = 30min = 210
Participant n	**800 ÷ 95 (1)** = 15 rem 7 You get 15 games and R15.7	320 ÷ 240 = 48 She can cut 48 bandanas from the piece of fabric	90 ÷ 2 = 45 We will drive 45 k/h in 2 ½ hours
Participant o	95 + 95 + 95 ....= 855 800 – 95 = 95c 95c or R95.00	40 + 40 She can get 40cm fabric	**90 + 90 = 180 (1)** 60h + 60h = 120 180km
Participant p	**R800 ÷ 95 (1)** = x R800 ÷ 95 = 9 rem 45 9 games R45 will be left of change	(320+ 249) – (40cm + 40cm) = x 560 – 80cm 48~~0~~ She can make 48 bandanas	**(90km x 2) + (90km ÷ 2)** **= 180 +45** **= 225km (4)** **It is 225km away from their place**
Participant q	**800 ÷ 95 (1) = 8 rem 40 (1)**	**(320cm ÷ 40cm)+(240cm ÷ 40cm)** **320cm ÷ 40cm = 8 (1)** **240cm ÷ 40cm = 6 (1)** 6 + 8 = 14 550 ÷ 50 = 11	**90 x 2 = 180 (1)** It is 180km
Participant t	800 x 95 = 76 000 You can get 76 games and 0c left over	320 – 240 = 120 + 80 = 200 She cuts 200 bandanas	250 x 90 = 22 500 Grandma's house is 225km away **(1?)**
Participant v	**800 ÷ 95 = 8 (3)** rem 5 You can buy 8 PC games and get R5.00 back	**(320 ÷ 4) x (240 ÷ 4) (2)** **320 ÷ 4** = 80 **240 ÷ 4** = 60 4800 She can make 480 bandanas	**90 x 2 = 180** **180 + 45 = 225** **Grandma's house is 205km away (3)**
Participant w	**95 x 8 = 760** **800 – 760 = 40** **He can buy 8 games and he will have R40 chanch (4)**	**240 ÷ 40 = 6 (1)** She can make 6 bandanas	**(90 x 2) + 45 =** **90 x 2 = 180** **90 ÷ 2 = 45** **180 + 45 = 225** **It is 225km away (4)**
Sub	**37/64 = 58%**	**10/64 = 16%**	**26/64 = 41%**
Total			**73/192 38%**

Table 73

Session 5b: Summative Assessment: Participant Understanding According to Matrix

CODE	Item 16				Item 17				Item 18			
Participant b	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant c	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant d	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant e	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant f	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant g	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant k	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant l	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant m	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant n	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant o	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant p	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant q	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant t	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant v	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Participant w	Uf	Sf	Rf		Uf	Sf	Rf		Uf	Sf	Rf	
	Uo	So	Ro		Uo	So	Ro		Uo	So	Ro	
	Uc	Sc	Rc		Uc	Sc	Rc		Uc	Sc	Rc	
Total	**Uf 15/16**	**Sf 14/16**	**Rf 7/16**	**36/48**	**Uf 6/16**	**Sf 6/16**	**Rf 1/16**	**13/48**	**Uf 10/16**	**Sf 10/16**	**Rf 6/16**	**26/48**
	Uo 10/16	**So 9/16**	**Ro 7/16**	**26/48**	**Uo 2/16**	**So 2/16**	**Ro 2/16**	**6/48**	**Uo 7/16**	**So 5/16**	**Ro 8/16**	**20/48**
	Uc 7/16	**Sc 6/16**	**Rc 6/16**	**19/48**	**Uc 0/16**	**Sc 0/16**	**Rc 0/16**	**0/48**	**Uc 4/16**	**Sc 4/16**	**Rc 3/16**	**11/48**
	32/ 48	**29/ 48**	**20/4 8**	**81/144**	**8/48**	**8/48**	**3/48**	**19/144**	**21/48**	**19/48**	**16/48**	**57/144**
											157/432	

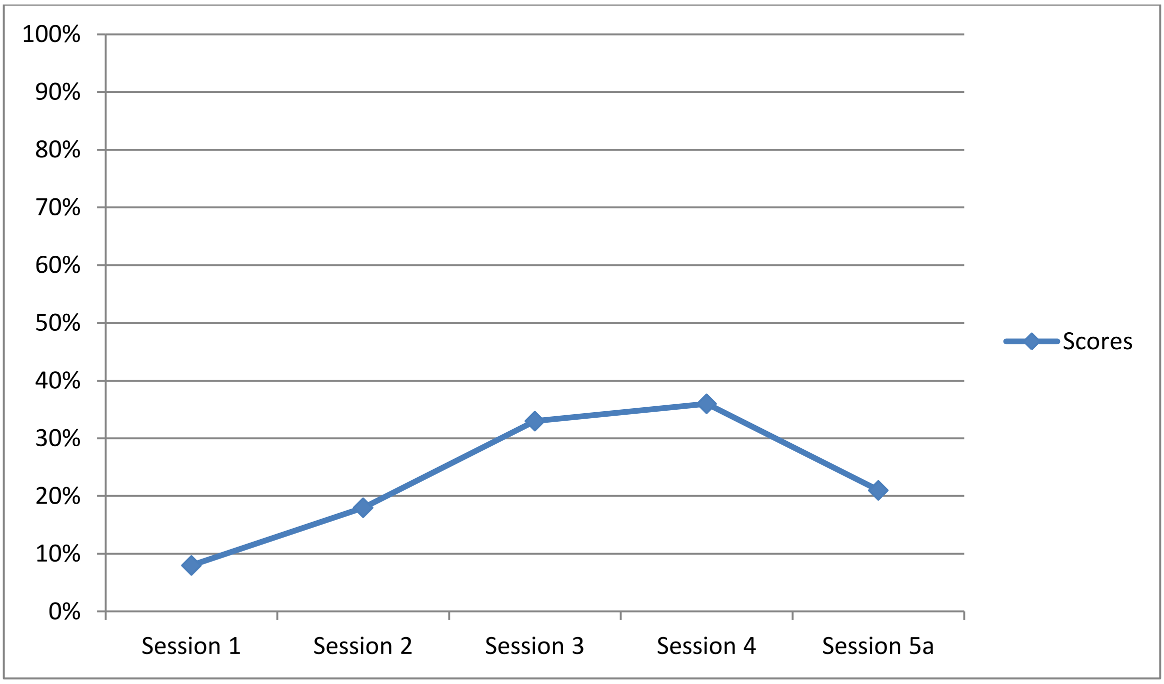


Figure 36. Group performance: Sessions 1-5a.

