An optimal investment planning framework for multiple distributed generation units in industrial distribution systems

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HIGHLIGHTS

- DG allocation for minimizing energy loss and enhancing voltage stability.
- Expressions to find the optimal power factor of DG with commercial standard size.
- A methodology for DG planning to recover investment for DG owners.
- Impact of technical and environmental benefits on DG investment decisions.
- Benefit-cost analysis to specify the optimal location, size and number of DG units.

ABSTRACT

This paper presents new analytical expressions to efficiently capture the optimal power factor of each Distributed Generation (DG) unit for reducing energy losses and enhancing voltage stability over a given planning horizon. These expressions are based on the derivation of a multi-objective index (*IMO*), which is formulated as a combination of active and reactive power loss indices. The decision for the optimal location, size and number of DG units is then obtained through a benefit–cost analysis. Here, the total benefit includes energy sales and additional benefits, namely energy loss reduction, network upgrade deferral and emission reduction. The total cost is a sum of capital, operation and maintenance costs. The methodology was applied to a 69-bus industrial distribution system. The results showed that the additional benefits are imperative. Inclusion of these in the analysis would yield faster DG investment recovery.

Keywords:
Distributed generation; Emission reduction; Loss reduction; Network upgrade deferral; Optimal power factor; Voltage stability

1. Introduction

For the reasons of energy security and economical and environmental benefits, there has been increased interest in the usage of Distributed Generation (DG) worldwide. DG can be defined as small-scale generating units located close to the loads that are being served [1]. It is possible to classify DG technologies into two broad categories: non-renewable and renewable energy re-sources [2]. The former comprises reciprocating engines, combus-tion gas turbines, micro-turbines, fuel cells, and micro-Combined Heat and Power (CHP) plants. The latter includes biomass, wind, solar photovoltaic (PV) and ocean-based power plants. From the utilities' perspective, DG units can bring multiple technical benefits to distribution systems such as loss reduction, voltage profile improvement, voltage stability enhancement, network upgrade deferral and reliability while supplying energy sales as a primary purpose [3–13]. In addition, DG units can participate into the competitive market to provide ancillary services such as spinning reserve, voltage regulation, reactive power support and frequency control [14–16]. However, inappropriate allocation and operations of these resources may lead to high losses, voltage rise and system instability as a result of reverse power flow [17,18].

DG planning by considering various technical issues has been discussed considerably over the last decade. Several approaches have been developed to place and size DG units for loss reduction due to its impact on the utilities' revenue. Typical examples are analytical methods [19–21], numerical approaches [22–24] and a wide range of heuristic algorithms such as Genetic Algorithm (GA) [25], Particle Swarm Optimization (PSO) [26] and artificial bee colony algorithm [27]. Moreover, in recent years, due to shar-ply increased loads and the demand for higher system security, DG

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Nomenclature

AE_y actual annual emission of the system with DG units (Ton CO_2)
$ALoss_y$ actual annual energy loss of the system with DG units (MW h)
$AVSM$ average voltage stability margin of the system
B present value benefit over a planning horizon ($)
BCR benefit and cost ratio
C present value cost over a planning horizon ($)
C_{DG} capital cost of DG ($/kW)
CE_y cost of each ton of generated CO_2 ($/Ton CO_2)
$CLoss_y$ loss value ($/MW h)
d discount rate
EI_y emission incentive ($/year)
LF load factor or average load level of the system over a planning horizon
LF_{base} load factor or average load level of the system over the base year
ILP, ILQ active and reactive power loss indices, respectively
IMO multi-objective index
IRR internal rate of return
LI_y N loss incentive ($/year) number
ND of buses
N_y network deferral benefit ($/kW)
planning horizon (years)
NPV net present value
OM_y annual operation, maintenance and fuel costs ($/year)
pf_{DGi} power factor of DG unit at bus i
P_{DGi}, Q_{DGi}, S_{DGi} active, reactive and apparent power sizes of DG unit, respectively at bus i
P_{Di}, Q_{Di} active and reactive power of load, respectively at bus i
P_i, Q_i net active and reactive power injections, respectively at bus i
P_{LDG}, Q_{LDG} total system active and reactive power losses with DG unit (MW), respectively
P_L, Q_L total system active and reactive power losses without DG unit (MW), respectively
R_y annual energy sales ($/year)
TE_y annual emission target level of the system without DG (Ton CO_2)
$TLoss_y$ annual energy loss target level of the system without DG (MW h)
VSM voltage stability margin
$|V_i|$, δ_i voltage magnitude and angle, respectively at bus i
Z_{ij} ijth element of impedance matrix ($Z_{ij} = r_{ij} + jx_{ij}$)
δ growth rate of demand a year
λ_{max} maximum loading
$\Delta AVSM$ an increase in the average voltage stability margin

allocation for voltage stability at the distribution system level has attracted the interest of some recent research efforts. For instance, DG units are located and sized using different methods: iterative techniques based on Continuous Power Flow (CPF) [8] and a hybrid of model analysis and CPF [28], power stability index-based method [29], numerical approach [30,31], simulated annealing algorithm [32] and PSO [33–35]. However, the cost–benefit analyses of DG planning have been ignored in the works presented above. Furthermore, a few recent studies have indicated that network investment deferral and emission reduction are other attractive options for DG planning. For instance, an optimal power flow-based method was successfully developed to place and size DG units for postponing network upgrade [4]. An immune-GA method was presented for placing and sizing DG units to reduce the total emission while minimizing the total cost as a sum of electricity purchased from the grid, installation, operation and network reinforcement costs [36]. An improved honey bee mating optimization approach was also proposed for locating and sizing DG units to reduce the total emission while minimizing the capital, fuel, operation and maintenance costs, voltage deviation and energy loss [5]. In addition, an planning framework was also developed for PV integration by reducing the installation, operation and maintenance costs and the energy imported from the grid [37]. It is obvious from the above review that numerous methodologies have developed for DG allocation in distribution systems with different applications. However, most of them have assumed that DG units operate at a pre-defined power factor. Depending on the nature of loads served, DG operation at optimum power factor may have positive impacts on system losses, voltage stability, and system capacity release.

