Investigation into mathematics instruction for learners with learning difficulties

by

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Summary

Teaching is a complex endeavour. Mathematics teachers have to take many aspects into account when teaching in order to assist all their learners to reach their full potential. They need to be aware and know their learners' capabilities, over and above their knowledge of the subject. Mathematics teachers who teach learners with learning difficulties need to know the intrinsic barriers to learning experienced by their learners. Keeping in mind the different intrinsic barriers to learning, teachers should also have knowledge of and apply different teaching approaches that will assist learners with learning difficulties to overcome such barriers to learning. In South Africa, research conducted on the instructional practice of mathematics teachers in special needs schools has so far been very limited.

The purpose of this study is to investigate the instructional practice and knowledge of teachers who teach mathematics in schools for learners with learning difficulties who experience intrinsic barriers to learning. Through this study I wanted to determine how mathematics teachers currently teach learners with learning difficulties. To accomplish this, a case study was conducted to investigate four mathematics teachers' basic knowledge of the types of intrinsic barriers to learning experienced by their learners, and their choices regarding the teaching approaches that underlie their instructional practice. This led to an investigation into how the chosen teaching approaches actually manifest in their classrooms. The four participants were selected from special needs schools for learners with learning difficulties. The data was collected by way of questionnaires and observations.

This study revealed that the number of years of teaching experience and training in special needs education had only a slight impact on how well teachers were informed on their learners' intrinsic barriers to learning. However, in terms of the teaching approaches, teaching experience and formal training in special needs education did seem to influence the use of teaching approaches. Hence, the more experience and training teachers had in special needs education, the more successful they were at finding the best teaching approaches to help learners to overcome their barriers to learning.

Due to the small sample used, the results from this study cannot be generalised. However, I hope that the findings will contribute to student-teacher training and in-service teacher training in both mainstream and special needs schools. Future research could possibly build on this study by investigating how student-teachers could be more effectively prepared for the challenges of both inclusive and special needs education.

Key words: mathematics; learners with learning difficulties; teachers; intrinsic barriers to learning; teaching approaches; special needs education; knowledge of the learner; teacher knowledge

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List of abbreviations

ADHD	Attention-deficit hyperactivity disorder
CRA	Concrete-representational-abstract
KCS	Knowledge of content and student
KCT	Knowledge of content and teaching
LCD	Lowest common denominator
LSEN	Learners with special educational needs
PCK	Pedagogical content knowledge
PGCE	Post graduate certificate in education
ZPD	Zone of proximal development

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Chapter 1

Introduction and contextualisation

1.1 Introduction

The type of knowledge that mathematics teachers need is complex (Hill, Ball & Schilling, 2008), even more so when they teach learners with learning difficulties, in either schools for learners with special educational needs (LSEN) or inclusive mainstream schools (Allsopp, Kyger & Lovin, 2007). For this reason the mathematics teacher's knowledge of the learner was chosen as the focus of this study. The government promotes inclusive education and wants LSEN schools to serve as resource centres for inclusive mainstream schools. Therefore the purpose of this study was to investigate, by way of a multiple-case study, the extent to which teachers are informed on the different types of intrinsic barriers to learning mathematics and the teaching approaches that can be applied to assist learners to overcome these barriers to learning. The extent to which teachers' knowledge of teaching was aligned with their actual instructional practice was also investigated.

In South Africa, schooling is divided into mainstream education and special needs education. Special needs schools are a remnant from the apartheid era. Special needs education accommodates learners with many different kinds of disability, inter alia physical, visual, hearing, mental and learning difficulties. Since 1994 the government has been committed to building an inclusive education system with the intention of accommodating as many learners as possible, including those experiencing different kinds of barriers to learning, in inclusive mainstream schools (Department of Education, 2001). Furthermore, the government intends that special needs schools will become resource centres for inclusive mainstream schools in their areas. Resources include the provision of professional expertise and support in terms of the curriculum, assessment and instructional practice (Department of Education, 2001). This implies that teachers in special needs schools must have specialised knowledge of how to adapt the curriculum, how to adapt their teaching approaches in order to teach learners with special needs, and how to assess them so as to make the outcomes of teaching effective.

Effective teaching consists of different elements. One of the key elements of effective teaching is the teacher's pedagogical content knowledge (PCK). Shulman (1986) was the first to introduce this term and others such as Ball, Thames and Phelps (2008) and Hill et al. (2008) expanded on his work. PCK encompasses the types of knowledge that teachers should have in order to teach effectively. One of the subcategories of PCK is knowledge of content and learner (Hill et al., 2008). Teachers need to really know their learners, including those with barriers to learning, and should be sensitive to their diverse needs (Department of Education, 2000). Knowing the learners includes having knowledge of the intrinsic barriers to learning that they experience, as well as knowing how to facilitate learning in order to help them to overcome those barriers. Allsopp et al. (2007) and other researchers (Dednam, 2011; Lerner & Johns, 2012; Mercer, Mercer & Pullen, 2014; Miller & Mercer, 1997; Miller & Hudson, 2006) have studied the intrinsic barriers to learning and the different ways in which teaching can be approached to assist learners to overcome them.

In South Africa very little research has been done on the role that teacher knowledge plays in special needs education. Landsberg's (2011) book found in the University of Pretoria library deals with inclusive education in South Africa and also discusses intrinsic and extrinsic barriers to learning. A search of the Eric data-base produced no other research on intrinsic barriers to learning and the teaching approaches used in special needs mathematics education in South Africa. Internationally, research on intrinsic barriers to learning and teaching approaches applied in special needs education to assist learners in overcoming these barriers was done mainly by Allsopp et al. (2007), Mercer et al. (2014), Lerner and Johns (2012) and Miller and Hudson (2006). More research was found that focused on a single intrinsic barrier, such as maths anxiety, memory difficulties or attention difficulties. To address this gap in the relevant literature, this study investigated a small group of special needs mathematics teachers' knowledge of intrinsic barriers to learning and the teaching approaches that they used to assist learners with learning difficulties to overcome those barriers.

1.2 Rationale

I majored in Mathematics and Remedial Education, and subsequently completed an honours degree in Learning Support. After teaching in a school for learners with learning difficulties for three years, I became a mathematics tutor for Grade 4 to 12 learners. The learners that I tutor complain that they do not understand mathematics and that their teachers make no special effort to help them to overcome their barriers to learning. I asked myself the following questions: What makes one teacher more effective than another in teaching mathematics, and do these effective teachers have special skills or knowledge that others lack? This led to the development of my particular interest in the instructional practice of teachers who teach mathematics to learners with learning difficulties.

The White Paper 6 on special needs education (Department of Education, 2001) emphasises the importance of special needs schools as resource centres for inclusive schools. This implies that teachers at special needs schools have specialised knowledge that mainstream schools lack. A question that should be asked relates to whether or not there should be inclusive schools and what special knowledge and skills special needs mathematics teachers possess to help learners to develop to their full potential. I sincerely hope that the findings of this study will be of value to both special needs and inclusive education in South Africa, as well as to policy makers and those responsible for training future teachers.

1.3 Problem statement

Learners experiencing learning difficulties often have multiple intrinsic barriers to learning. Those who teach these learners are confronted with these multiple intrinsic barriers to learning every day. Therefore, teachers in both LSEN and inclusive mainstream schools require specialised knowledge and skills in order to assist learners to overcome barriers to learning and to develop to their full potential. This study therefore aims to investigate the approaches used by mathematics teachers when teaching learners with learning difficulties, as well as their knowledge of the various types of intrinsic barriers to learning.

1.4 The purpose of the study

The purpose of this study is to investigate teaching approaches that may be effectively used by mathematics teachers in schools for learners with learning difficulties. To accomplish this, an in-depth study was conducted to determine the extent of teachers' knowledge of the different types of intrinsic barriers that affect learners' ability to learn mathematics, as well as their knowledge of the teaching approaches that should underlie their instructional practice. This led to an investigation into teachers' teaching approaches in their classrooms.

1.5 Research questions

The following primary and secondary research questions were formulated with the rationale, problem statement, and purpose taken into consideration.

Primary research question

How do mathematics teachers facilitate learning to help learners with learning difficulties to overcome their intrinsic barriers to learning?

Secondary research questions

1. Which different types of intrinsic barriers to learning are mathematics teachers who teach learners with learning difficulties aware of?
2. What are the teaching approaches used by mathematics teachers in order to assist learners with learning difficulties to overcome their intrinsic barriers to learning?
3. How do mathematics teachers think they should facilitate learning to help learners with learning difficulties to overcome their intrinsic barriers to learning?

1.6 Methodological considerations

The main focus of this study was the knowledge and instructional practice of mathematics teachers. The four participants were teachers at two LSEN primary schools in Pretoria that cater specifically for learners with learning difficulties. Three of them taught in the Intermediate Phase, while the fourth taught in the Senior Phase.

In order to answer the research questions, a qualitative research approach was followed. A multiple-case study was used to obtain an in-depth understanding of the teaching approaches used when teaching learners with learning difficulties. To collect the data required, two questionnaires (pre- and post-observation) were handed out to four participants and four lessons presented by each of the participants were observed using an observation schedule. The pre-observation questionnaire was compiled to investigate the knowledge of mathematics teachers concerning the intrinsic barriers to learning. The post-observation questionnaire was compiled to investigate how mathematics teachers think they should teach in order for them to assist learners to overcome their intrinsic barriers to learning. Data analysis for both the intrinsic barriers to learning mathematics and the teaching approaches was done by using categories identified from literature and set out in the conceptual framework (Figure 2.1).

1.7 Definition of terms

- Learning difficulty: The term learning disability is most frequently used in the literature. In this study preference is given to the term learning difficulty because of the negative connotation of the word disability, which implies an inability to do something. The word difficulty implies that although something might be difficult to do, it is not always impossible.
- Special needs education: This refers to schools that teach learners with different types of disability, such as learning, visual, hearing, physical and mental disabilities.

1.8 Possible contribution of the study

The government favours inclusive education. This implies the inclusion of not only learners of different races, classes, or religions, but also of learners with different mental and physical abilities. According to government policy, LSEN schools should serve as resource centres for inclusive mainstream schools in their areas (Department of Education, 2001). If special needs schools have resources and teachers with specialised skills and knowledge, should the goal be to have inclusive mainstream schools, or should there be special needs schools and mainstream schools? This study is an attempt to make a small contribution to both special needs education in mathematics and inclusive education in South Africa by using the knowledge that already exists in this field in other countries. It may also be useful for the training of mathematics teachers. The focus cannot be only on what is currently happening in the profession, but attention should also be given to the need to equip mathematics students aspiring to become mathematics teachers with the skills required to develop their learners to their full potential and to train and equip them to face the many challenges of either inclusive or special needs education.

1.9 The structure of the dissertation

The dissertation consists of five chapters. Chapter 1 contains the introduction and contextualisation, as summarised above. Chapter 2, which consists of the literature review and conceptual framework, provides an in-depth analysis and synthesis of the relevant literature and explains the conceptual framework on which this study is based. In Chapter 3, the methodology used in this study is explained. The selection of the participants, data collection and documentation, and data analysis are discussed, as well as the trustworthiness of the study and ethical considerations. Chapter 4 details the analysis of the findings based on the data obtained by way of the questionnaires and classroom observations. The findings are discussed in the light of the research questions, literature review and conceptual framework. Chapter 5 contains the conclusions that were arrived at and a discussion of their implications, as well as a reflection on the study, conclusions, recommendations and limitations of the study.

Chapter 2

Literature review and conceptual framework

2.1 Introduction

Teacher knowledge has been the subject of much research over the years, which is an indication of its importance in teaching. According to Pon (2001), a teacher's knowledge of the learner is a powerful predictor of a learner's success in mathematics. Researchers agree that successful mathematics teachers need to make a connection between subject content, the learners and pedagogy, which takes place within a specific context (Hill, Rowan & Ball, 2005; Kilpatrick, Swafford & Findell, 2001; Leinwand & Burill, 2001; Adler & Pillay, 2007). Shulman (1986) was the first person to coin the term pedagogical content knowledge (PCK), which links content with pedagogy. Teacher knowledge includes knowledge about the subject content, PCK and knowledge of the curriculum (Shulman, 1986). Hill et al. (2008) and Ball et al. (2008) elaborated on and refined Shulman's (1986) classification. They divided knowledge of teaching into two main categories, namely subject content knowledge and PCK. PCK incorporates the curricular knowledge referred to by Shulman (1986) and is further refined to include knowledge of content and students (KCS), and knowledge of content and teaching (KCT).

This study focuses on special needs education, especially on the teaching of mathematics to learners with learning difficulties. Greer and Meyen (2009), as well as Allsopp et al. (2007), maintain that special needs mathematics educators need additional knowledge. According to these researchers, this additional knowledge includes increased subject content knowledge, knowledge of barriers to learning, and the knowledge required to be able to refine instructional planning so as to be able to translate the curriculum into a workable curriculum for learners with learning difficulties. Therefore teachers need to understand the learners with learning difficulties and the barriers to learning they are faced with. Those who teach learners with learning difficulties should recognise each learner's individual difference and should be able to adapt their teaching in ways that can assist each learner with learning difficulties to develop to his or her full potential (Mercer et al., 2014).

2.2 Teaching learners with learning difficulties

Learning difficulties is a broad concept which includes a variety of difficulties. The term learning difficulties coined by Kirk in 1963 (Lerner & Johns, 2009), is used to describe a neurobiological disorder and was defined by the National Advisory Committee on Handicapped Children and the Individuals with Disabilities Education Improvement Act (IDEA-2004) (Department of Education, 2006, p. 46757) as: "… a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculation, including conditions such as perceptual disabilities, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia."

Mercer et al. (2014) postulate that special needs educators are confronted with learners with a variety of learning difficulties ranging from mild to severe. Learners with learning difficulties experience intrinsic barriers to learning that can impede their understanding or application of mathematical concepts. These barriers to learning mathematics include difficulties in respect of memory, attention span, processing, language and metacognitive thinking, as well as maths anxiety, learnt helplessness and passivity (Allsopp et al., 2007; Miller & Mercer, 1997; Dednam, 2011). Mathematics teachers and their learners with learning difficulties are confronted with these barriers every day in the classroom. Teachers simply cannot rely on only one type of teaching strategy and teaching approach if they want their learners with learning difficulties to reach their full potential.

Two teaching strategies used in special needs education that are often discussed and researched are explicit and implicit instruction (Mercer et al., 2014; Lerner & Johns, 2012; Allsopp et al., 2007). In the case of explicit instruction, the teacher provides an explanation or demonstrates a skill or concept in different contexts and then provides many opportunities for independent practise to ensure the eventual mastery of that particular skill or concept (Mercer et al., 2014). However, this form of instruction is associated with behaviourism as the teacher takes most of the responsibility for learning through teaching (Mercer et al., 2014). By contrast, implicit instruction emphasises the thinking processes that are involved in learning (Mercer et al., 2014).

Although these two teaching strategies are often seen as mutually exclusive, Mercer et al. (2014) suggest that both should be used to teach learners with learning difficulties. According to Mercer et al. (2014) explicit instruction can be equated with direct instruction (Table 2.1). Mercer et al. (2014) and Allsopp et al. (2007) believe that when learning is difficult for learners, the teacher should provide extensive support.

Table 2.1: The continuum of explicit and implicit instructional practice
(Adapted from Mercer et al., 2014; Pon, 2001)

Explicit instruction	**Interactive instruction**	**Implicit instruction**
Direct teacher assistance	Balance between direct and non-direct teacher assistance (collaborative)	Non-direct teacher assistance
Teacher regulation of learning	Shared regulation of learning	Learner regulation of learning
Emphasis on both complex and prerequisite skills including basic facts	Use of scaffolding to assist learners in acquiring basics and beginning to explore mathematical relationships	Emphasis on mathematical concepts, relationships and applications
Directed discovery	Guided discovery	Self-discovery
Behavioural		Holistic/Constructional

Implicit instruction is associated with constructivism as learners take responsibility for their own learning. Implicit instruction always takes place within an authentic context that enhances understanding and connects the classroom to real-life problem-solving (Slabbert, De Kock & Hattingh, 2009). The teacher facilitates learning by helping learners to discover new knowledge and then construct meaning (Table 2.1). The teacher only facilitates the learning as much as is needed through scaffolding (Slabbert et al., 2009; Mercer et al., 2014) and Socratic dialogue (Birnbacher & Krohn, 2004). Socratic dialogue takes place when teachers ask their learners questions in a way that will help them to solve a mathematical problem without directly pointing out the solution, or the path towards finding it. It is a method that guides learners towards independent problem-solving (Birnbacher & Krohn, 2004).

Furthermore, between explicit and implicit instruction, there is what Mercer et al. (2014) identify as interactive instruction (Table 2.1). Interactive instruction is based on the belief that knowledge is a collaborative enterprise and should include both explicit and implicit instruction.

This form of instruction is based on guided discovery through a dialogue that takes place between the teacher and the learner, in other words, a shared regulation of learning is necessary (Mercer et al., 2014). Moshman (1982) maintains that learning occurs when there is continuous interaction between the teacher and the learner. Interactive instruction has three key features, which include explicit teacher demonstration (I do it), guided practice (we do it) and independent practice (you do it) (Mercer et al., 2014; Miller & Hudson, 2006).

While researchers agree that teachers need to develop independent learners who possess the skills that are necessary to take responsibility for their own learning, they disagree on how this should be achieved (Mercer et al., 2014; Lerner & Johns, 2012; Allsopp et al., 2007; Slabbert et al., 2009). It may take learners with learning difficulties, especially those with multiple barriers to learning, longer than those without learning difficulties to become independent learners. Even in the case of learners with learning difficulties, explicit teaching alone will not produce independent learners.

2.3 Barriers to learning in mathematics

In order to be able to help their learners to develop to their full potential, mathematics teachers need to be well informed on the different types of barriers that could be experienced when learning mathematics. This knowledge should lead teachers to choose the most suitable teaching approaches to assist learners to overcome their particular barriers to learning. Barriers to learning mathematics can be divided into two main categories, namely intrinsic and extrinsic barriers (Dednam, 2011; Allsopp et al., 2007). Extrinsic and intrinsic barriers are interconnected in that extrinsic barriers can lead to or cause intrinsic barriers. Whereas extrinsic barriers to learning are caused by the learners' external environment, for example, family, community or school environment (Dednam, 2011; Allsopp et al., 2007), intrinsic barriers come from within the learners and can prevent them from learning and understanding mathematical concepts (Dednam, 2011; Allsopp et al., 2007; Miller & Mercer, 1997). This study will focus on the intrinsic barriers to learning only. Table 2.2 provides an overview of how different researchers classified and named the different intrinsic barriers to learning mathematics. Each researcher's view will be followed by a brief discussion of their classification of the intrinsic barriers to learning mathematics.

Table 2.2: Overview of the classification of the intrinsic barriers to learning mathematics

Allsopp et al. (2007)	**Dednam (2011)**	**Miller and Mercer (1997)**
Learnt helplessness	Emotional difficulties - lack of self-confidence when doing mathematics	Emotional and social characteristics - low self-esteem
Passive learning	Emotional problems - passivity	Emotional and social characteristics - emotional passivity
Maths anxiety	Emotional difficulties - anxiety	Emotional and social characteristics - maths anxiety
Low level of academic achievement		Emotional and social characteristics - history of academic failure
Memory difficulties		Information-processing factors - memory problems
Attention difficulties	Attention-deficit-related difficulties	Information-processing factors - attention deficits
Processing deficits	Perceptual skills difficulties	Information-processing factors - Visual-spatial deficits and auditory-processing difficulties
	Gross motor and fine motor difficulties	Information-processing factors - motor difficulties
Metacognitive thinking deficits		Metacognitive characteristics
	Difficulties with abstract and symbolic thinking and reading difficulties	Language disabilities

Allsopp et al. (2007) identify the following eight intrinsic barriers to learning mathematics (Table 2.2). Learnt helplessness, which occurs when learners expect to fail and rely on others for help because of continuous failure; passive learning, which occurs when learners do not actively make connections between their existing knowledge and new knowledge; memory difficulties, which results in learners struggling to retain and retrieve basic mathematical information, and consequently experiencing difficulties with multi-step sequencing and problem-solving (Allsopp et al., 2007); learners with attention difficulties whose attention is easily distracted often miss important information; metacognitive thinking deficits, which occur when learners have difficulty with monitoring their learning; processing deficits, which according to Allsopp et al.

(2007) cause learners to struggle with accurately perceiving what they hear, see, and/or feel and could lead to misperception regarding what they learn; low level of academic achievement caused by gaps in learners' knowledge of mathematics; and maths anxiety, which result in learners 'shutting down' when they are confronted with learning something new.

Dednam (2011) identifies seven intrinsic barriers to learning mathematics. The first three, which are emotional in nature, are: lack of self-confidence, passivity and maths anxiety. A lack of self-confidence develops when learners believe that they cannot do mathematical problems. Passivity means that these learners show little interest in trying to understand mathematics. Learners who struggle with maths anxiety are afraid of making mistakes when solving mathematical problems. The fourth intrinsic barrier to learning mathematics identified by Dednam (2011) is difficulty with abstract and symbolic thinking and reading difficulties and refers to learners who struggle to understand the relationship between numbers and objects and whose reading difficulties cause them to find word problems particularly challenging. Difficulties stemming from attention deficits constitute the fifth intrinsic barrier to learning mathematics and include hyperactivity and impulsivity, which is common among learners who have a short attention span and are easily distracted. Gross and fine motor difficulties constitute the sixth intrinsic barriers to learning mathematics and can develop when learners do not move around and perceive and manipulate objects around them. These learners may also have difficulties with writing. The final intrinsic barrier to learning mathematics identified by Dednam (2011) relates to inadequate perceptual skills. Learners with these difficulties, which could include visual and auditory perceptual difficulties, will struggle to give meaning to information perceived through the different senses.

Miller and Mercer (1997) identify ten intrinsic barriers to learning mathematics. The first four are of an emotional and social nature and include repeated academic failure in mathematics, which frequently results in low self-esteem, and passivity. Furthermore, maths anxiety can develop from a fear of failure and low self-esteem. The next four intrinsic barriers to learning mathematics are grouped under the category information-processing factors and include: attention deficits, memory problems, visual-spatial deficits and auditory-processing difficulties, and motor disabilities. According to Miller and Mercer (1997) learners with attention difficulties experience problems with maintaining attention during instruction. Memory problems include learners who are unable to retain new mathematical information, forget steps and struggle to solve multi-step word problems. Visual-spatial and auditory-processing difficulties include problems with differentiating between numbers, directional calculations and oral drills. Learners with motor disabilities find it difficult to write legibly. The last two intrinsic barriers to learning identified by Miller and Mercer (1997) are metacognitive problems and language disabilities. Learners with metacognitive difficulties lack the necessary awareness of the skills, approaches and resources required to perform mathematical tasks, whereas those with language difficulties will struggle with reading and will therefore find it difficult to solve complex word problems.

A total of ten intrinsic barriers to learning mathematics were identified from the literature. However, for the purpose of this study only eight of those were considered to be appropriate. The two barriers that were not selected are low levels of academic achievement, which are caused by the other barriers to learning, and motor difficulties, which are more physical in nature whereas the others are more cognitive in nature.

For the purpose of this study the three main classifications presented by Miller and Mercer (1997) will be used: emotional difficulties, information-processing difficulties, and metacognitive thinking difficulties. However, because this study is grounded in the constructivist paradigm, information-processing difficulties will be changed to cognitive difficulties because of the behaviouristic connotation of the information-processing learning model. Emotional difficulties include maths anxiety, passivity and learnt helplessness. Cognitive difficulties include attention difficulties, memory difficulties, processing difficulties and language difficulties. Metacognitive difficulties relate to metacognitive thinking. These eight intrinsic barriers to learning mathematics are interconnected as one barrier can lead to another. For example, a short attention span can result in memory difficulties, and problems with memorising information can lead to maths anxiety. Each of the eight intrinsic barriers to learning mathematics that were selected will now be discussed in detail.

2.3.1 Emotional difficulties

Learners who struggle with mathematics often lack the basic mathematical skills and have 'holes' or gaps in their mathematical knowledge. Once these gaps start to occur, they will have a compounding effect. Mathematics is one of the subjects where year after year the curriculum builds on previous understanding, therefore any gaps in the learners' understanding will become more significant each year and unless the resultant problems are addressed in a timely and effective manner, failure in mathematics will become almost inevitable (Allsopp et al., 2007). This could lead to emotional difficulties, which occur when learners feel helpless, develop maths anxiety, or lose any motivation to learn and understand mathematics.

2.3.1.1 Learnt helplessness

In 1967 two psychologists, Seligman and Maier conducted experiments with dogs that were repeatedly shocked but were unable to escape from the enclosure in which they were being kept. After two days, the dogs, stopped trying to escape when shocks were administered, even though the enclosure had been opened. This phenomenon was explained by Seligman and Maier (1967) as learnt helplessness. Learnt helplessness is when someone "has learned that outcomes are uncontrollable by his responses and is seriously debilitated by this knowledge" (Maier & Seligman, 1976, p. 4).

Maier and Seligman (1976) developed a model of learnt helplessness that affected three areas: motivation, cognition and emotions. First, when participants were faced with uncontrollable situations their motivation to learn was affected in a negative way. They lacked the motivation to respond because they had learnt that nothing they did could affect the outcome. Second, their cognition was affected. They found it difficult to believe that the environment had changed so they could now succeed when doing something - they still expected to fail. Finally, their emotions were negatively affected in that they became despondent and depressed (Maier & Seligman, 1976).

Following these laboratory experiments conducted during the 1960s and 1970s, more research was conducted during the 1980s and 1990s, but very little information could be found on research relating to learnt helplessness during the twenty-first century, in particular research focused on learnt helplessness in the area of mathematics. Kloosterman (1984) maintains that learnt helplessness in mathematics results from learners' idea that they are unable to understand the subject content, and that learners with learnt helplessness will attribute their success to extrinsic factors (e.g. assistance from others), and their failure to intrinsic factors (e.g. lack of ability). A lack of effort, and therefore motivation, plays a significant role in learnt helplessness (Kloosterman, 1984; Wieschenberg, 1994). It is encouraged by that 'inner voice' that tells you that no matter how hard you try, failure is inevitable (Dednam, 2011).

Learners with learning difficulties usually struggle academically and are used to academic failure, which results in low self-image and low self-esteem (Valas, 2001). When completing a task in mathematics, these anticipate failure even before trying (Allsopp, Kyger & Ingram, n.d.). They then learn to become more reliant on the teacher, peers or parents to assist them or complete assignments for them. They will stop trying and will avoid completing tasks in mathematics. This could, in turn, lead to passivity and even maths anxiety (Allsopp et al., n.d.; Miller & Mercer, 1997; Dednam, 2011; Lerner & Johns, 2009). Yates (2009) adds that learners with learnt helplessness display specific forms of behaviour that teachers can recognise. These behaviours include: a negative reaction to failure and learning, low levels of motivation, no persistency, and no effort made. When they are presented with a new mathematical problem to solve, they immediately complain that it is too difficult and that they cannot do it (Yates, 2009).

Certainly not all learners who experience repeated failure will develop learnt helplessness. Gentile and Monaco (1986) postulate that the development of learnt helplessness will depend on a learner's personality and how early in life the repeated failure started. They further suggest that the earlier in life learners feels that they have no control over situations the more likely they will develop learnt helplessness.

Learnt helplessness occurs when people perceive, through experience, that the results they obtain are not determined by what they do (Gentile & Monaco, 1986; Dednam, 2011). The earlier in life uncontrollable situations in a learner's life appear, and the more frequent they are, the more likely it is that the learner will develop learnt helplessness. Once learners have developed learnt helplessness, they will lack the motivation required to complete a mathematical task and will eventually stop trying.

2.3.1.2 Passivity

As in the case of learnt helplessness, most of the available research relating to passivity was done during the 1980s. Few recent articles on this topic could be found, and where passivity was indeed mentioned it was part of other research, such as on learnt helplessness. Passivity in learners can be defined as a lack of awareness of their own cognitive processes and of goal-orientated motivation to do a task (Wong, 1980). Miller and Mercer (1997) identify learnt helplessness and passivity as attributes of learning difficulties and further explain that a cycle develops: learnt helplessness leads to passivity, and passivity again leads to learnt helplessness.

