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RELATING TEACHERS' CONTENT KNOWLEDGE TO LEARNER PERFORMANCE IN PROBABILITY

A dissertation submitted in fulfilment of the requirements for the degree of Masters of Education (Med).

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DECLARATION

I declare that this research report is my own work, and that those who contributed to its compilation are acknowledged. This report is submitted as partial fulfilment of the requirements for the degree of Masters of Education at the University of Pretoria. It has never been before been submitted for any other degree or examination at any academic institution. The sources of the materials that were used in this report have been referred to properly.

Signed ________________________ March 2015

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Lastly and most important, my heavenly Father without whom nobody and nothing is possible.

ABSTRACT

The poor performance of South African learners in Mathematics in recent national and international benchmark tests is concerning. Literature have repeatedly shown a link between the mathematical content knowledge (MCK) of teachers and the performance of the learners they teach. In an attempt to update and improve Curriculum 2005 (1997-2010), the National Curriculum Statement (NCS) was introduced in 2011. NCS included the Curriculum and Assessment Policy Statement (CAPS), which stipulates content to be taught as well as how it should be assessed in terms of four cognitive levels. CAPS included some previously non-compulsory topics (for example probability) which used to be part of Paper 3 in Curriculum 2005. This study examined the MCK of teachers ($n = 8$) on the topic of probability in relation to the performance of the Grade 11 ($n = 89$) and Grade 12 ($n = 75$) learners on the same topic. The MCK of the teachers was examined by investigating their qualifications, years of experience in teaching mathematics, professional development sessions (organised by the Gauteng Department of Education) they were involved in, as well as examining written responses on examination-type questions on probability (Test 1). The learners' performance was examined by investigating their written responses on the topic of probability in the final Grade 11 examination (November 2013) and the Grade 12 preparatory examination (September 2014). The poor performance from both teachers and learners was a matter of concern regarding teachers' MCK. Even though the teachers were all qualified (all had a teaching diploma or a degree), they only managed to score and average of 73% on Test 1. The teacher with the most teaching experience performed the best in the test, showing a strong MCK. The teachers felt that the professional development sessions that they attended did not contribute to the improvement of their MCK on probability. The performance of the learners with regard to probability was consistent with the poor performance of learners in mathematics benchmark tests, and they only managed to score an average of 48% in Grade 11 and a shocking 30% in Grade 12.

ACRONYMS

ANA	Annual National Assessment
CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
FET	Further Education and Training
GDE	Gauteng Department of Basic Education
MCK	Mathematical Content Knowledge
NCS	National Curriculum Statement
NSC	National Senior Certificate
OBE	Outcomes-based Education
PCK	Pedagogical Content Knowledge
SCK	Subject Content Knowledge
TIMSS	Third International Mathematics and Science Study

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CHAPTER 1: CONCEPTUALISATION OF THE STUDY

In this study, I investigated how the level of teachers' mathematical content knowledge (MCK) relates to the performance of the learners they teach. Based on literature, the knowledge domains that successful teachers need include, among others, subject content knowledge (SCK), general pedagogical knowledge, knowledge of the curriculum and pedagogical content knowledge (PCK). It is vitally important that teachers know the mathematics content they teach. MCK in this study refers to the mathematical content knowledge that teachers possess. The development of mathematics teachers should start with solid MCK as a foundation. The PCK and knowledge of instruction of the subject will develop with time and experience. The content knowledge base of teachers is important, as several studies have shown a positive link between teachers' MCK and the performance of the learners they teach. With recent changes in the mathematics curriculum in South Africa, specifically the recent implementation of the Curriculum Assessment Policy Statement (CAPS), mathematics teachers are faced with the challenge of teaching some sections of work that have never been taught in South African schools. This leads to some interesting questions regarding the content knowledge and specifically the MCK of teachers, as well as the performance of the learners they teach, on the new topic of probability. The research question then is: What is the relationship between Grade 12 mathematics teachers' MCK levels and the performance of the learners they teach on the topic of probability?

Teachers in South Africa now have to adapt to a new curriculum, which is challenging enough, and they also have to prepare to teach a topic in which they may have very little background (Wessels, 2008). The MCK of teachers was investigated by looking at the following characteristics:

- The tertiary qualifications of the teachers
- The years of experience in teaching mathematics of the teachers
- The professional development sessions in which the teachers participated
- The written responses given by teachers in a test consisting of examination-type questions related to the topic of probability

Relating teachers' content knowledge to learner performance in probability.

To link the MCK levels of the teachers to the performance of the learners, the learners' performance was investigated by examining their written responses to probability questions in the Grade 11 final examinations and the Grade 12 preparatory examinations.

The written responses of the teachers and learners were examined using the cognitive levels suggested in CAPS for assessing the cognitive skills necessary to answer the questions in all formal assessment tasks.

1.1 RATIONALE

Teachers' SCK is critically important for teaching successfully (Adler & Davis, 2006; Ball, Thames, & Phelps, 2008). In this study, the SCK that was investigated was that of mathematical content knowledge (MCK). MCK is one of the most important factors influencing successful teaching (Ball et al., 2008; Hill, Rowan, & Ball, 2005) and forms the foundation of teaching. Several studies have shown the link between teachers' MCK and the performance of the learners they teach (for example,Hill et al., 2005; Kahan, Cooper, & Bethea, 2003; Wu, 2005). According to Ball, Lubienski and Mewborn (2001), teachers do not have the minimum SCK to teach mathematics effectively. This is the case in other countries, as investigated by Kanyongo, Schreiber and Brown (2007), and in South Africa specifically with mathematics teachers (Wessels & Nieuwoudt, 2010).

The introduction of a new curriculum puts strain on teachers, as shown by Wessels and Nieuwoudt (2011) when they investigated the introduction of Curriculum 2005 in South Africa. In 2011, Department of Basic Education (DBE) updated and replaced NCS with CAPS and this might have put further strain on teachers.

The content of CAPS was updated to include topics such as Euclidean geometry, statistics and probability and not all mathematics teachers are familiar with these topics. In this study, the focus was on the topic of probability.

The investigation of teachers' MCK was important in this study because of the effect it may have on the performance of the learners they teach. National and international benchmark tests completed in South Africa, such as the Annual National Assessment (ANA), Third

International Mathematics and Science Study (TIMSS) and the final Grade 12 mathematics examinations have shown that learners perform poorly in mathematics (DBE, 2012).

1.1.1 Curriculum change in South Africa

In 1994, after the abolishment of apartheid, a new approach in education, namely Outcomes-based Education (OBE), was introduced. OBE was introduced in all schools in South Africa with the National Senior Certificate (NSC) being the exit point for Grade 12 learners. This curriculum came under scrutiny and many shortcomings were highlighted (Spreen & Vally, 2010). After severe criticism and implementation problems, the DBE introduced a new approach, called Curriculum 2005, in 1997 (Mouton, Louw, & Strydom, 2012).

Curriculum 2005 consisted of subject statements, learning programme guidelines and subject assessment guidelines (DBE, 2011). In 2009, the Minister of Basic Education in South Africa appointed a task team to review and make recommendations to Curriculum 2005. The amendment and development of mathematics curricula in developing countries are necessary to ensure learners have the knowledge and skills that they need to participate in the world (Mhlolo, Venkat, & Schafer, 2012). The task team, using Curriculum 2005 as a starting point, developed a new curriculum, and so the NCS Grades R-12 was introduced in 2011. NCS includes the Curriculum and Assessment Policy Statement (CAPS), national policy pertaining to the programme and promotion requirements of the NCS Grades R-12, and the National Protocol for Assessment Grades R-12.

The CAPS document addresses, among other things, aspects of Curriculum 2005 that impacted negatively on the quality of teaching in schools, updates Curriculum 2005 by making some changes to the content, and aims to provide clear specifications of what is to be taught and learned (DBE, 2011). By building on the previous curriculum, CAPS has not only updated Curriculum 2005, but also aims to provide clearer specification of what is to be taught and learnt on a term-by-term basis (DBE, 2011). This CAPS document also ensures that the curriculum addresses all the knowledge and skills the learners need to be effective members of South Africa as a developing country (DBE, 2011). It is noted by Naidoo (2012) that, when implementing a new curriculum, teachers should be made aware of the intentions of the changes made in order for them to take ownership of the changes and implement them successfully. When curricula are updated to include new topics, it is possible that some teachers may have no or inadequate MCK and/or teaching experience on these topics (Selvaratnam, 2011).

NCS was introduced to schools in phases over a period of three years with full implementation in 2014. In the FET phase, NCS for Grade 10 was implemented in January 2012, Grade 11 in January 2013 and Grade 12 in January 2014. The first Grade 12 CAPS examinations were written in September 2014.

The table below gives a summary of the development of the South African curriculum after the abolishment of apartheid until present.

Table 1. Curricula in South Africa

Type of curriculum	Date
Outcomes-based Education	1994
Curriculum 2005	1997
National Curriculum Statement	2011

The Grade 10-12 phase of CAPS for mathematics included, in many cases, the same topics as those included in Curriculum 2005, but new topics were also introduced. Curriculum 2005 consisted of three papers testing mathematical content, namely Paper 1 and Paper 2, which were compulsory for all learners who took mathematics in this phase, and Paper 3, which was non-compulsory and included topics such as Euclidean geometry, statistics and probability. The amended curriculum, CAPS, which consists of Paper 1 and Paper 2 only, now includes Euclidean geometry, statistics and probability. For the purpose of this study, I only discuss the topic of probability further on.

In CAPS, probability is introduced on a very basic level in the intermediate phase (Grade 4-6). Learners in this phase are introduced to the concept by using simple scenarios, and asking learners to calculate probability of events. As learners move to the senior phase (Grade 7-9), the scenarios become more complicated by including determining probability in compound events using two-way contingency tables and predicting relative frequency in simple events. In the FET phase (Grade 10-12), CAPS builds on the knowledge learners possess with regard to probability. Learners in this phase need to be able to use:

- Venn diagrams and contingency tables in three events to solve probability problems
- Tree diagrams to calculate the probability of consecutive or simultaneous events which are not necessarily independent
- The probability identity: $P\ (A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$
- The addition rule $(P\ (A\ or\ B) = P(A) + P(B))$ for mutually exclusive events, the product rule $(P\ (A\ and\ B) = P(A) \times P(B))$ for independent events and the complementary rule $(P\ (not\ A) = 1 - P(A))$
- The fundamental counting principle to solve probability questions

Probability is a very important topic to include in the curriculum because of its usefulness in the everyday life context of learners. It is also important as it plays a vital role in the development of critical reasoning, and basic knowledge of probability is required in many professions (Wessels & Nieuwoudt, 2011). The questions related to this topic are often contextualised to include scenarios that learners may encounter in their everyday lives (Wessels & Nieuwoudt, 2011).

1.1.2 MCK of teachers concerning newly introduced topics

I have been a mathematics teacher for seventeen years and mathematics head of department for five years. In literature, learners' performance is often linked to the teacher teaching mathematics. With the poor performance of learners in mathematics, this puts most mathematics teachers in a poor light. I have been involved in the Grade 12 final examination marking process as a marker and a senior marker over the last 10 years. I noticed during examination marking sessions that there were schools that had no learners passing mathematics. The question that came to my mind was: Surely not all the learners from a school are unable to pass, where does the problem lie, is it with teachers or learners? On a personal level, it is very important for me to understand the knowledge that mathematics teachers possess, as it may influence the way in which they teach, hence influencing the performance of learners. With the introduction of CAPS, my experience is that mathematics teachers in the FET phase have different challenges. Some of these challenges include teaching topics that was previously part of Paper 3. In this study, I investigated the MCK of teachers in relation to one of the previously excluded topics, namely probability. Probability was chosen as it has been my experience that many teachers have had little or no exposure to this topic on a secondary or tertiary level.

Probability includes questions that may incorporate all the cognitive levels (discussed in detail in Chapter 3) mentioned in CAPS for inclusion in assessment (DBE, 2011). CAPS specifies that, for Grade 11 and 12, probability should be taught for two weeks in Term 3 and tested in the final examination with a mark allocation of 15±3. This ratio of time spent on a topic versus mark allocation in examination is consistent with the other topics included in CAPS. Probability questions on Grade 12 level were included for the first time in the preparatory examinations in September 2014.

This study focused on the MCK of teachers, specifically the MCK the teachers possess on the newly introduced topic of probability. The performance levels of learners are related to the knowledge and skills they possess. This performance is very important for learners, teachers and ultimately for South Africa as a developing country. In order to grow as a nation, it is essential to produce learners who have the skills to make a contribution to the community by not only knowing mathematics but also being able to use mathematics (Mullis et al., 2003).

1.2 Problem Statement

The performance of South African learners in benchmark tests (national as well as international) is very poor (Mullis et al., 2003). In an attempt to rectify this, various curriculum changes have been made over the last ten years. Studies have shown that changes in curricula can be a challenge for some teachers (Ramnarain & Fortus, 2013). When these changes include new topics being introduced, the challenges teachers face intensify. Research has shown the link between the performance of learners and the knowledge teachers possess and therefore these changes in curricula may influence both teachers and learners in the same way.

The MCK of teachers has been discussed and researched for at least the last four decades. The initial literature review showed that the performance of learners depends, among other things, on the MCK of the teachers teaching them (Wu, 2005; Hill et al., 2005; Betts, Zau, & Rice, 2003). To gain the necessary MCK that may lead to deep understanding of the content, teachers need MCK of all the mathematics topics in the school curriculum as well as an additional number of years of further studies in tertiary mathematics (Ball et al., 2008). It is also important to note that the specialised MCK that teachers need to teach is different from common MCK. Common MCK refers to the doing of the mathematics itself, while specialised MCK refers to the ability of teachers to teach the mathematics the learners need to learn (Ball et al., 2008; Copur-Gencturk & Lubienski, 2013). Copur-Gencturk and Lubienski (2013) confirm that teachers need a combination of common MCK and specialised MCK to have a successful practice. For the purpose of this study, only the common MCK was investigated.

The MCK involved in the entire curriculum for Grade 12 mathematics is vast, and therefore would have been too time-consuming to investigate. As mentioned, the focus of the study was on the topic of probability as it is newly included in CAPS.

In Curriculum 2005, probability was part of Paper 3, the non-compulsory mathematics NSC examination and was therefore not presented in all schools, or even fully at those schools that offered Paper 3, since only a few learners wrote Paper 3. With the introduction of CAPS, the topics previously included in Paper 3 were incorporated into Paper 1 and Paper 2. These topics, which include probability, are now compulsory and must be presented in all schools that present mathematics as a subject. The implication of this change is that not all teachers have the necessary experience to teach probability and this has led to the research question of this study.

1.3 Research Questions

The research in this study was done to build on the knowledge already available on the topic of teachers' content knowledge and the performance of learners (Mertens, 2005). In an attempt to contribute to this existing knowledge, the following research question for this study was established.

1.3.1 Primary research question

How does the level of teachers' MCK relate to the performance of the learners they teach on the topic of probability?

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1.3.2 Secondary research questions

1.3.2.1 What is the level of MCK of teachers in probability, based on the cognitive levels recommended by the CAPS?

1.3.2.2 What are the challenges that teachers experience in relation to probability?

1.3.2.3 How do the tertiary training and experience levels of mathematics teachers relate to the MCK they possess?

1.3.2.4 How can learners' performance in probability be described in terms of the cognitive levels recommended by CAPS?

1.4 DEFINITION OF TERMS

In this section, I define the terms used in this study.

1.4.1 Teachers

The teachers referred to in this study are teachers in four specifically selected government schools in the Tshwane South district of Gauteng, teaching mathematics to learners in Grade 11 and 12.

1.4.2 Learners

The learners referred to in this study are Grade 11 (in 2013) and Grade 12 (in 2014) learners from four specifically selected government schools in the Tshwane South district of Gauteng, taught by the teachers participating in the study.

1.4.3 Mathematical content knowledge

The term MCK in this study refers to the content knowledge that the teachers possess concerning the topic of probability.

1.4.4 Curriculum assessment policy statement

CAPS refers to the official curriculum document of the South African government outlining all the concepts to be taught and assessed at school level.

1.4.5 Cognitive levels

The levels, based on CAPS taxonomy, used to rank assessment questions based on the level of skills needed to answer questions correctly. These levels include knowledge, routine procedures, complex procedures and problem solving.

1.4.6 Test 1

Test 1 is a data collection instrument in the form of a question paper set by the researcher that included examination-type questions on probability. Test 1 included questions on all cognitive levels and was completed by the participating teachers.

1.4.7 FET phase

The FET phase is the last phase of basic education at a secondary school level in South Africa. It includes Grade 10, Grade 11 and Grade 12.

1.4.8 Interviews

The interview is a data collection instrument used in this study. Semi-structured individual interviews were conducted by the researcher with all the participating teachers.

1.4.9 Preparatory examination

The preparatory examination is an examination, normally written at the end of August and beginning of September, that precedes the final Grade 12 examination where learners are assessed on all the work in the CAPS document. It is used as a data collection instrument in this study.

1.5 Possible Contributions of the Study

The study investigated the MCK of teachers and the performance of the learners concerning the newly introduced topic of probability.

The analysis of the MCK levels of teachers regarding probability can assist in the development of training material for mathematics teachers in the Grade 10-12 phase by tertiary education facilities and the DBE.

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This study may be able to inform developers of training sessions on methods, time and frequency of development sessions of in-service teachers.

This study may also reveal some misconceptions related to the topic of probability with relation to teachers' and learners' understanding of the topic.

1.6 SIGNIFICANCE OF THE STUDY

My study sheds light on the MCK of teachers and the performance of learners on the topic of probability. The results indicate not only what teachers know about probability but also what level of mastery they have with regard to the cognitive skills needed to answer questions. The results from the written work of the learners give an indication of their level of performance. This performance level is linked to average scores and cognitive levels needed to solve the question.

1.7 STRUCTURE OF THE DISSERTATION

After the introduction of the study in Chapter 1, Chapter 2 of this report discusses some of the literature related to the MCK of teachers and the performance of learners on the topic of probability. Research conducted on the knowledge of teachers is widespread and includes national and international studies. In the literature review, topics such as the domains of teachers' knowledge, the MCK of teachers, the link between teachers' knowledge and learners' performance, cognitive levels involved in assessment and the professional development of teachers are discussed.

Chapter 3 discusses the conceptual framework of the study based on the cognitive levels linked to assessment as described in the CAPS document.

In Chapter 4, the report discusses the research design and methodologies employed in this study. This is where the sample, the data collection instruments and data analysis strategies are discussed.

The instruments used to gather data to answer the research questions of the study are discussed in Chapter 5. This chapter includes discussion of the instruments along with a detailed analysis of the instruments.

Chapter 6 includes the data presentation and analysis, incorporating the quantitative and qualitative approaches used in this study. The chapter ends by drawing conclusions on the analysis of the data.

The interpretations of the analysed data are done in Chapter 7. Conclusions are drawn in relation to the research question and finally recommendations are made for further studies within this field.

CHAPTER 2: LITERATURE REVIEW

In this chapter, I discuss some of the literature concerned with teachers' knowledge as well as the link it has to the performance of learners in mathematics. I started the literature review by broadly looking at the knowledge domains of teachers. The focus of the study, however, is specifically on the MCK of teachers and thus the first part of this chapter looks at this in some detail. As I was also interested in the performance of learners, specifically in mathematics and in Grade 11 and 12, the chapter continues by examining how learners perform. In order to measure the MCK of teachers and the performance level of learners, it was important to look at literature related to the cognitive levels used to rank assessments. How the current literature relates to the research questions of the study is discussed in this chapter under the following sub-headings:

- The domains of teachers' knowledge
- The MCK of teachers
- Teachers' MCK and the performance of learners
- Cognitive levels linked to assessment
- Professional development of teachers

2.1 DOMAINS OF TEACHERS' KNOWLEDGE

Teaching is a highly complex activity, with interaction between the teacher, the learner, the environment and the subject matter (Mason & Johnston-Wilder, 2004). Teachers' content knowledge is complex and multi-dimensional and has an impact on teaching and learning. Teaching as an activity can only be successful if teachers are effective (Walshaw, 2012). Various factors influence how successful and effective a teacher is. These include, among others, the knowledge they possess, the way they manage learners in a class environment and how they teach (Ball et al., 2001; Shulman, 1987; Walshaw, 2012). In order to get a complete picture of the knowledge of teachers, it was important to look at the domains of teachers' knowledge as found in literature. The knowledge teachers possess, according to Shulman (1987), involves different domains, such as content knowledge, general pedagogical knowledge, knowledge of the curriculum, knowledge of learners and pedagogical content knowledge (PCK). It must, however, be noted that these different domains are not separate entities, because in practice they are integrated.

As teacher knowledge consists of different domains, the content knowledge of teachers is multi-dimensional. Plotz, Froneman and Nieuwoudt (2012) devised a model on teachers' knowledge. According to them, knowledge includes understanding of procedures, concepts, representations and reasoning as part of an integrated content knowledge structure that teachers need to possess to teach effectively. To enhance their content knowledge structures, teachers need to reflect on their own practices and level of understanding (Ajayi, 2014; Hertzog & O'Rode, 2011). The development or expansion of a sound content knowledge structure is important when a curriculum changes, particularly when new topics are included. When Hill, Schilling and Ball (2004) comment on the diversity of domains of teachers' knowledge, they build their model on that of Shulman (1987). They mention factors such as overall intelligence, knowledge of teaching and knowledge of the content. The ability to solve mathematical problems, the construction of questions for learners and application of content knowledge to teaching are also mentioned as part of teachers' knowledge (Ball, 1990; Burgess, 2010; Hill et al., 2005; Shulman, 1987).

Figure 1 depicts the many domains that ultimately contribute to the knowledge that teachers possess. The content domains include knowledge of mathematics, pedagogy, the curriculum and the learners (Kanyongo & Brown, 2013; Kleickmann et al., 2013). There are, however, other factors such as values, intelligence and context involved in the knowledge that teachers need to possess. When considering the figure, it is important to note that it is simply a representation of the different domains contributing to teachers' knowledge. The figure depicts only some of the domains involved in teachers' knowledge and does not illustrate the relative relationship between or the importance of individual domains.

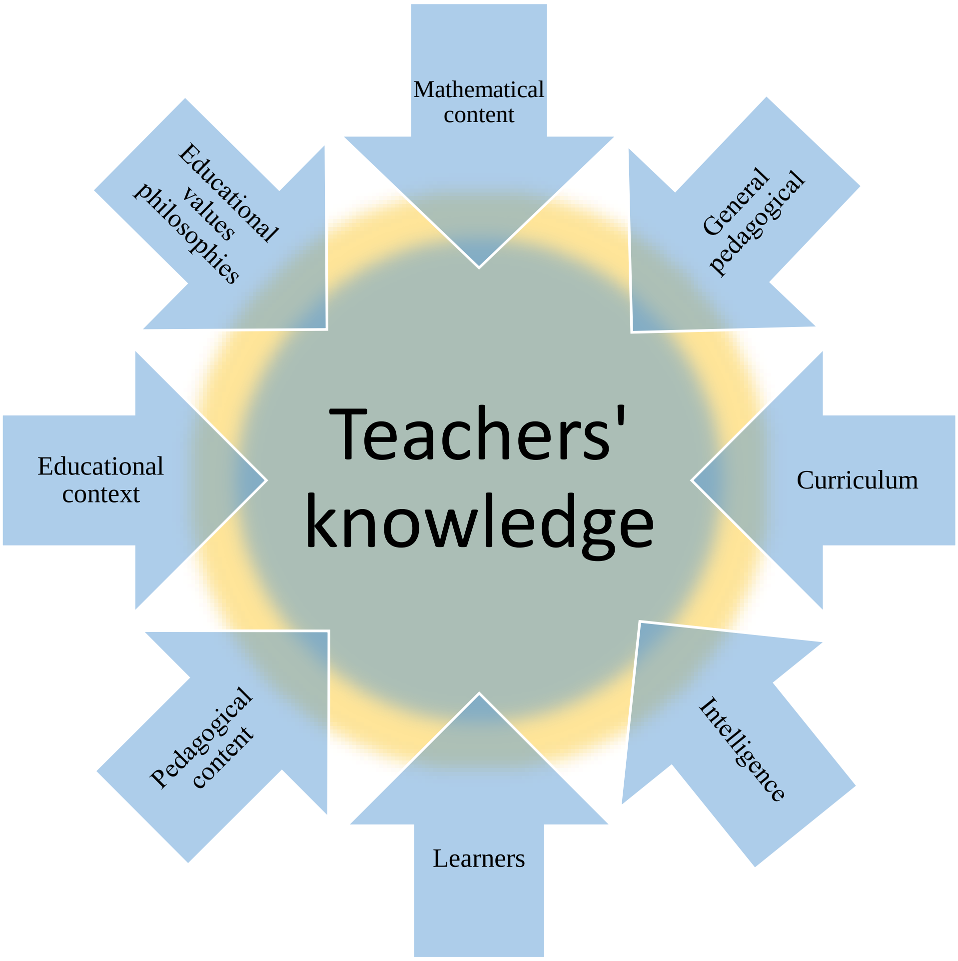


Figure 1. Domains of teachers' knowledge (as seen by Even (1993), Kanyongo & Brown, (2013), Kleickmann et al. (2013) and Shulman (1987))

When teachers' knowledge is mentioned in recent literature, there seems to be a focus on two of these domains: PCK and MCK. PCK has been the focus of many recent studies (Adler, Pournara, Taylor, Thorne, & Moletsane, 2009; Ball et al., 2008; Shulman, 1987; Wu, 2005). PCK is defined by Shulman (1987, p. 8) as the "special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding". In this study, however, the focus was on the MCK of teachers, as it is one of the factors directly linked to learners' achievement (Hill et al., 2005; Mogari, Kriek, Stols, & Iheanachor, 2009; Peressini, Borko, Romagnano, Knuth, & Willis, 2004). A study conducted in KwaZulu-Natal showed that mathematics teachers do not know the mathematics they are supposed to teach at Grade 12 level and therefore they fail to give learners the opportunity to engage with the content on a higher level (Bansilal, Brijlall, & Mkhwanazi, 2014). Similar results were reported by Spaull (2013) in a report commissioned by the Centre for Development and Enterprise – he reports that mathematics teachers have below basic levels of MCK.

Brijlall and Isaac (2011) revealed a positive connection between the teachers' MCK and their classroom practice. According to them, classroom practice includes the atmosphere in the class, the relationship and interaction between the teacher and the learners, as well as the way in which the teacher designs learning activities. A study conducted in Chicago, Europe, Lesotho, Botswana and Namibia indicated a strong connection between the MCK teachers possess and the classroom activities they design (Hawk, Coble, & Swanson, 1985; Kanyongo et al., 2007). The good pedagogical practices employed by teachers with a high MCK include individual teaching approaches for individual learners, the use of practical equipment, using contextual examples in problem solving and presentation of mathematical material to learners (Ball, Hill, & Bass, 2005; Baumert et al., 2010; Hawk et al., 1985; Kanyongo & Brown, 2013; Mogari et al., 2009).

Even though this study focused on MCK, it was important to get an understanding of the other domains contributing to the knowledge that teachers possess. This background will enable the reader to better comprehend the complexity and interactions of the domains needed to be an effective teacher. In the next section, I discuss the literature specifically related to the MCK of teachers.

2.2 MCK of Teachers

MCK is a competence that teachers need when teaching mathematics. Bransford, Brown and Cocking (2000) state that, to gain competence, one needs factual knowledge, understanding of these facts and the ability to organise the facts to enable retrieval and application thereof. The improvement of teachers' MCK has become an important topic of discussion in mathematics education (Adler et al., 2009; Hertzog & O'Rode, 2011; Wu, 2005).

According to Wu (2005, p. 6), "…the most important step in improving mathematics teaching is to bolster teachers' content knowledge by directly teaching them the mathematics they need in the classroom". Wu (2005) further attests that the biggest challenge in becoming a quality mathematics teacher is to have a good MCK. Wu (2005) elaborates on this by saying that it is vitally important that teachers must know the mathematics they teach. He explains that the education of mathematics teachers should start with solid MCK as a foundation, while PCK and instruction of the subject can follow over time through experience (Ball et al., 2008; Mogari et al., 2009). Wu (2005) highlights that a strong content foundation allows more effective pedagogical possibilities, which essentially means that teachers with a good MCK are able to create better pedagogical opportunities for the learners they teach.

Gaining MCK is a multi-faceted process that includes cognitive processes like problem solving, making connections and using various representations, as well as having procedural and conceptual knowledge and skills (Plotz et al., 2012). Plotz et al. (2012, p. 75) define problem solving as "engaging in a task for which the solution method is not known in advance". The understanding of mathematics is based on the connections made between different ideas, and the more connections are made in a network of ideas, the better the understanding (Plotz et al., 2012). In that way, the level of teachers' MCK fundamentally depends on their understanding of mathematics. The capability to present a concept in different ways also indicates a deep understanding of the concept and helps to conceptualise mathematical connections (Mhlolo et al., 2012). Being able to make connections implies the ability to make connections between mathematics and contexts outside of mathematics, and the interconnectedness inside mathematics (Blum, Galbraith, Henn, & Niss, 2007). Mathematics is therefore contextualised as it is placed in the context of real life rather than the context of mathematics. The skills of problem solving and making of connections develop as the teacher gains experience (Burgess, 2010; Even, 1993) or engages actively with the relevant resource materials and collaborates with other teachers (Wessels & Nieuwoudt, 2013).

In order to answer the research questions of this study, the MCK of the teachers needed to be measured. Some of the methods used in research include: written responses of teachers to examination-type questions (Bansilal et al., 2014), qualification level (Mogari et al., 2009) and the number of mathematics courses taken on tertiary level (Betts et al., 2003).

In my study, I approached the investigation of the MCK in two ways. I investigated the MCK of teachers by looking at their qualification level, as Betts et al. (2003) did in their study. In addition to this, I used a similar approach to that of Bansilal et al. (2014) by investigating the written responses of teachers on examination-type questions on the topic of probability that included all cognitive levels of assessment.

2.3 TEACHERS' MCK AND LEARNERS' PERFORMANCE

The fundamental aim of teaching mathematics is to empower learners with the knowledge they need for further education and training and to help them to make a meaningful contribution to the society in which they live (DBE, 2003). According to Barnes and Venter (2008), the aim of teaching mathematics is to equip learners with problem solving strategies, skills, knowledge and confidence to use mathematics to solve problems that the learners encounter in their community.

There seems to be a common thread in the motivation of studies linked to the MCK of teachers. This motive appears to be the global issue of learners' consistent poor performance in mathematics (Geary, 2011; Kalloo & Mohan, 2012; Mogari et al., 2009). It is thus important that, when investigating the importance of MCK of teachers, one must also take into account the influence that the teachers' MCK has on the achievement of the learners they teach (Hill et al., 2005). Research has found a connection between the MCK teachers possess and the performance of the learners they teach (Blömeke & Delaney, 2012; Tchoshanov, 2011). In similar investigations into the MCK of teachers in Chicago, Hawk et al. (1985) found that the learners (Grade 6 to Grade 12) of mathematics teachers with a good MCK achieved better results than learners of teachers who were taught by teachers with a lower MCK level. These results were confirmed elsewhere: in Lesotho (Mogari et al., 2009) and in Southern African primary schools in Lesotho, Namibia and Botswana (Kanyongo & Brown, 2013). A study done in Gauteng by Mji and Makgato (2006) to investigate the poor performance of South African Grade 12 learners in TIMSS showed that one of the factors that directly influences the performance of learners in mathematics is the MCK of their teachers. Peressini et al. (2004) concur with these findings and argue that teachers without a deep MCK base will not be in a position to provide the necessary learning opportunities for learners. Learners need opportunities where they can actively engage with the mathematics content in order to construct meaning and develop a deep understanding of mathematics (Fosnot, 1996; Mhlolo, 2013). To develop and facilitate these opportunities for learners, teachers must have a good MCK base.

Researchers have used different strategies to investigate the link between teachers' MCK and the performance of the learners they teach. To investigate this, I used various techniques previously mentioned in literature. These techniques include the examination of teachers' qualifications (Betts et al., 2003; Mogari et al., 2009; Kahan et al., 2003) and the number of years of experience of the teachers (Burgess, 2010; Even, 1993). In section 2.3.1, I look at research related to teachers' qualification and then in section 2.3.2, I investigate the research done on the number of years of teaching experience of teachers and how that influences the performance of learners.

2.3.1 Teachers' qualifications

Teachers' teaching qualifications have been shown to contribute to their students' academic achievement (Fakeye, 2012). Mogari et al. (2009) investigated the MCK of teachers by linking it to the level of their qualification. The qualifications of teachers can be investigated in more depth, as done by Kleickmann et al. (2013). In their study, they looked at the number and level of mathematics courses taken by teachers in their tertiary education and found that more courses on a higher level lead to better MCK. Similar work done by Betts et al. (2003) confirm that the number and level of mathematics courses completed by teachers influences their MCK. A study done in Nigeria suggested that teachers teaching a subject should be trained and certified in the subject they teach as they have specific knowledge related to that subject (Amuche & Musa, 2013).

2.3.2 Teachers' years of teaching experience

Some teachers are more experienced in their profession than others. Teaching experience has been shown to be an indicator of the MCK teachers possess. As teachers gain experience, they have been shown to gain MCK as well (Burgess, 2010). In an investigation done by Harris and Sass (2011) to promote student achievement, they found that experienced teachers are more effective teachers. For the purpose of this study, looking at probability as newly introduced topic, it is important to consider the experience levels of teachers (Burgess, 2010). Mogari et al (2009) found that teachers experience became constant between five and eight years. A recent study on conceptual understanding of science teachers on topics newly introduced into a curriculum showed that teachers lack the ability to use appropriate examples to link content knowledge to context (Ramnarain & Fortus, 2013). Both experienced and inexperienced teachers may struggle with conceptual understanding, especially when dealing with a new topic. Research investigated this when Wessels and Nieuwoudt (2010) showed that, even though teachers believe that they possess enough MCK to answer questions in probability, most teachers involved in the study could not apply MCK to answer contextual questions.

2.4 Cognitive Levels Linked to Assessment

CAPS specifies that all formal assessment tasks must be analysed according to the cognitive levels mentioned in the document (DBE, 2011). These cognitive levels are based on the cognitive domains suggested in the assessment frameworks and specification developed after the TIMSS study of 1999 (Mullis et al., 2003). The cognitive levels are discussed as part of the literature review as they are linked to the performance levels of teachers and learners involved in this study.

The first cognitive domain discussed by Mullis et al. (2003) is knowledge. Mullis et al. (2003) go from the assumption that all mathematical reasoning ultimately relies on knowledge. The more knowledge a learner can recall, the better chance they have of solving problems. CAPS refers to this knowledge as first level knowledge. Knowledge is described as straight recall and the identification of correct formulas on an information sheet (DBE, 2011). There is a similar description of skills involved in this cognitive level by Mullis et al. (2003) when they describe the skills as recall of definitions, vocabulary, units, properties of numbers and conversion of entities that are mathematically equivalent as well as the knowledge of algorithmic procedures (these include ×, +, − and ÷). For the purpose of my study, the cognitive level of knowledge includes questions on the recall of the product rule of independent events, the addition rule for mutually exclusive events and identification of the probability identity on the formula sheet (included in all mathematics examination papers).

The use of routine procedures to solve problems is the second cognitive level mentioned in CAPS (DBE, 2011). Procedures are the link between basic knowledge and the use of mathematics to solve routine problems (Mullis et al., 2003). The skills involved in this cognitive domain include the ability to recall a set of actions that will enable the solution of a set of similar problems by performing a set of actions (Mullis et al., 2003). CAPS describes the skills as the use of simple procedures and well-practiced calculations that require a few steps. In this study, assessments related to this cognitive domain include questions where the addition rule for mutually exclusive events and the product rule for independent events are recalled from memory. The correct identification of the probability identity on the formula sheet that is included in all mathematics examinations and using this identity to solve a problem without changing the subject is also classified as knowledge.

To engage in the next cognitive domain mentioned in CAPS, complex procedures, one needs to be able to solve problems that involve complex procedures. Questions that relate to this cognitive level often include problems set in a real life situation where there is often no obvious route to the solution (DBE, 2011). Mullis et al. (2003) link this cognitive domain to questions that require the use of equivalent representations for the same problem. The topic of probability therefore lends itself beautifully to this level of questions. In this study, the questions on this level include many real life scenarios and the use of Venn diagrams, tree diagrams and two-way contingency tables to represent some of these real life scenarios that are often part of probability questions.

The highest cognitive domain discussed in CAPS is the skill of problem solving. To engage in problem solving requires higher order reasoning and processes that lead to the solution of non-routine problems (DBE, 2011). Intuitive and inductive thinking and the capability to reason logically and systematically are needed to engage in the cognitive level of problem solving (Mullis et al. 2003). Questions on this cognitive level are unfamiliar and non-routine type questions. Even though the knowledge and skills involved in the solution to these questions are learnt procedures, the problems are solved using higher order reasoning and thinking (Mullis et al., 2003). Probability questions related to this cognitive domain include questions that are asked in a non-routine manner. An example of this would be the use of a two-way contingency table in conjunction with the theory on mutually exclusive events to calculate variables in a real life scenario.

2.5 Professional Development of Teachers

Professional development of teachers is a process where qualified teachers are engaged in various strategies to improve the subject knowledge and skills required to teach successfully (Ferreira, 2014; Meier, 2011). This statement is very valuable in this study where the focus is on the introduction of a new topic into the curriculum.

According to CAPS, learners should develop mathematical reasoning and creative skills to prepare them for the more abstract mathematics of higher education (DBE, 2011). With the established connection between teachers' MCK and learners' performance, mathematics teachers attempting to teach new topics efficiently may be under pressure, as mentioned by Wessels and Nieuwoudt (2011). To assist teachers in gaining and developing the necessary MCK to teach new topics effectively, in-service training and support are essential as part of teachers' professional development (Mosala & Junqueira, 2013).

MCK can be developed through, among other ways, discussion forums, workshops, engagement in professional communities and peer support groups (Wessels & Nieuwoudt, 2011). Professional development of teachers focuses not only on MCK development but also on the development of successfully applying various teaching strategies (Fricke, Horak, Meyer, & Van Lingen, 2008). Burgess (2010) suggests that teachers improve their knowledge through personal reflection, using teaching logs, as well as discussions with colleagues on specific topics. Miranda and Adler (2010) and Sproule (2011) also note that teachers' peer support groups further provide opportunities for teachers to improve their MCK. In recent studies conducted in China on co-learning between mathematics teachers and mathematics researchers, Huang, Su and Xu (2014) found that teachers improved their teaching skills when engaging in a process called the Chinese lesson study model. The Chinese lesson study is a model of professional development that includes cycles on collaboration of lesson plans, teaching lessons and observing teaching, post-lesson probing and reflection, and reviewing of lessons taught by teachers.

In South Africa, the DBE launched training sessions for all teachers, including mathematics teachers, in the FET phase in 2011, during the school holidays in June/July and October, in order to assist teachers in their preparation for the implementation of CAPS. These sessions took place in all districts of all the provinces. The sessions took place at schools in the districts and teachers were involved for three full days. A study done on the experiences of teachers on the implementation of NCS mathematics in Grade 10-12 highlights the fact that appropriate guidance from the DBE will assist teachers with the implementation of a new curriculum (Mosala & Junqueira, 2013). Grade 10-12 Physical Science and Mathematics teachers believe that they need training and resources to improve their own knowledge (Basson & Kriek, 2012).

Relating teachers' content knowledge to learner performance in probability.

According to Wu (2005), professional development programmes are not always effective due to various factors, such as the individual needs of the teachers and the time allocated to the professional development programme. One must also ask whether these sessions assist teachers in making connections between topics and in doing so contribute to the teachers' progression to a deeper understanding of mathematics. Any professional development opportunity needs to take place over an extended period of time to ensure effectiveness (Kleickmann et al., 2013) and address the personal needs of teachers (Singh, 2011). For professional development programmes to address the individual MCK of teachers, teachers participating in the programme need to be assessed as individuals in order to identify and address their personal needs (Wessels & Nieuwoudt, 2011).

These recommendations may be taken into account when examining the in-service training sessions presented to South African mathematics teachers. In this study, in-service training refers to the compulsory CAPS training hosted by the GDE. In order to answer the research question pertaining to the teachers' training, the participating teachers were questioned on their personal experiences related to these training sessions. The questions were aimed at establishing the impact of teachers' professional development, in the form of CAPS training sessions, on MCK.

2.6 Conclusion

In this chapter, it became apparent that teaching is a very complex activity that requires a wide variety of knowledge domains. These domains all contribute to the way teachers teach and the learning opportunities they can provide (Ball et al., 2008; Baumert et al., 2010; Shulman, 1987). Research, both nationally and internationally, shows that the concerns around mathematics teaching are not unfounded, as teachers seem not to possess enough MCK to teach effectively (Ball et al., 2008; Betts et al., 2003; Brijlall & Isaac, 2011). The level of MCK that teachers possess is one of the focal points of this study, and in order to answer the research questions it was important to measure these levels. Various methods have been used over the years by researchers and for this study, some of these methods were mimicked, such as examining written work as done by Bansilal et al. (2014) and examining the teachers' qualification as done by Fakeye (2012). I used the qualification level, years of teaching experience as well as a test including examination questions to try and gauge the levels of MCK that teachers have.

When examining the MCK of teachers by looking at written responses to examination-type questions, it is important to examine the questions by specifically looking at the level of the questions. CAPS gives very clear guidelines on the levels involved in assessment and these levels are all linked to cognitive skills needed to solve questions. The MCK of teachers in this study is described by using the cognitive levels needed to solve a series of examination-type questions set to include all the cognitive levels mentioned in this chapter.

The real concern in mathematics ultimately lies with the poor performance of learners in mathematics. The literature in this chapter points to the fact that the MCK of teachers is strongly linked to the performance of the learners they teach (Betts et al., 2003; Hill et al., 2005). It is also known that South African learners have been performing exceptionally poorly in mathematics benchmark tests. The performance of the learners in this study was measured using the same methods as those used to examine the MCK of teachers. The cognitive levels used to guide assessment tasks were used in the investigation of the learners' examinations.

In order to improve learner performance, the MCK of teachers needs to improve. This improvement can happen by using in-service professional development. In order for these sessions to be productive and effective, it is important for the teachers to actively engage in a programme over an extended period of time.

CHAPTER 3: CONCEPTUAL FRAMEWORK

After discussing the background, defining the research questions and investigating the literature concerning this study, it is now important to establish a conceptual framework to link the concepts involved in this study. The focus of this study is on the MCK of teachers and the performance of the learners they teach. The framework is constructed by acknowledging the different domains that contribute to the MCK of teachers. Secondly, to build on this knowledge, the cognitive levels involved in assessment are discussed. The cognitive levels mentioned by Mullis et al. (2003) in their report on the TIMSS results of 1999 were incorporated into CAPS to assist teachers in the setting of well-balanced formal assessment tasks. Well-balanced in this case refers to the fact that all cognitive levels are represented in the correct ratio when assessment tasks are designed.

MCK is seen as the most fundamental domain of knowledge that teachers should have (Kahan et al., 2003; Wu, 2005). According to Bransford et al. (2000) competent teachers need MCK which consists of three elements. The first is a deep foundation of factual knowledge. The second is a deep understanding of the content knowledge, which forms a bridge between knowing the factual content and using the factual content. The last element is the ability to organise the factual content knowledge in ways that facilitate easy retrieval and application of the knowledge to novel situations (Ball et al., 2005; Bransford et al., 2000; Shulman, 1987; Wu, 2005). The following figure is a graphic representation of the elements that MCK consists of and shows how these elements are linked.

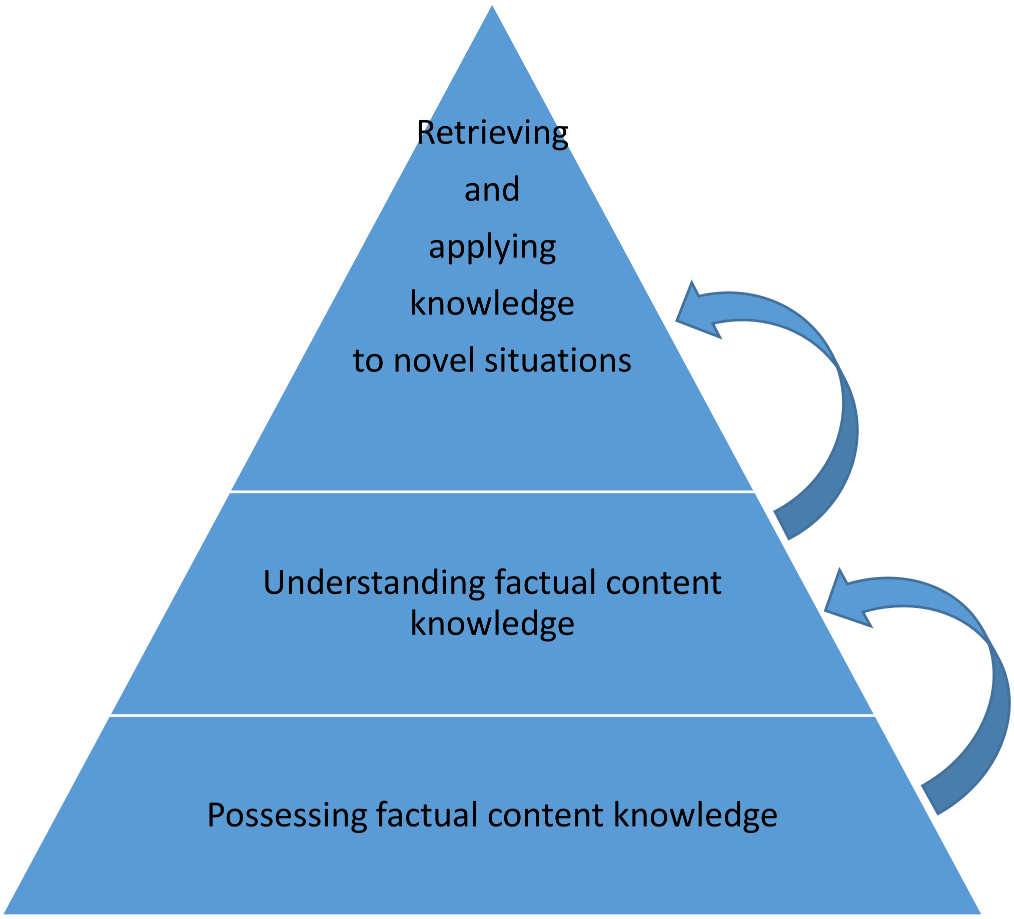


Figure 2. Mathematical Content Knowledge (Bransford et al., 2000)

The three elements are incorporated in the four cognitive domains prescribed by the DBE in the mathematics CAPS document (DBE, 2011). The cognitive levels for mathematics were incorporated into CAPS as a guideline for analysing the skills needed to answer questions in formal assessment tasks. These cognitive levels measure the level of knowledge needed to answer questions, and are used to ensure balanced assessment tasks.

The four levels recommended by Mullis et al. (2003) include knowing facts and procedures, using concepts, solving routine procedures, solving complex procedures and reasoning. These cognitive levels were used to investigate the MCK levels of the participants in this study and are represented in Figure 3. In this study, it was important to use the same method and cognitive levels to investigate both the MCK of the teachers and the performance of the learners in the written responses they gave to examination-type questions.

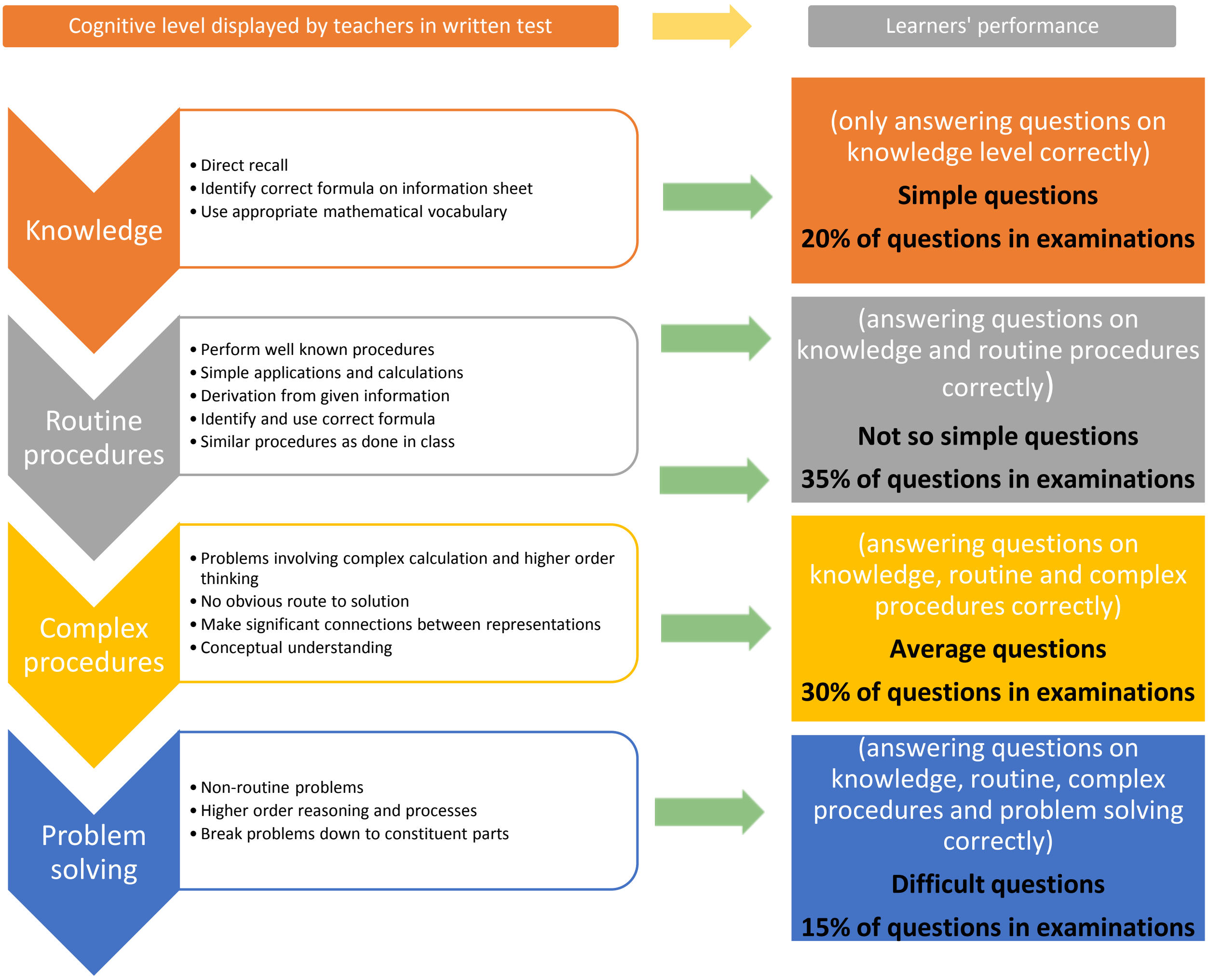


Figure 3. Cognitive levels for assessment (DBE, 2011)

3.1 KNOWLEDGE

Knowledge is the first cognitive level, which refers to knowing facts that are essentially the language that forms the foundation of mathematics (DBE, 2011; Mullis et al., 2003). This relates to the first level of knowledge mentioned by Bransford et al. (2000). Wu (2005) also emphasises the importance of the factual content knowledge of teachers. He goes as far as to say that, to improve the teaching of mathematics, teachers should be taught the mathematics they need to teach in the classroom. The CAPS document refers to knowledge skills as straight recall of facts and the appropriate use of mathematical vocabulary (DBE, 2011). The skill that is involved in choosing the correct formula to solve a problem without changing the subject of the equation is also classified under the cognitive level of knowledge. Factual knowledge alone, however, is not enough. In terms of this study, this cognitive level indicates a very basic level of knowledge. It is stipulated in CAPS that all formal assessments should include 20% questions based on the knowledge level (DBE, 2011).