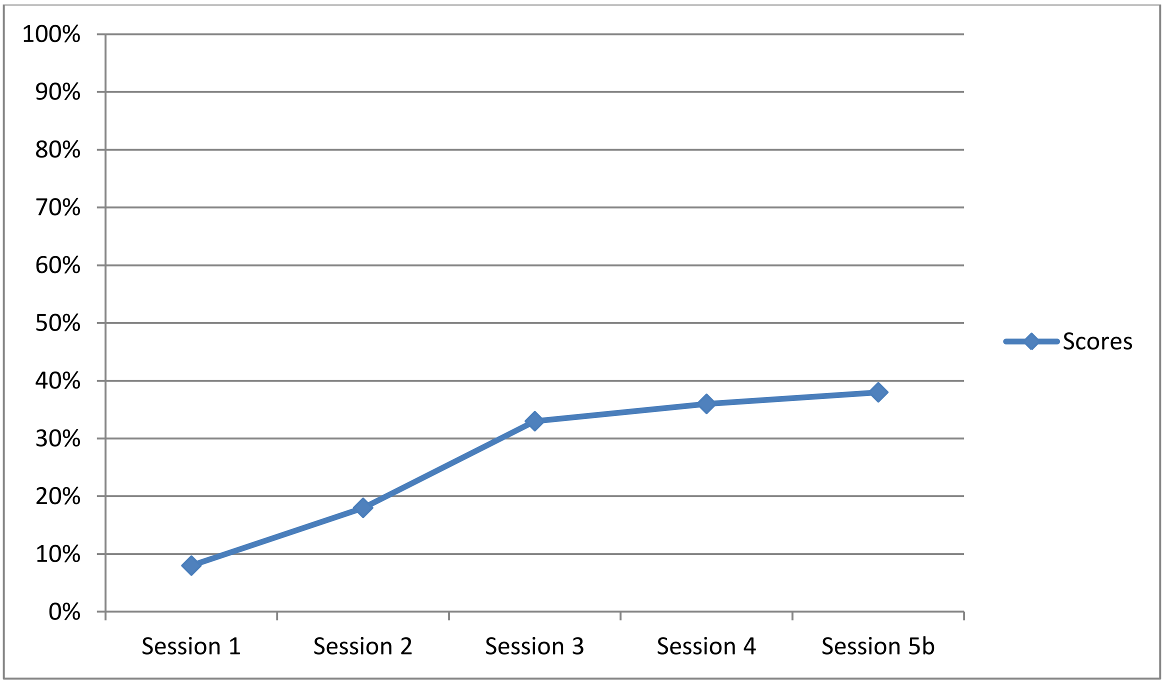


Figure 37. Group performance: Sessions 1-5b.

Table 74

Progressive Individual Performance: Scores and Understanding

Name		**Pre-test**	**Interven-tion 1**	**Interven-tion 2**	**Interven-tion 3**	**Post-test a**	**Post-test b**
Participant b	Score	Absent	17%	25%	17%	50%	42%
	Understand	Absent	3	4	4	9	11
Participant c	Score	Absent	8%	50%	50%	33%	67%
	Understand	Absent	2	12	13	6	17
Participant d	Score	8%	0	17%	58%	50%	33%
	Understand	2	0	2	13	12	9
Participant e	Score	0	8%	75%	33%	0	50%
	Understand	1	2	16	9	0	11
Participant f	Score	8%	8%	50%	8%	33%	8%
	Understand	1	2	6	7	7	2
Participant g	Score	0	0	Absent	33%	8%	25%
	Understand	0	0	Absent	8	2	6
Participant k	Score	Absent	8%	50%	0	0	50%
	Understand	Absent	2	8	0	0	9
Participant l	Score	25%	58%	50%	83%	Absent	50%
	Understand	6	15	15	23	Absent	14
Participant m	Score	8%	0	0	25%	17%	33%
	Understand	2	0	0	5	3	7
Participant n	Score	17%	42%	17%	17%	8%	8%
	Understand	3	5	4	4	2	2
Participant o	Score	0	0	0	33%	8%	8%
	Understand	1	0	1	7	2	3
Participant p	Score	17%	17%	58%	58%	17%	42%
	Understand	4	5	17	15	2	16
Participant q	Score	0	0	25%	0	0	42%
	Understand	1	0	6	1	2	9
Participant t	Score	8%	0	0	Absent	8%	8%
	Understand	2	0	0	Absent	2	0
Participant v	Score	8%	92%	58%	Absent	8%	67%
	Understand	8	26	16	Absent	2	20
Participant w	Score	8%	33%	Absent	100%	75%	75%
	Understand	1	12	Absent	26	21	20

Table 75

Individual Scores and Metacognitive Reporting: Participants Present: Session 1 & 5b

Metacognitive questionnaire: Questions with response options:

a. (i) Before you closed your eyes, did you understand the word sum completely?
1. Yes 2. No, because...

(ii) With eyes open still, what helped you to understand the problem better, reading it yourself, or listening to it while it was read to you?
1. Reading it 2. Listening 3. Both together

b. (iv) Is this a slower or a quicker way for you than drawing a picture?
1. Slower 2. Quicker 3. It became quicker

(v) Was it easy or difficult to let things move or change?
1. Easy 2. Difficult 3. It became easier

c. (i) When you opened your eyes, did you have a plan of what you would write down?
1. Yes 2. No 3. Not completely

(ii) Could you clearly remember the numbers that you had fixed in the virtual space of your mind?
1. Yes 2. No, not at all 3. I was not sure

* S-> Score to memorandum
* U-> Understanding according to matrix of understanding

No	**a. (i)**	**a. (ii)**	**b. (iv)**	**b. (v)**	**c. (i)**	**c. (ii)**	**Pre-test**	**Post-test**	**Difference**
Participant n	1	3	2	3	1	1	S 17% U 3	8% 2	**-9%** **-1**
Had an understanding of the problem before she closed her eyes. Found it easy to enter into the VSM and see the situation as a picture, but hard to attach numbers and words to the picture. She had the perception that she knew exactly what to do when she opened her eyes and it was easy to remember all the numbers.									
Participant t	1	3	3	2	1	2	S 8% U 2	8% 0	**0%** **-2**
Although she thought that she understood the problem, she could not see a picture in the VSM. She could not make things move or change and could not remember the numbers. She thought though that she knew what to do to solve the problem, from the first understanding, but not from the VSM.									
Participant f	2	2	3	2	1	2	S 8% U 1	8% 2	**0%** **+1**
Had not understood the question clearly before closing eyes. Found it quick to enter, but hard to manipulate objects in the VSM. Once opened eyes, could not remember numbers, was not sure what to write down or how to solve the problem.									
Participant o	1	2	3	3	1	1	S 0% U 1	8% 3	**+8%** **+2**
Had no understanding of the problem before she closed her eyes. Was dependant on the teacher to create the VSM and found it hard to write down anything in that space. She thought though that she knew what to do when she opened her eyes and it was not difficult to remember all the numbers.									
Participant m	1	3	1	1	3	1	S 8% U 2	33% 7	**+25%** **+5**
Had understood the question before entering into the VSM. Found it a slower way of dealing with a problem, yet she found it easy to form a picture and easy to manipulate things in the VSM. She was not completely sure what to do when she opened her eyes but could clearly remember the numbers that she saw in the VSM.									
Participant g	1	2	3	3	1	3	S 0% U 0	25% 6	**+25%** **+6**
Had understood the question before entering into the VSM. Found that it became easier and quicker to apply the method as time passed. She opened her eyes with a clear picture of what she was about to do, yet was not sure about the numbers that she saw in the VSM.									
Participant d	1	1	2	3	1	3	S 8% U 2	33% 9	**+25%** **+7**

<table>
<tr><td colspan="10">Had understood the question before entering into the VSM. Found it a quicker way of dealing with the problem and it became easier as time passed. She opened her eyes with a clear picture of what she was about to do, yet was not sure about the numbers that she saw in the VSM.</td></tr>
<tr><td>Participant l</td><td>1</td><td>1</td><td>1</td><td>2</td><td>1</td><td>1</td><td>S 25%
U 6</td><td>50%
14</td><td>+25%
+8</td></tr>
<tr><td colspan="10">Had understood the question from the start. Immediately saw the problem in terms of numbers and did not feel the need to make use of the VSM to be able to solve the problem, since the situation automatically turns to a sum in his mind. He does not see pictures, only numbers.</td></tr>
<tr><td>Participant p</td><td>1</td><td>3</td><td>2</td><td>1</td><td>1</td><td>1</td><td>S 17%
U 4</td><td>42%
16</td><td>+25%
+12</td></tr>
<tr><td colspan="10">She is confident that she understood the problem from the start and could easily access and easily manage the problem situation in the VSM. Then she knew and remember the numbers with certainty what to write down to solve the problem. Nothing of the VSM was difficult for her to deal with.</td></tr>
<tr><td>Participant q</td><td>1</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>S 0%
U 1</td><td>42%
9</td><td>+42%
+8</td></tr>
<tr><td colspan="10">Although she found it easier and easier to enter and manage the VSM, she is uncertain whether she is successful in doing so. She is pessimistic about how she understood the problem, she doubts if she could remember the numbers and whether her strategy of doing the sums were good enough.</td></tr>
<tr><td>Participant e</td><td>2</td><td>3</td><td>3</td><td>3</td><td>3</td><td>1</td><td>S 0%
U 1</td><td>50%
11</td><td>+50%
+10</td></tr>
<tr><td colspan="10">Although not certain what the problem required before entering into the VSM, she found that the process became easier and easier and she had a clear idea of the numbers and of what to do after opening her eyes.</td></tr>
<tr><td>Participant v</td><td>1</td><td>3</td><td>2</td><td>3</td><td>3</td><td>3</td><td>S 8%
U 8</td><td>67%
20</td><td>+59%
+12</td></tr>
<tr><td colspan="10">She had not understood the problem before she entered the VSM. She found it hard to imagine, could not see a complete picture and it vanished when she opened her eyes. She found it hard to make things move or change and could not remember the numbers once she opened her eyes.</td></tr>
<tr><td>Participant w</td><td>1</td><td>1</td><td>2</td><td>3</td><td>1</td><td>1</td><td>S 8%
U 1</td><td>75%
21</td><td>+67%
+20</td></tr>
<tr><td colspan="10">Had an understanding of the problem before he closed his eyes. Found it easy to enter into the VSM and see the situation as a picture and easy to see it as a sum. It was hard not to be distracted. Although he thinks it is a slow method, the process became easier. He was uncertain about the numbers when opening his eyes.</td></tr>
</table>