According to Scardamalia, Bereiter, McLean, Swallow and Woodruff (1989) passive learners reveal the following characteristics: mental activities are organised around topics rather than goals; learners' focus is usually superficial understanding, for example, one approach will be tried and if that does not work they will stop trying; and they will try to simply add information instead of transforming it and integrating it with existing knowledge. Learners will, therefore, attempt to simply memorise information without understanding it. Once a task has been completed, passive learners will make no attempt to check or revise their work (Scardamalia et al., 1989). All this will lead to academic failure or low levels of academic achievement.

Passive learners are cognitively passive because they do not actively participate in or self-regulate their own learning, which can lead to demotivation (Miller & Mercer, 1997; Mercer et al., 2014; Dednam, 2011). They also do not know how to direct their thinking when learning information, how to go about gaining more knowledge, or even what approaches to use when they have to memorise what they have learnt. Since they believe that they are unable to learn because of many past failures they lose interest in learning (Lerner & Johns, 2012). Repeated failure, demotivation and memorising facts without understanding can result in the development of maths anxiety (Allsopp et al., 2007).

Furthermore, passive learners will wait for teacher direction before starting a task (Lerner & Johns, 2009). These learners do not actively make connections between their prior knowledge and what they are currently learning and they fail to integrate what they are learning now with what they already know (Lerner & Johns, 2009; Allsopp et al., n.d.). When they need to solve a mathematical problem, these learners may not know which previously learnt approaches or pre-knowledge could be used to solve it (Allsopp et al., n.d.).

Passive learners lack the necessary motivation to learn or complete tasks, as well as the cognitive ability to make appropriate connections with previous knowledge. They also lack self-regulation in their learning and are unable to direct their thinking when learning. Passive learning is usually, but not in all cases, the result of repeated academic failure, learnt helplessness and even maths anxiety.

2.3.1.3 Maths anxiety

It was not until 1976, when a popular magazine published an article about maths anxiety, that researchers started studying maths anxiety as a psychological problem, rather than a skills deficit (Tobias & Weissbrod, 1980). Maths anxiety is emotion based and can be defined as a negative emotional reaction to mathematics, or a perceived idea of not being good in mathematics (Maloney & Beilock, 2012; Wadlington & Wadlington, 2008; Lerner & Johns, 2009). Feelings that are associated with maths anxiety include tension, helplessness, dislike, worry, fear, panic, paralysis and mental disorganisation (Philipp, 2007; Tobias & Weissbrod, 1980). Experiencing a little anxiety or tension when completing a mathematical task can be beneficial, but too much anxiety or tension will decrease and impede performance (Philipp, 2007). Maths anxiety can also intensify the impact of other barriers on learning mathematics (Allsopp et al., 2007).

Finding the origin of maths anxiety is complex and research suggests that it starts at an early age (Blazer, 2011; Jansen, Louwerse, Straatemeier, Van der Ven, Klinkenberg & Van der Maas, 2013; Maloney & Beilock, 2012). Maths anxiety can range from mild to severe (Wadlington & Wadlington, 2008). Wadlington and Wadlington (2008) posit that although maths anxiety is not a learning disability, it does interfere with a learner's ability to learn and understand mathematics. Furthermore, they distinguish between specific and global maths anxiety. Specific maths anxiety is localised to a specific mathematical situation, whereas global maths anxiety pertains to all mathematical situations.

Blazer (2011) postulates that maths anxiety develops as a result of personality, intellectual and environmental factors. Personality factors can include low self-esteem, struggling to handle frustration, shyness and intimidation, whereas intellectual factors include an inability to understand mathematical concepts. Environmental factors include high parent expectations and poor teaching which lead to a fear of failure and making mistakes (Miller & Mercer, 1997; Dednam, 2011; Jansen et al., 2013). A cycle emerges in which a learner who fails in mathematics develops maths anxiety, which will in turn lead to failure and low academic performance, or vice versa (Wadlington & Wadlington, 2008; Jansen et al., 2013).

Maths anxiety can cause learners to ‘freeze up’ when they have to complete a mathematical task (Lerner & Johns, 2012). Researchers, such as Miller and Mercer (1997) and Sparks (2011), agree that the brain’s emotional centre becomes so overworked when a learner is anxious that its working memory and critical thinking become impaired and cannot function as needed (Jansen et al., 2013; Maloney & Beilock, 2012), which leads to mental disorganisation and confused thinking (Blazer, 2011; Miller & Mercer, 1997). The result is that, in a sense, learners use up all the brainpower needed to solve a problem by worrying too much (Sparks, 2011).

Blazer (2011) divided the symptoms of maths anxiety into three categories: physical symptoms, which include increased heart rate, clammy hands, upset stomach, and light headedness; psychological symptoms, which include an inability to concentrate, feelings of helplessness and worry; and behavioural symptoms, which includes avoiding anything that has to do with mathematics or putting it off till the last minute.

Learners who find mathematics difficult and experience maths anxiety often become anxious when merely thinking about it. This will cause them to ‘shut down’, which will affect their confidence in their ability to learn mathematics (Allsopp et al., n.d.; Jansen et al., 2013). These learners will avoid solving mathematical problems, or they will do it quickly but inaccurately, just to get it over with so that their stress levels can begin to subside (Jansen et al., 2013).

Solving mathematical problems quickly and inaccurately will result in poor academic achievement, which will again lead to further maths anxiety (Jansen et al., 2013).

2.3.1.4 Summary

Learnt helplessness, passivity and maths anxiety are related emotional problems, which can develop into barriers to learning mathematics. Even though these barriers are caused by factors arising from either outside (for example ineffective teaching or high parent expectations) or inside (for example memory difficulties or attention difficulties), learners internalise these barriers over time. Cognitive and emotional changes occur inside the learners, which will affect them in a negative way when they are confronted with mathematical tasks. Such changes will eventually become intrinsic barriers to learning. Even if the outside factors that caused the negative emotions change, learners with learning difficulties will have internalised them to such an extent that, despite the change, the effects will continue to have a negative impact on their performance.

2.3.2 Cognitive difficulties

Learners with learning difficulties frequently exhibit cognitive difficulties, which may include attention, memory, language and processing difficulties. These four intrinsic barriers to learning are interlinked as any one of them can lead to the emergence of another. For example, learners with a short attention span will often experience difficulties with memorising information. Learners may therefore have to deal with a combination of these barriers.

2.3.2.1 Memory difficulties

Information can be seen as flowing from the sensory memory to the short-term and working memory, and then finally to the long-term memory (Swanson, Cooney & McNamara, 2004). Memory consists of different components that can be divided into: sensory memory, short-term memory, working memory and long-term memory (Swanson et al., 2004; Lerner & Johns, 2009). Sensory memory is the initial processing of information and short-term memory processes information for a little bit longer. Working memory focuses on information storage, as well as the active interpretation of new information supported by information from the long-term memory. Long-term memory is the permanent storage of information with unlimited capacity (Swanson et al., 2004; Lerner & Johns, 2009).

Working memory is an active system and in mathematics it is responsible for performing mental arithmetic (Gathercole & Alloway, 2008; Lerner & Johns, 2009). A learner needs to hold and manipulate information in the working memory, which serves as a mental workspace for performing mental calculations. Possible reasons for learners expriencing difficulty with storing information in the working memory include distractions created by themselves or from an outside source, difficulty with suppressing irrelevant information, and difficulty with updating information (overloading) stored in the working memory (Gathercole & Alloway, 2008; Swanson et al., 2004). Because of these problems experienced with the working memory, long-term memory can also be affected (Gathercole & Alloway, 2008).

When learners with learning difficulties are unable to effectively memorise information it can affect their performance in many different ways and could diminish their self-esteem (Gathercole & Alloway, 2008). Some learners may not remember basic mathematical operations, or they may not be able to represent and automatically recall mathematical facts. They may also not be able to solve complex mathematical problems or word problems (Allsopp et al., 2007; Miller & Mercer, 1997; Mastropieri, Scruggs, Davidson & Rana, 2004; Gathercole & Alloway, 2008). Moreover, Allsopp et al. (2007) maintain that there is a strong connection between language difficulties and memory difficulties. Difficulties with the working memory, which serves as a temporary memory system and form a liaison between memory storage and memory retrieval, is often highlighted in discussions of learning difficulties (Lerner & Johns, 2012; Andersson & Lyxell, 2007; Reimann, Gut, Frischknecht & Grob, 2013).

A teacher can explain a concept and think that learners understand. However, when learners are asked to apply their knowledge to solve a mathematical problem that requires an understanding of that particular concept, they are unable to do so. Allsopp et al. (2007) postulate that this is not only a memory storage problem, as Miller and Mercer (1997) suggest, but could also be a memory retrieval problem. Retrieval depends on how the information was stored in the first place, which in turn depends on how effective and organised the storage process was (Sousa, 2006). A teacher should therefore determine whether a learner perhaps has a memory storage or retrieval problem. Learners with a memory storage problem may find it hard to remember because they struggle to understand concepts, and concepts that are not understood cannot really be learnt and are therefore easily forgotten (Allsopp et al., 2007). The reason for this may be that learners only learn disconnected facts that will make it difficult to see patterns or relationships, or select relevant information. Learning disconnected facts serves no real purpose, therefore learners should be encouraged to learn with understanding (conceptual understanding) (Bransford, Brown & Cocking, 2000). Learners with a memory retrieval problem find it difficult to organise new information and make associations and will struggle to retrieve the learnt information from memory at a later stage (Allsopp et al., 2007).

Difficulties in respect of short-term memory and working memory, in particular, will lead to problems with the storage of information. In order to store information in the short-term memory, it is essential to pay attention and acquire a conceptual understanding of the information provided (Bransford et al., 2000). Problems with regard to long-term memory will lead to retrieval problems, so that learners will find it difficult to recall the information that they have learnt (Lerner & Johns, 2009).

2.3.2.2 Attention difficulties

The term attention deficit disorder (ADD) was first introduced by the American Psychiatric Association and in 1982, and was changed to attention-deficit hyperactivity disorder (ADHD) in 1987 (Mercer & Mercer, 2001). According to Lerner and Johns (2012) between 25% and 40% of learners with learning difficulties also have ADHD. ADHD is a chronic neurological condition that makes it difficult for learners to control their behaviour (Lerner & Johns, 2012). ADHD can be divided into three subcategories: inattention (inability to concentrate on at task), impulsivity (responds without thinking about the consequences), and hyperactivity (a constant motor activity that drives a learner to go from one interest to the next) (Lerner & Johns, 2012). Learners may experience problems in all three areas, or in only one or two (Lerner & Johns, 2012; Dednam, 2011). Harris, Reid and Graham (2004) suggest that this is actually a broader syndrome that entails difficulties or deficiencies in the self-regulation processes, such as cognitive control and social-emotional control.

Learners with ADHD are characterised by their inability to stay with one task, to focus their attention on a task, and to complete a task. They are also easily distracted, do not seem to listen to instructions, and their work is not neat. They are fidgety, restless and talk too much in class or speak out of turn (Lerner & Johns, 2009). They may process information slowly, lose their place easily and need to be told to stop doing something that does not involve the task at hand (Dednam, 2011). All this leads to academic underachievement in mathematics (Harris et al., 2004).

Solving mathematical problems requires a great deal of attention, especially in the case of complex problems, such as long division (Allsopp et al., 2007; Miller & Mercer, 1997). In order to be successful, a learner must be able to maintain attention on the current task, but must also be able to switch attention to a new task when required (Mercer & Mercer, 2001). Learners with attention difficulties struggle to screen out stimuli from the outside or focus too intently on stimuli that are irrelevant (Mercer & Mercer, 2001; Allsopp et al., 2007). Due to this distractibility they may miss important pieces of information (Allsopp et al., 2007). Another problem regarding attention is that these learners will not be able to identify and focus on important features that indicate the differences between objects, for instance the difference between a triangle and a rectangle. These learners may focus on the size or the colour of the example and not on how many sides there are (Allsopp et al., n.d.). Hyperactive, impulsive learners make many unnecessary mistakes and tend to give answers before thinking (Dednam, 2011).

Attention difficulties are complex and include not only inattention, but also hyperactivity, impulsivity and problems with regard to self-regulation. This condition, known as ADHD, is commonly linked to learners with learning difficulties and often results in poor academic achievements owing to learners' inability to focus on the task at hand.

2.3.2.3 Processing difficulties

Processing difficulties are closely related to perceptual skills. Perception is the process of recognising, interpreting and giving meaning to information perceived through the senses (Dednam, 2011). Perception can be divided into three subcategories, namely visual, auditory and tactual-kinaesthetic perception (Allsopp et al., 2007; Dednam, 2011; Lerner & Johns, 2012). Processing difficulties stem from a central nervous system dysfunction that impairs learners' ability to process information received through their senses (Allsopp et al., 2007).

Learners with learning difficulties find it difficult to accurately perceive what they see, hear and/or feel. Even though there is nothing wrong with their eyesight or hearing, their central nervous systems process information differently, leading to wrong interpretations of what they see, hear or feel (Allsopp et al., 2007). Visual perception, i.e. the ability to identify, organise and interpret what one sees through one's eyes is usually the primary way in which information is normally gathered when learning. Auditory perception is the ability to organise and interpret what is heard (Dednam, 2011; Lerner & Johns, 2012). Auditory processing difficulties include difficulties doing oral drills, being unable to count on from within a sequence (Miller & Mercer, 1997) and distinguishing between numbers that sound alike. Learners who experience auditory problems also cannot count 'in their heads', but tend to use their fingers when counting (Dednam, 2011). Tactual-kinaesthetic perception, also known as haptic perception (Lerner & Johns, 2012), is the ability to identify objects through touching them. Kinaesthetic perception enables a person to gain information via different parts of his or her body through movement (Dednam, 2011; Lerner & Johns, 2012).

Processing difficulties include either input pathways, as discussed, or output pathways, or a combination of the two. Examples of a combination of input-output difficulties are visual-motor, visual-spatial, auditory-motor, visual-speech or auditory-speech difficulties (Miller & Mercer, 1997; Dednam, 2011; Allsopp et al., 2007). In some cases a learner processes the information correctly, but the speed at which it is done is an issue (Allsopp et al., 2007).

Learners with visual-spatial difficulties cannot orientate themselves in space and therefore direction of objects, relationships between objects and their differences and similarities will not make sense to them (Dednam, 2011). Moreover, Miller and Mercer (1997) suggest that these learners will easily lose their place in their workbooks, will find it difficult to distinguish between numbers and operation symbols, will struggle when copying from a board, and will have difficulties with directional aspects in mathematics.

Learners who experience difficulties with processing will have problems processing information perceived through their different senses and may also process information slowly. These learners find it difficult to learn as their brains process auditory, visual or tactual-kinaesthetic information differently.

2.3.2.4 Language difficulties

Language, which can be defined as the understanding and use of signs and symbols in order to represent ideas (Gargiulo & Metcalf, 2013), can be divided into two categories, namely receptive and expressive language (Gargiulo & Metcalf, 2013; Lerner & Johns, 2012). Language disorders occur when there is a delay or difficulty in either of these two categories. According to Lerner and Johns (2012), receptive language is a prerequisite for the development of expressive language. Morin and Franks (2010) postulate that language difficulties are under-acknowledged as a factor in the successful understanding of mathematics, as language-processing problems (such as dyslexia) can make it difficult for learners to learn new mathematical vocabulary and use abstract thinking, which is essential when working with symbols, signs and operations (Wadlington & Wadlington, 2008; Dednam, 2011).

Receptive language refers to understanding what one hears (Gargiulo & Metcalf, 2013). Learners with receptive language difficulties may understand words when they are used in isolation but not necessarily when they are used in a sentence. These learners may understand a word in one context, but may be unable to understand the same word when it is used in another context (Lerner & Johns, 2012). Receptive language difficulties can therefore lead to reading difficulties. In understanding mathematics, learning difficulties that are aggravated by reading difficulties, not only affect learners' understanding of the mathematical text, but also their mathematical vocabulary, which in turn will impede their problem-solving skills (Allsopp et al., 2007). Mathematics has its own special language and some terms have one meaning in mathematics and another in other subjects (Wadlington & Wadlington, 2008). The use of language in mathematics is important for calculations and solving word problems (Miller & Mercer, 1997), as well as working with placeholders and phonological memory (Morin & Franks, 2010; Reimann et al., 2013).

Expressive language refers to language spoken in such a way that others can understand the meaning of the words (Gargiulo & Metcalf, 2013). Learners with expressive language difficulties struggle to speak, but have no problem with understanding others when they speak (Lerner & Johns, 2012). Phonological processing is one area of spoken language and refers to the ability to distinguish the sounds of language (phonemes) (Lerner & Johns, 2009). Difficulties with phonological processing may influence number sequencing and retrieval, may be a factor in increased response time, and may reduce the capacity of the working memory and hinder verbal fact retrieval (Morin & Franks, 2010; Reimann et al., 2013).

Not all learners with learning difficulties will have language difficulties, but in many cases learning difficulties are compounded by the presence of language difficulties (Lerner & Johns, 2012). Language difficulties include problems with expressive language, which includes phonological processing, and with receptive language, which includes difficulties with reading. If learners are unable to read or understand what they are reading, or to understand oral instructions, they will find it extremely difficult to complete mathematical tasks.

2.3.2.5 Summary

Cognitive difficulties include memory, attention, processing and language difficulties. All these barriers to learning mathematics stem from problems that occur in the pathways of a learner's brain. Many of these problems are interlinked, for example, attention difficulties may lead to memory difficulties, and language difficulties may be linked with processing difficulties. Most, if not all, of these difficulties cannot be readily overcome and will probably remain with learners for the rest of their lives. The challenge lies in how learners with learning difficulties will be taught to manage and cope with these difficulties, which will determine whether they will ultimately understand mathematics and experience success.

2.3.3 Metacognitive thinking difficulties

The term metacognition was coined by Flavell in 1976 (Kolencik & Hillwig, 2011; Butler, 1998). According to Flavell's broad definition of metacognition, it is a person's own knowledge of his or her own cognitive processes and products (Butler, 1998). Other researchers have since refined the concept and have agreed upon a basic definition that describes metacognition as an awareness of the skills, approaches and resources that are needed, and knowledge of how to regulate behaviour in order to perform and successfully complete a task (Harris et al., 2004).

Metacognition is higher-order thinking and assists in controlling cognitive processes and learners' ability to monitor their own learning (Rosenzweig, Krawec & Montague, 2011; Allsopp et al., 2007). Through metacognition learners construct their own meaning from information received and use it to sort and classify this information (Gargiulo & Metcalf, 2013). It can be explained as thinking about one's own thinking (Gargiulo & Metcalf, 2013; Lerner & Johns, 2012).

Metacognition consists of three related elements: metacognitive knowledge, metacognitive experience, and metacognitive skill (Rosenzweig et al., 2011; Whitebread & Pino Pasternak, 2012). Metacognitive knowledge is knowledge that is gradually accumulated about one's own mental processing and is stored in a person's memory regarding approach selection and how to execute tasks (Rosenzweig et al., 2011; Whitebread & Pino Pasternak, 2012). Metacognitive experience refers to self-judgement before performing a task, while it is being executed and after it has been completed. This self-judgement includes self-interpretation or self-evaluation of a person's familiarity with the task, comprehension of the task, perception of task difficulty, effort required to complete the task, and confidence in one's ability to complete it successfully (Rosenzweig et al., 2011; Miller & Mercer, 1997). Metacognitive skill is the regulation of mental processing and the procedures and approaches that one uses while executing a task in order to monitor and control one's understanding, and the ability to go back and repair it if necessary. These approaches can include self-observation, self-evaluation, self-monitoring, self-instruction and self-questioning (Rosenzweig et al., 2011; Miller & Mercer, 1997; Allsopp et al., 2007; Whitebread & Pino Pasternak, 2012). Metacognition helps learners to control and regulate their own learning, as well as helping them to know what to do, when to do it and how to do it (Kolencik & Hillwig, 2011).

Rosenzweig et al. (2013) posit that learners with learning difficulties are often impulsive, use trial and error to solve problems and do not evaluate their answers. They also struggle with complex problems, mathematical vocabulary and fact retrieval. According to Allsopp et al.

(2007), the problem is not that learners with learning difficulties do not use approaches, but that they do not necessarily know when and how to use them when solving mathematical problems. Learners with learning difficulties struggle to evaluate their progress. They are uncertain about when to employ different approaches and are unable to judge whether an approach has been successful, or whether changes are required. Metacognition is therefore important if a learner wants to apply mathematics (Allsopp et al., 2007).

The intrinsic barriers to learning mathematics can be classified into the following three main categories: emotional difficulties (learnt helplessness, passivity and maths anxiety); cognitive difficulties (memory, attention, processing and language difficulties); and metacognitive thinking difficulties. Learners with learning difficulties may show any combination of these intrinsic barriers to learning and their severity will differ from one learner to the next. A teacher who teaches learners with learning difficulties will probably have to deal with these intrinsic barriers to learning at some time or other. It is therefore imperative for teachers to be fully aware of every learner's intrinsic barriers to learning in order to be able to help them to reach their full potential in mathematics.

2.4 Possible teaching approaches that could assist learners with learning difficulties

Teachers should not only be aware of the intrinsic barriers to learning mathematics that learners with learning difficulties experience in the mathematics classrooms, but should also know which teaching approaches they should employ in order to assist learners to overcome their intrinsic barriers to learning and to reach their full potential. Teaching approaches that might help learners with learning difficulties to acquire mathematical understanding are summarised in Table 2.3. It is important to note that these teaching approaches are universal and could be used in all teaching situations, and not just for learners with learning difficulties. However, the researchers who focus specifically on mathematics education for learners with learning difficulties stress the importance of using these approaches all the time and in combination with one another (Allsopp et al., 2007; Lerner & Johns, 2012; Mercer et al., 2014).

Two teaching approaches mentioned by all five researchers (see Table 2.3) are scaffolding and concrete-representational-abstract (CRA) sequencing. These researchers all agree that one should start explaining a new concept by using the concrete level, followed by the representational level and only then using abstract symbols (Allsopp et al., 2007; Lerner and Johns, 2012; Miller & Hudson, 2006; Mercer et al., 2014; Miller & Mercer, 1997). Although it is generally agreed that scaffolding is an important teaching approach when teaching learners with learning difficulties, researchers do not necessarily agree on how scaffolding should take place. The last teaching strategy in Table 2.3 is mentioned by all the researchers, but the contents differ. Allsopp et al. (2007), Mercer et al. (2014), and Miller and Mercer (1997) suggest that feedback should be given in order to motivate learners. Miller and Mercer (1997), Mercer et al. (2014) and Miller and Hudson (2006) emphasise the frequent monitoring of learner progress while teaching so as to determine the level of learner understanding. With the exception of Allsopp et al. (2007), all the researches mentioned the importance of setting attainable goals.

Table 2.3: An overview of possible teaching approaches that could assist learners with learning difficulties

Allsopp et al. (2007)	**Lerner and Johns (2012)**	**Miller and Hudson (2006)**	**Mercer et al. (2014)**	**Miller and Mercer (1997)**
Teaching within authentic context		Integrate real-world applications	Teaching generalisation of a maths concept in other contexts	Teaching generalisation of a maths concept in other contexts
Building meaningful connections	Link new maths information to prior knowledge			Consider background knowledge
Demonstration and scaffolding	Scaffolding instruction	Consider appropriate structures for teaching specific concepts	Teacher demonstration	Scaffolding and implementing demonstration
Concrete-representational-abstract (CRA) sequencing	Progression from concrete learning to abstract learning	Use various modes of representation	Using CRA sequence	Using CRA sequence
Teaching learners' problem-solving strategies	Develop learners' problem-solving skills		Develop learners' problem-solving skills	Develop learners' approaches to problem-solving
Provide immediate corrective feedback	Goal-setting	Goal-setting and frequently monitoring learner progress	Goal-setting, monitoring of progress and feedback	Goal-setting, monitoring of progress and feedback

Allsopp et al. (2007) and Miller and Hudson (2006) encourage the use of authentic real-life examples as a teaching approach as it makes mathematics meaningful. Mercer et al. (2014) and Miller and Mercer (1997) suggest that a teacher should assist learners to generalise mathematical concepts to other real-life contexts. Allsopp et al. (2007), Lerner and Johns (2012) and Miller and Hudson (1997) postulate that it is necessary to assist learners to build meaningful connections by linking new mathematical knowledge to prior knowledge when teaching learners with learning difficulties as these learners do not make these connections themselves. All the above-mentioned researchers, with the exception for Miller and Hudson (2006), include the development of the learners' problem-solving skills as a teaching approach since learners with learning difficulties do not necessarily know where to start when solving a mathematical problem.

Lerner and Johns (2012) posit that understanding mathematics is a gradual process and cannot be defined by either knowing it or not knowing it. Understanding mathematics lies on a continuum where knowledge is slowly built while unsystematic thinking changes to systematic thinking (Lerner & Johns, 2012). The teaching approaches that are employed should assist learners in building their own mathematical knowledge (Allsopp et al., 2007; Mercer et al., 2014, Lerner & Johns, 2012). Therefore, the goal of the teaching approaches employed by teachers should be to assist learners become independent thinkers and to help them to eventually solve mathematical problems on their own.

Researchers disagree on how this goal should be achieved. On the one hand, researchers who focus on special needs education insist that it is necessary to follow a behaviouristic approach, at least at first, which requires the teacher to first demonstrate and explain a concept, then to guide the learner, and finally to encourage the learner do the task independently (Mercer et al., 2014; Allsopp et al., 2007; Lerner & Johns, 2012; Miller & Hudson, 2006). They are not against the idea of learners constructing their own knowledge, but argue that because of the barriers to learning a teacher should first assist learners who really struggle and should gradually change to a more constructivist way of teaching. On the other hand, researchers who focus more on a constructivist approach believe that learning can take place only when learners construct their own knowledge and maintain this can only be achieved through facilitation of learning (Slabbert et al., 2009). Bransford et al. (2000) posit that if learners are allowed to first struggle with a concept they learn much more than those who did not have that opportunity. This helps to prepare learners for learning with understanding.

Many of the teaching approaches discussed below are closely related and sometimes the lines that separate them are not easily defined. This overlap is evident in the discussion of each of these approaches, where one reference may be made to another.

2.4.1 Authentic context

Researchers agree that using a real-life authentic context is one of the most important aspects in the facilitation of learning (Slabbert et al., 2009; Engelbrecht, 2012; Allsopp et al., 2007). It is important to take learners' interests and experiences into consideration when teaching new concepts. An authentic and meaningful learning context can be created by using examples from the learners' environment and interests. This will help to arouse and retain the learners' interest in what they are learning and will give meaning to their learning (Allsopp et al., 2007).

A mathematics teacher who wants to use a context to develop a mathematical concept needs to consider various aspects. First, the context that the teacher chooses should be age and culturally appropriate (Allsopp et al., 2007). Teaching within an authentic context demonstrate to learners the relevance of mathematics in their daily lives and this will motivate and help them to generalise the concepts to other contexts when necessary (Miller & Hudson, 2006; Wadlington & Wadlington, 2008). Second, the context should not distract learners' attention from the mathematical concepts being taught, but should nevertheless be interesting and relevant (Allsopp et al., 2007). Third, technology can be used successfully in that computers can simulate real-life situations in which problems need to be solved (Miller & Hudson, 2006).

Learners with learning difficulties usually struggle with the generalisation of a concept. Generalisation means the ability to effectively solve a problem by using a concept in different contexts (Mercer et al., 2014). The generalisation of a concept can be achieved when using different examples from many different contexts in teaching a specific mathematical concept (Mercer et al., 2014). In order to use this approach, from a constructivist perspective, learners could be asked to think of a context in which the mathematical concept that is being taught could be used, thus developing their ability to generalise (Allsopp et al., 2007).