Recently, a few studies have presented DG allocation while considering the optimal power factor, to which the active and reactive power injections of each DG are optimized simultaneously. For instance, a rule of thumb for DG operation was developed for minimizing power losses [20]. For this rule, it is recommended that the power factor of DG should be equal to the system load factor. A PSO-based method was presented to identify the location, size and power factor of DG for minimizing power losses [26]. In [21], three different analytical approaches were presented to determine the location, size and power factor of renewable DG (i.e., biomass, wind and solar PV) for minimizing energy losses. A dual index-based analytical approach was proposed to find the location, size and power factor of DG for minimizing power loss and improving loadability [38]. Finally, a self-correction algorithm was proposed to specify the size and power factor of PV and battery energy storage units for minimizing energy losses and enhancing voltage stability [39]. The above review shows that a few works have discussed the optimal power factor of DG units. However, the size of DG units obtained from the existing studies may not match the standard sizes available in the market. Furthermore, a comprehensive benefit–cost study on multiple DG allocation with optimal power factor while considering the issues of energy loss and voltage stability has not been reported in the literature.

This paper aims at expending the previous preliminary study in [40] where analytical expressions were developed based on a single objective to identify the optimal power factor of each DG unit for minimizing energy losses. In this paper, analytical expressions are presented based on a multi-objective index (*IMO*) to determine the optimal power factor for reducing energy losses and enhancing voltage stability in industrial distribution systems over a given planning horizon. Here, new analytical expressions are developed to efficiently determine the optimal power factor of each DG unit with a commercial standard size to ease the computational burden. In this study, it is assumed that DG units are owned and operated by distribution utilities. To make the work comprehensive, in addition to the analytical expressions presented to specify the optimal power factor, a benefit–cost analysis is carried out in the paper to determine the optimal location, size and number of DG units. The total benefit as a sum of energy sales, energy loss reduction, network upgrade deferral and emission reduction is compared to the total cost including capital, operation and maintenance costs.

The rest of the paper is structured as follows: Section 2 describes the modeling of loads and DG units. Section 3 presents active and reactive power loss indices and a combination of both known as the *IMO*. Methods of assessing the energy loss and voltage stability, and analyzing the benefit and cost with DG units are also presented in this section. Section 4 presents analytical expressions to capture the optimal power factor of DG units and a computational procedure for DG allocation. Section 5 presents and discusses a case study on a 69-bus test distribution system. Finally, the key contributions and conclusions of the work are summarized in Section 6.

2. Load and DG modeling

2.1. Load modeling

The system considered under the study is assumed to follow the industrial load duration curve as shown in Fig. 1, including four discrete load bands (maximum, normal, medium and minimum) that change as the load grows over a planning horizon. The load factor or average load level of the system over the base year, LF_{base} can be defined as the ratio of the area under load curve to the total duration (four load bands: 8760 h). That means $LF_{base} = \sum_{t=1}^{8760} \frac{p.u.\ load(t)}{8760}$ where $p.u.\ load\ (t)$ is the demand in p.u. at period t.

Assuming the growth rate of demand a year (δ), the load factor or average load level of the system over a given planning horizon (Ny), LF can be calculated as:

$$LF = \frac{1}{Ny} \sum_{y=1}^{Ny} \sum_{t=1}^{8760} \frac{p.u.\ load(t)}{8760} \times (1+\delta)^y \tag{1}$$

The dependence of loads on the voltage and time at period t can be expressed as [41]:

$$P_i(t) = P_{oi}(t) \times V_i^{n_p}(t);\ Q_i(t) = Q_{oi}(t) \times V_i^{n_q}(t) \tag{2}$$

where P_i and Q_i are respectively the active and reactive power injections at bus i, P_{oi} and Q_{oi} are respectively the active and reactive loads at bus i at nominal voltage; V_i is the voltage at bus i; n_p = 0.18 and n_q = 6.0 are respectively the active and reactive industrial load voltage exponents [41].

2.2. DG modeling

Given the fact that most of the DG units are normally designed to operate at unity power factor under the standard IEEE 1547 [42]. Consequently, inadequacy of reactive power support for voltage regulation may exist in distribution systems, given a high DG penetration. It is likely that shortage of reactive power support may be an immediate concern at the distribution system level in the future. Conventional devices such as switchable capacitors, voltage regulators and tap changers are actually employed for automatic voltage regulation, but they are not fast enough to compensate for transient events [43,44]. As fast response devices, synchronous machines-based DG technologies (e.g., gas turbine engine) are allowed to control reactive power for voltage regulation.

As reported in [45], gas turbine engines use a turbine spun by the gases of combustion to rotate an electric generator. For DG application, gas turbine engines have smaller sizes than any other source of rotating power and provide higher reliability than reciprocating engines. They also have superior response to load variations and excellent steady state frequency regulation when compared to steam turbines or reciprocating engines. Moreover, they can operate on a wide range of fuels such as natural gas, waste gas, methane, propane and diesel. In addition, gas turbine engines require lower maintenance and produce lower emissions than reciprocating engines.

For the above reasons, gas turbine engine-based DG units are adopted in this study. As synchronous machines, the DG units are capable of delivering active power and injecting or absorbing reactive power. It is assumed that the units offer a constant energy supply at rated capacity. Given P_{DGi} and Q_{DGi} values which correspond to the active and reactive power of DG unit injected at bus i, the power factor of DG unit at bus i (pf_{DGi}) can be expressed as follows [21]:

$$pf_{DGi} = \frac{P_{DGi}}{\sqrt{P^2_{DGi} + Q^2_{DGi}}} \tag{3}$$

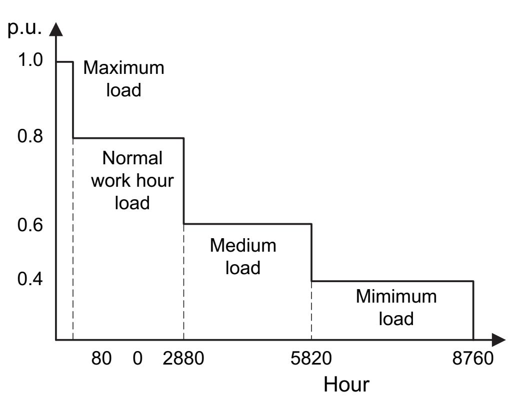

Fig. 1. Load duration curve.