3.2 Routine Procedures

The second cognitive level mentioned in the CAPS document is routine procedures (DBE, 2011). These procedures enable the making of connections between elements of knowledge (Mullis et al., 2003). Routine procedures include skills like performing familiar and rehearsed procedures. The derivation of prescribed formulas and the proof of prescribed theorems are also seen as routine procedures. To master these skills on this cognitive level, one must be able to choose the correct formula from a formula sheet and use this formula correctly. Calculations involving a few steps are also part of routine procedures. CAPS prescribes that 35% of the questions in all formal assessment must be on the level of routine procedures (DBE, 2011).

3.3 Complex Procedures

Complex procedures, the third cognitive level, involve higher order thinking skills. These skills include the ability to solve problems that involve complex calculations or reasoning. The questions on this cognitive level often have no obvious route to get to the correct solution. Significant connections between different representations that require conceptual understanding must be made to answer questions on this level correctly (DBE, 2011). These complex procedures form a bridge between knowing the factual content and using the factual content to solve problems (Bransford et al., 2000). When formal assessment tasks are set, it is prescribed by CAPS that 20% of the questions in the assessment task should be on the complex procedures level (DBE, 2011).

3.4 Problem Solving

The cognitive level of problem solving is closely related to what is referred to as reasoning by Mullis et al. (2003). This cognitive level includes problems set in real-life situations and/or concerned with pure mathematics, and involves the capacity for logical and systematic thinking to solve non-routine problems (Mullis et al., 2003). CAPS describes the skills involved in problem solving as using higher order thinking to solve non-routine questions that may require the ability to break the problem down into its constituent parts (DBE, 2011). Problem solving is therefore related to the last level mentioned by Bransford et al. (2000) as the ability to organise the factual content knowledge in ways that facilitate easy retrieval and application of the knowledge to novel situations. Formal assessment tasks need to include 15% questions based on this cognitive level according to CAPS (DBE, 2011).

3.5 Reasons for Using the Chosen Framework

For the purpose of this study, it was important to use some method to measure not only the MCK levels of teachers but also the performance of learners. To aid the validity of the results of the study, I used the same method to measure the MCK of the teachers as I used to measure the performance of learners when examining their written responses to the examination-type questions they answered.

CAPS gives very clear guidelines on the cognitive levels that should be incorporated in all formal assessment tasks. These levels describe the cognitive skills involved in solving the questions that needed to be answered by the teachers and the learners. These skills are described in detail in the CAPS document and examples of questions on each level are also given to help avoid any misunderstandings.

3.6 Conclusion

In the examination of the MCK levels of teacher and the performance of learners, it is important to establish a framework that will enable the measurement of these levels. In this study, the different domains of teachers knowledge were acknowledged and the cognitive levels recommended by CAPS were used to devise a framework. The cognitive levels mentioned in CAPS were derived from a report on the international benchmark test, TIMSS, by Mullis et al. (2003).

Knowledge is the foundation level and deals with the most basic skills needed to solve questions. Knowledge is followed by routine procedures. These routine procedures are easy calculations which may involve a few steps or the identification and use of correct formulas. The cognitive level of complex procedures requires conceptual understanding of the question.

The calculations involved in this cognitive level may include complex calculations. On the top cognitive level, problem solving skills are required to solve questions. It may require higher order reasoning and the questions are often non-routine type questions.

When considering the cognitive levels, one can see the progressive building of skills and knowledge from a very basic level of calculations to the deep understanding necessary to engage in problem solving skills.

The instruments that involved the written responses of teachers were analysed using the conceptual framework presented in this chapter, and this analysis is discussed in Chapter 5 of this report.

CHAPTER 4: RESEARCH DESIGN AND METHODOLOGY

This chapter outlines the research strategy and methodology of the investigation of the MCK of teachers on the topic of probability[1] and how it relates to the performance of the learners they teach. The discussion begins with the research strategy, mainly directed by the paradigmatic assumptions related to this study, which involves both quantitative and qualitative approaches. The sample and data collected for the study are then discussed in detail along with a brief discussion on the individual data collection instruments and the analysis of the data. Reliability, validity and ethical considerations related to this study are discussed, as it is of vital importance to acknowledge ethics and the considerations around it in any educational study. The chapter concludes with a summary of the data collection strategy of the study.

4.1 RESEARCH STRATEGY

4.1.1 Research methods

The methodology of this study involved a mixed methods research approach as research methods are not isolated but often overlap (Verma & Mallick, 1999). The research paradigm in this study is a post-positivistic paradigm. This paradigm evolved from the positivist paradigm in the twentieth century because of the problem of over-quantification (Guba & Lincoln, 1994). This paradigm involves a combination of quantitative and qualitative research methods that enables the researcher to enrich empirical data by including sense-data gathered from participants to shed light on the participants' perspectives (Scotland, 2012). The incorporation of some positivistic aspects such as quantification is important, but to avoid over-quantification also involves an interpretive position as part of the epistemological assumptions where subjectivity plays a role (Scotland, 2012).

[1] Probability was chosen as part of this study for a number of reasons. Probability was previously, in Curriculum 2005, excluded from the compulsory section of the mathematics curriculum. It is therefore seen as a new section of work in CAPS. The questions related to this topic are often contextualised and seen as challenging to both teachers and learners.

The quantitative branch of a mixed methods study refers to the objective part of the study concerned with numerical and empirical data (Maree, 2007). When quantitative studies are done, strategies of enquiry are used to collect data by using predetermined instruments (Creswell, 2003). In this study, the quantitative investigation used two methods, namely descriptive and correlation methods. As Hoy (2010, p. 1) states: "Measurement and statistics are central to quantitative research because they are the connections between empirical observations and mathematical expressions of relations."

The descriptive nature of this study attempts to describe existing conditions without analysing relationships (Fraenkel & Wallen, 2000). Descriptive methods employ descriptive statistics that encapsulate information into an easily understandable, quantitative form that describes the available data (Maree, 2007; Murray, 1998; Opie, 2004). The descriptive statistics used in this study include the calculation of percentages, ranges, means, medians, standard deviations and the distribution of the data to enable commenting on the symmetry of the data. The symmetry of the data is discussed by commenting on the skewedness of the data. The skewedness of the distribution is calculated by finding the difference between the mean and the median of the data. Table 2 shows the meaning of the calculated values related to the distribution of the data.

Table 2. Symmetry of distribution of data

Mean - median	Symmetry of distribution	Explanation
> 0	Positively skewed	Data spread more to the right of the median
$= 0$	Symmetrical	Data spread symmetrically around the median
< 0	Negatively skewed	Data spread more to the left of the median

The results from the descriptive statistics are only applicable to this sample and no conclusions can be drawn about the wider population. In this study, the descriptive statistics assisted in the answering of the research questions related to this study (Opie, 2004).

Correlation methods were also incorporated in the quantitative methodology of this study to establish to which degree statistical relationships between variables exist (Fraenkel & Wallen, 2000). In order to investigate the possible correlation, scatter plots (See Figure 4 for examples of correlation using scatter plots) were used to illustrate the relationship between two quantitative variables and Pearson's correlation coefficients (*r*) were calculated. These correlation coefficients (*r*) measure the strength of the relationship between two quantitative variables. This correlation coefficient is a number between -1 and 1. Table 3 shows the value of *r*, and the meaning of this value (Pike et al., 2012).

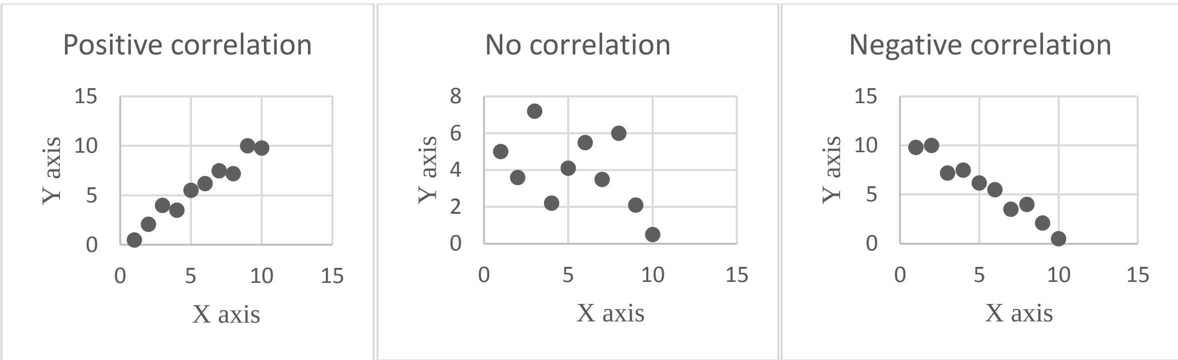


Figure 4. Correlation using scatter plots

Table 3. Meaning of Pearson's correlation coefficient (*r*) values

Value of *r*	Meaning
$r = 1$	Perfect positive correlation
$0.9 \leq r < 1$	Very strong positive linear correlation
$0.7 \leq r < 0.9$	Significant positive linear correlation
$0.3 \leq r < 0.7$	Weak positive linear correlation
$- 0.3 \leq r < 0.3$	No significant linear correlation
$- 0.7 \leq r < - 0.3$	Weak negative linear correlation
$- 0.9 \leq r < - 0.7$	Significant negative linear correlation
$- 1 \leq r < - 0.9$	Very strong negative linear correlation
$r = -1$	Perfect positive correlation

Qualitative research leads to a better explanation of inexplicable results from the quantitative research (Scotland, 2012). The results of qualitative research therefore enhance the quantitative research results, which may lead to a better understanding of the problem being investigated (Basit, 2010). The need for incorporating some qualitative methods became evident when quantitative studies in education left interpreters of findings uncertain and confused (Husén, 1988). Qualitative research has a strong constructivist perspective. With this approach, there is collaboration between the researcher and the participants where they strive to examine and give meaning to issues related to the study (Creswell, 2003). The qualitative branch of a research project also provides contextual information (Guba & Lincoln, 1994). The qualitative strategies used in this study mostly involve semi-structured interviews with teachers and the interpretation of written responses of the teachers and learners. During the interviews, open-ended questions gave me the opportunity to understand the multiple meanings of individual experiences of the teachers. By scrutinising the written responses, it was possible to gain extra insight into the results given by teachers and learners. The fact that certain participants did not attempt some of the questions and that some teachers were not willing to write Test 1 also contributed to the explanation of results.

4.2 Sample and Data Collection

The participants in this study were chosen purposefully. The sample consisted of four schools in the Tshwane South district of Gauteng. The reason for me choosing this district was that it is in close proximity to my place of work and residence and was therefore logistically more practical. The schools were chosen according to the pass rate percentages they achieved in the Mathematics 2013 NSC examination. Pass rate here refers to the percentage of learner who wrote the examination and managed to score above 30% in that examination. The sample consisted of two groups. Group 1: two schools with a pass rate of higher than 90% and Group 2: two schools with a pass rate of lower than 50%. These schools were identified using an annual report published by the DBE (2014) in January 2014. This report lists the subject performance, as percentage of learners achieving above 30% (therefore passing the subject), for Accounting, Agricultural Sciences, Business Studies, Economics, English (first additional language), Geography, History, Life Sciences, Mathematical Literacy, Physical Sciences and

Mathematics. The rationale behind this choice was to investigate the two extremities of the range of possible pass rates.

Schools in both groups were approached and four schools agreed to participate. Two schools were allocated to the two groups of the study. In Group 1, School A and School B, both with a pass rate of higher than 90%, took part in the study. School C and D fell into Group 2, the category of schools with a pass rate of lower than 50%. It must also be noted here, as mentioned in Chapter 1, that the two schools in Group 1 are situated in an urban area and the schools in Group 2 are situated in a peri-urban area of Tshwane South district and that the home languages of the learners and teachers include Afrikaans, English, Sesotho and isiZulu.

From the participating schools, the research design specified that two teachers from each school should be included into the sample. Mathematics teachers from each participating school were asked to take part in the study. The teachers' target group included teachers who taught Grade 11 mathematics in 2013 and Grade 12 mathematics in 2014 to the same group of learners. This, however, was not possible. Two teachers from School A and four teachers from School B voluntarily participated in the study. The teachers from School A were labelled as Teacher 1 and Teacher 2. The teachers from School B were labelled as Teacher 3, Teacher 4, Teacher 5 and Teacher 6. In both School C and School D there was only one teacher per school who taught mathematics to learners in the FET phase and they both agreed to participate in the study. These teachers were labelled Teacher 7 and Teacher 8 respectively. The sample therefore included eight teachers in total.

The learners' sample included learners taught by the participating teachers in either Grade 11 or Grade 12 or both Grade 11 and 12 (This information can be seen in Table 4). Teacher 1 taught 19 learners in Grade 11 and Grade 12. Teacher 2 only taught six learners in Grade 11 and five of those learners in Grade 12. Teacher 3 taught 64 learners in Grade 11 and 11 of those learners were also taught by her in Grade 12. Teacher 4 did not teach any Grade 11 learners but taught 15 learners in Grade 12. Teachers 5, 6 and 8 did not teach any Grade 11 learners. Teacher 7 had the opportunity to teach all 25 learners in both Grade 11 and 12. The schools from Group 1 contributed to the study by giving the Grade 11 final and the Grade 12 preparatory examinations to be examined as part of the data collection. The schools in Group 2 did not have any of the Grade 11 examinations, as they had been destroyed before the start of the study. From Group 2, School C contributed to the data by giving me access to the Grade 12 preparatory examination. However, this was not the case with School D. No data were gathered from the learners of School D as the school had some policy changes after the start of the research and no longer allowed any research at their facility. In this study, learners were passive participants as their contribution only included written work done as part of their school-based assessment. The written work included in this study was the probability sections of the Grade 11 final examination (2013) and the Grade 12 preparatory examination (2014). Table 4 shows a summary of the number of schools, teachers and learners involved in the study.

Table 4. Number of schools, teachers and learners

Group	School code and geographical description	Pass rate	Teachers	Number of Grade 11 learners who gave written responses to Grade 11 final examination	Number of learners who were taught by the same teacher in Grade 11 and Grade 12
Group 1	School A Urban	> 90%	Teacher 1	19	19
			Teacher 2	6	5
	School B Urban	> 90%	Teacher 3	64	11
			Teacher 4	0	15
			Teacher 5	0	0
			Teacher 6	0	0
Group 2	School C Peri-urban	< 50%	Teacher 7	0	25
	School D Peri-urban	< 50%	Teacher 8	0	0

The data collected enable the description and explanation of the status of the phenomena that are being investigated (McMillan & Schumacher, 2001); in this case, the MCK of Grade 12 teachers teaching probability according to CAPS requirements for the first time and the performance of the learners taught by these teachers. An overview of the data collection process can be seen in Table 5, where there is also an indication of how the data are linked to the research questions of this study. It is important to note here that a pilot study was not conducted, as the instruments used were already moderated and standardised by the GDE at the start of the investigation (Grade 11 final examination) and for Test 1 the instrument was scrutinised by three experts.

Table 5. Data collection

Date	Phase	Instrument	Participants	Purpose	Research question
November 2013	Phase 1	Grade 11 final examination set by School A	Learners	Performance levels	See section 1.3.2.2
November 2013		Grade 11 final examination set by GDE	Learners	Performance levels	See section 1.3.2.4
June 2014	Phase 2	Test 1	Teachers	MCK levels	See section 1.3.2.1
July 2014	Phase 3	Semi-structured interviews	Teachers	Personal information, information on education and training	See section 1.3.2.2 1.3.2.3
September 2014	Phase 4	Grade 12 preparatory examination	Learners	Performance levels	See section 1.3.2.4

Phase 1 of the data collection started with the learners writing the Grade 11 final mathematics examination Paper 1 in November 2013 (as set by School A and the GDE respectively). In July 2014, phase 2 was completed when the teachers wrote Test 1. Phase 2 was followed by semi-structured interviews conducted as the third phase of the data collection process. The Grade 12 students contributed to phase 4 of the data collection in September 2014 by completing the Grade 12 preparatory examination that was written by the learners and marked by the teachers. The data from the Grade 11 final examinations and the Grade 12 preparatory examinations were gathered after the teachers had completed their marking process. The written work that contributes to the formal assessment of the learners is seen as evidence and therefore photographs of all the written responses were taken at the schools. This evidence is also allowed to be destroyed by the school six months after the end of each academic year. By taking the photographs, I had access to all the written responses at all times. The data collection instruments are now discussed in the order that they were conducted during the data collection process.

4.2.1 Grade 11 final mathematic examination Paper 1

As part of the prescribed assessment program, all Grade 11 mathematics learners must write a final mathematics examination. This was the first phase of the data collection process. The final Grade 11 examinations for all subjects are written at public schools according to a timetable published by the GDE. The GDE also monitors the examination process at schools to ensure that examination regulations are met. This examination was chosen as a data collection instrument in order to gain information on the performance of learners in probability. The assessment of Grade 11 mathematics consists of two papers, Paper 1, which includes the topic of probability, and Paper 2, which includes topics like trigonometry and Euclidean geometry. Both these papers are allocated 150 marks and are completed in a maximum of three hours each. The rationale behind choosing this examination as an instrument includes the fact that it is standardised, as it is set and moderated at provincial level by the Gauteng Department of Basic Education (GDE).

The same examinations are written by all public schools in Gauteng, with the exception of some schools that choose to write papers set by themselves and moderated by the GDE. Of the four schools involved in this study, only School A and School B took part in this section of the study as the other two schools had destroyed these examination papers before the data collection for this study started.

4.2.1.1 Grade 11 final examination set by School A

School A chose to set their own examination, which was then moderated by the GDE. Twenty-five learners from this school participated in the study. The questions relating to probability in this examinations included a Venn diagram and a question on dependent, independent and mutually exclusive events.

The Venn diagram question was introduced to the learners by making them aware of the fact that there were a number of students that were not included in the three events related to the Venn diagram. Learners were then asked to represent the real world scenario in the form of a Venn diagram. In the last question related to the diagram, learners were required to use the information in their diagram to answer a question on the probability of an event happening.

<u>Question 7.1</u>

7.1 A survey was done at a local library to show the different reading preferences for 80 students.

- 44 read the National Geographic magazine
- 33 read the Getaway magazine
- 39 read the Leadership magazine
- 23 read both the National Geographic and the Leadership magazines
- 19 read both the Getaway and the Leadership magazines
- 9 read all three magazines
- 69 read at least one magazine

7.1.1 How many students do not read any of these magazines? (1)

7.1.2 Let the number of students that read National Geographic and Getaway, but not Leadership be represented by x. Represent this information in a Venn diagram. (5)

7.1.3 Show that $x = 5$. (3)

7.1.4 What is the probability (correct to 3 decimal places) that a random student will read at least 2 of the 3 magazines? (3)

In the second probability question, the learners were expected to use their knowledge to solve a question related to dependent, independent and mutually exclusive events. The learners needed to recall the fact that the probability identity: $P\ (A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$ is part of a formula sheet provided, and that it would be appropriate to use this identity in this question. The complementary rule of probability, $P(not\ A) = 1 - P(A)$, was needed to solve the last question of this examination paper. It was expected that learners would be in a position to use this rule even though it is not included in the formula sheet.

Question 7.2

7.2 A smoke detector system in a large warehouse uses 2 devices: A and B. The probability that device A will detect any smoke is 0.95 and device B is 0.98. The probability that both will detect smoke at the same time is 0.94.

7.2.1 What is the probability that device A or B will detect smoke? (3)

7.2.2 What is the probability that smoke will not be detected? (1)

The total mark allocation for the probability question was 16 marks, which is within the recommendation made in CAPS with relation to the mark allocation rewarded to probability. The papers were photographed and the probability question was marked and analysed by me.

4.2.1.2 Grade 11 final examination set by GDE

Schools B, C and D wrote this examination. Sixty-four learners from School B agreed to participate in the study by allowing me access to their written responses. This paper was very similar to the paper set by School A. It also included questions on a Venn diagram and dependent, independent and mutually exclusive events.

A real world scenario was set in the first question and learners were asked to use this information to set up a Venn diagram. The questions that followed the drawing of the diagram involved the probability identity: $P\ (A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$. Along with the question paper, learners received a formula sheet and the probability identity was included in this formula sheet. Learners could use the formula sheet to assist them in the solution of the probability problems included in this question. In conjunction with the formula, learners were required to use the calculated values of their Venn diagram to solve the question correctly.

Question 7

A school organises a camp for 103 Grade 8 learners. The learners were asked to write down what food they prefer. They could choose between beef (B), vegetables (V) and chicken (C). The following information was gathered.

- 2 learners do not eat beef, vegetables or chicken.
- 5 learners only eat vegetables.
- 2 learners eat only beef.
- 12 learners do not eat chicken at all.
- 3 learners eat only chicken.
- 66 learners eat chicken and beef.
- 75 learners eat chicken and vegetables.

Let the number of learners that eat beef, vegetables and chicken be *x*.

7.1 Draw a Venn diagram to represent the above information. (7)

7.2 Calculate *x*. (2)

7.3 Calculate the probability that a learner that is chosen at random eats:

7.3.1 Only beef and chicken but not vegetables. (2)

7.3.2 Only two of the food types, beef, chicken and vegetables. (2)

The last question of the Grade 11 mathematics examination set by the GDE focused on calculating the probability of independent events using the product rule: $P(A\ and\ B) = P(A) \times P(B)$. This rule is not included in the information that is given to learners in the formula sheet that is included in the examinations. Learners needed to recall the formula by memory and then do the necessary substitutions and calculations to obtain the correct solution.

Question 8

Events X and Y are independent. (4)

P(X) = $\frac{1}{3}$ and P(X and Y) = $\frac{1}{12}$, calculate P(Y).

Seventeen marks were allocated to probability in this question paper, which is within the regulations stipulated by CAPS for marks allocated to probability. The papers were photographed and the probability questions were marked and analysed by me.

4.2.2 Test 1

The second phase of data collection, which involved the teachers, commenced by asking the participating teachers to complete Test 1, a test that consisted of examination-type questions related to probability. Test 1 was administered in order to gain information on the MCK of teachers in probability. Test 1 was set in a collaborative effort between myself and another experienced mathematics teacher, and moderated by my supervisor to ensure standardisation. In the design of the instrument it was important to include questions on all four cognitive levels[2], which include knowledge, routine procedures, complex procedures and problem solving, as described in the conceptual framework of this study and recommended by the DBE in the CAPS document (DBE, 2003).

Of the eight teachers participating in the study, only six teachers gave written responses to Test 1. Right form the onset of the study, the two teachers from School C and School D did not feel comfortable with writing a test. The participating teachers completed the questions under examination conditions at their own schools after school hours. Teachers took approximately 30 minutes to complete the written responses.

The first question of Test 1 involved calculations related to dependent, independent and mutually exclusive events. In order to answer the first sub-question correctly, the teachers needed to know the theory behind mutually exclusive events, by stipulating that mutually exclusive events cannot occur simultaneously. The second sub-question involved the product rule for independent events. The probability identity was used to solve the last sub-question. This probability identity was on the formula sheet that is included in all examinations, but was not made available to the teachers when they completed Test 1.

[2] See Chapter 3

Question 1

Three events, A with a probability of $P(A) = 0.3$, B with a probability of $P(B) = 0.4$ and C with a probability of $P(C) = 0.2$.

- A and B are independent
- B and C are independent
- A and C are mutually exclusive

Calculate the probability of the following events occurring:

1.1 Both A and C occur. (1)

1.2 Both B and C occur. (1)

1.3 At least one of A or B occur. (4)

Question 2 involved the setting up of a tree diagram and then questions on the probability of events around this diagram followed. The teachers should have been in a position to understand the real world problem that was set out in this question and should have then used this information to set up a tree diagram of consecutive events to represent the scenario. After setting up the diagram, the teachers had to use the diagram to calculate the probability of events related to the diagram. These calculations could be done independently of any probability rules or identities. To complete the question correctly, the teachers were expected to again use the information from the tree diagram and link it to the number of working days there are in a year.

Question 2

In Johannesburg, South Africa, the probability of a sunny day is $\frac{6}{7}$ and the probability of a rainy day is $\frac{1}{7}$. If it is a sunny day, the probability that Thapelo will wear a dress to work is $\frac{7}{10}$, the probability that Thapelo will wear a skirt to work is $\frac{1}{5}$ and the probability that she will wear pants to work is $\frac{1}{10}$. If it is a rainy day, then the probability that Thapelo will wear a dress to work is $\frac{1}{9}$, the probability that she will wear a skirt to work is $\frac{5}{9}$ and the probability that Thapelo will wear pants to work is $\frac{1}{3}$.

2.1 Draw a tree diagram to represent the above information. Indicate on your diagram the probabilities associated with each branch as well as all the outcomes. (5)

2.2 For a day selected at random, what is the probability that:

2.2.1 It is rainy and Thapelo will wear a dress. (1)

2.2.2 Thapelo wears pants. (2)

2.3 If Thapelo works 245 days in a year, on approximately how many occasions does she wear a skirt to work? (4)

The last question of Test 1 involved a Venn diagram. This question was very similar to the Venn diagram questions that were part of both the Grade 11 final examination questions. The teachers were required to set up a Venn diagram (See Appendix C1) and then link some probability questions to the diagram. In order to complete the diagram correctly, the teachers needed to calculate a variable, as was the case in the two Grade 11 examinations. Once the diagram was set out, some questions were asked on the probability of certain events happening in the sample space. In order to answer these question, the teachers needed to extract information from the diagram they set up.

<u>Question 3</u>

A school has 174 Grade 12 learners, a survey is done among the learners and the following is found:

- 37 learners take Life Science
- 60 learners take Physical Science
- 111 learners take Mathematics
- 29 learners take Life Science and Mathematics
- 50 learners take Physical Science and Mathematics
- 13 learners take Life Science and Physical Science
- 45 learners do not take Life Science, Physical Science or Mathematics
- x learners take Life Science, Physical Science and Mathematics

3.1 Draw a Venn diagram to represent the information above. (6)

3.2 Show that $x = 13$. (2)

3.3 If a learner is selected at random, calculate the probability that the learner has the following combination of subjects:

3.3.1 Physical Science and Mathematics, but not Life Science. (2)

3.3.2 Only one of Life Science, Physical Science or Mathematics. (2)

All the written responses given by the teachers were photographed and marked according to a memorandum and any deviations from the memorandum were checked for correctness and added to the memorandum where necessary. The analysis of this data is discussed in the data analysis section of this chapter.

4.2.3 Semi-structured interviews

The next phase of the data collection, phase 3, consisted of the individual semi-structured interviews conducted with the teachers. The interview schedule included open-ended questions (See Appendix D) and interviews were conducted in either English or Afrikaans depending on the preference shown by the participants. I recorded the interviews with permission from the participating teachers. The recording of interviews is essential in ensuring accurate transcription. The transcription was done by the researcher. The interviews that were conducted in Afrikaans were translated, by the researcher, to make them accessible to more readers and to incorporate them in the study. The interviews were semi-structured and consisted of three parts, which included information on the participating school, information on the teachers (personal information and information related to their education and professional development) and responses to questions arising from the analysis of the written responses given by the teachers in Test 1. Even though the interviews were time-consuming, rich data were gathered during this stage of the data collection process.

The first part was aimed at getting information on the school and the learners taking mathematics at the school. The following questions were asked:

- What is the name of the school and in which area is the school located (urban or peri-urban)?
- How many Grade 12 learners take mathematics at the school and how many teachers teach mathematics to Grade 12 learners?

The questions then related to the teachers on a more personal level by asking them about their years of teaching experience, education and professional development. All the teachers involved in this study answered all the questions in this section. The questions in this section of the interview were aimed at gaining data on the training of teachers. The questions started by asking teachers about their secondary school training and if they had mathematics on higher or standard grade when they attended school, if they completed school when the senior certificate was the exit certificate for secondary school. If they completed their school career during the NSC phase (Curriculum 2005[3]), the question changed to whether they had mathematics or mathematical literacy as a subject when they exited school. The questions then shifted to the tertiary training of teachers. They were asked whether they qualified as teachers by graduating with a diploma or degree in education and whether they had any training in probability during their tertiary training. The last questions in this part related to the teachers' professional development. Professional development in this study was seen as the in-service training teachers participated in with the focus on the CAPS training sessions presented by the GDE.

The final part of the semi-structured interviews was only designed after the written responses to Test 1 had been marked and analysed. The questions were specifically designed according to the responses given by teachers in Test 1. It was important to understand why teachers struggled with certain questions in Test 1 and whether teachers had any misconceptions related to probability. The aim of this final part of the semi-structured interviews was to enrich the data gathered using the written responses of teachers in Test 1.

All the interviews were conducted with the teachers in private at the participating schools, after school hours. On average, the duration of the interviews was between 30 and 45 minutes each.

4.2.4 Grade 12 preparatory examination 2014

In the fourth and final phase of this study, the preparatory mathematics examination Paper 1 was chosen as a data collection instrument in order to gain more information on the

[3] Curriculum 2005 was the first curriculum introduced in the democratic South Africa, see Chapter 1.

performance of learners in probability. The learners' performance in the probability section of the preparatory examination paper 2014 was compared to their performance in the probability section of the final Grade 11 mathematics examination they wrote in November 2013 when they completed Grade 11.

The preparatory examination, which is similar to the Grade 11 final mathematics examination with regard to standardisation, mark and time allocation, was written by all Grade 12 learners in September 2014. The only difference between the Grade 11 and Grade 12 examination is the content. Grade 12 CAPS stipulates that probability must include probability questions linked to the fundamental counting principle along with all the probability content included in Grade 11 CAPS.

The probability question in the Grade 12 preparatory examination included questions on dependent, independent and mutually exclusive events, a two-way contingency table as well as the fundamental counting principle. The question was the last question in Paper 1 and contributed 15 marks to the total for Paper 1.

The first question in the Grade 12 preparatory examination related to dependent, independent and mutually exclusive events. For learners to solve this question successfully, they needed to correctly identify the probability identity from the formula sheet that was provided to them as part of the question paper. They could simply copy the identity, substitute given values and calculate the answer without changing the subject of the identity.

<u>Question 13.1</u>

13.1 Tebo writes an Art and a Music examination. He has a 40% chance of passing the Music examination, a 60% chance of passing the Art examination and a 30% chance of passing both the Music and Art examination.

Calculate the probability that Tebo will pass the Music or Art examination. (3)

This question was followed by a question that incorporated a two-way contingency table with independent events. The solution to the two-way contingency table was linked to the product rule for independent events. This rule was not included in the formula sheet and therefore learners needed to recall this rule in order to solve the question. If the rule was used correctly to solve one of the variables, the other variables could be solved using the available values in the contingency table.

Question 13.2

13.2 A survey was conducted asking 60 people with which hand they write and what colour hair they have. The results are summarized in the table below.

		HAND USED TO WRITE WITH		
		Right	Left	**Total**
HAIR COLOUR	Light	*a*	*b*	**20**
	Dark	*c*	*d*	**40**
	Total	**48**	**12**	**60**

The survey concluded that the 'hand used for writing' and 'hair colour' are independent events.

Calculate the values of *a*, *b*, and *c*. (5)

The rest of question 13 was based on the fundamental counting principle. CAPS states that learners in Grade 12 need to be able to solve probability questions related to the fundamental counting principle. Probability was, however, not included here and therefore question 13.3 was not included as part of the data for this study.

13.3 The digits from 1 to 9 are used to make 5-digit codes.

13.3.1 Determine the number of 5-digit codes possible, if the digits are arranged in any order without repetition. (2)

13.3.2 Determine the number of 5-digit codes possible, if the code formed has to be an even number and the digits may not be repeated. (3)

13.3.3 Determine the number of 5-digit codes possible, if the code formed only uses even digits and repetition of digits is allowed. (2)

Seventy-five learners from three schools (School A, School B and School C) participated in this part of the study. School D had some policy changes since the start of the study and no longer allowed any research in their school.

4.3 DATA ANALYSIS

In this section, I discuss how the data gathered during data collection were categorised and analysed. Data collection took place between July and September of 2014 at four schools in the Tshwane South district in Gauteng.

The process of data analysis started with the categorisation of the participants to ensure that the research outcomes are comprehensible and that the research questions are addressed (Murray, 1998). By categorising the data, one may be able to make useful comparisons. The categorising of data began on a subliminal level when the intent of the study and the research questions were asked (Murray, 1998). These categories had to be clearly defined to ensure correct capturing of data (Murray, 1998).

The categories in this study included the teachers, the learners participating in the study and the cognitive levels[4] mentioned in the CAPS document. The categories enabled the comparison between the teachers and the learners and the identification of possible links between the MCK levels of teachers and performance levels of the learners they teach. Following the categorisation of the participants, the process of data analysis continued. All the written responses to probability examination-type questions in this study, completed by the participating teachers and learners, were photographed, marked and coded by me in order to generate useful units for analysis (Maree, 2007). The written work also served as a vignette of the responses given by the participants. During the marking process, a mark, expressed as a percentage, was awarded so that it could be linked to a performance level. The percentages scored on the probability questions were captured in tables for easy analysis. These data were coded using an interval scale to enable measurement of differences in quantity or magnitude

[4] See Chapter 3

(Maree, 2007). Table 6 shows how the percentages scored by teachers and learners were linked to a performance level.

The performance levels used in this study included poor, below average, average and above average. For learners to pass mathematics at the end of Grade 12 they need to score 30% or more (national pass percentage in South Africa). Scores below 30% in this study were classified as poor. The next level is below average (30% - 49%). I classified the performance as below average since most universities do not recognise an average of below 50% for scientific fields of study.

The next two levels (average and above average) I classify as acceptable levels in scientific fields of study. To define the average performance level, I calculated the average score for the Grade 12 mathematics examinations starting from 2010 and ending in 2013. This average score refers to the average mark scored by the learners in the mentioned examinations. This average is 51.8% and therefore the performance level of average lies between 50% and 65% (DBE, 2012). A score below this range is classified as below average and a score above this is classified as above average. Even though I used the performance levels in Table 6, I classified exceptional teachers as those who scored above 80% in Test 1.

Table 6. Performance level and percentage scored

Performance level	Percentage mark scored in tests
Poor	0% - 29%
Below average	30% - 49%
Average	50% - 64%
Above average	65% - 100%

Another coding process was implemented to categorise the answers for teachers and learners according to the level of correctness of the answers. Coding, according to Basit (2003), is a process where elements are labelled for the allocation of units of meaning to descriptive information. The codes used in this study included: correct (C), almost correct (AC), somewhat correct calculations or methods (SC), no correct calculations (NC) and did not attempt (DNA), and are referred to as rank scores in this study. The data gathered in this manner are ordinal in nature, and were captured in tables. These tables are available in Appendix H.

Table 7 summarises the categories included in this study. In Chapter 5 these instruments are analysed in detail.

Table 7: Questions, mark allocation and cognitive level distribution of data collection instruments

Data collection instrument	Participants	Question	Mark allocation	Cognitive level	
Grade 11 final examination 2013 (Set by School A, moderated by GDE)	25 learners from School A	7	16 marks	Knowledge	2 marks
				Routine procedures	9 marks
				Complex procedures	3 marks
				Problem solving	2 marks
Grade 11 final examination 2013 (Set and moderated by GDE)	64 learners from School B	7-8	17 marks	Knowledge	2 marks
				Routine procedures	8 marks
				Complex procedures	4 marks
				Problem solving	3 marks
Test 1	6 teachers	1-3	30 marks	Knowledge	2 marks
				Routine procedures	8 marks
				Complex procedures	14 marks

Data collection instrument	Participants	Question	Mark allocation	Cognitive level	
				Problem solving	6 marks
Grade 12 preparatory examination 2014	75 learners	13	15 marks	Knowledge	0 marks
				Routine procedures	10 marks
				Complex procedures	3 marks
				Problem solving	2 marks

The data analysis for this study is discussed according to the process followed during data collection, starting with the Grade 11 final examinations set by School A and the GDE respectively, followed by Test 1 and the individual semi-structured interviews, and lastly, looking at the Grade 12 preparatory examination.

4.3.1 Grade 11 final examination (November 2013)

4.3.1.1 Grade 11 final examination set by School A (November 2013)

The analysis of the written responses of 25 learners to the Grade 11 final examination set by School A started after the teachers had marked the papers and had processed the learners’ marks. The written responses were then photographed, marked and analysed. The analysis of the photographs was done using both quantitative and qualitative strategies.

The quantitative analysis included the calculation of percentages scored by the learners. These scores were calculated for the whole of Paper 1 as well as for the probability question alone. The data gathered here were used to comment on the distribution of the data by looking at symmetry or skewedness. Descriptive statistics were employed to analyse the mean, median, standard deviation and the range of the data. The relationship between the variables was shown using a scatter plot. Pearson’s correlation coefficient (*r*) was also incorporated to calculate the possible correlation between the variables involved in this examination. The percentages were used to analyse the performance levels achieved by the learners. The performance levels included poor (0%-29%), below average (30%-49%), average (50%-64%) and above average (65%-100%). The written responses were also categorised as rank scores to indicate the correctness of the solutions. These rank scores included correct (C), almost correct (AC), some correct procedures or calculations (SC), no correct procedures of calculations (NC) and did not attempt (DNA). The results from the rank scores were tabulated (See Appendix H). The quantitative analysis further included the difficulty level of each question according to the cognitive levels mentioned in CAPS. These levels are knowledge, routine procedures, complex procedures and problem solving.

Qualitatively, the written responses were scrutinised to identify possible response patterns and misconceptions. The written responses to the probability questions were marked by me and the mark for the whole paper was taken as it had been calculated by the teacher who marked the examination. Variations of written responses given by learners on each question are shown and commented on.

Analysis of this data may lead to some insight into the research questions linked to the performance levels of the learners.

4.3.1.2 Grade 11 final examination set by GDE

This examination paper was written by Schools B, C and D, but the examination papers were only available at School B as School C and D had destroyed these papers before the start of the study. The analysis of the data from this paper was done in the same way as for the Grade 11 final examination set by School A by using both quantitative and qualitative methods.

After the analysis of both the Grade 11 examinations involved in this study, the two examinations were analysed together. Correlation calculations were done to analyse possible similarities between the two examinations as well as the similarities between the percentages scored by learners in these examinations.

4.3.2 Test 1

In order to analyse the written responses given by six teachers (two teachers form School A and four teachers from School B) in Test 1, I marked the work according to the memorandum. The marking was done and the responses were analysed using two methods. The responses were marked and a mark was allocated that could be converted to a percentage. This percentage was used to rank the performance of the teachers according to performance levels involved in the study, namely poor, below average, average and above average. A mark below 30% was classified as poor; a mark between 30% and 50 % was classified as below average. An average performance lay between 50% and 65%, and any percentage higher than 65% was classified as above average. The responses to every sub-question were also classified according to rank scores. These rank scores grouped responses according to the level of correctness. The rank scores used included: correct (C), almost correct (AC), some correct procedures or calculations (SC), no correct procedures of calculations (NC) and did not attempt (DNA). The analysis involved in this instrument was qualitative as the sample size was only six.

The written responses were scrutinised during and after the marking process to look for possible patterns or misconceptions that teachers may have. This analysis was done on the data from Test 1, but later this analysis was also used in conjunction with the analysis of the written responses given by learners. This may lead to the identification of possible links between the written responses of teachers and the written responses of the learners they teach.

4.3.3 Semi-structured interviews

The analysis of the data gathered during the interview process was done after the transcription of all the interviews. The transcription process also involved the translation of the interviews that were conducted in Afrikaans. The information from these interviews was summarised in table form to enable easy analysis. This table includes information related to the school at which the teacher was teaching, the gender of the teacher along with the years of teaching experience that the teachers have. The information related to the teachers' tertiary qualification and professional development programmes they were involved in is also included in this table.

4.3.4 Comparing Grade 11 and Grade 12 learners' work to those of the teachers

The data gathered during the Grade 11 final examination and the Grade 12 preparatory examination were analysed alongside the data gathered from the teachers in Test 1. All the written responses for the teachers and the learners were already marked and percentage scores and rank scores were also calculated. The analysis included comparisons between the written responses of the learners and those of their teachers. The analysis was done on the questions that were similar in the examinations of the learners and that of the teachers. Extracts of their written work were compared using the photographs taken during the data collection process.

The section that follows discusses the issues of validity and reliability that relate to this study. These issues must be considered to ensure the trustworthiness of the research (Maree, 2007).

4.4 Reliability and Validity

"Reliability and validity are the technical terms that refer to the objectivity and credibility of research" (Peräkylä, 2011, p. 366). When designing a research project, it is very important to consider the factors involved in the study that may threaten the reliability and validity of the study.

Reliability, according to Maree (2007), refers to the repeatability of the results of quantitative research (Oluwatayo, 2012). When the same instrument used in quantitative research is administered to a different sample, it should yield the same results. Reliability therefore refers to how accurately the notes and data of the researcher relate to what actually happened (Oluwatayo, 2012).

According to Oluwatayo (2012), three types of reliability must be considered in educational research, namely stability, equivalence and internal consistency.

Stability or repeatability measures the consistency over time with a similar sample. If an instrument is reliable, it will yield the same results when it is repeatedly administered to a similar sample. Equivalence in reliability refers to the administration of a second instrument that is similar to the first concerning the items, content and difficulty level. This test for reliability was not conducted with the sample of teachers as it was not practical due to time limitations. To ensure internal consistency, the data gathered from the administration of the instrument must be collated and analysed using appropriate statistics. Cronbach alpha was used to measure the internal consistent of the Grade 11 probability questions and the grade 12 preparatory final examinations probability questions with the help of a statistician.

A high value of Cronbach signifies that the items are measuring the underlying (or latent) construct. George and Mallery (2003) provide the following rules of thumb that if it is > 0.9 – excellent, > 0.8 – good, > 0.7 – acceptable, >0.6 questionable, >0.5 poor and < 0.5 unacceptable. However, the generally agreed lower limit for Cronbach's alpha is 0.7, although it may decrease to 0.6 in exploratory research (Hair, Black, Babin, & Anderson, 2014). In this research 0.6 is also used as an acceptable level. The following reliability was obtained.

Reliability results of questions

Aspect	No. of items	Cronbach's alpha	Acceptable level
Grade 11 probability questions	6	0.603	Acceptable
Grade 12 probability questions	5	0.726	Acceptable

The reliability of the instruments was acceptable. It can be observed that the Grade 12 probability questions were more reliable than the Grade 11.

Validity concerns the extent to which the research measures what it set out to measure (Maree, 2007; Oluwatayo, 2012). Validity types that are relevant to this study include content, face and construct validity, and are discussed below.

Content validity "hinges on the extent to which meaningful and appropriate inferences of decisions are made on the basis of scores derived from the instrument used in a research" (Oluwatayo, 2012, p. 391). To ensure content validity in this study, Test 1 was assessed in a collaborative effort between the researcher, an experienced mathematics teacher and the supervisor of the study to ensure that this instrument showed evidence of comprehensive coverage of all the items related to the topic of probability. This collaboration not only catalysed a process of critical thinking but also ensured reflection on the design process and the purpose of the instrument used in the study (Wilson, 2009). The same group of experts also examined the content validity of the memoranda for all the examinations and Test 1 in the same way.

The next type of validity that is important in this study is *face validity*, which refers to the way in which the instrument is presented to the participants. Factors that should be considered when assessing the face validity of an instrument is the clarity of the items within the instrument, the language used (grammar and spelling), legibility and attractiveness of the instrument (Oluwatayo, 2012). The supervisor assessed the face value of Test 1 by regarding all the considerations mentioned by Oluwatayo (2012), while the GDE has set standards on the factors that influence the face validity of their examinations (Grade 11 final and Grade 12 preparatory examinations).

Construct validity is described by Oluwatayo (2012, p. 393) as follows: "…construct validity shows the degree to which inferences are legitimately made from the operationalisations in one's study to the theoretical constructs on which those operationalisations are based". In this study, construct validity refers to the fact that it was possible to measure the MCK of teachers by looking at their qualification level, their teaching experience and the written responses they gave to Test 1. The way in which the questions in the examinations and Test 1 were categorised according to the cognitive levels mentioned in the conceptual framework[5] should be legitimate.

4.5 Ethical Considerations

When doing any kind of research, ethics always have to guide the process and those involved in the process. Ethical principles such as trust, transparency, truthfulness, respect and kindness must be considered during the decision making process that guides research in social sciences (Menter, Elliot, Hulme, Lewin, & Lowden, 2011; Punch, 2003). The consequences of all actions taken during the research process must be taken into consideration (Punch, 2003). The educational well-being of the learners must always be the highest priority, and it is important to always choose the path that will lead to the good of all the individuals involved in the study.

[5] See Chapter 3

Ultimately, when ethics are discussed it is most important to follow the most virtuous way of engaging in research (Punch, 2003).

In the process of questioning the acceptability of the research approach followed in this study, the proposed participants and their involvement were examined. To initiate this process, the proposed study was presented to the Ethical Committee of the University of Pretoria for ethical clearance. This was granted in April 2014 and permission was given to continue with the fieldwork involved in the study. It is also necessary to request permission to conduct research from the GDE when research is conducted in GDE schools; this process was followed and permission was granted in April 2014.

Meetings were scheduled with the principals of the schools that were chosen as part of the sample. During these meetings, I discussed the intended research with the headmasters. It was important to be truthful and as informative as possible to ensure that the expectations for the study were clear. When the headmasters agreed that the research could be done at their schools, a similar discussion was held with all the mathematics teachers of the participating schools. Again, it was important to highlight the expectations and explain that all participation was voluntary and that any party could withdraw from participating at any time. Meetings involving the Grade 12 learners and parents (some of the learners were under the age of 18 and therefore needed parental consent to participate) then took place to inform them of the proposed study.

All willing participants were issued consent letters. These letters served a double purpose. The letters explained what the study entailed, so if they had any questions after the meetings they could revisit the details by reading the letters. It also gave the proposed participants a second chance to consider if they were completely willing to participate in the study. In the end, all participants signed consent letters (See Appendix G).

The photographs of the written responses of the examinations and Test 1, and the tape recording of the interviews also had to be considered during the ethical process to ensure that no participant felt threatened or uncomfortable with their contribution to the study at any point during the study.

4.6 LIMITATIONS OF THE STUDY

Some of the limiting factors of this study included time, finding schools willing to participate and, more specifically, participants willing to engage in all the elements of the study.

The biggest limitation of this study was time. The teachers involved in the study had very little free time available to spend on the study because of the responsibilities they had at their schools. These responsibilities included academic duties and extramural activities.

Probability was taught for the first time on Grade 12 level in the third term of 2014. According to CAPS, two weeks of teaching time must be allocated to this topic. The time from introduction of the topic until the writing of the preparatory examination was only four weeks. There was no time to reinforce the principles related to probability before the learners were examined on the topic for the first time. The limited time between introducing a topic and writing examinations on the topic for the first time should also be considered when looking at the limitations of the study.

I found it very difficult to find schools willing to participate in the study. Many principals would not allow any research to be conducted in their schools. They were concerned about the schedules of the teachers and the possible interference with the programme of the learners. The use of personal network recruitment in the end proved invaluable. A larger number of participants would have ensured better-rounded results.

When designing the study, I went out from the assumption that most schools would have more than one teacher teaching Grade 11 and 12 mathematics. This was only the case in the urban schools. In both the peri-urban schools, only one teacher was responsible for teaching mathematics to all the Grade 11 and 12 learners.