Mathematics teachers should, therefore, strive to use real-life situations with which their learners are familiar, but which will not distract their attention from the mathematical concepts that they want to teach. Since learners with learning difficulties struggle to understand that one mathematical concept can be used in many different contexts, the mathematics teacher should use one mathematical concept in different contexts in order to teach learners how concepts can be generalised.

2.4.2 Building meaningful connections

Every learner arrives in a classroom with prior knowledge that can either facilitate or impede the learning of new ideas. It can also influence how they will organise and interpret new knowledge. This will in turn affect the way in which they will remember, reason, solve problems and acquire new knowledge (Bransford et al., 2000). The building of meaningful connections can be interpreted two ways. First, the teacher assists learners to make connections between what they already know and the new mathematical concept that is being learnt (Allsopp et al., 2007; Nel & Nel, 2012). A teacher should therefore take note of the possible incompleteness of learners' prior knowledge and then build on their incomplete understanding to assist them to achieve a more mature understanding (Bransford et al., 2000). This could be done at the beginning of the lesson and could be used in conjunction with the teaching approach of teaching within an authentic context (Mercer et al., 2014; Allsopp et al., 2007).

Second, building meaningful connections can also mean that a learner makes a connection between internal and external representations. Hiebert and Carpenter (1992) maintain that when mathematical concepts are communicated to learners, it is done either verbally or by way of written symbols, pictures or physical objects. Internal representation refers to the thinking about mathematical ideas that produce networks of knowledge. These internal representations should therefore be structured in a learner's mind (Hiebert & Carpenter, 1992). A connection should then be made between the internal and external representation in order to understand mathematics. An effective way to connect the internal and external representations of knowledge is to make use of the concrete-representational-abstract (CRA) sequencing (Hiebert & Carpenter, 1992; Allsopp et al., 2007). The CRA sequencing will be discussed in detail in paragraph 2.4.4.

Miller and Hudson (2006) suggest that the building of meaningful connections can be linked with advanced organisers. An advanced organiser was developed by Ausubel and is a teaching tool that assists learners to organise information by connecting new information to a larger cognitive structure (Ausubel, 2010). Ausubel (2010) believes that an advanced organiser should be introduced to learners prior to presenting them with the learning material. The mathematics teacher can introduce the advanced organiser by first reviewing prerequisite knowledge and linking new concepts to what the learners already know and have experienced. They can then identify what the learners will be doing, in other words, provide them with a focal point in order to capture their attention. Finally, they can be provided with a rationale for the lesson, i.e. an explanation of how it relates to their lives (Mercer et al., 2014; Allsopp et al., 2007; Miller & Hudson, 2006).

Assisting learners to build meaningful connections between prior knowledge and new knowledge, and internal and external representations are important. The teacher should be aware of any incomplete knowledge that learners have and build on that in order to promote their understanding of mathematics. This can be done just before the new concept is introduced by using an advanced organiser.

2.4.3 Scaffolding

The term scaffolding was coined by Wood, Bruner and Ross (1976). Scaffolding can be viewed as a process whereby a learner (of less experience) is assisted by an adult (with more experience) to complete a task that is beyond the learner's ability, but can be completed with the assistance of an adult or a more experienced person (Wood et al., 1976; Gultig & Stielau, 2012). The key to successful scaffolding is to offer a supporting structure that can initiate and sustain interest in order for learners to become involved, after which the support is gradually removed so as to shift the responsibility for learning from the teacher to the learner (Larkin & Ellis, 2004; Van de Pol, Volman & Beishuizen, 2010; Mercer et al., 2014; Allsopp et al., 2007; Cagiltay, 2006). Gultig and Stielau (2012) postulate that although learners must build their own towers of knowledge, the teacher's task is to provide the scaffolding.

Since the coining of the term many different types of scaffolding have been identified, for instance incidental scaffolding, strategic scaffolding, actual scaffolds, prop scaffolds, localised scaffolds, step-by-step or foothold scaffolds, hints and slots scaffolds (Anghileri, 2006), technology scaffolding (Bransford et al., 2000), and online scaffolding, which includes conceptual, metacognitive, procedural and strategic scaffolding (Calgitay, 2006). Researchers agree on the definition of scaffolding, but differ with regard to how the scaffold should be constructed.

Two groups of researchers identified general scaffolding strategies. Some elements correspond but others differ. Wood et al. (1976), who coined the term, identified six elements of scaffolding, whereas Tharp and Gallimore (1988) identified six interdependent scaffolding strategies. The last three elements identified by Wood et al. (1976) and the first three identified by Tharp and Gallimore (1988) are similar. The first of these is demonstration, where the teacher offers behaviour for imitation or demonstrates the solution to a task. The second element is frustration control (Wood et al., 1976) or contingency management (Tharp & Gallimore, 1988), where the teacher responds to learners' state of mind. The third element is marking critical features (Wood et al., 1976) or feeding back (Tharp & Gallimore, 1988), where the teacher confirms and verifies. The other three elements identified by Wood et al. (1976) are recruitment, during which learners' interest in the task is aroused; reduction in degrees of freedom which involves the simplification of the task; and direction maintenance, when the teacher uses verbal prompts to keep the learners on the right track. The other three elements identified by Tharp and Gallimore (1988) are instructing by calling for specific action; questioning by calling for linguistic response; and cognitive structuring by providing explanations and belief structures that organise and justify.

Scaffolding can also be linked to Vygotsky's zone of proximal development (ZPD). How much support a teacher will provide to the learner will depend on each learner's ZPD. The ZPD can be divined as "the distance between the actual developmental zone as determined by independent problem-solving and the level of potential development as determined through problem-solving under adult guidance" (Vygotsky, 1978, p. 86). In other words, the ZPD is where the learner still needs assistance from someone else to complete a task. The instruction in the ZPD should therefore not be too easy or too difficult, but just sufficiently challenging to promote the development of new skills (Lui, 2012). This can be achieved by using scaffolding, where temporary support and guidance is provided to assist a learner in mastering a new concept that is beyond his or her current level of development (Gultig & Stielau, 2012).

2.4.3.1 Scaffolding in mathematics

Anghileri (2006) suggests a hierarchy of three levels of scaffolding for learning mathematics. Anghileri's (2006) view of scaffolding is more elaborate than those of Wood et al. (1976) and Tharp and Gallimore (1988). Level one is called environmental provisions which include the provision of manipulatives, puzzles and also how the teacher organises the classroom. This level of scaffolding does not involve direct interaction between the teacher and learners. The second level involves explaining, reviewing and restructuring and requires teacher-learner interaction. This level has many aspects, such as explaining and reviewing important aspects of the task; prompting and probing; parallel demonstration; learners explaining and justifying; restructuring and simplifying the task for further clarification; identifying meaningful contexts; rephrasing learners' explanations; and negotiating meaning with the learners. This level is a combination of some of the elements of Wood et al. (1976) and Tharp and Gallimore (1988). Level three involves the development of conceptual thinking and includes the development of representational tools through words and symbols or other visual means, such as graphics. It also includes helping learners to make connections between mathematical concepts, and finally generating conceptual discourse, which goes beyond explanation and justification and includes reflection on the concept being taught at the time (Anghileri, 2006). This level links to Tharp and Gallimore's (1988) cognitive structuring.

2.4.3.2 Scaffolding in special needs education literature

Researchers who researched the use of scaffolding in special needs education view scaffolding in terms of an apprenticeship model consisting of three phases. According to Miller and Hudson (2006) and Van de Pol et al. (2010) the scaffolding process follows a sequence of 'I do' or teacher responsibility, followed by 'we do' or joint responsibility, and then 'you do' or learner responsibility. The 'I do' or teacher responsibility does not mean that learners are not involved in the learning process. According to this view of scaffolding, once the teacher has prepared learners through demonstration, their contribution to the learning process increases and the responsibility of learning shifts from the teacher to the learner (Mercer, Lane, Jordan, Allsopp & Eisele, 1996).

According to special needs education researchers, the 'I do' part of the scaffolding process is demonstration (Allsopp et al., 2007; Nel & Nel, 2012; Miller & Hudson, 2006). Wood et al. (1976), Tharp and Gallimore (1988) and Anghileri (2006) also included demonstration as an element of scaffolding. However, the way they view demonstration and where it fits into the sequence of scaffolding differs. According to Mercer et al. (1996), the goal of demonstration is for learners to become active, strategic and independent learners. The following demonstration principles are suggested to assist learners with learning difficulties to understand new mathematical concepts: link new concepts to previous knowledge; incorporate multi-sensory methods when teaching; simplifying the mathematical problem by breaking it into smaller, manageable parts; cue key features of a specific mathematics concept; demonstrate and think aloud; provide examples and non-examples for a specific concept; ask questions to make learners think through the problem and to check for understanding; explain and elaborate when necessary; while using demonstration, provide immediate and specific corrective feedback; do frequent short reviews of concepts already done; provide generous amounts of specific positive reinforcement; and summarise and synthesise at the end (Allsopp et al., 2007; Larkin & Ellis, 2004; Wadlington & Wadlington, 2008; Lerner & Johns, 2009; Nel & Nel, 2012; Mercer et al., 2014).

The second phase of the scaffolding process is the 'we do' part, also referred to as guided practice (Mercer et al., 2014; Miller & Hudson, 2006). The amount of guidance that a teacher will give depends on learner understanding and different learners may need different amounts of assistance and support (Mercer at al., 1996). Elements of guided practice are: asking leading questions; the teacher repeating or rephrasing lesson content; doing a task partially in order for learners to do the rest through guidance; monitoring learners' progress and giving corrective feedback until learners are ready to work independently; determining the amount of assistance necessary to support learners; allowing learners to take risks when solving a problem; assisting learners to self-monitor their progress and self-evaluate their performance; fading teacher support as necessary (Miller & Hudson, 2006; Mercer et al., 2014; Allsopp et al., 2007). This phase corresponds with Wood et al.'s (1976) direction maintenance, Tharp and Gallimore's (1988) instruction by calling for specific action and questioning by calling for linguistic response, and with Anghileri's (2006) second level of scaffolding.

The last phase of the scaffolding process is the 'you do' part, during which the learner works independent from the teacher. In other words, it is the learner who now demonstrate his skill and understanding in solving the mathematical problem (Miller & Hudson, 2006). This last part of scaffolding consists of the following elements: explaining the rationale and importance of the assignment; ensuring that learners understand the assignment before they start working; praising learners for independent work done; circulating among learners to monitor progress; assisting learners who are struggling by asking leading questions; and holding learners accountable for independent work (Mercer et al., 2014). This phase links with frustration control (Wood et al., 1976) or contingency management (Tharp & Gallimore, 1988), which occurs when the teacher responds to learners' state of mind, and with Anghileri's (2006) third level of scaffolding or Tharp and Gallimore's (1988) cognitive control.

Learners with learning difficulties experience multiple barriers to learning, as previously discussed. These barriers to learning include lack of motivation, memory difficulties, maths anxiety and learnt helplessness. Since many learners with learning difficulties struggle to link different mathematical concepts, as they find it difficult to know where to start with a task and mathematics does not make sense to them, teaching by using scaffolding is important. Not all researchers agree on how scaffolding should be used. For the purpose of this study, I will use the three stages to which special needs education researchers adhere. The reason for choosing these three stages is because they are easily identified when observing a lesson. The other views of scaffolding are much more elaborate and will be more difficult to identify when doing observations.

2.4.4 Concrete-representational-abstract (CRA) sequencing

According to Lerner and Johns (2012), the concrete level represents actual material that learners can manipulate, such as blocks, sticks, plastic pieces, marbles and so on. The representational or semi-concrete level makes use of pictures, tallies, etc. which learners can use to solve mathematical problems. These pictures or tallies represent the actual concrete objects used on the concrete level. The abstract level represents symbols such as numbers and operational signs to solve a mathematical problem without the assistance of real objects or pictures (Lerner & Johns, 2012). Lerner and Johns (2012) propose that mathematics learning lies on a continuum, with concrete knowledge at the one end and abstract knowledge at the other end, and representational knowledge in between.

When teachers represent mathematical concepts in multiple ways, it helps to generalise a concept and ensure meaningfulness (Miller & Hudson, 2006). An effective way to do this, according to Miller and Hudson (2006), is by using CRA sequencing. Teachers should start at the concrete level and move through to the abstract level when teaching a new concept, as the concrete level of understanding is the most important level for conceptual understanding (Allsopp et al., 2007; Mercer & Mercer, 2001; Wadlington & Wadlington, 2008). Hands-on learning materials, for example concrete objects, encourage learners to explore ideas and give them an opportunity to be involved in their own learning (Lerner & Johns, 2012). Wadlington and Wadlington (2008) suggest that the teacher should first show a mathematical concept concretely before making use of pictures and diagrams (representational level). The abstract level should only be used after multiple exposures to the concrete and representational level of a mathematical concept (Wadlington & Wadlington, 2008).

Since learners with learning difficulties have problems with abstract thinking, explaining a concept in a concrete and then in a representational manner can assist them to better understand a mathematical concept. One should frequently make use of the CRA sequencing when teaching mathematics to learners with learning difficulties.

2.4.5 Problem-solving strategies

Problem-solving in mathematics is an important skill to master, but it is also a complex endeavour. Unfortunately, many learners with learning difficulties have a problem in this area (Lerner & Johns, 2012). In order for learners with learning difficulties to solve mathematical problems correctly, they need to have a mathematical knowledge base, should be able to apply existing knowledge to new and unfamiliar situations, and have the ability to actively engage in the thinking processes (Mercer et al., 2014).

Polya (1957) maintains that problem-solving consists of four stages. First, one has to recognise and understand the problem and what is required to solve it. Second, one has to understand how the different items in the problem are connected in order to plan a procedural approach. Third, one has to decide on the mathematical knowledge needed to solve the problem, after which the plan must be carried out to solve it, and the steps checked. Finally, one has to look back at the solution and review it to ensure that it makes sense.

Mercer et al. (2014) suggest using scaffolding as a way to teach the problem-solving stages to solve mathematical problems. The teacher starts by demonstrating how the four problem-solving processes should be used and provides ample opportunities to practice, using approaches in relevant contexts (Allsopp et al., 2007; Miller & Mercer, 1997; Polya, 1957). Teachers should emphasise both the actions and the thinking that are needed to solve a problem by verbalising their thinking (Allsopp et al., 2007; Miller & Mercer, 1997). Following demonstration, they should ask the learners to verbalise how they solved the particular problem so that the teacher can determine whether they used the appropriate cognitive approaches in solving the problem (Rosenzweig et al., 2011). The teachers should provide guidance by asking appropriate and probing questions so that learners can learn how to combine thinking and language with the calculation skills and concepts that are required for solving word problems (Lerner & Kline, 2006).

The following should be considered when assisting learners with learning difficulties to develop problem-solving skills: encourage learners to take risks and remind them to self-monitor and self-evaluate their performance, which will help them to independently solve mathematical problems in the future (Allsopp et al., 2007; Polya, 1957). Also encourage the use of multiple ways of solving mathematical problems (Lerner & Johns, 2012) and in the beginning guide them through the four problem-solving processes.

2.4.6 Formative assessment

Formative assessment is on-going assessment designed to make the way learners think visible and benefits both the teachers and the learners (Bransford et al., 2000). It is a diagnostic tool that can highlight learning problems (Gultig & Stielau, 2012). It is ungraded, but provides learners with opportunities to revise and improve their thinking and helps teachers to identify problems in learners' thinking (Bransford et al., 2000). It takes place throughout learning and therefore supports learning. The goal of formative assessment is to establish whether the learning outcomes are being achieved (Reddy, 2004). Formative assessment could include clarifying and sharing of learning intentions through goal-setting, monitoring the progress of learners as you teach through asking questions, providing feedback, assisting learners to be actively involved in their own learning by doing self-assessment, and encouraging learners to assist their fellow learners by doing peer assessment (William, 2008).

Goal-setting in mathematics is important as it can improve a learner's feeling of self-worth and fosters motivation (Mercer et al., 2014). According to Mercer et al. (2014), teachers need to be clear about the instructional goals and should communicate these goals in such a way that learners are clear about what needs to be achieved, what they need to do to achieve the goals, what they will learn when the goals are achieved, and why it is important to learn the specific concept. Setting a time when the goal must be achieved also helps learners to focus. When setting goals, the teacher must ensure that they are sufficiently challenging, i.e. not too easily attainable (Mercer et al., 2014). It is also important to involve the learners in the setting of goals, with the teacher providing the necessary guidance.

It is important for teachers to monitor learners' progress in order to determine whether they need to change their teaching approach or spend more or less time on a particular concept (Mercer et al., 2014; Lerner & Johns, 2009). During a lesson teachers need to constantly monitor the progress of learners and give appropriate corrective feedback (Mercer et al., 2014). In order to monitor learners' progress, teachers need to verify and, if necessary, clarify learners' understanding by asking open-ended and probing questions (Allsopp et al., 2007). Mercer et al. (2014) posit, and other researchers (Allsopp et al., 2007; Miller & Mercer, 1997) agree, that timely feedback is even more important when teaching learners with learning difficulties because these learners are hesitant when doing tasks, or make many errors, and timely feedback will motivate and encourage them to continue.

The forms of formative assessment most commonly recommended, and the ones that will be focused on in this study are goal-setting, monitoring the progress of learners through questioning, and giving immediate feedback. It is important for mathematics teachers to regularly ascertain whether learners understand what is being taught.

2.4.7 Conclusion

The six teaching approaches that were discussed are not only useful when teaching mathematics to learners with learning difficulties, but can be applied in any type of school. Teachers who teach learners with learning difficulties are confronted with multiple barriers to learning mathematics at a time and should therefore use as many of these approaches as possible in their lessons. By using a combination of these approaches in teaching, they will succeed in assisting learners with learning difficulties to overcome their barriers to learning mathematics. A key element for the teacher to keep in mind when planning a mathematics lesson is to make sure that learners are involved in the learning process. The challenge and goal for mathematics teachers should be to develop independent learners, and these strategies may assist them in achieving that goal.

2.5 Conceptual framework

Learners with learning difficulties have many intrinsic barriers to learning and mathematics teachers need to know what these barriers are and which teaching approaches they can employ to help those learners to overcome them. The conceptual framework is based on eight intrinsic barriers to learning mathematics identified from the relevant literature, as discussed in paragraph 2.3. Six possible teaching approaches were also identified from the literature, and were discussed in paragraph 2.4. Moreover, in the conceptual framework possible links between the eight intrinsic barriers to learning mathematics and the six possible teaching approaches were made. The eight intrinsic barriers to learning mathematics and the six possible teaching approaches will be used as categories for my fieldwork (Figure 2.1).

The eight intrinsic barriers can be divided into three categories, namely emotional difficulties, cognitive difficulties and metacognitive thinking difficulties. The first category, emotional difficulties, includes learnt helplessness, passivity and maths anxiety. Learners who display learnt helplessness have become dependent on others for assistance because of continuous failure. Passivity develops because of learners' inability to make connections with previous knowledge, and when learners become anxious about mathematics they will 'shut down', which causes maths anxiety. These three emotional barriers to learning are interlinked and one can cause the other. The second category, cognitive difficulties, includes memory difficulties, attention difficulties, processing difficulties and language difficulties. Memory difficulties include problems with retaining and retrieving mathematical information. When learners are easily distracted and struggle to focus on relevant information they have attention difficulties. Processing difficulties occur when information from the outside is distorted when it is heard, seen and/or felt. Learners who struggle with abstract or symbolic language or thinking have language difficulties. The third category, metacognitive thinking difficulties, refers to learners who find it difficult to monitor their own learning. See paragraph 2.3 for an in-depth discussion.

Intrinsic barriers to learning mathematics in learners with learning difficulties (Allsopp et al., 2007; Miller & Mercer, 1997; Dednam, 2011)
Emotional difficulties
1. Learnt helplessness – because of continuous failure in mathematics, learners expect to fail and rely heavily on someone to assist them
2. Passivity – learners are unable to make connections with mathematical concepts previously learnt
3. Maths anxiety – learners ‘shut down’ when confronted with learning something new in mathematics
Cognitive difficulties
4. Memory difficulties – difficulties with retaining and retrieving mathematical information
5. Attention difficulties – they can miss important bits of mathematical information due to distractibility
6. Processing difficulties – learners find it difficult to perceive what they see, hear and/or feel. Information coming from the outside is distorted
7. Language difficulties – learners struggle to distinguish between relevant and irrelevant language in word problems and with abstract and symbolic language and thinking
Metacognitive thinking difficulties
8. Metacognitive thinking difficulties – difficulty with monitoring their own learning

Possible teaching approaches (Allsopp et al., 2007; Lerner & Johns, 2012; Miller & Hudson, 2006; Mercer et al., 2014; Miller & Mercer, 1997)
1. Authentic context
2. Building meaningful connections
3. Concrete-representational-abstract sequencing (CRA)
4. Scaffolding
5. Problem-solving strategies
6. Formative assessment

Possible link between intrinsic barriers to learning mathematics and teaching approaches
Learnt helplessness: scaffolding
Passivity: authentic context, scaffolding and formative assessment
Maths anxiety: authentic context, building meaningful connections, CRA sequencing, scaffolding and formative assessment
Memory difficulties: authentic context, building meaningful connections, CRA sequencing and problem-solving strategies
Attention difficulties: scaffolding, CRA sequencing and problem-solving strategies
Processing difficulties: CRA sequencing
Language difficulties: CRA sequencing, scaffolding and problem-solving strategies
Metacognitive thinking difficulties: scaffolding, problem-solving strategies and formative assessment

Figure 2.1: Intrinsic barriers to learning mathematics and possible teaching approaches to accommodate learners with learning difficulties

Teachers should not only have knowledge of the eight intrinsic barriers to learning mathematics. Even more important for teachers to know is what teaching approaches to use to assist learners with learning difficulties to overcome their intrinsic barriers to learning. There are six possible teaching approaches that teachers can implement to assist learners to overcome their intrinsic barriers to learning mathematics. These six teaching approaches are: authentic context, building meaningful connections, CRA sequencing, scaffolding, problem-solving strategies, and formative assessment. All six strategies are discussed in detail in paragraph 2.4.

It is important to make the connection between the intrinsic barriers to learning and the possible teaching approaches to accommodate them. The knowledge that teachers have of the intrinsic barriers to learning mathematics should influence the instructional decisions teachers make every day in their classrooms. The connection between the knowledge of the intrinsic barriers to learning mathematics and teaching approaches was predominantly made by Allsopp et al. (2007) and summarised by Allsopp et al. (n.d.). These connections will now be discussed.

2.5.1 Emotional difficulties

2.5.1.1 Learnt helplessness

Learnt helplessness develops when learners experience continuous failure in mathematics. When learners continuously fail mathematics, it affects their motivation, cognition and emotions (Maier & Seligman, 1976). Learners with learning difficulties do not want to attempt something new as they expect to fail and would rather rely on the teacher to assist them than try doing their work on their own (Allsopp et al., 2007; Miller & Mercer, 1997; Dednam, 2011). See paragraph 2.3.1.1 for an in-depth discussion on learnt helplessness.

A teacher can assist learners to overcome learnt helplessness by using scaffolding (Allsopp et al., 2007). Scaffolding, in terms of special needs education, is when a teacher provides support in the beginning, when a new concept is taught, through teacher demonstration and guided practice (Miller & Hudson, 2006; Lerner & Johns, 2012; Mercer et al., 2014). Through teacher demonstration, a teacher can use advanced organisers that provide learners with cues to identify important aspects of a concept, thus providing structure and assisting learners to build meaningful connections between concepts (Miller & Hudson, 2006). Learners can use these advanced organisers to master a concept instead of relying on the teacher for support (Allsopp et al., 2007; Mercer et al., 2014; Miller & Hudson, 2006). The ultimate goal of scaffolding is to help learners to achieve independent learning (Mercer et al., 2014). Scaffolding can be used in bridging the gap where assistance is initially given to help learners to overcome a barrier to learning. It can help them to arrive at a stage where, through guided practice, they will build enough confidence to become independent learners.

2.5.1.2 Passivity

Passive learners lack motivation to learn and usually do not actively look for and make connections between what they already know and the new concept being taught (Lerner & Johns, 2012). These learners view mathematics as a series of disconnected concepts and will therefore only memorise information without attempting to understand new mathematical concepts (Allsopp et al., 2007). They do not know where to start when solving a problem and also have difficulty to self-regulate their own learning (Miller & Mercer, 1997; Dednam, 2011). See paragraph 2.3.1.2 for a detailed discussion.

Different teaching approaches can help learners to develop a less passive attitude towards mathematics. First, Mercer et al. (2014) suggest that scaffolding be used as a tool to assist learners to become more proactive, as the goal of scaffolding is to assist learners to think for themselves. Second, teaching concepts within an authentic context and assisting learners to build meaningful connections will assist learners with learning difficulties to start looking for and recognise the connections between what they are learning and their reality (Allsopp et al., 2007). Third, by intentionally assisting learners to make connections to previously learnt concepts the teacher will help them to make a connection between different concepts, so that they will develop the ability to use more than one approach to solve a problem. It will also help them to understand the interconnectedness of approaches and concepts (Nel & Nel, 2012). When learners realise that mathematics is not a series of disconnected concepts, they will start looking for the connections themselves (Allsopp et al., 2007). Lastly, formative assessment with goal-setting and regular feedback will assist passive learners to become motivated to continue in their efforts to complete the task (Mercer et al., 2014).

2.5.1.3 Maths anxiety

Learners with learning difficulties sometimes suffer from maths anxiety. Maths anxiety occurs when learners ‘shut down’ when they are confronted with a mathematics task that they feel they cannot complete (Jansen et al., 2013). These learners experience mathematics as difficult, undoable and irrelevant (Allsopp et al., 2007). Maths anxiety reduces learners' working memory so that they become unable to block out distractions or retain information (Blazer, 2011; Sparks, 2011; Maloney & Beilock, 2012). For a more in-depth discussion on maths anxiety, see paragraph 2.3.1.3.

According to Blazer (2011), maths anxiety can be overcome by teaching within an authentic context to show learners that mathematics is relevant. Teaching mathematics by using the CRA sequencing can also assist these learners. The use of concrete objects that they can manipulate will eventually assist the learners' understanding of abstract concepts, which could reduce anxiety (Blazer, 2011). As with learnt helplessness, learners with maths anxiety can also benefit from scaffolding. Initial assistance by the teacher can help to make learners less anxious about starting a task. Through teacher demonstration, a teacher can use advanced organisers that cue learners to important aspects of a concept and therefore provide structure (Allsopp et al., 2007). Teachers should provide timely feedback (Jansen et al., 2013) and should praise learners for both small and large successes (Wadlington & Wadlington, 2008). They should encourage learners to value not only correct answers, but any progress in learning mathematics, and should promote cooperation, rather than competition among learners (Wadlington & Wadlington, 2008).

2.5.2 Cognitive difficulties

2.5.2.1 Memory difficulties

Learners with learning difficulties, especially memory difficulties, often struggle to retain and retrieve mathematical information and to follow instructions (Gathercole & Alloway, 2008). They also experience difficulties when attempting to solve mathematical problems that require multi-step sequencing (Miller & Mercer, 1997; Allsopp et al., n.d.). Problems with retaining information are related to not understanding the concepts and having an inefficient way of storing information (Swanson et al., 2004). Problems experienced with retrieving information are related to problems with organisation and making associations when storing the information (Sousa, 2006). In other words, the problem stems from an inability to link new knowledge to existing knowledge (Allsopp et al., 2007). See paragraph 2.3.2.1 for a detailed discussion.

When assisting learners with memory difficulties to build meaningful connections, the teacher helps them to link new knowledge to existing knowledge, which is something these learners struggle to do themselves (Allsopp et al., 2007). When a teacher teaches within an authentic context, learners are better able to remember a concept because the context in which it is taught makes sense to them. If the amount of material is reduced by assisting the learners to organise the information and to store it effectively, they are able to retrieve information more easily (Gathercole & Alloway, 2008). Using CRA sequencing helps learners with memory difficulties as they will have an opportunity to see and touch a mathematical concept on a concrete level, which will help them remember it better. Teaching learners problem-solving strategies such as mnemonics, acronyms, pictures and concept maps may help them to store the information learnt more effectively and retrieve it more successfully (Allsopp et al., 2007; Gathercole & Alloway, 2008). The teacher can assist learners with poor memory by repeating important information and by encouraging them to use memory aids such as pictures and charts (Gathercole & Alloway, 2008).