3. Problem formulation

3.1. Active power loss index

The total active power loss (P_L) in a system with N buses can be expressed as [46]:

$$P_L = \sum_{i=1}^{N}\sum_{j=1}^{N}[\alpha_{ij}(P_iP_j + Q_iQ_j) + \beta_{ij}(Q_iP_j - P_iQ_j)] \tag{4}$$

where

$$\alpha_{ij} = \frac{r_{ij}}{V_iV_j}\cos(\delta_i - \delta_j);\ \beta_{ij} = \frac{r_{ij}}{V_iV_j}\sin(\delta_i - \delta_j)$$

$V_i\angle\delta_i$ is complex voltage at bus i; $r_{ij} + jx_{ij} = Z_{ij}$ is the ijth element of impedance matrix [Z_{bus}]; P_i and P_j are respectively active power injections at buses i and j; and Q_i and Q_j are respectively the reactive power injections at buses i and j.

The active and reactive power injected at bus i where a DG unit is installed can be expressed as [20]:

$$P_i = P_{DGi} - P_{Di} \tag{5}$$

$$Q_i = Q_{DGi} - Q_{Di} \tag{6}$$

where P_{DGi} and Q_{DGi} are respectively the active and reactive power injections from DG at bus i; P_{Di} and Q_{Di} are respectively the active and reactive power of a load at bus i.

Substituting Eqs. (5) and (6) into Eq. (4), we obtain the total active power loss with DG unit (P_{LDG}) as follows:

$$P_{LDG} = \sum_{i=1}^{N}\sum_{j=1}^{N}[\alpha_{ij}((P_{DGi} - P_{Di})P_j + (Q_{DGi} - Q_{Di})Q_j) + \beta_{ij}((Q_{DGi} - Q_{Di})P_j - (P_{DGi} - P_{Di})Q_j)] \tag{7}$$

Finally, the active power loss index (ILP) can be defined as Eq.(7) divided by Eq. (4) as follows [38]:

$$ILP = \frac{P_{LDG}}{P_L} \tag{8}$$

3.2. Reactive power loss index

The total reactive power loss (Q_L) in a system with N buses can be expressed as [46]:

$$Q_L = \sum_{i=1}^{N}\sum_{j=1}^{N}\left[\gamma_{ij}(P_iP_j + Q_iQ_j) + \xi_{ij}(Q_iP_j - P_iQ_j)\right] \tag{9}$$

where

$$\gamma_{ij} = \frac{x_{ij}}{V_iV_j}\cos(\delta_i - \delta_j);\ \xi_{ij} = \frac{x_{ij}}{V_iV_j}\sin(\delta_i - \delta_j)$$

Substituting Eqs. (5) and (6) into Eq. (9), we obtain the total reactive power loss with DG unit (Q_{LDG}) as follows:

$$\begin{aligned} Q_{LDG} = \sum_{i=1}^{N}\sum_{j=1}^{N}\Big[&\gamma_{ij}((P_{DGi} - P_{Di})P_j + (Q_{DGi} - Q_{Di})Q_j) \\ &+ \xi_{ij}((Q_{DGi} - Q_{Di})P_j - (P_{DGi} - P_{Di})Q_j)\Big] \end{aligned} \tag{10}$$

Finally, the reactive power loss index (ILQ) can be defined as Eq. (10) divided by Eq. (9) as follows [38]:

$$ILQ = \frac{Q_{LDG}}{Q_L} \tag{11}$$

3.3. Multi-objective index

The multi-objective index (IMO) is a combination of the ILP and ILQ impact indices, which are respectively related to energy loss and voltage stability by giving a weight to each impact index. This IMO index can be expressed by Eq. (12) which is subject to the constraint on the pre-specified apparent power of DG capacity (S_{DGi}) through a relationship between P_{DGi} and Q_{DGi}. That means:

$$IMO = \sigma_1 ILP + \sigma_2 ILQ \tag{12}$$

subject to

$$S^2_{DGi} = P^2_{DGi} + Q^2_{DGi}$$

where $\sum_i^2 \sigma_i = 1.0 \wedge \sigma_i \in [0, 1.0]$. This can be performed as all impact indices are normalized with values between zero and one [25]. When DG unit is not connected to the system (i.e., base case system), the IMO is highest at one.

The weights are intended to give the corresponding importance to each impact index for DG connection and depend on the required analysis (e.g., planning and operation) [25,39,47–49]. Determining the appropriate weights will also rely on the experience of engineers and the concerns of distribution utilities. DG integration in distribution networks has a significant impact on the energy loss and voltage stability. Currently, the energy loss is one of the major concerns at the distribution system level due to its impact on the utilities' profit, while the voltage stability is less important than the energy loss. Hence, the weight for the energy loss should be higher than that for the voltage stability. In future, if the impor-tance of voltage stability is increased due to a rise in load demands and system security concerns, the weights can be adjusted based on the priority. Considering the current concerns mentioned above and referring to previous papers [25,39,47–49], this study assumes that the active power loss related to energy loss receives a signifi-cant weight of 0.7, leaving the reactive power loss related to volt-age stability at a weight of 0.3.

The lowest IMO implies the best DG allocation for energy loss reduction and voltage stability enhancement. The objective function defined by the IMO in Eq. (12) is subject to technical constraints described below.

3.4. Technical constraints

The maximum DG penetration, which is calculated as the total capacity of DG units, is limited to less than or equal to a sum of the total system demand and the total system loss.

$$\sum_{i=2}^{N} P_{DGi} \leqslant \sum_{i=2}^{N} P_{Di} + P_L; \quad \sum_{i=2}^{N} Q_{DGi} \leqslant \sum_{i=2}^{N} Q_{Di} + Q_L \tag{13}$$

where P_L and Q_L are respectively calculated using Eqs. (4) and (9).

The voltage at each bus is maintained close to nominal.

$$V_i^{min} \leqslant V_i \leqslant V_i^{max} \tag{14}$$

where V_i^{min} and V_i^{max} are respectively the lower and upper bounds of the voltage at bus i, V_i = 1 p.u. (substation voltage).

The thermal capacity of circuit n (S_n^{max}) is less than the maximum apparent power transfer (S_n).

$$|S_n| \leqslant S_n^{max} \tag{15}$$

3.5. Energy loss and voltage stability

3.5.1. Energy loss

The total active power loss of a system with DG unit at each period t, P_{loss} can be obtained from Eq. (7). Here, the total period duration of a year is 8760 h, which are calculated as a sum of all the total period durations of all the load levels throughout a year as shown in Fig. 1. The total annual energy loss in a distribution system can be calculated as $ALoss_y = \sum_{t=1}^{8760} P_{loss}(t)$. Hence, the total energy loss over a given planning horizon (Ny), E_{Loss} can be expressed as:

$$E_{Loss} = \sum_{y=1}^{Ny}\sum_{t=1}^{8760} P_{loss}(y, t) \times \Delta t \tag{16}$$

where Δt is 1 h, which is the time duration of period t.