Table 76

Individual Scores and Metacognitive Reporting: Participants Absent: Session 1

<table>
<tr><th>No</th><th>a. (i)</th><th>a. (ii)</th><th>b. (iv)</th><th>b. (v)</th><th>c. (i)</th><th>c. (ii)</th><th>Process</th><th>Post-test</th><th>Difference</th></tr>
<tr><td>Participant b</td><td>1</td><td>3</td><td>2</td><td>3</td><td>1</td><td>1</td><td>S 19%
U 4</td><td>42%
11</td><td>+23%
+7</td></tr>
<tr><td colspan="10">Had understood the question before entering the VSM. Had no difficulty seeing the picture and adding the numbers to work with. The numbers were still usable after opening his eyes. He found the process hard in the beginning, but easier later on.</td></tr>
<tr><td>Participant c</td><td>1</td><td>3</td><td>2</td><td>2</td><td>3</td><td>3</td><td>S 36%
U 9</td><td>67%
17</td><td>+34%
+8</td></tr>
<tr><td colspan="10">Had understood the question before entering into the VSM. Found it a quicker way of dealing with the problem and found no difficulty to construct the picture, but to move or change things was hard. However, when she opened her eyes, the picture had disappeared and she was uncertain.</td></tr>
<tr><td>Participant k</td><td>1</td><td>3</td><td>1</td><td>3</td><td>1</td><td>1</td><td>S 19%
U 3</td><td>50%
9</td><td>+31%
+6</td></tr>
<tr><td colspan="10">Had understood the question before entering into the VSM. Found it a slower way of dealing with the problem and it became easier as time passed. He opened her eyes with a clear memory of the numbers and knew what to do.</td></tr>
</table>

Table 77

Summary of Participant Responses to Metacognitive Questionnaire

Class of metacognition	**Type of metacognition**	**Response option 1**	**Response option 2**	**Response option 3**
Metacognitive experience	Time spent	Slow (3)	Became faster (6)	Quick (7)
	Effort spent	Difficult (4)	Became easier (6)	Easy (2)
	Memory	No memory (2)	Uncertain (5)	Clear (9)
Metacognitive knowledge	Goal	Goal unclear (2)		Goal clear (14)
	Strategy	Unclear plan (5)		Clear plan (11)
	Preference for modality	Reading (3)	Listening (3)	Both reading and listening (10)

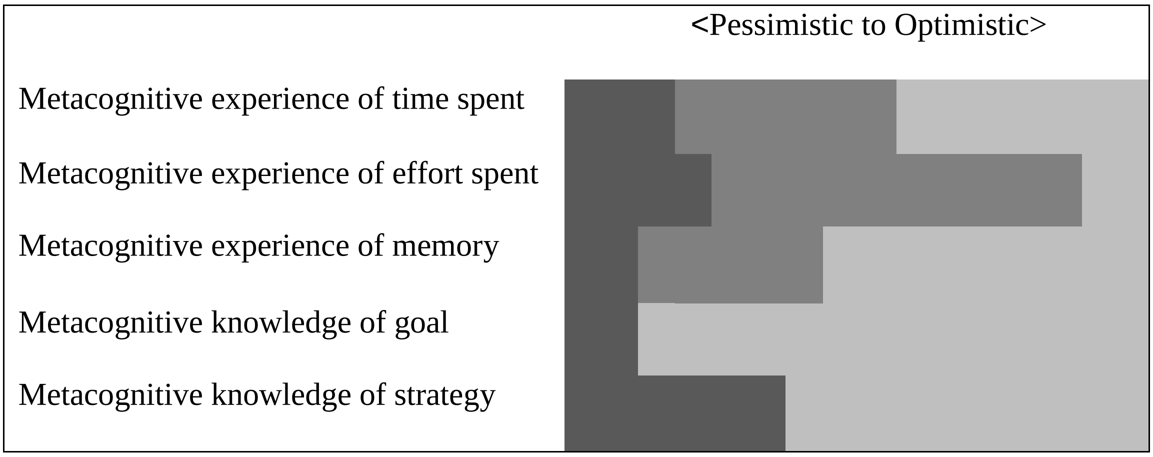


Figure 38. Metacognitive knowledge and experience from pessimistic to optimistic.

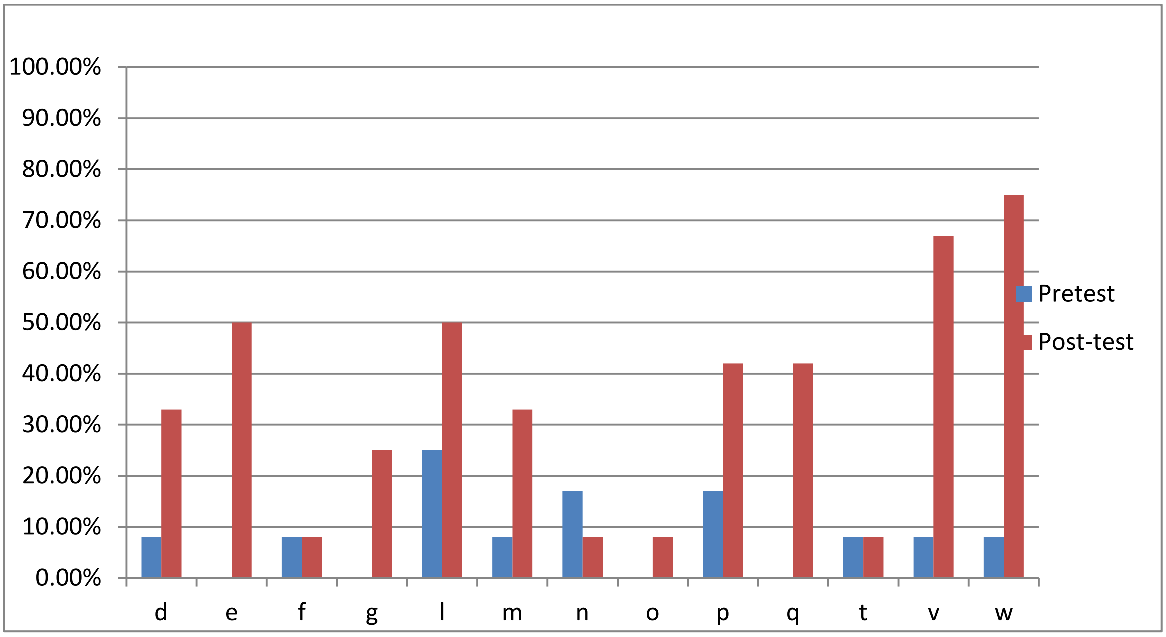


Figure 39. Individual scores: Participants present at both Sessions 1 and 5b.