2.5.2.2 Attention difficulties

Learners with learning difficulties often have attention difficulties. They are easily distracted and will not pay attention to what the teacher is explaining (Lerner & Johns, 2009). When a task requires multiple steps, they may not be able to complete it or may miss steps because they have not listened to the teacher (Miller & Mercer, 1997). They focus too much on outside stimuli and are not always able to filter out stimuli coming at them (Allsopp et al., 2007). Learners with attention difficulties can be impulsive and tend to act without first thinking things through (Dednam, 2011). For further information, see paragraph 2.3.2.2.

The teacher can use scaffolding by using visual, auditory, tactile, and kinesthetic cueing to highlight important and relevant features of a concept, thus incorporating CRA sequencing (Mercer et al., 2014; Allsopp et al., 2007). This multi-method teaching can assist learners with attention difficulties, because if they miss one cue then they have an opportunity to understand the concept when another cue is used. Allsopp et al. (2007) suggest that the teacher can assist these learners by teaching procedures connected to concepts. When learners are distracted and miss a step, the teacher can then use this type of cueing to assist them in completing the task.

Teaching problem-solving skills to learners with learning difficulties will help them to monitor their own progress and self-evaluate their performance (Polya, 1957). Outside distractions should also be kept to a minimum in order for these learners to focus on the work and not the environment.

2.5.2.3 Processing difficulties

Learners who have processing problems will find it difficult to 'correctly' perceive what they see, hear and/or feel. The problem is not related to their sight or hearing, but is caused by the way in which their central nervous systems process information (Allsopp et al., 2007). This leads to difficulties with auditory and visual discrimination and they struggle to distinguish between similar sounds or letters and will hear or read 'incorrectly' (Dednam, 2011). Processing difficulties also include visual-spatial and auditory-processing difficulties (Miller & Mercer, 1997; Dednam, 2011). For more information, see paragraph 2.3.2.3.

By using CRA sequencing, teachers can assist learners with processing difficulties. When a teacher teaches using concrete objects, learners can see and touch the objects while listening to the teacher's explanation. If this is combined with movement, where learners enact a concept with their bodies, the processing difficulties can be overcome by integrating information across modalities. Movement combined with touching/feeling and seeing an object while hearing the teacher explain it incorporates all the senses (Allsopp et al., 2007; Miller & Hudson, 2006).

2.5.2.4 Language difficulties

Many learners with learning difficulties experience language difficulties which include reading difficulties. Their language difficulties may cause them to confuse mathematical terms such as minus, take away and borrow (Wadlington & Wadlington, 2008; Allsopp et al., 2007). Word problems are especially difficult for learners with language difficulties as they not only struggle to read the problem, but also to understand the underlying mathematical language structure (Lerner & Johns, 2012). Learners with language difficulties may also experience problems when trying to communicate their understanding of mathematical concepts (Lerner & Johns, 2009; Dednam, 2011). See paragraph 2.3.2.4 for an in-depth discussion.

Since learners with language difficulties have an inadequate vocabulary, it may be useful to start a lesson by teaching vocabulary before teaching the mathematical concept. Their problem-solving skills can be developed by giving learners a chance to communicate their ideas of what they need to do by way of a class discussion (Miller & Hudson, 2006). Scaffolding can be used where a teacher assists learners and points out the important information that needs to be focused on in solving word problems. Through guided practice, teachers can help learners with learning difficulties to translate the language in the problem to mathematical symbols (Allsopp et al., 2007). By using CRA sequencing, i.e. by using concrete objects to first explain a concept, the teacher can help learners with learning difficulties to understand the concept without language. Once the concept has been grasped in its concrete form, the representational and abstract explanation can be more easily understood.

2.5.3 Metacognitive thinking difficulties

Metacognition refers to higher-order thinking and helps to control a person's cognitive processes (Rosenzweig et al., 2011; Allsopp et al., 2007). It is through metacognition that learners construct their own meaning when confronted with new information. They also use metacognition to sort and classify new information and incorporate it with previously learnt knowledge (Gargiulo & Metcalf, 2013). Learners who experience difficulties in the area of metacognitive thinking struggle to monitor their own learning, which includes evaluating whether they are learning, implementing approaches when needed and knowing if the approach implemented was successful or not (Allsopp et al., 2007; Kolencik & Hillwig, 2011). For more information see paragraph 2.3.3.1.

Creating a classroom climate where teaching for thinking is set as a priority is the first important step in teaching metacognition (Kolencik & Hillwig, 2011). A classroom climate for thinking will include posing problems, raising questions, creating dilemmas and inviting learners to engage in problem-solving; encouraging experimentation and risk taking; and remaining non-judgemental while learners explore ideas (Kolencik & Hillwig, 2011). Furthermore, learners with metacognitive thinking problems can be assisted through scaffolding, in which case teachers demonstrate metacognitive thinking by thinking aloud. In other words, the teacher should emphasise both the actions and the thinking that are involved in solving a problem

(Allsopp et al., 2007; Kolencik & Hillwig, 2011). To help these learners to become metacognitive thinkers, teachers should demonstrate problem-solving strategies and show learners how to organise themselves. As learners are asked to explain their thinking processes, either during or after a task, the teacher can determine how successful a learner's metacognitive processes are and what needs to be done to rectify them (Rosenzweig et al., 2011). Teachers should also assist learners to self-monitor and self-evaluate their performance by asking questions (Allsopp et al., 2007).

2.5.4 Summary

Teachers who teach learners with learning difficulties have a significant challenge in that their learners have to cope with multi-faceted difficulties. It would be impossible to help all these diverse learners by using a single teaching approach. Teachers must therefore be flexible and should use as many different teaching approaches as possible in order to make sure that they reach all the learners in their classrooms and develop each learner to his or her full potential. Learners with learning difficulties usually struggle with mathematics and they largely depend on their teachers to assist them. The main goal of the mathematics teacher should be to encourage, motivate and assist learners by using the teaching approaches that are best suited to helping them become independent learners. One way of reaching this goal is through interactive instruction.

2.5.5 Interactive instruction

Pon (2001) posits that teachers teaching learners with learning difficulties should be highly flexible risk takers. They should not be content with using only explicit instruction as is usually the case when teaching learners with learning difficulties. The main aim for the teacher teaching learners with learning difficulties should be to develop independent learners who can solve problems on their own (Mercer et al., 2014; Pon, 2001; Lui 2012). Mercer et al. (2014) recommended active instruction as an effective way to teach learners with learning difficulties. The teacher should direct learners in a way that will provide them opportunities to question or expand upon what was previously learnt (Pon, 2001). Vygotsky's ZPD can be connected to Mercer et al.'s (2014) interactive instruction, where the teacher targets the ZPD of learners by creating a balance between teacher guidance and learner responsibility. Learner involvement is important and learners must be allowed to ask questions and explain their answers.

Some characteristics of a classroom where teachers use a constructivistic framework for teaching and use interactive instruction is the following (Pon, 2001): the teacher is only one of many sources that learners can learn from; use real-life situations that challenge them to use existing knowledge; allow learners' responses to drive the direction of the lesson; ask thoughtful, open-ended questions and give learners thinking time afterwards; encourage learner autonomy and initiative; be willing to let go of classroom control; use raw data, manipulatives, and so on; do not separate knowing from the process of finding out; and insist that learners express themselves clearly when giving an answer, because once they can communicate their understanding the teacher can know that they have truly learnt.

In order to achieve a balance between the teacher's guidance and learners taking responsibility for their own learning is difficult. One way in which this can be achieved is through the ZPD of Vygotsky by means of scaffolding (Lui, 2012). The ZPD can be explained as the difference between the actual development level of a learner and the level of potential development (Vygotsky, 1978). In other words, the ZPD bridges the gap between what a learner can do independently and what a learner is capable of when assisted by someone else by means of scaffolding (Siyepu, 2013). Lui (2012) suggests that the most beneficial instruction will be within the ZPD just beyond learners' current level of independent capabilities (Wass, Harland & Mercer, 2011). A mathematical problem given to learners should therefore be neither too easy nor too difficult, but just challenging enough to help them develop new skills by building on the skills they already have (Lui, 2012). This can be a challenge in a classroom situation as each learner's ZPD can be different. In order for it to work in a classroom setting, the teacher should make the problem difficult so that it is challenging even for the most advanced learner. Through scaffolding, the teacher then helps each learner, giving more guidance or support to some and less to others. Once a learner, assisted by the teacher, has mastered the task, the assistance can be withdrawn and the learner will then be able to complete the task independently (Siyepu, 2013).

The role of the teacher should be one of responsive guidance, focused on developing learners' own thinking (Anghileri, 2006; Mercer et al. 2014). This can be done by the scaffolding of learning, where the teacher adjusts his or her support as the learner's understanding of mathematics develops and ultimately removes it when the learner can solve a problem independently (Anghileri, 2006).

2.5.6 Summary

This chapter began by explaining why it is important for mathematics teachers who teach learners with learning difficulties to understand and have knowledge of their learners. This knowledge of the learner includes being aware of the intrinsic barriers to learning mathematics with which learners have to cope. In addition to having knowledge of the learner, the mathematics teacher should also know which teaching approaches can be implemented to assist learners with learning difficulties to overcome their intrinsic barriers to learning. Eight intrinsic barriers to learning, as well as six possible teaching approaches, were identified and discussed. The eight intrinsic barriers discussed were learnt helplessness, passivity, maths anxiety, memory difficulties, attention difficulties, processing difficulties, language difficulties, and metacognitive thinking difficulties. The six possible teaching approaches discussed were authentic context, building meaningful connections, scaffolding, CRA sequencing, problem-solving strategies and formative assessment. Subsequently the possible connection between the eight intrinsic barriers to learning and the six teaching approaches was discussed and a theoretical framework for this study was provided. In the next chapter the methodology for this study will be discussed.

Chapter 3

Methodology

3.1 Introduction

This chapter describes the methodological approach used for this study. The research paradigm for this study will be discussed first, followed by a description of the research approach and design. This will include a detailed description on how the participants were selected. Next the three stages in which data were collected will be elaborated on, followed by a short discussion on the types of instruments that were used. The analysis of the data will then be discussed. Finally, consideration will be given to quality criteria and ethical issues.

3.2 Research paradigm

The paradigm that underpinned this study was constructivism. Plato said that knowledge is justified true belief. In other words, knowledge is where truth and belief overlap (Ichikawa & Steup, 2012). From the point of view of constructivism, truth and belief is constructed by each person individually and will therefore be unique to each person (Liu & Matthews, 2005). This construction can either be through one's own cognitive processes (cognitive constructivism), or through the social environment and interaction with that environment (social constructivism) (Terre Blanche & Durrheim, 2006).

This study is an investigation into how mathematics teachers' instructional practice can facilitate learning to help learners with learning difficulties to overcome their intrinsic barriers to learning. From a constructivist point of view, our understanding of reality is shaped by our experiences (Lincoln, Lynham & Guba, 2011) and therefore each learner's reality is different, because learners all differ in respect of their backgrounds, their levels of competency, their barriers to learning and their level of understanding mathematics. The knowledge of how to assist these learners with these diverse backgrounds, competencies and barriers is therefore also unique.

Even though knowledge in constructivism is relative and locally constructed through one's own experiences, it can also be constructed by collective experiences (Lincoln et al., 2011). Each learner's combination of intrinsic barriers to learning mathematics will be different and unique to each teacher's circumstances, but common barriers to learning mathematics do exist and these can be focused on. Allsopp et al. (2007) also indicate that similar teaching approaches can exist when teaching different concepts and teaching learners with different learning difficulties.

This study is grounded in a dialectical constructivist paradigm and therefore the methodological paradigm for this study is interpretivism. Interpretivism argues that we cannot understand and interpret a situation without understanding the perceptions, beliefs and attitudes that are involved in that situation ('Methodological paradigms,' n.d.). We therefore need to understand and interpret phenomena in context (Lincoln et al., 2011; Terre Blanche & Durrheim, 2006). For this reason I made use of qualitative research.

3.3 Research approach and design

3.3.1 Research design

This qualitative study intended to investigate the teaching approaches as well as knowledge of the participants. The aim was therefore not to manipulate or control behaviour, but rather to let events unfold and to investigate them as they happened, with no interference by the researcher. Since a case study normally attempts an in-depth investigation of a matter, the emphasis will be on quality rather than on quantity (Yin, 2009; Rule & John, 2011).

The case study design that was employed was a multiple-case study design, which allows for more than one perspective (Cohen, Manion & Morrison, 2011). Therefore the unit of analysis was individuals. Each participant's knowledge of the intrinsic barriers to learning mathematics and the teaching approaches used in the classroom were investigated. According to Rule and John (2011) a multiple-case study allows for some breadth and depth of focus and can be used to a limited degree for generalisation if common factors can be found. A multiple-case study also provided more data to work with, to base ones findings on and to make recommendations. This multiple-case study was cross-sectional as the participants' years of teaching experience and special needs training were taken into account during the sampling process.

3.3.2 Selection of participants and sampling procedures

The sampling approach for this study was purposive maximum variation sampling. Purposive maixmum variation sampling implies that the selection of the participants should be based on specific and diverse criteria (Flyvbjerg, 2011; Cohen et al., 2011). A structured questionnaire was designed to cover different criteria. The two main criteria included the participants' years of teaching experience in general and in LSEN schools, and formal training in special needs education (learning support or remedial education) (Addendum C).

Permission was obtained from three LSEN schools (public and private) in Pretoria to send out these questionnaires (Annexure A). Participants' answers were then used as the basis for selection (Table 3.1). All the teachers that taught mathematics in these three LSEN schools were women. Four primary school teachers, three from the Intermediate Phase and one from the Senior Phase, were selected. The first teacher taught Grade 6 Mathematics and had less than three years' teaching experience and no special needs training for teaching learners with learning difficulties; the second teacher taught Grade 5 Mathematics and had less than three years' teaching experience, but had received special needs training for teaching learners with learning difficulties; the third teacher taught Grade 7 Mathematics, had received no special needs training for teaching learners with learning difficulties and had more than ten years' teaching experience; the fourth teacher taught Grade 4 Mathematics, had received special needs training for teaching learners with learning difficulties and had more than ten years' teaching experience.

Table 3.1: Selection criteria for participants

Selection criteria	**Teacher 1** **School A** **Grade 6**	**Teacher 2** **School B** **Grade 5**	**Teacher 3** **School A** **Grade 7**	**Teacher 4** **School A** **Grade 4**
Less than three years' teaching experience	Yes	Yes	No	No
More than ten years' teaching experience	No	No	Yes	Yes
Special needs training	No	Yes	No	Yes
No special needs training	Yes	No	Yes	No

3.4 Instruments for data collection

Three instruments were used for data collection. The first was a pre-observation questionnaire, which consisted of two open-ended questions that probed the knowledge of the mathematics teacher concerning the barriers to learning of her own learners and the barriers to learning mathematics of learners with learning difficulties in general (Addendum D). The teachers had to list the barriers that they were aware of. The aim of this questionnaire was therefore to investigate the teachers' knowledge of the intrinsic barriers to learning mathematics with which learners with learning difficulties have to cope.

The second instrument was a structured observation schedule. After the first four observations the observation schedule was changed to include a section to take notes. The observation schedule included three aspects namely teaching approaches, teacher continuum and types of questions asked (Addendum E). Originally, when the observation schedule was designed and the observations were done, eight categories were used. Upon further study and research, demonstration and scaffolding were collapsed into one category, namely scaffolding. The category 'considering the language of mathematics' was not included as a category in the conceptual framework as it was not considered to be a suitable teaching approach for this study. A category of 'other teaching approaches' was added for when any other type of teaching approach that was not listed was observed. The aim of the observations was to determine which teaching approaches teachers used to help their learners with learning difficulties to overcome their barriers to learning.

The third instrument was a post-observation questionnaire, which consisted of three open-ended questions (Addendum F). The purpose of the first two questions was to determine the scope of the participants' knowledge of the teaching approaches for mathematics teachers who teach learners with learning difficulties. The purpose of the last question about the teacher continuum was to determine how the teachers viewed teaching. Therefore the aim of the post-observation questionnaire was to investigate how teachers think they should facilitate learning in order to help learners to overcome their intrinsic barriers to learning.

3.5 Data collection procedure

Before data was collected, a pilot study was conducted in order to test the validity of my questionnaires and observation schedule. This was done at a private inclusive mainstream school. One lesson was observed and three teachers were asked to complete two questionnaires. Their answers indicated that the questions were not clear enough. A colleague, Mr Gouws, who taught remedial education at the University of Pretoria for many years, was consulted and the necessary changes were made to the questions.

The data collection for my research was divided into three stages (Table 3.2): a semi-structured pre-observation questionnaire; structured observations; and a semi-structured post-observation questionnaire.

Table 3.2: Overview of data collection per participant

SECONDARY RESEARCH QUESTIONS	STAGES	DATA COLLECTION METHODS	DOCUMENTATION
Which different types of intrinsic barriers to learning are mathematics teachers who teach learners with learning difficulties aware of?	Stage 1	Semi-structured pre-observation questionnaire	Questionnaire on teacher's knowledge of learners' intrinsic barriers to learning
What are the teaching approaches used by mathematics teachers in order to assist learners with learning difficulties to overcome their intrinsic barriers to learning?	Stage 2	Observation	Observation per participant for four lessons by using a structured observation schedule
How do mathematics teachers think they should facilitate learning to help learners with learning difficulties to overcome their intrinsic barriers to learning?	Stage 3	Semi-structured post-observation questionnaire	Questionnaire about teacher's knowledge of how to help learners with learning difficulties to overcome their barriers to learning

3.5.1 Data collection: Stage 1

After selecting the four participants from those who had completed the structured questionnaire, I gave each participant a semi-structured pre-observation questionnaire in order to investigate their knowledge of the types of intrinsic barriers to learning mathematics that learners with learning difficulties have (Addendum D). The purpose of this questionnaire was to determine how much the mathematics teachers knew about the learners that they teach every day. The aim of this questionnaire was to answer the first secondary research question: Which different types of intrinsic barriers to learning are mathematics teachers who teach learners with learning difficulties aware of?

3.5.2 Data collection: Stage 2

During the second stage I observed the participants as they taught their lessons. I was presented as an observer and did not take part in the lessons. A structured observation sheet was used to record the observed lessons (Addendum E). After having observed the first four lessons I found it difficult to immediately categorise what I had observed. I then decided to only write down as much as I could about the lesson and to do the categorisation after the observation. After each lesson, when possible, photos were taken of the blackboard and/or any teaching support material that the teacher had used. I observed four lessons presented by each participant, which allowed me to gain first-hand experience of what was happening in the classroom. The main aim of the observations was to determine which types of teaching approaches the participants used in the facilitation of a mathematics lesson. This enabled me to answer the second secondary research question: What are the teaching approaches used by mathematics teachers in order to assist learners with learning difficulties to overcome their intrinsic barriers to learning?

3.5.3 Data collection: Stage 3

During the third stage, I gave each participant a second semi-structured questionnaire. This questionnaire investigated the mathematics teacher's knowledge of how to facilitate learning for learners with learning difficulties (Addendum F). The reason for giving them the questionnaire after the observations was to ensure that they would not be influenced by the questions on the questionnaire when I observed their lessons. I wanted to observe how they taught before asking them how they thought they ought to teach. The purpose of this questionnaire was to determine which types of teaching approaches the teachers thought they should use to help learners with learning difficulties to overcome their intrinsic barriers to learning mathematics. Furthermore, I wanted to compare the teachers' knowledge of how to facilitate the learning of learners with learning difficulties with the way they actually facilitated learning. The aim of this questionnaire was to deal with my third secondary question: How do mathematics teachers think they should facilitate learning to help learners with learning difficulties to overcome their intrinsic barriers to learning?

3.6 Data analysis and interpretation

This study was based on the constructivist paradigm and I used interpretivism to conduct the analysis and interpretation of the data collected. Each teacher had her own context, background, years of teaching experience and training, and the data had to be interpreted with that in mind (Cohen et al., 2011). The analysis of the two questionnaires and observation schedules was done deductively by using the categories that were identified from the literature and set out in my conceptual framework (Figure 2.1). For the pre-observation questionnaire about the teachers' knowledge of the intrinsic barriers to learning mathematics, their answers were categorised according to the following eight categories: learnt helplessness, passivity, maths anxiety, memory difficulties, attention difficulties, processing difficulties, language difficulties and metacognitive thinking difficulties. For the post-observation questionnaire, which focused on the teachers' knowledge of the teaching approaches, the answers were categorised into six categories: authentic context, building meaningful connections, scaffolding, CRA sequencing, problem-solving strategies and formative assessment. The words and actions of the teacher during the observed lessons were categorised either immediately or later according to the same six categories included in the post-observation questionnaire.

Each participant's pre-observation questionnaire and the observation schedule were analysed and interpreted separately. The post-observation questionnaires were compared to the appropriate observation schedules to determine whether the participants' knowledge of how learners with learning difficulties should be taught corresponded with the way in which they were observed to facilitate learning. Finally, a comparison was made between the four participants' data by way of a cross-case analysis to enable me to draw conclusions and make recommendations for future research.

The results for each participant were organised, discussed and interpreted according to the research instruments. These research instruments were in turn linked to the secondary research questions (Table 3.2). The reason for this organisation of the results was my paradigmatic perspective of constructivism. Each participant's unique background and the context in which she taught were respected in the organisation of the results and the analysis of the data. For the cross-case analysis three tables were used. Table 4.1 compared the intrinsic barriers to learning of which each participant was aware of. Table 4.2 compared the teaching strategies used by the participants during observations, and Table 4.3 compared the participants' view on how learners with learning difficulties should be taught.

3.7 Quality criteria

Several quality criteria need to be considered when doing qualitative research. One such a quality criteria is validity. According to Cohen et al. (2011), validity is basically achieved when an instrument measures what it is supposed to measure and is a matter of degree rather than an ultimate state. Different types of validity exist that are all unique but not exclusive to qualitative research.

The first type of validity applicable to this study is internal validity, also referred to as credibility. Internal validity means that the findings must accurately describe what you intend to research through neutrality, credibility, dependability, consistency and applicability of the interpretations and conclusions (Cohen et al., 2011). The purposive maximum variation sampling that was used included participants with various types of training and teaching experiences, which helped to achieve internal validity. For this study four teachers were chosen, two with and two without special needs training, and two with less than three years' teaching experience and two with more than ten years' teaching experience in LSEN schools. Questionnaires were handed out in the same order and the research for each participant was done in the same manner. Another requirement for internal validity was to make sure that my research was peer reviewed. This was ensured as every step of my research, from the research proposal defence to input and advice from my supervisor and co-supervisor, was peer reviewed. Categorising of the data was also member-checked. Construct validity was the second type of validity that applied to this study. This relates to the naming and choosing of the categories for the measuring instruments. The categories for analysis should also be meaningful to the participants and be confirmed in the relevant literature (Cohen et al., 2011). My literature study revealed that Allsopp et al. (2007) already had categories set out with which, for the most part, other researchers in the field of special needs education agreed. These categories were used for the coding of the questionnaires. In the observation schedule, the chosen categories used for my data analysis were on the schedule.

Another quality criterion that had to be considered for this study was triangulation, which can be described as looking at a particular phenomenon from different angles by using more than one data collection instrument (Cohen et al., 2011). When triangulation is achieved, concurrent validity is also achieved. In this study two questionnaires and an observation schedule were used as the data collection instruments. Theoretical triangulation occurs when one draws on competing viewpoints instead of using only one viewpoint (Cohen et al., 2011). Theoretical triangulation was achieved by using various researchers' views on the intrinsic barriers to learning mathematics and their different opinions on the teaching approaches that ought to be used when teaching learners with learning difficulties.

Reliability in qualitative research is controversial and some researchers doubt whether it can be achieved at all (Cohen et al., 2007). I achieved reliability in my observations by observing each teacher during four lessons presented on different days. This ensured that I obtained a general idea of how they normally taught. Reliability in the questionnaires was achieved by conducting a pilot study and refining the questionnaires.

3.8 Ethical considerations

Permission for this research was obtained from the Ethics committee of the Univeristy of Pretoria (Addendum G), as well as from the Gauteng Department of Education. Informed consent was obtained from the teachers who participated and from their principals (Addendum A). No informed assent from learners was obtained as no video or audio recordings were made. No photos of learners or teachers were taken, but after the lesson photos were taken of learning and teaching support material. Learners were only informed about the purpose of my visit and were assured that only their teachers would be observed (Addendum B). Participation in this study was voluntary and no one was forced to participate. Participants were informed that they were free to withdraw at any time if they wanted to.

Confidentiality, anonymity and privacy are the main concerns when doing research (Cohen et al., 2011). This was accomplished by assigning a pseudonom to each participant and the schools where they taught. The schools were referred to as ‘School A’ and ‘School B’, and the four participants as Teacher 1, Teacher 2, Teacher 3 and Teacher 4.

3.9 Summary

In this chapter I discussed constructivism as the paradigm for this study, which means that the interpretation of the data collected will be subjective and interpretive in nature. A qualitative research approach was used and an investigative case study was conducted with four teachers who taught learners with learning difficulties at two primary schools in Pretoria. The data collection was done in three stages that involved a semi-structured pre-observation questionnaire, observations and a semi-structured post-observation questionnaire. The questionnaires were used to determine the extent of the teachers’ knowledge of learners’ barriers to learning mathematics and how the participants thought they should teach learners with learning difficulties to overcome their barriers to learning. Observations were done to determine which teaching approaches were used in the classroom. Categories obtained from the relevant literature were used to interpret all the data. Finally, quality criteria and the ethical considerations that were taken into account were discussed. In the next chapter the data will be presented and the findings discussed.

Chapter 4

Presentation and discussion of the findings

4.1 Introduction

Data collection was done in three stages, as explained in Chapter 3. The data relating to each participant will be presented separately and will be interpreted separately. The data provided by each participant in the pre-observation questionnaire will be recorded and discussed first, after which data recorded during the observations will be discussed. Finally, the data obtained by way of the post-observation questionnaire will be recorded and discussed. This will be followed by a cross-case analysis.

Four teachers were selected by using purposive maximum variation sampling. Data collection started when each teacher received a questionnaire. This questionnaire was designed to investigate the teachers' knowledge of the types of intrinsic barriers to learning mathematics experienced by their learners and by learners with learning difficulties in general. The categories that were used to interpret the data were selected from the literature (see Chapter 2). Each mathematics teacher was visited four times over a period of three week in order to do classroom observations with the purpose of investigating which types of teaching approaches mathematics teachers used to teach their learners with learning difficulties. An observation schedule that contained six categories chosen from literature was used. This is discussed in detail in Chapter 2. Finally, a post-observation questionnaire was given to the teachers after the observation of their lessons. The aim of this questionnaire was to investigate how the participants thought they should teach learners with learning difficulties in order to help them to overcome their barriers to learning mathematics. The teachers' answers given in response to the questions in the questionnaires and words used while they were teaching are given in italics. The italicised sections therefore represent translated and paraphrased answers and are not necessarily direct quotes.

4.2 Teacher 1

Teacher 1 was a female teacher who was 27 years old and had just fewer than three years' special needs teaching experience. She had a BEd degree but no training in special needs education. While studying, Teacher 1 was a teaching assistant in the Computer Technology class at the special needs school in Pretoria where she taught Grade 6 Mathematics and Grade 6 and 7 Natural Science.