3.5.2. Voltage stability

The static voltage stability can be analyzed using the relationship between the receiving power (P) and the voltage (V) at a certain bus in a distribution power system, as illustrated in Fig. 2. This curve is known as a P–V curve and obtained using the CPF technique [50]. The critical point (CP) or voltage collapse point in the curve represents the maximum loading (λ_{max}) of the system. The voltage stability margin (VSM) is defined as the distance from an operating point to the critical point. As shown in Fig. 2, the scaling factor of the load demand at a certain operating point (λ) varies from zero to λ_{max}. When DG unit is properly injected in the system, the loss reduces. Accordingly, the V_1 and CP_1 enhance to V_2 and CP_2, respectively. Hence, the maximum loadability increases from λ_{max1}

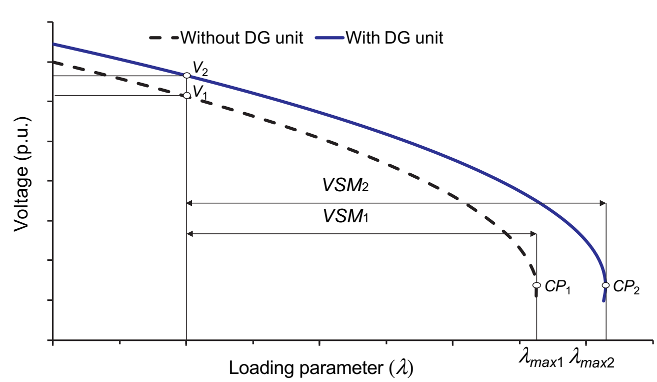

Fig. 2. DG impact on maximum loadability and voltage stability margin.

to λ_{max2} as defined by (17) [28,31,38,39] and the voltage stability margin subsequently improves from VSM_1 to VSM_2.

$$P_D = \lambda P_o; \quad Q_D = \lambda Q_o \tag{17}$$

where P_{oi} and Q_{oi} correspond to the initial active and reactive power demands, respectively.

3.6. Benefit and cost analysis

3.6.1. Utility's benefit

The present value benefit (B) in \$ given to a utility to encourage DG connection over a planning horizon from owning and sitting its own DG units can be expressed as follows:

$$B = \sum_{y=1}^{Ny} \frac{R_y \pm LI_y \pm EI_y}{(1+d)^y} + ND \sum_{i=2}^{N} P_{DGi} \tag{18}$$

where all annual values are discounted at the rate d; R_y is annual energy sales (\$/year) in year; ND is the network deferral benefit (\$/kW); P_{DGi} is the total DG capacity connected at bus i (kW); and N_y is the planning horizon (years). The loss incentive LI_y (\$/year) can be written as [3,4]:

$$LI_y = CLoss_y(TLoss_y - ALoss_y)$$

where $CLoss_y$ is the loss value (\$/MW h), $ALoss_y$ is the actual annual energy loss of the system with DG units (MW h), and $TLoss_y$ is the target level of the annual energy loss of the system without DG unit (MW h). When DG units are integrated into the grid for primary energy supply purposes, the environmental benefit as a result of reducing the usage of fossil fuel energy resources could be obtained. The emission incentive EI_y (\$/year) including the emission produced by the electricity purchased from the grid and DG units can be formulated as [36]:

$$EI_y = CE_y(TE_y - AE_y)$$

where CE_y is the cost of each ton of generated CO_2 (\$/Ton CO_2); AE_y is the actual annual emission of a system with DG units (Ton CO_2); TE_y is the target level of the annual emission of the system without DG unit (Ton CO_2).

3.6.2. Utility's cost

The present value cost (C) in \$ incurred by a distribution utility over a planning horizon can be expressed as [3,4]:

$$C = \sum_{y=1}^{Ny} \frac{OM_y}{(1+d)^y} + C_{DG} \sum_{i=2}^{N} P_{DGi} \tag{19}$$

where OM_y is the annual operation, maintenance and fuel costs (\$/year) in year y; C_{DG} is the capital cost of DG (\$/kW).

3.6.3. Benefit–cost ratio analysis

The benefit–cost ratio (BCR) can be expressed as follows:

$$BCR = \frac{B}{C} \tag{20}$$

where the B and C are calculated using Eqs. (18) and (19), respectively. The decision for the optimal location, size and number of DG units is obtained when the BCR as given by Eq. (20) is highest.

4. Proposed methodology

The reactive power of DG units can be utilized for loss reduc-tion, voltage profile and stability enhancement, and network investment deferral. It could be highlighted that lack of attention to reactive power support at the DG planning stage would poten-tially result in an increase in investment costs used to add reactive power resources and other devices at the operation stage. However, redundancy of reactive power can lead to reverse power flow, thereby leading to high losses, voltage rise, system instability, etc. Consequently, it becomes necessary to study the optimal power factor of DG units to which the active and reactive power injections of each DG are optimized simultaneously [20].

4.1. Optimal power factor

In practice, the choice of the best DG capacity may follow commercial standard sizes available in the market or be limited by energy resource availability. Given such a pre-specified DG capacity, the DG power factor can be optimally calculated by adjusting the active and reactive power sizes at which the IMO as defined by Eq. (12) can reach a minimum level. Using the Lagrange multiplier method, the constrained problem defined by Eq. (12) can be mathematically converted into an unconstrained one as follows:

$$L(P_{DGi}, Q_{DGi}, \lambda_i) = \sigma_1 ILP + \sigma_2 ILQ + \lambda_i \left(S^2_{DGi} - P^2_{DGi} - Q^2_{DGi}\right) \tag{21}$$

where λ_i is the Lagrangian multiplier.