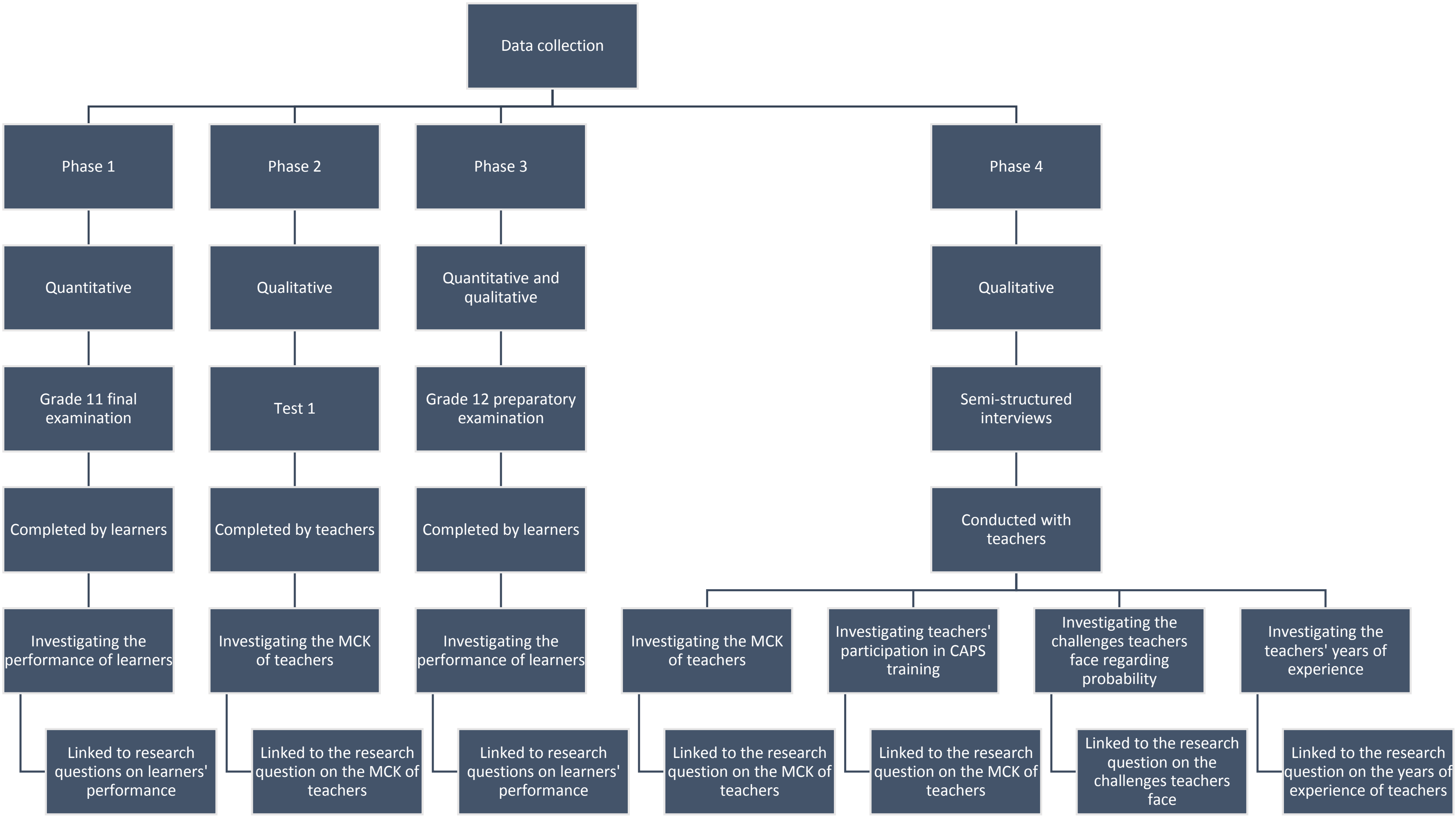
Some of the teachers who agreed to participate in the study were unwilling to complete all of the instruments involved in the study. All the teachers were willing to participate in the interviews, but neither of the teachers from the peri-urban schools completed the written test, Test 1.

Relating teachers' content knowledge to learner performance in probability.

4.7 SUMMARY

This chapter discussed how quantitative and qualitative research approaches were used in a mixed method research strategy to gather data from teachers and learners. The teachers were involved in writing Test 1 and participated in semi-structured individual interviews. The learners, being passive participants, wrote two examinations, one in Grade 11 and one in Grade 12. The data gathered using these instruments were recorded, coded and analysed. The results for the analysis of the data were then used to answer the research questions related to the study. Ultimately, it was the goal of the study to gain insight into the MCK levels of the teachers and the performance levels of the learners they teach with regard to the newly introduced topic of probability. Figure 3 shows a flow diagram incorporating all the elements involved in the study.

Figure 5: Data collection strategy



CHAPTER 5: DESCRIPTION OF DATA COLLECTION INSTRUMENT AND TECHNIQUES

This chapter discusses the instruments used during the data collection process of this study. The analysis was done in terms of the conceptual framework.

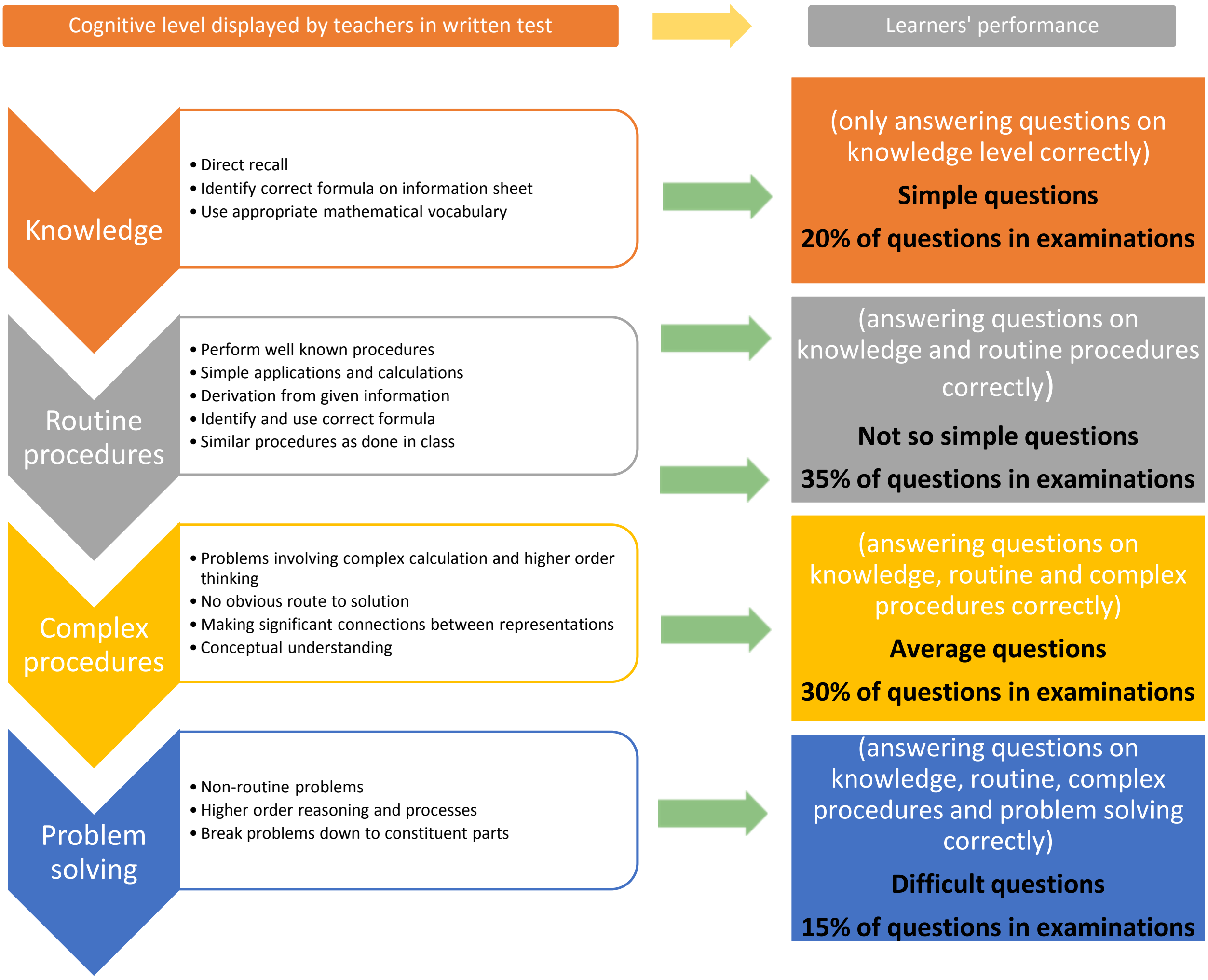


Figure 6. Assessment tool based on cognitive levels (DBE, 2011)

The conceptual framework is based on the cognitive levels recommended by CAPS and deduced from recommendations made by Mullis et al. (2003) in a report on the results of TIMSS. These cognitive levels give an indication of the difficulty level of the questions as well as the skills that are required to solve the questions. Knowledge is the first cognitive level mentioned in CAPS. The questions related to this level need to make a contribution of 20% to the total mark allocation of all formal assessments. Knowledge as a cognitive level includes questions that require straight recall and the use of appropriate vocabulary. Thirty-five percent of the marks allocated to formal assessment must be on the routine procedures cognitive level. Routine procedures include the ability to perform well-known procedures that may include the identification and use of formulas without changing the subjects of the formulas. Routine procedures as a cognitive level are followed by complex procedures. These complex procedures include questions based on a real world context where there is no obvious route to the solution, and 20% of the total marks allocated to formal assessments should be on this level. The last cognitive level is problem solving and this level must make up the last 15% of the total mark allocation of formal assessments. Problem solving as a cognitive level requires higher order reasoning to solve non-routine problems.

In the discussion on the analysis of the instruments involved in this study, the analyses of the Grade 11 examinations, Test 1 and the Grade 12 preparatory examination are done according to the cognitive levels and the mark allocation to each level. The semi-structured interview schedule is analysed by discussing the three sections of the interviews separately.

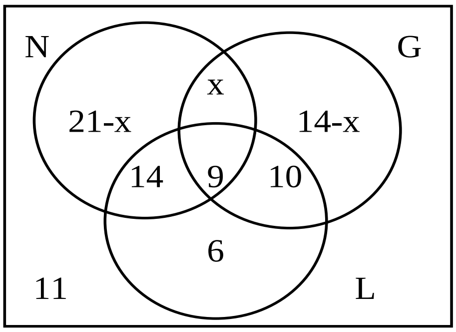
5.1 Grade 11 Final Examinations

At the end of 2013, all Grade 11 learners wrote a final mathematics examination. These examinations are set by the GDE or the school itself. Of the four participating schools, Schools B, C and D wrote the GDE examination and School A chose to set and write their own paper. In the next section, the individual questions from the two instruments are analysed according to the cognitive levels involved and the marks awarded to each of these cognitive levels. In the CAPS document, it is mentioned that the mark allocation for the topic of probability should be between 12 and 18 marks out of the 150 total marks allocated to Paper 1.

5.1.1 Grade 11 Final examination set by School A

Twenty-five learners from School A completed this question paper. The question related to probability was the last question (question 7) of Paper 1 of the final Grade 11 mathematics examination. Question 7 consisted of two subsections, namely section 7.1 and section 7.2. These sections of question 7 are discussed separately to include detail on the question that was asked, the memorandum of the question along with the mark allocation and the cognitive level involved in the solution of the question. Question 7 made a contribution of 16 marks to the total of 150 marks for Paper 1.

- **Question 7.1**

<table>
<tr><td colspan="4">A survey was done at a local library to show the different reading preferences for 80 students.
• 44 read the National Geographic magazine
• 33 read the Getaway magazine
• 39 read the Leadership magazine
• 23 read both the National Geographic and the Leadership magazines
• 19 read both the Getaway and the Leadership magazines
• 9 read all three magazines
• 69 read at least one magazine</td></tr>
<tr><th>Question number</th><th>Solution</th><th>Mark allocation</th><th>Cognitive level</th></tr>
<tr><td>7.1.1
How many students do not read any of these magazines?</td><td>11 students do not read any magazines</td><td>(1)</td><td>Knowledge</td></tr>
<tr><td>7.1.2
Let the number of students that read National Geographic and Getaway, but not Leadership be represented by x. Represent this information in a Venn diagram.</td><td>
</td><td>(3)

(2)</td><td>Complex procedures / Problem solving</td></tr>
<tr><td>7.1.3
Show that $x = 5$</td><td>$21 - x + x + 14 - x + 14 + 9 + 10$
$+ 6 + 11 = 80$
$\therefore x = 5$</td><td>(3)</td><td>Routine procedures</td></tr>
<tr><td>7.1.4
What is the probability (correct to 3 decimal places) that a random student will</td><td>$P(at\ least\ 2) = \frac{14 + 9 + 10 + 5}{80}$
$P\ (at\ least\ 2) = 0.475$</td><td>(3)</td><td>Routine procedures</td></tr>
</table>

read at least 2 of the 3 magazines?			

Question 7.1 started with a real life scenario that included three events in a sample space. In order to get the learners to engage in the question, a simple question on the lowest cognitive level of knowledge was given in question 7.1.1. The learners needed to build on this in question 7.1.2 and draw a Venn diagram using the problem solving skills of breaking a problem down in its constituent parts, and then use complex procedures to solve the problem that was based in a real world context. Routine procedures were needed to solve questions 7.1.3 and 7.1.4. To answer question 7.1.4 correctly, learners need to identify and use a formula that was given to them on a formula sheet, and the calculations involved in the solution of question 7.1.3 are seen as well-known procedures. One mark was allocated to the knowledge level, six marks to routine procedures, three marks to complex procedures and lastly, two marks were allocated to the problem solving level.

- **Question 7.2**

Question 7.2 was a question based on events where the learners need to use the rules concerning dependent, independent and mutually exclusive events in order to solve the question correctly.

A smoke detector system in a large warehouse uses 2 devices: A and B. The probability that device A will detect any smoke is 0.95 and device B is 0.98. The probability that both will detect smoke at the same time is 0.94.			
Question number	Solution	Mark allocation	Cognitive level
7.2.1 What is the probability that device A or B will detect smoke?	$P\ (A\ or\ B)$ $= P(A) + P(B)$ $- P(A\ and\ B)$ $P\ (A\ or\ B)$ $= 0.95 + 0.98$ $- 0.94$ $P\ (A\ or\ B) = 0.99$	(3)	Routine procedures

7.2.2 What is the probability that smoke will not be detected?	$P\ (not\ A\ or\ B)$ $= 1 - P(A\ and\ B)$ $P(not\ A\ or\ B)$ $= 1 - 0.99$ $P(not\ A\ or\ B)$ $= 0.01$	(1)	Knowledge

Question 7.2 was a question that engaged learners in the identification and use of dependent, independent and complementary events. Again, these questions were set in real life contexts, but here it was not seen as a complex procedure as learners only had to identify and use the identity $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$ in question 7.2.1 and it was therefore seen as a routine procedure. In question 7.2.2 the complementary rule was used to calculate the solution on the knowledge level as it required straight recall of this probability identity along with a simple calculation. Three marks were awarded to routine procedures and one mark to knowledge.

Table 7 gives a summary of the cognitive levels involved in this Grade 11 examination. The cognitive level is mentioned along with the number of marks allocated to each level. The allocated marks are then represented as a percentage of the total marks allocated to the probability question as a whole. Lastly, the percentages recommended by CAPS are stipulated.

Table 8. Cognitive levels and mark allocation for 2013 Grade 11 final examination of School A

Cognitive level	Mark allocation	Percentage of total	Recommended % by CAPS
Knowledge	2	12.5 %	20 %
Routine procedures	9	56.25 %	35 %
Complex procedures	3	18.75 %	30 %
Problem solving	2	12.5 %	15 %

The mark allocation of the Grade 11 final examination set by School A is compared with the recommended mark allocation per cognitive level in CAPS. The marks allocated to questions on the knowledge level were less than recommended by CAPS; only 12.5% of the marks were allocated to this cognitive level. The majority of the marks in this question were awarded to question on the routine procedures level. Of the sixteen marks allocated to the probability question, nine marks were allocated to routine procedures, which is 56.25% of the total. This was much higher than the recommended 35% mentioned in CAPS. When examining the questions based on complex procedures, 18.75% of the marks were allocated to this cognitive level. Problem solving was awarded two of the sixteen marks allocated to question 7. The spread of the allocated marks over the four cognitive levels did not follow the recommendations made in CAPS. Too few marks were awarded on the cognitive levels of knowledge, complex procedures and problem solving. The majority of the marks went to routine procedures. By examining the probability question of School A, it is clear that the question was too easy, as the mark allocation to the two higher cognitive levels was much lower than recommended in CAPS.

5.1.2 Grade 11 Final examination set by the GDE

Sixty-four learners from School B completed the Grade 11 final examination paper set by the GDE in 2013. The questions related to probability included question 7 and question 8 of Paper 1, which were the last two questions of the paper, and 17 marks of the 150 marks for Paper 1 were allocated to these questions. These two questions are analysed separately to indicate the questions that were asked along with the memorandum of these questions. The cognitive levels and the marks allocated to each sub-question with relation to the cognitive levels are also discussed.

- **Question 7**

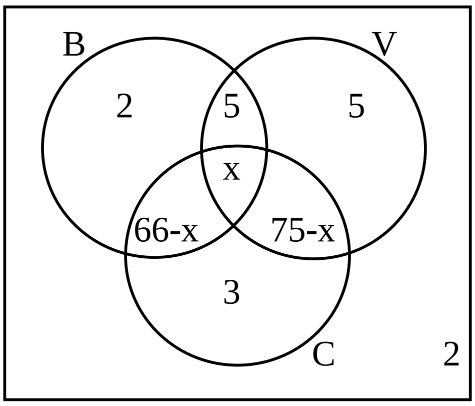
In question 7, three events were linked in a real world context and learners were asked to draw a Venn diagram to show the relationship between the three events and to do some probability calculations linked to the diagram.

A school organises a camp for 103 Grade 8 learners. The learners were asked write down what food they prefer. They could choose between beef (B), vegetables (V) and chicken (C).

The following information was gathered.

- 2 learners do not eat beef, vegetables or chicken.
- 5 learners only eat vegetables.
- 2 learners eat only beef.
- 12 learners do not eat chicken at all.
- 3 learners eat only chicken.
- 66 learners eat chicken and beef.
- 75 learners eat chicken and vegetables.

Let the number of learners that eat beef, vegetables and chicken be x.

<table>
<tr><th>Question number</th><th>Solution</th><th>Mark allocation</th><th>Cognitive level</th></tr>
<tr><td>7.1
Draw a Venn diagram to represent the above information.</td><td>
</td><td>(4)

(3)</td><td>Complex procedures / Problem solving</td></tr>
<tr><td>7.2
Calculate x.</td><td>$2+5+5+66-x+x+75-x$
$+3+2=103$
$\therefore x=55$</td><td>(2)</td><td>Routine procedures</td></tr>
<tr><td>7.3.1
Calculate the probability that a learner that is chosen at random eats: only beef and chicken but not vegetables.</td><td>$P(B \text{ and } C)=\frac{11}{103}$
$P\ (B \text{ and } C)=0.107$</td><td>(2)</td><td>Routine procedures</td></tr>
</table>

7.3.2 Calculate the probability that a learner that is chosen at random eats: only two of the food types, beef, chicken and vegetables.	$P\ (only\ 2) = \frac{11+20+5}{103}$ $P\ (only\ 2) = 0.35$	(2)	Routine procedures

Question 7.1 involved the drawing of a Venn diagram that involved three events in a sample space. The question was set in a real life context that required conceptual understanding and therefore fell within the cognitive level of complex procedures. In order to set the question out correctly, learners needed to break the problem down into constituent parts and therefore the learners needed to engage in the cognitive level of problem solving in question 7.1. Question 7.2 was a routine procedure that required the setting up of a linear equation. Questions 7.3.1 and 7.3.2 both required only routine procedures to solve, as learners needed to choose a formula from an information sheet and use it correctly. In question 7, three marks were allocated to problem solving skills, four marks to complex procedures and six marks to questions on the routine procedures level. This question was set out in a very similar way to the Venn diagram question in the Grade 11 final examination set by School A.

Question 8

Events X and Y are independent. $P(X) = \frac{1}{3}$ and $P(X\ and\ Y) = \frac{1}{12}$, calculate P(Y)			
Question number	Solution	Mark allocation	Cognitive level
8. Calculate P(Y).	$P\ (X\ and\ Y) = P(X) \times P(Y)$ $\frac{1}{12} = \frac{1}{3} \times P(Y)$ $P\ (Y) = \frac{1}{4}$	(2) (2)	Knowledge / Routine procedures

Question 8 was a single question that incorporated the use of the product rule for independent events, $P(A\ and\ B) = P(A) \times P(B)$. In order to calculate the probability of Y in this question, learners had to use the product rule and make a simple calculation. This question therefore needed a combination of knowledge and routine procedures to solve successfully.

The probability questions from the Grade 11 final examination set by the GDE were analysed by examining the mark allocation and distribution of marks with regard to the cognitive levels mentioned in CAPS (DBE, 2011). The results from this analysis are summarised in Table 8.

Table 9. Cognitive levels and mark allocation for 2013 GDE Grade 11 final examination

Cognitive level	Mark allocation	Percentage of total	Recommended % by CAPS
Knowledge	2	11.76 %	20 %
Routine procedures	8	47.06 %	35 %
Complex procedure	4	23.53 %	30 %
Problem solving	3	17.65 %	15 %

The mark distribution with regard to the allocation of marks to the four cognitive levels recommended by CAPS shows that the majority of marks were awarded to the routine procedures cognitive level. Only 11.76% of the marks on probability in this examination were asked on a knowledge level, where the recommended percentage is 20%. Four marks, or 23.53% of the total marks allocated to probability, were awarded to complex procedures. The highest cognitive level was awarded three of the seventeen marks allocated to probability, which is higher than recommended in CAPS.

When comparing the two sets of probability questions from the Grade 11 final examination set by School A and the Grade 11 examination set by the GDE with regard to the mark allocated to each cognitive levels, there are a few similarities. Both examinations essentially had a first section that included a Venn diagram and another question on dependent, independent and mutually exclusive events. The mark allocations were also similar when examining the cognitive levels of knowledge and routine procedures. The knowledge level was awarded fewer marks, and routine procedures were awarded more marks than recommended by CAPS. The GDE examination, however, allocated more marks to the highest cognitive level of problem solving, and therefore one can say that the Grade 11 examination set by the GDE was slightly more difficult than the Grade 11 examination set by School A.

5.2 TEST 1

As discussed in Chapter 4, Test 1 was designed in a collaborative effort between me and another experienced mathematics teacher. In the design of this instrument, it was vital to ensure the inclusion of all specifications stipulated by CAPS concerning probability. CAPS gives very clear guidelines on what specific content needs to be included in the learning plan when dealing with the topic of probability. The probability content in Test 1 therefore included probability questions based on the specifications listed in CAPS and included questions on dependent, independent and mutually exclusive events, Venn diagrams, two-way contingency tables and tree diagrams. To ensure that the research questions are answered, it was important to include questions that incorporated the whole spectrum of cognitive levels in the recommended ratio mentioned in CAPS.

The questions asked in Test 1 are now analysed individually by referring to the content that is included in the curriculum statement and the cognitive levels, along with the mark allocation awarded to each cognitive level. The content covered in Test 1 included questions on dependent, independent and mutually exclusive events, Venn diagrams and tree diagrams. The curriculum statement along with a description of each of the curriculum statements and cognitive levels mentioned is specified in CAPS (DBE, 2011).

- **Question 1**

The first question of Test 1 was a question of probability related to dependent, independent and mutually exclusive events.

Three events, A with a probability of $P(A) = 0.3$, B with a probability of $P(B) = 0.4$ and C with a probability of $P(C) = 0.2$.

- A and B are independent
- B and C are independent
- A and C are mutually exclusive

Calculate the probability of the following events occurring:			
Question number	Solution	Mark allocation	Cognitive level
1.1 Both A and C occur.	$P(A \text{ and } C) = 0$	(1)	Knowledge
1.2 Both B and C occur.	$P(B \text{ and } C) = P(B).P(C)$ $P(B \text{ and } C) = (0.4)(0.2)$ $P(B \text{ and } C) = 0.08$	(1)	Knowledge
1.3 At least one of A or B occur.	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ or } B) = 0.3 + 0.4 - 0.12$ $P(A \text{ or } B) = 0.58$	(4)	Routine procedures

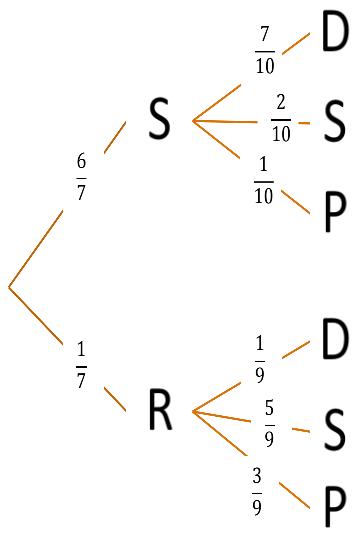
Question 1 covered the content related to the theory and rules concerned with dependent, independent and mutually exclusive events. The cognitive level involved in the solution of questions 1.1 and 1.2 was that of knowledge as it required straight recall of mutually exclusive events and the identity $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. In question 1.3, routine procedures were used to calculate the correct solution by choosing the correct formula and doing simple arithmetic to solve a single variable. Two marks were allocated on the cognitive level of knowledge and on routine procedures, four marks were allocated.

In responding to this question it was expected that all teachers would have the MCK to solve question 1 with 100% accuracy as it only involved the lower two cognitive levels of knowledge and routine procedures.

- **Question 2**

The second question posed to the teachers in Test 1 was a question that included the setting up of a tree diagram and answering questions related to the tree diagram.

In Johannesburg, South Africa, the probability of a sunny day is $\frac{6}{7}$ and the probability of a rainy day is $\frac{1}{7}$. If it is a sunny day, the probability that Thapelo will wear a dress to work is $\frac{7}{10}$, the probability that Thapelo will wear a skirt to work is $\frac{1}{5}$ and the probability that she will wear pants to work is $\frac{1}{10}$. If it is a rainy day, then the probability that Thapelo will wear a dress to work is $\frac{1}{9}$, the probability that she will wear a skirt to work is $\frac{5}{9}$ and the probability that Thapelo will wear pants to work is $\frac{1}{3}$.

Question number	Solution	Mark allocation	Cognitive level
2.1 Draw a tree diagram to represent the above information. Indicate on your diagram the probabilities associated with each branch as well as all the outcomes.		(3) (2)	Complex procedures/ Problem solving
2.2.1 For a day selected at random, what is the probability that: It is rainy and Thapelo will wear a dress	$P(Rainy, Dress) = \frac{1}{36}$	(1)	Complex procedures
2.2.2 For a day selected at random, what is the probability that: Thapelo wears pants	$P(Pants) = \frac{6}{7} \times 0.1 + \frac{1}{7} \times \frac{3}{9}$ $P(Pants) = \frac{2}{15}$	(2)	Complex procedures

2.3 If Thapelo works 245 days in a year, on approximately how many occasions does she wear a skirt to work?	$P(Skirt) = \frac{6}{35} + \frac{5}{63}$ $P(Skirt) = \frac{79}{315}$ Thapelo wears a skirt for $\frac{79}{315} \times 245$ days = 61 days.	(2) (2)	Complex procedures / Problem solving

In question 2, the content related to the representation of the probability of consecutive or simultaneous events that were not necessarily independent by using a tree diagram (DBE, 2011). In the analysis of the cognitive levels involved, question 2 mostly involved complex procedures as the sub-questions were based on a real world context and a conceptual understanding was necessary to represent the given information in a different form, in this case the setting up of a tree diagram. The solutions to questions 2.1 and 2.3 involved some problem solving skills. Problem solving requires a higher order reasoning where the problem needs to be broken down in constituent parts in order to get to the correct solution. These problem solving skills were needed in the initial set up of the tree diagram in question 2.1 and in the calculation that included the extra dimension of number of days in a year in question 2.3. There were eight marks allocated to the complex procedures level and four marks involving problem solving skills.

The question related to the tree diagram was seen as a higher level question as it mostly involved the two higher cognitive levels of complex procedures and problem solving. Despite the difficulty level of the question, teachers were expected to do well in this question.

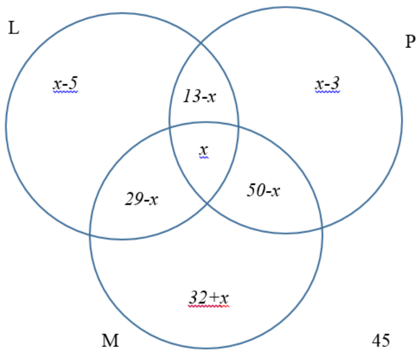
- **Question 3**

The last question of Test 1 was a question on a Venn diagram. The question required the teachers to set out a Venn diagram that included three events, and then probability questions related to the Venn diagram were asked.

A school has 174 Grade 12 learners, a survey is done among the learners and the following is found:

<table>
<tr><td colspan="4">

- 37 learners take Life Science
- 60 learners take Physical Science
- 111 learners take Mathematics
- 29 learners take Life Science and Mathematics
- 50 learners take Physical Science and Mathematics
- 13 learners take Life Science and Physical Science
- 45 learners do not take Life Science, Physical Science or Mathematics

x learners take Life Science, Physical Science and Mathematics
</td></tr>
<tr><th>Question number</th><th>Solution</th><th>Mark allocation</th><th>Cognitive level</th></tr>
<tr><td>3.1
Draw a Venn diagram to represent the information above.</td><td>
</td><td>(4)

(2)</td><td>Complex procedures / Problem solving</td></tr>
<tr><td>3.2
Show that x=13.</td><td>$x - 5 + 13 - x + x - 3 + x + 29 - x + 50 - x + 32 + x + 45 = 174$
$x = 13$</td><td>(2)</td><td>Routine procedures</td></tr>
<tr><td>3.3.1
If a learner is selected at random, calculate the probability that the learner has the following combination of subjects: Physical Science and</td><td>$P(M \text{ and } P \text{ not } L) = \frac{37}{174} = 0.21$</td><td>(2)</td><td>Complex procedures</td></tr>
</table>

Mathematics but not Life Science.			
3.3.2 If a learner is selected at random, calculate the probability that the learner has the following combination of subjects: Only one of Life Science, Physical Science or Mathematics	$P(only\ M\ or\ P\ or\ L)$ $= \frac{8+10+45}{174}$ $= 0.36$	(2)	Routine procedures

CAPS states that learners should be able to use a Venn diagram to solve probability problems for three events in a samples space (DBE, 2011). Question 3 was based on this curriculum statement. The questions included three of the cognitive levels, namely routine procedures, complex procedures and problem solving. In question 3.1, the teachers needed to engage in the cognitive level of complex procedures as the question was set in a real world context. Teachers also needed the ability to break the problem down into constituent parts in order to set the question out correctly. In order to do this, teachers needed to engage in the cognitive level of problem solving. Routine procedures were followed in the calculation and solution of questions 3.2 and 3.3.2 as teachers had to set up and solve a linear equation that was a well-known procedure. Question 3.3.1 was on the complex procedures level, as it required conceptual understanding in the interpretation of the Venn diagram. In question 3, four marks were allocated to routine procedures, five marks to complex procedures and only three marks to the problem solving level.

Table 10 gives a summary of the cognitive levels and mark allocation to each cognitive level found in Test 1. The mark allocation per cognitive level in Test 1 did not comply with the ratios recommended by CAPS. Test 1 included more questions on the higher cognitive levels of complex procedures and problem solving, as it was important to investigate the MCK of teachers on the higher cognitive levels. The reasoning behind this is linked to the literature review of this study, where studies have shown that the MCK of teachers should include higher order cognitive level skills such as problem solving (Ball et al., 2008; Wu, 2005).

Table 10. Cognitive levels and mark allocation for Test 1

Cognitive level	Mark allocation	Question number	% of probability question	% of total
Knowledge	2	1.1, 1.2	6.67 %	20 %
Routine procedures	8	1.3, 3.2, 3.3.2	26.67%	35 %
Complex procedures	13	2.1, 2.2.1, 2.2.2, 2.3, 3.3.1	43.33 %	30 %
Problem solving	7	2.1, 2.3, 3.1	23.33 %	15 %

The research of Ball et al. (2008) states that these higher levels of knowledge are necessary for teachers to ensure that they have the necessary skills to assign the correct work to learners and to give constructive comments on learners' work. These skills may enable learners to engage in mathematics and, in doing so, improve their skills and knowledge.

5.3 Semi-structured Interviews

Semi-structured individual interviews were conducted with all eight teachers who participated in this study. The interviews were conducted after the six teachers wrote Test 1. Two teachers were not willing to complete Test 1. Test 1 was written by the teachers, after which it was marked by myself according to the memorandum (see Appendix C2). This process was necessary as the information gathered from the teachers' written responses led to some additional questions being included in the interviews. The interviews were conducted in both English and Afrikaans, depending on the preference of the teachers. All the interviews were voice recorded to ensure no misinterpretation during the transcription of the interviews. The recordings of the interviews were transcribed, and those conducted in Afrikaans were translated into English to enable the incorporation of the data into this report.

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Analysis of the semi-structured interview schedule is done in this section to show how it is integrated in the answering of some of the research questions of this study. The interviews consisted of three parts. The first part included questions related to administration information regarding the school and the learners taking mathematics, followed by questions on biographical information of the teachers as well as a question on their years of teaching experience. The information gathered included:

- The name of the school
- The region of the school and if it is in in an urban or peri-urban area
- The number of Grade 12 learners in the school
- The number of Grade 12 learners with mathematics as a subject
- The number of teachers teaching mathematics on a Grade 12 level
- The gender of the participating teachers
- The years of teaching experience of the teachers

The information regarding the teachers' years of experience assisted in answering the research question on how the years of experience could be linked to the MCK levels they displayed in the written responses in Test 1. This section was then followed by questions on the training of the teachers.

The second part of the semi-structured interviews asked questions related to the academic history of the teachers. These questions started on the secondary education level and then progressed to ask questions on the tertiary education level of the teachers. The questions addressed the following matters:

- The highest mathematics qualification at school level, which included choices such as senior certificate Higher Grade or Standard Grade, NSC Mathematics or Mathematical Literacy
- The tertiary qualification of the teacher, which included whether the teacher had a diploma or a degree, and at what institution this qualification was achieved
- The mathematics courses taken by the teacher on tertiary level and specifically if any of these included probability
- The professional development the teacher was involved in, specifically asking about CAPS training by the DBE

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With the changing curricula, it was important to establish whether the teachers had any training on probability at school level. In order to investigate MCK more specifically, the teachers were also asked whether their tertiary qualification included any mathematics courses that included probability. It was also important to investigate whether the teachers were involved in professional development programmes, as this may also have influenced their MCK levels. Teachers were therefore asked about their involvement in the training sessions hosted by the GDE before and during the introduction of CAPS. The questions related to the CAPS training sessions gathered information on the number and duration of the sessions the teachers attended. The teachers were also given the opportunity to discuss their personal experiences and opinions on the usefulness of the training sessions.

The final part of the interviews only involved some of the teachers who wrote Test 1. The questions asked here were specifically designed around the written responses that teachers gave in Test 1. The focus here was not on the teachers who performed well in Test 1 (Teacher 1 who scored 100% and Teacher 2 who scored 97%), but they were asked a few questions related to their written responses and overall performance. The four remaining teachers, who did not do so well (Teacher 3 who scored 73%, Teacher 4 who scored 53%, Teacher 5 who scored 43% and Teacher 6 who scored 77%), were interviewed in more detail. The questions in part 3 of the semi-structured interviews asked teachers specific questions on the incorrect written responses they gave in Test 1. These question were incorporated to illuminate possible misconceptions and insight into the MCK levels that these teachers possess. This last section of the interviews also included a question on the challenges, if any, that the teachers faced in relation to probability as a topic in general. The data gathered in the first two sections of these interviews were captured and organised in tabulated form and are discussed in the next chapter.

5.4 GRADE 12 PREPARATORY EXAMINATION

The last instrument used to measure the performance of learners on the topic of probability was the Grade 12 preparatory examination written in September 2014. All the Grade 12 learners ($n = 75$) involved in this study completed the same examination set by the GDE. In the analysis of this examination, the first thing to note is that the topic of probability in CAPS for Grade 12 contains more content than in Grade 11. In Grade 11, the curriculum statement includes questions on dependent, independent and mutually exclusive events, Venn diagrams, two-way contingency tables and tree diagrams. In Grade 12, all of these are included but the use of the fundamental counting principle to solve probability questions is added to the content.

The questions related to the topic of counting and probability was the last question in the Grade 12 preparatory examination Paper 1, Question 13, and 15 marks of the 150 marks for Paper 1 were allocated to this question.

- **Question 13**

Question 13 consisted of two sections. The first two questions (question 13.1 and 13.2) related to probability and the last question (question 13.3) was based on the fundamental counting principle. This last question did not include any questions related to probability. For the sake of comprehensiveness, this question is discussed here, but it was not included as part of the data gathered for this study.

- **Question 13.1**

This question was based on the probability concerned with dependent, independent and mutually exclusive events.

<table>
<tr><td colspan="4">Tebo writes an Art and a Music examination. He has a 40% chance of passing the Music examination, a 60% chance of passing the Art examination and 30% chance of passing both the Music and Art examination.</td></tr>
<tr><th>Question number</th><th>Solution</th><th>Mark allocation</th><th>Cognitive level</th></tr>
<tr><td>13.1
Calculate the probability that Tebo will pass the Music or Art examination.</td><td>$P(M \text{ or } A) = P(M) + P(A) - P(M \text{ and } A)$
$P(M \text{ or } A) = 0.4 + 0.6 - 0.3$
$P(M \text{ or } A) = 0.7$</td><td>(3)</td><td>Routine procedures</td></tr>
</table>

Question 13.1 used the identity $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$, and was regarded as a routine procedure as learners needed to identify the correct formula from a formula sheet provided and use the formula directly to get to the answer.

- **Question 13.2**

In this question, a contingency table was used to test learners' knowledge on probability. The basic principle involved in this question was related to the concept of independent events.

A survey was conducted asking 60 people with which hand they write and what colour hair they have. The results are summarized in the table below.

		HAND USED TO WRITE WITH		
		Right	Left	**Total**
HAIR COLOUR	Light	a	b	**20**
	Dark	c	d	**40**
	Total	**48**	**12**	**60**

The survey concluded that the 'hand used for writing' and 'hair colour' are independent events.

Question number	Solution	Mark allocation	Cognitive level
13.2 Calculate the values of a, b, and c.	$P(Right\ and\ Light)$ $= P(Right)$ $\times\ P(Light)$ $\frac{a}{60} = \frac{48}{60} \times \frac{20}{60}$ $\therefore a = 16$ $b = 4, \quad c = 32\ and\ d = 8$	(2) (3)	Complex procedures / Problem solving

In question 13.2, at first glance it looks like the simple use of the addition rule for mutually exclusive events, $P(A\ and\ B) = P(A) \times P(B)$, but because of the way the contingency table was set up using four variables (a, b, c and d), this question was not classified as a routine procedure. To solve this question, learners needed a combination of problem solving skills and complex procedures. Question 13.2 was a non-routine problem that needed conceptual understanding of mutually exclusive events to solve the question successfully. The rest of Question 13 related to the fundamental counting principle.

- **Question 13.3**

Question 13.3 related to the fundamental counting principle. This principle is included in the Grade 12 CAPS and can be linked to probability questions. In this question paper it was not the case, and therefore this section of question 13 was not included in the data analysis of this study. I chose to discuss it here as the marks awarded to this question made a contribution to the total mark awarded to the topic of probability.

The digits from 1 to 9 are used to make 5-digit codes.			
Question number	Solution	Mark allocation	Cognitive level
13.3.1 Determine the number of 5-digit codes possible, if the digits are arranged in any order without repetition.	$9 \times 8 \times 7 \times 6 \times 5$ $= 15120$	(2)	Knowledge
13.3.2 Determine the number of 5-digit codes possible, if the code formed has to be an even number and the digits may not be repeated.	$8 \times 7 \times 6 \times 5 \times 4$ $= 6720$	(3)	Routine procedures
13.3.3 Determine the number of 5-digit codes possible, if the code formed only uses even digits and repetition of digits is allowed.	4^5 $= 1024$	(2)	Routine procedures

The curriculum statement states that learners should be able to answer questions by applying the fundamental counting principle to solve probability questions. In this case, the fundamental counting principles were not linked to probability, and were therefore not related to this study. However, I discuss the analysis of this question as it was part of question 13. Question 13.3 consisted of three questions that were all classified as routine procedures questions. In all three questions, the learners were asked to solve questions that were similar to those they would have encountered in class.

To complete the analysis of the probability question in the preparatory examination, Table 11 shows the cognitive levels and mark allocation for question 13.

Table 11. Cognitive levels and mark distribution for 2014 preparatory examination

Cognitive level	Mark allocation	Percentage of total	Recommended % by CAPS
Knowledge	0	0 %	20 %
Routine procedures	10	66.67 %	35 %
Complex procedures	3	20 %	30 %
Problem solving	2	13.33 %	15 %

The questions on probability had no questions on the fundamental cognitive level of knowledge. The majority of marks were allocated to the routine procedures level where 10 of the 15 marks for Question 13 were awarded. The questions on the cognitive levels of complex procedures and problem solving were three and five marks respectively, which are both below the recommended mark allocation mentioned in CAPS.

5.5 Summary

In the above discussion, the strategies related to the analysis of the instruments used in the study were considered. The analysis included quantitative as well as qualitative strategies, as the research approach is one of mixed methods. In the discussion on the analysis of the data collection instruments, it is important to incorporate the conceptual framework of the study, and therefore the discussion around the analysis included the cognitive levels as described in CAPS. The quantitative analysis focused specifically on these levels by analysing not only the difficulty level of the questions but also the marks allocated to each of the levels. The qualitative analysis in this study enriched the analysis that was done quantitatively and was therefore essential in the analysis process (Scotland, 2012).

CHAPTER 6: PRESENTATION AND ANALYSIS OF DATA

In this chapter, the data collected for this study are presented and analysed quantitatively and qualitatively in order to answer the main research question: **How does the level of teachers' MCK relate to the performance of the learners they teach on the topic of probability?** In order to answer the main research question, the analysis of the written responses of learners and teachers along with the responses given by teachers in the semi-structured interviews provides some insight.

Four sub-questions were used to answer the main research question.

- What is the level of MCK of teachers in probability, based on the cognitive levels recommended by the CAPS?
- What are the challenges that teachers experience in relation to probability?
- How do the tertiary training and experience levels of mathematics teachers relate to the MCK they possess?
- How can learners' performance in probability be described in terms of the cognitive levels recommended by CAPS?

The quantitative data discussion is presented and analysed using descriptive and correlation methods. This quantitative analysis is done on the written responses of the learners in the Grade 11 final examination and the Grade 12 preparatory examination. Descriptive statistics are used in this section to describe the data using the range, mean, median and standard deviation. The distribution of the data is also considered by investigating the symmetry or skewedness of the data as well as drawing histograms. The correlation, or lack thereof, is analysed using scatter plots to indicate relationships and the calculation of the Pearson correlation coefficient (r) to indicate possible statistical significance of the relationships between elements. The elements discussed in this study include the possible relationship between:

- The total mark scored in Paper 1 and the total mark scored in the probability question in the Grade 11 final examination

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- The total mark scored in Paper 1 and the total mark scored in the probability question in the Grade 12 preparatory examination
- The probability question scores in Grade 11 final examination and Grade 12 preparatory examination

The percentages scored by the learners in the examinations are analysed according to the performance levels[6] and the cognitive levels[7]. The data are also analysed according to the level of correctness of the solutions by using rank scores[8].

The sub-question for the quantitative analysis was:

- How can learners' performance in probability be described in terms of the cognitive levels recommended by CAPS?

The qualitative presentation and analysis include the data collected form Test 1 and the semi-structured interviews with the teachers as well as the written responses given by learners in the Grade 11 final and Grade 12 preparatory examinations. The results are represented using extracts from the written responses given by teachers in Test 1 and learners in the Grade 11 and Grade 12 examinations. The semi-structured interviews were transcribed and excerpts from the interviews are included in the qualitative analysis. The data gathered during the individual semi-structured interview process are used in the interpretation and discussion of the findings related to the other research sub-questions. Finally, the written responses given by teachers in Test 1 are compared (to investigate possible similarities or differences) to the written responses given by learners in the Grade 11 and Grade 12 examinations where there were similarities in the questions they answered.

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[6] Performance level refers to the average scores learners achieved in the examinations and include poor, below average, average and above average as seen in Chapter 4.

[7] Cognitive levels are recommended by CAPS and include knowledge, routine procedures, complex procedures and problem solving.

[8] Rank scores include correct, almost correct, some correct, none correct and did not attempt the solution.

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The sub-questions for the qualitative analysis were the following:

- What is the level of MCK of teachers in probability, based on the cognitive levels recommended by the CAPS?
- What are the challenges that teachers experience in relation to probability?
- How do the tertiary training and experience levels of mathematics teachers relate to the MCK they possess?

6.1 Presentation and Analysis of Quantitative Data

The quantitative data of this study are analysed using descriptive statistics, discussions on the distribution of the data and correlation methods (see Appendix H). The data used in this analysis include the written responses given by learners in the Grade 11 final examinations and the Grade 12 preparatory examination.

The descriptive statistics used in the analysis of the quantitative data include the calculation of:

- The mean, median and standard deviation

The discussion on the distribution of the data in the quantitative analysis includes the calculation or drawing of:

- The range of the data and the skewedness along with the symmetry of the distribution
- Histograms

The possible correlation is discussed using:

- Scatter plots to show the relationship between elements
- Pearson's correlation coefficient to show the significance or the possible relationship between elements

The quantitative analysis assisted with the answer to the research question on the performance of the learners in probability. The discussion includes the two Grade 11 final examinations, the Grade 12 preparatory examination and a discussion on the comparison between the probability questions answered by the same learners in Grade 11 and Grade 12.

6.1.1 Grade 11 final examination

The discussion on the analysis of the two Grade 11 final examinations starts with descriptive statistics, followed by the analysis of the distribution and lastly the correlation of elements. Eighty-nine Grade 11 learners from School A ($n = 25$) and School B ($n = 64$) participated in this part of the study.

6.1.1.1 Grade 11 final examination set by School A.

Twenty-five Grade 11 learners participated by writing this examination. The learners' written responses are discussed. The written responses were marked to get percentage scores to give insight into the performance levels of the learners. The responses were also marked by me to generate rank scores to investigate the level of correctness of the solutions and link this to the cognitive levels involved in this study.

Learners did not perform well in this examination and scored an average of 53.2% for Paper 1 in total and an even lower 51% for the probability questions. In order to answer the research questions that are related to the performance levels of the learners in probability, the analysis of the data includes the grouping of learners into the four performance levels. Table 12 shows the performance levels of the Grade 11 learners who wrote the examination set by School A. The table indicates that ten of the learners performed at an average level and six at an above average level. Six learners performed poorly and three performed at a below average level, which is disturbing.

Table 12. Performance levels of learners on probability in the Grade 11 examination set by School A

Performance level	Number of learners achieving at this level
Poor (0%-29%)	6
Below average (30%-49%)	3
Average (50%-64%)	10
Above average (65%-100%)	6

The values related to descriptive statistics for the Grade 11 final examination set by School A are summarised in Table 13.

Table 13. Descriptive statistics and distribution of the data for the Grade 11 examination set by School A

	Whole of Paper 1	Probability question
Mean	53.20%	51.25%
Median	51.00%	50.00%
Mode	50.00%	50.0%
Standard deviation	18.51%	25.90%
Skewness	0.30	0.052
Kurtosis	-0.386	-0706
Maximum	90.0	100.0
Minimum	19.0	6.25
Range	71.0	93.75
Coefficient of variation	34.79%	50.54%
Distribution	Positively skewed	Positively skewed

The mean value of the probability question indicates an average performance level. The standard deviation is very high in the probability question, which indicates that the data items are far spread out. This is supported by a coefficient of variation of 50.54% for the probability question as compared to 34.79% of the whole Paper 1. By examining the distribution of the data, the spread of the data related to the probability question is almost symmetrical but the difference between the mean and the median is one unit and therefore the data are more spread to the right of the median. The values in the data set are spread over a large range, with the minimum score achieved by a learner in School A being 6% and the maximum being 100%.

The following histogram and box plot shows the distribution for percentages scored for the whole paper along with the percentages scored for the probability question.

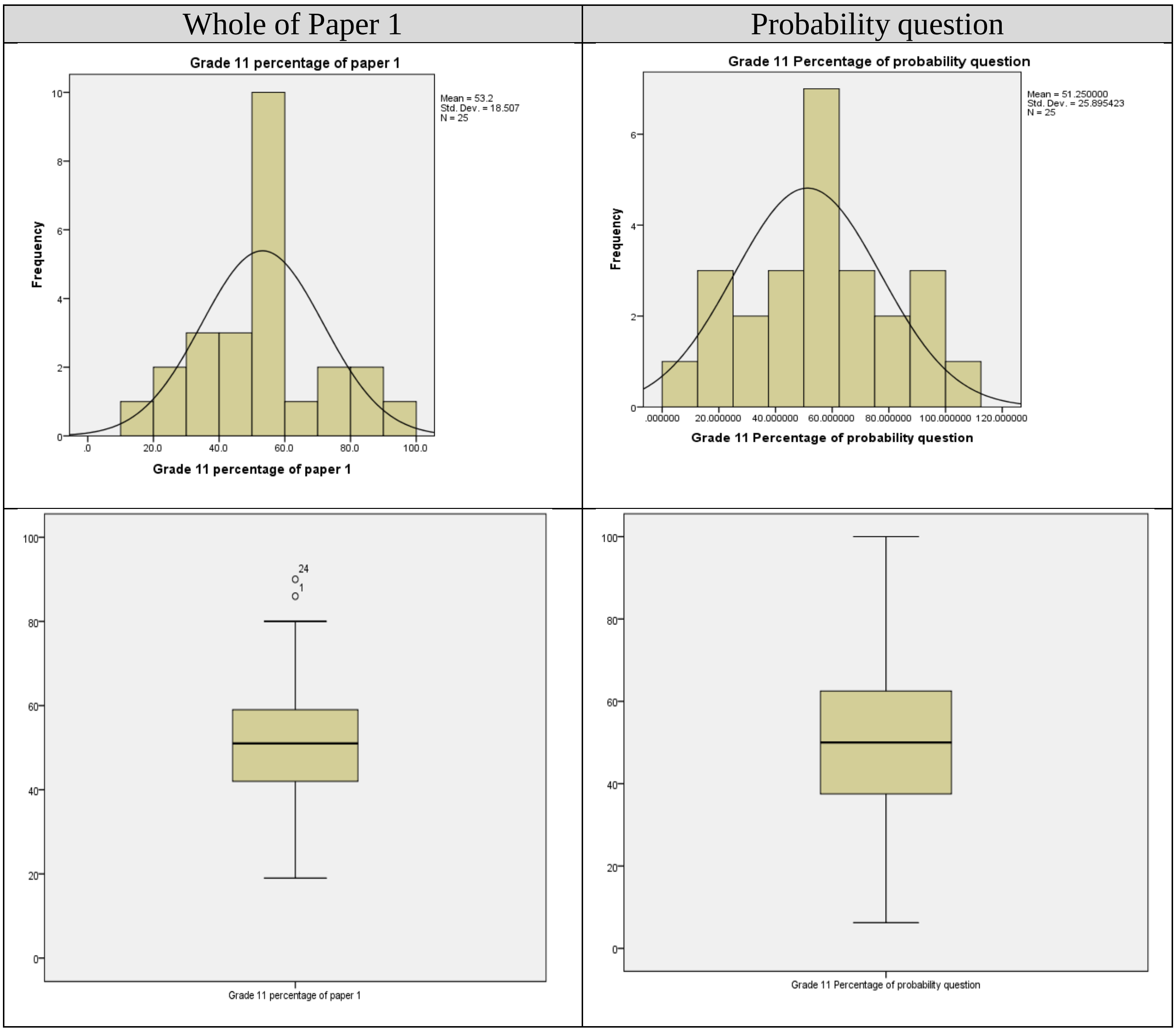

Figure 7. Histogram and boxplot showing distribution of marks for Grade 11 final examination set by School A

From the histogram, it can be seen that in general the learners performed better in the whole paper than in the probability question alone. In the probability question there were more learners who got less than 20% as compared to the whole paper. Both histograms showed that the data is almost symmetrical and this is supported by the box plots. Although the box plot for the whole paper had outliers to the right, the data is almost symmetrical. A proper test was done to confirm normality. The Shapiro Wilk test was used to test whether data was normally distributed. The test resulted in p-values of 0.518 and 0.429 for the whole paper and probability question respectively. Since the p-values were more than 0.05, the null hypothesis of normality was not rejected. Thus, data was normally distributed.