4.2.1 Pre-observation questionnaire

The pre-observation questionnaire was given to each teacher to investigate their basic knowledge of the intrinsic barriers to learning. Teacher 1 mentioned five of the eight categories that were identified from the literature and discussed in Chapter 2, paragraph 2.3 and summarised in Figure 2.1. The first category mentioned was memory difficulties and Teacher 1 explained that *learners struggle to work methodically*. She explained that *with a long method such as long division, learners forget the 'recipe' and will swap the steps around*. She further indicated that *learners forget previous years' work and therefore they cannot build on already learnt concepts*. Moreover, *learners' knowledge of their times tables is below average and they do basic calculations on their fingers or make use of calculators*. For the category attention difficulties Teacher 1 mentioned that *learners become distracted when they are working on word problems and therefore they may only do half of a problem*.

In terms of processing difficulties, Teacher 1 explained that *learners struggle to do assignments from two books. If the question is in one book they are not able to keep their place in one book and write down their answer next to the correct question number in the second book*. Furthermore, *learners change digits around so that 81 will become 18*. Language difficulties were also identified. According to Teacher 1, *learners struggle to find answers from a passage because many learners do not understand what they read and their vocabulary is below average*. Furthermore, *learners do not understand words such as how, when and which* and *also learners have problems with abstract thinking*. With regard to the last category, metacognitive thinking difficulties, Teacher 1 mentioned that *learners have planning and organisational difficulties*.

Teacher 1 did not mention any of the three subcategories of emotional difficulties, which include learnt helplessness, passivity and maths anxiety. The teacher was knowledgeable about the remaining five categories, i.e. memory difficulties, attention difficulties, processing difficulties, language difficulties and metacognitive thinking difficulties.

4.2.2 Observation

4.2.2.1 Background

There were ten and nine learners in the Afrikaans and English Grade 6 classes respectively. Both classes remained in their classrooms and the teacher came to them. Five learners in the Afrikaans class used computers to do their schoolwork and had a teaching assistant. Learners who were allowed to use computers experienced difficulties with writing as they had cerebral palsy or low muscle tone, which prevented them from writing properly. The teaching assistant was not friendly and did not have much patience with the learners. The English class did not have a teaching assistant and none of the learners worked on computers. Learners did not have textbooks, but the teacher selected sections from the textbook to produce worksheets to be completed by the learners and to prepare for tests and exams. The teacher used the overhead projector, blackboard and worksheets to teach. Learners were also allowed to use calculators as many of them struggled with mental calculations.

4.2.2.2 Observation: teaching approaches

Four lessons were observed over a period of three weeks on 6, 13, 14 and 20 May 2014 in order to investigate the types of teaching approaches that mathematics teachers use in their classrooms. The data was recorded by using an observation schedule. After each observation I classified the actions and words of the teacher into the categories identified on my observation schedule. The identification of the categories was based on the literature, as discussed in detail in Chapter 2, paragraph 2.4 and summarised in Figure 2.1. I took photos of the teaching material and the work explained on the blackboard. This was done at the end of each lesson as I did not want to stop or disturb the class while the teacher was teaching. The photos were not considered essential to the study. The teacher also provided me with copies of the worksheets given to the learners.

The first lesson observed was a revision lesson about the order of operations. The second lesson dealt with the concept of money and the third was a repetition of the second lesson as the learners did not understand the concepts that had been previously explained. The fourth lesson observed dealt with profit and loss.

Each of the six teaching approaches that were identified as categories will now be discussed separately. During the observation of the four lessons I noticed that the teacher repeated information and certain concepts many times. Repetition can be viewed as a behaviourist teaching approach as it encourages learners to remember facts through conditioning and, unlike the chosen teaching approaches, does not assist a teacher to teach for understanding.

Authentic context

Teacher 1 did not use authentic contexts to teach the mathematical concepts for Lessons 1 and 3. However, she did use authentic context for Lessons 2 and 4. In Lesson 2, Teacher 1 used the example of selling cupcakes or fudge and asked the learners to explain how they would calculate the profit or loss. She started Lesson 4 by asking the learners to 'create' a business. Referring back to this business and other examples of businesses mentioned throughout the lesson, she explained the concept of profit and loss. The products that had to be sold were lollypops and chocolates.

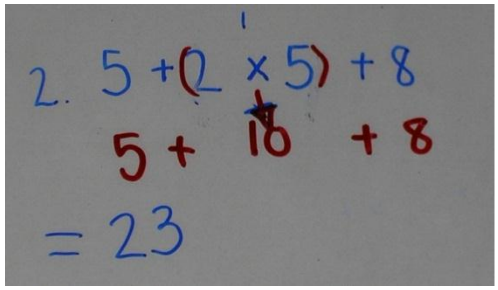
Building meaningful connections

Teacher 1 did not build any meaningful connections in her teaching of Lessons 1, 2 and 4. However, some meaningful connections were built in Lesson 3, during which she referred to the concept of Th H T U to explain the place holder concept as learners did not understand how to write, for example, 10 cents in rands (R0,10). The teacher changed the Th H T U to R100 R10 R1 T U and wrote 113 cents as rands in the follows way:

R100	R10	R1,	T	U
		1,	1	3

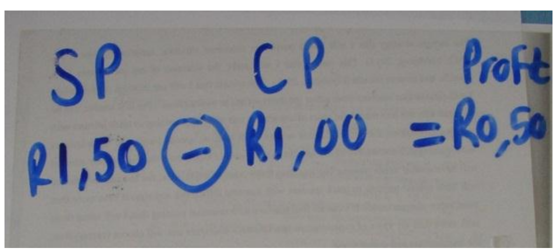
Scaffolding

In Lesson 1, Teacher 1 provided the sequence for the order of operations as the learners could not recall it. Without any further explanation she asked the learners to complete some examples such as: 5 + (2 x 5) + 8. Once the learners had completed the examples, Teacher 1 provided the answers. She then proceeded to show and explain the steps on the overhead projector (Picture 4.1).



Picture 4.1: Explaining the order of operations

In Lesson 2, Teacher 1 demonstrated to the learners how to write 1 rand as R1. She asked *how many cents there were in R1* and showed them how 1 cent is written in rand (R0,01), after which she asked them whether what she had written was correct. She explained how one can calculate how many 20-cent coins there are in R1: R1 ÷ 0,20. The teacher then explained how each question on the worksheet should be completed by doing an example of each. In Lesson 3 the teacher used Th H T U to explain how cents can be converted to rands. She copied the amount R16,53 from the worksheet and asked the learners *how many rands there were. How many 10-cent pieces? Why only 5 10-cent pieces? Do you have loose cents?* In Lesson 4, Teacher 1 demonstrated how to calculate profit and loss by using the formula: selling price (SP) – cost price (CP) = profit/loss by doing a few profit/loss problems on the overhead projector. *I bought a lollypop for R1 and sold it for R1,50 – Did I make a profit? The profit is R0,50. How do we work out the profit?* (Picture 4.2).



Picture 4.2: Explaining how to calculate profit

Teacher 1 wrote the above formula in column form, with the SP first, followed by the CP and then the profit/loss column. However, on the worksheet the columns were inverted and the cents were not in rand form, so that the learners had to convert first. The different order of the columns confused the learners. The teacher had to assist them and had to re-explain how to solve the problem with the columns changed around. She further explained that one always has to convert all the amounts to cents or rands before subtracting.

Concrete-Representational-Abstract (CRA) sequencing

Teacher 1 generally taught in an abstract way in all four lessons. During Lesson 1 she wrote the order of operations on the board: ( ), second x ÷, then + -. In Lesson 2 she used only the blackboard to explain, for example: How many 10-cent coins are needed to make R1 or how to write 5c or 50c in rand form. However, in Lesson 3 she used the representational level by using the Th H T U and drew pictures of money notes and coins on the blackboard to explain the concept of money. In Lesson 4, Teacher 1 only gave learners the formula for profit/loss. In all the lessons she wrote the problem in symbols on the blackboard, for example, in Lesson 3: R0,80 + R0,20 + R0,10 = R1,10. Teacher 1 did not use any concrete objects to explain the concepts.

Problem-solving strategies

In Lessons 1, 3 and 4 problem-solving was explained, but no in-depth problem-solving strategies were developed. In Lesson 2 no problem-solving strategies were explained. Teacher 1 gave learners only the order of operations and the formula for profit/loss (see scaffolding for the formula), which helped them to complete the worksheets for Lessons 1 and 4. Whenever the learners were uncertain about how to proceed, she referred them back to the order of operations and the formula. In Lesson 3 the teacher gave R100 R10 R1 T U as a method to convert cents to rands and rands to cents.

Formative assessment

No goal-setting took place in the lessons observed. In all four lessons Teacher 1 gave her learners worksheets to complete. During each lesson he moved around in the classroom to answer learners' questions and to check the individual progress of each learner. In Lesson 3 she asked an open-ended question and also some closed questions, such as: W*hy are there only 5 10-cent coins? Are you now going to be okay?* In Lesson 4 all the learners struggled to complete the worksheet and Teacher 1 took the time to explain to all the learners how the worksheet should be completed. The questions asked included the following: W*hat is the difference between the cost price and selling price? How do we work out the profit? What do you see? Are you going to be able to do the worksheet now?* Individual feedback was given during all the lessons when the teacher monitored the progress of the learners.

4.2.2.3 Discussion and interpretation

Teacher 1 did not use any authentic, real-life context while teaching, except in Lesson 4 she used the idea of starting an imaginary business after which the learners really became involved in the lesson. This was in stark contrast with the other lessons where no authentic context was used. During the other lessons learners were not as excited about or involved in the lessons being presented. Furthermore, Teacher 1 did not help learners to make meaningful connections, except in Lesson 3, where she used hundreds, tens, and units to explain the conversion of rands to cents, thus connecting the new concept to a concept that learners were already familiar with.

Teacher 1 started her lessons by demonstrating and explaining the relevant concepts before giving the learners one or two mathematical problems to solve and assisting them when necessary. Afterwards she handed out worksheets, which the learners had to complete on their own. However, when learners struggled the teacher was quick to assist them, but did not ask any probing questions. A significant part of the lesson time was spent on teacher demonstration and explanation. Learners were not given time to first think about a concept. For example, when the teacher was asked if the answer would be different if the problem was solved in a different way, she told the learners what would happen instead of allowing them to try to solve the problem in both ways and to discover the results for themselves. Too much assistance from the teacher will keep learners dependent on her and will not help them to develop as independent learners. Mercer et al. (2014) and other researchers agree that the goal of scaffolding is to help learners to become responsible for their own learning and to eventually be able to do their work with little or no teacher support (Larkin & Ellis, 2004; Van de Pol et al., 2010; Mercer et al., 1996).

Teacher 1 used mainly the abstract level to teach, except in Lesson 3 when she used the representational level by drawing coins on the board. She did not assist learners to develop problem-solving skills, but only provided formulae, such as the order of operations sequence and the formula for profit/loss (see scaffolding for these formulae). Furthermore, she did not teach any of the four steps of problem-solving as set out by Polya (1957) (see Chapter 2, paragraph 2.4.6 for a detailed discussion). Formative assessment was only done to a limited degree as no goal-setting was done. However, learners' progress was monitored and immediate feedback was given. Predominantly closed questions (yes/no or one answer questions) were asked, which did not provide the teacher with any indication of whether or not the learners understood the work.

From the observations one may conclude that Teacher 1 did not use a wide variety of teaching approaches. When she did use the different teaching approaches, she did so to a limited degree. She used mainly the abstract level to teach and taught exclusively through demonstrating and explaining the work. She did not use authentic contexts readily and she was not skilled in or focused on asking probing questions in order to determine whether the learners understood the concepts. Furthermore, Teacher 1 did not concentrate on assisting learners to develop problem-solving strategies. The reason for this could be that she had only about three years' teaching experience and no formal training in the area of special needs education. If learners with learning difficulties are to benefit from such teaching approaches, teachers should not only employ such approaches, but should also consider the extent to, and the way in which such teaching approaches are used.

4.2.3 Post-observation questionnaire

The four participants were asked to complete the post-observation questionnaire, which was designed to determine their views on how learners with learning difficulties should be taught. Teacher's 1 view was that *explicit teaching is the best way to teach, because there are only some 'strong' learners in a class. Most learners, about 80%, do not have the motivation or the competence to think for themselves.* She felt that *when one does explicit teaching, learners are full of self-confidence and have the tools to be able to start a task. Otherwise learners are lost. If I do implicit teaching, each learner will come and ask me if it is correct, or they will ask me what I suggest they do. I will give separate worksheets as homework for those that can benefit from implicit teaching.*

Teacher 1 mentioned five of the six categories of teaching approaches identified from the literature, which were discussed in Chapter 2, paragraph 2.4 and summarised in Figure 2.1. She mentioned that *revision needs to be done of previous years' work and the previous day's concepts and content before explaining the new concept*. Revision can be recognised as a behaviouristic teaching approach as the teacher continuously reminds learners of concepts previously learnt. Revision was not included as a teaching approach in this study as the focus was on constructivism.

The first category mentioned by Teacher 1 was authentic context. She mentioned that one *needs to make the work part of the learners' world*. Therefore, *concepts with which the learners are familiar with should form part of the teacher's lesson.* With regard to the category CRA sequencing, the teacher wrote that *it can be done by using pictures*. Moreover, *learners are allowed to rather use calculators than counting on their fingers*. For the category scaffolding the teacher suggested that *many examples must be done on the board, before leaving learners on their own to complete the worksheet.* She further explained the importance of a *teacher-learner teaching approach as it is important to correct learners as soon as they make mistakes.*

For the category problem-solving Teacher 1 explained that when learners needed to complete a task that required a lengthy procedure she would *make it easier for them by giving them a 'recipe' or a rhyme*. For the category of formative assessment she mentioned that she would *move around in the classroom while they complete the worksheet,* and she would then *point out any mistakes they made*.

Teacher 1 did not mention the category of building meaningful connections. However, she did make some mention of the other five categories. She knew that learners struggle to understand certain concepts that were not concretely represented, but her solution for this was to use pictures only. No mention was made of the possible use of concrete objects and the representational level was also not mentioned. The teacher's idea of scaffolding was to assist learners as soon as they struggle and to demonstrate first before giving learners time to practise. She did not understand the concept of problem-solving strategies as she suggested giving them 'recipes' and rhymes.

Lastly, Teacher 1's view of formative assessment included only learner monitoring and giving feedback. Goal-setting was not mentioned.

4.2.4 Comparison between the results of the observations and the post-observation questionnaire

A link can be seen between the answers that Teacher 1 gave in the post-observation questionnaire and her instructional practice. The way she described her teaching corresponded closely with how she taught in the classroom. She did not use concrete objects to teach, but rather taught in an abstract way. She monitored the learners closely and corrected their mistakes immediately. Even though she had indicated that one should teach within an authentic context, her teaching did not reflect this. She also mentioned that learners should be given 'recipes' or rhymes to help them to solve complex problems, even though those were not used, she did gave them formulae to solve the mathematical problems.

4.2.5 Conclusion regarding Teacher 1

Teacher 1's view that learners with learning difficulties require explicit teaching was confirmed by the observations and the post-observation questionnaire. This teacher explained the mathematical concepts and she controlled learning through constant repetition of information and rote learning by giving, for example, the rule of operations. When learners struggled with a task, she helped them immediately and showed them where they had made a mistake. In other words, the teacher did the thinking for the learners and did not use the opportunities that were available for constructivism. Learners were not given a chance to explain how they thought a problem should be solved or why they had done their calculations in a particular way. Mainly closed questions were asked. Even though Teacher 1 used the six teaching approaches, she lacked an in-depth understanding of them. The way she actually taught was very close to how she explained her views on how she believed teaching should take place. She knew most of the intrinsic barriers to learning that learners with learning difficulties face every day. The only intrinsic barriers to learning mathematics that were not identified were the three emotional difficulties, i.e. learnt helplessness, passivity and maths anxiety. Her inadequate knowledge and failure to use many of the teaching approaches could possibly be linked to Teacher 1's inexperience as a teacher (less than three years) and the fact that she had not had any specific special needs or remedial training.

4.3 Teacher 2

Teacher 2 was a 30-year-old female with two years' special needs teaching experience. She had completed a BCom degree, followed by a PGCE diploma (majoring in Inclusive Education) and was busy with a BEd Honours degree (Remedial Education and Learning Disabilities). She taught Grade 5 Mathematics and Grade 6 Afrikaans at a Pretoria school for learners with learning difficulties.

4.3.1 Pre-observation questionnaire

Each of the four participants was asked to complete the pre-observation questionnaire in order to determine how familiar they were with the intrinsic barriers to learning mathematics. Teacher 2 identified six of the eight categories that were identified from the relevant literature, and discussed in Chapter 2, paragraph 2.3 and summarised in Figure 2.1. The first category mentioned was learnt helplessness and Teacher 2 explained *that learners lack confidence as they come from mainstream schools where they developed feelings of failure*. With regard to the category passivity, she wrote *that learners lack motivation and have backlogs in understanding basic mathematical concepts which will lead to an inability to connect current concepts with previous knowledge*. *Learners dislike mathematics due to prior failure*, which falls in the category maths anxiety.

With reference to the category attention difficulties, the teacher mentioned that *learners struggle to concentrate and focus for longer than a few minutes at a time and become bored quickly if activities and teaching methodology are the same every day.* Also, *touch and noise sensitivity may cause some learners to be overstimulated and therefore too distracted to work.* Examples of processing difficulties mentioned were that *learners change or swop digits which lead to miscalculations, and that the work speed can sometimes be too fast for learners*. *Medication can have negative side effects and tends to make them lethargic and their brains not to function 'properly'*. Language difficulties included *problems with understanding long division or fractions as these concepts can be too abstract for learners.*

Teacher 2 was familiar with most of the intrinsic barriers to learning mathematics and although she mentioned all three categories of emotional difficulties, no reference was made of memory difficulties, which fall under cognitive difficulties, and metacognitive thinking difficulties.

4.3.2 Observation

4.3.2.1 Background

Two classes were involved in the study, an English class with 10 learners and an Afrikaans class with 14 learners. Except for some posters against the back wall, little information was displayed in the classroom. The reason for this was to keep distractions to a minimum. The learners used textbooks and also completed worksheets compiled by the teacher. An overhead projector, the blackboard, and manipulatives were sometimes used to teach mathematics.

4.3.2.2 Observation: teaching approaches

Four lessons were observed during a period of three weeks on 6, 13, 19 and 23 May 2014, in order to gain information on the types of teaching approaches used by mathematics teachers. The data was recorded by using an observation schedule. After each observation the teacher's actions and words were classified according to the categories that had been identified during the literature research, as discussed in Chapter 2, paragraph 2.4 and summarised in Figure 2.1. I took photos of the teaching material and work explained on the blackboard. I only took photos at the end of the lesson, as I did not want to stop or disturb the teaching taking place. The photos were not considered essential to the study. The teacher also gave me copies of the learners' worksheets.

The main theme at the time of the observations was fractions. Lesson 1 was about the concept of fractions and equivalent fractions and Lesson 2 dealt with the addition and subtraction of fractions with different denominators. During Lesson 3 multiplication with fractions and natural numbers was explained, and Lesson 4 was about mixed fractions.

Each of the six teaching approaches that were identified as categories will now be discussed separately. While observing the four lessons, I noticed that the teacher repeated information and certain concepts many times. She also revised concepts that had been dealt with during previous lessons. Repetition and revision can be viewed as behaviourist teaching approaches as, unlike the chosen teaching approaches, they condition learners to remember facts and do not support teaching for understanding.

Authentic context

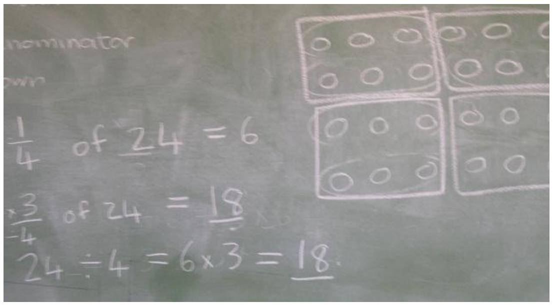
Teacher 2 used authentic contexts in Lessons 1 and 3, but not in Lessons 2 and 4. In Lesson 1 she used the example of slices of pizza and a bar of chocolate to revise fractions. She started Lesson 3 by drawing 24 circles representing sweets on the blackboard. She then asked learners: *How many would half of the 24 sweets be?*

Building meaningful connections

Teacher 2 used the building of meaningful connections in Lesson 1, 2 and 4, but no such connections were evident in Lesson 3. In Lesson 1 she asked the learners where they had seen equivalent fractions before. They indicated the fraction wall and diagrams, for example a chocolate drawn on the blackboard. In Lesson 2, Teacher 2 first revised the addition and subtraction of fractions with the same denominator before teaching addition and subtraction with different denominators. In Lesson 4 she asked the learners to explain what whole numbers and fractions were before explaining the concept of mixed numbers.

Scaffolding

In Lesson 1, Teacher 2 demonstrated how fractions can be created by folding and cutting a paper plate. She explained what the learners should do, for example: *hold the board like this and cut it neatly like this* or *write a half here, like this*. She started with simple concepts and slowly increased the degree of difficulty of the questions, for example: *How much is a half and a quarter? If you take away a half and an eight, how much is left over?* Every time she used the cut paper plates to help the learners to visualise the answer. The learners were not allowed to use their own initiative. In Lesson 2 she did various mathematical problems with the learners, reminding them of the following two golden rules: *The denominators have to be the same, and everything you do at the bottom must also be done at the top*. Example: $\frac{2}{5}$ x $\frac{2}{2}$ $+\frac{1}{10}=\frac{4}{10}+\frac{1}{10}=\frac{5}{10}$. Learners were given an opportunity to solve a problem with the teacher. When a learner gave the wrong answer, he or she was corrected by the teacher. Thereafter the teacher handed out a worksheet that learners had to complete on their own. When they struggled to complete the worksheet she assisted each learner individually by pointing out their mistakes. When learners did not know how to continue, she was quick to help. In Lesson 3 she demonstrated how circles can be drawn on the blackboard to calculate the answer (Picture 4.3). She also showed them how to do this mathematically (Picture 4.3).



Picture 4.3: Explaining how to multiply with fractions

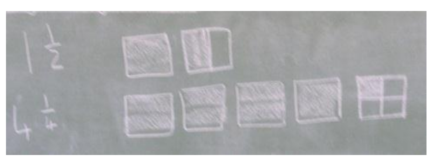
Teacher 2 controlled the learning as she did all the explaining. Open-ended questions were asked, for example: *How many groups must you divide the circles into?* These questions helped the learners to think for themselves. A rule was given to assist them to solve mathematical problems of this type: The big number ÷ by the bottom number x by the top number, for example: $\frac{2}{3}$ of 24 = (24 ÷ 3) x 2 = 8 x 2 = 16. When the learners did not understand how to divide the drawn circles on the board, she made the problem easier for them by using $\frac{1}{4}$ instead of $\frac{2}{3}$. In Lesson 4 the teacher asked one learner to assist her in demonstrating what mixed numbers looked like. The learner helped to hold up the paper plates. The teacher explained that mixed numbers are formed by putting whole numbers and fractions together. She demonstrated this couple of times by using different examples of how a diagram of mixed numbers can be draw and how to write a mixed number when given a diagram. Learners then completed a worksheet on their own. Even though she asked open-ended questions, she controlled the learning by telling them what to do and how to do it.

CRA sequencing

Some form of CRA was used in all the lessons except Lesson 2. For Lesson 1, learners had to bring their own paper plates. Each paper plate was cut up into halves, quarters and eights (concrete level) (Picture 4.4).

Picture 4.4: Using paper plates to explain the concept of fractions

Teacher 2 started the lesson by drawing a pizza and a bar of chocolate on the blackboard (representational level) and writing the appropriate fraction on each pieces in symbols (abstract level). In Lesson 3 she used the representational level by drawing circles on the blackboard (Picture 4.3), and also the abstract level by writing the symbols that each of the circles represented. No concrete objects were used. In Lesson 4 she used the paper plates from Lesson 1 to demonstrate the concept of mixed numbers (Picture 4.4). She also explained the correct fractions by drawing diagrams on the blackboard and then writing the correct mixed numbers in symbol form (Picture 4.5).



Picture 4.5: Representation of mixed numbers

Problem-solving strategies

No problem-solving strategies were taught in Lessons 1 and 4 as only the recognition of basic fractions and mixed numbers was taught. Only basic problem-solving strategies were developed in Lessons 2 and 3. In Lesson 2 the teacher only gave learners the two golden rules: make the denominators the same and whatever you do at the bottom must also be done at the top. The golden rules were also written on the worksheet to serve as a reminder. In Lesson 3, Teacher 2 gave learners the option of either using the drawn circles to calculate the answer, or using the formula: the bigger number ÷ by the bottom number x the top number.

Formative assessment

Teacher 2 did not set any goals for the learners in the observed lessons. Learner monitoring and feedback were done during all the lessons. Lesson 1 was a practical lesson and many questions were asked to make sure that the learners understood the work and to monitor their progress. Open-ended and closed questions were asked throughout the lesson. Examples of closed questions: *Do you agree that this represents one whole pizza? What makes a half? How many eights make a whole?* Examples of open-ended questions: *Two over two, four over four and eight over eight what do you observe? The bigger the number below the line is, the smaller the fraction is. Why?* In Lessons 2 and 3, Teacher 2 moved around the classroom and assisted learners individually. In Lesson 2 she asked many open-ended questions, such as: *What must I do? Help me! Why? Why do I want 9? Why do you want to change that fraction?* In Lesson 3 open-ended questions were also asked, for example: *How did you get 12? How are you going to divide it into three groups?* Explaining mixed numbers took most of the lesson time in Lesson 4. The teacher asked open-ended and closed questions such as: *Is this a mixed number? Why? How will I draw this mixed number? Why not?*

4.3.2.3 Discussion and interpretation

Teacher 2 taught within an authentic context to a limited degree. The examples that were used were basic and brief. In Lessons 2 and 4 no authentic contexts were used. The teacher assisted learners in building meaningful connections. In Lesson 1 she asked learners to make the connections themselves by asking them where they had seen equivalent fractions before. In the other lessons the teacher made the connections for the learners.

Most of lesson time was taken up by demonstrating and explaining the mathematical concepts. The teacher also invited learners to solve problems with her on the overhead projector and then gave them worksheets to complete on their own. She was in control of the lessons and she controlled all the aspects of learning. Learners were told what to do, when to do it, and how to do it. Learners were not allowed to use their own initiative to solve the problems. This does not correspond with what Mercer et al. (2014) and other researchers (Larkin & Ellis, 2004; Van de Pol et al., 2010; Mercer et al., 1996) regard as the right way to use scaffolding. These researchers posit that scaffolding should be used to produce independent learners. By controlling how learners solve problems and not allowing them to use their own initiative, they are not given an opportunity to develop as independent learners and continue to depend on the teacher for solving mathematical problems.

Teacher 2 used paper plates as concrete objects to demonstrate fractions. She made ample use of the representational level in the CRA sequencing; drawing pictures of pizzas and chocolates, drawing circles representing sweets, and colouring in blocks to represent mixed numbers. These concrete objects and pictures were always accompanied by the mathematical symbols. This allowed learners to make a connection between the abstract concept and how that concept was physically represented. The problem-solving strategies that were taught were basic and not according to the way Polya (1957) suggested (see Chapter 2, paragraph 2.4.6 for a detailed discussion). Polya (1957) recommended four steps for problem-solving, but Teacher 2 only gave the learners rules to follow when solving a specific problem, for example: The bigger number ÷ by the bottom number x the top number. Formative assessment was done mainly by asking predominantly open-ended questions to monitor learner progress and feedback was then given. By asking open-ended questions, the teacher indicated that she was interested in finding out what the learners thought about the concepts dealt with. No goal-setting, as recommended by Mercer et al. (2014), was done.

From the observation, it can be concluded that Teacher 2 made use of all the expected teaching approaches as listed in Table 2.3. Some of the teaching approaches, such as using an authentic context and problem-solving strategies, were used to a limited degree only. She used the CRA sequencing by using the representational level together with the abstract level. However, few concrete objects were used. Scaffolding was used, but she controlled the learning excessively by telling the learners how to solve the mathematical problems instead of enabling them to become independent learners. Teacher 2 did well by asking not only closed questions, but also open-ended questions that were used to gage learner understanding. Much emphasis was placed on the repetition of concepts while teaching, and revision was done before a new concept was introduced. Most or all of the teaching approaches were used during lessons, even though some were used only briefly and others more elaborately. Teacher 2 had only two years' teaching experience, but had undergone some formal training in special needs education and inclusive education, which was evident from the fact that she made use of the teaching approaches. However, she lacked the skills that would have enabled her to use all the teaching approaches to the fullest extent.