The frequencies of incidents of understanding are calculated here in percentage points, each item out of a potential number of 9 incidents of understanding. This was done for the sake of comparability.

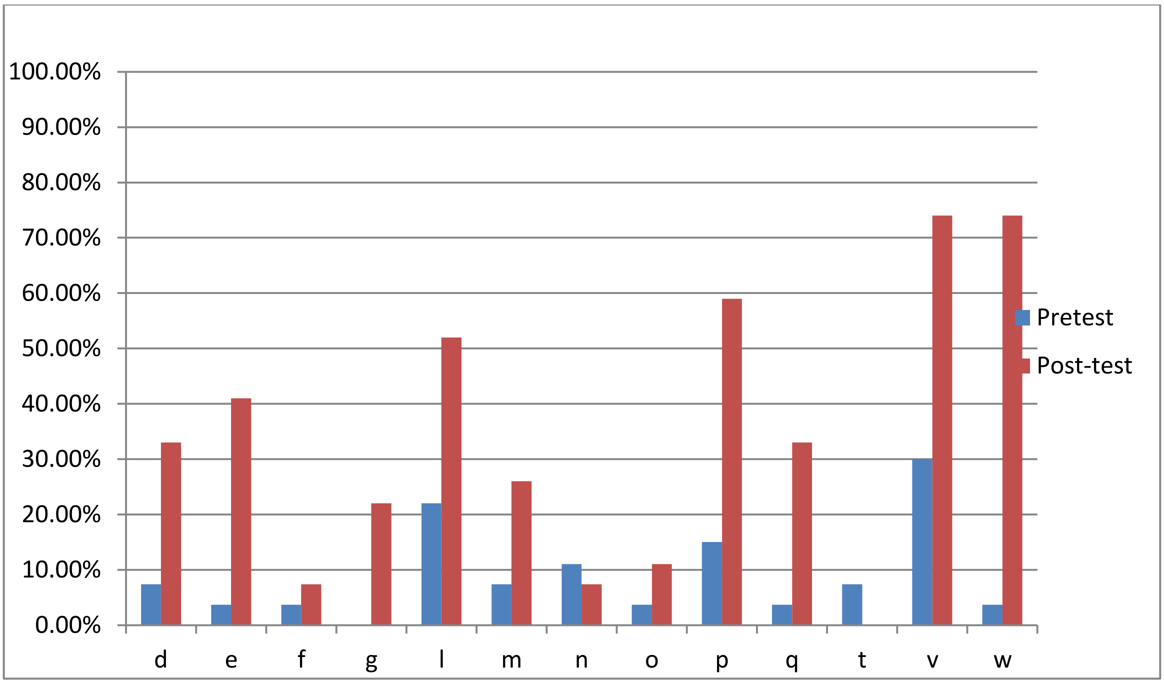


Figure 40. Individual understanding: Participants present at both Sessions 1 and 5b.

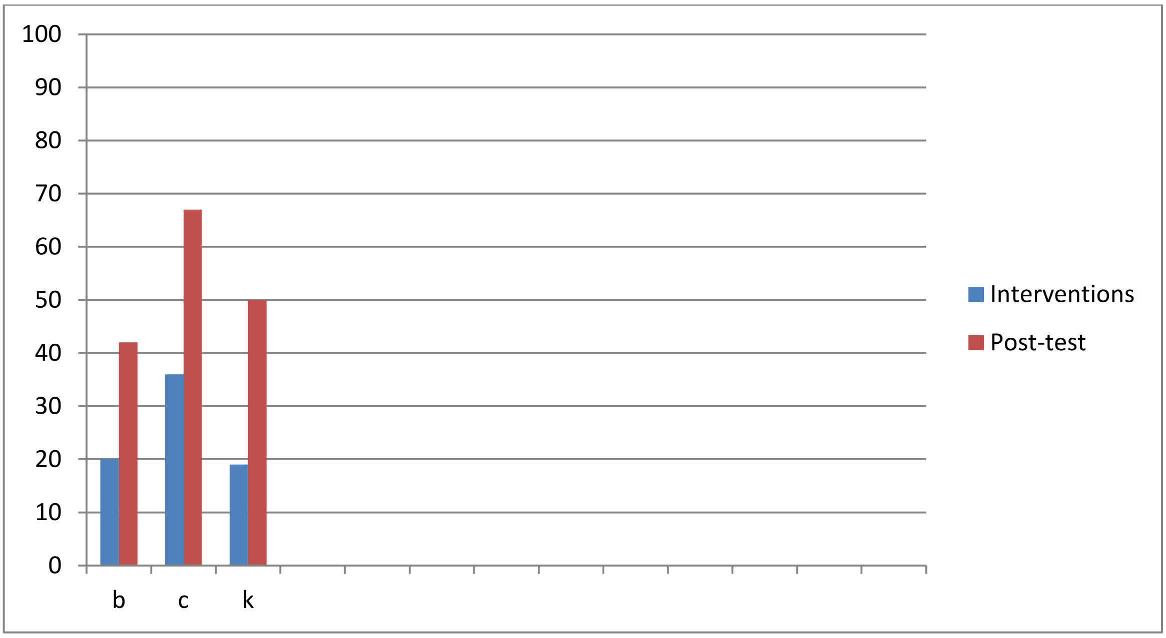


Figure 41. Individual scores: Participants absent at Session 1.

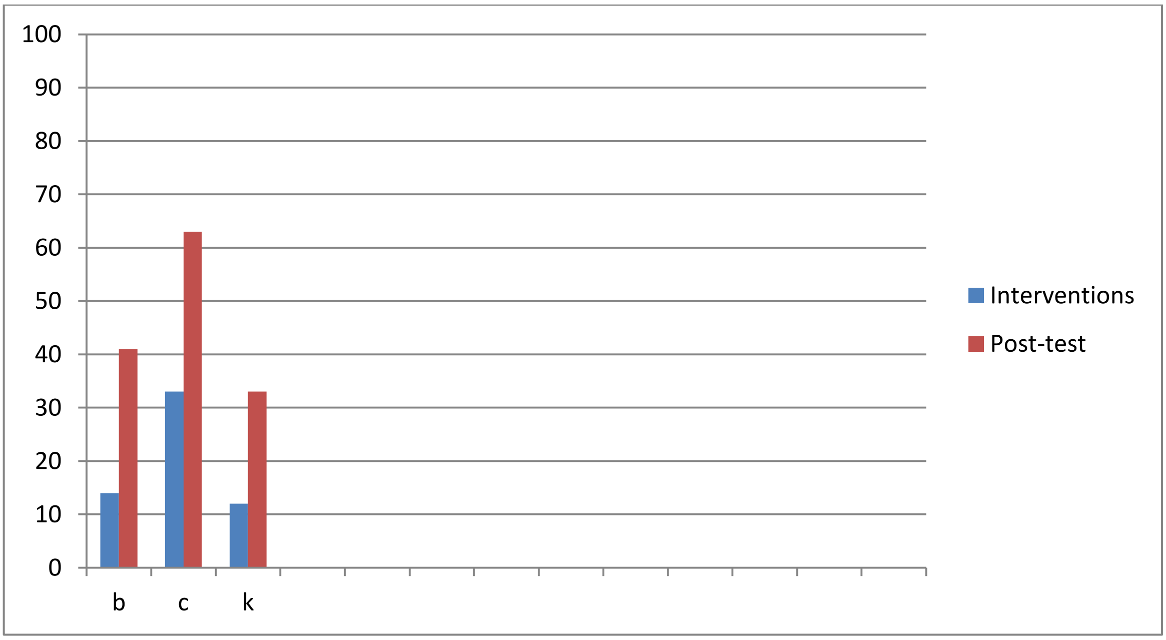


Figure 42. Individual understanding: Participants absent at Session 1.