Substituting Eqs. (8) and (11) into Eq. (21), we obtain:

$$L = \frac{\sigma_1}{P_L} P_{LDG} + \frac{\sigma_2}{Q_L} Q_{LDG} + \lambda_i \left(S^2_{DGi} - P^2_{DGi} - Q^2_{DGi}\right) \tag{22}$$

The necessary conditions for the optimization problem, given by Eq. (22), state that the derivatives with respect to control variables P_{DGi}, Q_{DGi} and λ_i become zero.

$$\frac{\partial L}{\partial P_{DGi}} = \frac{\sigma_1}{P_L} \frac{\partial P_{LDG}}{\partial P_{DGi}} + \frac{\sigma_2}{Q_L} \frac{Q_{LDG}}{\partial P_{DGi}} - 2\lambda_i P_{DGi} = 0 \tag{23}$$

$$\frac{\partial L}{\partial Q_{DGi}} = \frac{\sigma_1}{P_L} \frac{\partial P_{LDG}}{\partial Q_{DGi}} + \frac{\sigma_2}{Q_L} \frac{Q_{LDG}}{\partial Q_{DGi}} - 2\lambda_i Q_{DGi} = 0 \tag{24}$$

$$\frac{\partial L}{\partial \lambda_i} = P^2_{DGi} + Q^2_{DGi} - S^2_{DGi} = 0 \tag{25}$$

The derivative of Eqs. (7) and (10) with respect to P_{DGi} and Q_{DGi} are given as:

$$\frac{\partial P_{LDG}}{\partial P_{DGi}} = 2\sum_{j=1}^{N}[\alpha_{ij} P_j - \beta_{ij} Q_j] = 2\alpha_{ii} P_i + 2A_i \tag{26}$$

$$\frac{\partial Q_{LDG}}{\partial P_{DGi}} = 2\sum_{j=1}^{N}[\gamma_{ij} P_j - \xi_{ij} Q_j] = 2\gamma_{ii} P_i + 2C_i \tag{27}$$

$$\frac{\partial P_{LDG}}{\partial Q_{DGi}} = 2\sum_{j=1}^{N}[\alpha_{ij} Q_j + \beta_{ij} P_j] = 2\alpha_{ii} Q_i + 2B_i \tag{28}$$

$$\frac{\partial Q_{LDG}}{\partial Q_{DGi}} = 2\sum_{j=1}^{N}[\gamma_{ij} Q_j + \xi_{ij} P_j] = 2\gamma_{ii} Q_i + 2D_i \tag{29}$$

where

$$A_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} (\alpha_{ij} P_j - \beta_{ij} Q_j); \; B_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} (\alpha_{ij} Q_j + \beta_{ij} P_j)$$

$$C_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} (\gamma_{ij} P_j - \xi_{ij} Q_j); \; D_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} (\gamma_{ij} Q_j + \xi_{ij} P_j)$$

Substituting Eqs. (26) and (27) into Eq. (23), we obtain:

$$\frac{2\sigma_1}{P_L}[\alpha_{ii}P_i + A_i] + \frac{2\sigma_2}{Q_L}[\gamma_{ii}P_i + C_i] - 2\lambda_i P_{DGi} = 0 \tag{30}$$

Substituting Eq. (5) into Eq. (30), we obtain:

$$P_{DGi} = \frac{P_{Di}Y_i - \frac{\sigma_1}{P_L}A_i - \frac{\sigma_2}{Q_L}C_i}{Y_i - \lambda_i} \tag{31}$$

where

$$Y_i = \frac{\sigma_1\alpha_{ii}}{P_L} + \frac{\sigma_2\gamma_{ii}}{Q_L}$$

Similarly, substituting Eqs. (28) and (29) into Eq. (24), we obtain:

$$\frac{2\sigma_1}{P_L}[\alpha_{ii}Q_i + B_i] + \frac{2\sigma_2}{Q_L}[\gamma_{ii}Q_i + D_i] - 2\lambda_i Q_{DGi} = 0 \tag{32}$$

Substituting Eq. (6) into Eq. (32), we obtain Eq. (33), where Y_i is given in Eq. (31).

$$Q_{DGi} = \frac{Q_{Di}Y_i - \frac{\sigma_1}{P_L}B_i - \frac{\sigma_2}{Q_L}D_i}{Y_i - \lambda_i} \tag{33}$$

Substituting Eqs. (31) and (33) into Eq. (25), we obtain:

$$Y_i - \lambda_i = \pm\frac{1}{S_{DGi}} \times \sqrt{\left(P_{Di}Y_i - \frac{\sigma_1 A_i}{P_L} - \frac{\sigma_2 C_i}{Q_L}\right)^2 + \left(Q_{Di}Y_i - \frac{\sigma_1 B_i}{P_L} - \frac{\sigma_2 D_i}{Q_L}\right)^2} \tag{34}$$

Substituting Eq. (34) into Eqs. (31) and (33), we obtain:

$$P_{DGi} = \pm\frac{\left(P_{Di}Y_i - \frac{\sigma_1 A_i}{P_L} - \frac{\sigma_2 C_i}{Q_L}\right)S_{DGi}}{\sqrt{\left(P_{Di}Y_i - \frac{\sigma_1}{P_L}A_i - \frac{\sigma_2}{Q_L}C_i\right)^2 + \left(Q_{Di}Y_i - \frac{\sigma_1}{P_L}B_i - \frac{\sigma_2}{Q_L}D_i\right)^2}} \tag{35}$$

$$Q_{DGi} = \pm\frac{\left(Q_{Di}Y_i - \frac{\sigma_1 B_i}{P_L} - \frac{\sigma_2 D_i}{Q_L}\right)S_{DGi}}{\sqrt{\left(P_{Di}Y_i - \frac{\sigma_1}{P_L}A_i - \frac{\sigma_2}{Q_L}C_i\right)^2 + \left(Q_{Di}Y_i - \frac{\sigma_1}{P_L}B_i - \frac{\sigma_2}{Q_L}D_i\right)^2}} \tag{36}$$

It is observed from Eq. (35) that P_{DGi} can be positive or negative, depending on the characteristic of system loads. However, the load power factor of a distribution system without reactive power compensation is normally in the range from 0.7 to 0.95 lagging (inductive load). P_{DGi} is assumed to be positive in this study, i.e., DG unit delivers active power. Q_{DGi} can be positive or negative, as given by Eq. (36). Q_{DGi} can be positive with inductive loads or negative with capacitive loads (i.e., DG unit injects or absorbs reactive power).