4.3.3 Post-observation questionnaire

The post-observation questionnaire that was given to each participant had been designed to determine their views on how learners with learning difficulties should be taught. Teacher 2 was of the opinion that *a combination of explicit and implicit teaching should take place. Many learners with learning difficulties struggle to work independently and they need considerable guidance. They do, however, need to be taught how to think for themselves.* She believed that she *should therefore explain the work properly and move around the classroom to give guidance and share ideas on how to find solutions.*

Teacher 2 identified three of the six categories of teaching approaches and also mentioned that the teacher should *use a variety of teaching methods, set up small victories daily, work slowly but smartly, should do many examples in class, and revisit basic mathematical skills which many learners with learning difficulties still lack.* The three categories that were not mentioned were building meaningful connections, problem-solving strategies and formative assessment.

The first category that was mentioned was teaching within an authentic context. It was indicated that the teacher should *use contextual examples with which the learners are familiar with.* With regard to scaffolding, Teacher 2 mentioned that *these learners need guidance, but that they need to be taught to think for themselves.* For the category CRA sequencing Teacher 2 recommended the *use of concrete teaching resources whenever possible.*

Teacher 2 did not mention many of the expected categories and referred to only three of the six teaching approaches. Only the concrete level was mentioned to assist learners who struggle to think abstractly. No reference was made to the use of the representational level. The teacher felt that guidance was important and that repetition and revision of concepts were necessary.

4.3.4 Comparison between the results of the observation and the post-observation questionnaire

There was a marked difference between how Teacher 2 thought she should teach her learners and the way she actually taught during observation. She used more teaching approaches than she had mentioned on the questionnaire. Much guidance was given and work was explained to the learners. She also assisted learners who needed help. Although Teacher 2 indicated that learners should be encouraged to think independently, her teaching did not reflect this view as she controlled both the learning and the way in which the learners did their work.

Teacher 2 mentioned the importance of authentic contexts when teaching and adhered to this requirement to a limited degree. She also mentioned the need for repetition and revision, which was demonstrated in her teaching. A notable difference between the answers given in the questionnaire and what was observed in the classroom was that Teacher 2 mentioned that one should use concrete objects while teaching, but that during observation she showed a preference for using the representational level together with the abstract level. She built meaningful connections during the lessons, used limited problem-solving strategies and formative assessment. However, she had made no mention of any of these teaching approaches when she completed her questionnaire.

4.3.5 Conclusion regarding Teacher 2

Even though Teacher 2 viewed teaching as guiding learners and helping them to think independently, her teaching did not reflect this as explicit teaching was the main mode of teaching. She controlled all the learning and how learners did their work. However, she did encourage the learners to think about what had been taught by asking open-ended questions. Many of the teaching strategies used had not been mentioned in the questionnaire, but she did use all six teaching strategies during the observation period. It may be said that how a teacher teaches in the classroom is more important that the information obtained by way of a questionnaire. Some of the teaching strategies used were only brief and basic, and the teacher had no real understanding of how all the teaching strategies could be implemented fully in order to benefit the learners. However, she did have a fair idea of the intrinsic barriers to learning that her learners with learning difficulties face every day. The only barriers that were not identified were memory difficulties and metacognitive thinking difficulties.

Her knowledge of the intrinsic barriers to learning and the use of the teaching approaches can possibly be attributed to Teacher 2's formal training in remedial education and learning disabilities. The lack of depth with which the teaching approaches were applied could possibly be linked to her lack of practical experience in the classroom (two years' teaching experience).

4.4 Teacher 3

Teacher 3, a 57-year-old female, had taught in a mainstream school for 22 years, which was followed by a period of 11 years in a special needs school. She had a teaching degree and no specific special needs training and taught Grade 7 Mathematics in a special needs school in Pretoria.

4.4.1 Pre-observation questionnaire

In the pre-observation questionnaire completed by the participants to inform the researcher on their basic knowledge of the intrinsic barriers to learning mathematics, Teacher 3 mentioned six of the eight intrinsic barriers to learning that were identified from the literature, as discussed in Chapter 2, paragraph 2.3 and summarised in Figure 2.1. The first category mentioned was passivity. The teacher explained that *learners do not have confidence to attempt a given task.* Referring to the category memory difficulties, she stated that *learners have poor memory, both short- and long-term memory.* Furthermore, *learners also do not remember steps that you teach them for solving problems, such as for the addition and subtraction of fractions.* With regard to the category attention difficulties, she noted that *learners have a short attention span.*

Learners turn numbers around and also struggle to work vertically to solve problems because of a poor number concept. These examples indicate processing difficulties. The teacher also identified language difficulties as *learners read poorly, they have poor language and vocabulary, and they find it difficult to visualise word problems.* Furthermore, *learners have* *little insight into what you ask and what they read.* Metacognitive thinking difficulties were also mentioned and she stated that *learners have a problem with time management. They either do not complete a problem or they rush and make many mistakes.*

Teacher 3 failed to identify two of the subcategories of emotional difficulties, namely learnt helplessness and maths anxiety. She was knowledgeable about the remaining six categories of intrinsic barriers to learning, which include passivity, memory difficulties, attention difficulties, processing difficulties, language difficulties and metacognitive thinking difficulties.

4.4.2 Observation

4.4.2.1 Background

The observation was done in two Grade 7 classes - an English class with 13 learners and an Afrikaans class with eight learners. Two of the learners in the English class used computers for their schoolwork as they experienced difficulties with writing. The learners remained in their classrooms while the teachers change classes for teaching. Teacher 3 used the whiteboard, worksheets, and sometimes manipulatives to teach. Learners did not have textbooks, but completed and learnt from worksheets created by the teacher. One learner in the Afrikaans class had an assistant to help him with organisation, etc.

4.4.2.2 Observation: teaching approaches

Five lessons were observed during a period of three weeks on 8, 12, 14, 20 and 22 May 2014. Since the lesson presented on 12 May was a repetition of the lesson presented to the English class on 8 May I decided to observe an extra lesson. An observation schedule as used to record the data and after each observation the teacher's actions and words were classified into the categories identified from the literature, which were discussed in Chapter 2, paragraph 2.4 and summarised in Figure 2.1. Whenever possible, photos were taken of the teaching material and work explained on the blackboard at the end of a lesson as I did not want to disturb or interrupt the teaching process. The teacher also provided me with copies of the worksheets used during the lessons.

The mathematics that was taught during this time focused on fractions. Lesson 1 was an introduction to equivalent fractions and consisted of work done during the previous year. Lesson 2 dealt with how to change fractions into equivalent fractions. The focus of Lesson 3 was the addition and subtraction of fractions with different denominators, and in Lesson 4 the learners were taught how to do multiplication and division of fractions.

Each of the six teaching approaches that were identified as categories will now be discussed separately. During the observation of the four lessons I noted that the teacher repeated information and concepts many times. She also revised concepts that had been taught previously or work done during previous lessons. Repetition and revision can be viewed as behaviourist teaching approaches as they condition learners to remember facts and do not support teaching to promote understanding. These behaviouristic teaching approaches were not used as this study focused on constructivist teaching approaches.

Authentic context

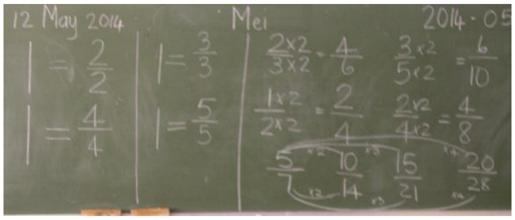
Teacher 3 used authentic contexts in the first three lessons. In Lesson 4, when multiplication and division of fractions were taught, no authentic contexts were used. The teacher started Lesson 1 by calling learners to the front and giving them R10 in different formats, for example one learner was given a R10 note, another received two R5 coins, and the last one received a R5 coin, two R2 coins and a R1 coin. The teacher pretended that they were a family and that the money was pocket money. She also used the example of a cake that had to be cut into slices to demonstrate division. In Lesson 2 she used the example of a sandwich that had to be divided, and also the marks awarded for a test. During Lesson 3 the learners had to complete worksheets containing word problems. She started the lesson by using the example of cooking porridge, explaining that one had to follow a recipe as it was necessary to know what steps had to be followed.

Building meaningful connections

Teacher 3 built meaningful connections in Lessons 1 and 4, but not in Lessons 2 and 3. In Lesson 1 she talked about *equal rights*, which she equated to the amount that she gave to each of the learners (R10). *To be fair, each learner must receive the same value, even if the amounts appear to be different.* This was equated to equivalent fractions. In Lesson 4 the teacher reminded learners of the addition and subtraction of fractions and explained *that one of the first steps was to change the mixed numbers into improper fractions. This also applies when we multiply fractions, but we do not have to calculate the LCD* (lowest common denominator) *when we multiply fractions*.

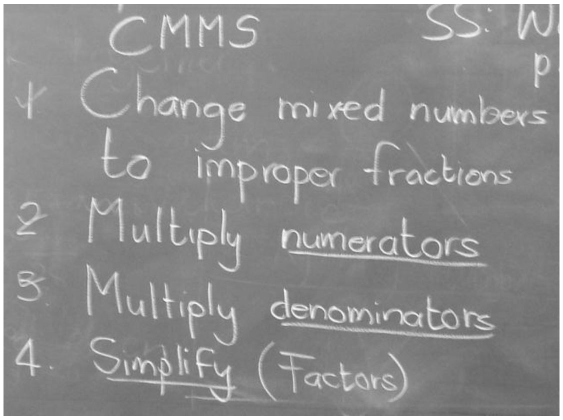
Scaffolding

In Lesson 1, Teacher 3 explained the concept of equivalent fractions by using various wooden manipulatives (Picture 4.8), but explained equivalent fractions by telling each learner how many wooden blocks they had to put on their wooden boards. She sometimes made a wrong statement or gave an incorrect answer to see whether the learners noticed the mistake, for example: *I gave him more money than I have her, because she has four coins and he has one.* Another example, $\frac{3}{7}+\frac{2}{7}+\frac{2}{7}=\frac{7}{21}$, was given to evaluate whether they had listened and had understood the concept. However, learners were not given an opportunity to do something on their own without the teacher first explaining it to them. In Lesson 2 she explained and demonstrated how to calculate equivalent fractions by multiplying with 1: $\frac{2}{2}$ or $\frac{3}{3}$ or $\frac{4}{4}$ and so on (Picture 4.6). She explained the concept and when learners had to complete the worksheet and struggled, she told them that they had to learn to do it themselves and encouraged them to draw a picture of the word problems to try to solve them without assistance.



Picture 4.6: Explanation of equivalent fractions

In Lesson 3, Teacher 3 explained how two fractions with different denominators had to be added together and gave a step-by-step demonstration to show the learners how to solve the problem. Example: $\frac{2}{3}+\frac{4}{9}$. While she was explaining the problem, she asked the learners many questions, for example: C*an I add the two fractions? What do I need to do now?* She then showed that the LCD was needed first: M_3: 3, 6, **9**, 12, 15 and M_9: **9**, 18, 27, therefore the LCD is 9. Using a coloured pen, she added the multiplication: $\frac{2}{3} \times \frac{3}{3} + \frac{4}{9}$. Afterwards one of the learners was invited to be the teacher. He had to explain what he was doing, and was also allowed to ask the other learners what he should do when he did not know how to continue. The teacher helped him from time to time by telling him what type of questions he should ask. In Lesson 4 the teacher broke down the concept of multiplication of fractions into four steps and suggested that the learners memorise the letters CMMS (Picture 4.7) to remind them of the steps.

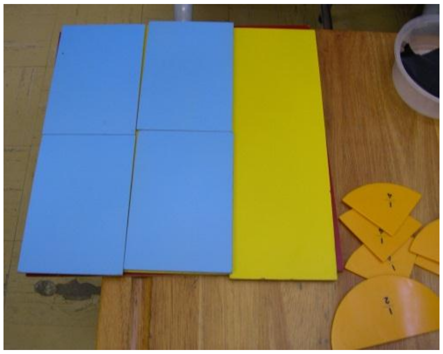


Picture 4.7: Four steps followed when multiplying fractions

In this lesson the concept was taught for the first time, therefore, the teacher did all the explaining. She explained the steps that had to be followed and first did an easy and then a more difficult example while using the four steps that had been explained at the start of the lesson.

CRA sequencing

The teacher used CRA sequencing in Lessons 1 and 2, but not in Lessons 3 and 4. In the last two lessons she used only the abstract level to explain the concepts. In Lesson 1 she used real money to demonstrate that the same amount may look different, but that the value remained the same. Teacher 3 gave each learner different wooden or plastic pieces that represented fractions that they had to manipulate before she drew diagrams on the board and wrote the equivalent fraction in symbols next to each diagram. The learners could therefore not only listen to her and look at what she was doing, but could at the same time manipulate the pieces they had been given (Picture 4.8).



Picture 4.8: Wooden and plastic pieces used to demonstrate equivalent fractions

In Lesson 2 the teacher used the representational level by drawing a diagram to re-explain equivalent fractions and also wrote the problem on the whiteboard in symbols. A sandwich that needed to be divided was drawn on the whiteboard. No manipulatives were used during this lesson.

Problem-solving strategies

In Lesson 1, Teacher 3 gave the learners a rule for calculating equivalent fractions: *What you do at the top you need to do at the bottom, therefore you need to always multiply by one.* In Lesson 2 she asked them to give the steps that are followed when solving word problems. The steps that were given were: *You write the word problem, then you do the calculation and then you write an answer sentence.* She also advised learners to first draw a picture or diagram as this is useful when solving a word problem. In Lesson 3 the learners were reminded of the steps for adding or subtracting mixed numbers: *First you circle the operation as a reminder that you need to add or subtract, then you add or subtract the whole numbers. Then make the denominators the same by finding the LCD. Then add or subtract the fractions and if necessary, you simplify the fraction.* In Lesson 4 the teacher explained the four steps to solving problems of multiplication with fractions by using the acronym of CMMS. *Change mixed numbers into improper fractions, multiply numerators, multiply denominators and simplify* (Picture 4.7).

Formative assessment

Teacher 3 set no goals for the learners in any of the lessons. Formative assessment was done in Lessons 1, 2 and 3, but not in Lesson 4, during which there were many interruptions and she had to start the lesson late. In Lesson 1 she did monitor the learners' progress by asking many open-ended and closed questions that encouraged them to think for themselves, for example: *What did I write? Why? What does it mean? Do you understand?* In Lesson 2 learners had to complete a worksheet with word problems. Teacher 3 assisted them individually to monitor their progress. When they asked her questions, she did not give them the answers but asked whether they had drawn a diagram or picture to illustrate the word problem before she would assist them. In Lesson 3 she gave them feedback on their homework. To monitor their progress the teacher asked many open-ended questions, for example: *How do I make the denominators the same? Can I add the denominators? Why? Why does the denominator remain the same?*

4.4.2.3 Discussion and interpretation

For the most part Teacher 3 used authentic contexts to teach the concepts, but she only helped the learners to make meaningful connections in Lessons 1 and 4. She did most of the explaining and demonstrating of concepts and she controlled the learning that took place. In Lesson 1, each learner was given some pieces of wood to work with, but she told them how many pieces they should place on their boards. Teacher 3's teaching method resembled that of Mercer et al.'s (2014) interactive instruction or guided practice (see Chapter 2, paragraph 2.5.4). She asked many open-ended questions, which encouraged independent thinking. When Teacher 3 explained something, she normally first asked learners what they thought she should do. One learner also had an opportunity to 'teach' the class. The teacher sometimes gave wrong answers or made an incorrect statement to check whether the learners were paying attention and understood the concept being taught. However, much time was spent on demonstrating and explaining concepts, with the result that little time was left for independent practise. The teacher attempted to encourage learners to think independently and to use the tools that she had given them. She therefore succeeded to some degree in doing scaffolding in the three phases described by Miller and Hudson (2006) and Van de Pol et al. (2010).

In the first lesson Teacher 3 applied the complete CRA sequencing and used concrete objects (wooden boards) to teach the concept of equivalent fractions. She then drew diagrams and wrote the equivalent symbols on the whiteboard. She also used real money to explain equivalence. In Lesson 2 she used the representational and abstract levels and in Lessons 3 and 4 the abstract level only. Teacher 3 taught her learners problem-solving by providing them with specific steps, but did not use the four steps recommended by Polya (1957) (see Chapter 2, paragraph 2.4.6). Formative assessment was done by asking many open-ended questions to check for learner understanding. This was the main form of formative assessment that was used. She did much repetition and revised concepts frequently during the observed lessons.

From the observations one might conclude that Teacher 3 did not use all the teaching approaches to their fullest extent, but did use authentic contexts and helped the learners to build meaningful connections in some of the lessons. She used scaffolding and encouraged interaction and learner participation. Her solution to teaching problem-solving skills was to give the learners specific steps that could be learnt and followed. The way she asked questions was encouraging as she really attempted to promote independent thinking. This teacher had many years of experience as a teacher in special needs education, which was evident from the way she approached teaching. However, one would have expected her to use the teaching approaches more effectively.

4.4.3 Post-observation questionnaire

In the post-observation questionnaire completed by the participants to gain and understanding of how they thought learners with learning difficulties should be taught, Teacher 3 indicated that she believed that *a combination of explicit and implicit teaching is needed, therefore guided learning.* According to her *it is important that learners must discover and learn for themselves. This is the only way they can master mathematics, but the teacher must lead them and help them. Learners can also sometimes explain things better to each other.* She further believed that *too many methods are not good for learners, but you still need to teach all the possible methods to the advanced learners.*

Teacher 3 mentioned only two of the six teaching approaches, as identified from the literature and discussed in Chapter 2, paragraph 2.4 and summarised in Figure 2.1. She recommended doing *different mental exercises before you start with the actual lesson.* The first category mentioned was CRA sequencing, which *can be done by encouraging learners to draw a picture to illustrate their understanding, for instance of a word problem.* For scaffolding the teacher mentioned that *you need to start with easy concepts. It is important that learners must discover* *and learn for themselves. This is the only way they can master mathematics, but the teacher must lead them and help them.*

Teacher 3 failed to mention the majority of the categories. Categories not mentioned were authentic context, building meaningful connections, problem-solving strategies and formative assessment. It is possible that she did not put much effort into completing the questionnaire. As far as CRA sequencing is concerned, no mention was made of the concrete level. Her view about guided learning showed that she did have a good understanding of how scaffolding should take place.

4.4.4 Comparison between the results of the observation and the post-observation questionnaire

The method of teaching demonstrated by Teacher 3 during observation did not reflect the views given in her response to the questions in the questionnaire. Although teaching within an authentic context, building meaningful connections, problem-solving strategies and formative assessment were not mentioned in the questionnaire, she did use them to a limited degree when she taught.

The way she taught showed some similarities to the three categories mentioned in the questionnaire. The teacher mentioned using revision and did in fact do so. She taught the learners according to her view of scaffolding. Although she mentioned only the representational level in the questionnaire, she did use concrete objects in some of the lessons observed.

4.4.5 Conclusion regarding Teacher 3

The way Teacher 3 taught did not fully correspond with her views about how learners with learning difficulties should be taught, as expressed in the questionnaire. She mentioned that learners should discover and learn independently, but this was not reflected in her teaching. She controlled the learning and learners were given specific steps for solving problems. The teacher did, however, ask many open-ended questions, which encouraged learners to think about the problems. She believed that learners needed much guidance and assistance, and this was confirmed by how she taught. Her desire was to create independent learners who can solve problems on their own, but it was not clear whether she actually achieved that. When problems had to be solved, she did the thinking for the learners by giving them the steps for solving each type of problem and they then had to memorise these steps and apply them in the right context. The teacher was familiar with and knowledgeable about the intrinsic barriers to learning and identified six of the eight barriers to learning. Only learnt helplessness and maths anxiety were omitted.

Teacher 3 had over 30 years' teaching experience, which included 11 years in special needs education. Her experience can account for her knowledge of the intrinsic barriers to learning mathematics and the fact that she employed most of the teaching approaches in her lessons. One would normally expect a teacher with so much experience to use the teaching approaches in a more intensive way and to have more thorough knowledge and understanding of the different teaching approaches.

4.5 Teacher 4

Teacher 4 was a 53-year-old female. Her teaching experience included eight years in a mainstream school and 22 years in a special needs school. In addition to her teaching degree in Education she had a diploma in Minimum Brain Dysfunction, which she had obtained through two years' part-time study. At the time of the observations she taught all their subjects to a Grade 4 class at a special needs school in Pretoria.

4.5.1 Pre-observation questionnaire

In her response to the pre-observation questionnaire that was completed by all the participants to determine their basic knowledge of the intrinsic barriers to learning mathematics, Teacher 4 mentioned seven of the eight categories that had been identified in the relevant literature and were discussed in Chapter 2, paragraph 2.3 and summarised in Figure 2.1. The first category mentioned was learnt helplessness. Teacher 4 stated that *if many instructions are given learners give up before trying*, which can be categorised as learnt helplessness. Maths anxiety became a problem *when learners felt overwhelmed*. Learners with learning difficulties *have poor memories and their concentration is weak. Learners forget that division by two is halving the* *number and they follow an explanation only up to a certain point and then they become tired*. The former difficulties referred to memory difficulties and the latter to attention difficulties.

Processing difficulties were explained as *the reversing of digits, for example, they write 19 as 91. They also have directional problems. For example, when multiplying a 3-digit number by a 2-digit number, they become confused. They confuse + and x. They also have problems with perseveration, where they write the same answer for a few consecutive sums. These learners also struggle to find their place on a worksheet after looking at the board for a while.* Furthermore, *one learner might give an oral answer correctly, but when she writes it down she will write 9, even though she meant 8.* Language difficulties mentioned were *problems with reading and abstract thinking when solving word problems.* With reference to the category metacognitive thinking difficulties Teacher 4 mentioned that *learners struggle to check their work again once they have completed a task. Some learners cannot come to a point where they realise that they do not know what to do and that they can receive help if they ask.* The only category that was not mentioned by Teacher 4 was passivity, but she had a good in-depth knowledge of each of the other seven categories.

4.5.2 Observation

4.5.2.1 Background

Teacher 4's Grade 4 class consisted of 10 English-speaking learners, who remained in the same classroom for the duration of the school day. Three of the learners used computers to complete their work as they had muscle-tone problems. All the walls were covered with colourful posters (Picture 4.9) containing all kinds of information.

Picture 4.9: Classroom with many posters on the walls

The teacher used an overhead projector, the blackboard, worksheets, posters and manipulatives to teach. Learners did not use textbooks, but worked on and learnt from worksheets created by the teacher.

4.5.2.2 Observation: teaching approaches

Over a period of two weeks four lessons were observed on 6, 8, 13 and 15 May 2014 in order to investigate the types of teaching approaches used by mathematics teachers in their classrooms. The data was recorded by using an observation schedule. After each observation I classified the actions and words of the teacher into the identified categories on my observation schedule. These categories, identified from the literature, was discussed in Chapter 2, paragraph 2.4 and summarised in Figure 2.1. Whenever possible, I took photos of the teaching material and work explained on the blackboard at the end of a lesson. The photos were taken afterwards as I did not want to disturb the learning process. The teacher provided me with the worksheets used by the learners.

The main theme of the mathematics taught during the time of observation was division and the last lesson dealt with fractions. Lesson 1 dealt with division with a remainder in a specific context, and in Lesson 2 the learners were taught how to use the division method without a remainder. In Lesson 3, division with a remainder was taught, and Lesson 4 was an introduction to fractions.

Each of the six teaching approaches that were identified as categories will now be discussed separately. During the observation of the four lessons, the teacher repeated information and concepts many times. She also revised concepts taught previously or work done during the previous lessons. Repetition and revision can be viewed as behaviourist teaching approaches as they condition learners to remember facts and, unlike the chosen teaching approaches, do not assist a teacher in teaching for understanding.

Authentic context

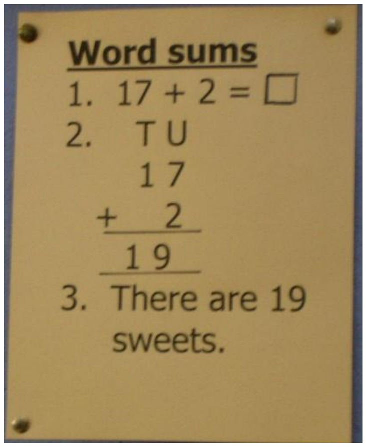
In Lesson 1 learners gathered around in a group and the teacher used sweets that they had to divide equally into small plates. She used the context of a family, so the division of the sweets needed to be fair. Some learners were asked to divide an uneven number of sweets into each plate. In Lesson 2 the teacher asked learners how they would divide 17 sweets among 5 learners, and 40 sweets among 6 learners. This time real sweets were not used. In Lesson 3, Teacher 4 used word problems to explain division. For example: *There are 72 Jelly Tots that must be shared among five learners. How many Jelly Tots will each one receive?* Another example: *You need to study all the Natural Science work. You have four days to study. How many pages do you have to study each day?* The teacher began Lesson 4 by using an apple (Picture 4.13) and saying: *I have two learners, but only one apple. This is a problem, what do I do?* She also used a real Twix chocolate to demonstrate fractions and the example of a pizza that had to be cut into equal slices.

Building meaningful connections

The teacher built meaningful connections in all four lessons. She started Lesson 1 by reminding the learners that multiplication and division go together, and that the one is the opposite of the other. She then explained that if 'big' numbers were used, multiplication could be used to calculate the answer. Example: 31 ÷ 5 = □. *Count in 5s to determine how many 5s you will need to come as close as possible to 31.* In Lesson 2 learners revised the 3x, 4x, 5x and 6x tables because the teacher explained division by counting in these numbers. She further explained the connection between the 3x and 6x tables: *for the 6x table you skip one answer from the 3x table.* She then asked the learners which other two tables also work in the same manner. In Lesson 3 the learners were reminded of the *'garage sums'* they had done the previous year, which would be used to solve these problems. In Lesson 4 the teacher revised the terms diagonal, horizontal and vertical lines as the learners needed to know these lines in order to be able to divide whole objects into smaller parts to create fractions.

Scaffolding

In Lesson 1 the teacher guided learners by asking questions, for example: *What will you do if you do not know how many sweets each will receive?* However, she did not really fade teacher guidance. Learners were never really left to do anything on their own, except at the end when they had to complete the worksheets. When learners were asked to solve mathematical problems on the overhead projector, the teacher always guided them by asking: *What do you have to do* *first?* In Lesson 2, Teacher 4 demonstrated the division method called *'garage sums'*. She asked questions such as: *Where do you 'park' the bigger number? Where do you 'park' the answer?* and used the abacus to demonstrate how you count in the number by which you are dividing. The teacher guided learners every step of the way. She started with easy problems without a remainder, gradually progressing to bigger numbers with a remainder. In Lesson 3 she broke down word problems into the following three steps (Picture 4.10): *Start with a number sentence, then use the method* (division in this case) *and then write your answer in words.*



Picture 4.10: Poster explaining the three steps for solving word problems

The teacher explained how to get to the first step (the number sentence): *Look at key words that will indicate the operation required and the numbers*. She guided the learners by asking questions to determine their level of understanding. The teacher provided individual assistance to any learner who still struggled with the concept. In Lesson 4 she demonstrated fractions by cutting an apple after first asking learners where she should cut it. She also drew a picture of a pizza on the overhead projector and asked in how many pieces it was divided. She guided the learners by asking them questions such as: *How many pieces did I eat*? She then took out flash cards containing fraction diagrams and asked the learners to identify the fractions represented (Picture 4.14). The teacher explained the work but also guided the learners by asking questions.