APPENDIX D

Lesson Plan (format adapted from Wiggins & McTighe, 2011)

Mathematics: Grade 6: Understanding the Concept: Average **Problem situation.** It takes Sello 10 minutes by bicycle from home to Zap Store, which is 2,4km from his house. At what average speed (in m/s) is Sello cycling?
Stage 1: Desired Results

Specification of Content: Grade 6: CAPS requirements (DBE, 2012a)
• The idea of *average* mainly occurs within the two content areas, "Data Handling" and "Measurement". As an element of mathematical problems, *average* is a sub-concept of the broad concept *rate* (pp. 256, 261, 274, 291), where *rate* is defined as "comparing quantities of different kinds" (p. 15).
• The concept *average* uses the representational form "... per..." (pp. 233, 256, 261) or ".../...", for example, km per hour or R/kg.
• For the calculation of *rate*, a distinction is made between calculating the total, if given the rate per object, or calculating the rate per object, or calculating rate and then applying it to generate more information (p. 210).
• Situations where the concept *average* are used, are in contexts of volume and capacity (p. 256), mass (p. 261), length (p. 274) and statistics (pp. 233, 269), e.g. "average body temperature" (p. 186), "average time taken from... to..." (pp. 188, 266, 268), "...rate per country" (p. 233) and "average rainfall" (p. 269). Applications include price per object in money contexts (pp. 256, 261).
• The computational algorithms used to calculate average, are learned within the content area "Numbers, Operations and Relationships", under the topic "Division". The specific requirements at Grade 6 (p. 264) are listed below:
    o division of at least whole 4-digit by 3-digit numbers;
    o computations involving whole numbers and decimal fractions using any of the following methods:
        ➢ reciprocal relationships;
        ➢ long division;

<table>
<tr><td colspan="2">

- building up and breaking down;
- rounding off and compensating;
- using a calculator.

- The additional skills required in this lesson are:
  - the ability to interpret graphical data (p.10);
  - the ability to locate a position and direction on a map (p. 288); and
  - problem solving.

</td></tr>
<tr><td>

Teacher objectives

To create or confirm the understanding of:

- Speed as a relation between measures of distance and time;
- Actual speed as the changing rate at which an object is moving;
- Average speed as a calculated value typifying the relation between distance and time;
- Distance as dividend and time as divisor to calculate average speed;
- The position of units of distance and time in representing speed;
- The difference in terms, *changing speed* / *constant speed*; *actual speed* / *average speed*; and *acceleration* /*deceleration*.

</td><td>

Essential questions

- How do time, distance and speed relate to each other?
- How does actual travelling speed change or vary over a distance?
- How does changing speed differ from constant speed?
- How does constant speed differ from average speed?
- How can we accurately calculate average speed?
- How do we represent actual- and average speed in symbols?
- How do we represent actual and average speed graphically?
- How does actual speed differ from average speed?

</td></tr>
</table>

Student objectives (learning outcomes)

To have an understanding which enables them to:

- Decide when it would be appropriate to establish average speed, using distance and time (use-application dimension of understanding, Usiskin, 2012);
- Accurately calculate average speed, using distance as the dividend and time as the divisor (skill-algorithm dimension of understanding, Usiskin, 2012); and
- Represent average speed in the form of (a) numbers together with the units of distance and time in the correct notation, as m/s and (b) graphic images on two axes (representation/metaphor dimension of understanding, Usiskin, 2012).

To use the metacognitive skill of visual imagery, enabling them to:

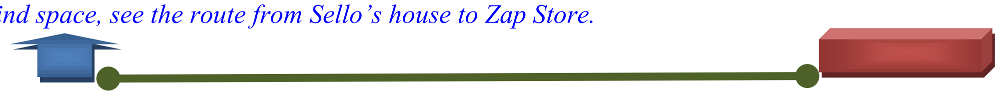
- Reconstruct the situation as a visual object in the virtual space of their minds;
- Attach the relevant number values to objects in the mental picture; and
- Move and manipulate these objects so as to make change(s) observable.

Stage 2: Teaching and Learning Plan

Part A: Teacher Guidelines for the Lesson: Sello goes to town.

Teacher reads out to the class: *It takes Sello 10 minutes by bicycle from home to Zap Store, which is 2,4km from his house. At what average speed is Sello cycling? Now close your eyes, relax, listen and follow the instructions as I am guiding you:*

In your mind space, see the route from Sello's house to Zap Store.



In your mind space, write down the distance in metres. Remember it is 2.4km. Write that in metres.

Now also in your mind space, write down the time it takes him in seconds. Remember it is 10 min.

In your mind space, see Sello cycling over the whole distance from beginning to end.

In your mind space, see Sello riding bit, by bit, by bit of the route each second... take enough time to see him riding, second... by second... by second...

Now you can open your eyes and do ***Question 1*** *on your worksheets:*

Question 1: **a. How many metres are in 2,4km?**

b. How many seconds are in 10 minutes?

c. Calculate how many metres Sello rides in a second.

Figure 43. Distance and time that Sello cycles from his house to Zap Store.

On your worksheets, look carefully at Options A and B. Think for yourselves which option shows Sello's speed in the best way. If you decide it is Option A, tick that box. If you decide it is Option B, tick that box. If you decide it is both, tick both boxes. If you decide that none of them show his speed in a good way, don't tick any box.

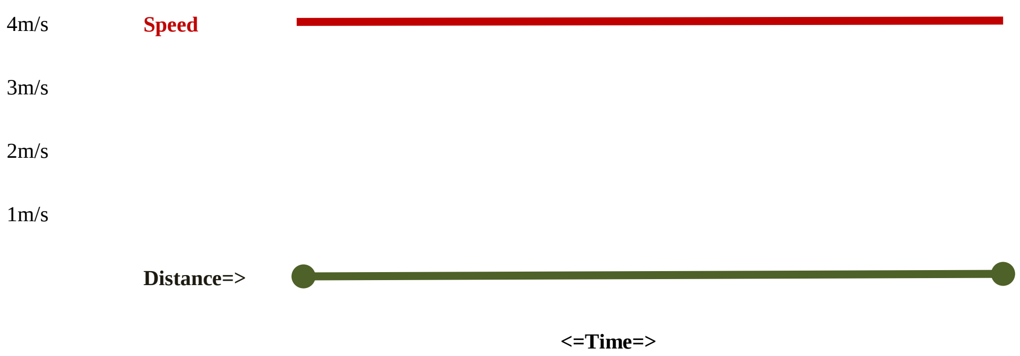


Figure 44. Option A: Sello went at a speed of 4m/s.

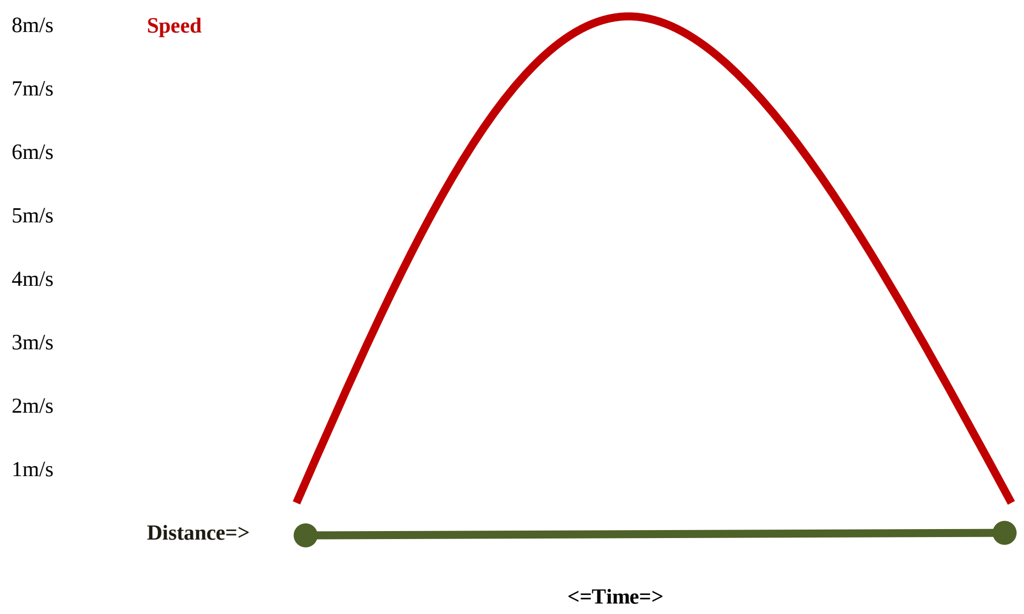


Figure 45 . Option B: Sello went at a speed of 4m/s.