Given a S_{DGi} value is pre-defined, the optimal P_{DGi} and Q_{DGi} values are respectively calculated using Eqs. (35) and (36) to minimize the *IMO* as defined by Eq. (12), after running only one load flow for the base case system. Accordingly, the optimal power factor (pf_{DGi}) value is specified using Eq. (3). Any power factors rather than the optimal pf_{DGi} value will lead to a higher *IMO*.

4.2. Computational procedure

DG units are considered to be placed at an average load level (*LF*) defined by Eq. (1) over a given planning horizon, which has the most positive impact on the *IMO*. This also reduces the computational burden and the search space. The energy loss given by Eq. (16) is calculated by a multiyear multi-period power flow analysis over the planning horizon. The computational procedure is explained for each step as follows:

Step 1: Set the apparent power of DG unit (S_{DGi}) and the maximum number of buses to connect DG units.
Step 2: Run load flow for the system without DG unit at the average load level over the planning horizon (*LF*) using Eq. (1).
Step 3: Find the optimal power factor of DG unit for each bus using Eq. (3). Place this DG unit at each bus and find the *IMO* for each case using Eq. (12).
Step 4: Locate the optimal bus for DG at which the *IMO* is minimum with the corresponding optimal size and power factor at that bus.
Step 5: Run multiyear multi-period load flow with the DG size obtained in *Step 4* over the planning horizon. Calculate the energy loss and its corresponding *BCR* using Eqs. (16) and (20), respectively.
Step 6: Repeat *Steps 3–5* until "the maximum number of buses is reached". These buses are defined as "a set of candidate buses". Continue to connect DG units to "these candidate buses" by repeating *Steps 3–5*.
Step 7: Stop if any of the violations of the constraints (Section 3.4) occurs or the last iteration *BCR* is smaller than the previous iteration one. Obtain the results of the previous iteration.

5. Case study

5.1. Test systems

The proposed methodology was applied to an 11 kV 69-bus radial distribution system with four feeders that are fed by a 6 MVA 33/11 kV transformer, as depicted in Fig. 3 [51]. Its complete data can be found in [52]. The total active and reactive power of the system at the average load level defined by Eq. (1) is 3.35 MW and 2.30 MVAr, respectively. The proposed methodology was simulated in MATLAB environment.

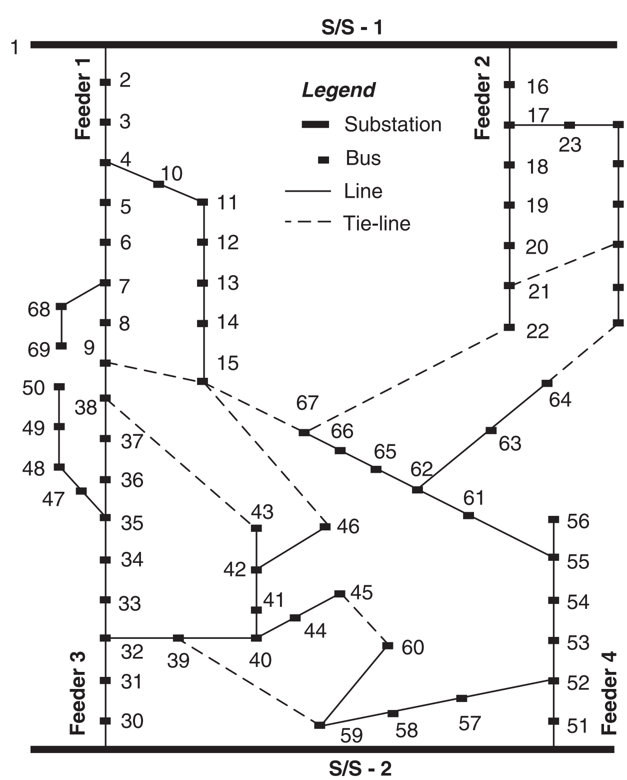

Fig. 3. Single line diagram of the 69-bus test distribution system without DG [51,52].

5.2. Assumptions and constraints

Operating voltages are limited in the range of 0.94–1.06 p.u. and feeder thermal limits are 5.1 MVA (270 A) [3]. It is assumed that the time-varying voltage dependent industrial load as defined in Eq. (2) is considered in this simulation. The loading at each bus fol-lows the industrial load duration curve across a year shown in Fig. 1 over a planning horizon of 15 years with a yearly demand growth of 3%. As given by Eq. (1), the load factor or average load level over the planning horizon (*LF*) is 0.75. All buses are candidate for DG investment and more than one DG units can be installed at the same bus. The substation transformers are close to their ther-mal rating and would need replacing in the near future, while the conductors exhibit considerable extra headroom for further de-mand. For the reasons of simplicity, DG units are connected at the start of the planning horizon and operating for the whole time a year (8760 h) at rated capacity throughout the planning horizon. That means the average utilization factor of DG units is 100%. Nat-ural gas engine-based DG technology is used. Its size (S_{DG}) is pre-specified at 0.8 MVA. The input data given in Table 1, are employed for benefit and cost analyses.

5.3. Numerical results

The total load of the system is 4.07 MVA. Given a pre-defined DG size of 0.8 MVA each and the constraint of DG penetration as defined by Eq. (13), the maximum number of DG units is limited to be five with a total size of 4 MVA. To compare the benefits brought to the utility, five scenarios (1, 2, 3, 4 and 5 DG units) have been analyzed.

5.3.1. Location, size and power factor with respect to indices

Fig. 4 presents the 69-bus system with DG units. The optimal locations are identified at buses 62, 35, 25, 4 and 39 where five DG units (i.e., DGs 1, 2, 3, 4 and 5, respectively) are optimally placed. Table 2 shows a summary of the results of the location, power factor and size of DG units for the five scenarios as men-tioned earlier over the planning horizon of 15 years. As each DG unit is pre-defined at 0.8 MVA, its power factor is adjusted such that the *IMO* index obtained for each scenario is lowest. The opti-mal power factor for each location is quite different, in the range of 0.82–0.89 (lagging). The total size is increased from 0.8 to 4 MVA with respect to the number of DG units increased from one to five. It has been found from the simulation that three sce-narios (i.e., 3, 4 and 5 DG units) satisfy the technical constraints. When less than three DG units are considered, the violation of the voltage constraint (i.e., the operating voltages are under 0.94 p.u.) occurs at several buses in the system.