The rank scores generated when marking this examination were compiled in a table (See Appendix I). The rank scored table was analysed by using the cognitive levels of the conceptual framework of this study, and the results are reflected in Table 14. It is encouraging to see that all learners attempted all the questions regardless of the cognitive level involved in the solution. It is, however, a concern that 40 % of the learners could not do any correct calculations on the cognitive level of knowledge. A surprising result, however, is that learners could answer the questions on the two higher cognitive levels of complex procedures and problem solving with more success than they had in answering the routine procedures questions.

Table 14. Number of learners achieving specific rank scores for cognitive levels in the Grade 11 final examination set by School A

	Knowledge	Routine procedures	Complex procedures	Problem solving
Did not attempt the question	0	0	0	0
No correct procedures or calculations	10	12	2	2
Some correct procedures and calculations	0	5	5	6
Almost correct procedures and calculations	0	2	4	3
Correct procedures and calculations	15	6	14	14

The analysis of the relationship between the marks that the learners achieved in the whole of Paper 1 and the marks they achieved in the probability question was done by calculating the Pearson's correlation coefficient and drawing scatter plots of this data. In this case, $r = 0.78$ $(p - value = 0.000)$. The p-value was <0.01 which was significant at the 1% thus it was highly significant. This significant positive correlation indicates that, if learners performed well in the probability question, there also performed well in the whole paper and vice versa. The scatter plot in Figure 8 shows the marks scored by the learners in the whole of Paper 1 versus in the probability question. Some of the points are not visible as the values coincided.

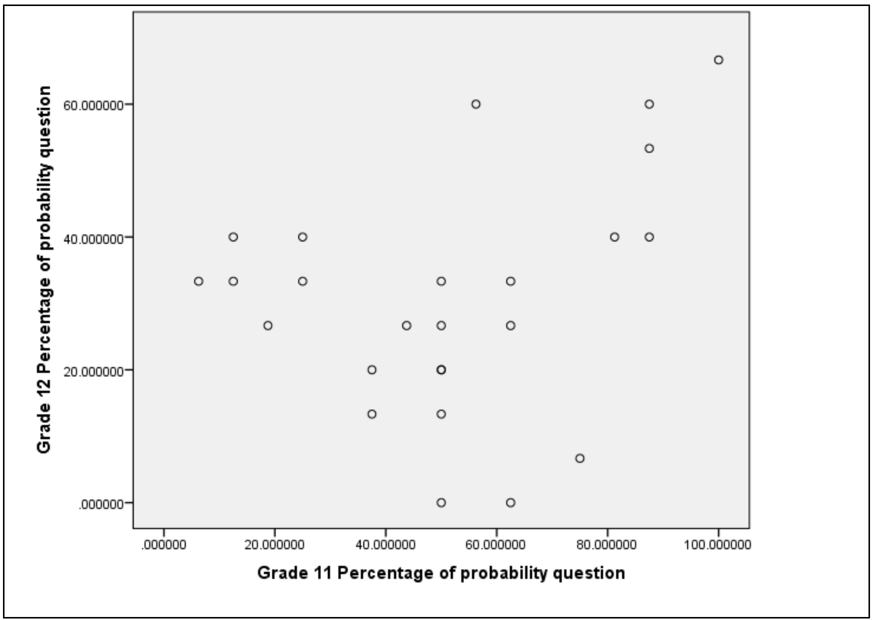

Figure 8. Scatter plots of Grade 11 final examination whole paper vs. probability question set by School A

The results from the analysis above show that learners did not perform well in the section of probability in general. The performance level of the majority of students was only average. The analysis of the performance of the learners with regard to the cognitive levels shows some mixed results. The questions related to the cognitive level of knowledge were not answered as well as one would expect. The learners either answered the question correctly or incorrectly. When examining the higher cognitive levels of complex procedures and problem solving, one would have expected that learners would be less successful in their attempts to solve these questions. The results, however, show that more learners attempted and managed to get some, almost and completely correct solutions.

6.1.1.2 Grade 11 final examinations set by GDE

The written responses of 64 learners from School B were marked. The results from the marking process were used to generate data in two forms. The marks allocated to the whole examination paper and the marks allocated to the probability question alone were converted to percentage scores. The percentage scores were used to analyse the data using descriptive statistics, as well as to describe the distribution of the data. The relationship between elements was analysed using a scatter plot and the significance of this relationship was measured by calculating Pearson's correlation coefficient (r). The sub-questions were also used to generate rank scores (See Appendix I). These rank scores comment on the level of correctness of the solutions given by learners and include correct, almost correct, some correct and no correct procedures or calculations. A rank score of did not attempt was also incorporated. The rank scores also enabled the analysis of the data according to the cognitive levels involved in the study.

Table 15 shows the performance levels of the learners in the Grade 11 examination set by the GDE. Of the 64 learners who wrote this test, 22 learners performed in accordance with the performance level of poor and 18 with the performance level of below average. This constitutes 63% of the learners and is very concerning.

Table 15. Performance levels of learners on probability in the Grade 11 examination set by GDE

Performance level	Number of learners achieving on this level
Poor (0%-29%)	22
Below average (30%-49%)	18
Average (50%-64%)	18
Above average (65%-100%)	6

The learners from School B scored an average of 51.98% for Paper 1 and 42.28% for the probability question. The average scored on the probability question was almost 10% lower than the average scored for Paper 1. This difference between the averages is remarkable and shows that the learners did not perform well in probability. When the difference between the mean and the median was calculated, it showed that the data are negatively skewed and therefore more data are spread to the left of the median. The standard deviation of the data concerning the probability question is 18.45, which shows quite a wide spread of the data around the mean of 42.28%. The coefficient of variation of the whole paper was 31.59% whilst for the probability question was 43.64%. The percentage scored were more spread out in the probability question than in the whole paper. The percentages scored in the probability question ranged from 0% to 100%, which is very high.

Table 16. Descriptive statistics and distribution of the data for the Grade 11 examination set by GDE

	Whole of Paper 1	Probability question
Mean	51.98%	42.28%
Median	50.00%	47.06%
Mode	43.00%	52.94%
Standard deviation	16.42	18.45
Skewness	0.427	0.232
Kurtosis	-0.518	0.629
Maximum	93.00	100.00
Minimum	25.00	0.000
Range	68	100
Coefficient of variation	31.59%	43.64%
Distribution	Positively skewed	Negatively skewed

The distribution of the data is shown in the histogram and boxplots below, which includes the marks scored for the whole of Paper 1 and the marks scored for the probability question. The histogram of the probability question confirms the concerning result that learners are performing better in the whole Paper 1 than in the probability question.

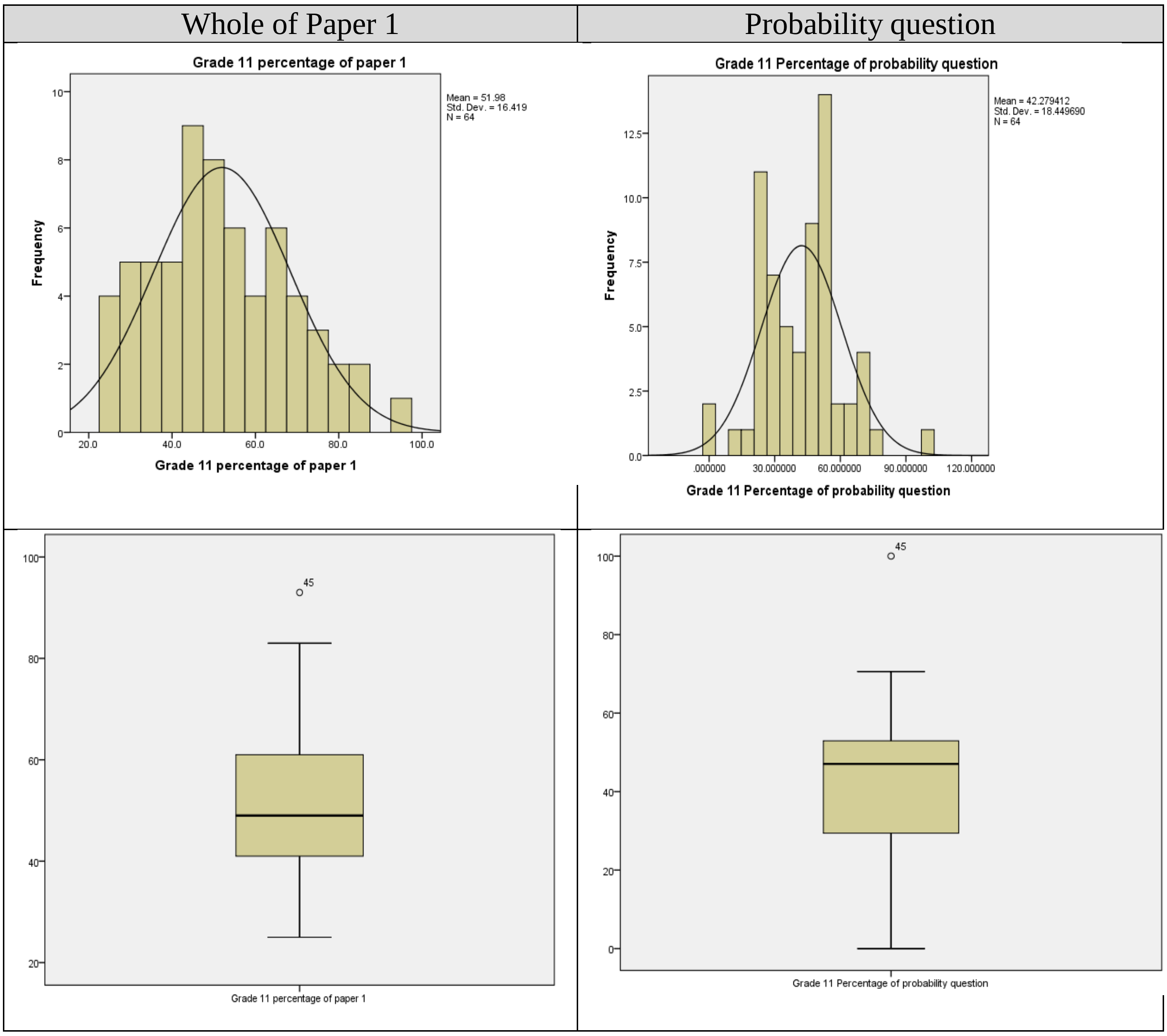

Figure 9. Histograms and boxplots showing distribution of marks for Grade 11 final examination set by GDE

Both histograms and boxplots showed that the data is almost symmetrical although on the boxplot there is an outlier to the right. The Shapiro Wilk test gave p-values of 0.063 and 0.144 for the whole paper and probability question respectively. Since the p-values were more than 0.05, the null hypothesis of normality was not rejected. Thus, data was normally distributed.

The data generated by categorising the results in the form of rank scores were captured in tables (See Appendix I). The rank scores were linked to the cognitive levels involved in this study, as shown in Table 17 below. In the analysis of Table 17, it is encouraging to see that all learners attempted all the questions, but in doing so many still did not manage to achieve any marks on the knowledge and routine procedures questions. The routine procedures involved the use of the product rule and the complementary rule, which required straight recall and simple calculations. It is concerning that 53 of the 64 learners did not manage to recall simple rules of probability. It is, however, encouraging, when considering the questions that included complex procedures and problem solving skills, that learners not only attempted the questions but most of the learners showed some level of understanding by answering some or almost all of the question on these levels correctly.

Table 17. Number of learners achieving specific rank scores for cognitive levels in the Grade 11 final examination set by GDE

	Knowledge	Routine procedures	Complex procedures	Problem solving
Did not attempt the question	0	0	0	0
No correct procedures or calculations	28	53	2	2
Some correct procedures and calculations	3	1	37	54
Almost correct procedures and calculations	2	1	24	7
Correct procedures and calculations	31	9	1	1

The analysis of the relationship between the percentages that learners scored in the whole of Paper 1 and the probability question was done by drawing a scatter plot that can be seen in Figure 8.

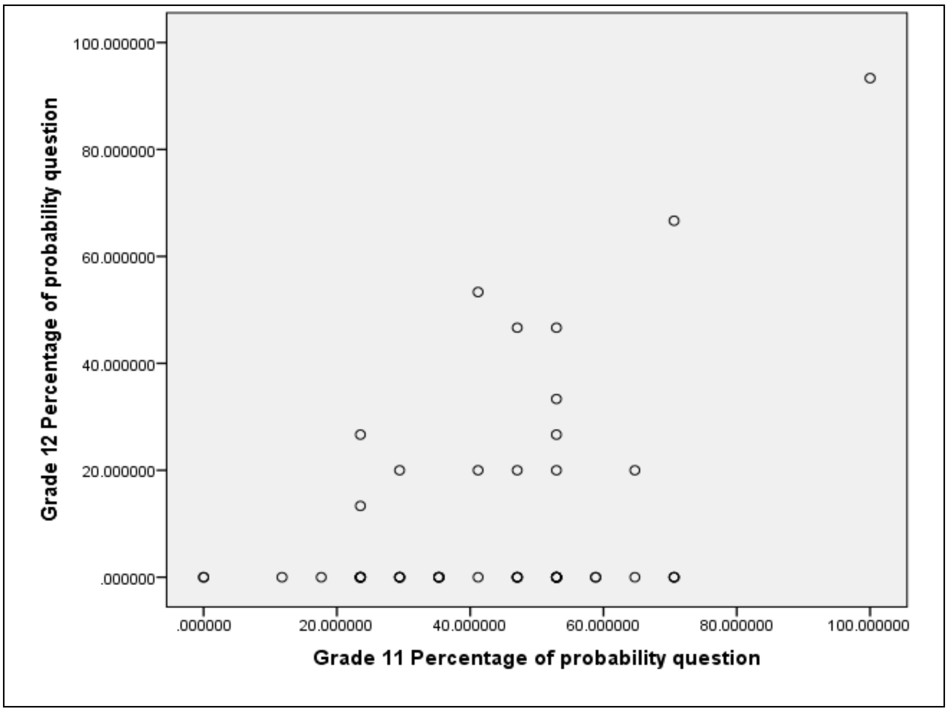

Figure 10. Scatter plot of Grade 11 final examination set by the GDE

To analyse the significance of this relationship, the correlation was calculated using Pearson's correlation coefficient. The value of the correlation coefficient is, $r = 0.507$ $(p - value = 0.000)$. The p-value was <0.01 which was significant at the 1% thus it was highly significantly different from zero. This significant positive correlation indicates that, if learners performed well in the probability question, there also performed well in the whole paper and vice versa. 0.507, which is an indication of a weak positive linear correlation. The weak positive correlation in this test shows that the marks scored by the learners in the whole paper are not really an indication of the marks they scored in probability. Next, I compare the results found in the two Grade 11 examinations.

6.1.1.3 Comparing Grade 11 set by School A and Grade 11 set by GDE

The two Grade 11 papers used in this study were analysed individually in the previous two sections. In this section, the two papers are compared with regard to the probability question. The comparison remarks on the difficulty level (using the cognitive levels), the average scores, the performance levels and the distribution of the marks (using a histogram).

The two papers both involved all four cognitive levels. Table 18 below shows the marks allocated to each of the cognitive levels. The mark allocation is given as percentages, as the number of marks allocated to probability in the two examinations were not the same. When comparing the percentages, it can be seen that the Grade 11 examination set by the GDE contained more questions on the cognitive levels of complex procedures and problem solving than was the case with the examination set by School A.

Table 18. Cognitive levels involved in the two different Grade 11 examinations

	Mark allocation as percentages	
Cognitive level	Grade 11 set by School A	Grade 11 set by GDE and written by School B
Knowledge	12.5 %	11.76 %
Routine procedures	56.25 %	47.06 %
Complex procedures	18.75 %	23.53 %
Problem solving	12.5 %	17.65 %

The performance levels in Table 19 show that the learners writing the examination set by School A achieved better results (average of 51%) than the learners from School B (average of 42%). This difference in the percentages scored could be explained by considering the cognitive levels involved in the different examinations. The examination set by the GDE was more difficult than the examination set by School A.

Table 19. Performance of learners in the two different Grade 11 examinations

	Percentage learners achieving at this level	
Performance level	Grade 11 set by School A	Grade 11 set by GDE
Poor (0%-29%)	24%	34.4%
Below average (30%-49%)	12%	28.2%
Average (50%-64%)	40%	28.2%
Above average (65%-100%)	24%	9.4%

Relating teachers' content knowledge to learner performance in probability.

6.1.1.4 *Comparing the Grade 11 probability questions by socio demographic factors of teacher characteristics*

The comparative analysis was further done by determining using independent t-tests and ANOVA (Analysis of Variance) whether the performance differed by school, caps training, and performance level of teacher and by teacher.

Independent t-tests

Since the data was normally distributed, the independent t- tests was used to determine whether performance differed by the socio demographic variables which had two categories The two categorical variables was gender, whether one received CAPS training and performance level of teachers. The teachers performed at average or above average level. The aim was to determine whether the performances of the learners were affected by these factors. The question was whether there was an association between learners' performance and characteristics of the teacher. The test 5% level of significance was used and if a p-value less than 0.05 was obtained then the null hypothesis of equal means would be rejected. The assumptions of the test were

Assumption 1: The observations are independent of each other.

Assumption 2: The data are normally distributed.

The null hypothesis to be tested was

H_o: The means are equal

H_1: The means are different

All the groups had p-values greater than 0.05 as shown in the table below.

Table 20. Independent T-test to determine difference in means

Independent T-test to determine difference in means by CAPS training, school and performance level of teacher on probability question					
Group	**Categories**	**Mean**	**T-value**	**p-value**	**Decision**
CAPS training	Yes	51.98	-1.831	0.071	Null hypothesis is not rejected
	No	53.20			
School	School A	53.20	1.831	0.071	Null hypothesis is not rejected
	School B	51.98			
Performance level of teacher	Average	57.29	1.517	0.133	Null hypothesis is not rejected
	Above average	43.90			
$^{*}P<0.05$ and $^{**}p<0.01$					

Relating teachers' content knowledge to learner performance in probability.

The mean for those who did not receive training was higher than for the one who received training although the difference was not significant as evidence by a p-value of 0.071. The same pattern was observed as one looked at the difference by school. The results are similar since all the school B were the ones that received CAPS training whilst school A did not. The performance level of teachers did not impact on the performance of learners. It can be noted that the teachers who performed above average had their learners having a lower mean than those who performed on the average.

The 95% confidence level error bars were constructed. Overlapping of error bars indicates that the groups are not different as indicated by the plots below.

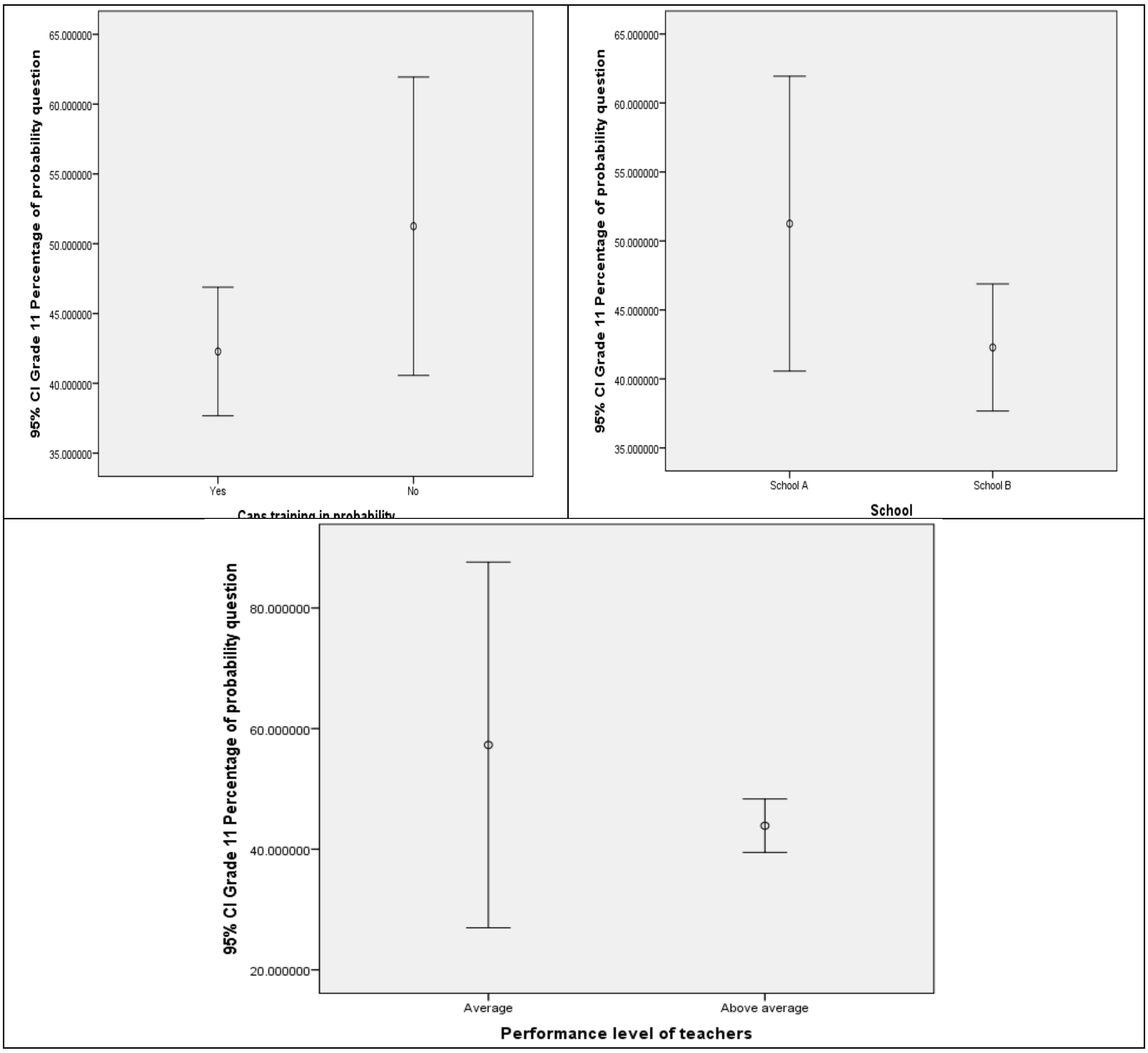

Figure 11. The 95% confidence interval error bars of Grade 11 probability

Although the error bars overlap signifying no significant difference, it can be noted that there is more variability in those who did not receive CAPS training, School A and the teachers who performed averagely. One can conclude that consistent in performance can be reached if one receives CAPS training and if the performance of the teacher is above average

ANOVA tests

The ANOVA test was used to determine mean differences differed by teacher. The teachers who performed were teacher 1, 3 and 4. The assumptions of the ANOVA tests are:

Assumption 1: The observations are independent

Assumption 2: The data is normally distributed

Assumption 3: The groups have equal variances

The null hypotheses to be tested were:

H_o: The means are equal

H_1: At least one pair of mean differ

The 5% level of significance was used and a p-value less than 0.05 led to the rejection of the null hypothesis. The analysis is shown below.

Table 21. Anova tests to determine mean difference

ANOVA test to determine mean difference by teacher on probability question					
Group	**Category**	**Mean**	**F-value**	**p-value**	**Decision**
Teacher	1	42.28	2.003	0.141	Null hypothesis is not rejected
	3	49.34			
	4	57.29			
$^{*}P \leq 0.05$ and ** $p<0.01$					

There was no difference in performance of learners by teacher as supported by p-value = 0.141 leading to non-rejection of the null hypothesis. However it can be noted that teacher 3 had the highest mean learner performance of 57.29. The 95% confidence interval error bar is shown in Figure 12.

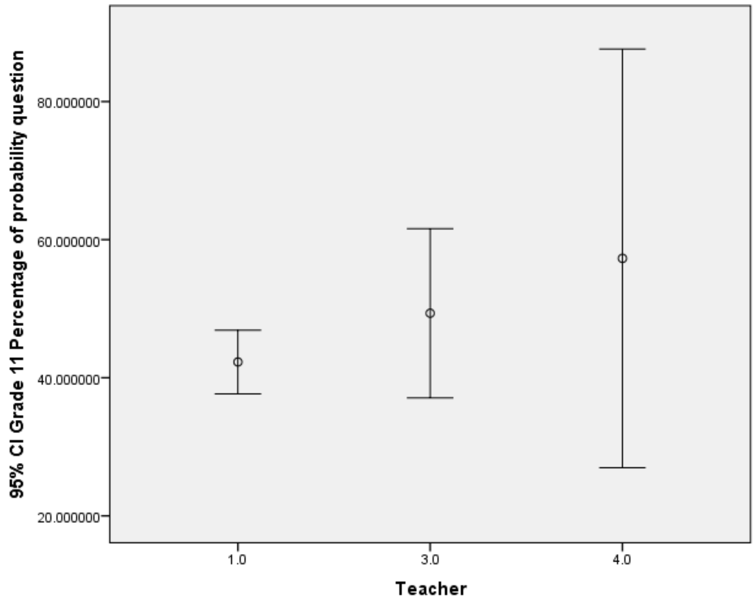

Figure 12. The 95% confidence interval error bars of Grade 11 probability by teacher

Although the error bars overlap signifying no significant difference, it can be noted that there is less variability in teacher 1.

6.1.2 Grade 12 preparatory examination

The written work of 75 Grade 12 learners was marked to generate percentage scores and rank scores. It is important to note that the probability question in this paper included some questions on the fundamental counting principle. However, these questions were not linked to probability and were therefore not included in the analysis of this research. This examination is discussed and analysed in the same way as the Grade 11 final examinations. The percentage scores were analysed using descriptive statistics to determine the mean, median and standard deviation. The distribution of the data is discussed using a histogram and using the range and the difference between the mean and the median to describe the skewedness of the data. In order to analyse the relationship between the marks scored for the whole of Paper 1 and the probability question, a scatter plot was drawn. The significance of the relationship was analysed by calculating Pearson's correlation coefficient.

In order to incorporate the conceptual framework of the study, analysis was done according to the performance levels of the learners with regard to the cognitive level of each question. The rank scores generated during the marking process are used to analyse the data in this regard. The rank scores include did not attempt, no correct procedures or calculations, some correct procedures and calculations, almost correct procedures and calculations and lastly, correct procedures and calculations.

The performance levels of the learners with regard to the percentage scores they achieved in the probability question in the Grade 12 preparatory examination is concerning. Forty-eight learners performed poorly and another twelve learners performed below average. In total, 80% of the learners fell within these two levels.

Table 22. Performance levels of learners on probability in the Grade 12 preparatory examination

Performance level	Number of learners achieving at this level
Poor (0%-29%)	48
Below average (30%-49%)	12
Average (50%-64%)	5
Above average (65%-100%)	10

The analysis of the data using descriptive statistics makes use of the percentages scored by learners in the probability question of this examination and can be seen in Table 23. The learners scored an average of 43.07% for Paper 1 overall, and an average of 30% for the probability questions. This poor average mark for probability is very concerning as the questions covered the same content that was tested in Grade 11. The new content included in Grade 12, that of the fundamental counting principle, did not include any calculations regarding probability and was therefore not included in the probability data. The standard deviation of the percentage scores in the probability question is 31.56 and shows that the data are wide spread around the mean. Similar to the results found in Grade 11, the range on the probability question is very high and marks ranged between 0% and 100%. The difference between the mean and the median is positive and therefore indicates that the data are spread to the right of the median.

Table 23. Descriptive statistics and distribution of the data for the Grade 12 preparatory examination

	Whole of Paper 1	Probability question
Mean	31.96%	16.98%
Median	31.00%	0%
Mode	0%	0%
Standard deviation	24.08	21.54
Skewness	0.461	1.239
Kurtosis	-0.488	1.153
Maximum	91.00	93.33
Minimum	0.00	0.00
Range	91.00	93.33
Coefficient of variation	75.34%	126.86%
Distribution	Positively skewed	Positively skewed

The distribution of the data is shown in the histogram and box plot in Figure 13. The distribution of the percentage marks scored by the Grade 12 learners in the whole Paper 1 can be seen alongside the distribution of the marks for the probability question in the preparatory examination. It is clear from the histogram that the probability question was not answered well. The large number of learners ($n = 39$) who achieved marks in the range of 0-9% in the probability question is very concerning. Both plots showed that data was positively skewed.

Whole of Paper 1	Probability question

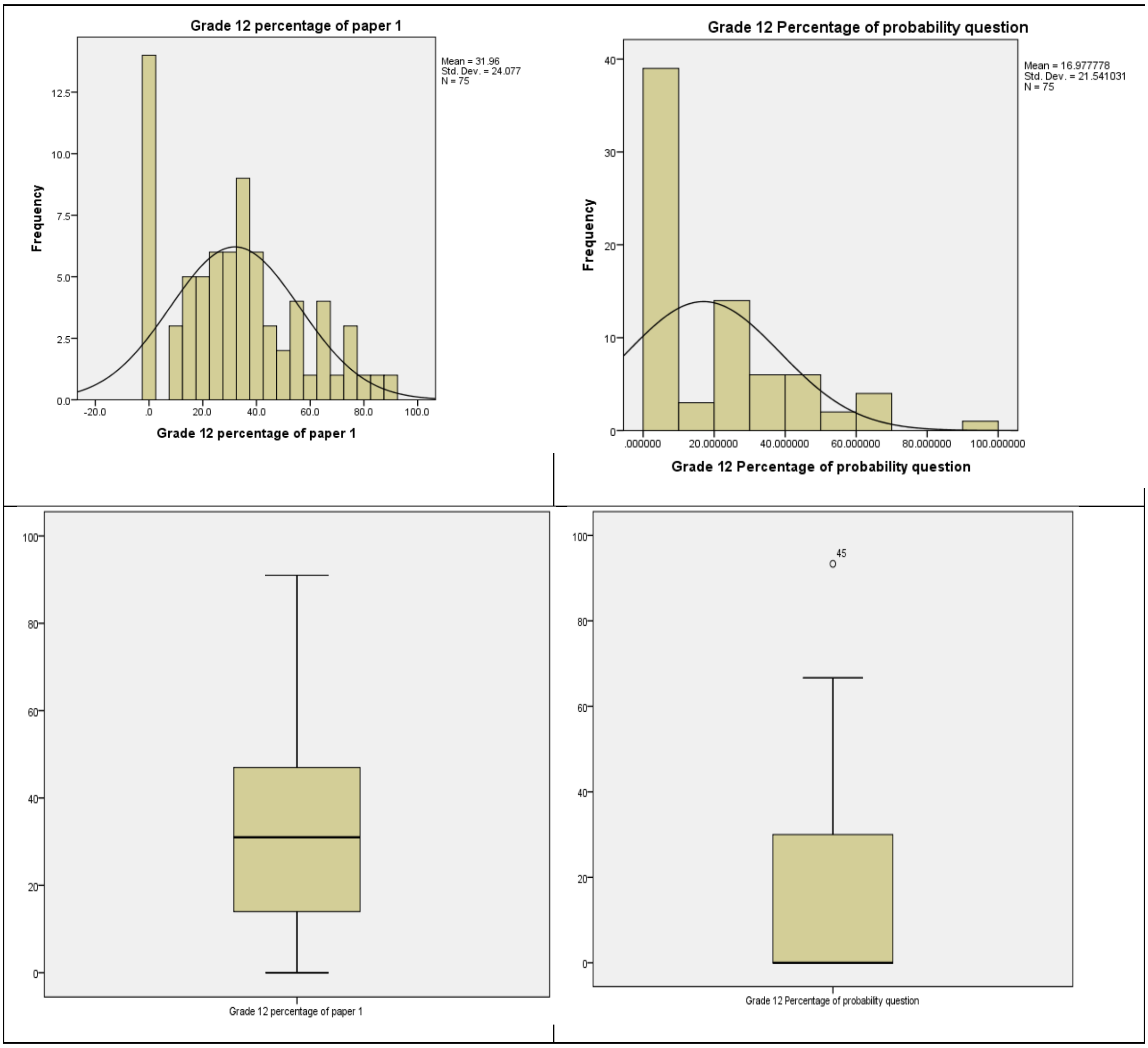

Figure 13. Distribution of Grade 12 preparatory examination percentage scores

The Shapiro Wilk test gave p-values of 0.003 and 0.000 for the whole paper and probability question respectively. Since the p-values were less than 0.05, the null hypothesis of normality was rejected. Thus data was not normally distributed. However parametric tests were used for comparing since the sample size was large and the central limit theorem was applied. The central limit theorem states "that as the sample size (i.e., the number of values in the sample) gets large enough, the sampling distribution of the mean is approximately normally distributed. This is true regardless of the shape of the distribution of the individual values in the population" (Levine, Krehbiel, & Berenson, 2013).

The rank scores generated during the marking process were used to analyse the learners' performance with regard to the cognitive levels and the results are summarised in Table 24 below. In the analysis of the rank scores, it is noted that the probability question did not include any questions on the cognitive level of knowledge. It seems that learners could either complete the routine procedures question with absolute accuracy or with no level of accuracy, as only two learners showed some and only five learners showed almost correct procedures and calculations. The analysis of the questions on the cognitive level of complex procedures and problem solving yielded results that were similar. Almost half of the participants could not engage in any correct procedures or calculations. Eleven learners could answer the questions on these cognitive levels correctly, and only two and one learners respectively could almost get the correct answer on the complex procedures and problem solving cognitive levels.

Table 24. Number of learners achieving specific rank scores for cognitive levels in the Grade 12 preparatory examination

	Knowledge	Routine procedures	Complex procedures	Problem solving
Did not attempt the question	No questions on this cognitive level was included in the probability question of the Grade 12 preparatory examination	14	14	14
No correct procedures or calculations		21	41	41
Some correct procedures and calculations		2	7	8
Almost correct procedures and calculations		5	2	1
Correct procedures and calculations		33	11	11

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The analysis of the relationship between the percentage scores of the whole Paper 1 and the probability question in this paper was done graphically by using a scatter plot. To establish the significance of this relationship, Pearson's correlation coefficient was calculated. Figure 14 is the scatter plot of the percentage scores of the whole Paper 1 and the probability question of this examination paper.

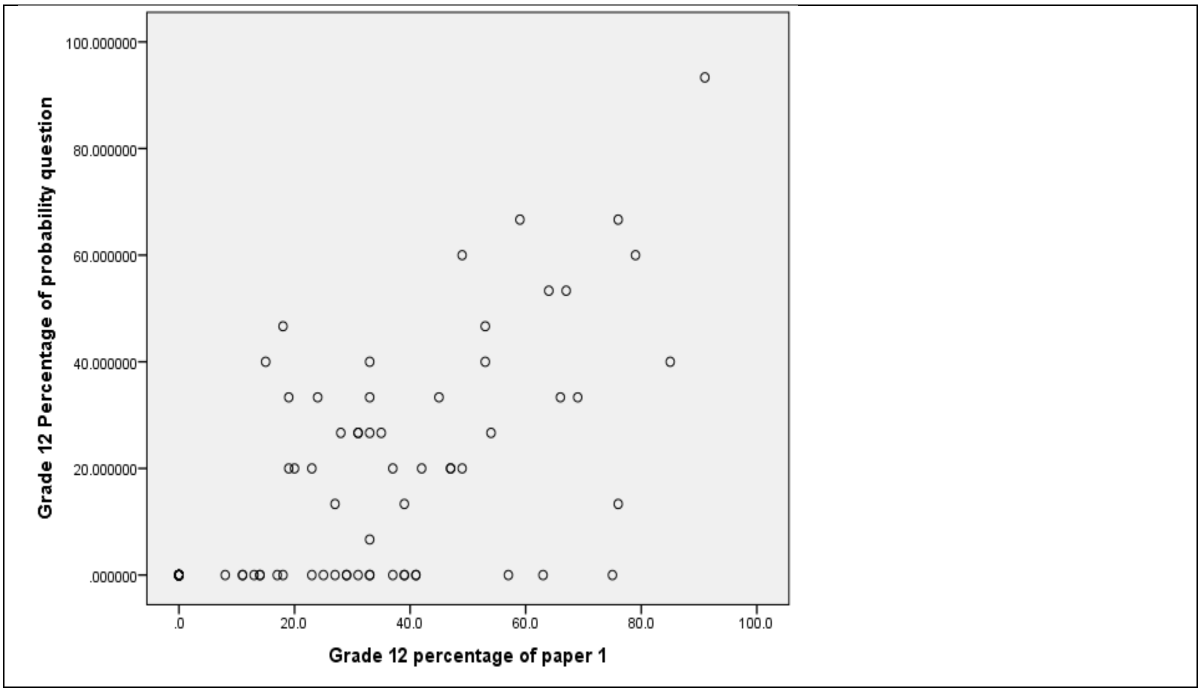


Figure 14. Scatter plot of Grade 12 preparatory examination

Pearson's correlation coefficient was calculated to be 0.57 (p-value = 0.000), which indicates a weak positive linear correlation which was significantly different from zero. The weak correlation implies that the marks scored by the learners in the probability paper cannot be used to predict the mark that they scored for the whole paper.

6.1.2.1 Comparing the Grade 12 probability questions by socio demographic factors of teacher characteristics

The independent t-tests and ANOVAs were used to determine whether the performance differed by school, caps training, performance level of teacher and by teacher on the Grade 12 probability questions.

Independent t-tests

All the composite variables had p-values greater than 0.05 except the composite variable knowledge creation. The *t-value = -2.619* with a *p-value = 0.011* as shown in Table 25.

Table 25. Independent T-test to determine difference in means

Independent T-test to determine difference in means by CAPS training, school and performance level of teacher on probability question					
Group	**Categories**	**Mean**	**T-value**	**p-value**	**Decision**
CAPS training	Yes	10.13	-4.334	0.000**	Null hypothesis is rejected
	No	30.67			
School	School A	30.67	4.334	0.000**	Null hypothesis is rejected
	School B	10.13			
Performance level of teacher	Average	32.22	1.836	0.070	Null hypothesis is not rejected
	Above average	15.65			
*P<0.05 and ** p<0.01					

The mean for those who did not receive training was higher than for the one who received training and the difference was significant as evidence by a p-value of 0.000. The same pattern was observed as one looked at the difference by school. The results are similar since all the school B were the ones that received CAPS training whilst school A did not. The performance level of teachers did not impact on the performance of learners. It can be noted that the teachers who performed above average had their learners having a lower mean than those who performed on the average.

The 95% confidence level error bars were constructed. Non-overlapping of error bars indicates that the groups are different as indicated by the plots below.

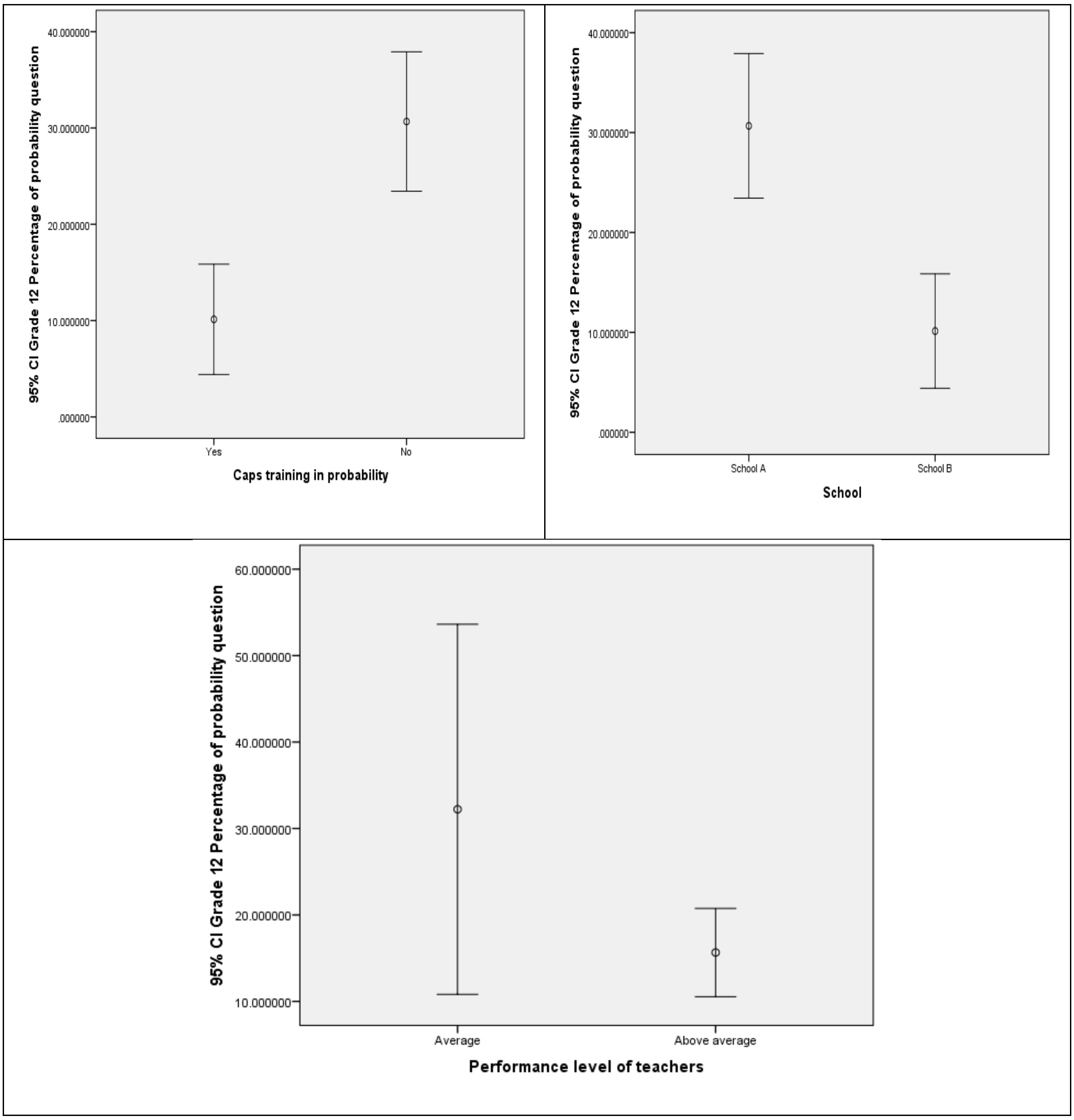


Figure 15. The 95% confidence interval error bars of Grade 11 probability

The error bars for CAPS training and school do not overlap signifying significant difference. The error bar for performance levels of teachers overlap signifying no significant difference between teachers who performed averagely and those who performed above average. It can be noted that there is more variability in those learners with teachers who performed averagely. One can conclude that consistent in performance can be reached if the performance of the teacher is above average

Relating teachers' content knowledge to learner performance in probability.

ANOVA tests

The ANOVA test was used to determine mean differences differed by teacher.

The analysis is shown below.

Table 26. ANOVA test to determine mean differences

ANOVA test to determine mean difference e by teacher on Grade 12 probability question					
Group	**Category**	**Mean**	**F-value**	**p-value**	**Decision**
Teacher	1	10.13	9.295	0.000**	Null hypothesis is rejected
	3	30.18			
	4	32.22			
*P≤0.05 and ** p<0.01					

There was difference in performance of learners by teacher as supported by p-value = 0.000 leading to rejection of the null hypothesis. The difference was highly significant. It can be noted that teacher 1 had the lowest mean learner performance of 10.13. The 95% confidence interval error bar is shown in Figure 16.

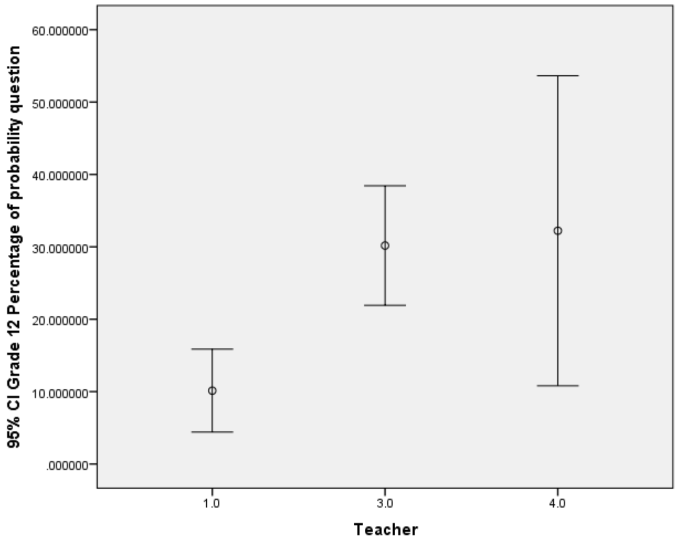


Figure 16. The 95% confidence interval error bars of Grade 12 probability by teacher

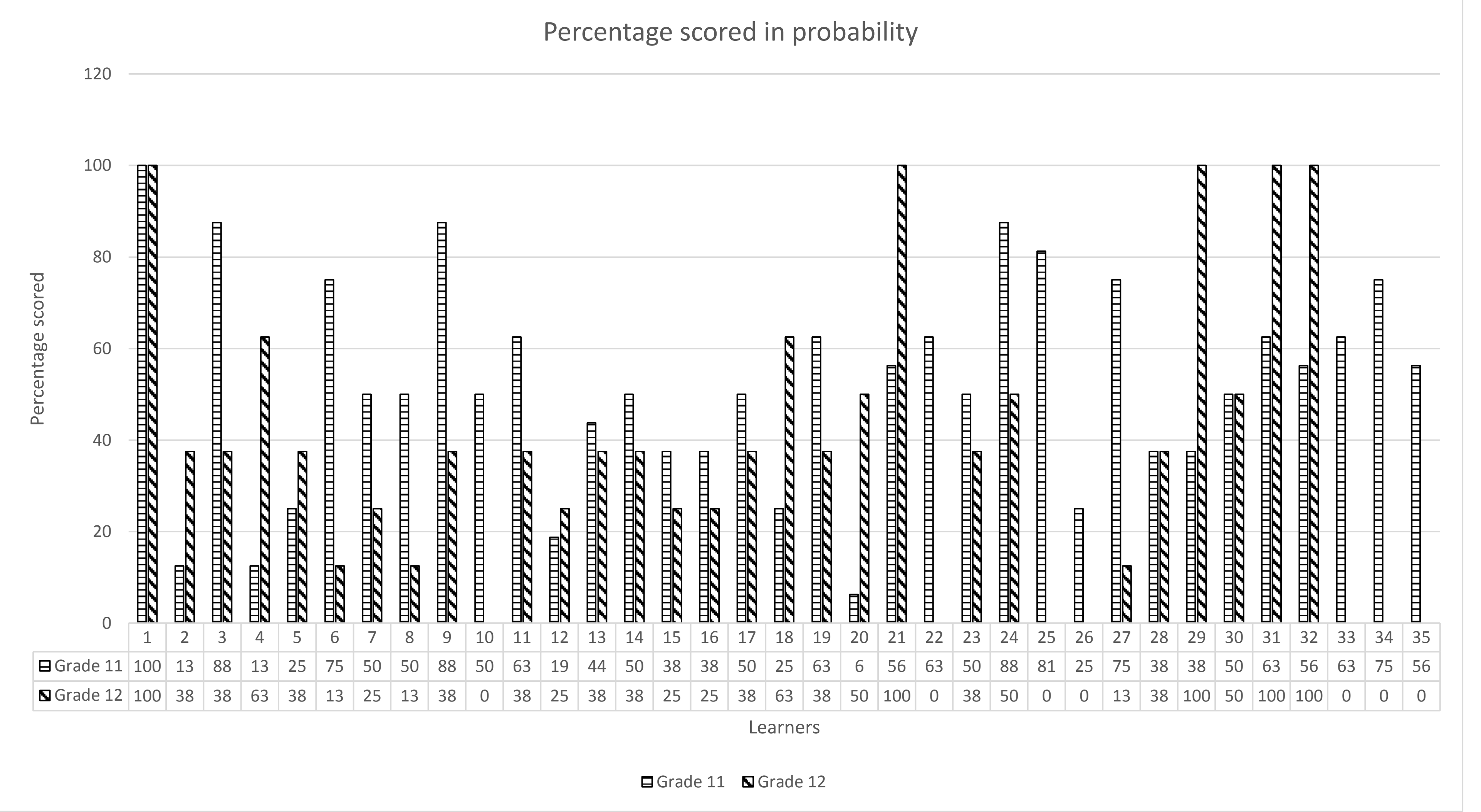
There is less variability in learner performance of teacher 1 and more variability in learner performance of teacher 4.

In section 6.1.3, I discuss the learners who participated in the study by giving written responses in the Grade 11 final examination and in the Grade 12 preparatory examination.

In section 6.1.3, I discuss the learners who participated in the study by giving written responses in the Grade 11 final examination and in the Grade 12 preparatory examination.

6.1.3 Learners participating in Grade 11 and Grade 12 examinations

To follow the progress of learners, it is important to consider the 35 learners who gave data in the Grade 11 final examination and the Grade 12 preparatory examination. The percentages scored by each of the 35 learners in the Grade 11 and the Grade 12 examinations are represented in a histogram shown in Figure 11 below. In the Grade 11 examination, only one learner managed to score 100% for probability and in Grade 12, five learners scored 100%. What is concerning was that there were 7 learners who did not manage to score a single mark for probability in Grade 12 even though they scored between 25% and 81% in Grade 11. This is surprising, as the average for this specific group of learners was 37% in Grade 11 and 50% in Grade 12.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Grade 11	100	13	88	13	25	75	50	50	88	50	63	19	44	50	38	38	50	25
Grade 12	100	38	38	63	38	13	25	13	38	0	38	25	38	38	25	25	38	63

	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Grade 11	63	6	56	63	50	88	81	25	75	38	38	50	63	56	63	75	56
Grade 12	38	50	100	0	38	50	0	0	13	38	100	50	100	100	0	0	0

Figure 17. Marks scored in probability for Grade 11 and Grade 12

The data from the 35 learners were used to draw a scatter plot (Figure 18) to investigate possible correlations between Grade 11 and Grade 12 marks for the probability question. Pearson's correlation coefficient (*r*) was calculated to be -0.411, which shows a significant weak correlation between the marks scored for probability in Grade 11 and the marks scored for probability in Grade 12. The value for *r* shows that the marks learners scored for probability in the Grade 11 examination gives no strong indication of the marks they will score in the Grade 12 examination. However the correlation is significantly different from zero.