Discuss: *Why would someone say that Option A is a true picture of his speed?*

Discuss: *Why would someone say that Option B is a true picture of his speed?*

Close your eyes, see Sello riding from his house to Zap Store. Relax, listen and imagine as I am guiding you. While Sello is cycling, look in your mind picture and see if there is any point at which Sello is going at 0m/s? Is there any time that Sello is going at less than 4m/s? Is there any time that Sello is going at more than 4m/s? Now open your eyes, and on your worksheets, answer **Questions 2, 3 and 4**.

As you know, very few roads are really straight, so, on your worksheets, look at the real route from Sello's house to Zap Store:

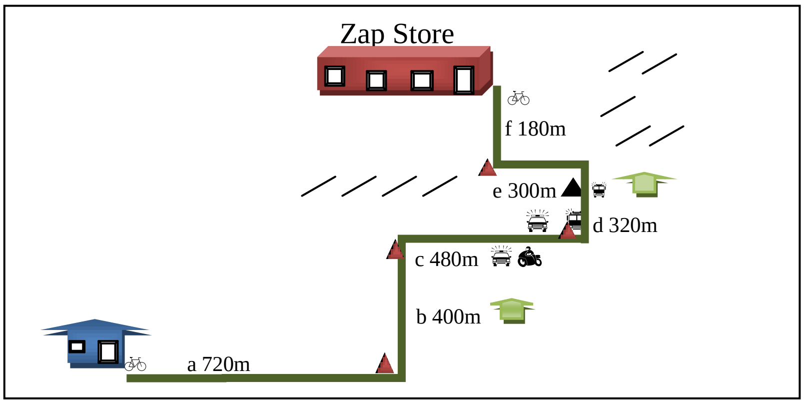


Figure 46 . Sello's real route from his house to Zap Store.

You can see that he cycled a certain distance, stopped at the corner, turned, cycled again, stopped and so on until he reached the store. The time it took him, is like this:

- *Segment a took him 120 sec*
- *Segment b took him 80 sec*
- *Segment c took him 120 sec*
- *Segment d took him 80 sec*
- *Segment e took him 100 sec*
- *Segment f took him 100 sec*

Now on your worksheets, do **Question 5** *, where you need to calculate Sello's average speed for each of the separate segments of his trip.*

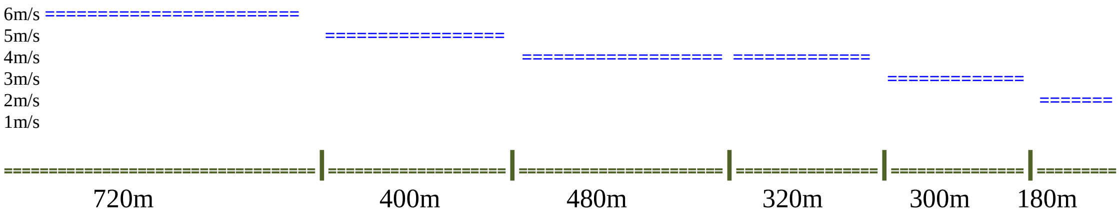
Discuss: *Why does the average speed differ for the various segments of the trip?*

Discuss: *Does the average speed differ from the actual (real) speed that he rode?*

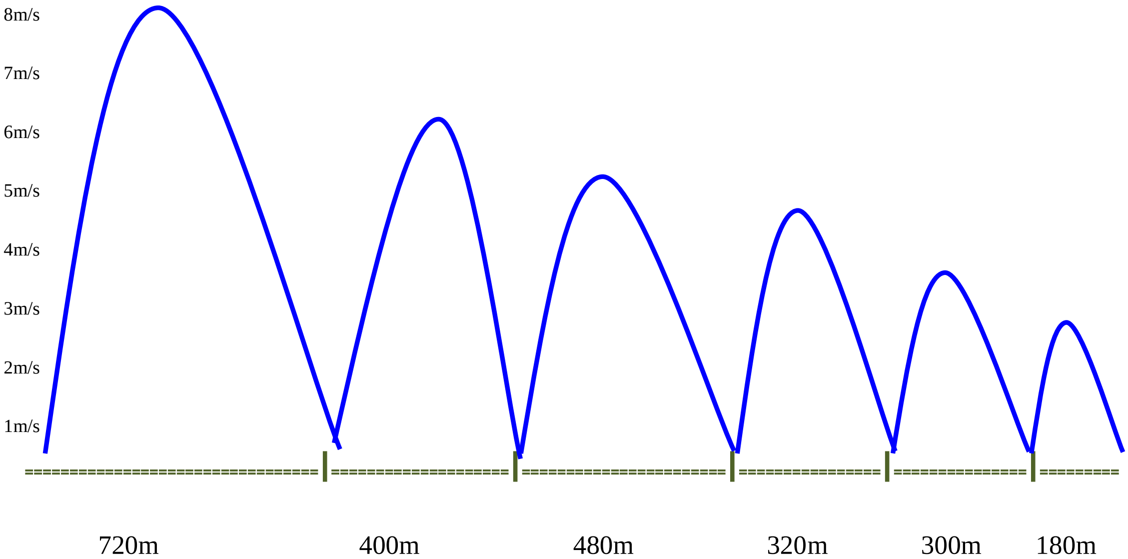
Do **Question 6:** *What was his lowest average speed, was it the same, or was it lower than his lowest average speed?* **Class to discuss.**

Do **Question 7:** *What was his highest speed, was it the same, or was it higher than his highest average speed?* **Class discuss.**

Look at the next two pictures. It is the same route, stretched out in one straight line.



*On your worksheets, at this picture (***Question 8***), above the distance line at the correct height, draw Sello's* ***average*** *speed, separately for each section.*



*On your worksheets, above the distance line at the approximate height (***Question 9***), draw the* ***actual*** *speed at which Sello cycled, separately for each section.*

On the worksheets, do **Question 10** *to find the total distance of all segments together.*

Then complete **Question 11** *to calculate Sello's average speed over the total distance.*

Complete **Question 12** *for both rows, and in both cases, illustrate by a single continuous line drawing each time, the terms that appear in that row.*

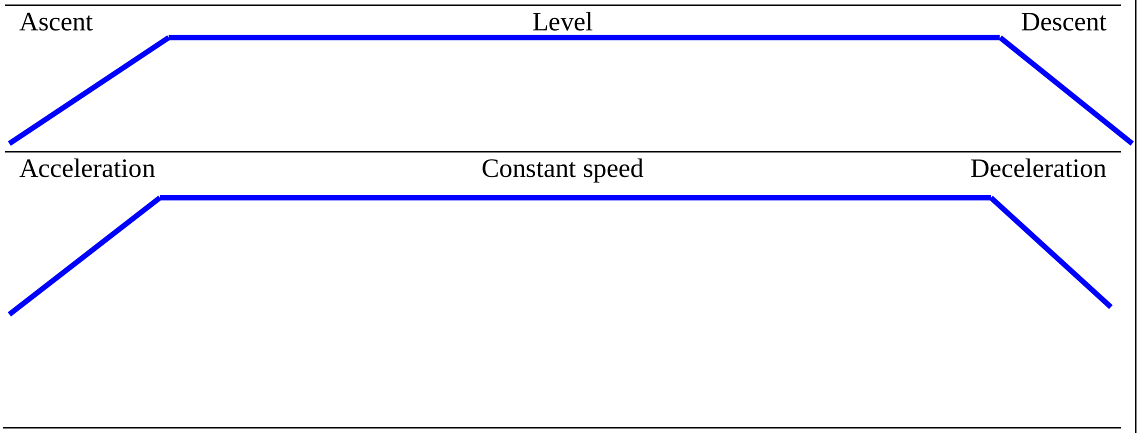


Figure 47. Continuous line drawings of two sets of terms.