Fig. 5 shows a comparison of the *ILP*, *ILQ* and *IMO* indices with different numbers of DG units over the planning horizon, which are related to the active power loss index, reactive power loss in-dex and a multi-objective index. As shown in Fig. 5, the indices re-duce when the number of DG units is increased from one to five. However, when the number of DG units is further increased, the total penetration of DG is higher than the total demand as previ-ously mentioned along with an increase in the values of indices. Substantial reductions in the indices are observed in three scenar-ios (i.e., 3, 4 and 5 DG units) when compared to one and two DG units. For each scenario, the *ILP* is lower than the *ILQ*. This indicates that the system with DG units can benefit more from minimizing the active power loss than to the reactive power loss.

Table 1
Economic input data.

Gas engine-based DG capacity [3]	0.8 MVA
Investment cost [3]	\$976/kW
Operation and maintenance and fuel costs [3]	\$46/MW h
Electricity sales [3]	\$76/MW h
Loss incentive [3]	\$78/MW h
Network upgrade deferral benefit for deferral of transformer upgrades [3]	\$407/kW of DG
Emission factor of grid [36]	0.910 Ton CO_2/MW h
Emission factor of 1 MVA gas engine [36]	0.773 Ton CO_2/MW h
Emission cost [36]	\$10/Ton CO_2
Discount rate [3]	9%

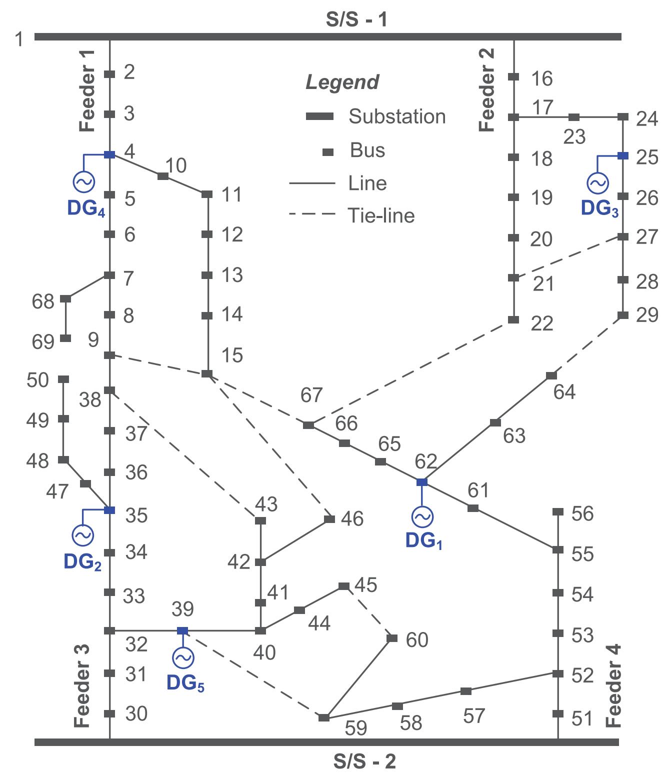

Fig. 4. Single line diagram of the 69-bus test distribution system with DG units.

5.3.2. DG impact on energy loss and voltage stability

Fig. 6 presents the total energy loss of the system for different scenarios without and with DG units over the planning horizon. For each scenario, the total energy loss for each year is estimated as a sum of all the energy losses at the respective load levels of that year. As shown in Fig. 6, the system energy loss with no DG units increases over the planning horizon due to the annual demand growth of 3%. A significant reduction in the energy loss over the planning horizon is observed for the scenarios with DG units when compared to the scenario without DG units. The lowest energy loss is achieved for the scenario with five DG units. As the amount of the power generation from three DG units is still not sufficient, the system energy loss for the scenario with three DG units reduce insignificantly when compared to that with four or five DG units.

Fig. 7 shows the impact of DG allocation on the voltage stability of the system with and without DG units over the planning horizon of 15 years. For each year, the simulation has been implemented at the maximum demand, where the voltage stability margin (*VSM*) i s worst when compared to the other loading levels. In each year, as defined in Fig. 2, the *VSM* of the system with 3–5 DG units signif-icantly enhances when compared to that of the system without DG units. For example, when three DG units generate an amount of 2.4 MVA at buses 62, 35 and 25 in the first year found in Table 2,

Table 2
Location, size and power factor of DG units.

Scenarios	DG location	DG size (MVA)	DG power factor (lag.)	Total DG size (MVA)	Permissible constraints?
1 DG	62	0.8	0.87	0.8	No
2 DGs	62	0.8	0.87	1.6	No
	35	0.8	0.89		
3 DGs	62	0.8	0.87	2.4	Yes
	35	0.8	0.89		
	25	0.8	0.89		
4 DGs	62	0.8	0.87	3.2	Yes
	35	0.8	0.89		
	25	0.8	0.89		
	4	0.8	0.85		
5 DGs	62	0.8	0.87	4.0	Yes
	35	0.8	0.89		
	25	0.8	0.89		
	4	0.8	0.85		
	39	0.8	0.82		

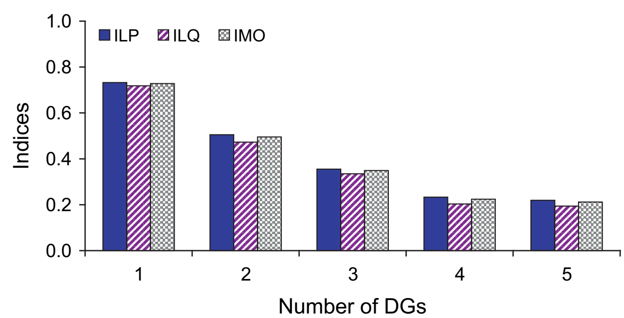

Fig. 5. Indices (*ILP*, *ILQ* and *IMO*) for the system with various numbers of DG units over planning horizon.

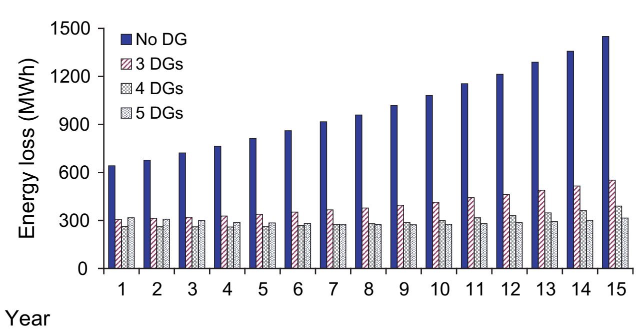

Fig. 6. Losses of the system with and without DG units over planning horizon.