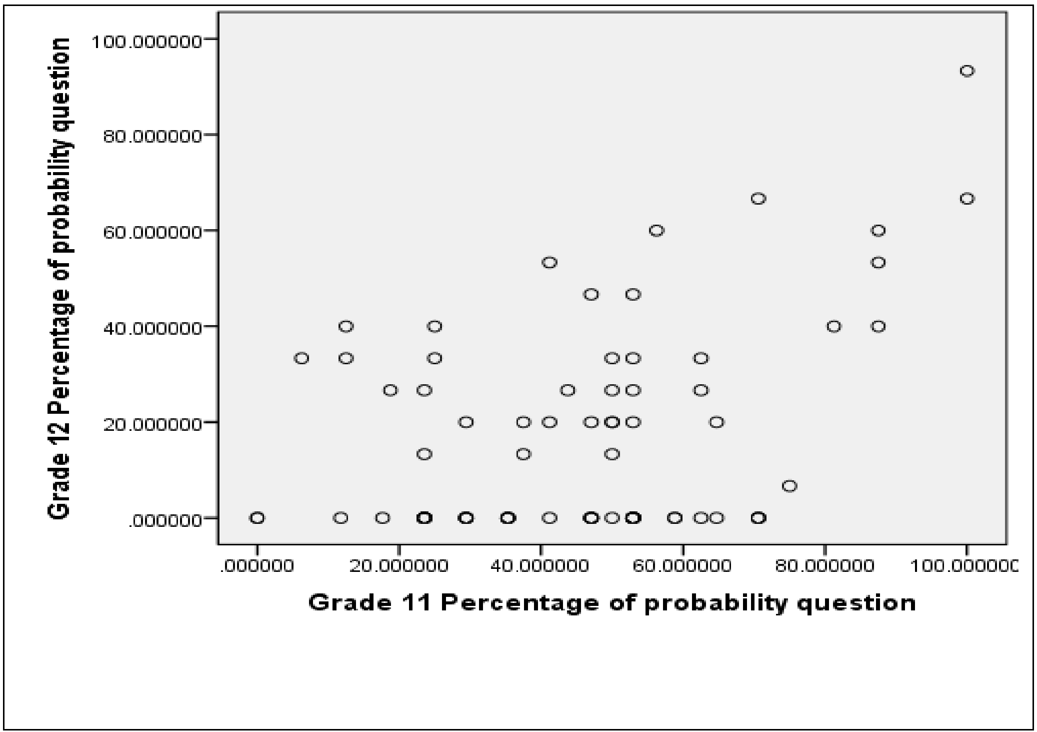


Figure 18. Scatter plot of Grade 12 preparatory examination and Grade 11 final examination Figure 19. Scatter plot of Grade 11 final examination and Grade 12 preparatory examination

When considering all the quantitative analysis done on the written responses of the learners in this study, the results are consistent with the results found in other studies done by, among others, Geary (2011) and Kalloo and Mohan (2012) remarking on the poor performance of learners in mathematics in general, and in this study, poor performance specifically in terms of probability.

6.2 QUALITATIVE DATA PRESENTATION AND ANALYSIS

The description and analysis of Test 1 and the semi-structured interviews conducted with the participating teachers are discussed in detail in this section, along with the analysis of the written responses given by the learners in the Grade 11 final examination and the grade 12 preparatory examination. The results from the qualitative data analysis may contrast or agree with the results found in the quantitative analysis. The analysis was done according to what teachers and learners wrote in their written responses as well as what was said by the teachers during the individual interviews. The results include selected extracts from the written responses and excerpts from the interviews.

6.2.1 Teachers' written and verbal responses to Test 1

Out of the eight teachers participating in the study, two did not write the test, but only responded to the first two sections of the semi-structured interviews. I marked Test 1 according to the memorandum, after which it was photographed. The marking process generated data as percentages and data as rank scores. The photographs were used to give extracts of the written responses given in Test 1. The semi-structured interviews were all recorded and transcribed, and in some cases translated.

The rank scores included correct (C), almost correct (AC), somewhat correct (SC), none correct (NC) and did not attempt (DNA) procedures and calculations. These rank scores were used in conjunction with the cognitive levels of knowledge, routine procedures, complex procedures and problem solving skills to analyse teachers' responses. Table 23 shows the results as rank scores combined with a description of each question and the cognitive levels involved in each question. The table only shows the information of the six teachers who agreed to write Test 1; the two teachers who were not willing to write the test are excluded.

Table 27. Question, cognitive level and rank scores of teachers in Test 1

Question number and topic	Cognitive level	T1	T2	T3	T4	T5	T6
1.1 Dependent and independent events	Knowledge	C	NC	NC	NC	NC	NC
1.2 Dependent and independent events	Knowledge	C	C	NC	NC	C	C
1.3 Dependent and independent events	Routine procedures	C	C	C	NC	SC	C

Question number and topic	Cognitive level	T1	T2	T3	T4	T5	T6
2.1 Tree diagram	Complex procedures	C	C	C	C	SC	C
2.2.1 Probability of event	Complex procedures / problem solving	C	C	C	C	NC	C
2.2.2 Probability of event	Complex procedures	C	C	C	C	NC	C
2.3 Probability of event and contextualising	Complex procedures	C	C	C	AC	DNA	AC
3.1 Venn diagram	Complex procedures / Problem solving	C	C	AC	SC	SC	AC
3.2 Calculating variables in Venn diagram	Routine procedures	C	C	SC	AC	NC	C
3.3.1 Probability of event	Complex procedures	C	C	DNA	DNA	SC	AC
3.3.2 Probability of event	Routine procedures	C	C	DNA	DNA	SC	AC

It is apparent from Table 23 that some teachers struggled to answer the questions on the cognitive level of knowledge correctly. Knowledge as a cognitive level is closely linked to the ability to directly recall theory and formulas related to dependent, independent and mutually exclusive events. This is a most surprising result, as the questions on this cognitive level involved the formulas and rules used to answer questions on dependent, independent and mutually exclusive events. Most teachers could not recall the formulas and rules and therefore could not apply them to the low-level questions.

In the section that follows, the analysis of the results of the written work of the teachers in Test 1 (consisting of three questions) is done with reference to the cognitive level involved in each question (knowledge, routine procedures, complex procedures and problem solving). The results are presented with selected extracts (one correct and one incorrect written response given by the teachers) from Test 1 and excerpts from the interviews. The six teachers who participated in this part of the study are referred to as T_1, T_2, T_3, T_4, T_5, and T_6. The researcher is referred to as R. The discussion follows the questions as they were asked in Test 1. These discussions focused mainly on the incorrect responses given by teachers but the interviews conducted with T_1 was done with a different focus, as she did not give a single incorrect answer.

The discussion works through the questions asked in Test 1 in numerical order. The question is given, followed by extracts from the written responses and then relevant excerpts related to the questions follow.

6.2.1.1 Question 1

Three events, A with a probability of $P(A) = 0.3$, B with a probability of $P(B) = 0.4$ and C with a probability of $P(C) = 0.2$.

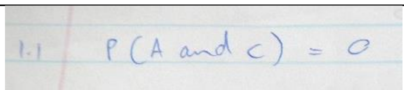
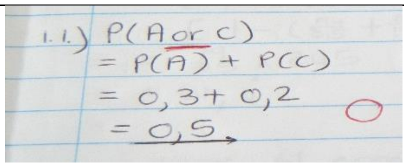
- A and B are independent
- B and C are independent
- A and C are mutually exclusive

Calculate the probability of the following events occurring:

- Question 1.1

In the first question, teachers were asked to find the probability of both A and C occurring. The question stated that A and C were mutually exclusive and therefore $P(A \; and \; B) = 0$. The result for this question is concerning. The cognitive level involved in this question was knowledge and all teachers were expected to solve this question correctly. This was, however, not the case, as only one teacher answered this question correctly. Table 28 shows extracts from the written responses of two teachers.

Table 28. Extracts from written responses given by teachers to question 1.1

Question	Both A and C occur.
Extract of correct response by T_1	
Extract of incorrect response by T_2	

The incorrect solution given by T_2 is very surprising as this was the only question that T_2 did not complete correctly. The following is an excerpt from the conversation with T_2 during the interview.

R: 'Can you please explain to me why you answered this question in the way you did?'

T_2: (Considering her response very carefully, and then responding.) *'O no! I cannot believe I did that. Now I see the question actually asked for the chances of A and C and not A or C.'*

R: 'So, can you explain how you would answer the question now?'

T_2: 'Yes, the question says that the two events are mutually exclusive and thus the probability of both events happening will be zero. Am I right?'

From the above excerpts it is clear that T_2 misread the question and therefore could not get the correct answer. When she was asked about her response, she seemed embarrassed. When reconsidering the question, she was able to rectify the mistake and got the correct solution. What is surprising was that she still wanted confirmation that her revised answer was correct, which may indicate that she is not fully confident in her own content knowledge.

- Question 1.2

The second question asked for the calculation of the probability of both B and C occurring. The question was solved using the product rule of probability $P(B \text{ and } C) = P(B) \times P(C)$. As was the case in question 1.1, teachers needed to recall the rule, substitute given values and make a simple calculation. This question was answered better than Question 1.1. Four of the six teachers managed to get the correct answer. Table 29 shows an extract form a correct answer given by T_6 and an incorrect answer given by T_3.

Table 29. Extracts from written responses given by teachers to question 1.2

Question	Both B and C occur.
Extract of correct response by T_6	1.2 P(B and c) = 0,4 × 0,2 = 0,08
Extract of incorrect response by T_3	1.2) P(B en c) = 0,4 + 0,2 = 0,6

In the interview process, T_3 was asked about the response she gave in this question. An excerpt from the interview is given below.

R: '…let's have a look at what you wrote here (indicating question 1.2 with a pencil). *Can you tell me what you were thinking when you answered this question?'*

T_3: 'I really struggled to do this question. We haven't done probability in such a long time, I cannot remember the correct formula to use.'

R: 'When last did you do probability?'

T_3: Let me think…, it think it was last year just before the exams started.'

R: 'Okay, how long have you been teaching probability?'

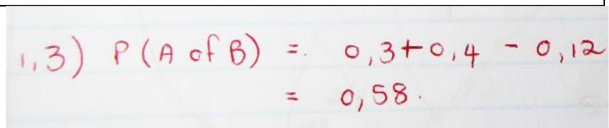
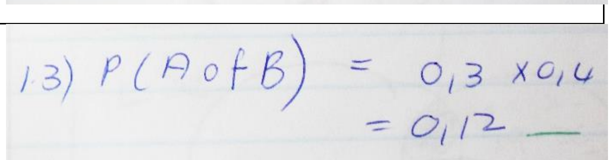
T_3: 'When we started with CAPS, so not very long. I think that's why I still don't know the formulas so well. I forget these things when I am not teaching them at the moment. I normally pick it up quickly when I'm teaching it to my kids.'

During this conversation, it became apparent that T_3 could not recall basic formulas and therefore did not display the basic MCK needed to answer the questions. This is concerning, as it seems that she has not spent enough time engaging in the topic so that her MCK could grow and she could recall the fundamental rules involved in probability as suggested by Plotz et al. (2012).

- Question 1.3

The probability of at least one of A or B occurring was asked in question 1.3. The probability identity, $P\ (A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$, was used to solve this question. Of all the questions related to probability of dependent, independent and mutually exclusive events, the teachers answered question 1.3 with the highest level of accuracy. Table 30 shows extract from the written responses given by teachers in answering the question. T_4 incorrectly used the product rule in her calculation.

Table 30. Extracts from written responses given by teachers to question 1.3

Question	At least one of A or B occur.
Extract of correct response by T_3	
Extract of incorrect response by T_4	

In the semi-structured interviews, T_4 was asked about the way she responded to question 1.3. The following is an excerpt from the interview.

R: 'When I went through your test, it looked like you struggled to answer these questions, do you agree?'

T_4: 'Yes, it's not that I don't understand it, it's more like I have forgotten half of this stuff. It's difficult to remember these things when there is no formula sheet.'

R: 'So you think you would have done better if I included a formula sheet?'

T_4: 'Absolutely!'

The conversation with T_4 was interesting, as she did not answer any of the sub-questions in question 1 correctly. Her response with regard to the formula sheet was significant, as one would feel that teachers would at least have the necessary MCK to recall formula without a formula sheet. It is also worth noting that the formula sheet included in examinations (See

Appendix H) only provide two probability formulas, of which the identity probability is one. The product and addition rules of probability were not included in the formula sheet.

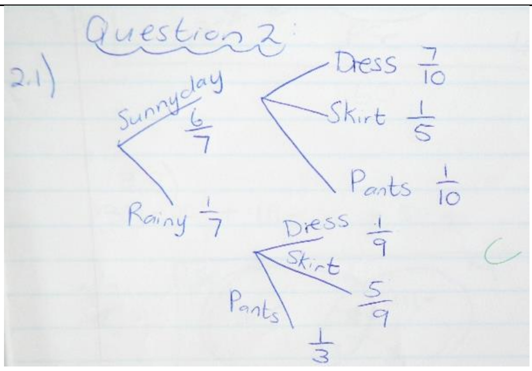
6.2.1.2 Question 2

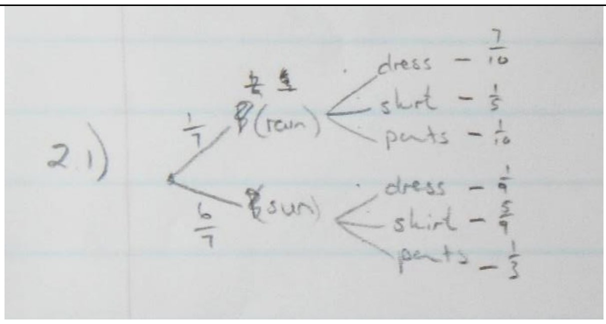
In Johannesburg, South Africa, the probability of a sunny day is $\frac{6}{7}$ and the probability of a rainy day is $\frac{1}{7}$. If it is a sunny day, the probability that Thapelo will wear a dress to work is $\frac{7}{10}$, the probability that Thapelo will wear a skirt to work is $\frac{1}{5}$ and the probability that she will wear pants to work is $\frac{1}{10}$. If it is a rainy day, then the probability that Thapelo will wear a dress to work is $\frac{1}{9}$, the probability that she will wear a skirt to work is $\frac{5}{9}$ and the probability that Thapelo will wear pants to work is $\frac{1}{3}$.

- Question 2.1

In question 2.1, teachers had to use the given information to draw a tree diagram. This question was answered with good success. All but one of the teachers managed to get to the correct solution. Table 31 shows an extract from the response given by T_4; even though she could not complete question 1 with any accuracy, she managed a correct answer for this diagram.

Table 31. Extracts from written responses given by teachers to question 2.1

Question	Draw a tree diagram
Extract of correct response by T_4	

Extract of incorrect response by T_5	

In the following excerpt from the interview with T_4, she makes some interesting remarks on tree diagrams.

R: 'Now, this question was answered well. Why would you say you managed to get this question correct?'

T_4: 'I love doing tree diagrams, it's so nice to figure out where what should go.'

Considering her response, it seems that she enjoyed this type of question. A possible reason is that she understands the principles behind the diagram. One might conclude that teachers would be more inclined to spend time on a topic they understand and enjoy doing and neglect to build MCK on the topics that they do not understand or enjoy as much.

When considering T_5's response to the tree-diagram, it is noted that she changed some information around. The information belonging to the 'rain' branch was linked to the 'sun' branch and vice versa. The following is an excerpt from the interview conducted with T_5.

R: 'If we look at your tree diagram, it looks like you doubted yourself, do you agree?'

T_5: 'I can't really remember, but yes it does look like it.'

R: 'Do you see that you got this mixed up?' (Pointing to the second branch of the tree diagram.)

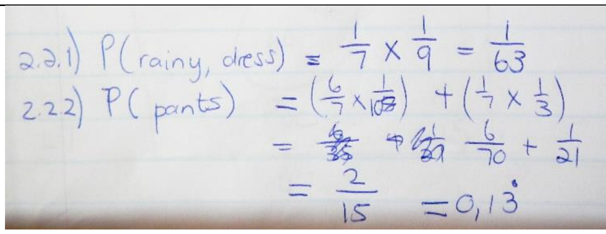
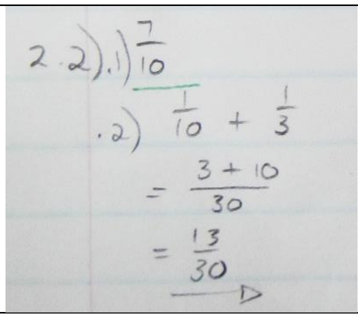
T_5: ' O, sorry. I think I didn't read it properly.'

It seems that T_5 did not draw her diagram with enough care and in doing so she only got some of the diagram correct. Another reason may be that she did not read the question with enough attention, as she mentioned in the excerpt above.

- Question 2.2

The tree diagram drawn in the previous question now needed to be used to answer the next two questions. The questions read as follow: For a day selected at random, what is the probability that it is a rainy day and Thapelo will wear a dress and for a day selected at random, what is the probability that Thapelo wears pants? The next two questions follow from the tree diagram. If teachers drew the tree diagram accurately, they should have been able to read the answers of the diagram. All the teachers who drew the tree diagram correctly also completed question 2.2.1 correctly. Only T_5 did not get the correct answer.

Table 32. Extracts from written responses given by teachers to questions 2.2.1 and 2.2.2

Question	2.2.1 For a day selected at random, what is the probability that it is rainy and Thapelo will wear a dress ? 2.2.2 For a day selected at random, what is the probability that it is rainy and Thapelo wears pants?
Extract of correct response by T_4	
Extract of incorrect response by T_5	

In the conversation with T_5, the discussion on the written solution to question 2.2.1 and question 2.2.2 came as part of her answer to the previous question on her attempt to draw the tree diagram.

R: 'Do you see that you got this mixed up?' (Pointing to the second branch of the tree diagram.)

T_5: 'O, sorry. I think I didn't read it properly. O no! Look at what I did here (pointing to question 2.2.1). *I must have been fast asleep on that day.'*

R: 'What do you mean by that?'

T_5: 'I cannot believe that I simply wrote down the value of the last part of the diagram, I should have multiplied these two, (indicating the two consecutive values on the branches leading to rainy day and dress) *and here* (pointing to 2.2.2) *I did it again. I'm so sorry.'*

R: 'It's okay, you do not need to apologise.'

T_5: 'You know, after that day that we wrote the test, I realised that I could not do most of the things you asked, so I have tried to refresh my memory on probability a bit.'

R: ' How did you do that?'

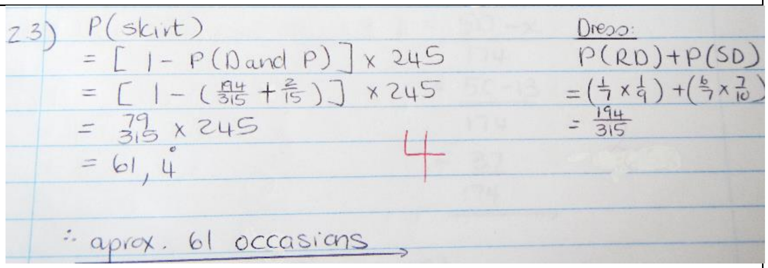
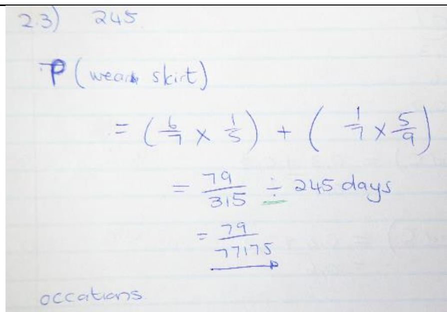
T_5: 'I went back to the test, and spent some time on it with my textbook, and I think I will do much better on that test now. You know, I have only taught this topic once, last year, so I tend to forget some of the things.'

From the response of the teacher, it seems that she understood that she did not possess the necessary MCK on probability. What is encouraging is that she made an attempt to rectify this by spending time on the topic and seems to feel more confident in her ability to solve problems on this topic now.

- Question 2.3

In question 2.3, the teachers needed to link the information from the tree diagram to the real world. The question asked: If Thapelo works 245 days in a year, on approximately how many occasions does she wear a skirt to work? Table 33 shows extracts from the written responses given by the teachers to this question.

Table 33. Extracts from written responses given by teachers to question 2.3

Question	If Thapelo works 245 days in a year, on approximately how many occasions does she wear a skirt to work?
Extract of correct response by T_2	
Extract of incorrect response by T_4	

The correct response to question 2.3 by T_2 was interesting. She chose to calculate the probability of wearing a skirt by using the complementary rule. When questioned on this. she responded in the following manner.

R: 'Why did you choose to answer question 2.3 by using the complementary rule rather than just calculating the probability of Thapelo wearing a skirt?

T_2: 'The reason why I would have probably done this is that out of habit I use what I have found.'

R: 'What do you mean by that?'

T_2: 'I suppose it is part of my teaching style. Questions are progressive, so if there is a chance that I can use what I have already worked out, then I will do so. So I have already calculated the probability of wearing pants in 2.2.2, so I calculated the probability of wearing a dress and then put them together to calculate the probability of wearing a skirt.'

The experience of T_2 was demonstrated in the answer. She understood the way in which questions are set, and how there is often a progression of calculations in a question.

The response of T_4 in this question was concerning. She made an incorrect calculation and then the answer did not make any sense. In questioning her on this, she responded in the following manner.

R:'I see that you did not complete this question (indicating question 2.3), *can you remember why?'*

T_4: 'Let me see, yes, I remember. When I made the calculation, the answer that I got made no sense, surely the answer should be at least a day? I don't know how to get the answer.'

This response raises some serious questions on the MCK of T_4. It is disconcerting that she used the incorrect operation, with division used instead of multiplication (with the 245 days), and confirmed that she did not know how to answer the question.

6.2.1.3 Question 3

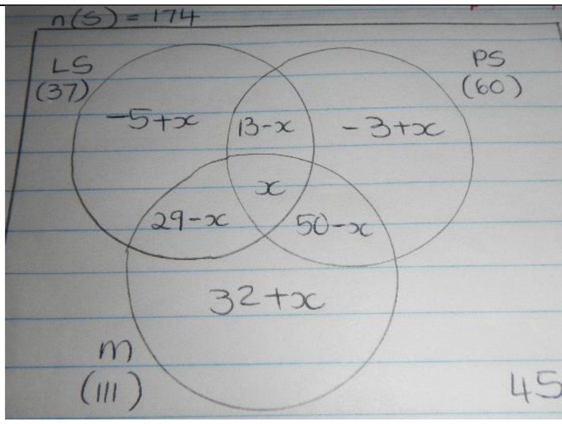
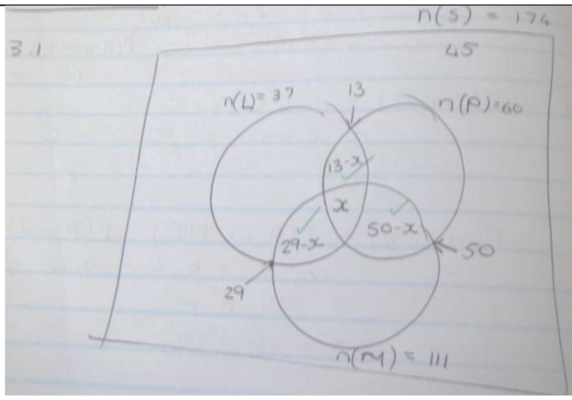
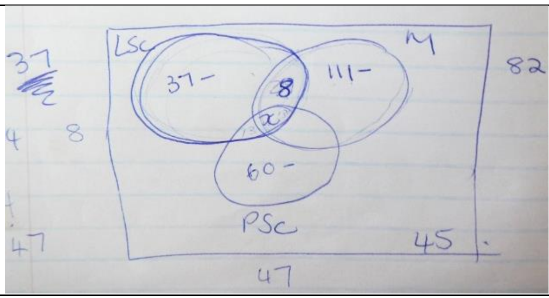
A school has 174 Grade 12 learners, a survey is done among the learners and the following is found:

- 37 learners take Life Science
- 60 learners take Physical Science
- 111 learners take Mathematics
- 29 learners take Life Science and Mathematics
- 50 learners take Physical Science and Mathematics
- 13 learners take Life Science and Physical Science
- 45 learners do not take Life Science, Physical Science or Mathematics
- x learners take Life Science, Physical Science and Mathematics

- Question 3.1

In question 3.1, teachers were asked to draw a Venn diagram of three events in a sample space. This question involved the cognitive levels of complex procedures and problem solving. All the teachers understood the initial setup using three circles to indicate the events involved in this question, but only two teachers, T_1 and T_2, could complete the rest of the question correctly.

Table 34. Extracts from written responses given by teachers to question 3.1

Question	Draw a Venn diagram to represent the information above.
Extract of correct response by T_2	
Extract of partially correct response by T_6	
Extract of partially correct response by T_4	

The incorrect responses given by teachers were all very similar. T_3, T_4, T_5, and T_6 all drew the three intersecting circles and correctly allocated *x* in the section that included all three events. T_6 went further and completed the intersections where two events overlapped correctly. This was, however, not the case for T_4, as can be seen in the extract in Table 30. In questioning T_4 about her inability to answer the question, she said the following:

R: 'I see here, (indicating question 3.1) *that you struggled with this question. Is that right?'*

T₄: 'Yes, I suppose you are right.'

R: 'Can you maybe explain to me what your thought process was when you completed this question?'

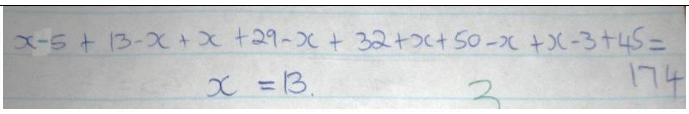
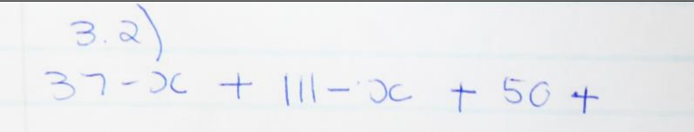
T₄: 'Well, when I tried to work out the values that should go here (pointing to the sections of the circles outside the intersecting parts) *I kept on getting a minus x and I just could not understand that, so eventually I just left it.'*

From the responses given by T_4, she seemed dejected. What is concerning is the fact that she did not attempt to use the information to do a calculation involving *x* but simply stopped halfway through her calculation.

- Question 3.2

Question 3.2 asked teachers to prove that $x = 13$. This was a simple calculation if the Venn diagram was set up correctly; therefore, it was not surprising that only the two teachers who correctly set up the Venn diagram could complete this question correctly. Table 35 shows of the solutions given by T_1 and T_4.

Table 35. Extracts from written responses given by teachers to question 3.2

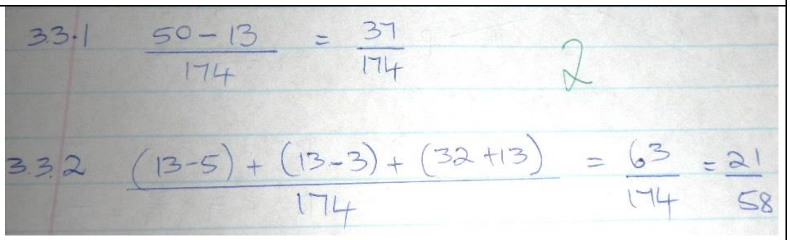
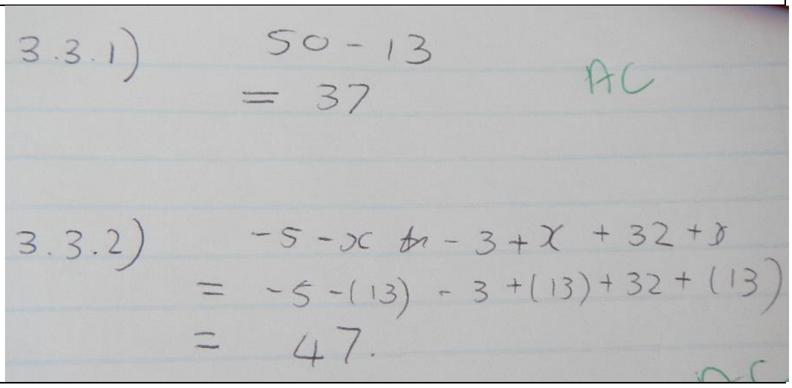
Question	Show that $x = 13$
Extract of correct response by T_1	
Extract of incorrect response by T_4	

- Question 3.3.1 and question 3.3.2

These questions could be solved even if teachers failed to calculate question 3.2 correctly. By giving the value of *x* it was possible to go back to the Venn diagram and complete it by substitution of *x*. Apart from T_1 and T_2, who completed the whole question 3 correctly, T_5 and

T_6 attempted to solve this question. The rest of the teachers did not answer questions 3.3.1 and question 3.3.2. However, T_6 still did not manage to answer question 3.3 with accuracy. Table 36 shows extracts of the solutions given by T_1 and T_6.

Table 36. Extracts from written responses given by teachers to question 3.3

<table>
<tr><td>Question</td><td>Show that $x = 13$</td></tr>
<tr><td>Extract of correct response by T_1</td><td>
</td></tr>
<tr><td>Extract of partially correct response by T_6</td><td>
</td></tr>
</table>

Even though T_6 attempted the questions, she failed to calculate the probability; instead, she only calculated the values that would complete the Venn diagram. In the interview with T_6, she was asked about her response to this question.

R: 'I see you did some calculations here, can you please explain what you did?'

T_6: 'Once the value of x was given, I used it to work out the missing values in the Venn diagram, so that I could work out what was asked. Why did you not give me all the marks here?'

R: 'If you look at the question you will see that it actually asks for the probability.'

T_6: 'Oh, so I should have just gone on and divided by the total number.'

R: 'Yes, that's right, why do you think you didn't do that?'

T_6: 'Well, I suppose I just wanted to finish the test, maybe I was in a hurry that day, I can't remember.'

When looking at the answers T_6 gave, she was inquisitive about the results she got for her test, which shows that she was interested in finding out what she did wrong. Without prompting, she knew what she had to do to get to the correct answer. It seems that even though not all the allocated marks were awarded, she possessed the necessary MCK to get the correct answer.

In the interview with T_1, the only teacher scoring full marks for Test 1, the questions were not related to specific questions in Test 1, but the focus was more on how T_1 managed to be so successful in answering probability questions. The following is an excerpt from the interview.

R: 'Congratulations, you did really well in the test.'

T_1: 'Thank you, I was not always so good in probability.'

R: 'So, how did you get this good?'

T_1: 'Time and effort, I suppose. When the syllabus changed to include Paper 3, I was asked to present this work to some learners at my school. I was fine with geometry (referring to Euclidean geometry) *because I have been teaching that forever. Probability was a different story. I had no background on this and had to essentially teach myself, so that I could teach the kids. It took a lot of time and effort, but now I am happy I did it.'*

R: 'When you say you taught yourself, how did you go about it?'

T_1: 'I got my hands on as many textbooks as possible. At first, I just read through the sections on probability and then I started doing the questions to see if I understood the concepts. I was lucky enough to have a colleague who had some knowledge, I think she studied BSc or something and she helped me when I could not get to the answers in the textbook.'

R: 'What was the most difficult thing for you?'

T_1: 'I think just understanding the basic concepts involved in the identity, learning and memorising the rules also took a while.'

Considering these responses, time spent actively engaging in a topic and experience seemed to be the determining factors for this teacher to gain the MCK she needed to be confident and competent in the topic of probability.

6.2.2 Teachers' responses to questions in semi-structured interviews related to their background

All eight teachers participated in this stage of the research and the data gathered during these individual interviews are tabulated. Table 37 summarises the data, starting with the codes allocated to each teacher and school. The years of teaching experience of the individual teachers are indicated along with a list of their tertiary qualifications. The last section in the table indicates whether the participating teachers were involved in any professional development program, with special attention on the CAPS training sessions hosted by the GDE that were compulsory for all teachers in the FET phase during 2012 and 2013.

Table 37: Background information from teachers' semi-structured interviews

Teacher code	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
School	A	A	B	B	B	B	C	D
Gender	Female	Female	Female	Female	Female	Female	Male	Male
Teaching experience (years)	36	16	8	13	5	3	20	8
Tertiary qualification	TTHD	HDIPED	BEd, BA (Hons)	BSc, HED	BEd	BEd	UDE	BSc (Ed)
Professional development	CAPS training	CAPS training	CAPS training	CAPS training	CAPS training	CAPS training	CAPS training	CAPS training

The participating teachers were all qualified. Three teachers had a teaching diploma and four had a degree in teaching, one of whom also had an Honours degree, and one teacher had a professional degree as well as a postgraduate teaching diploma. All the teachers participated in professional development by attending the CAPS training sessions hosted by the GDE. Their experience levels varied considerably from 3 to 36 years of teaching experience.

From the data above, it appears that even though the participating teachers were all qualified to teach mathematics and attended professional development sessions, they still seemed to lack the necessary MCK related to probability. The results seem to be consistent with the research conducted by Bansilal et al. (2014) in KwaZulu-Natal that showed that the MCK that teachers possess is not always enough. When the teachers were asked about their inability to answer the knowledge level questions in Test 1, they remarked that they would have been able to answer the questions correctly if they were supplied with a formula sheet. This is of great concern, as it seems that teachers are not engaging in the topic of probability to the degree that they are able to recall the basic rules related to the topic. This lack of basic knowledge raises the question about the efficiency of the professional development sessions, as all the teachers attended the sessions hosted by the GDE, but their knowledge level is still below average or average.

When looking at the experience level of the teachers in relation to their results in Test 1, the results seem to be consistent with the work of Burgess (2010), as the teachers with the most teaching experience performed the best in Test 1. The teachers with less experience remarked that they did not feel as comfortable with probability as a topic, as it was a new topic for them. They also said that if they were provided with the correct answers to probability questions then they would be able to generate a process that would lead to the answer. This remark reinforces the statement that the teachers lack the basic MCK to solve probability questions.

6.2.3 Teachers' and learners' written responses

In this section, the written responses of the learners and the teachers are analysed. For this analysis, only the questions that were similar for the teachers and the learners are discussed. The discussion is done by quoting the question and then showing extracts from the responses by the teacher and her learners.

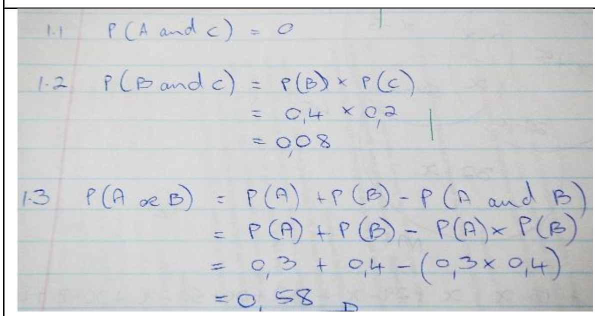
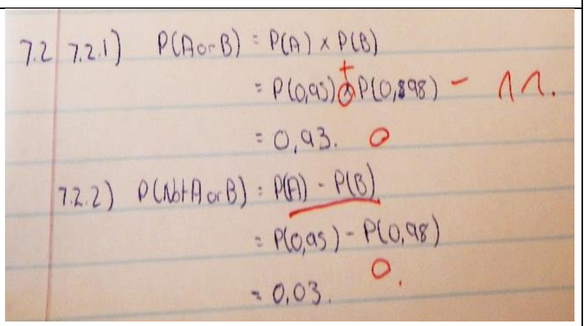
The questions that are discussed here include the questions related to the use of the probability identity and probability rules to solve questions on dependent, independent and mutually exclusive events and Venn diagrams.

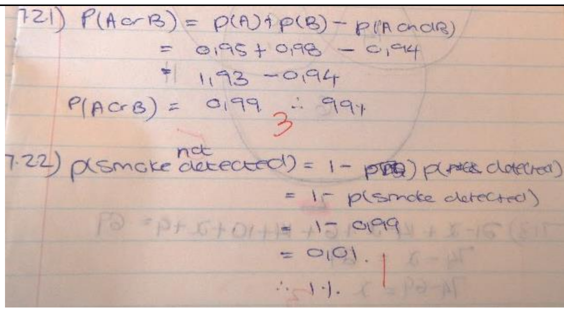
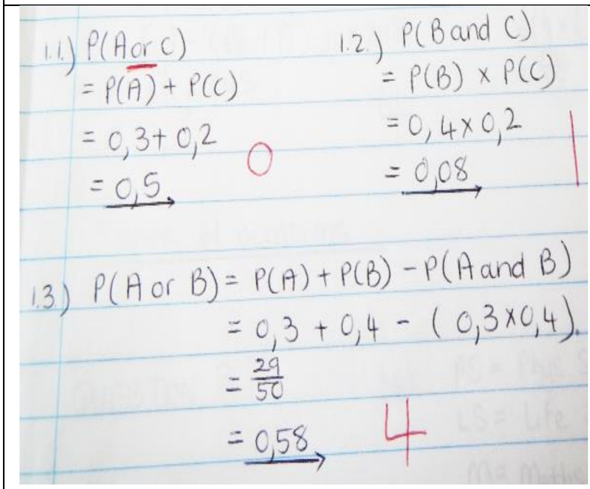
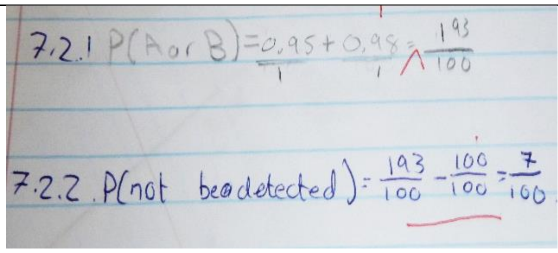
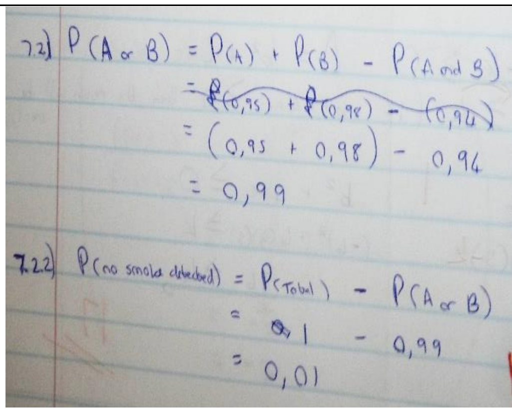
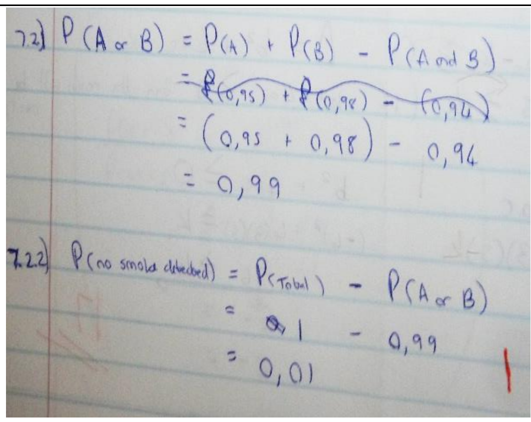
6.2.3.1 *Dependent, independent and mutually exclusive events (probability identity and rules)*

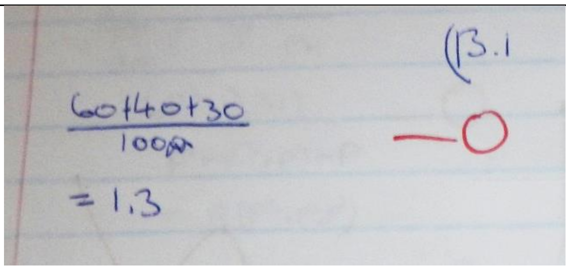
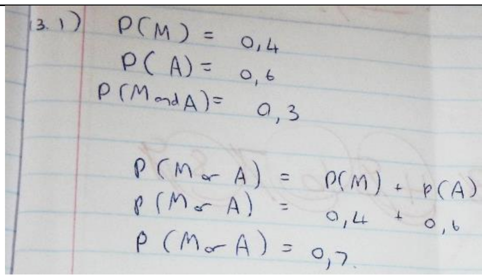
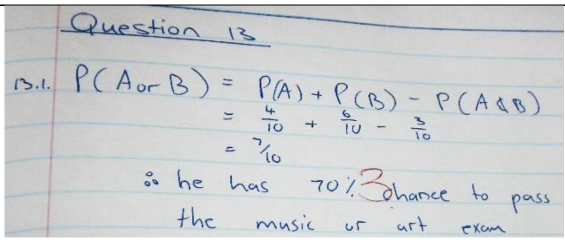
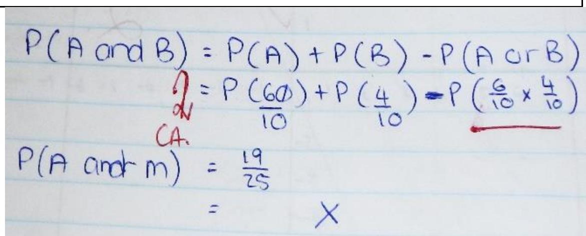
Test 1 and all the examinations written by learners included questions related to the use of the probability identity and the probability rules.

In Table 38, the question related to the probability identity and probability rule that was included in the Grade 11 examination set by School A is discussed. The table includes extracts from the written responses given by the two teachers form School A, along with the extracts from the Grade 11 examination set by the school and written by their learners.

Table 38. Responses to questions related to probability identity

<table>
<tr><th colspan="2">Question</th></tr>
<tr><td>Test 1</td><td>Grade 11 examination set by School A</td></tr>
<tr><td>Three events, A with a probability of $P(A) = 0.3$, B with a probability of $P(B) = 0.4$ and C with a probability of $P(C) = 0.2$.
• A and B are independent
• B and C are independent
• A and C are mutually exclusive
Calculate the probability of the following events occurring:
1.1 Both A and C occur.
1.2 Both B and C occur.
1.3 At least one of A or B occur.</td><td>A smoke detector system in a large warehouse uses 2 devices: A and B. The probability that device A will detect any smoke is 0.95 and device B is 0.98. The probability that both will detect smoke at the same time is 0.94.
7.2.1 What is the probability that device A or B will detect smoke?
7.2.2 What is the probability that smoke will not be detected?</td></tr>
<tr><td colspan="2">Responses to question on dependent, independent and mutually exclusive events</td></tr>
<tr><td>T_1</td><td>Learners taught by T_1</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

	 7.21) P(A or B) = P(A) + P(B) − P(A and B) = 0,95 + 0,98 − 0,94 = 1,93 − 0,94 P(A or B) = 0,99 ∴ 99% 3 7.22) P(smoke not detected) = 1 − P(smoke detected) = 1 − 0,99 = 0,01. ∴ 1%.
T_2	Learners taught by T_2
 1.1) P(A or C) = P(A) + P(C) = 0,3 + 0,2 = 0,5 0 1.2) P(B and C) = P(B) × P(C) = 0,4 × 0,2 = 0,08 1.3) P(A or B) = P(A) + P(B) − P(A and B) = 0,3 + 0,4 − (0,3×0,4) = 29/50 = 0,58 4	 7.2.1 P(A or B) = 0,95 + 0,98 = 193/100 7.2.2 P(not be detected) = 193/100 − 100/100 = 7/100  7.2) P(A or B) = P(A) + P(B) − P(A and B) = (0,95 + 0,98) − 0,94 = 0,99 7.22) P(no smoke detected) = P(Total) − P(A or B) = 1 − 0,99 = 0,01  7.2) P(A or B) = P(A) + P(B) − P(A and B) = (0,95 + 0,98) − 0,94 = 0,99 7.22) P(no smoke detected) = P(Total) − P(A or B) = 1 − 0,99 = 0,01
Question	
Grade 12 preparatory examination	
Tebo writes an Art and a Music examination. He has a 40% chance of passing the Music examination, a 60% chance of passing the Art examination and a 30% chance of passing both the Music and Art examination.	

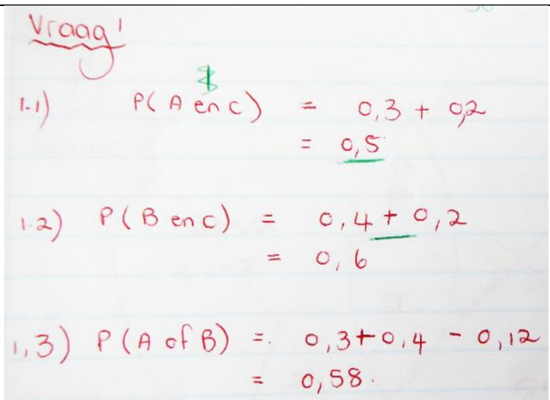
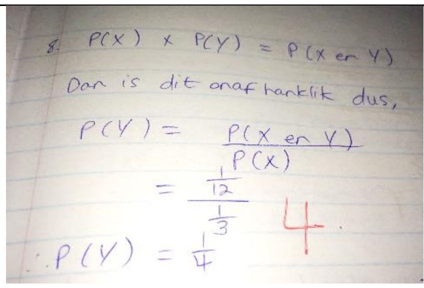
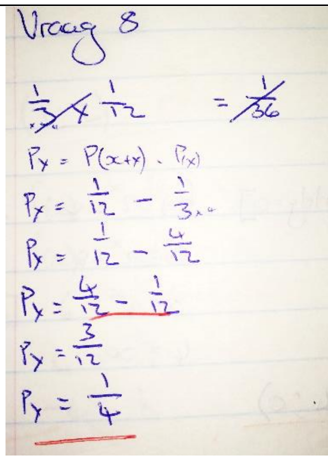
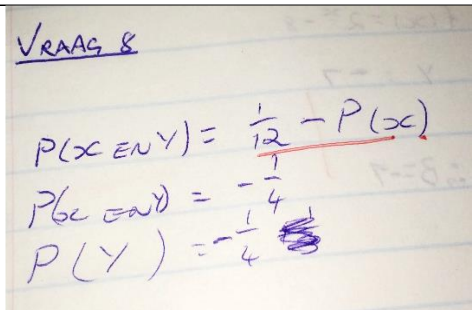
<table>
<tr><td colspan="2">Calculate the probability that Tebo will pass the Music or Art examination.</td></tr>
<tr><td colspan="2">Responses given by learners from School A</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

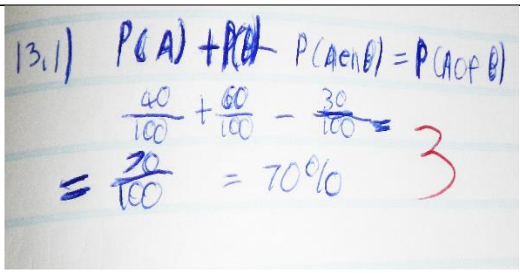
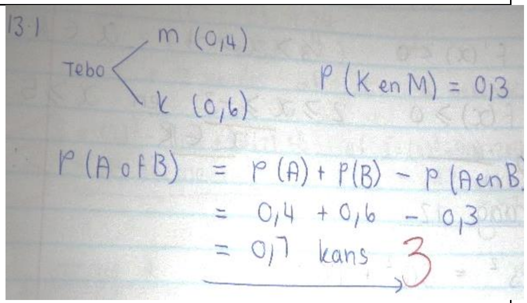
T_1 and T_2 were the best performing teachers in Test 1, with T_1 scoring full marks and T_2 making a single error. Looking at the responses of their respective learners, again there is a wide variety of responses ranging from no correct solutions to perfectly correct solutions.

The first section of Table 39 shows an extract from the written responses given by T_3 in Test 1 and extracts from some of the written responses given by her Grade 11 learners in the Grade 11 final examination set by the GDE. The second section includes the written responses related to the probability identity and probability rules in the Grade 12 preparatory examination.

Table 39. Responses to questions related to probability identity

<table>
<tr><td colspan="2">Question</td></tr>
<tr><td>Test 1</td><td>Grade 11 examination set by GDE</td></tr>
<tr><td>Three events, A with a probability of $P(A) = 0.3$, B with a probability of $P(B) = 0.4$ and C with a probability of $P(C) = 0.2$.
• A and B are independent
• B and C are independent
• A and C are mutually exclusive</td><td>Event X and Y is independent.
$P(X) = \frac{1}{3}$ and $P(X\ and\ Y) = \frac{1}{12}$, calculate P(Y)</td></tr>
</table>

<table>
<tr><td>Calculate the probability of the following events occurring:
1.1 Both A and C occur.
1.2 Both B and C occur.
1.3 At least one of A or B occur.</td><td></td></tr>
<tr><td colspan="2">Responses to question on dependent, independent and mutually exclusive events</td></tr>
<tr><td>T_3</td><td>Learners taught by T_3</td></tr>
<tr><td rowspan="3">
</td><td>
</td></tr>
<tr><td>
</td></tr>
<tr><td>
</td></tr>
<tr><td colspan="2">Question</td></tr>
<tr><td colspan="2">Grade 12 preparatory examination</td></tr>
<tr><td colspan="2">Tebo writes an Art and a Music examination. He has a 40% chance of passing the Music examination, a 60% chance of passing the Art examination and a 30% chance of passing both the Music and Art examination.</td></tr>
</table>

<table>
<tr><td colspan="2">Calculate the probability that Tebo will pass the Music or Art examination.</td></tr>
<tr><td colspan="2">Responses given by learners from School A</td></tr>
<tr><td>
</td><td>
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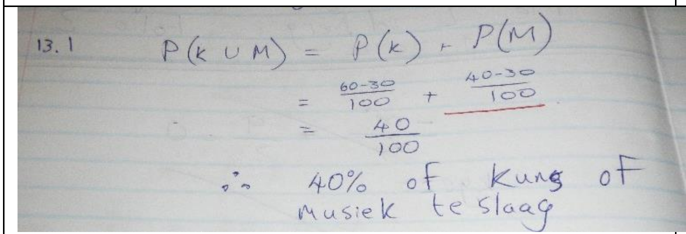
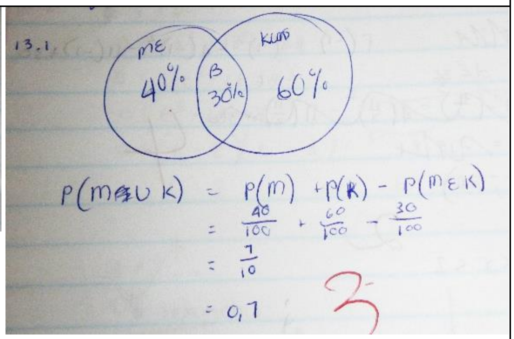
<tr><td>
</td><td>
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</table>

T_3 was not able to answer all the questions related to the probability identity correctly. This was also the case for many of the learners she taught. There were, however, learners who had the knowledge and applied the correct rule (product rule for independent events) to solve question 8 correctly. In the examination of the incorrect responses, one of the learners used the complementary rule and the other attempted the addition rule with no success. The teacher failed to use the addition rule for mutually exclusive events in question 1.1 and the product rule for independent events in question 1.2. In question 1.3, she correctly used the probability identity.