Part B: Learner Worksheet: Sello goes to town.

It takes Sello 10 min by bicycle from home to Zap Store, which is 2,4km from his house. We are going to find out at what average speed is Sello cycling.

Question 1: **a. How many metres are in 2,4km?**____________________

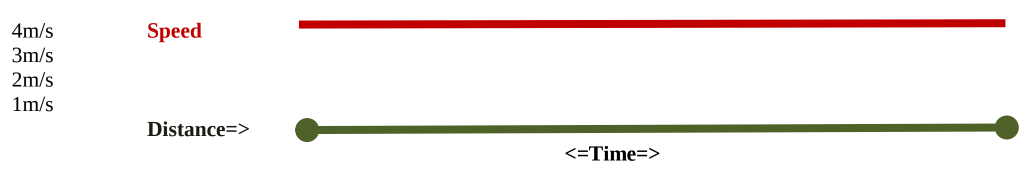
b. How many seconds are in 10 minutes? _________________

c. Calculate how many metres Sello rides in a second.

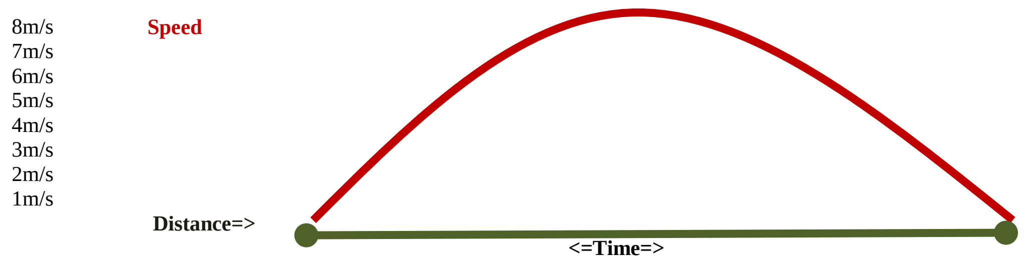
____

____

Option A: This is the speed at which Sello went Option A shows his speed best ☐



Option B: This is the speed at which Sello went Option B shows his speed best ☐

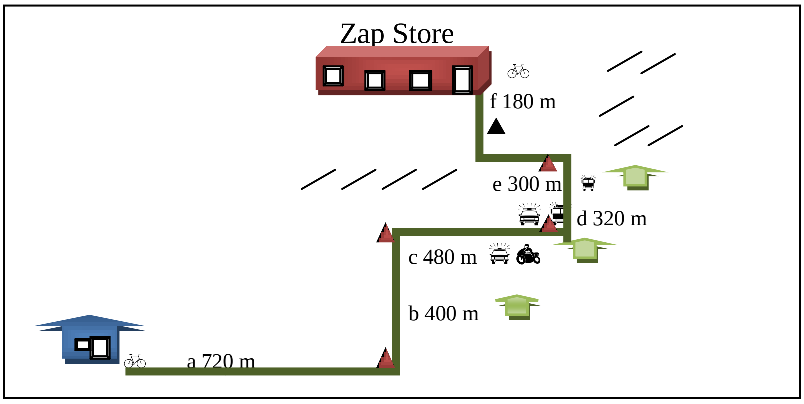


Question 2: Is there any point at which Sello is going at 0m/s? Yes☐ No ☐

Question 3: Is there any time that Sello is going at less than 4m/s? Yes☐ No ☐

Question 4: Is there any time that Sello is going at more than 6m/s? Yes☐ No ☐

Surprise! The real route from Sello's house to Zap Store actually looks like this:



The time it really takes him to cycle each segment, is given in the table below.

Question 5: In m/s, what is his average speed for each segment?

The time it takes Sello to cycle each distance	**My calculation of Sello's average speed for each segment**	**Answer**
a. 720m took him 120 sec		
b. 400m took him 80 sec		
c. 480m took him 120 sec		
d. 320m took him 80 sec		
e. 300m took him 100 sec		
f. 180m took him 100 sec		

Question 6: What was Sello's lowest actual speed? Was it the same, or lower than his lowest average speed?

Question 7: What was Sello's highest actual speed? Was it the same, or higher than his highest average speed?

____

Look at the route below. It is the same route, stretched out in one straight line.

6m/s
5m/s
4m/s
3m/s
2m/s
1m/s

720m | 400m | 480m | 320m | 300m | 180m

Question 8: Above the distance line at the correct height, draw the __average__ speed at which Sello rode, separately for each section.

8m/s
7m/s
6m/s
5m/s
4m/s
3m/s
2m/s
1m/s

720m | 400m | 480m | 320m | 300m | 180m

Question 9: Above the distance line at the approximate height, draw Sello's __actual__ speed, separately for each section.

Question 10: What is the total distance of all segments together?

____

Question 11: What was Sello's average speed over the total distance?

____

Question 12: In both rows below, and in both cases, illustrate by a single continuous line drawing, the terms that appear in that row.

Ascent	Level	Descent
Acceleration	Constant speed	Deceleration

Stage 3: Assessment Evidence

Marks are allocated, either according to the "Allocation of marks" memorandum, as set out in the first column, or considering flexible methods. Explanations accompany the allocation of marks in the right hand column.

Allocation of marks	**Explanation**
Suggested answers are in blue OR consider flexible methods; Correct calculations are allocated a red mark: 10 Understanding is allocated a green mark: 20 **Total: 30** 1. a. How many metres are in 2,4km? 2,4km = 2 400m (1) b. How many seconds are in 10 minutes? 10 min = 600 sec (1) c. Calculate how many metres Sello rides in a second. 2 400m (1) ÷ (1) 600 sec (1) = 4 (1) m/s OR metres in each second (1) 2. Is there any point at which Sello is going at 0m/s? Yes (1) 3. Is there any time that Sello is going at less than 4m/s? Yes (1)	1. a. and b. Correct conversion from km to m (1); from min to sec (1); c. Understanding speed as a relation between distance (dividend) and time (divisor); Correct calculation of 2 400 ÷ 600 = 4; Understanding that answer is represented in the relation of metres to seconds. ***Check that all learners understand up to now.*** 2. Understanding that riding starts and ends with standing still. 3. and 4. Understanding that higher and lower speeds make up and down for each other to find the average speed.

4. Is there a time that Sello is going at more than 4m/s? Yes (1) 5. In m/s, what is his average speed for each segment? a. 720m ÷ 120sec = 6m/s (1) b. 400m ÷ 80sec = 5m/s (1) c. 480m ÷ 120sec = 4m/s (1) d. 320m ÷ 80sec = 4m/s (1) e. 300m ÷ 100sec = 3m/s (1) f. 180m ÷ 100sec = 1.8m/s (1) 6. What was Sello's lowest speed, was it the same, or lower than his lowest average speed? Lower than 1.8m/s (1) 7. What was Sello's highest speed, was it the same, or higher than his highest average speed? Higher than 6m/s (1) 8. Above the distance line at the correct height, draw the **average** speed at which Sello rode, separately for each section (see teacher guidelines) (½), (½), (½), (½), (½), (½) 9. Above the distance line at the approximate height, draw Sello's **actual** speed, separately for each section) (½), (½), (½), (½), (½), (½) 10. What is the total distance of all segments together? 720m + 400m + 480m + 320m + 300m + 180m = 2400m (1) 11. What was his average speed over the total distance? 2 400m ÷ 600 sec (1) = 4m/s 12. In both rows below, and in both cases, illustrate by a single continuous line drawing, the terms that appear in that row (see teacher guidelines) (2), (2)	5. Based on the understanding in question 1, correct calculations of the average speed(s) for the various sections, earn the marks. The correct representation in m/s is not marked again. 6. and 7. Understanding that the lowest/highest average per segment are not the lowest/highest actual speeds that were reached (see 3. and 4.). 8. Understanding that the calculated average per segment is constant for the segment but may vary between segments. 9. Understanding that the actual speed is continuously varying for each segment, in direct relation to the specific segment average. 10. Correct calculation of the sum of the segment distances towards the total distance. 11. Understanding that the average for the total distance uses the total time as divisor. 12. Understanding that new terms "accelerate, decelerate, constant" correspond with familiar words like "ascending, descending, level".