CRA sequencing

Teacher 4 started Lesson 1 by teaching on the concrete level, using sweets that learners had to manipulate by dividing them into equal groups on small plates (Picture 4.11). She then used an abacus to demonstrate how many times, for example, 5 goes into 31 (Picture 4.12).

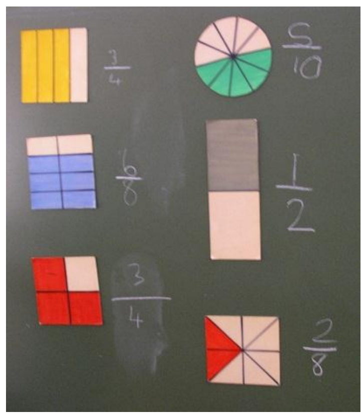
Picture 4.11: Plates and sweets used to teach division

Picture 4.12: Abacus used to teach division

Teacher 4 used the abacus to count in 5s and asked the learners how many groups of 5 there were. She then wrote the problem as a number sentence on the overhead projector, which represented the abstract level. In Lesson 2 she used the abacus for counting in the number that was used to divide by (concrete level). She pointed at the poster that showed the division method and used the overhead projector to explain to learners how the division problems should be done (abstract level). In Lesson 3 only the abacus was used to help struggling learners to count in the number they were dividing by. Word problems were written on the overhead projector. The teacher pointed at a poster that illustrated the method for doing word problems (Picture 4.10). In Lesson 4, Teacher 4 used a real apple and chocolate bars to demonstrate fractions (Picture 4.13).

Picture 4.13: The apple and chocolate bars used to explain fractions

The teacher used a drawing of a pizza that was divided in half and wrote the fraction in symbols on the blackboard. She also used flashcards that demonstrated different fractions and asked the learners to identify each fraction (Picture 4.14).



Picture 4.14: Flashcards used to demonstrate fractions

Problem-solving strategies

Teacher 4 used basic problem-solving strategies in the first three lessons, but not in Lesson 4. While demonstrating how to solve a problem in Lesson 1, she always repeated the steps. She demonstrated, for example, how multiplication (counting in the number) can be used to determine how many times you had to count to arrive at the answer. This was repeated many times to help the learners to remember the steps. In Lesson 2 she explained division by using an example of a garage. *You park the bigger number 'inside' and the answer 'on the roof'*. When learners had a chance to practise division problems, she kept asking them: *Where do you 'park' the bigger number and where do you 'park' the answer?* In Lesson 3, Teacher 4 explained the sequence in which word problems must be solved as follows: *First identify the numbers that you will use in the sum, then look at the key words and decide which operation you must use. You* *then write the number sentence, solve the problem by using the appropriate method, and then you write your answer in words. To know what words you need to use in your answer sentence, you need to look at the question.*

Formative assessment

The teacher set no goals for the learners in any of her lessons. She did, in all the lessons, monitored learner progress by asking open-ended questions. Examples of open-ended questions were: *What do I need to do next? Who can give me another example where this is also true? What will the number sentence be?* When learners had to complete worksheets, she always walked around in the classroom, checked the work of each learner and assisted those who had questions. She always gave feedback on the work that had been completed.

4.5.2.3 Discussion and interpretation

Teacher 4 used nearly all the teaching approaches in all four lessons. She taught every lesson within an authentic context, using sweets, a pizza, an apple, or a story. She succeeded in building meaningful connections with work done previously and did much revision and repetition of both old and new concepts while teaching. When scaffolding was applied she made use of the three phases suggested by Miller and Hudson (2006) and Van de Pol et al. (2010) (for an in-depth discussion, see Chapter 2, paragraph 2.4.3). However, Teacher 4 never faded teacher guidance. She kept assisting learners and asking them questions to guide them towards the next step.

The teacher made excellent use of the CRA sequencing in all the lessons and used real-life objects like sweets to explain division, an apple and bars of chocolate to explain fractions, and an abacus. She made use of both the representational and abstract levels while teaching. In Lesson 4 she actually followed the complete sequence by starting with the concrete level, followed by the representational level and then the abstract level. Her teaching of problem-solving strategies was basic and she did not use the four steps for problem-solving that Polya (1957) suggested (see Chapter 2, paragraph 2.4.6). She assisted learners in solving problems by providing steps or using the example of a ‘garage’. Formative assessment was done by asking many open-ended questions while teaching and assisting learners individually when they struggled to complete tasks on their own.

Based on the observations, one may conclude that Teacher 4 made use of all six teaching approaches in her lessons. She taught by using authentic contexts, made good use of the CRA sequencing and effectively helped learners to make meaningful connections. She used the three phases of scaffolding and successfully guided learners in the learning process. However, she did control the learning process by providing learners with steps for solving word problems and told them how to do their work and when to do it. She did not succeed in fading guidance. The types of questions asked did encourage learners to think about the concepts. Teacher 4 had 30 years' teaching experience, of which 22 years had been at special needs schools. She also had formal training in minimal brain dysfunction. Her teaching experience and training was evident in the way she approached teaching.

4.5.3 Post-observation questionnaire

The post-observation questionnaire was given to each participant to determine how they thought learners with learning difficulties should be taught. Teacher 4's view of teaching was *to always try to let learners discover for themselves. This will make learning easier if they show insight in the problem. However, sometimes this can be a problem because you only have a limited time period to cover the work. Then you must switch to explicit teaching.*

Teacher 4 mentioned four of the six categories of teaching approaches identified from the literature, as discussed in Chapter 2, paragraph 2.4 and summarised in Figure 2.1. She also mentioned the use of *different teaching styles and learning styles when teaching. Include tactile (handling of counters), visual (overhead projector or posters) and auditory (loud counting) ways of teaching. Practise on white boards using bigger writing so that mistakes can be easily erased.*

The first category mentioned was building meaningful connections, which means that one has *to start from the beginning as some learners forget what they have done previously and then to build on former knowledge is not possible.* For the category scaffolding the teacher mentioned that *the teacher should allow learners to ask other learners to explain to them if they got stuck.*

Furthermore s*tart with easy examples and slowly make them more difficult.* In the category CRA sequencing and problem-solving strategies she mentioned that one should *allow learners to draw the problem when solving word problems. Encourage them to have a scrap sheet at hand for working out.* She added that it is necessary to *use concrete objects. If learners cannot remember something you can refer them to the classroom posters that explain the concept and in that way they can help themselves when they struggle*. The categories that were omitted were authentic context and formative assessment.

Teacher 4 mentioned four of the six teaching approaches, which were only basic explanations of each category. For building meaningful connections she only mentioned that it was necessary to repeat previous work done as learners forget easily. The teacher's view of scaffolding was limited and she mentioned that learners should be allowed to assist one another. With regard to CRA sequencing she had a sound knowledge of how it should be used as she mentioned both the concrete and the representational levels. She was not very skilled in teaching problem-solving strategies.

4.5.4 Comparison between the results of the observation and the post-observation questionnaire

No real link could be found between the information provided in response to the questionnaire and the actual teaching observed in the classroom. The teacher used more teaching strategies than she had mentioned in the questionnaire and also used the teaching strategies more intensely than indicated. She used authentic contexts and built meaningful connections when teaching, despite the fact that building meaningful connections was only briefly mentioned in the questionnaire.

Two similarities between the observation and the questionnaire content were the CRA sequencing and problem-solving strategies. The way in which CRA sequencing was used in the classroom was much better than had been explained in the questionnaire. The problem-solving strategies that were mentioned were basic, and so was the teaching of problem-solving strategies in the classroom. Although formative assessment was not mentioned in the questionnaire, it was applied while teaching.

4.5.5 Conclusion regarding Teacher 4

Teacher 4's view of teaching was that one should teach in an implicit way (learner discovery), as far as possible, but she mentioned that time constraints sometimes make it necessary to switch to explicit teaching. This was demonstrated by her actual teaching. In Lessons 1 and 4 in particular, she first asked the learners to solve a problem, before she explained how it should be done. However, as the lessons progressed she took control of the learning and a more explicit teaching style prevailed. During explicit teaching she always asked open-ended questions to encourage the learners to think about the problem. She guided them throughout the lessons but even when learners were working through problems on their own she constantly reminded them of the questions they should ask themselves.

Teacher 4 had a sound knowledge of the intrinsic barriers to learning and could name seven of the eight barriers. This knowledge and the way she taught could possibly be linked to her many years of teaching at a special needs school and her formal training in minimal brain dysfunction.

4.6 Cross-case analysis

The data collected from each of the four teachers was recorded and discussed separately in paragraph 4.5. For each teacher the pre-observation questionnaire, the observation, and the post-observation questionnaire were discussed in that order, followed by a comparison between the observation and the post-observation questionnaire. A comparison between the four teachers will now be drawn. This comparison will follow the same sequence as when the teachers were discussed individually.

4.6.1 Comparison of the pre-observation questionnaires

The data obtained from the pre-observation questionnaires addressed the first secondary research question for this study namely: Which different types of intrinsic barriers to learning are mathematics teachers who teach learners with learning difficulties aware of? Table 4.1 contains a summary of the categories mentioned by each teacher in the questionnaire. All four teachers were well informed with regard to the intrinsic barriers to learning mathematics experienced by learners with learning difficulties. Teacher 2 identified the three subcategories of emotional difficulties, unlike the other three teachers. A possible explanation for their failure to mention all the emotional difficulties could be that they did not regard emotional difficulties as intrinsic barriers to learning. Kloosterman (1984) explained that emotional difficulties are usually caused by extrinsic barriers, such as parents' high expectations, which are internalised by the learner with the result that emotional difficulties, such as learnt helplessness, passivity and maths anxiety, become intrinsic barriers to learning. Table 4.1 indicates that Teacher 4, who had the most teaching experience and had received training that was relevant to special needs education, could name seven of the eight barriers to learning, whereas Teacher 1, who had the least teaching experience and no special needs training, could name only five. Teacher 2 and Teacher 3 could each name six of the eight barriers to learning.

Table 4.1: Pre-observation questionnaires

	Teacher 1	Teacher 2	Teacher 3	Teacher 4
Emotional difficulties				
Learnt helplessness	No	Yes	No	Yes
Passivity	No	Yes	Yes	No
Maths anxiety	No	Yes	No	Yes
Cognitive difficulties				
Memory difficulties	Yes	No	Yes	Yes
Attention difficulties	Yes	Yes	Yes	Yes
Processing difficulties	Yes	Yes	Yes	Yes
Language difficulties	Yes	Yes	Yes	Yes
Metacognitive thinking difficulties				
Metacognitive thinking difficulties	Yes	No	Yes	Yes

In this study even the teacher with the least experience was familiar with most of the barriers to learning mathematics. There was, however, a slight difference and one might possibly conclude that, in the case of the four participants, the more teaching experience they had in special needs education, the more knowledge they had of the intrinsic barriers to learning mathematics. Teacher 2 and Teacher 3 identified the same number of barriers to learning, therefore it can be concluded, for the purpose of this study, that having formal special needs training could make up for a teacher's lack of teaching experience.

4.6.2 Comparison of the observations

The data collected through observation was used to address the second secondary question of this study: What are the teaching approaches used by mathematics teachers in order to assist learners with learning difficulties to overcome their intrinsic barriers to learning? Interpreting the data and comparing the teachers should be done with caution. The underpinning philosophy of this study is constructivism, which implies that each teacher created her own experiences in the classroom. Even though all the learners involved experienced learning difficulties, each of the teachers taught mathematics to different learners in different grades. Furthermore, each teacher had to deal with learners with different combinations and degrees of learning difficulties, and these factors could have influenced the way they taught.

Table 4.2 summarises each teacher's use of the teaching approaches in the lessons observed. The two teachers with the least experience (Teachers 1 and Teacher 2) used fewer authentic contexts than the two more experienced teachers (Teacher 3 and Teacher 4). Teacher 4 used authentic contexts every time she taught, while Teacher 1 almost never built meaningful connections and the other two teachers (Teachers 2 and Teacher 3) used them fairly frequently.

Even though it was indicated that scaffolding was used, it should be noted that all the teachers used it to a limited degree. Observation was done according to Miller and Hudson (2006) and Van de Pol et al.'s (2010) views of scaffolding, which means that it included three phases namely demonstration, working together (guidance) and learners working independently. All the teachers did well as far as demonstration and explanation were concerned, but little time was allocated to working together and allowing learners to do work on their own and in their own way. Learning was controlled by all the teachers. Teacher 4 did allow learners to attempt solving a problem before she taught a concept and provided more guidance than the other teachers, but she did not fade her guidance. In other words, she built the scaffold to assist the learners, but the scaffolding was never removed.

Table 4.2: Observations

	Teacher 1	Teacher 2	Teacher 3	Teacher 4
Authentic context				
Lesson 1	No	Yes	Yes	Yes
Lesson 2	Yes	No	Yes	Yes
Lesson 3	No	Yes	Yes	Yes
Lesson 4	Yes	No	No	Yes
Building meaningful connections				
Lesson 1	No	Yes	Yes	Yes
Lesson 2	No	Yes	No	Yes
Lesson 3	Yes	No	No	Yes
Lesson 4	No	Yes	Yes	Yes
Scaffolding				
Lesson 1	Yes	Yes	Yes	Yes
Lesson 2	Yes	Yes	Yes	Yes
Lesson 3	Yes	Yes	Yes	Yes
Lesson 4	Yes	Yes	Yes	Yes
CRA sequencing (C - concrete, R - representational, A - abstract)				
Lesson 1	A	CRA	CRA	CA
Lesson 2	A	A	RA	CA
Lesson 3	RA	RA	A	CA
Lesson 4	A	CRA	A	CRA
Problem-solving strategies				
Lesson 1	No	No	No	No
Lesson 2	No	No	No	No
Lesson 3	No	No	No	No
Lesson 4	No	No	No	No
Formative assessment (G - goal-setting, M - monitoring progress, F - giving feedback)				
Lesson 1	MF	MF	M	MF
Lesson 2	MF	MF	MF	MF
Lesson 3	MF	MF	MF	MF
Lesson 4	MF	MF	No	MF

With regard to CRA sequencing, it was noted that only Teacher 4 used concrete objects all the time. Most of the teachers made use of diagrams and pictures to teach their learners. However, Teacher 1 taught only in an abstract way, except in Lesson 3 where the representational level was used. Problem-solving strategies were taught, but not in four stages, as recommended by Polya (1957) (see Chapter 2, paragraph 2.4.6). These stages could help learners to become independent problem solvers. However, the teachers taught their learners how to approach a problem by providing them with formulae or 'steps' that could be used to solve specific problems. They believed that if learners remembered the 'steps' or formulae for solving a problem, they would eventually be able to do it on their own. However, this approach does not ensure teaching for understanding as it enables learners to simply follow the 'steps' to solve a problem without any real understanding. This is not conducive to building understanding and becoming independent learners. The teachers used formative assessment to determine whether their learners understood the concepts by asking probing and open-ended questions. Only Teacher 1 asked mostly closed questions. None of the teachers set goals, but all of them monitored their learners' work by offering individual assistance. Generally, feedback was given immediately by all the teachers, with the exception of Teacher 3, who did not give feedback in Lessons 1 and 4.

From the above discussion one can conclude that Teacher 4, who had the most teaching experience, used nearly all the teaching approaches in all the lessons that were observed. The intensity and duration of the use of the teaching approaches were also considerably higher than in the case of the other teachers. It can also be concluded that Teacher 2, who had little teaching experience but had undergone further training in special needs education, and Teacher 3, who had many years of teaching experience but no further training in special needs education, used almost the same number of teaching approaches. The observations of Teacher 2 and Teacher 3 revealed that there was a slight difference between the intensity and duration of the use of these teaching approaches as Teacher 3 used the teaching approaches more often and with more intensity during the lessons than Teacher 2. As can be expected, Teacher 1, who had the least teaching experience and no special needs training, made the least use of the different teaching approaches and did not use many in combination with other approaches.

4.6.3 Comparison of the post-observation questionnaires

The data obtained from this questionnaire addressed the third and final secondary question of this study: How do mathematics teachers think they should facilitate learning to help learners with learning difficulties to overcome their intrinsic barriers to learning? Table 4.3 contains a summary of how the participants thought they should teach learners with learning difficulties. Any comparison and interpretation of the data should be done with caution as the way in which the teachers actually taught did not reflect the answers given on the questionnaires. For instance, Teacher 1 listed almost all of the six teaching approaches, but did not use them all during teaching, while the other three teachers omitted many of the teaching approaches in the questionnaire but actually did use them while teaching. One can therefore not assume that these teachers who did not mention all the teaching approaches were not familiar with them.

Table 4.3: Post-observation questionnaires

	Teacher 1	**Teacher 2**	**Teacher 3**	**Teacher 4**
Authentic context	Yes	Yes	No	No
Building meaningful connections	No	No	No	Yes
Scaffolding	Yes	Yes	Yes	Yes
CRA sequencing	Yes	Yes	Yes	Yes
Problem-solving strategies	Yes	No	No	Yes
Formative assessment	Yes	No	No	No

It should therefore be clear that it would not be appropriate to draw any conclusions and make comparisons between the different teachers' post-observation questionnaires. How teachers approach their teaching in the classroom is far more important than what they put on paper, and how they teach will determine whether ultimately they succeed in helping their learners with learning difficulties to reach their full potential in the mathematics classroom.

4.6.4 Conclusion regarding the cross-case analysis

The answers provided in the pre-observation questionnaires, as well as the data from the observations were compared. The findings suggested that, in the case of this study, teachers with more teaching experience were aware of more intrinsic barriers to learning mathematics than those with less teaching experience. The findings further suggest that, as seen in this study, training in special needs education and many years' teaching experience contributed in equal measure to the teachers' ability to get to know their learners. One could therefore conclude that the more teaching experience and training in special needs education teachers have, the better their knowledge of the intrinsic barriers to learning should be and the more effectively they should be able to use the teaching approaches. In this study it was evident that the participant with less teaching experience and no training in special needs used fewer teaching approaches than the others, but she was nevertheless familiar with the intrinsic barriers to learning. The findings further suggested, in the case of this study, that formal special needs training equips teachers to better assist learners with learning difficulties to reach their full potential.

4.7 Summary

In this chapter the collected data was discussed and interpreted according to the categories set out in the conceptual framework. Data was collected from four teachers by means of two questionnaires and observations in their classrooms. Each teacher's data was recorded, interpreted and discusses separately, and finally a cross-case analysis was done by comparing the questionnaires and the observations. However, it was decided not to interpret the data obtained by way of the post-observation questionnaire in the cross-case analysis. In the next chapter the final conclusions and implications will be discussed.

Chapter 5

Conclusions and implications

5.1 Introduction

In this chapter final comments are made and conclusions are drawn with the literature in mind. Each research question will be discussed in the light of the findings and the relevant literature. The limitations of the study and its implications for instructional practice will be discussed and suggestions will be made regarding possible future research. Finally, a reflection will be done on this study.

5.2 Summary of chapters

In Chapter 1 the study was introduced and its aim and purpose were explained. The purpose of this study was to investigate which teaching approaches mathematics teachers in special needs schools use to help learners with learning difficulties to overcome their barriers to learning. Teachers' knowledge of the barriers to learning mathematics with which learners with learning difficulties have to cope with and their knowledge of the different teaching approaches were also examined. The rationale for doing the study was discussed and three secondary research questions were formulated.

Chapter 2 consists of an in-depth literature review. The eight intrinsic barriers to learning mathematics and the six teaching approaches were discussed in detail. Based on the analysis of the literature a conceptual framework was compiled and used as the basis for the data collection. The conceptual framework was then discussed and links were made between the eight intrinsic barriers to learning mathematics and the six teaching approaches.

Chapter 3 contains a discussion of the methodology used for this study. This was followed by a discussion of the constructivist paradigm that underpins this study, the use of a qualitative multiple-case study design, and how data collection took place. Four mathematics teachers who teach learners with learning difficulties at two LSEN schools in Pretoria were selected and were asked to complete two questionnaires. Observation was also done to determine how these teachers use teaching approaches. Finally, quality criteria and ethical considerations were discussed.

In Chapter 4 the data was recorded. The data provided by the participants was recorded, collected and analysed done in the same manner. Each participant was discussed individually, after which a cross-case analysis was done. An inductive approach was used to analyse the data by using the categories that were set out in the theoretical framework. The findings were discussed and compared to the content of the literature and conclusions were drawn.

5.3 Discussion of research questions

This study aimed to investigate the knowledge of the teachers concerning the intrinsic barriers to learning mathematics and how mathematics teachers think they should teach learners with learning difficulties to assist learners to overcome those barriers. Furthermore, this study also investigated the teaching approaches that teachers use to assist their learners with learning difficulties to overcome their barriers to learning. For this reason the primary research question was formulated as follows: How do mathematics teachers facilitate learning to help learners with learning difficulties to overcome their intrinsic barriers to learning? To address this question, three secondary questions were formulated to guide this study. These three questions were:

1. Which different types of intrinsic barriers to learning are mathematics teachers who teach learners with learning difficulties aware of?
2. What are the teaching approaches used by mathematics teachers in order to assist learners with learning difficulties to overcome their intrinsic barriers to learning?
3. How do mathematics teachers think they should facilitate learning to help learners with learning difficulties to overcome their intrinsic barriers to learning?

5.3.1 Question 1: Which different types of intrinsic barriers to learning are mathematics teachers who teach learners with learning difficulties aware of?

To be able to answer this question four mathematics teachers were asked to complete a semi-structured pre-observation questionnaire. Eight intrinsic barriers to learning mathematics that were identified from the literature and set out in the conceptual framework were used as categories for coding the data (Figure 2.1). These categories included learnt helplessness, passivity, maths anxiety, memory difficulties, attention difficulties, processing difficulties, language difficulties and metacognitive thinking difficulties.

The findings based on this study indicated that all four teachers knew their learners and were aware of the intrinsic barriers to learning mathematics (Table 4.1). There were only slight differences between the four teachers' level of awareness of the barriers to learning. Teacher 1 was aware of five of the eight barriers to learning mathematics. She did not mention emotional barriers, which include learnt helplessness, passivity and maths anxiety. Teacher 2 and Teacher 3 were aware of six of the eight barriers to learning mathematics. Teacher 2 did not mention memory difficulties and metacognitive thinking difficulties, and Teacher 3 omitted learnt helplessness and maths anxiety. Teacher 4 was aware of seven of the eight intrinsic barriers to learning mathematics. The only barrier she did not mention was passivity. Therefore Teacher 1 was the least informed on the intrinsic barriers to learning (five of the eight), while Teacher 4 was the best informed and mentioned seven of the eight barriers that had been identified in the literature. Teacher 2 and Teacher 3 were aware of six of the eight intrinsic barriers to learning.

It was interesting to note that the category of emotional difficulties was the one that most of the teachers did not identify. Only Teacher 2 identified all the emotional difficulties, namely learnt helplessness, passivity and maths anxiety. A possible explanation for this omission could be that Teacher 1, Teacher 3 and Teacher 4 did not consider emotional difficulties to be intrinsic barriers to learning. According to Kloosterman (1984), emotional difficulties are generally caused by extrinsic barriers to learning, such as parents' high expectations or poor teaching. However, over time the learners internalise this and the result is that emotional difficulties become intrinsic barriers to learning.

According to the Education White Paper 6 on special needs education, the government expects teachers in special needs education to have specialised knowledge, particularly knowledge of their learners, as they need to serve as resource centres for other mainstream schools (Department of Education, 2001). Researchers such as Ball et al. (2008), Hill et al. (2008) and Allsopp et al. (2007) also postulate that knowledge of the learner is important.

The first secondary question posed in this study was: Which different types of intrinsic barriers to learning are mathematics teachers who teach learners with learning difficulties aware of? The participants were aware of the intrinsic barriers to learning mathematics. Teacher 4, who had the most teaching experience, as well as training in special needs education, was aware of the highest number of intrinsic barriers to learning mathematics. One could conclude that, in the case of this study, training in special needs education made up for the lack of teaching experience as Teacher 2 was familiar with the same number of intrinsic barriers to learning as Teacher 3, who had more than ten years' teaching experience but no special needs training.

5.3.2 Question 2: What are the teaching approaches used by mathematics teachers in order to assist learners with learning difficulties to overcome their intrinsic barriers to learning?

In order to answer this question, each teacher was observed during four lessons. An observation schedule was used for this purpose. The observation schedule contained six teaching approaches that had been identified from the literature and were included in the conceptual framework (Figure 2.1). These six teaching approaches were used as categories for coding the data. They included authentic context, making meaningful connections, scaffolding, CRA sequencing, problem-solving strategies and formative assessment.

Before discussing the findings of the observations it is important to note that the findings should be interpreted with caution. The underlying philosophy of this study is constructivism and therefore the view is that each participant was in a unique situation with learners with different degrees and combinations of learning difficulties. Teachers teaching such diverse learners have to cater for a variety of needs, which could affect the way in which they facilitate learning in their classrooms. Furthermore, even though constructivism was the underlying philosophy of this study, the teaching that was observed in School A and School B was more behaviouristic in nature. There may be many reasons for this, but examining the reasons falls outside the scope of this study. Research done by Mercer et al. (2014), Lerner and Johns, (2012) and Allsopp et al. (2007) on LSEN education suggests that learners with learning difficulties struggle to learn new concepts without assistance from and demonstration by their teachers. These researchers further suggest that much guidance should be given, especially in the beginning, and that these learners should be taught 'steps' to solve problems. Not surprisingly the teaching observed for this study was in line with the viewpoint of these researchers.

The six possible teaching approaches that were identified in the literature and were used as categories for this study were discussed in Chapter 2 from a constructivist point of view. However, teaching approaches can also be used from a behaviouristic point of view. A constructivist example is when scaffolding is viewed as learners building their own tower of knowledge while the teacher provides the scaffolding needed to build the knowledge (Gultig & Stielau, 2012). One way in which this can be done is through Socratic dialogue. Socratic dialogue occurs when a teacher asks questions that will help the learners to solve a mathematical problem without directly pointing out the solution or the paths towards it (Birnbacher & Krohn, 2004). Scaffolding can be done in a behaviouristic way. This starts with the 'I do' part, when the teacher demonstrates and explains, followed by the 'we do' part, when the teacher and the learners solve problems together, and finally the 'you do' part, which is when the learners practise solving similar problems on their own (Van de Pol et al., 2010). The findings in this study indicated that the four participating teachers used scaffolding in the behaviouristic way. The teachers first demonstrated and explained the concepts, then allowed learners to practise solving problems together with the teacher, and finally gave them homework to complete on their own. Learners who struggled to complete the work on their own were further assisted by the teacher.

Another teaching approach that can be used in both a constructivist and a behaviourist way is when learners are assisted to develop problem-solving skills. From a constructivist point of view, Polya (1957) suggests that problem-solving consists of four stages: the learner recognises and understands the problem and identifies what is required to solve it; the learner understands how the different items in the problem are connected and plans a procedural approach; the learner decides on the mathematical knowledge needed to solve the problem and solves it; and the learner considers the answer and decides whether it makes sense. From a behaviouristic viewpoint, problem-solving can be taught by teaching learners the steps that will help them to solve a word problem (Allsopp et al., 2007). The findings from this study indicated that all four teachers developed problem-solving strategies in a behaviouristic way. They gave the learners steps, recipes or rules to follow in order to solve different kinds of mathematical problems. When the learners struggled to solve the problems, which happened often, the teachers always reminded them of the steps, recipes or rules that they had been taught.

The results for the use of CRA sequencing were surprising. Only Teacher 4 used concrete objects every time she taught. Teacher 2 used them in Lessons 1 and 4, Teacher 3 used them in Lesson 1 only, and Teacher 1 never made used of concrete objects (Table 4.2) in the observed lessons. The representational level was also rarely used. Teacher 2 used it in three of her four lessons, Teacher 3 in two out of four lessons, and Teacher 1 and Teacher 4 used it only once. Researchers who studied the use of CRA sequencing, specifically in LSEN education, all agree that when teaching a mathematical concept to learners with learning difficulties it is important to start on the concrete level, proceed to the representational level and only then progress to the abstract level to ensure that they understand a concept well and remember it better (Lerner & Johns, 2012; Mercer et al., 2014; Allsopp et al., 2007; Miller & Mercer, 1997; Miller & Hudson, 2006). This sequence was followed in only four of the sixteen lessons observed.