In general the learners either scored full marks (31 learners) or no marks (28 learners) for question 8.

More learners managed to answer the questions on the probability identity and probability rules correctly in the Grade 12 preparatory examination than had been the case in the Grade 11 examinations. Interestingly, one learner used a Venn diagram to solve this question correctly.

Considering both School A with T_1 and T_2 and School B with T_3, there does not seem to be any comparison between how the teacher answered her question and how her learners answered their papers.

In Grade 12, more learners managed to score marks on the question related to the probability identity and the probability rules than in Grade 11. Even though there were learners who still did not manage to get the correct answer, they showed some understanding by at least applying the probability identity to the question. Only five learners did not manage to do any correct calculations in this question.

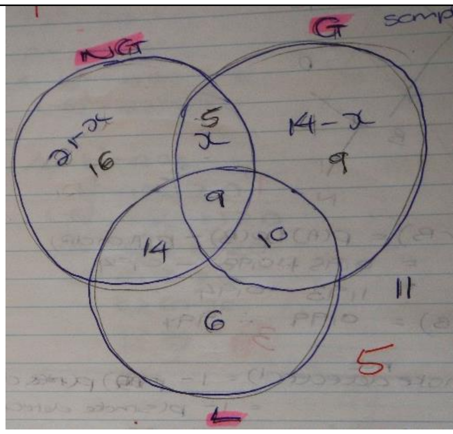
6.2.3.2 Venn diagram

Analysis of the results concerning the Venn diagram showed that learners seemed to understand the basic concept of a Venn diagram but failed to complete the question correctly. The results showed that the learners' answers corresponded with the answer of their teachers in the Venn diagram question. Teachers as well as learners failed to engage in the problem solving skills needed to correctly set up the Venn diagram and were therefore not able to use complex procedures to find the missing variable needed to complete the diagram successfully.

Table 40 shows the Venn diagram question in Test 1 for T_1 and T_2 along with some extracts from the written responses given to the Grade 11 final examination set by School A by some of the learners taught by these teachers.

Table 40. Responses to questions related to Venn diagram in School A

Question	
Test 1	Grade 11 examination set by School A
A school has 174 Grade 12 learners, a survey is done among the learners and the following is found: • 37 learners take Life Science • 60 learners take Physical Science • 111 learners take Mathematics • 29 learners take Life Science and Mathematics	A survey was done at a local library to show the different reading preferences for 80 students. • 44 read the National Geographic magazine • 33 read the Getaway magazine • 39 read the Leadership magazine

<table>
<tr>
<td>• 50 learners take Physical Science and Mathematics
• 13 learners take Life Science and Physical Science
• 45 learners do not take Life Science, Physical Science or Mathematics
• x learners take Life Science, Physical Science and Mathematics
3.1 Draw a Venn diagram to represent the information above. (6)
3.2 Show that $x = 13$. (2)
3.3 If a learner is selected at random, calculate the probability that the learner has the following combination of subjects:</td>
<td>• 23 read both the National Geographic and the Leadership magazines
•19 read both the Getaway and the Leadership magazines
• 9 read all three magazines
• 69 read at least one magazine
7.1.1 How many students do not read any of these magazines?
7.1.2 Let the number of students that read National Geographic and Getaway, but not Leadership be represented by x. Represent this information in a Venn diagram.
7.1.3 Show that $x = 5$.
7.1.4 What is the probability (correct to 3 decimal places) that a random student will read at least 2 of the 3 magazines?</td>
</tr>
<tr>
<td colspan="2">Responses to question on Venn diagram</td>
</tr>
<tr>
<td>T_1</td>
<td>Learners taught by T_1</td>
</tr>
<tr>
<td></td>
<td>
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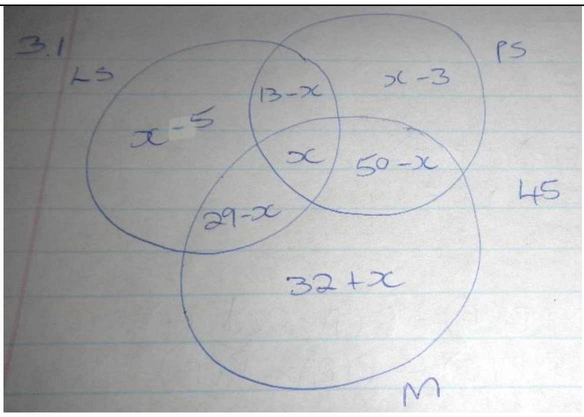
<td>
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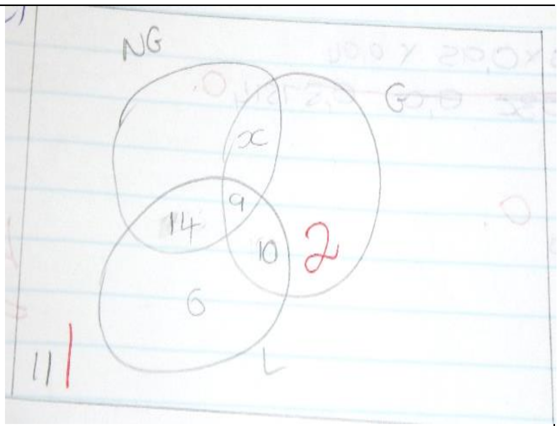
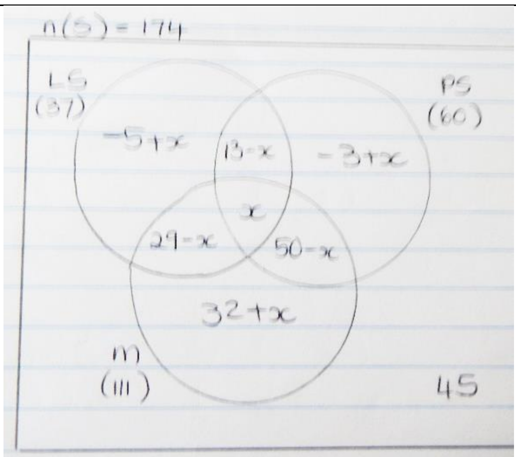
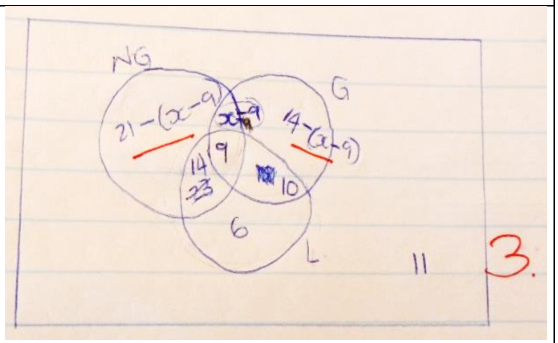
<td>
</td>
</tr>
<tr>
<td>T_2</td>
<td>Learners taught by T_2</td>
</tr>
<tr>
<td rowspan="2">
</td>
<td>
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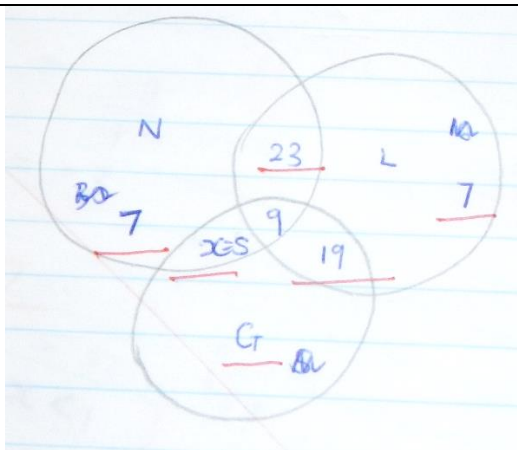
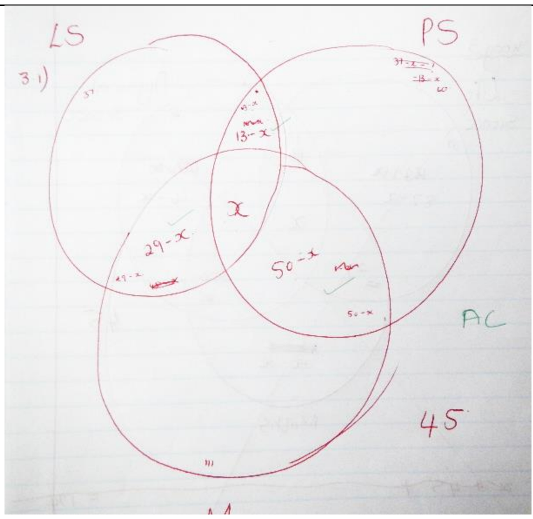
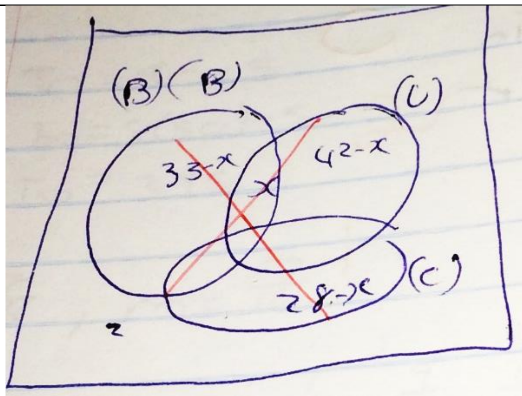
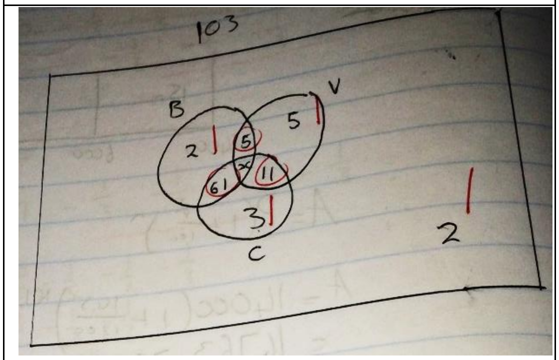
<td>
</td>
</tr>
</table>

The extracts from the written responses by the teachers and the learners in the Grade 11 examination set by School A show that, even though 14 of the 25 participating learners managed to draw the Venn diagram correctly, only 2 learners did not show any correct calculations or procedures in setting up the diagram. The other nine learners all managed some correct calculations and procedures. Both T_1 and T_2 managed to set their diagrams up correctly, which shows some similarities between teachers' and learners' written responses.

Table 41 quotes the Venn diagram questions form Test 1 and the Grade 11 examination set by the GDE, along with extracts form the written responses given by T_3 and some of the learners she taught in Grade 11.

Table 41. Responses to questions related to Venn diagram in Test 1 and in the Grade 11 examination set by GDE

Question	
Test 1	Grade 11 examination set by GDE
A school has 174 Grade 12 learners, a survey is done among the learners and the following is found: • 37 learners take Life Science • 60 learners take Physical Science • 111 learners take Mathematics • 29 learners take Life Science and Mathematics • 50 learners take Physical Science and Mathematics • 13 learners take Life Science and Physical Science • 45 learners do not take Life Science, Physical Science or Mathematics • x learners take Life Science, Physical Science and Mathematics 3.1 Draw a Venn diagram to represent the information above. (6) 3.2 Show that $x = 13$. (2) 3.3 If a learner is selected at random, calculate the probability that the learner has the following combination of subjects:	A school organises a camp for 103 Grade 8 learners. The learners were asked write down what food they prefer. They could choose between beef (B), vegetables (V) and chicken (C). The following information was gathered. • 2 learners do not eat beef, vegetables or chicken. • 5 learners only eat vegetables. • 2 learners eat only beef. • 12 learners do not eat chicken at all. • 3 learners eat only chicken. • 66 learners eat chicken and beef. • 75 learners eat chicken and vegetables. Let the number of learners that eat beef, vegetables and chicken be x. 7.1 Draw a Venn diagram to represent the above information. 7.2 Calculate x. 7.3 Calculate the probability that a learner that is chosen at random eats: 7.3.1 only beef and chicken but not vegetables.

<table>
<tr><td></td><td>7.3.2 only two of the food types, beef, chicken and vegetables.</td></tr>
<tr><td colspan="2">Responses to question on Venn diagram</td></tr>
<tr><td>T_3</td><td>Learners taught by T_3</td></tr>
<tr><td rowspan="2">
</td><td>
</td></tr>
<tr><td>
</td></tr>
</table>

The extracts from the learners as well as their teacher show poor comprehension of the concept of Venn diagrams. Neither the teacher nor her learners in Grade 11 could complete the Venn diagram accurately. This then leads to a further problem in the solution of the rest of the questions related to the diagram.

6.2.4 Teachers' results in Test 1 and the performance of the learners they taught

Even though this study did not include enough teachers to analyse their percentage scores quantitatively, they are useful in the qualitative part of the study. In order to further discuss the link between the MCK of teachers and the learners' performance, I discuss the teachers in conjunction with the learners they taught. The data can be seen in Table 38 below. The poor performance of learners is again concerning. The results shown in Table 38 for eight teachers are difficult to explain.

It was interesting that T_1 (most experience teacher with 36 years of experience) scored 100% in Test 1, which indicates that she had good MCK. However, the learners she taught only managed to score an average of 49% in Grade 11 and an average of 30% in Grade 12. Similarly, T_2 (16 years of experience) scored 97% for Test 1, while her learners scored an average of 57% in Grade 11 and an average of 26% in Grade 12. For T_3 (8 years of experience, not as experienced) the average score for Test 1 was 73%, while her learners scored an average of 42% in Grade 11 and an average of 38% in Grade 12. These results reveal that there is no substantial link between the teachers' MCK and the performance of the learners when examining the topic of probability for this group of learners. The aspect discussed here (MCK versus learners performance) indicates that the teachers' MCK is not the only factor influencing the performance of learners. There might be other factors influencing learners' performance that are not discussed in this study. The majority of the learners performed poorly, even though some teachers showed that they possessed the necessary MCK.

Table 42: Summary of teachers' and learners' performance

Teacher number	Average scored in Test 1	Average scored by learners in Grade 11 probability	Average scored by learners in Grade 12 probability
Teacher 1	100 %	49%	30%
Teacher 2	97 %	57%	26%
Teacher 3	73 %	42%	38%
Teacher 4	53 %	No data available	48%
Teacher 5	43 %	No data available	No data available
Teacher 6	77 %	No data available	No data available
Teacher 7	Did not write	No data available	3%
Teacher 8	Did not write	No data available	No data available

All the teachers who wrote Test 1 outperformed the learners they taught, even though their performance was still not as high as one would expect from qualified mathematics teachers. The fact that the teacher teaching the learners who only managed to score an average of 3% did not want to write Test 1 may be an indication of his lack of knowledge of the topic. Even though T_1 and T_2 performed very well in Test 1, the performance of their learners did not reflect this.

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6.3 CONCLUSION

To gain insight into the MCK of teachers in this study, four strategies were employed. The MCK of the teachers was investigated in Tests 1 and was measured with regard to performance and cognitive levels needed to answer the questions correctly. The tertiary qualifications of the teachers were examined along with their number of years of teaching experience. Professional development of the teachers with reference to the CAPS training sessions they attended was also explored as it is seen as a process whereby teachers could gain MCK.

The teachers wrote Test 1 and the data gathered in this way gave an indication of the level of MCK the teachers had on the topic of probability. Four of the teachers did not perform as well as one would expect (scoring more than 80% in Test 1). In many cases, teachers responded incorrectly to questions on the lowest cognitive level of knowledge. It is clear that some teachers are unable to make the necessary connections needed to gain MCK. The results form Test 1 clearly indicate that these four teachers did not have enough MCK on the topic of probability. All four teachers were qualified to teach mathematics but were never exposed to probability in their training (three had a BEd and one had a BSc). It was therefore not surprising that the teachers did not perform well in probability (scores ranging between 43% and 77%).

The two top performing teachers in Test 1, scoring 100% and 97% respectively, both had a teaching diploma rather than a degree and at least 16 years of experience. From the interviews it was clear that the teachers performed exceptionally well (more than 80% in Test 1) because of various reasons. The top performing teacher (100%) was the only teacher who had taught probability before, when it was part of Paper 3 in Curriculum 2005, and hence had a high level of MCK. The other exceptional teacher, scoring 97%, did so because she solved each and every question from the textbook in detail and understood it before presenting it in class, another aspect that shows a high level of MCK. When considering these results, experience and time spent on actively engaging with content have a positive impact on teachers' MCK (Plotz et al., 2012).

When examining the professional development, in the form of training sessions, that the participating teachers were involved in, it is noted that all the teachers attended training sessions on probability. The sessions that they attended were hosted by the GDE. When examining the comments made by the teachers with regard to these sessions, it is clear that most of the sessions were not beneficial to the teachers. Teachers elaborated on this statement by adding that, in their opinion, the sessions did not contribute to any gains in their MCK. This viewpoint is strengthened when considering the results that the teachers obtained in Test 1 and results found by Wessels (2008) when she remarked that probability is still a new topic and that teachers need assistance in the expansion of their MCK on this topic. It seems that the needs of the teachers are not taken into consideration when the training sessions are designed. For training to be effective, it needs to be designed according to the individual needs of teachers and the teachers should engage in these sessions over an extended period of time (Kleickmann et al., 2013). For teachers to improve their knowledge, it is imperative for them to engage actively with resources and other teachers (Wessels & Nieuwoudt, 2011). The CAPS sessions were lecture-style sessions and active engagement of teachers was not possible.

The performance of the learners in probability in this study seems to be consistent with the poor performance of learners in benchmark tests (Spaull, 2013). Most learners performed at a poor and below average level and only managed to score an average of 47.9% in Grade 11 and a shocking 30% in Grade 12 for probability. This poor performance is very concerning. Teachers, learners, schools and education departments should consider this and investigate possible action.

When looking at the link between the MCK of teachers and the performance of learners, the answers are not as clear-cut. Most of the learners involved in this study performed at a poor and below average performance level. The highest average was 57% for a group of learners in Grade 11. The same learners then only manage to score 26% in Grade 12. This group of learners were taught by the same teacher in Grade 11 and Grade 12 and the specific teacher scored 97% for Test 1. Similar results were found in the case of the teacher who scored 100% for Test 1.

The information gained during this study gives a clear indication of how teachers are struggling with the newly introduced topic of probability. The hope is that these results may lead to introspection on the side of the teachers, so that they are aware of their shortcomings. The GDE and other developers of training sessions can urgently implement some intervention strategies to enable the development of teachers' MCK.

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The learners and their performance were one of the motivational factors of this study. The results found in this study confirm the consistent poor performance of learners in mathematics. This study could not make a clear link between the performance of the learners and the MCK of the teachers who taught them. This leads to some questions on other factors that may influence the performance of learners.

CHAPTER 7: INTERPRETATIONS AND CONCLUSIONS

In this chapter, the results for the investigation into the MCK of teachers and the performance of learners in probability are interpreted. The primary research question of this study: **How does the level of teachers' MCK relate to the performance of the learners they teach on the topic of probability?** is answered by examining the MCK of teachers and the performance of the learners based on the cognitive levels. I start this chapter with an overview of the research question. Recommendations for further studies are then discussed, followed by the limitations of this study along with some suggestions for the direction of future studies.

7.1 OVERVIEW OF THE RESEARCH

The investigation aimed to investigate the MCK of teachers and the performance of their learners in Grade 11 and Grade 12 on the topic of probability. From the motivation of the study, as discussed in Chapter 1, the problems of teachers' possible lack of MCK and learners' consistent poor performance were emphasised. Some of the available literature related to this study was discussed in Chapter 2. The MCK of the teachers and the performance of the learners were measured with regard to the four cognitive levels (knowledge, routine procedures, complex procedures and problem solving) specified in CAPS and the performance levels (poor, below average, average and above average). The cognitive levels and performance levels were used in the development of the conceptual framework discussed in Chapter 3. The discussion in Chapter 4 included the research methodology, which involved a mixed methods approach. The instruments used to gather data were discussed and analysed in Chapter 5, and in Chapter 6 the results from the study were presented and analysed.

For the data collection, four investigations were conducted in the following manner. Participating teachers wrote Test 1, which consisted of examination-type questions on the topic of probability and the teachers were involved in semi-structured individual interviews after completing Test 1. The other two investigations were conducted with the learners. The learners were passive participants as only their written work from the Grade 11 final examination and the Grade 12 preparatory examination were included in the investigations.

7.2 ANSWERING THE RESEARCH SUB-QUESTIONS FOR THIS STUDY

The purpose of this study was to investigate the MCK of teachers as it may have an influence on the performance of the learners they teach. This led to the primary research question: **How does the level of teachers' MCK relate to the performance of the learners they teach on the topic of probability?**

In order to address the primary research question in the study, the following secondary questions were formulated.

- ***What is the level of MCK of teachers in probability, based on the cognitive levels recommended by the CAPS?***
- ***What are the challenges that teachers experience in relation to probability?***
- ***How do the tertiary training and experience levels of mathematics teachers relate to the MCK they possess?***
- ***How can learners' performance in probability be described in terms of the cognitive levels recommended by CAPS?***

In the section that follows, the four sub-question are addressed individually. The results from the analysis of the data, as seen in Chapter 6, are used in this discussion.

7.2.1 What is the level of MCK of teachers in probability, based on the cognitive levels recommended by the CAPS?

In order to address this research question, teachers' performance in probability was explored by analysing the written responses they gave in Test 1. In order to correctly solve the probability questions, skills are needed in all four of the cognitive levels. The results revealed that the teachers' performance in probability was not as good as it should be in order to teach the topic properly. The performance of the teachers in Test 1 on the cognitive level of knowledge was very poor as the probability identity and rule could not be recalled or used in the appropriate situations. The teachers in general showed more skill on the cognitive level of complex procedures where problems were based in real world contexts that involved conceptual understanding. The questions related to the level of problem solving proved to be a major obstacle for teachers. This result implies that teachers do not have the skills to answer non-routine problems requiring higher order reasoning. The lack of skills on this cognitive level was very visible in the questions related to Venn diagrams that included three events where one of the sections was unknown.

This lack of MCK of mathematics teachers found in this study concurred with the results found by Spaull (2013) and Bansilal et al. (2014), who showed evidence that teachers' MCK levels on the content that they are teaching are low. In a study conducted by Blum et al. (2007) the results showed, just like in this study on the Venn diagram specifically (see section 6.2.3) , that teachers struggle to solve questions where a link between mathematics and a context outside of mathematics is necessary to solve the question successfully.

7.2.2 What are the challenges that teachers experience in relation to probability?

This research question was answered by analysing the written responses given by teachers in Test 1 in conjunction with their responses to questions related to these written responses in the semi-structured interviews.

The responses given by teachers with regard to the challenges they face relating to probability were all linked to the level of knowledge they possess. Most of the teachers had no formal training on this topic and felt that their knowledge on this topic was not at the same level as their knowledge on some of the other topics that they taught.

Another challenge mentioned by teachers was the limited time they have available to work on the improvement of their MCK. The time that they have spent engaging with the content involved in probability has been limited. This limitation is linked to the fact that probability was only recently introduced to the curriculum and therefore teachers have not had as much time engaging with probability as they have had on other topics that have been in the curriculum for an extended period of time. This is then also linked to the experience level of the teacher.

Most of the teachers realise that their knowledge on the topic of probability is not as good as it should be. They do however feel that they will improve their knowledge if they engage with the topic over an extended period of time. This statement is in accordance with research done by Ball et al. (2008).

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7.2.3 How do the tertiary training and experience levels of mathematics teachers relate to the MCK they possess?

The tertiary training of the teachers involved in this study, specifically looking at training related to education, included two basic qualifications: teaching diplomas and teaching degrees. To answer the question on the MCK levels of the teachers and how this relates to their tertiary training, their qualifications were examined in conjunction with their performance levels in Test 1.

All the teachers were qualified to teach mathematics and the teachers with the teaching diplomas outperformed the teachers with teaching degrees in Test 1. If the assumption was made that a degree is a better qualification than a diploma, then this study could not find that the qualification level of the teacher is related to the MCK of the teacher. The results found in this study therefore do not correspond with the results found by Betts et al. (2003) which revealed that the MCK of teachers influences the performance of their learners.

In the consideration of the teaching experience of teachers and the MCK they possess, the years of teaching mathematics was considered in relation to the performance levels of the teachers in Test 1. In this regard, the study found that the teachers with the most years' experience in teaching had the highest level of MCK. The MCK of teachers improves with the time spent teaching the subject (in this case the probability topic) and engaging actively with the content. These results concur with the results presented by Burgess (2010) that teachers gain skills over time and these skills help teachers to improve their MCK. That was the case with the two teachers who performed exceptionally well as a result of their continuous exposure to the content and interrogating the content.

7.2.4 How can learners' performance in probability be described in terms of the cognitive levels recommended by CAPS?

The performance of the learners with regard to probability was investigated using written responses given by the learners during the Grade 11 final examination and the Grade 12 preparatory examination. The results were analysed and discussed along with the four cognitive levels as knowledge, routine procedures, complex procedures and problem solving.

The performance of the learners on probability was below average in all the examinations. The poor performance in Grade 11 (an average of 47%) was followed by an even weaker performance in Grade 12 (an average of 30%).

When considering the four cognitive levels mentioned by CAPS, the results were somewhat surprising. The performance on each cognitive level is discussed below.

- Knowledge

The questions related to knowledge include the use of formulas, in this case the probability identity or probability rules, along with simple calculations to solve probability questions related to dependent, independent and mutually exclusive events. The majority of learners, 50 of the 89 participating in Grade 11 (56%) and 60 of the 75 participating in Grade 12, (80%) displayed poor and below average performance for this cognitive level.

- Routine procedures

The routine procedures in probability questions include simple calculations of well-rehearsed procedures. The questions on this level were answered better than the questions on the knowledge level, but the performance was still low. Only 20% of the Grade 11 learners answered the routine procedures questions with almost correct or completely correct calculations. In Grade 12, 50% of the learners could employ routine procedures to solve the probability questions correctly.

- Complex procedures

The questions related to complex procedures include problems set in real world contexts, which require conceptual understanding. Surprisingly, more learners managed to do some correct calculations and procedures in the questions related to this cognitive level than on the two cognitive levels of knowledge and routine procedures. The questions in the Grade 11 examination paper that related to complex procedures mostly involved the setting up of Venn diagrams. The majority of Grade 11 learners (75 of the 89, therefore 84%) understood the basic concept of three events relating to each other in this way. The questions related to complex procedures in the Grade 12 examination included a two-way contingency table. In this case, the Grade 12 learners did not perform well since only 20 of the 75 Grade 12 learners

(27%) showed some correct calculations. The learners seemed to be more at ease with questions on Venn diagrams than they were with questions on two-way contingency tables.

- Problem solving

The highest cognitive level involves higher order reasoning and processes that require the ability to break down a problem into its constituent parts. The results of the questions involving problem solving were as expected as problem solving seems to be, in most cases, challenging for learners in general. Few learners, 17% of Grade 11 learners (15 of 89) and 15% of the Grade 12 learners (11 of 75), responded correctly to questions on problem solving.

In conclusion of the discussion related to the performance of the learners with relation to the four cognitive levels, several things were surprising. It was surprising that the learners did not perform well on the knowledge level. The learners could not recall the probability identity and the rules of probability when answering questions on dependent, independent and mutually exclusive events. This type of question occurred in every examination. The fact that learners are not able to recall, identify and use appropriate formula is very concerning. For the other three cognitive levels, routine procedures, complex procedures and problem solving, the results were as expected since questions pitched at these levels are normally challenging.

7.3 Answering the Main Research Question

In the discussion above, the sub-questions were answered and now the main research question is answered. The main research question of this study was: **How does the level of teachers' MCK relate to the performance of the learners they teach on the topic of probability?**

The results in this study have shown that their where teachers who could not solve problems on probability because of, as they indicated in section 6.2.1, their inadequate content knowledge. The number of teachers involved in this study did not allow for quantitative examination of the data, but the results of the qualitative analysis show that teachers' MCK level on probability is low. Teachers were more competent in solving questions on the levels of routine and complex procedures. They struggled to answer the questions on the levels of knowledge and problem solving mainly because they could not recall the formulas. They had expected that they would be provided with formula sheets. It is surprising that teachers are relying on formula sheets.

The marks scored by the six teachers in Test 1 were 43%; 53%; 73%; 77%; 97% and 100%. This low performance, with the exception of the two teachers who performed exceptionally well, concurs with the results found by, among others, Bansilal et al. (2014), Mogari et al. (2009) and Amuche and Musa (2013), that teachers' MCK levels are generally low.

The performance of the learners in this study was poor. They only managed to score an average of 47% in Grade 11 and an average of 30% in Grade 12 for probability. Most of the learners were more able to answer questions that were at the routine procedures level. Learners' poor performance here is consistent with the poor performance shown by South African learners in benchmark tests (Spaull, 2013).

In continuation of the answering of the main research question, the performance of the learners with relation to the MCK of their teachers was discussed. The analysis of the written work of the learners in conjunction with the written work of their learners was done. In some cases, the responses of the learners were very similar to the responses given by the teachers. This was, however, not conclusive, as many learners taught by the top performing teacher (T_1) in Test 1, performed poorly and some of the learners taught by the teachers performing poorly in Test 1 managed to score at an above average level.

From the data gathered in this study, it is not possible to conclude with certainty that there is a link between teachers' MCK level and the performance of the learners they teach on probability.

7.4 RECOMMENDATIONS OF THE STUDY

Some recommendations from this study may be used to inform curriculum developers and those involved in the design and implementation of professional development of teachers where MCK is concerned (Burton & Bartlett, 2009).

The interesting results from this study show that teachers with a teaching diploma outperformed the teachers with teaching degrees in Test 1. It must, however, be noted that the teachers with the teaching diplomas also had more experience in teaching. This result may provide some insight when curriculum developers of tertiary institutions structure their courses. It may be worthwhile to investigate the content of the two diploma courses that these top performing teachers were involved in and compare it to the content of some of the degrees offered in educational training at present.

Analysis of the teachers' responses in the semi-structured interviews showed that teachers did not feel that the professional development sessions (CAPS training hosted by GDE) made any positive contribution to their MCK. These remarks should be taken into consideration when recommendations are made to the developers of training sessions for teachers. According to the results of this research, teachers all attended, but looking at their responses in Test 1 and the semi-structured interview, this had no impact on their MCK on probability. The training sessions conducted during 2012 and 2013 should be analysed and programmes should be devised that will ensure the improvement of the MCK of teachers by considering factors such as time allocated and the frequency of the sessions, content that is included, and the style in which the content is presented.

The inconclusive results concerning the link between the teachers' MCK and the performance of the learners in this study may motivate studies related to other factors that may be linked the performance of learners. Investigations regarding factors involved in the motivation of learners to perform well may lead to some interesting results.

7.5 Limitations of the Study

This study was limited in that only four schools and eight teachers from a single province were used. The fact that only three examinations were used in this study is also seen as a limitation. Because of these factors, the results found cannot be generalised.

A major limitation of the study was time. Time was a limitation in more than one way. The teachers who participated in the study were all very involved in their schools on academic and extramural levels and therefore could not spend much time on the study. Time was also a factor when considering that probability as a topic was only included in Grade 11 examinations in 2013 and in Grade 12 examinations in 2014 and was only taught in Term 3 in Grade 12 (2014). This did not leave much time for the gathering of data at the end of 2013.

A further limitation with regard to the participating teachers was that only six of the eight participating teachers completed Test 1, which decreased the already small sample even more. Because of the small sample size with regard to the teachers, it was not possible to do quantitative analysis on any of the results gathered in Test 1.

The incomplete data set with the withdrawal of one school limited this study with regards to possible comparisons between the two groups of schools mentioned in section 4.2. The fact that no teachers from Group 2 wrote Test 1 was a serious limitation to the study as I was not able to comment on their individual MCK relating to probability at all.

7.6 LAST REFLECTIONS

This investigation has not just helped me to gain knowledge on the MCK of teachers and the performance of their learners, but in the process of looking for answers to my specific questions, I have found answers to questions that I never knew I had.

Reading through articles published by many established researchers has given me the opportunity to engage in some of the practices they found to be useful. This enabled me to use my classroom for informal research and in doing so I believe that I not only improved my own skill and knowledge, but also made a contribution to the success of the learners I teach.

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Appendix A1

Grade 11 final mathematics examination School B

Question 7

7.1 A survey was done at a local library to show the different reading preferences for 80 students.

- 44 read the National Geographic magazine
- 33 read the Getaway magazine
- 39 read the Leadership magazine
- 23 read both the National Geographic and the Leadership magazines
- 19 read both the Getaway and the Leadership magazines
- 9 read all three magazines
- 69 read at least one magazine

7.1.1 How many students do not read any of these magazines? (1)

7.1.2 Let the number of students that read National Geographic and Getaway, but not Leadership be represented by x. Represent this information in a Venn-diagram. (5)

7.1.3 Show that x=5. (3)

7.1.4 What is the probability (correct to 3 decimal places) that a random student will read at least 2 of the 3 magazines? (3)

7.2 A smoke detector system in a large warehouse uses 2 devices: A and B. The probability that device A will detect any smoke is 0,95 and device B is 0,98. The probability that both will detect smoke at the same time is 0,94.

7.2.1 What is the probability that device A or B will detect smoke? (3)

7.2.2 What is the probability that smoke will not be detected? (1)

APPENDIX A2

Grade 11 final mathematics examination GDE

QUESTION 7

A school organises a camp for 103 Grade 8 learners. The learners were asked write down what food they prefer. They could choose between beef (B), vegetables (V) and chicken (C).

The following information was gathered.

- 2 learners do not eat beef, vegetables or chicken.
- 5 learners only eat vegetables.
- 2 learners eat only beef.
- 12 learners do not eat chicken at all.
- 3 learners eat only chicken.
- 66 learners eat chicken and beef.
- 75 learners eat chicken and vegetables.

Let the number of learners that eat beef, vegetables and chicken be x.

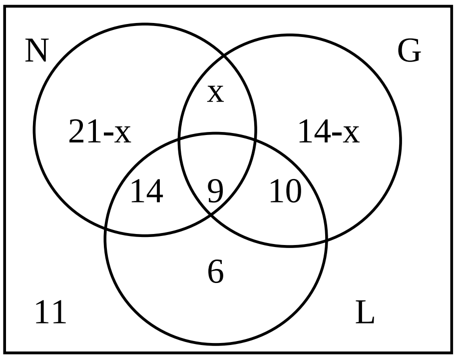
7.1 Draw a Venn-diagram to represent the above information. (7)
7.2 Calculate x. (2)
7.3 Calculate the probability that a learner that is chosen at random eats:
7.3.1 only beef and chicken but not vegetables. (2)
7.3.2 only two of the food types, beef, chicken and vegetables. (2)

QUESTION 8

Event X and Y is independent. (4)
$P(X) = \frac{1}{3}$ and $P(X \text{ and } Y) = \frac{1}{12}$, calculate P(Y)

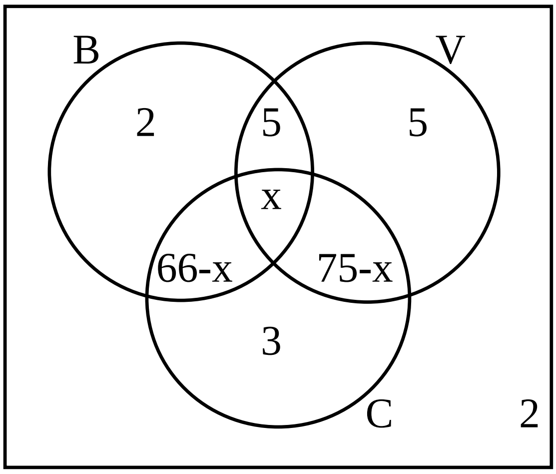
APPENDIX B1

Grade 11 final mathematics examination memorandum School B

Question number	Solution	Mark allocation	Cognitive level
7.1.1	11	(1)	Knowledge
7.1.2		(3) (2)	Complex procedures / Problem solving
7.1.3	$21 - x + x + 14 - x + 14 + 9 + 10 + 6 + 11 = 80$ $\therefore x = 5$	(3)	Routine procedures
7.1.4	$P(at\ least\ 2) = \frac{14 + 9 + 10 + 5}{80}$ $P\ (at\ least\ 2) = 0.475$	(3)	Routine procedures
7.2.1	$P\ (A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$ $P\ (A\ or\ B) = 0.95 + 0.98 - 0.94$ $P\ (A\ or\ B) = 0.99$	(3)	Routine procedures
7.2.2	$P\ (not\ A\ or\ B) = 1 - P(A\ and\ B)$ $P(not\ A\ or\ B) = 1 - 0.99$ $P(not\ A\ or\ B) = 0.01$	(1)	Knowledge

APPENDIX B2

Grade 11 final mathematics examination memorandum GDE

Question number	Solution	Mark allocation	Cognitive level
7.1		(4) (3)	Complex procedures / Problem solving
7.2	$2+5+5+66-x+x+75-x+3+2 = 103$ $\therefore x = 55$	(2)	Routine procedures
7.3.1	$P(B\ and\ C) = \dfrac{11}{103}$ $P\ (B\ and\ C) = 0.107$	(2)	Routine procedures
7.3.2	$P\ (only\ 2) = \dfrac{11+20+5}{103}$ $P\ (only\ 2) = 0.35$	(2)	Routine procedures

8	$P\ (X\ and\ Y) = P(X) \times P(Y)$ $\dfrac{1}{12} = \dfrac{1}{3} \times P(Y)$ $P\ (Y) = \dfrac{1}{4}$	(2) (2)	Knowledge / Routine procedures

APPENDIX C1

Test 1

Question 1

Three events, A with a probability of P(A)=0.3, B with a probability of P(B)=0.4 and C with a probability of P(C)=0.2.

- A and B are independent
- B and C are independent
- A and C are mutually exclusive

Calculate the probability of the following events occurring:

1.1 Both A and C occur. (1)
1.2 Both B and C occur. (1)
1.3 At least one of A or B occur. (4)

Question 2

In Johannesburg, South Africa, the probability of a sunny day is $\frac{6}{7}$ and the probability of a rainy day is $\frac{1}{7}$. If it is a sunny day, the probability that Thapelo will wear a dress to work is $\frac{7}{10}$, the probability that Thapelo will wear a skirt to work is $\frac{1}{5}$ and the probability that she will wear pants to work is $\frac{1}{10}$. If it is a rainy day, then the probability that Thapelo will wear a dress to work is $\frac{1}{9}$, the probability that she will wear a skirt to work is $\frac{5}{9}$ and the probability that Thapelo will wear pants to work is $\frac{1}{3}$.

2.1 Draw a tree diagram to represent the above information. Indicate on your diagram the probabilities associated with each branch as well as all the outcomes. (5)
2.2 For a day selected at random, what is the probability that:
 2.2.1 It is rainy and Thapelo will wear a dress (1)
 2.2.2 Thapelo wears pants (2)
2.3 If Thapelo works 245 days in a year, on approximately how many occasions does she wear a skirt to work? (4)

Question 3

A school has 174 Grade 12 learners, a survey is done among the learners and the following is found:

- 37 learners take Life Science
- 60 learners take Physical Science
- 111 learners take Mathematics
- 29 learners take Life Science and Mathematics
- 50 learners take Physical Science and Mathematics
- 13 learners take Life Science and Physical Science
- 45 learners do not take Life Science, Physical Science or Mathematics
- x learners take Life Science, Physical Science and Mathematics

3.1 Draw a Venn-diagram to represent the information above. (6)
3.2 Show that x=13. (2)

3.3 If a learner is selected at random, calculate the probability that the learner has the following combination of subjects:

3.3.1 Physical Science and Mathematics but not Life Science. (2)

3.3.2 Only one of Life Science, Physical Science or Mathematics (2)

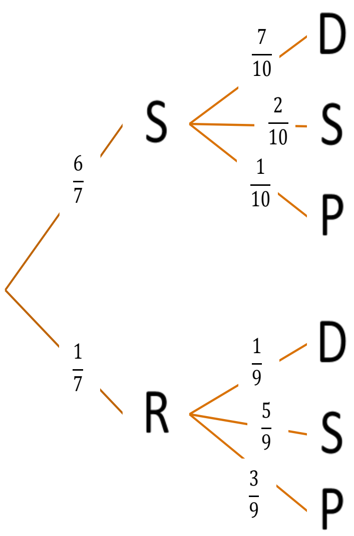
Appendix C2

Test 1 Memorandum

Question 1

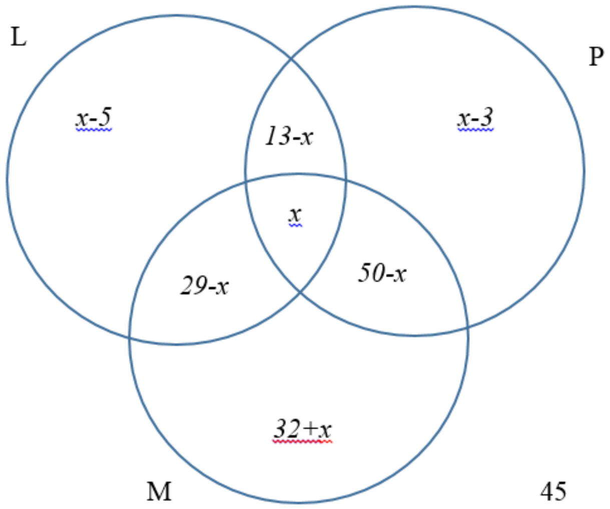
Question Number	Solution	Mark allocation	Cognitive Level
1.1	$P(A\ and\ C) = 0$	(1)	Knowledge
1.2	$P(B\ and\ C) = P(B).P(C)$ $P(B\ and\ C) = (0.4)(0.2)$ $P(B\ and\ C) = 0.08$	(1)	Knowledge
1.3	$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$ $P(A\ or\ B) = 0.3 + 0.4 - 0.12$ $P(A\ or\ B) = 0.58$	(4)	Routine procedures

Question 2

Question Number	Solution		Mark allocation	Cognitive Level
2.1			(3) (2)	Complex procedures/ Problem solving
2.2	2.2.1	$P(Rainy, Dress) = \frac{1}{36}$	(1)	Complex procedures
	2.2.2	$P(Pants) = \frac{6}{7} \times 0.1 + \frac{1}{7} \times \frac{3}{9}$ $P(Pants) = \frac{2}{15}$	(2)	Complex procedures

2.3	$P(Skirt) = \frac{6}{35} + \frac{5}{63}$ $P(Skirt) = \frac{79}{315}$ Thapelo wears a skirt for $\frac{79}{315} \times 245$ days = 61 days.	(2) (2)	Complex procedures /Problem solving

Question 3

Question Number	Solution	Mark allocation	Cognitive Level
3.1		(4) (2)	Complex procedures / Problem solving
3.2	*x-5+13-x+x-3+x+29-x+50-x+32+x+45=174* *x=13*	(2)	Routine procedures
3.3.1	$P(M\ and\ P\ not\ L) = \frac{37}{174} = 0.21$	(2)	Complex procedures
3.3.2	$P(only\ M\ or\ P\ or\ L) = \frac{8+10+45}{174} = 0.36$	(2)	Routine procedures

APPENDIX D

INTERVIEW SCHEDULE

Administration only	
School Name:	
Region:	
# Gr 12 learners:	
# Gr 12 learners taking Math:	
# of teachers teaching Gr 12 Math:	
Teacher's code:	

Interview schedule

Part 1

Personal information

1 Gender: Male: ☐ Female: ☐

2 Years of Mathematics teaching experience ☐

Part 2

Academic History

3 What is your highest Mathematics qualification on school level? (Mark the appropriate option(s) with a x.)

3.1 Senior certificate, Mathematics HG ☐
3.2 Senior certificate, Mathematics SG ☐
3.3 National senior certificate Mathematics ☐
3.4 National senior certificate Mathematical literacy ☐
3.5 Other ☐
3.6 If you marked other in 3.5, please specify your highest Mathematics qualification on school level.

____

4 Please list you tertiary qualifications.

Qualification	Institution	Year obtained

____

5 Please list the Mathematics course included in the qualifications you obtained.

Course name	Year level	Content

6 Did you attend any of the CAPS training sessions hosted by the Department of Education?

Yes ☐ No ☐

7 If you answered yes in 6, please indicate how many sessions you attended. ☐

8 If you answered yes in 6, did you find it useful

Yes ☐ No ☐

9 Please elaborate on your answer in number 8.

____

____

Thank you for your time

APPENDIX E1

Grade 12 preparatory mathematics examination

QUESTION 13

13.1 Tebo writes an Art and a Music examination. He has a 40% chance of passing the Music examination, a 60% chance of passing the Art examination and a 30% chance of passing both the Music and Art examination.
Calculate the probability that Tebo will pass the Music or Art examination. (3)

13.2 A survey was conducted asking 60 people with which hand they write and what colour hair they have. The results are summarized in the table below.

		HAND USED TO WRITE WITH		
		Right	Left	**Total**
HAIR COLOUR	Light	*a*	*b*	**20**
	Dark	*c*	*d*	**40**
	Total	**48**	**12**	**60**

The survey concluded that the 'hand used for writing' and 'hair colour' are independent events.
Calculate the values of *a*, *b*, and *c*. (5)

13.3 The digits from 1 to 9 are used to make 5-digit codes.

13.3.1 Determine the number of 5-digit codes possible, if the digits are arranged in any order without repetition. (2)

13.3.2 Determine the number of 5-digit codes possible, if the code formed has to be an even number and the digits may not be repeated. (3)

13.3.3 Determine the number of 5-digit codes possible, if the code formed only uses even digits and repetition of digits is allowed. (2)

APPENDIX E2

Grade 12 preparatory mathematics examination memorandum

Question number	Solution	Mark allocation	Cognitive level
13.1	$P(M\ or\ A) = P(M) + P(A) - P(M\ and\ A)$ $P(M\ or\ A) = 0.4 + 0.6 - 0.3$ $P(M\ or\ A) = 0.7$	(3)	Routine procedures
13.2.	$P(Right\ and\ Light) = P(Right) \times P(Light)$ $\frac{a}{60} = \frac{48}{60} \times \frac{20}{60}$ $\therefore a = 16$ $b = 4, \quad c = 32\ and\ d = 8$	(2) (3)	Complex procedures / problem solving
13.3.1	$9 \times 8 \times 7 \times 6 \times 5$ 15120	(2)	Knowledge
13.3.2	$8 \times 7 \times 6 \times 5 \times 4$ 6720	(3)	Routine procedures
13.3.3	4^5 1024	(2)	Routine procedures

APPENDIX F

Mathematics/P1 NSC DBE/November 2014

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

GAUTENG

APPENDIX G

LETTER OF INFORMED CONSENT FOR PARENTS.

FACULTY OF EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

Groenkloof Campus

Pretoria 0002

Republic of South Africa

Tel: +27 12 420 –5734

Fax: +27 12 420-5621

Cell: 071 887 8624

Date: ……………………….

Dear Parent(s)/Guardian(s):

RE: <u>REQUEST TO PARTICIPATE IN A RESEARCH PROJECT</u>

I am a mathematics teacher and head of department of Mathematics at Sutherland High School. I have enrolled for my Master's Degree at the University of Pretoria at the Department of Science, Mathematics and Technology Education under the supervision of Dr B. Mofolo-Mbokane and co-supervision of Dr J. Botha. I hereby request you to grant permission for your child in Grade 12 to participate in my research project.

The topic of my research is:

The mathematical content knowledge levels of mathematics teachers and the performance of the learners they teach regarding probability

The aim of this study is to identify possible connections between the mathematics content knowledge of teachers and the performance of the learners they teach in mathematics, with the focus on probability. The findings of the study may contribute to solutions for the current poor performance of learners in probability.

Relating teachers' content knowledge to learner performance in probability.

To ensure minimum disruption to the learners' schedule, the learners will complete the normal preliminary examination as scheduled by the Department of Basic Education in September 2014, as they would have done in any case as part of the compulsory assessment process for Grade 12 learners. I am asking for permission to make photocopies of the section of probability, without revealing the learners' names for anonymity. I also ask to make photocopies of the learners' written responses in the final Grade 11 mathematics examination scripts for November 2013. I will only be in contact with the learners directly when I explain the purpose of the study and hand out the consent letters, I will ask teachers to provide me with the written responses of the learners.

The photocopied responses as discussed will be marked, analysed and interpreted. The learners' written responses will serve as examples in the publication of the results of the study. The learners will remain anonymous at all times and no names will be revealed. This will ensure that the learners' identity are kept private at all times.

The teachers involved in the study will write two tests on probability, their written responses will be marked and analysed. The teachers will be asked to engage in interviews and discussion forums to help develop their mathematical content knowledge on probability as it is a new section in the syllabus.

Learners' and teachers' participation in this study is completely voluntary and their names will not be revealed. Their identity will remain confidential at all times. The learners may withdraw from the study at any time without any consequences.

Yours sincerely

Suzelle de Kock

________________________ ________________________

Suzelle de Kock Dr B Mofolo-Mbokane

I ____ give permission for my child

____ to participate in this research study.

Relating teachers' content knowledge to learner performance in probability.

I ____ give permission that my child's results from the mathematics preliminary examination in Grade 12 and the final Grade 11 examination may be used in this study. I am aware that the findings of this research will be used to promote teaching and learning and will be published. I am aware that my child will remain anonymous.

Signed: _____________________________________ Date: ____________________________

LETTER OF INFORMED CONSENT FOR TEACHERS

FACULTY OF EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

Groenkloof Campus

Pretoria 0002

Republic of South Africa

Tel: +27 12 420 –5734

Fax: +27 12 420-5621

Cell: 071 887 8624

Date: ………………………

Dear Teacher

RE: <u>REQUEST TO PARTICIPATE IN A RESEARCH PROJECT</u>

I am a mathematics teacher and head of department of Mathematics at Sutherland High School. I am enrolled for my Master's Degree at the University of Pretoria at the Department of Science, Mathematics and Technology Education under the supervision of Dr B. Mofolo-Mbokane and co-supervision of Dr J. Botha. I hereby request permission for you to participate in my research project.