Stage 3: Generic Learning Plan to Apply to Other Problems
"Ke na le plane" (Ek sien 'n plan; I see a way). **In the virtual space of the mind, *ke na le plane*.** **1. *Start with the problem situation that is described in words.*** Read the problem and say in your mind: I must find out_________________________ **2. *Take this problem situation into the virtual space of your mind.*** Close your eyes and keep them closed. Create a base for the situation. Fix everything that will remain the same throughout the event, onto the base. If a name was given in the problem to any of these objects, label them likewise. If a number value was given in the problem to any of these objects, label them likewise. Select those objects that will move or change during the event. Put the objects that will move or change, in one corner, ready to be fetched. If any of these objects were given a number value, labe them likewise. All the bits of information given in the problem, are now in one screen. Still with your eyes closed, slow play the event from start to finish. Keep your eyes closed. Watch the changes that happen as you slow play the event. Keep on asking the question that you need to answer. **3. *Return this problem as a mathematical calculation*** Open your eyes and write down in numbers what you have seen in the virtual space of your mind, checking with the problem that you are using the numbers accurately. Do your calculations and arrive at an answer, checking for precision. Repeat the question and give your answer, checking if it is a reasonable answer.

APPENDIX E

Letters to Participants

Letter to the Principal of the School

10 April 2014

INVITATION TO PARTICIPATE IN A RESEARCH ON EXPLORATION OF A METACOGNITIVE STRATEGY IN THE CONCEPTUALISATION OF MULTIPLICATIVE STRUCTURES FOR GRADE 6

Dear Sir

I am a doctoral student from the University of Pretoria investigating a metacognitive strategy that could benefit learners in the understanding of division as it applies in everyday contexts, or as it is popularly known as “word-sums”. The study is benevolent and can only benefit learner’s approach towards mathematics problems for the rest of their school careers.

As an English medium primary school, representative of South Africa’s population and following the CAPS guidelines, your school is ideally suited for such a research. However, the school’s participation is voluntary and should you choose not to participate or wish to withdraw your school’s participation at any time during the study, you are free to do so. The identity of the school is kept confidential except if you wish to have it published.

Should I gain your permission, you are requested to hand out a letter to Grade 6 learners requesting volunteer participants to attend five sessions on a Friday afternoon from 14:30-16:00 during which the use of division in equal sharing, in rate and in area problems will be taught and assessed according to the CAPS requirements, using a metacognitive strategy.

The recording of research data will take two forms:

a. I will keep a journal of all planning, lessons and events.

b. Learner work and assessments will be kept in a portfolio per individual, identified by a code name to protect their identity. Audio-taped responses will be transcribed and kept securely with the learner portfolios.

I will use the research results, in the form of a thesis, to meet the requirements for a doctoral degree in Assessment and Quality Assurance at the faculty of Education, University of Pretoria. The examiners and the academic community as well as learners and their parents will have access to the findings of the study before it is published. After publication the findings of the study could be used at different forums and for mathematics education development and by the Department of Education.

I am doing the study under the supervision of Dr Caroline Long (caroline long@up.ac.za) and Professor Sarah Howie (sarah howie@up.ac.za): Department of Science, Mathematics and Technology Education, Centre for Evaluation and Assessment, University of Pretoria. If you have any questions concerning the research study, please call any one of the above or myself on +27 83 484 8996 or e-mail me at ryna.duplooy@gmail.com.

If you as the representative of your school will allow Grade 6 learners to be approached for participation in this research, please sign this letter as a declaration of your consent.

PRINCIPAL'S SIGNATURE ……………………………………………..

DATE: ……………………………………………..

RESEARCHER'S SIGNATURE ……………………………………………..

DATE: ……………………………………………..

Signed: **Student** ____________________ **Date** ________________

Mrs MC Du Plooy

Supervisor ____________________ **Date** ________________

Dr MC Long

Letters to Parents

10 April 2014

INVITATION TO PARTICIPATE IN A MATHEMATICS RESEARCH STUDY

Dear Grade 6 Parent

I am a doctoral student from the University of Pretoria conducting a study on how learners can use their own brain power to cope better with division word sums. The experience and what they have learned here, may only benefit them now and for later years in Mathematics.

I have selected your child's school as my research site and the principal has granted consent for me to request volunteer participants from this year's Grade 6 learners. As participation is voluntary, learners can choose not to participate or to withdraw from the study at any time.

The study will be done during five Friday afternoons at school from 14:30 to 16:00 in 2014, the dates which will be announced later. I will assess and teach three mathematics problems per day while I am guiding them. Afterwards they will tell me individually how they worked the problems out. Children will be identified by code names to protect their identity. Their written work and the audio recordings of their comments will be kept safe and strictly confidential.

I will use the research results, in the form of a thesis, to meet the requirements for a doctoral degree in Assessment and Quality Assurance at the faculty of Education, University of Pretoria. The examiners and the academic community as well as learners and their parents will have access to the findings of the study before it is published. After publication the findings of the study could be used at different forums, for mathematics education development and by the Department of Education.

I am doing the study under the supervision of Dr Caroline Long (caroline long@up.ac.za) and Professor Sarah Howie (sarah howie@up.ac.za): Department of Science, Mathematics and Technology Education, Centre for Evaluation and Assessment, University of Pretoria. If you have any questions concerning the research study, please call me on +27 83 484 8996 or e-mail me at ryna.duplooy@gmail.com

Even with the reassurance that participation is voluntary and that your children and their identity are safe in this study, it is still required that the learners' parents or legal guardians give their consent should their child choose to participate. If you agree to the above arrangements, please sign this letter to declare your consent to the participation of your child in the study. .

NAME OF LEARNER

……………………………………………..

SIGNATURE OF PARENT/GUARDIAN

……………………………………………..

DATE

……………………………………………..

RESEARCHER'S SIGNATURE

……………………………………………..

DATE

……………………………………………..

Signed: **Student** ____________________ **Date** ________________

Mrs MC Du Plooy

Supervisor ____________________ **Date** ________________

Dr MC Long

Letters to Grade 6 Learners

10 April 2014

INVITATION TO GRADE 6 LEARNERS TO TAKE PART IN A MATHEMATICS RESEARCH

Dear Grade 6 learner,

I am doing a study to help learners to use their own very powerful brain power to do mathematics word sums, especially in division. There can be no harm in participating, you can only benefit from it, even for later years in mathematics. Will you please participate?

We are going to work at school for five Friday afternoons from 14:30 to 16:00. I will let you know the dates of these Fridays. It is your own choice to participate. Should you choose not to participate or wish to withdraw your participation at any time during the study, you are free to do so. I will give you a code name to keep your identity safe.

I can only work with 20 learners, therefore the first 20 learners that sign and return this form and whose parents also sign their letters, will be part of the study group. If you wish to take part, please complete the part below:

My **name** is ________________________________ and I want to participate in the study about really using my own brain power to do mathematics.

This is my **signature** _____________________________ and I signed this letter on the **date** _______________________________

Researcher signature ________________ **Date** ____________

Ryna du Plooy

Signed: **Student** _______________ **Date** _____________

Mrs MC Du Plooy

Supervisor _______________ **Date** _____________

Dr MC Long