In this study the teaching approach known as formative assessment included the categories goal-setting, monitoring the learners' progress while they are working, and giving feedback. None of the four teachers set goals or encouraged the learners to set goals for themselves in the lessons that were observed (Table 4.2). According to Mercer et al. (2014) the setting of goals in mathematics is important for learners as it fosters motivation (Mercer et al., 2014). It is also important for learners to be involved in setting goals under the guidance of their teacher. All four teachers monitored their learners' progress by either moving around in the classroom and checking learners' work individually, or asking open-ended and probing questions. In only two of the sixteen lessons were feedback not given. Another teaching approach, authentic context, was used all the time by the two most experienced teachers (Teacher 3 and Teacher 4), except in one lesson in which it was not used at all. The two least experienced teachers (Teacher 1 and Teacher 2) each used it in two of their four observed lessons. The teaching approach that involves the building of meaningful connections was used mostly by the teachers with the special needs training, who each used it in three of their four lessons. It was used less by the teachers with no special needs training, with Teacher 1 using it only once and Teacher 3 using it in two of her four lessons.

The second secondary question of this study was: What are the teaching approaches used by mathematics teachers in order to assist learners with learning difficulties to overcome their intrinsic barriers to learning? The teacher with the most teaching experience, who also had special needs training (Teacher 4), used all six teaching approaches in all four lessons observed. She was the only teacher who used the concrete level to teach all four of her lessons. The intensity and duration of the use of these teaching approaches by Teacher 4 also differed significantly from how they were used by the other three teachers, in other words, she used them constantly and more intensely. As was expected the teacher with the least teaching experience and no special needs training used the six teaching approaches rarely or not at all. Teacher 2 (who undergone training in special needs education but had less than three years' teaching experience) and Teacher 3 (who had more than ten years' teaching experience, but no special needs training) made roughly the same use of the teaching approaches. From these results one can conclude that experience and training in special needs education are beneficial when teaching learners with learning difficulties. Teachers who lack teaching experience can benefit from formal training in special needs education as it will enable them to effectively help learners to overcome their intrinsic barriers to learning.

5.3.3 Question 3: How do mathematics teachers think they should facilitate learning to help learners with learning difficulties to overcome their intrinsic barriers to learning?

To find an answer to this question, four teachers who taught mathematics to learners with learning difficulties were asked to complete a semi-structured post-observation questionnaire. The six teaching approaches that were used on the observation schedule, i.e. authentic context, building meaningful connections, scaffolding, CRA sequencing, problem-solving strategies and formative assessment, were also used as categories for coding the data.

No definite answer could be given to the secondary research question as the way in which the four teachers taught did not reflect the answers provided by them. For instance, although Teacher 2 mentioned only three of the six approaches and Teacher 3 only two, they both used all six teaching approaches when they were observed (Table 4.3). One possible reason for this outcome is that the teachers received the questionnaire after the observations and right before the start of the exams, when they were preparing exam papers, and did not have enough time to properly consider their answers. They may not have put in enough time to complete it to the best of their ability. It is therefore best not to draw any conclusions in this regard and to leave this research question unanswered.

5.4 Concluding remarks concerning this study

The results of this investigation show that the more teaching experience a teacher had, the better equipped she was to use the teaching approaches. Formal special needs training also prepared teachers more effectively for using the teaching approaches. Knowledge of the learners' intrinsic barriers to learning is, to some extent, influenced by a teacher's experience and formal special needs training. The teacher who had many years' teaching experience but had not received any special needs training was nearly on equal footing with the teacher who had less than three years' teaching experience and special needs training.

From the literature it has emerged that knowing the learner is an important aspect of teaching learners with learning difficulties, particularly now, as many mainstream schools are becoming inclusive schools. As discussed in Chapter 1, government has suggested that LSEN schools should function as resource centres for mainstream inclusive schools (Department of Education, 2001). Many learners in mainstream schools have intrinsic barriers to learning and the findings of this study confirmed that teachers who teach learners with learning difficulties have a sound knowledge of these intrinsic barriers to learning. Teachers in inclusive mainstream schools would also benefit from being properly informed on the intrinsic barriers to learning mathematics.

The six teaching approaches that were identified from the literature should not be considered to be for exclusive use in LSEN schools. These six teaching approaches can be found in all schools and all teachers who would like to help their learners to reach their full potential should use them. The use of some teaching approaches is optional in mainstream schools, but essential in special needs education. Their combination and the intensity with which they are used are also important in special needs education. The findings, based on this research, suggest that teachers with many years of teaching experience in LSEN schools use these teaching approaches more intensely and more often than those with less teaching experience. However, some of the teaching approaches such as problem-solving strategies and scaffolding, were underutilised and were not well understood and implemented by any of the participants. From the results of the study one can conclude that all the participants controlled the learning and therefore made use of explicit instruction. There may be many reasons for this, but those reasons were not explored in this study. Mercer et al. (2014) maintained that interactive instruction (guided practice) is the best way to teach learners with learning difficulties; however, this teaching method was not observed during the data collection process.

5.5 Limitations of this study

The findings of this research and the generalisability of the results were subject to certain limitations relating to the participants. Using the constructivist paradigm, a case study was done with four teachers who taught mathematics to learners with learning difficulties. Each teacher's situation was unique as their learners had different combinations of barriers to learning. Furthermore, all the participants were female and white and taught in two public schools. The results could have been different if male teachers or people from different racial backgrounds, or even if private schools, had been included. Another limitation concerned the data collection by means of questionnaires and observations. The questionnaires were open-ended and little guidance was given. For this reason the results of the two questionnaires may not be reliable. This applies in particular to the post-observation questionnaire, which might not have provided a true account of what the teachers actually knew. Due to time constraints, only four lessons presented by each participant were observed. Although at the time four observed lessons were considered to be enough to give me a fair indication of how the teachers taught, it is possible that the results might have been different if more lessons had been observed.

5.6 Recommendations for future research

This study was done in LSEN schools. Further research could examine more closely the similarities and/or differences between the teaching approaches used by teachers who teach in LSEN schools and those teaching in mainstream schools. Furthermore, this was a broad-based study that focused on eight intrinsic barriers to learning and six teaching approaches. Future research could focus on only one of the six teaching approaches for a more in-depth investigation and to determine whether there are any differences in how that particular approach is applied in LSEN and in mainstream schools. In the light of the fact that the third secondary research question could not be answered, this question could possibly be researched on its own to probe teachers' knowledge of and beliefs concerning teaching approaches by making use of questionnaires and interviews. Future research could also focus on the learner and investigate how learners with learning difficulties cope in classes and what they do to overcome their intrinsic barriers to learning.

5.7 Implications

The findings of this study suggest that formal special needs training, such as remedial training, can be beneficial to all teachers. Teacher 2, who had received formal special needs training, but had little teaching experience, compared well with Teacher 3, who had more than ten years' teaching experience, but no formal special needs training. From the results of this study it can be deduced that if inclusive education is to be successful, the teachers should be equipped with the knowledge required for successful special needs education. A possible recommendation is that, since inclusive education is fast becoming the norm, all students registered for teaching courses should complete courses in learning support, which includes remedial training, and that regular in-training courses be required for teachers teaching in inclusive mainstream schools.

5.8 Final reflections

From the time when I decided to complete a master's degree, I was determined to find a topic that really interested me. It was this interest in the topic that kept me focused on my studies. I have learnt so much about myself and have grown personally, in my work as a mathematics tutor, and academically. I always felt that LSEN schools were being neglected by researchers. This opinion was substantiated when I started doing research for the literature review and discovered how little research had been done in South Africa on mathematics teaching in LSEN schools, compared to mainstream schools. LSEN schools are a vital part of the education system as many learners with various difficulties find it hard to cope in mainstream schools. These learners need the teacher's focused attention and small classes to be able to excel. I was also surprised to find that private schools for learners with learning difficulties outnumbered their public counterparts.

The four teachers that participated in my research were open to sharing their experiences and time with me, and for that I am extremely grateful. The observations were challenging for me as I had to remain focused on what I had to observe and had to guard against becoming distracted. I realised that collecting data is not easy and that much thought had to go into how you formulate the questions for the questionnaires and how to record the data while observing.

It is my hope that my study will contribute to mathematics teachers' instructional practice in both LSEN and mainstream schools, as well as to student-teacher and in-service teacher training.

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Addendums

Addendum A: Letters of permission and consent

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

FACULTY OF EDUCATION

1 November 2013

GAUTENG DEPARTMENT OF EDUCATION

Dear Sir / Madam

Request from GDE for permission to do research in schools, during and after school hours

I am currently enrolled as a Master student at the University of Pretoria. The title of my proposed dissertation is as follows: **Facilitating mathematics teaching for learners with learning difficulties: a case study.** My research is about mathematics teachers who teach in schools for learners with learning difficulties. I propose to investigate what teachers' knowledge is concerning the learning barriers of their learners as well as how they teach these learners in the classroom to overcome these barriers to learning. I hope, at the end of my research, to be able to make a contribution to the improvement of teachers' classroom practice in special needs education, especially in the field of mathematics.

In order to collect data for this research, I would like to choose a purposive sample of four mathematics teachers (intermediate and/or senior phase) teaching in schools for learners with learning difficulties. In order to select my sample I intend to hand out questionnaires to three schools in Pretoria (Transvalia, New Hope and Prospectus Novus). These questionnaires will enquire about their experience in special needs schools and their training to teach in special needs schools (attached).

After the selection of four teachers I intend to observe each of the four teachers teaching a class for 5 periods. I would like to video record (or audio record if I cannot get permission for video recording) the lessons with permission from all parties involved (GDE, principals, teachers and parents/guardians). A letter of assent from the learners will also be obtained. Interviews will only be conducted for clarification, after school hours. The questionnaires will be given to the teachers to complete in their own time. All participation from schools and teachers will be voluntary and all information, as well as the video and audio recordings, will be treated with confidentiality.

I am aware of the fact that research may not take place during school hours, however I have also taken note that in exceptional circumstances permission may be granted. I only intend to do the observations during school hours which will not disrupt the normal flow of the class. I would not take part in class activities at all.

I would also like to ask permission to video record the lessons which will be done from the back of the classroom focused only on the teacher. If a learner's face is by some chance recorded, it will be blurred out and the files will be password protected and only my supervisor and I will have access to the files.

I, therefore, formally request your permission to conduct my research at schools in Pretoria next year. I trust that my request will meet with your favourable response.

Yours faithfully

______________________________	______________________________
Researcher: Ms E Louw	Date

Researcher: Ms E Louw
University of Pretoria
Groenkloof campus
elizma.louw@up.ac.za
Cell: 072 266 0752

______________________________	______________________________
Supervisor: Dr G Stols	Date

FACULTY OF EDUCATION

17 March 2014

Dear Sir / Madam / Doctor

I am currently enrolled as a Master student at the University of Pretoria. My research is aimed at investigating how mathematics teachers help learners to overcome their barriers to learning. I hereby request permission to use your school for a pilot study for my research. I would like to invite two **mathematics** teachers, one in the intermediate and one in senior phase to participate in this pilot study. The pilot study is aimed at verifying whether my questionnaires and observation schedule are valid.

I intend to observe two mathematics teachers teaching a class for 1 lesson. The teachers will also be required to complete 2 questionnaires in order to establish the validity of the questions. An interview will be done afterward to receive feedback. The questionnaires and interview will be done outside schools hours at a time and place convenient for the teachers.

All participation is voluntary. Confidentiality and anonymity will be guaranteed at all times.

If you are willing to allow two members of your staff to participate in this study, please sign this letter as a declaration of your consent.

Yours sincerely

______________________ ______________________

Researcher: Ms E Louw Date

Groenkloof campus
University of Pretoria
elizma.louw@up.ac.za
Cell: 072 266 0752

______________________ ______________________

Supervisor: Prof G Stols Date

I, the undersigned, hereby grant consent to Ms E Louw to conduct her pilot study in this school for her Master's research.

School principal's name: ____________________________

School principal's signature: _________________________ Date: ________________

E-mail address: ___________________________________

Contact number: __________________________________

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

FACULTY OF EDUCATION

17 March 2014

Dear Sir / Madame

I am currently enrolled as a Master student at the University of Pretoria. My research is aimed at investigating how mathematics teachers help learners to overcome their barriers to learning. You are invited to participate in a pilot study for this research. The pilot study is aimed at verifying whether my questionnaires and observation schedule are valid.

Your participation in this research is voluntary and confidential. It is proposed that you form part of the pilot study by being observed, teaching a mathematics class for 1 lesson. Before the observation you will be asked to complete a questionnaire (15 minutes). Afterward another questionnaire needs to be completed (30 minutes). An interview will also be required where you would have an opportunity to give me feedback on the questionnaires and my observation schedule (1 hour).

All participation is voluntary. Confidentiality and anonymity will be guaranteed at all times.

If you are willing to participate in this study, please sign this letter as a declaration of your consent, i.e. that you participate willingly.

Yours sincerely

______________________ ______________________

Researcher: Ms E Louw
Groenkloof campus
University of Pretoria
elizma.louw@up.ac.za
Cell: 072 266 0752

Date

______________________ ______________________

Supervisor: Prof G Stols

Date

I, the undersigned, hereby grant consent to Ms E Louw to observe my classes. I declare that I will willingly complete two questionnaires and participate in interviews for her pilot study for her Master's research.

Participant's name: ___________________________

Participant's signature: ________________________ Date: ________________

E-mail address: ______________________________

Contact number: _____________________________

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

FACULTY OF EDUCATION

17 March 2014

Dear Sir / Madam / Doctor

I am currently enrolled as a Master student at the University of Pretoria. My research is aimed at investigating how mathematics teachers help learners to overcome their barriers to learning. I hereby request permission to use your school for my research. I would like to invite **mathematics** teachers in the intermediate and/or senior phase to participate in this research. This research will be reported upon in my Master's dissertation at the University of Pretoria.

Before the start of the research I need to select my sample (4 teachers). For this I need all mathematics teachers in the intermediate and senior phase at your school to fill in a very short questionnaire concerning their years of experience and training. If one or more of your teachers are selected as part of my study, only then will I start with data collection.

I intend to observe a mathematics teacher teaching a class for about 5 lessons. I would like to video record the lessons if consent can be obtained from all parties involved (principal, teacher, parents and learners). This will allow a clear and accurate record of the teacher's instructional practice. The video recording will be done from the back of the classroom, focused on the teacher only. If by some chance a learner's face is filmed it will be blurred out and the files will be password protected. If all the relevant parties do not give permission for the video recording of the lessons, then permission for audio recordings will be obtained.

The process will be as follows: during the second term of this year, should you look favourably upon my request, I would like to observe the lessons taught by a willing mathematics teacher for about 5 lessons on different days. Before and after the observation a questionnaire will be given to the teacher to complete in their own time outside school hours. Interviews will only be conducted for clarification of the observations and questionnaires if deemed necessary. The interview will be scheduled at a time and place convenient for the teacher, after school hours.

The learners will not take part in the research but will be in attendance of the class together with the researcher. The learners will receive an assent letter to inform them about the research that will be conducted. The parents/guardians will receive a letter of informed consent for the video/audio recording of the lessons.

All participation is voluntary and once committed to the research the teacher(s) or school may still withdraw at any time. Confidentiality and anonymity will be guaranteed at all times by using pseudonyms for the school and the teacher. Your school and the teacher will, therefore, not be identifiable in the findings of my research. Only my supervisor and I will have access to the video/audio recordings which will be password protected. The data collected will only be used for academic purposes.

After the successful completion of my Master's degree I will give feedback to the school in the form of a written report and if the school is willing I would like to do a presentation of my findings to all mathematics teachers at your school.

For any questions before or during the research, please feel free to contact me. If you are willing to allow members of your staff to participate in this study, please sign this letter as a declaration of your consent.

Yours sincerely

______________________ ______________________

Researcher: Ms E Louw Date

Groenkloof campus
University of Pretoria
elizma.louw@up.ac.za
Cell: 072 266 0752

______________________ ______________________

Supervisor: Prof G Stols Date

I, the undersigned, hereby grant consent to Ms E Louw to conduct her research in this school for her Master's research. I, the undersigned, hereby also grant consent to Ms E Louw to video record / audio record the lessons (please circle preferred choice or both).

School principal's name: ____________________________

School principal's signature: _________________________ Date: ________________

E-mail address: ___________________________________

Contact number: __________________________________

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

FACULTY OF EDUCATION

17 March 2014

Dear Sir / Madame

I am currently enrolled as a Master student at the University of Pretoria. You are invited to participate in a research project aimed at investigating how mathematics teachers help learners to overcome their barriers to learning. This research will be reported upon in my Master's dissertation at the University of Pretoria.

Your participation in this research is voluntary and confidential. It is proposed that you form part of this study's data collection phase by being observed, teaching one mathematics class for 5 lessons. I would like to video record the lessons if consent can be obtained from all parties involved (teacher and parents). This will allow a clear and accurate record of your instructional practice. If permission is not granted from all parties involved then I would like to audio record the lessons.

The process will be as follows: during the second term of this year, should you be willing to participate, I would like to observe the lessons taught by you for one class for 5 lessons. You will not be required to do anything extra than what you normally do during teaching; no extra preparation is needed. Before and after the observation a questionnaire will be given to you to complete in your own time after school hours. The questionnaire beforehand will take about 15 minutes of your time and the questionnaire afterwards will take about 30 to 45 minutes of your time. No extra preparation needs to be done to complete the questionnaires. Interviews will only be conducted for clarification of the observations and questionnaires if deemed necessary and will be audio recorded. This will not take more than 30 minutes at a time (if necessary). The interview will be scheduled at a time and place convenient for you, outside normal school hours.

Should you declare yourself willing to participate in this research, you will form part of a group of 4 teachers that I will focus on for my research. Confidentiality and anonymity will be guaranteed at all times. This will be done by allocating pseudonyms to you and the school. You may decide to withdraw at any time without giving any reasons for doing so. You and your school will not be identifiable in the findings of my research and only my supervisor and I will have access to the video/audio recordings which will be password protected. You will have access to the transcription of the interviews and before the recording of my data in my dissertation it will be made available to you for checking. The data collected will only be used for academic purposes.

After the successful completion of my Master's degree I will give feedback to the school in the form of a written report and if the school is willing I would like to do a presentation of my findings to all mathematics teachers at your school.

If you are willing to participate in this study, please sign this letter as a declaration of your consent, i.e. that you participate willingly and that you understand that you may withdraw at any time.

Yours sincerely

_____________________ _____________________

Researcher: Ms E Louw Date
Groenkloof campus
University of Pretoria
elizma.louw@up.ac.za
Cell: 072 266 0752

_____________________ _____________________

Supervisor: Prof G Stols Date

I, the undersigned, hereby grant consent to Ms E Louw to observe my classes. I declare that I will willingly complete two questionnaires and participate in interviews if necessary for her Master's research. I, the undersigned, hereby also grant consent to Ms E Louw to video record / audio record the lessons (please circle preferred choice or both).

Participant's name: ___________________________

Participant's signature: ________________________ Date: ________________

E-mail address: ______________________________

Contact number: _____________________________

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

FACULTY OF EDUCATION

17 March 2014

Dear Parent / Guardian

I am currently enrolled as a Master's student at the University of Pretoria. My research is focused on the way mathematics teachers help learners overcome their barriers to learning through the way they teach. In order to do this I will be observing how a mathematics teacher teaches for 5 lessons. I would like to video record (or otherwise audio record) these lessons as it will help me to have an accurate record of the teacher's teaching practice and for the analysis that will take place afterwards.

The video recordings are for the sole purpose of investigating how the teacher teaches mathematics. Your child does not form part of my research but, if his/her class is chosen, your child will be present in the class during the research. The video recordings will be taken from the back of the classroom and I will only focus on and film the teacher. If it does happen that your child is filmed his/her face will be blurred out. Your child's confidentiality and anonymity will be protected at all times and only my supervisor and I will have access to the recordings. The video/audio recordings will also be password protected. The video recordings will only be used for the completion of my Master's degree and not for any other purpose.

If you have any questions or concerns, please do not hesitate to contact me. If you are willing for your child to be present during the video/audio recorded lessons, please sign this letter as a declaration of your consent.

Yours sincerely

______________________ ______________________

Researcher: Ms E Louw Date

Groenkloof campus
University of Pretoria
elizma.louw@up.ac.za
Cell: 072 266 0752

______________________ ______________________

Supervisor: Prof G Stols Date

I, the undersigned, hereby grant consent / do not grant consent (circle the appropriate one) to Ms E Louw to video record / audio record (please circle preferred choice or both) the lessons where my child will be present for her Master's research.

Parent's/guardian's name: ______________________________

Parent's/guardian's signature: ___________________________ Date: _____________

Parent's/guardian's telephone number: ____________________

Child's name: _____________________________________

Grade (e.g. 6A): __________________________________

Addendum B: Information letter to learners

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

FACULTY OF EDUCATION

16 March 2014

Dear Learner / Beste leerder

I am a student from the University of Pretoria and will be coming to look at how your mathematics teacher is teaching you. I will be there for 5 lessons. I will not be teaching you, but I will only sit there and watch how your teacher teaches. You will not be involved in any way and you do not have to do anything.

Ek is ‘n student van die Universiteit van Pretoria en gaan kom kyk hoe julle wiskunde onderwyser(es) vir julle klasgee. Ek sal daar wees vir 5 lesse. Ek gaan nie vir julle klas gee nie, maar ek gaan net daar sit en kyk hoe julle onderwyser(es) klasgee. Jy gaan geensins betrokke wees nie en hoef niks te doen nie.

______________________________ ____________________

Researcher / Navorser: Me E Louw
Groenkloof campus
University of Pretoria
elizma.louw@up.ac.za
Cell: 072 266 0752

Date / Datum

______________________________ ____________________

Supervisor / Studieleier: Prof G Stols

Date / Datum

Addendum C: Questionnaire for sampling purposes

QUESTIONNAIRE FOR SAMPLING

To be completed by teachers teaching mathematics in the intermediate and/or senior phase(s).

Thank you for being willing to complete this questionnaire. Your answers will be treated confidentially. This short questionnaire serves as a way for me to select the participants for my master's research. The purpose of this questionnaire is to explore:

- the years of teaching experience you have, especially in special needs education
- the years of training in special needs education
- type of training.

It is important to answer the questions truthfully.

Age: ___ 21-30 ___ 31-40 ___ 41-50 ___ 51-60 ___ 60+

Gender: ___ Male ___ Female

Years of teacher experience (interrupted or uninterrupted):

___ 0-3 ___4-6 ___7-9 ___ 10-12 ___13+

Years of special needs teacher experience:

___ 0-3 ___4-6 ___7-9 ___ 10-12 ___13+

Do you have any formal special needs training (including remedial training)? ___ Yes ___ No

If yes, what type of training? ____

____

How long was this training? ____

Will you be willing to participate in research on teacher knowledge and practice? Please see attached consent letter of what the research entails (not to be completed at this stage.)

__Yes __No

Please return in the enclosed envelope.

COMPLETE THE FOLLOWING ONLY IF YOU ARE INTERESTED IN BEING INVOLVED IN THE RESEARCH

If you are willing to participate in the research please provide the following details and return it in the enclosed envelope.

Name and surname: ______________________________________

Telephone number: ______________________________________

Email address: ____

Name of school: ____

For any questions please feel free to contact me.

Elizma Louw

Cell: 072 266 0752

Email: elizma.louw@up.ac.za

Addendum D: Pre-observation questionnaire

QUESTIONNAIRE FOR TEACHER'S KNOWLEDGE OF INTRINSIC BARRIERS TO LEARNING

Thank you for being willing to complete this questionnaire. The purpose of this questionnaire is to determine your knowledge of the barriers/problems that learners with learning difficulties experience. Your answers will be treated confidentially.

____

School: A B C Date: ________________

Teacher: 1 2 3 4

Please list as many problems/barriers that you know of or that your learners struggle with in learning and understanding mathematics. The barriers should only be about problems/barriers that the learner experience from within him/herself. No external problems/barriers should be listed. Only those problems/barriers that hinder a learner internally or dysfunctions in his/her brain to understand and do mathematics should be listed.

1.____

____

2.____

____

3.____

____

4.____

____

5.____

____

6.____

____

7.____

____

8.____

9.

10.

11.

12.

Addendum E: Observation schedule

CLASSROOM OBSERVATION SCHEDULE OF INSTRUCTIONAL PRACTICE

Date: ____________ Time: _____________ Grade: ____________ Topic: ________________

Type of lesson: New / Review / Other _________________________

Size of class: _______________

School: A B Teacher: 1 2 3 4

Teaching strategies observed during lesson	
Using authentic context (real-life examples; generalisation)	
Building meaningful learner connections (connect to pre-knowledge)	
Using CRA sequencing (objects, pictures, diagrams, letters)	
Using demonstration (multi-sensory, cueing)	
Using scaffolding (guided practice, fading of teacher demonstration)	
Using problem-solving strategies (model through thinking aloud, guided questions)	
Consider the language of mathematics (vocabulary first, learner expresses)	
Goal-setting, progress monitoring and feedback	
Other teaching strategies	

Teaching continuum	
Explicit instruction (teacher directed, teacher do all)	
Interactive instruction (guided instruction, balance between other 2)	
Implicit instruction (learner-directed, learner do all through discovery)	

Open / Closed / Prompting / Learners' questions: ____

Notes

Addendum F: Post-observation questionnaire

QUESTIONNAIRE ON TEACHER'S KNOWLEDGE OF CLASSROOM PRACTICE

Thank you for being willing to complete this questionnaire. The purpose of this questionnaire is to determine what your knowledge is of how to teach in order for learners with learning disabilities to overcome their problems/barriers to learning. Your answers will be treated confidentially.

School: A B C Date: ___________________

Teacher: 1 2 3 4

Briefly explain what aspects you are taking into consideration when planning a mathematics lesson? List each aspect and briefly explain each aspect.

1.____

____

____

2.____

____

____

3.____

____

____

4.____

____

____

Referring to the first questionnaire, list and briefly explain what methods you think you can implement in your day to day teaching of mathematics to help your learners with learning difficulties overcome their intrinsic barriers to learning. Other intrinsic barriers to learning maybe not mentioned (by you) on the first questionnaire, and which you can consider are learnt helplessness, passive learning, maths anxiety, memory difficulties, attention difficulties, processing difficulties, motor disabilities, language difficulties, and metacognitive thinking difficulties.

1.____

2.____

3.____

4.____

5.____

6.____

7.____

8.____

9.____

10.____

11.____

Do you think that you as a mathematics teacher teaching learners with learning difficulties should rather do: direct teaching (giving and explaining all the concepts, you are responsible to teach them), indirect teaching (let learners discover and learn for themselves, the learner is responsible to learn themselves), or a combination of the two (you and the learner are both responsible for the learning (guided learning)? Choose one and then briefly explain why you chose it.

Addendum G: Ethical clearance certificate

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA
Faculty of Education

RESEARCH ETHICS COMMITTEE

CLEARANCE CERTIFICATE	**CLEARANCE NUMBER :** SM 13/11/01
DEGREE AND PROJECT	MEd Investigation into mathematics instruction for learners with learning difficulties
INVESTIGATOR(S)	Elizma Louw
DEPARTMENT DATE	Science, Mathematics and Technology Education
CONSIDERED	1 December 2014
DECISION OF THE COMMITTEE	APPROVED

Please note:
For Masters applications, ethical clearance is valid for 2 years
For PhD applications, ethical clearance is valid for 3 years.

CHAIRPERSON OF ETHICS COMMITTEE	Prof Liesel Ebersöhn
DATE	1 December 2014
CC	Jeannie Beukes Liesel Ebersöhn Dr BLK Mofolo-Mbokane Prof GH Stols

This ethical clearance certificate is issued subject to the following condition:

1. It remains the students' responsibility to ensure that all the necessary forms for informed consent are kept for future queries.

Please quote the clearance number in all enquiries.