The topic of my research study is:

The mathematical content knowledge levels of mathematics teachers and the performance of the learners they teach regarding probability

The aim of this study is to identify possible connections between the mathematics content knowledge of teachers and the performance of the learners they teach in mathematics, with the focus on probability. The outcome of the study may lead to some solutions to the current poor performance of learners in probability. In the process of this study, I am hoping to support participating teachers and empowering them by making a contribution to their content knowledge, preparation, applications of various methods in class, as well as building peer relationships.

You will be asked to complete some questions relating to probability prepared by myself and administered at a neutral venue. This should not take more than 30 minutes of your time. Your written responses will be marked and analysed by myself. A discussion on the written responses will follow in the form of an individual interview at your convenience. During this interview some question related to your education and training, bio-graphical information as well as some probing questions on the questions you answered on probability will be asked. This interview will be tape recorded and will last approximately 60 minutes.

Apart from my interest in the content knowledge that teachers possess, I would like to make a contribution to your professional development. As part of this research I request you to participate in a bi-monthly discussion forum with other teachers participating in the study at a venue convenient for all. The discussions will take place after school at a time convenient for all participating teachers, and will last 60 minutes per session. The discussion forums will give you the opportunity to develop your mathematical content knowledge and to engage with other teachers experiencing the same challenges in teaching probability.

Lastly, I would also like you to complete the section of the mathematics preliminary examination in September 2014, pertaining to probability. The completion of these questions will take place at the same time and under the same conditions as the learners who will also be writing the examination.

All the sessions will take place after school hours, to ensure minimum disruption to the school activities, learners' and your own programmes.

I will also ask you to assist me in finding the written responses of the learners taught by yourself in the Grade 11 final mathematics examination of 2013 and the mathematics preliminary examination for Grade 12 in 2014. I will make photocopies of the written responses and return all the documents to you.

It is important to understand that if you agree to participate in the study all information will be treated as sensitive and confidential as your identity will not be revealed at any time.

Relating teachers' content knowledge to learner performance in probability.

Participation in this study is therefore completely voluntary and anonymous. You may withdraw from the study at any point during the study, with no consequences.

Yours Sincerely
Suzelle de Kock

__________________________ __________________________

Suzelle de Kock Dr B Mofolo-Mbokane

I ____ volunteer to participate in this research by writing a content knowledge test, the probability section of the preliminary examination, participating in an individual interview and being part of bi-monthly discussion forums. I am aware that the findings of this research will be used to promote teaching and learning and will be published. I am aware that my identity will be protected and I will remain anonymous.

Signed: ___________________________________ Date: ___________________________

LETTER OF INFORMED ASSENT FOR LEARNERS

FACULTY OF EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

Groenkloof Campus

Pretoria 0002

Republic of South Africa

Tel: +27 12 420 –5734

Fax: +27 12 420-5621

Cell: 071 887 8624

Date: ………………………

Dear Grade 12 learner

RE: <u>REQUEST TO PARTICIPATE IN A RESEARCH PROJECT</u>

I am a mathematics teacher and head of department of Mathematics at Sutherland High School. I am enrolled for my Master's Degree at the University of Pretoria at the Department of Science, Mathematics and Technology Education under the supervision of Dr B. Mofolo-Mbokane and co-supervision of Dr J. Botha. I hereby request your assent to take part in this study by giving me permission to access some of your written responses from Grade 11 and Grade 12.

The topic of my study is:

The mathematical content knowledge levels of mathematics teachers and the performance of the learners they teach regarding probability

This study investigates the possible links or connections between what mathematics teachers know and the results of the learners that those teachers teach. The study will only focus on the topic of probability. At the end of the study there may be some answers to the fact of why some learners perform poorly in probability.

Relating teachers' content knowledge to learner performance in probability.

If you want to take part in this study, I will look at two of your mathematics examinations. I will make photocopies of the probability section of your final Grade 11, 2013 mathematics examination and the probability section of the preliminary examination you are due to write in September 2014. I will only be in direct contact with you when I explain the purpose of the study and hand out the assent letters, and will not disturb your programme or take any of your time. In agreeing to participate, it is important to understand that your work may be used when the results of the study is published, but your identity will never be revealed. You will remain anonymous at all times.

Your mathematics teachers will also be asked to participate in the study.

Taking part in this study is completely voluntary and you will remain nameless. Your identity will remain confidential at all times and you may decide at any point in time to stop participating without any consequences.

Yours Sincerely
Suzelle de Kock

________________________ ________________________
Suzelle de Kock Dr B Mofolo-Mbokane

I ____ volunteer to participate in this research. I am aware that the findings of this research will be used to promote teaching and learning and will be published. I am aware that my identity will be protected and I will remain anonymous.

Signed:____________________________________ Date: ________________________

LETTER OF INFORMED CONSENT FOR GAUTENG DEPARTMENT OF EDUCATION

FACULTY OF EDUCATION

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

Groenkloof Campus

Pretoria 0002

Republic of South Africa

Tel: +27 12 420 –5734

Fax: +27 12 420-5621

Cell: 071 887 8624

Date: ……………………….

Dear Sir/Madam

RE: REQUEST TO PARTICIPATE IN A RESEARCH PROJECT

I am a mathematics teacher and head of department of Mathematics at Sutherland High School. I am enrolled for my Master's Degree at the University of Pretoria at the Department of Science, Mathematics and Technology Education under the supervision of Dr B. Mofolo-Mbokane and co-supervision of Dr J. Botha. I hereby request you to give permission for this study to be conducted at the schools I will identify.

The topic of my research study is:

The mathematical content knowledge levels of mathematics teachers and the performance of the learners they teach regarding probability

As a Mathematics teacher for the past 13 years and head of department mathematics at a high school, I have come to notice that there may be a link between the teacher teaching mathematics and the performance of the learners taught by these teachers. This brought forth the question of the qualification levels of teachers and the mathematical content knowledge they have, and how teachers can be assisted in the development of their content knowledge.

I request permission to conduct research in four schools in one of the Tshwane districts in Gauteng. Participants in my study will include eight mathematics teachers and one Grade 12 mathematics class taught by each participating teacher (in total 8 classes).

The four schools will be chosen purposively according to their Grade 12 mathematics pass rate in the NCS examination 2013. Two schools with a pass rate of below 50% and two schools with a pass rate of above 90% will be chosen.

The teacher participants will include two teachers from each school, randomly chosen, but willing to participate and teaching mathematics in the FET phase. The learner participants will be Grade 12 learners, randomly chosen, willing to participate and taught by the participating teachers. The learner participants will be passive participants, I will only work with photocopies of their work, and not with the learners personally.

The Grade 12 learners will not be disrupted during the year, as mentioned, learners will be passive participants. I will make photocopies of their written responses of the final mathematics examination in Grade 11 in 2013 and the preliminary examination in September 2014. The participating learners will write the mathematics preliminary examination as scheduled by the Department of Education. The photocopied written responses from the learners will be marked an analysed and quantitatively by myself as part of the data collection strategy of the study. My only interaction with the learners will be when I explain the purpose of my study to them, and hand out consent letters. The learners will each receive a consent letter to themselves and a consent letter to their parents/guardians. The teachers will be asked to collect these letters from the learners a week after distribution.

Regarding the teachers, the research will include a test (set by myself and moderated by my supervisor on the topic of probability), an individual interview, bi-monthly discussion forums and a second test (probability section of mathematics preliminary examination 2014).

The test will be written at a convenient venue, under examination conditions and should take 30 minutes to complete. The written responses of the teachers will be marked and analysed qualitatively and quantitatively by myself . According to the written responses I will set up a list of questions that I will ask to the individual teachers during an interview.

The interviews will take place at a venue chosen by the individual participant and will apart from the mathematical content related questions also include some personal questions on the teachers training and experience. The interviews will be tape recorded to ensure accurate transcription of the events and is expected to last between 30 and 60 minutes each.

The next phase of the research will be in the form of discussion forums. These sessions will involve all the participating teachers. The teachers will come together at a venue that will be convenient for all participants and during these sessions content, pertaining to probability will be discussed. Each of these sessions will last one hour. Teachers will have the opportunity to share experiences and knowledge and build professional relationships.

The last data collection involving the teachers will be the writing of the probability section of the preliminary examination in September 2014. The writing of this examination will take place at the participating schools under examination conditions and will take approximately 30 minutes to finish, depending on the mark allocation awarded to probability.

All the data collection will take place after school hours to ensure minimum disruption to the programmes and activities of the teachers, learners and schools involved in the study. Consent letters will go out to all the learners, parents/guardians, principals and teachers involved.

Kind regards
Suzelle de Kock

____________________ Suzelle de Kock	____________________ Dr B Mofolo-Mbokane

LETTER OF INFORMED CONSENT FOR PRINCIPAL

100
1908 - 2008
UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

FACULTY OF EDUCATION
DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

Groenkloof Campus
Pretoria 0002
Republic of South Africa
Tel: +27 12 420 –5734
Fax: +27 12 420-5621

Cell: 071 887 8624

Date: ....

Dear Principal

RE: <u>REQUEST TO PARTICIPATE IN A RESEARCH PROJECT</u>

I am a mathematics teacher and head of department of Mathematics at Sutherland High School. I am enrolled for my Master's Degree at the University of Pretoria at the Department of Science, Mathematics and Technology Education under the supervision of Dr B. Mofolo-Mbokane and co-supervision of Dr J. Botha. I hereby request you to give permission for this study to be conducted at your school.

The topic of my research study is:

The mathematical content knowledge levels of mathematics teachers and the performance of the learners they teach regarding probability

As a Mathematics teacher for the past 13 years and head of department mathematics at a high school, I have come to notice that there may be a link between the teacher teaching mathematics and the performance of the learners taught by these teachers. This brought forth the question of the qualification levels of teachers and the mathematical content knowledge they have, and how teachers can be assisted in the development of their content knowledge.

I request permission to conduct research at your school. Participants in my study will include eight mathematics teachers, two from your school, and one Grade 12 mathematics class taught by each participating teachers.

The teacher participants will include two teachers from each school, randomly chosen, but willing to participate and teaching mathematics in the FET phase. The learner participants will be Grade 12 learners, randomly chosen, willing to participate and taught by the participating teachers. The learner participants will be passive participants I will only work with photocopies of their work, and not with the learners personally.

The Grade 12 learners will not be disrupted during the year, as mentioned, learners will be passive participants. I will make photocopies of their written responses of the final mathematics examination in Grade 11 in 2013 and the preliminary examination in September 2014. The participating learners will write the mathematics preliminary examination as scheduled by the Department of Education. The photocopied written responses from the learners will be marked and analysed quantitatively by myself as part of the data collection strategy of the study. My only interaction with the learners will be when I explain the purpose of my study to them, and hand out consent letters. The learners will each receive a consent letter to themselves and a consent letter to their parents/guardians. The teachers will be asked to collect these letters from the learners a week after distribution.

Regarding the teachers, the research will include a test (set by myself and moderated by my supervisor on the topic of probability), an individual interview, bi-monthly discussion forums and a second test (probability section of mathematics preliminary examination 2014).

The test will be written at a convenient venue, under examination conditions and should take 30 minutes to complete. The written responses of the teachers will be marked and analysed qualitatively and quantitatively by myself . According to the written responses I will set up a list of questions that I will ask to the individual teachers during an interview. These interviews will take place a venue chosen by the individual participant and will apart from the mathematical content related questions also include some personal questions on the teachers training and experience. The interviews will be tape recorded to ensure accurate transcription of the events and is expected to last 60 minutes each. The next phase of the research will be in the form of discussion forums.

The discussion sessions will involve all the participating teachers. The teachers will come together at a venue that will be convenient for all participants and during these sessions content, pertaining to probability will be discussed. Teachers will have the opportunity to share experiences and knowledge and build professional relationships. Each session will take one hour.

The last data collection involving the teachers will be the writing of the probability section of the preliminary examination in September 2014. The writing of this examination will take place at the participating schools under examination conditions and is expected to last 30 minutes, depending on the mark allocation of the probability section of the paper.

All the data collection will take place after school hours to ensure minimum disruption to the programmes and activities of the teachers, learners and schools involved in the study. Consent letters will go out to all the learners, parents/guardians, principals and teachers involved as well as the Gauteng Department of Basic Education.

I need permission to use the preliminary mathematics examination questions on probability and the result of this part of the examination paper as one of my data collection instruments.

Kind regards
Suzelle de Kock

__________________________ __________________________
Suzelle de Kock Dr B Mofolo-Mbokane

APPENDIX H

Correlations Analysis

Correlations: Question 7

Correlations

		Q7.1.1	Q7.1.2	Q7.1.3	Q7.1.4	Q7.2.1	Q7.2.2
Q7.1.1	Pearson Correlation	1	-.601**	-.218*	-.329**	.357**	-.259
	Sig. (2-tailed)		.000	.040	.002	.001	.211
	N	89	89	89	89	89	25
Q7.1.2	Pearson Correlation	-.601**	1	.421**	.549**	-.111	-.048
	Sig. (2-tailed)	.000		.000	.000	.300	.820
	N	89	89	89	89	89	25
Q7.1.3	Pearson Correlation	-.218*	.421**	1	.455**	-.107	.276
	Sig. (2-tailed)	.040	.000		.000	.320	.182
	N	89	89	89	89	89	25
Q7.1.4	Pearson Correlation	-.329**	.549**	.455**	1	-.064	.109
	Sig. (2-tailed)	.002	.000	.000		.549	.603
	N	89	89	89	89	89	25
Q7.2.1	Pearson Correlation	.357**	-.111	-.107	-.064	1	.572**
	Sig. (2-tailed)	.001	.300	.320	.549		.003
	N	89	89	89	89	89	25
Q7.2.2	Pearson Correlation	-.259	-.048	.276	.109	.572**	1

Sig. (2-tailed)	.211	.820	.182	.603	.003	
N	25	25	25	25	25	25

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

Correlations: Question 13

Correlations

		Q13.1	Q13.2	Q13.3.1	Q13.3.2	Q13.3.3
Q13.1	Pearson Correlation	1	.283**	.478**	.404**	.326**
	Sig. (2-tailed)		.004	.000	.000	.001
	N	104	104	104	104	104
Q13.2	Pearson Correlation	.283**	1	.459**	.465**	.314**
	Sig. (2-tailed)	.004		.000	.000	.001
	N	104	104	104	104	104
Q13.3.1	Pearson Correlation	.478**	.459**	1	.524**	.419**
	Sig. (2-tailed)	.000	.000		.000	.000
	N	104	104	104	104	104
Q13.3.2	Pearson Correlation	.404**	.465**	.524**	1	.587**
	Sig. (2-tailed)	.000	.000	.000		.000
	N	104	104	104	104	104
Q13.3.3	Pearson Correlation	.326**	.314**	.419**	.587**	1

Sig. (2-tailed)	.001	.001	.000	.000	
N	104	104	104	104	104

**. Correlation is significant at the 0.01 level (2-tailed).

Correlations

Correlations

		Grade 11 Percentage of probability question	Grade 12 Percentage of probability question
Grade 11 Percentage of probability question	Pearson Correlation	1	.411**
	Sig. (2-tailed)		.000
	N	89	75
Grade 12 Percentage of probability question	Pearson Correlation	.411**	1
	Sig. (2-tailed)	.000	
	N	75	75

**. Correlation is significant at the 0.01 level (2-tailed).

Correlations

Correlations

		Grade 11 percentage of paper 1	Grade 11 Percentage of probability question		
Grade 11 percentage of paper 1	Pearson Correlation	1	.601**		
	Sig. (2-tailed)		.000		
	N	89	89		
Grade 11 Percentage of probability question	Pearson Correlation	.601**	1		
	Sig. (2-tailed)	.000			
	N	89	89		
Grade 12 percentage of paper 1	Pearson Correlation	.363**	.323**		
	Sig. (2-tailed)	.001	.005		
	N	75	75		
Grade 12 Percentage of probability question	Pearson Correlation	.386**	.456**		
	Sig. (2-tailed)	.001	.000		
	N	75	75		

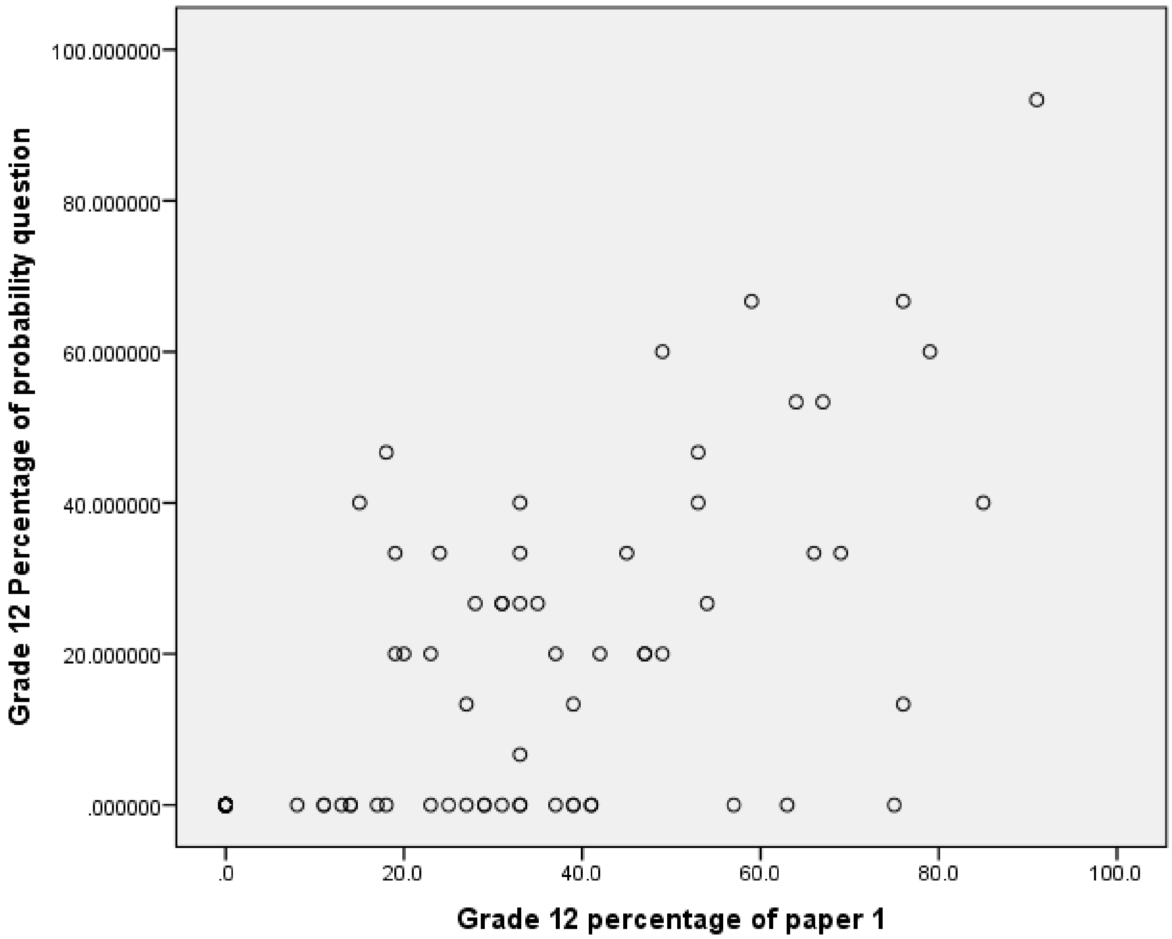
Correlations

		Grade 12 percentage of paper 1	Grade 12 Percentage of probability question
Grade 11 percentage of paper 1	Pearson Correlation	.363**	.386**
	Sig. (2-tailed)	.001	.001

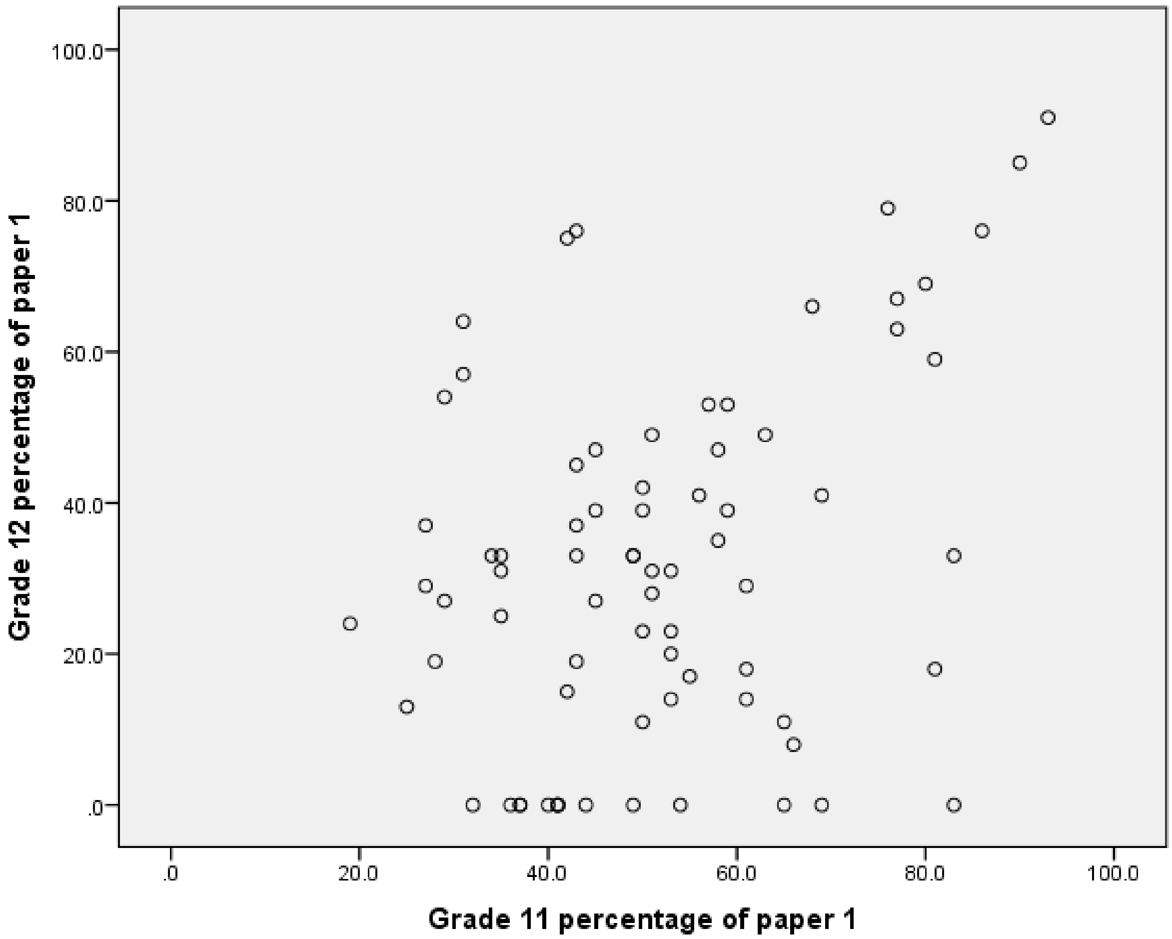
	N	75	75
Grade 11 Percentage of probability question	Pearson Correlation	.323**	.456**
	Sig. (2-tailed)	.005	.000
	N	75	75
Grade 12 percentage of paper 1	Pearson Correlation	1	.574**
	Sig. (2-tailed)		.000
	N	75	75
Grade 12 Percentage of probability question	Pearson Correlation	.574**	1
	Sig. (2-tailed)	.000	
	N	75	75

**. Correlation is significant at the 0.01 level (2-tailed).

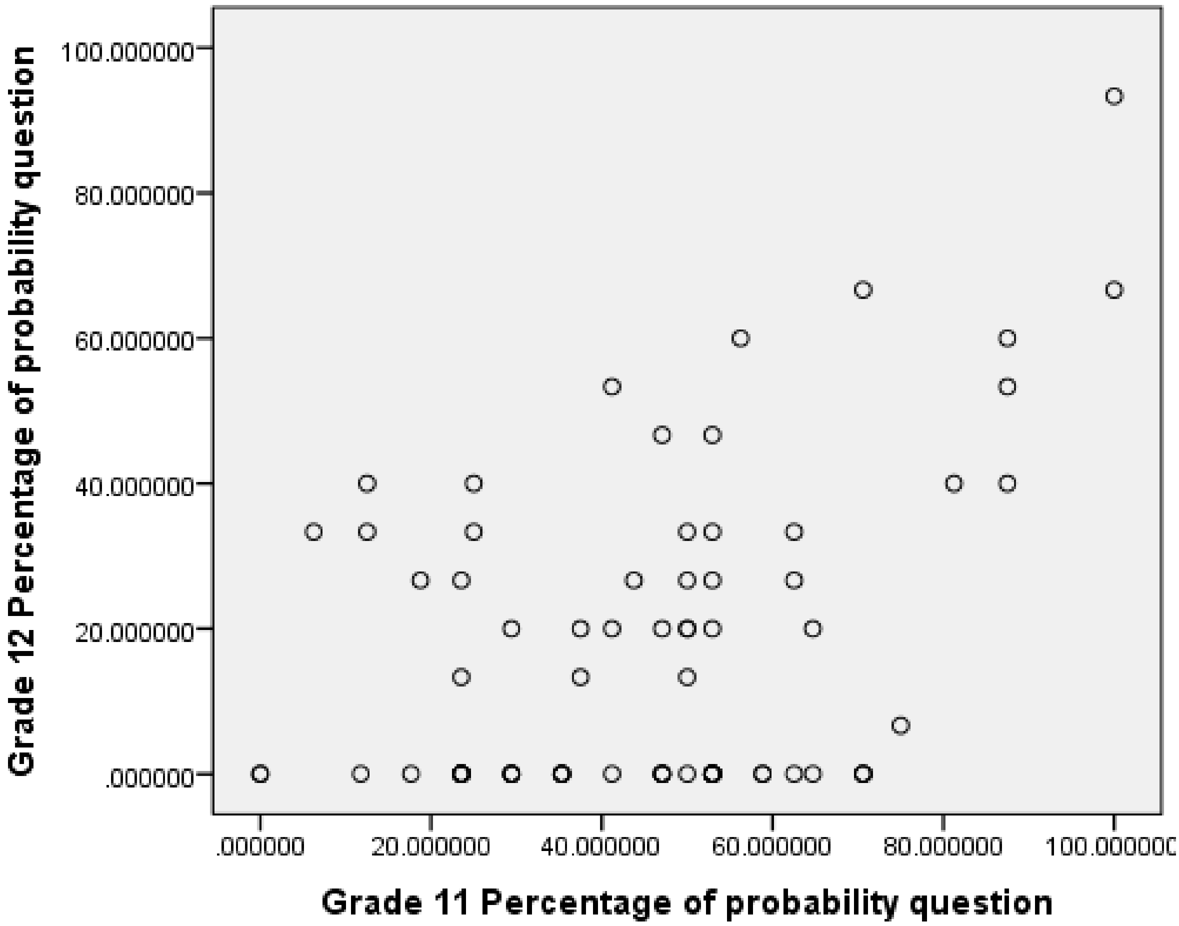
Graph


Grade 12 Percentage of probability question
100.000000
80.000000
60.000000
40.000000
20.000000
.000000
.0
20.0
40.0
60.0
80.0
100.0
Grade 12 percentage of paper 1

Graph


100.0
80.0
60.0
40.0
20.0
.0
Grade 12 percentage of paper 1
.0
20.0
40.0
60.0
80.0
100.0
Grade 11 percentage of paper 1

Graph



Comparing groups

T-Test: CAPS

Group Statistics

	Caps training in probability	N	Mean		
Grade 11 percentage of paper 1	Yes	64	51.984		
	No	25	53.200		
Grade 11 Percentage of probability question	Yes	64	42.27941176		
	No	25	51.25000000		
Grade 12 percentage of paper 1	Yes	50	26.380		

	No	25	43.120		
Grade 12 Percentage of probability question	Yes	50	10.13333333		
	No	25	30.66666667		

Group Statistics

	Caps training in probability	Std. Deviation	Std. Error Mean
Grade 11 percentage of paper 1	Yes	16.4186	2.0523
	No	18.5068	3.7014
Grade 11 Percentage of probability question	Yes	18.449689927	2.306211241
	No	25.895422697	5.179084539
Grade 12 percentage of paper 1	Yes	23.9982	3.3939
	No	20.4212	4.0842
Grade 12 Percentage of probability question	Yes	20.168901321	2.852313379
	No	17.533037598	3.506607520

Independent Samples Test

		Levene's Test for Equality of Variances								
		F	Sig.							
Grade 11 percentage of paper 1	Equal variances assumed	.107	.745							

	Equal variances not assumed									
Grade 11 Percentage of probability question	Equal variances assumed	3.086	.082							
	Equal variances not assumed									
Grade 12 percentage of paper 1	Equal variances assumed	.911	.343							
	Equal variances not assumed									
Grade 12 Percentage of probability question	Equal variances assumed	.133	.716							
	Equal variances not assumed									

Independent Samples Test

		t-test for Equality of Means						
		t	df					
Grade 11 percentage of paper 1	Equal variances assumed	-.303	87					
	Equal variances not assumed	-.287	39.600					
Grade 11 Percentage of probability question	Equal variances assumed	-1.831	87					

	Equal variances not assumed	-1.582	33.953						
Grade 12 percentage of paper 1	Equal variances assumed	-2.986	73						
	Equal variances not assumed	-3.152	55.602						
Grade 12 Percentage of probability question	Equal variances assumed	-4.334	73						
	Equal variances not assumed	-4.543	54.565						

Independent Samples Test

		t-test for Equality of Means				
		Sig. (2-tailed)	Mean Difference			
Grade 11 percentage of paper 1	Equal variances assumed	.763	-1.2156			
	Equal variances not assumed	.775	-1.2156			
Grade 11 Percentage of probability question	Equal variances assumed	.071	-8.970588235			
	Equal variances not assumed	.123	-8.970588235			
Grade 12 percentage of paper 1	Equal variances assumed	.004	-16.7400			
	Equal variances not assumed	.003	-16.7400			
Grade 12 Percentage of probability question	Equal variances assumed	.000	-20.533333333			

	Equal variances not assumed	.000	-20.533333333			

Independent Samples Test

		t-test for Equality of Means		
		Std. Error Difference	95% Confidence Interval of the Difference	
			Lower	
Grade 11 percentage of paper 1	Equal variances assumed	4.0142	-9.1943	
	Equal variances not assumed	4.2323	-9.7720	
Grade 11 Percentage of probability question	Equal variances assumed	4.899064840	-18.708010084	
	Equal variances not assumed	5.669349782	-20.492683185	
Grade 12 percentage of paper 1	Equal variances assumed	5.6054	-27.9116	
	Equal variances not assumed	5.3103	-27.3795	
Grade 12 Percentage of probability question	Equal variances assumed	4.737800966	-29.975757041	
	Equal variances not assumed	4.520175650	-29.593590081	

Independent Samples Test

	t-test for Equality of Means
	95% Confidence Interval of the Difference
	Upper

Grade 11 percentage of paper 1	Equal variances assumed	6.7631
	Equal variances not assumed	7.3408
Grade 11 Percentage of probability question	Equal variances assumed	.766833613
	Equal variances not assumed	2.551506715
Grade 12 percentage of paper 1	Equal variances assumed	-5.5684
	Equal variances not assumed	-6.1005
Grade 12 Percentage of probability question	Equal variances assumed	-11.090909626
	Equal variances not assumed	-11.473076585

T-Test: School

Group Statistics

	School	N	Mean	Std. Deviation	
Grade 11 percentage of paper 1	School A	25	53.200	18.5068	
	School B	64	51.984	16.4186	
Grade 11 Percentage of probability question	School A	25	51.25000000	25.895422697	
	School B	64	42.27941176	18.449689927	
Grade 12 percentage of paper 1	School A	25	43.120	20.4212	
	School B	50	26.380	23.9982	

Grade 12 Percentage of probability question	School A	25	30.66666667	17.533037598	
	School B	50	10.13333333	20.168901321	

Group Statistics

	School	Std. Error Mean
Grade 11 percentage of paper 1	School A	3.7014
	School B	2.0523
Grade 11 Percentage of probability question	School A	5.179084539
	School B	2.306211241
Grade 12 percentage of paper 1	School A	4.0842
	School B	3.3939
Grade 12 Percentage of probability question	School A	3.506607520
	School B	2.852313379

Independent Samples Test

		Levene's Test for Equality of Variances								
		F	Sig.							
Grade 11 percentage of paper 1	Equal variances assumed	.107	.745							

	Equal variances not assumed										
Grade 11 Percentage of probability question	Equal variances assumed	3.086	.082								
	Equal variances not assumed										
Grade 12 percentage of paper 1	Equal variances assumed	.911	.343								
	Equal variances not assumed										
Grade 12 Percentage of probability question	Equal variances assumed	.133	.716								
	Equal variances not assumed										

Independent Samples Test

		t-test for Equality of Means						
		t	df					
Grade 11 percentage of paper 1	Equal variances assumed	.303	87					
	Equal variances not assumed	.287	39.600					
Grade 11 Percentage of probability question	Equal variances assumed	1.831	87					

	Equal variances not assumed	1.582	33.953					
Grade 12 percentage of paper 1	Equal variances assumed	2.986	73					
	Equal variances not assumed	3.152	55.602					
Grade 12 Percentage of probability question	Equal variances assumed	4.334	73					
	Equal variances not assumed	4.543	54.565					

Independent Samples Test

		t-test for Equality of Means				
		Sig. (2-tailed)	Mean Difference			
Grade 11 percentage of paper 1	Equal variances assumed	.763	1.2156			
	Equal variances not assumed	.775	1.2156			
Grade 11 Percentage of probability question	Equal variances assumed	.071	8.970588235			
	Equal variances not assumed	.123	8.970588235			
Grade 12 percentage of paper 1	Equal variances assumed	.004	16.7400			
	Equal variances not assumed	.003	16.7400			
Grade 12 Percentage of probability question	Equal variances assumed	.000	20.53333333 3			

	Equal variances not assumed	.000	20.533333333			

Independent Samples Test

		t-test for Equality of Means		
		Std. Error Difference	95% Confidence Interval of the Difference	
			Lower	
Grade 11 percentage of paper 1	Equal variances assumed	4.0142	-6.7631	
	Equal variances not assumed	4.2323	-7.3408	
Grade 11 Percentage of probability question	Equal variances assumed	4.899064840	-.766833613	
	Equal variances not assumed	5.669349782	-2.551506715	
Grade 12 percentage of paper 1	Equal variances assumed	5.6054	5.5684	
	Equal variances not assumed	5.3103	6.1005	
Grade 12 Percentage of probability question	Equal variances assumed	4.737800966	11.090909626	
	Equal variances not assumed	4.520175650	11.473076585	

Independent Samples Test

		t-test for Equality of Means
		95% Confidence Interval of the Difference
		Upper
Grade 11 percentage of paper 1	Equal variances assumed	9.1943

	Equal variances not assumed	9.7720
Grade 11 Percentage of probability question	Equal variances assumed	18.708010084
	Equal variances not assumed	20.492683185
Grade 12 percentage of paper 1	Equal variances assumed	27.9116
	Equal variances not assumed	27.3795
Grade 12 Percentage of probability question	Equal variances assumed	29.975757041
	Equal variances not assumed	29.593590081

T-Test: Performance level

Group Statistics

	Performance level of teachers	N	Mean		
Grade 11 percentage of paper 1	Average	7	45.429		
	Above average	83	52.277		
Grade 11 Percentage of probability question	Average	6	57.29166667		
	Above average	83	43.89617293		
Grade 12 percentage of paper 1	Average	6	39.167		
	Above average	69	31.333		
	Average	6	32.22222222		

Grade 12 Percentage of probability question	Above average	69	15.65217391		

Group Statistics

	Performance level of teachers	Std. Deviation	Std. Error Mean
Grade 11 percentage of paper 1	Average	26.5948	10.0519
	Above average	16.8894	1.8539
Grade 11 Percentage of probability question	Average	28.890057402	11.794316546
	Above average	20.304941745	2.228756905
Grade 12 percentage of paper 1	Average	26.9103	10.9861
	Above average	23.9298	2.8808
Grade 12 Percentage of probability question	Average	20.403340322	8.329628806
	Above average	21.261708407	2.559608905

Independent Samples Test

		Levene's Test for Equality of Variances								
		F	Sig.							
Grade 11 percentage of paper 1	Equal variances assumed	.588	.445							
	Equal variances not assumed									

Grade 11 Percentage of probability question	Equal variances assumed	.486	.488							
	Equal variances not assumed									
Grade 12 percentage of paper 1	Equal variances assumed	.003	.953							
	Equal variances not assumed									
Grade 12 Percentage of probability question	Equal variances assumed	.217	.642							
	Equal variances not assumed									

Independent Samples Test

		t-test for Equality of Means						
		t	df					
Grade 11 percentage of paper 1	Equal variances assumed	-.982	88					
	Equal variances not assumed	-.670	6.415					
Grade 11 Percentage of probability question	Equal variances assumed	1.517	87					
	Equal variances not assumed	1.116	5.363					

Grade 12 percentage of paper 1	Equal variances assumed	.762	73					
	Equal variances not assumed	.690	5.709					
Grade 12 Percentage of probability question	Equal variances assumed	1.836	73					
	Equal variances not assumed	1.902	5.985					

Independent Samples Test

		t-test for Equality of Means				
		Sig. (2-tailed)	Mean Difference			
Grade 11 percentage of paper 1	Equal variances assumed	.329	-6.8485			
	Equal variances not assumed	.526	-6.8485			
Grade 11 Percentage of probability question	Equal variances assumed	.133	13.39549374 0			
	Equal variances not assumed	.312	13.39549374 0			
Grade 12 percentage of paper 1	Equal variances assumed	.448	7.8333			
	Equal variances not assumed	.517	7.8333			
Grade 12 Percentage of probability question	Equal variances assumed	.070	16.57004830 9			
	Equal variances not assumed	.106	16.57004830 9			

Independent Samples Test

		t-test for Equality of Means		
		Std. Error Difference	95% Confidence Interval of the Difference	
			Lower	
Grade 11 percentage of paper 1	Equal variances assumed	6.9746	-20.7090	
	Equal variances not assumed	10.2214	-31.4728	
Grade 11 Percentage of probability question	Equal variances assumed	8.832913285	-4.160877856	
	Equal variances not assumed	12.003052117	-16.841556859	
Grade 12 percentage of paper 1	Equal variances assumed	10.2771	-12.6489	
	Equal variances not assumed	11.3575	-20.3042	
Grade 12 Percentage of probability question	Equal variances assumed	9.025032206	-1.416814939	
	Equal variances not assumed	8.714029710	-4.765437456	

Independent Samples Test

		t-test for Equality of Means
		95% Confidence Interval of the Difference
		Upper
Grade 11 percentage of paper 1	Equal variances assumed	7.0119
	Equal variances not assumed	17.7757
Grade 11 Percentage of probability question	Equal variances assumed	30.951865336

	Equal variances not assumed	43.632544339
Grade 12 percentage of paper 1	Equal variances assumed	28.3156
	Equal variances not assumed	35.9709
Grade 12 Percentage of probability question	Equal variances assumed	34.556911557
	Equal variances not assumed	37.905534075

Oneway

Descriptives

		N	Mean	Std. Deviation	Std. Error				
Grade 11 percentage of paper 1	1.0	64	51.984	16.4186	2.0523				
	3.0	19	53.263	18.8321	4.3204				
	4.0	6	53.000	19.1625	7.8230				
	Total	89	52.326	16.9322	1.7948				
Grade 11 Percentage of probability question	1.0	64	42.27941176	18.449689927	2.306211241				

	3.0	19	49.34210526	25.421339806	5.832055328				
	4.0	6	57.29166667	28.890057402	11.794316546				
	Total	89	44.79923992	21.047852538	2.231067907				
Grade 12 percentage of paper 1	1.0	50	26.380	23.9982	3.3939				
	3.0	19	44.368	18.6554	4.2798				
	4.0	6	39.167	26.9103	10.9861				
	Total	75	31.960	24.0773	2.7802				
Grade 12 Percentage of probability question	1.0	50	10.13333333	20.168901321	2.852313379				
	3.0	19	30.17543860	17.122423541	3.928153362				
	4.0	6	32.22222222	20.403340322	8.329628806				
	Total	75	16.97777778	21.541030988	2.487344008				

Descriptives

		95% Confidence Interval for Mean			
		Lower Bound	Upper Bound	Minimum	
Grade 11 percentage of paper 1	1.0	47.883	56.086	25.0	
	3.0	44.186	62.340	19.0	
	4.0	32.890	73.110	35.0	

	Total	48.759	55.893	19.0	
Grade 11 Percentage of probability question	1.0	37.67081634	46.88800719	.000000	
	3.0	37.08941169	61.59479884	12.500000	
	4.0	26.97341079	87.60992254	6.250000	
	Total	40.36546128	49.23301856	.000000	
Grade 12 percentage of paper 1	1.0	19.560	33.200	.0	
	3.0	35.377	53.360	19.0	
	4.0	10.926	67.407	11.0	
	Total	26.420	37.500	.0	
Grade 12 Percentage of probability question	1.0	4.40139500	15.86527167	.000000	
	3.0	21.92269462	38.42818257	.000000	
	4.0	10.81022972	53.63421473	.000000	
	Total	12.02163665	21.93391890	.000000	

Descriptives

		Maximum
Grade 11 percentage of paper 1	1.0	93.0
	3.0	86.0
	4.0	90.0
	Total	93.0
Grade 11 Percentage of probability question	1.0	100.000000

	3.0	100.000000
	4.0	87.500000
	Total	100.000000
Grade 12 percentage of paper 1	1.0	91.0
	3.0	79.0
	4.0	85.0
	Total	91.0
Grade 12 Percentage of probability question	1.0	93.333333
	3.0	66.666667
	4.0	60.000000
	Total	93.333333

Test of Homogeneity of Variances

	Levene Statistic	df1	df2	Sig.
Grade 11 percentage of paper 1	.175	2	86	.840
Grade 11 Percentage of probability question	1.128	2	86	.329
Grade 12 percentage of paper 1	.639	2	72	.531
Grade 12 Percentage of probability question	.136	2	72	.873

ANOVA

		Sum of Squares	df	Mean Square		
Grade 11 percentage of paper 1	Between Groups	26.882	2	13.441		

	Within Groups	25202.669	86	293.054		
	Total	25229.551	88			
Grade 11 Percentage of probability question	Between Groups	1734.849	2	867.425		
	Within Groups	37250.215	86	433.142		
	Total	38985.064	88			
Grade 12 percentage of paper 1	Between Groups	4793.846	2	2396.923		
	Within Groups	38105.034	72	529.237		
	Total	42898.880	74			
Grade 12 Percentage of probability question	Between Groups	7046.066	2	3523.033		
	Within Groups	27291.119	72	379.043		
	Total	34337.185	74			

ANOVA

		F	Sig.
Grade 11 percentage of paper 1	Between Groups	.046	.955
	Within Groups		
	Total		
Grade 11 Percentage of probability question	Between Groups	2.003	.141
	Within Groups		
	Total		
Grade 12 percentage of paper 1	Between Groups	4.529	.014

	Within Groups		
	Total		
Grade 12 Percentage of probability question	Between Groups	9.295	.000
	Within Groups		
	Total		

Post Hoc Tests

Multiple Comparisons

Tukey HSD

Dependent Variable	(I) Teacher	(J) Teacher	Mean Difference (I-J)	Std. Error			
Grade 11 percentage of paper 1	1.0	3.0	-1.2788	4.4725			
		4.0	-1.0156	7.3090			
	3.0	1.0	1.2788	4.4725			
		4.0	.2632	8.0166			
	4.0	1.0	1.0156	7.3090			

		3.0	-.2632	8.0166			
Grade 11 Percentage of probability question	1.0	3.0	-7.062693498	5.437351705			
		4.0	-15.012254902	8.885841750			
	3.0	1.0	7.062693498	5.437351705			
		4.0	-7.949561404	9.746142239			
	4.0	1.0	15.012254902	8.885841750			
		3.0	7.949561404	9.746142239			
Grade 12 percentage of paper 1	1.0	3.0	-17.9884*	6.1999			
		4.0	-12.7867	9.9394			
	3.0	1.0	17.9884*	6.1999			
		4.0	5.2018	10.7731			
	4.0	1.0	12.7867	9.9394			
		3.0	-5.2018	10.7731			
Grade 12 Percentage of probability question	1.0	3.0	-20.042105263*	5.246952896			
		4.0	-22.088888889*	8.411584436			
	3.0	1.0	20.042105263*	5.246952896			

		4.0	-2.046783626	9.117210905			
	4.0	1.0	22.088888889*	8.411584436			
		3.0	2.046783626	9.117210905			

Multiple Comparisons

Tukey HSD

Dependent Variable	(I) Teacher	(J) Teacher	Sig.	95% Confidence Interval	
				Lower Bound	
Grade 11 percentage of paper 1	1.0	3.0	.956	-11.945	
		4.0	.989	-18.447	
	3.0	1.0	.956	-9.388	
		4.0	.999	-18.856	
	4.0	1.0	.989	-16.416	
		3.0	.999	-19.382	
Grade 11 Percentage of probability question	1.0	3.0	.400	-20.03056359	
		4.0	.215	-36.20463820	
	3.0	1.0	.400	-5.90517659	
		4.0	.694	-31.19372755	
	4.0	1.0	.215	-6.18012840	
		3.0	.694	-15.29460475	

Grade 12 percentage of paper 1	1.0	3.0	.013	-32.826	
		4.0	.407	-36.573	
	3.0	1.0	.013	3.151	
		4.0	.880	-20.580	
	4.0	1.0	.407	-10.999	
		3.0	.880	-30.983	
Grade 12 Percentage of probability question	1.0	3.0	.001	-32.59871345	
		4.0	.028	-42.21885233	
	3.0	1.0	.001	7.48549708	
		4.0	.973	-23.86539861	
	4.0	1.0	.028	1.95892545	
		3.0	.973	-19.77183136	

Multiple Comparisons

Tukey HSD

Dependent Variable	(I) Teacher	(J) Teacher	95% Confidence Interval
			Upper Bound
Grade 11 percentage of paper 1	1.0	3.0	9.388
		4.0	16.416
	3.0	1.0	11.945
		4.0	19.382
	4.0	1.0	18.447

		3.0	18.856
Grade 11 Percentage of probability question	1.0	3.0	5.90517659
		4.0	6.18012840
	3.0	1.0	20.03056359
		4.0	15.29460475
	4.0	1.0	36.20463820
		3.0	31.19372755
Grade 12 percentage of paper 1	1.0	3.0	-3.151
		4.0	10.999
	3.0	1.0	32.826
		4.0	30.983
	4.0	1.0	36.573
		3.0	20.580
Grade 12 Percentage of probability question	1.0	3.0	-7.48549708
		4.0	-1.95892545
	3.0	1.0	32.59871345
		4.0	19.77183136
	4.0	1.0	42.21885233
		3.0	23.86539861

*. The mean difference is significant at the 0.05 level.

Homogeneous Subsets

Grade 11 percentage of paper 1

Tukey HSD[a,b]

Teacher	N	Subset for alpha = 0.05
		1
1.0	64	51.984
4.0	6	53.000
3.0	19	53.263
Sig.		.981

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 12.770.

b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.

Grade 11 Percentage of probability question

Tukey HSD[a,b]

Teacher	N	Subset for alpha = 0.05
		1
1.0	64	42.27941176
3.0	19	49.34210526
4.0	6	57.29166667

Sig.		.168

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 12.770.

b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.

Grade 12 percentage of paper 1

Tukey HSD[a,b]

Teacher	N	Subset for alpha = 0.05
		1
1.0	50	26.380
4.0	6	39.167
3.0	19	44.368
Sig.		.130

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 12.537.

b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.

Grade 12 Percentage of probability question

Tukey HSD[a,b]

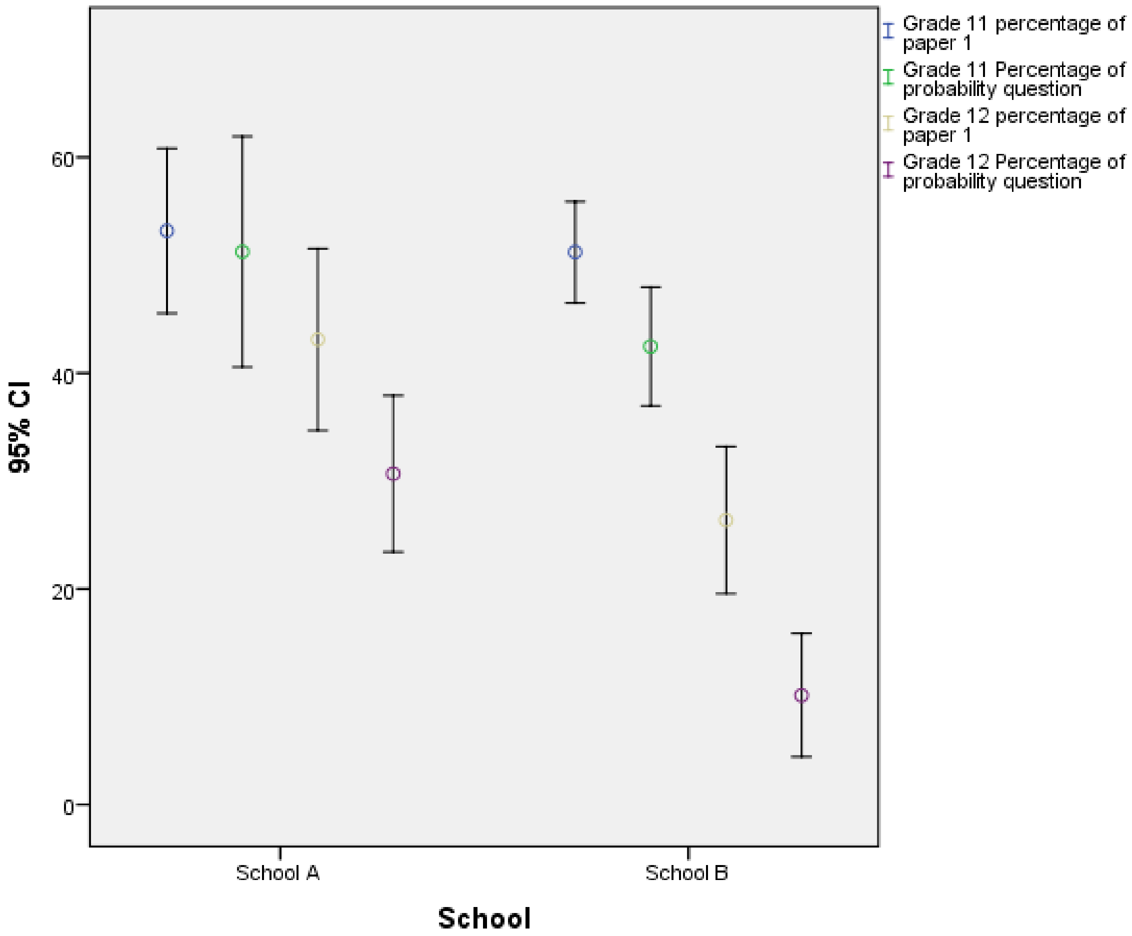
Teacher	N	Subset for alpha = 0.05	
		1	2
1.0	50	10.13333333	
3.0	19		30.17543860
4.0	6		32.22222222
Sig.		1.000	.963

Means for groups in homogeneous subsets are displayed.

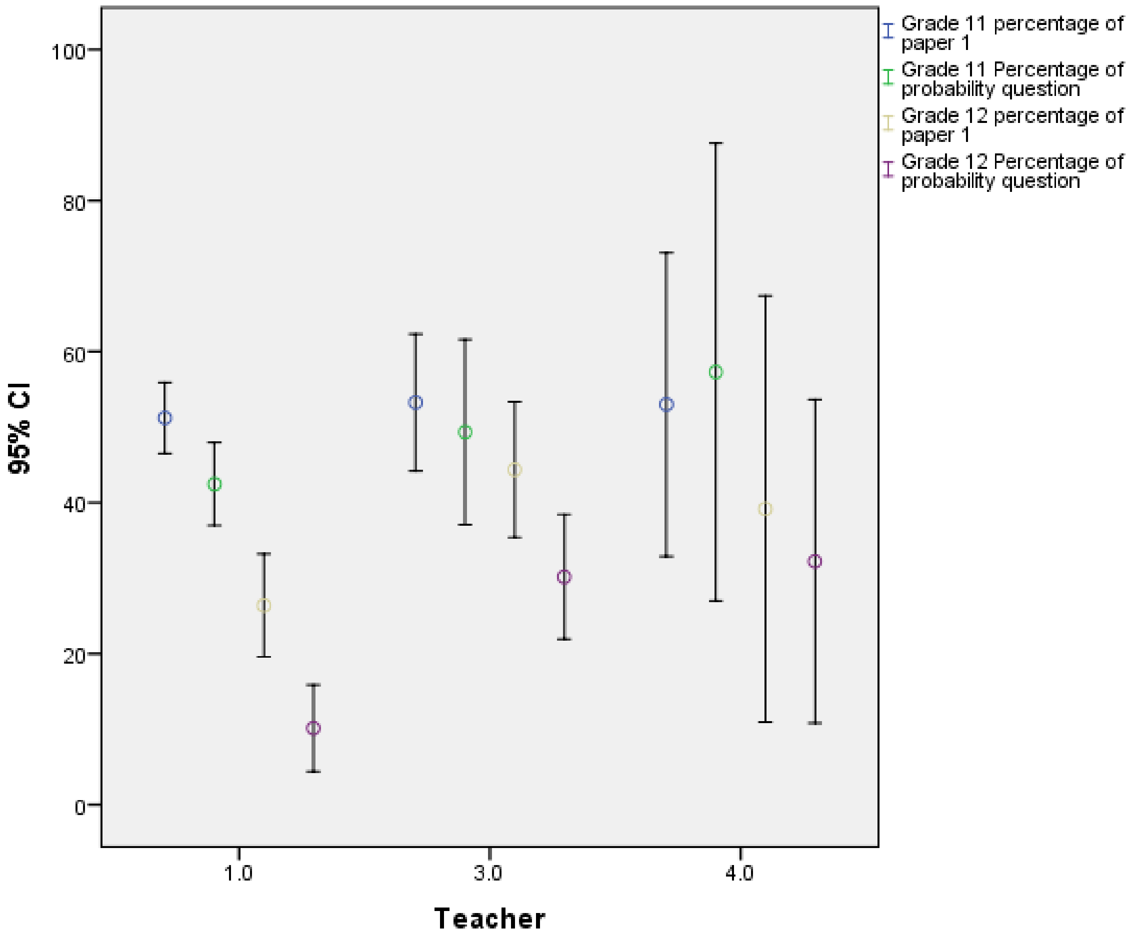
a. Uses Harmonic Mean Sample Size = 12.537.

b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.

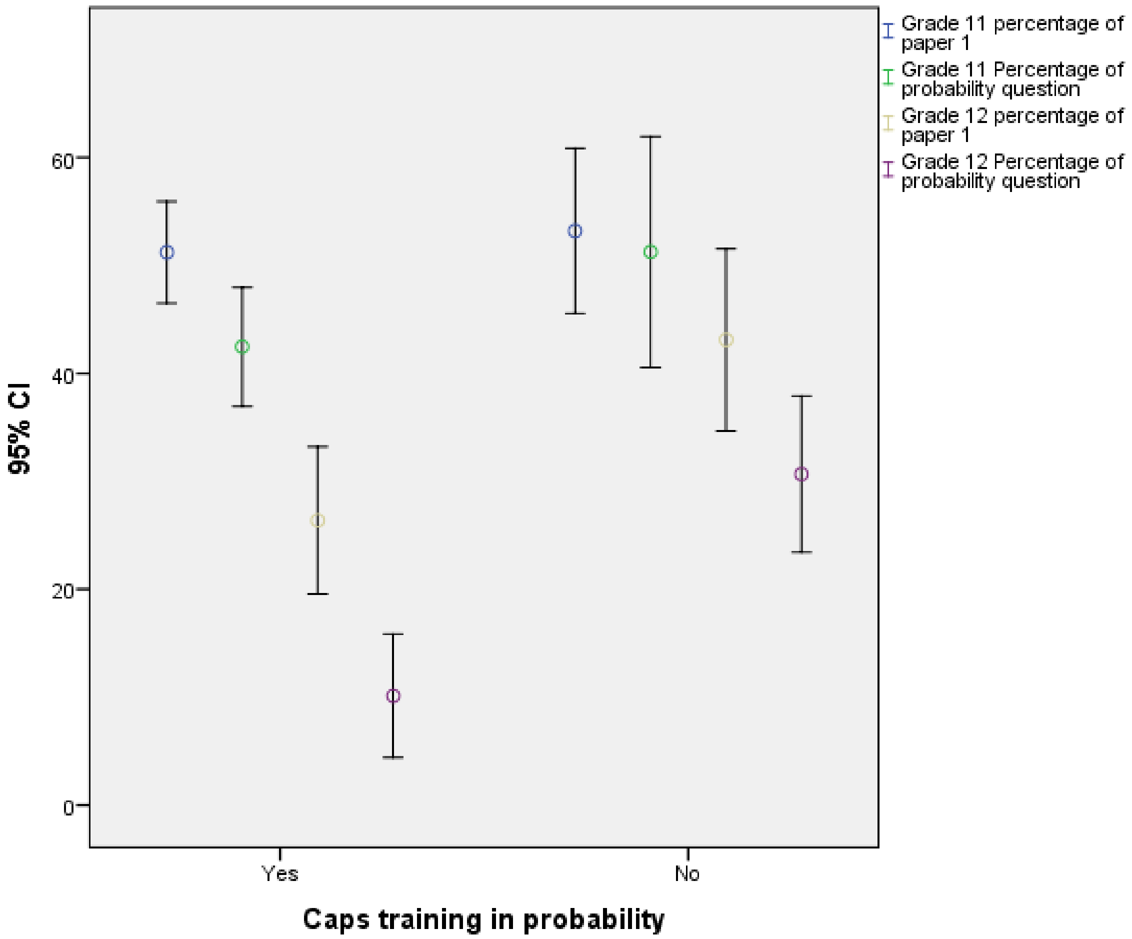
Graph


95% CI
60
40
20
0
School A
School B
School
Grade 11 percentage of paper 1
Grade 11 Percentage of probability question
Grade 12 percentage of paper 1
Grade 12 Percentage of probability question

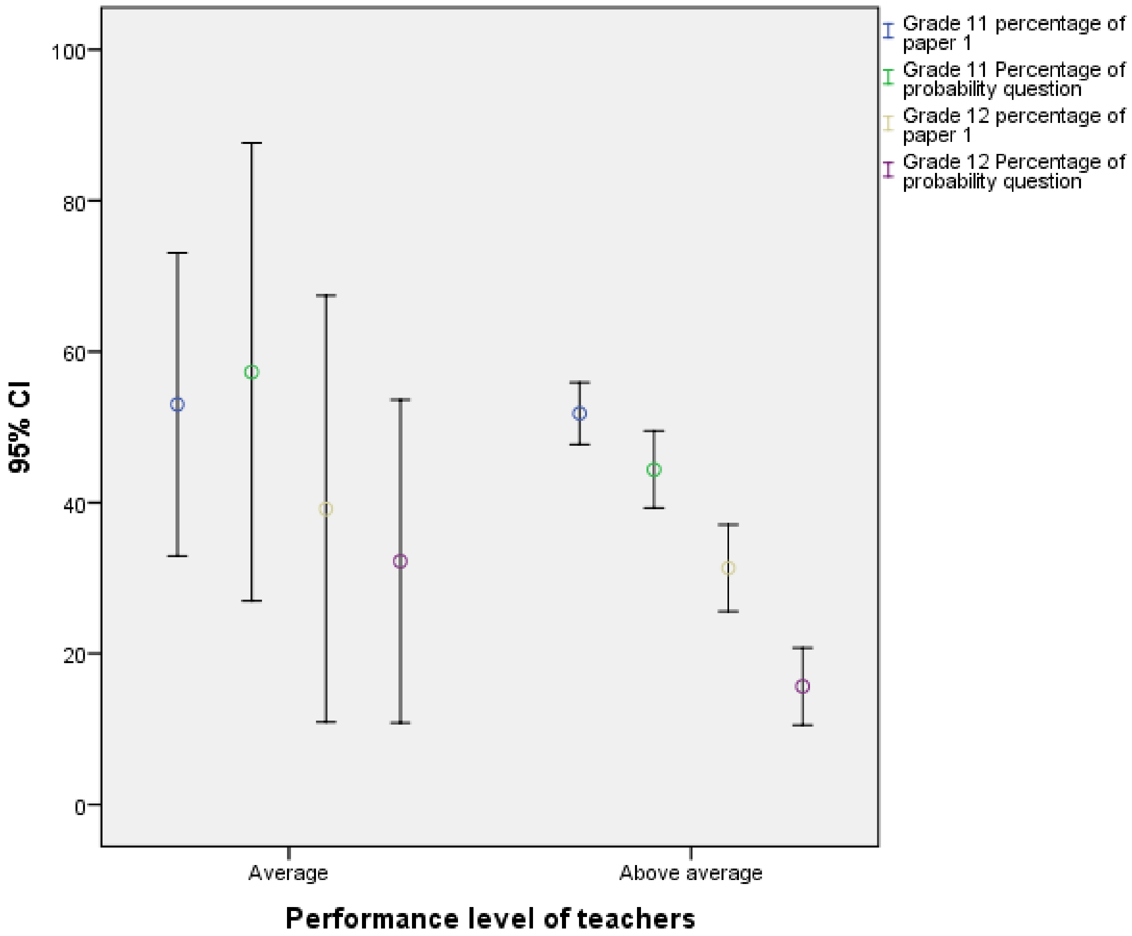
Graph


100
80
60
40
20
0
95% CI
1.0
3.0
4.0
Teacher
Grade 11 percentage of paper 1
Grade 11 Percentage of probability question
Grade 12 percentage of paper 1
Grade 12 Percentage of probability question

Graph

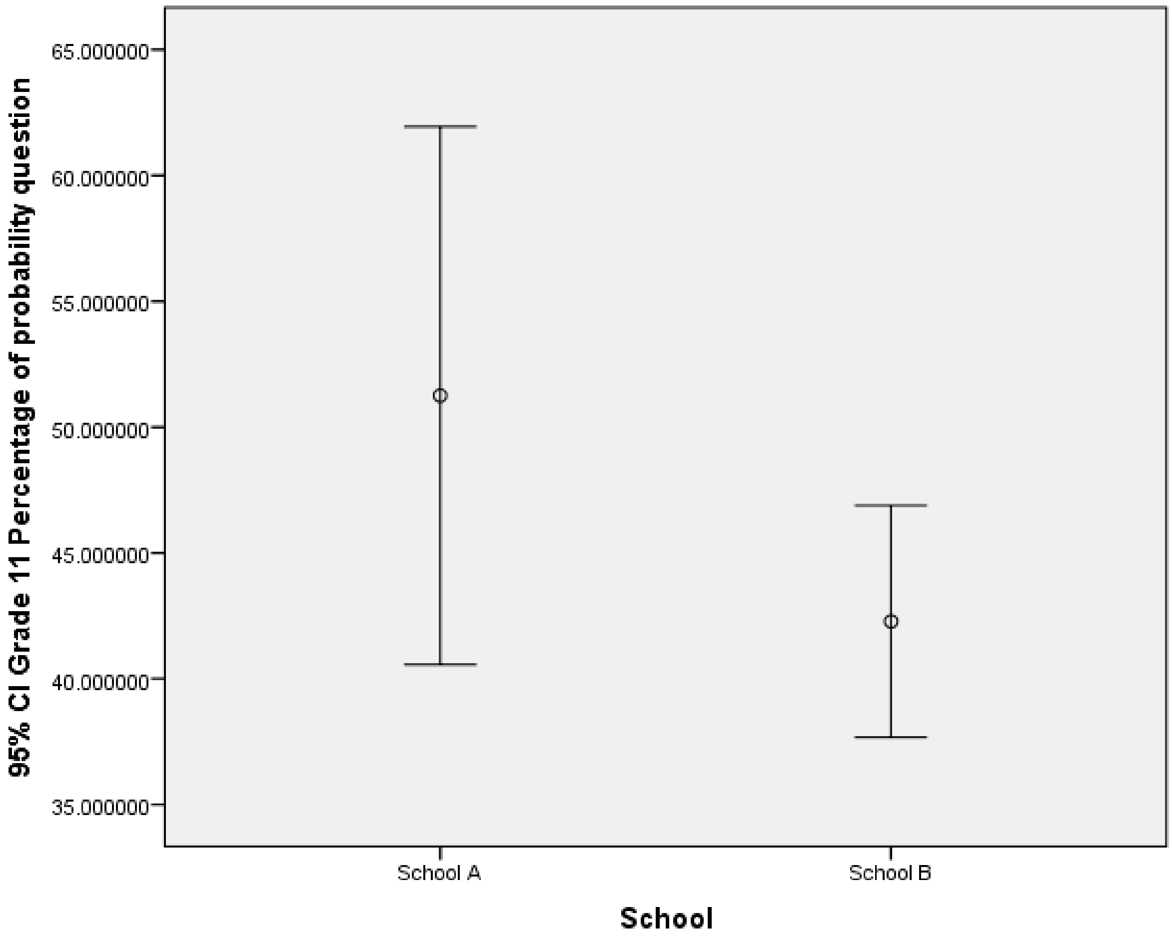

60
40
20
0
95% CI
Yes
No
Caps training in probability
Grade 11 percentage of paper 1
Grade 11 Percentage of probability question
Grade 12 percentage of paper 1
Grade 12 Percentage of probability question

Graph

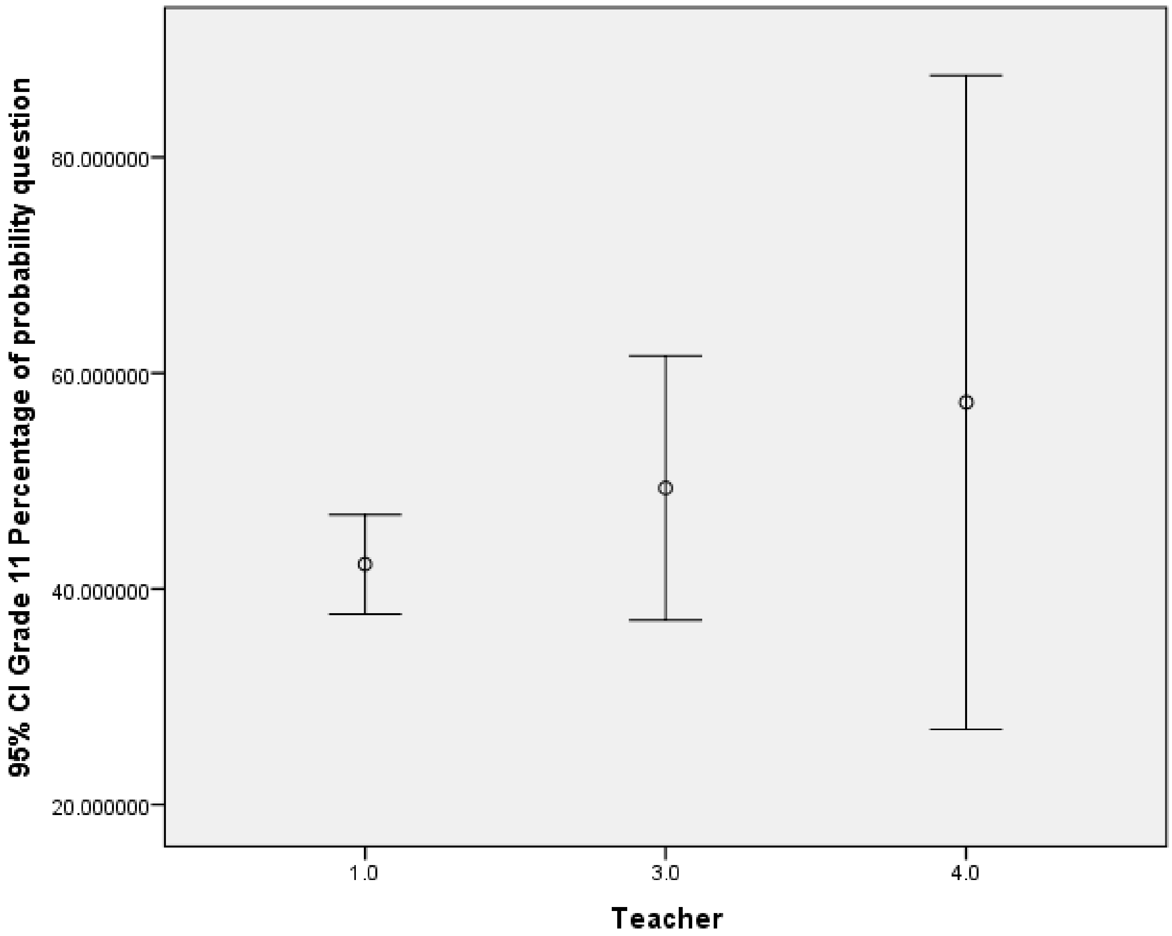


100
80
60
40
20
0
95% CI
Average
Above average
Performance level of teachers
Grade 11 percentage of paper 1
Grade 11 Percentage of probability question
Grade 12 percentage of paper 1
Grade 12 Percentage of probability question

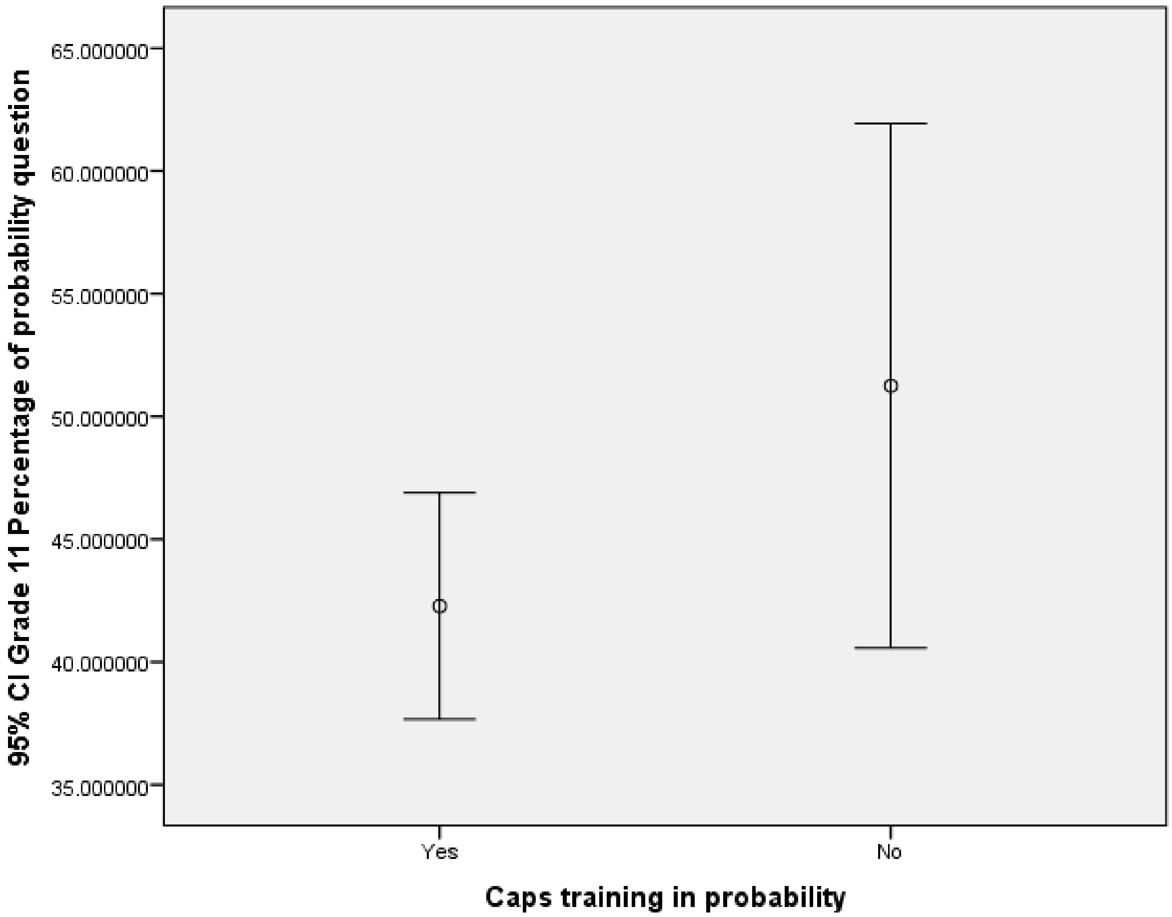
Graph


95% CI Grade 11 Percentage of probability question
65.000000
60.000000
55.000000
50.000000
45.000000
40.000000
35.000000
School A
School B
School

Graph

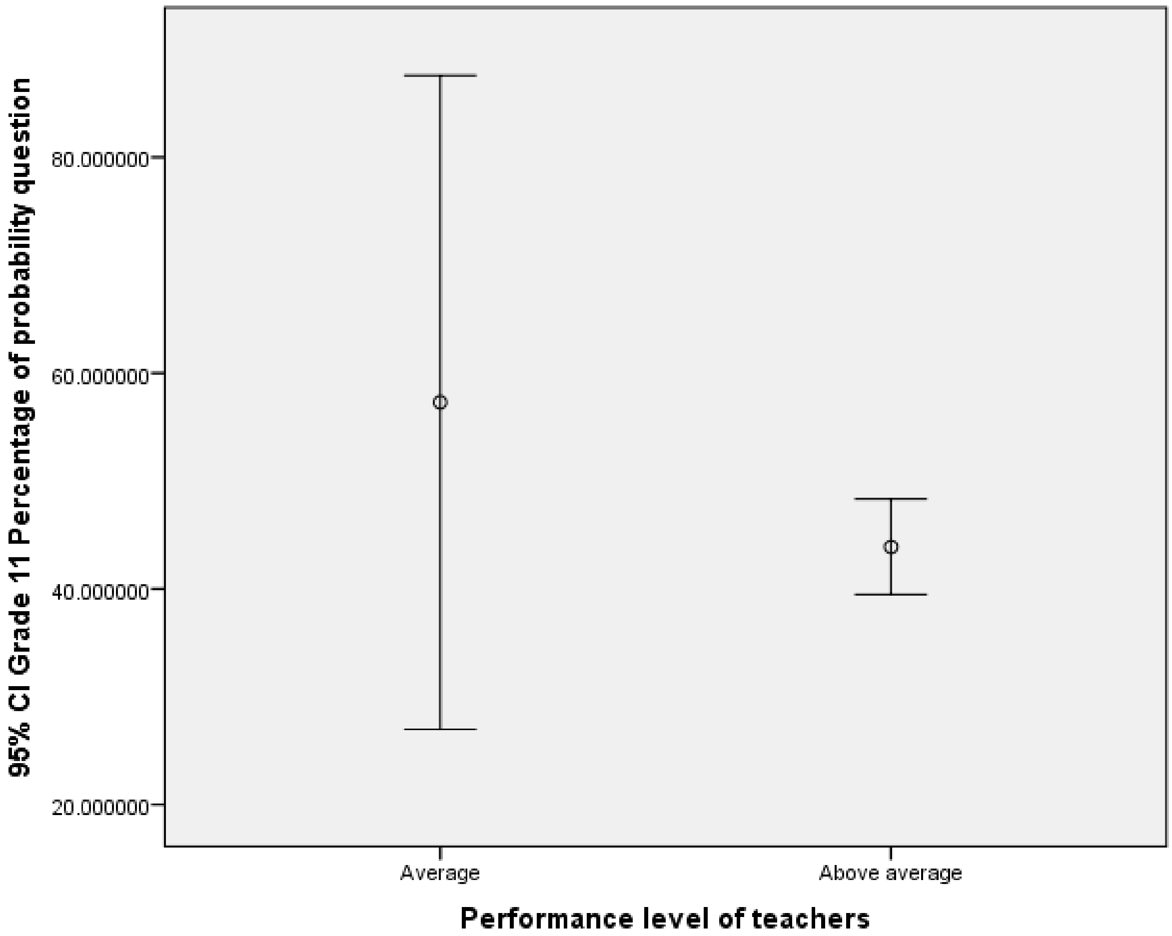

95% CI Grade 11 Percentage of probability question
80.000000
60.000000
40.000000
20.000000
1.0
3.0
4.0
Teacher

Graph

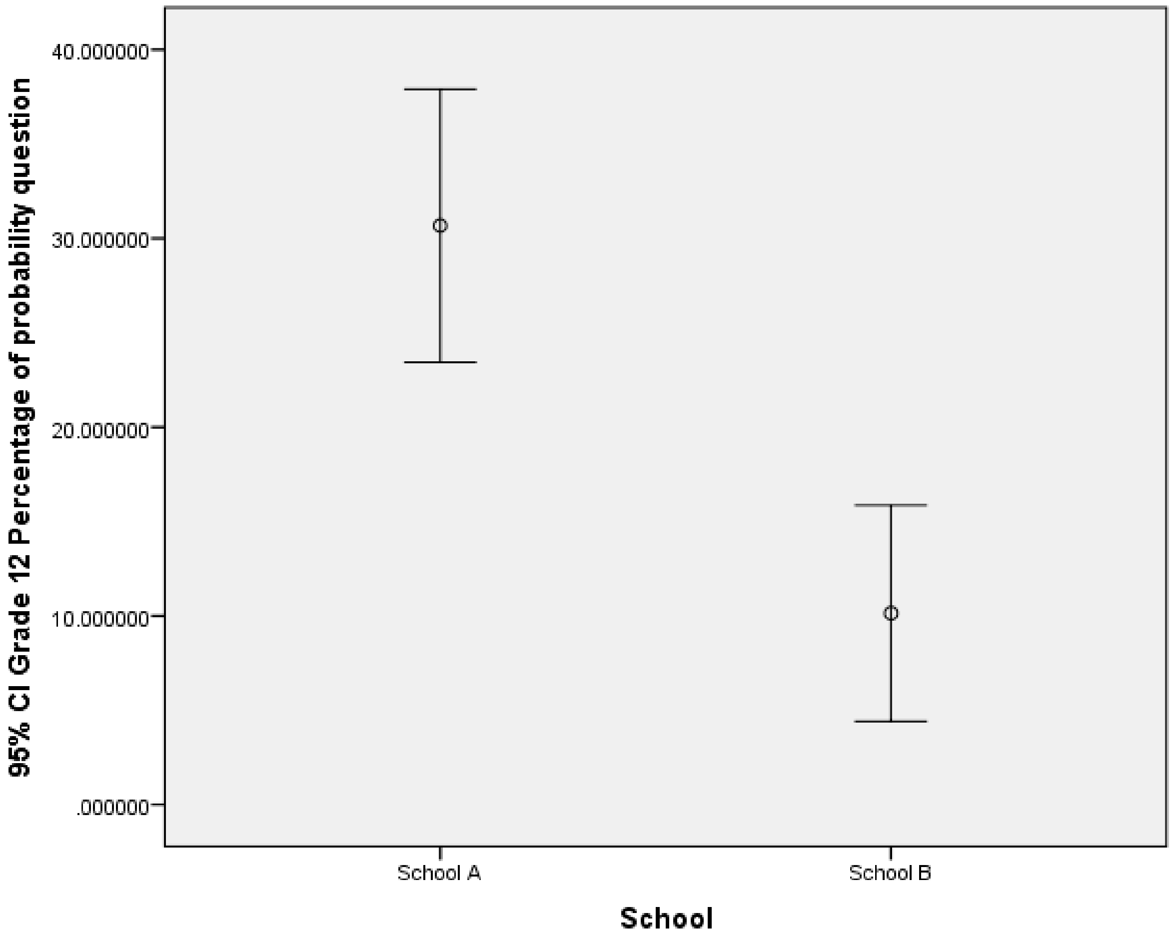

95% CI Grade 11 Percentage of probability question
65.000000
60.000000
55.000000
50.000000
45.000000
40.000000
35.000000
Yes
No
Caps training in probability

Graph

Relating teachers' content knowledge to learner performance in probability.

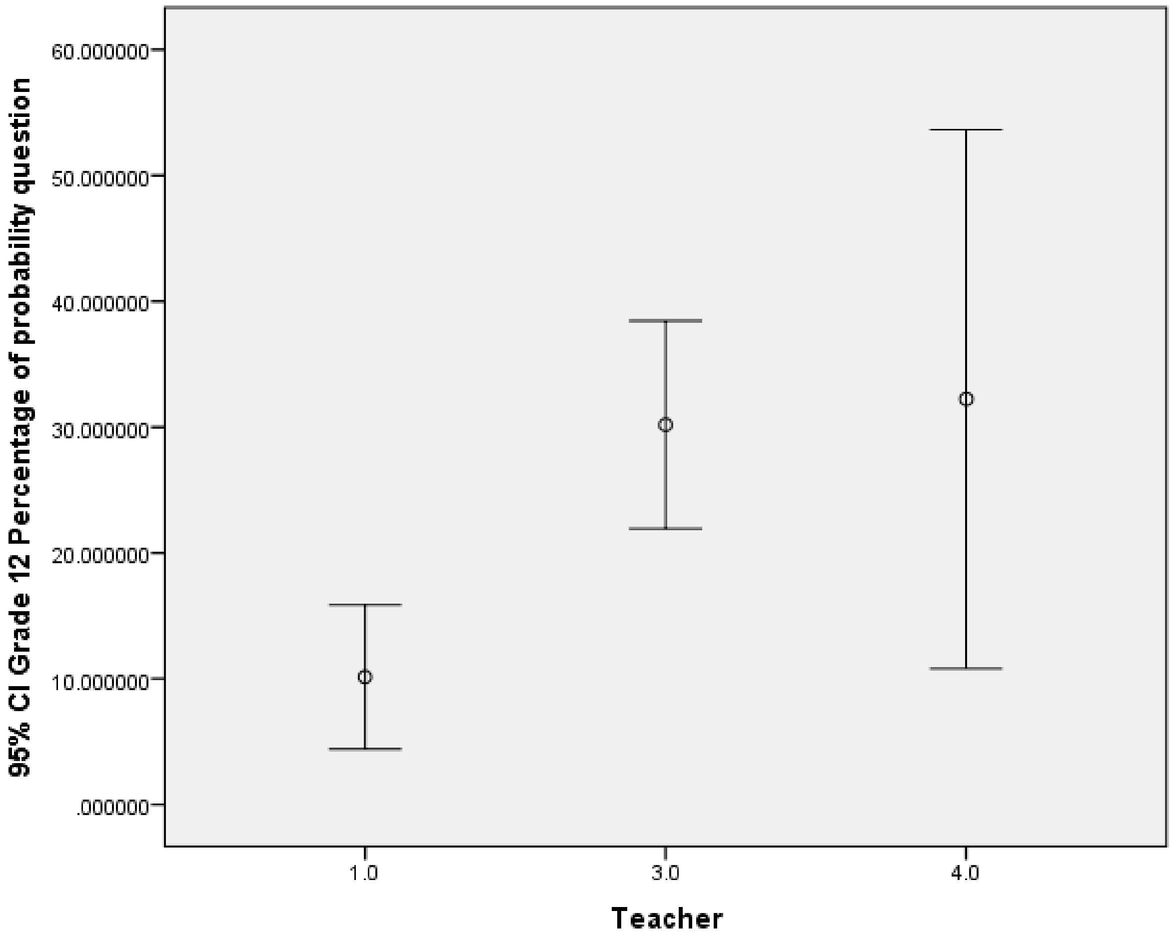


Graph


95% CI Grade 12 Percentage of probability question
40.000000
30.000000
20.000000
10.000000
.000000
School A
School B
School

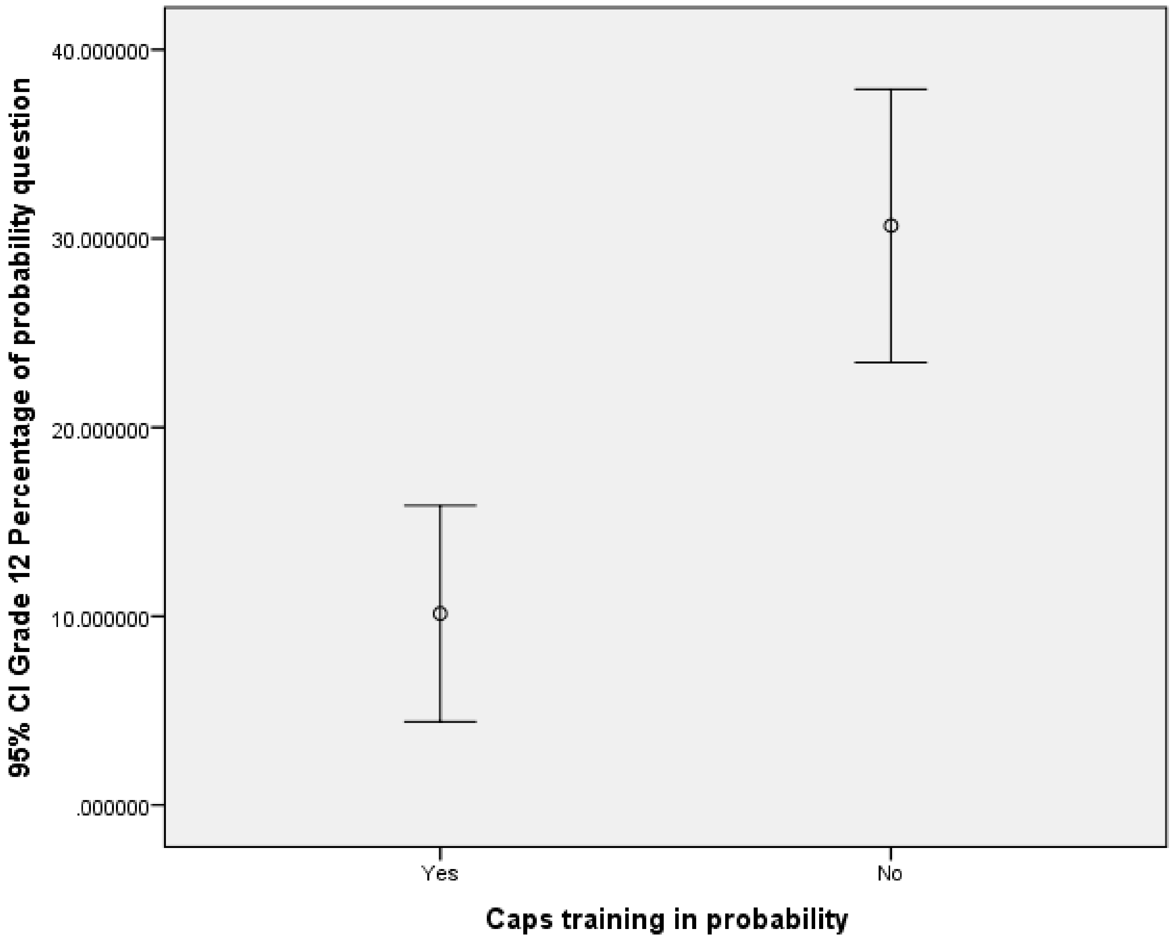
Graph

Relating teachers' content knowledge to learner performance in probability.



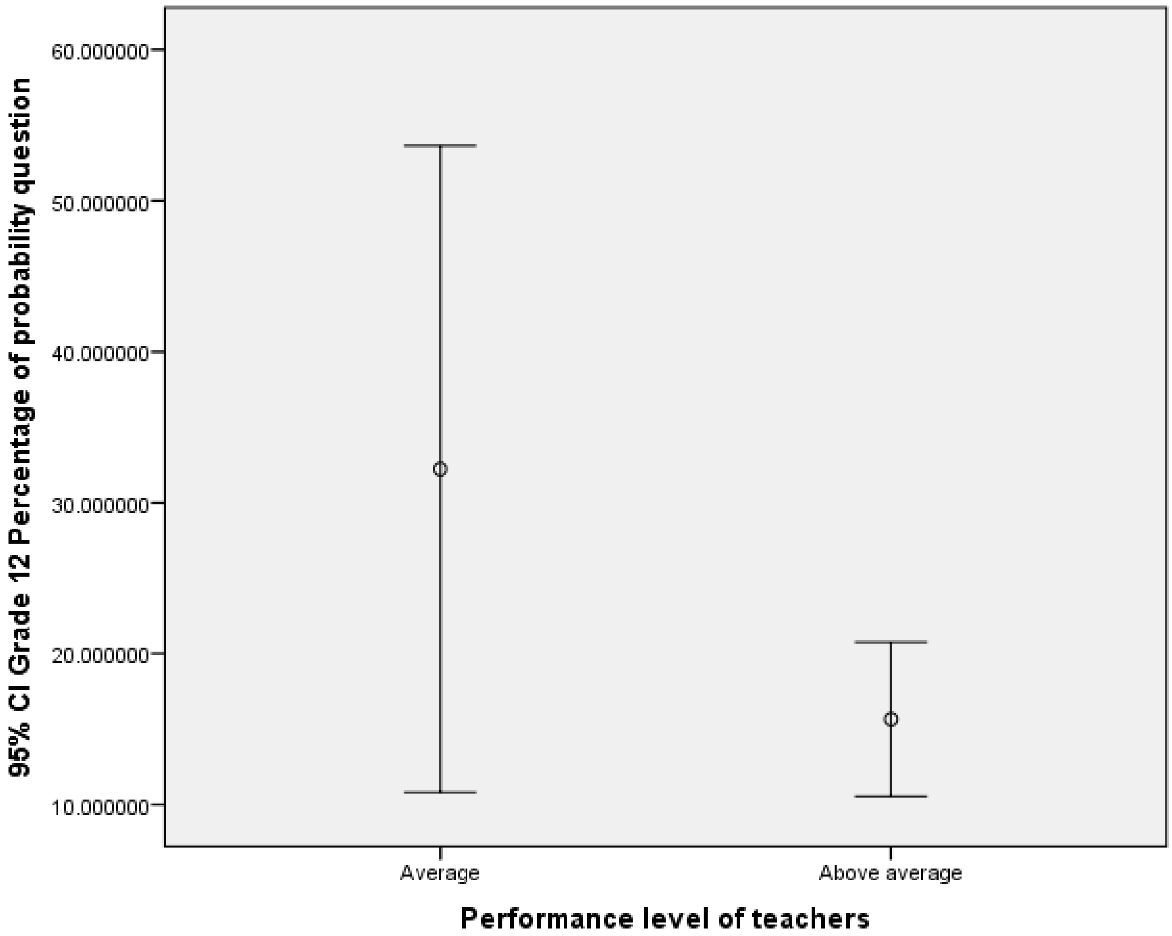
Graph

Relating teachers' content knowledge to learner performance in probability.



Graph

Relating teachers' content knowledge to learner performance in probability.



APPENDIX I

Rank scores: Grade 11 Final Examination

School	Teacher	Learner	Question 7.1.1	Question 7.1.2	Question 7.1.3	Question 7.1.4	Question 7.2.1	Question 7.2.2
B	T 3	L1	C	C	C	C	C	C
B	T 3	L2	C	NC	NC	SC	NC	NC
B	T 3	L3	C	C	C	C	SC	C
B	T 3	L4	C	NC	NC	NC	NC	C
B	T 3	L5	NC	SC	AC	NC	NC	C
B	T 3	L6	C	C	C	AC	NC	C
B	T 3	L7	C	C	NC	AC	NC	NC
B	T 3	L8	C	C	NC	SC	SC	NC
B	T 3	L9	C	C	C	SC	C	C
B	T 3	L10	C	C	A	NC	NC	NC
B	T 3	L11	C	C	NC	AC	AC	NC
B	T 3	L12	C	SC	NC	NC	NC	NC
B	T 3	L13	NC	SC	NC	SC	C	C
B	T 3	L14	C	AC	NC	NC	C	C
B	T 3	L15	C	C	NC	NC	NC	NC
B	T 3	L16	C	C	NC	NC	NC	NC
B	T 3	L17	C	AC	NC	AC	SC	NC
B	T 3	L18	C	SC	NC	SC	NC	NC
B	T 3	L19	C	C	C	SC	NC	NC
B	T 4	L20	NC	SC	NC	NC	NC	NC
B	T 4	L21	C	AC	C	SC	SC	NC
B	T 4	L22	C	C	C	SC	NC	NC
B	T 4	L23	C	SC	C	SC	NC	NC
B	T 4	L24	C	C	C	SC	C	C
B	T 4	L25	C	C	C	C	SC	NC

School	Teacher	Learner	Q7.1	Q7.2	Q7.3.1	Q7.3.2	Q8

A	T 1	L26	AC	NC	NC	NC	NC
A	T 1	L27	AC	C	NC	NC	C
A	T 1	L28	AC	NC	C	NC	NC
A	T 1	L29	AC	NC	C	NC	NC
A	T 1	L30	NC	NC	NC	NC	NC
A	T 1	L31	SC	NC	NC	NC	NC
A	T 1	L32	AC	NC	C	NC	NC
A	T 1	L33	AC	NC	NC	C	SC
A	T 1	L34	AC	NC	NC	C	C
A	T 1	L35	SC	NC	C	NC	C
School	**Teacher**	**Learner**	**Q7.1**	**Q7.2**	**Q7.3.1**	**Q7.3.2**	**Q8**
A	T 1	L36	AC	NC	NC	NC	SC
A	T 1	L37	AC	NC	NC	NC	C
A	T 1	L38	AC	NC	NC	NC	NC
A	T 1	L39	AC	NC	NC	NC	NC
A	T 1	L40	AC	NC	NC	NC	C
A	T 1	L41	AC	NC	NC	NC	NC
A	T 1	L42	AC	NC	NC	NC	NC
A	T 1	L43	AC	NC	NC	NC	C
A	T 1	L44	AC	C	NC	NC	NC
A	T 1	L45	C	C	C	C	C
A	T 1	L46	AC	NC	C	NC	NC
A	T 1	L47	AC	NC	NC	NC	C
A	T 1	L48	AC	NC	NC	NC	C
A	T 1	L49	AC	NC	A	C	C
A	T 1	L50	AC	NC	NC	NC	C
A	T 1	L51	AC	NC	NC	NC	NC
A	T 1	L52	AC	NC	NC	NC	NC
A	T 1	L53	AC	NC	C	NC	C
A	T 1	L54	AC	C	NC	NC	C
A	T 1	L55	AC	NC	NC	NC	C

A	T 1	L56	AC	NC	NC	NC	NC
A	T 1	L57	NC	NC	NC	NC	NC
A	T 1	L58	SC	C	NC	NC	C
A	T 1	L59	SC	NC	NC	NC	SC
A	T 1	L60	AC	NC	NC	NC	NC
A	T 1	L61	AC	SC	NC	NC	C
A	T 1	L62	AC	NC	NC	NC	C
A	T 1	L63	AC	NC	NC	NC	C
A	T 1	L64	AC	NC	C	C	NC
A	T 1	L65	AC	C	NC	NC	C
A	T 1	L66	SC	NC	NC	NC	NC
A	T 1	L67	AC	NC	SC	NC	AC
A	T 1	L68	AC	NC	NC	NC	NC
A	T 1	L69	SC	NC	NC	NC	C
A	T 1	L70	AC	NC	NC	NC	C
A	T 1	L71	AC	NC	NC	NC	C
A	T 1	L72	AC	NC	C	NC	C
A	T 1	L73	AC	NC	C	NC	C
A	T 1	L74	AC	NC	C	NC	NC
A	T 1	L75	AC	NC	C	NC	NC
A	T 1	L76	AC	NC	NC	NC	NC
School	**Teacher**	**Learner**	**Q7.1**	**Q7.2**	**Q7.3.1**	**Q7.3.2**	**Q8**
A	T 1	L77	AC	C	C	NC	C
A	T 1	L78	AC	NC	NC	NC	C
A	T 1	L79	AC	NC	NC	NC	AC
A	T 1	L80	AC	NC	NC	NC	NC
A	T 1	L81	AC	NC	NC	NC	C
A	T 1	L82	AC	NC	C	C	NC
A	T 1	L83	AC	NC	NC	NC	NC
A	T 1	L84	SC	NC	NC	NC	C
A	T 1	L85	AC	NC	NC	NC	NC

A	T 1	L86	AC	NC	NC	NC	C
A	T 1	L87	AC	NC	NC	NC	C
A	T 1	L88	AC	NC	NC	NC	C
A	T 1	L89	AC	NC	NC	NC	NC

Rank codes: Grade 12 Preparatory Examination

School	Teacher	Learner	Question 13.1	Question 13.2	Question 13.3.1	Question 13.3.2	Question 13.3.3
B	T 3	L1	C	C	C	NC	NC
B	T 3	L2	C	NC	C	NC	NC
B	T 3	L3	C	NC	C	AC	C
B	T 3	L4	NC	C	SC	NC	NC
B	T 3	L5	C	NC	SC	AC	NC
B	T 3	L6	SC	NC	NC	NC	NC
B	T 3	L7	AC	NC	NC	NC	NC
B	T 3	L8	NC	SC	C	NC	NC
B	T 3	L9	C	NC	C	S	C
B	T 3	L10	NC	NC	NC	NC	NC
B	T 3	L11	C	NC	SC	S	NC
B	T 3	L12	AC	NC	NC	AC	NC
B	T 3	L13	C	NC	NC	NC	AC
B	T 3	L14	C	NC	C	NC	NC
B	T 3	L15	AC	NC	NC	NC	NC
B	T 3	L16	AC	NC	SC	NC	NC
B	T 3	L17	C	NC	SC	NC	NC
B	T 3	L18	NC	C	NC	NC	NC
B	T 3	L19	C	NC	NC	NC	AC
B	T 4	L21	C	AC	SC	NC	NC
B	T 4	L22	C	C	SC	NC	NC
B	T 4	L23	NC	NC	NC	NC	NC
B	T 4	L24	C	NC	NC	NC	NC
B	T 4	L25	C	AC	C	NC	NC
A	T 1	L26	NC	NC	NC	NC	NC

A	T 1	L40	NC	NC	NC	NC	C
A	T 1	L42	SC	NC	NC	NC	NC
A	T 1	L46	C	NC	NC	NC	NC
A	T 1	L48	C	C	C	C	C
A	T 1	L53	C	SC	C	SC	NC
A	T 1	L62	NC	NC	NC	NC	NC
A	T 1	L67	NC	NC	NC	NC	NC
A	T 1	L87	C	SC	C	AC	C
C	T 2	L90	C	C	NC	AC	C
C	T 2	L94	C	C	C	AC	NC
C	T 2	L95	C	C	NC	S	C
C	T 2	L96	C	NC	NC	NC	NC
C	T 2	L97	C	NC	NC	S	NC
C	T 2	L98	C	C	C	C	NC
C	T 2	L99	NC	NC	C	SC	C
C	T 2	L100	C	NC	NC	SC	NC
C	T 2	L101	C	SC	C	C	C
C	T 2	L102	NC	SC	NC	NC	NC
C	T 2	L103	C	SC	NC	S	C
C	T 2	L104	AC	C	C	C	C
C	T 2	L105	C	NC	C	S	C
C	T 2	L106	NC	SC	NC	C	C
C	T 2	L107	C	NC	C	NC	C
C	T 2	L108	NC	C	C	S	C
C	T 2	L109	NC	NC	C	C	NC
C	T 2	L110	NC	NC	NC	NC	NC
C	T 2	L112	C	NC	NC	NC	NC
C	T 5	L113	DNA	DNA	DNA	DNA	DNA
C	T 5	L114	NC	NC	NC	NC	NC
C	T 5	L115	NC	NC	NC	NC	NC
C	T 5	L116	NC	NC	NC	NC	NC
C	T 5	L117	DNA	DNA	DNA	DNA	DNA
C	T 5	L118	DNA	DNA	DNA	DNA	DNA
C	T 5	L119	DNA	DNA	DNA	DNA	DNA

C	T 5	L120	DNA	DNA	DNA	DNA	DNA
C	T 5	L121	NC	NC	NC	NC	NC
C	T 5	L122	DNA	DNA	DNA	DNA	DNA
C	T 5	L123	DNA	DNA	DNA	DNA	DNA
C	T 5	L124	C	NC	NC	NC	NC
C	T 5	L125	DNA	DNA	DNA	DNA	DNA
C	T 5	L126	DNA	DNA	DNA	DNA	DNA
C	T 5	L127	DNA	DNA	DNA	DNA	DNA
C	T 5	L128	NC	NC	NC	NC	NC
C	T 5	L129	DNA	DNA	DNA	DNA	DNA
C	T 5	L130	C	NC	NC	NC	NC
C	T 5	L131	DNA	DNA	DNA	DNA	DNA
C	T 5	L132	C	NC	NC	NC	NC
C	T 5	L133	DNA	DNA	DNA	DNA	DNA
C	T 5	L134	DNA	DNA	DNA	DNA	DNA
C	T 5	L135	NC	NC	NC	NC	NC