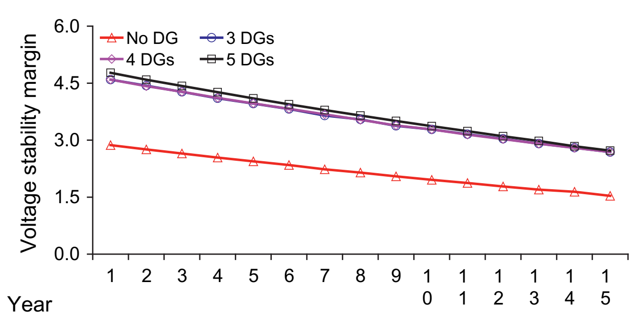

Fig. 7. Voltage stability margin curves for all scenarios over planning horizon.

the *VSM* increases to 4.5918 from the base case value of 2.8681 (without DG units). A similar trend has been found for years 2–15 as shown in Fig. 7. Furthermore, as shown in Fig. 7, a significant increase in the voltage stability margin is found for the scenarios with four or five DG units when compared to three DG units. This is due to the fact that the *ILP*, which is related to the reactive power loss of the system, significantly reduces for the sce-nario with four or five DG units when compared to three DG units, as shown in Fig. 5. In addition, it is observed from Fig. 7 that the *VSM* values with and without DG units reduce with respect to a yearly demand growth of 3%. Hence, the lowest *VSM* values are found in the year 15.

Table 3 shows a summary of the results of energy losses with-out and with DG units for each scenario over the planning horizon of 15 years. The energy savings due to loss reduction is beneficial. A maximum energy savings is achieved for the scenario with five DG units when compared to three and four DG units. Table 3 also presents a summary of the results of voltage stability with and without DG units over the planning horizon. The average voltage stability margin of the system (*AVSM*) is calculated as a sum of the *VSM* values of all years divided by the total planning horizon. An increase in the average voltage stability margin (Δ*AVSM*) is observed after 3–5 DG units are installed in the system. It is observed from Table 3 that the *VSM* for each scenario increases with respect to an increase in the number of DG units installed in the system as well as a reduction in the overall energy loss of the system.

5.3.3. Benefit and cost analysis

Table 4 presents the results of the total benefit and cost for three scenarios (i.e., 3, 4 and 5 DG units) without and with addi-tional benefits over the planning horizon of 15 years. The addi-tional benefit includes the loss incentive (*LI*), emission incentive (*EI*) and network upgrade deferral (*ND*). The total benefit (*B*) is a sum of all the additional benefits and the energy sales (*R*). The total cost (*C*) is a sum of the operation, maintenance and fuel cost (*OM*) and the DG capacity cost ($C_{DG}\,\Sigma P_{DGi}$). Table 5 shows a comparison of the results for three different scenarios without and with addi-tional benefits over the planning horizon. The results include the benefit–cost ratio (*BCR*), net present value ($NPV = B - C$), payback period, and internal rate of return (*IRR*).

For exclusion of the additional benefits, it is observed from Ta-ble 5 that the *BCR* is the same for all the scenarios at 1.288. The best solution is five DG units as the *NPV* is highest at k$4162. This solution generates an *IRR* of 16.48% and a payback period of 5.6 years. For inclusion of the additional benefits, it is seen from Table 4 that the energy sales (*R*) accounts for around 89–90% of the *B*, leaving the total additional benefit (*LI*, *EI* and *ND*) at roughly 10–11%. It is obvious that the *R* has a significant impact on the *BCR* when

Table 3
Energy loss and voltage stability losses over planning horizon.

Scenarios	Energy loss (GW h/15 years)	Energy savings (GW h/15 years)	Voltage stability	
			AVSM	$\Delta AVSM$
Base case	14.91		2.1667	
3 DGs	5.97	8.94	3.5745	1.4078
4 DGs	4.46	10.45	3.5758	1.4091
5 DGs	4.35	10.56	3.6855	1.5188

Table 4
Analysis of the present value benefit and cost for different scenarios over planning horizon.

Number of DGs	Without additional benefits			With additional benefits		
	3 DGs	4 DGs	5 DGs	3 DGs	4 DGs	5 DGs
R (k$)	11,421	15,090	18,624	11,421	15,090	18,624
LI (k$)	–	–	–	336	392	387
EI (k$)	–	–	–	244	316	379
$ND\ \Sigma P_{DGi}$ (k$)	–	–	–	860	1137	1403
Total benefit, B (k$)	11,421	15,090	18,624	12,862	16,934	20,793
R/*B* (%)				88.80	89.11	89.57
$(LI + EI + ND\ \Sigma P_{DGi})/B$ (%)				11.20	10.89	10.43
OM (k$)	6804	8990	11,095	6804	8990	11,095
$C_{DG}\ \Sigma P_{DGi}$ (k$)	2065	2728	3367	2065	2728	3367
Total cost, C (k$)	8869	11,718	14,462	8869	11,718	14,462

Table 5
Comparison of different scenarios over planning horizon.

Number of DGs	Without additional benefits			With additional benefits		
	3 DGs	4 DGs	5 DGs	3 DGs	4 DGs	5 DGs
$BCR = B/C$	1.288	1.288	1.288	1.450	1.445	1.438
$NPV = B - C$ (k$)	2552	3372	4162	3993	5216	6331
Payback period (years)	5.60	5.60	5.60	2.80	2.82	2.86
Internal rate of return, IRR (%)	16.48	16.48	16.48	32.65	32.38	31.93

compared to the total additional benefit. However, the additional benefits, particularly the *LI* play a critical role in decision-making about the total number of DG units or the amount of DG capacity installed. This factor has an impact on the *BCR*. As shown in Table 5, the *BCR* slightly drops from 1.450 to 1.438 when the number of DG units is increased from three to five, respectively. The best solution is three DG units with the highest *BCR* of 1.450. This solution generates an *NPV* of k$3993, an *IRR* of 32.65% and a payback period of 2.80 years. In general, in the absence of the additional benefits, the optimal number of DG units is five, while in the presence of the additional benefits, this figure is three. Inclusion of the additional benefits in the study can lead to faster investment recovery with

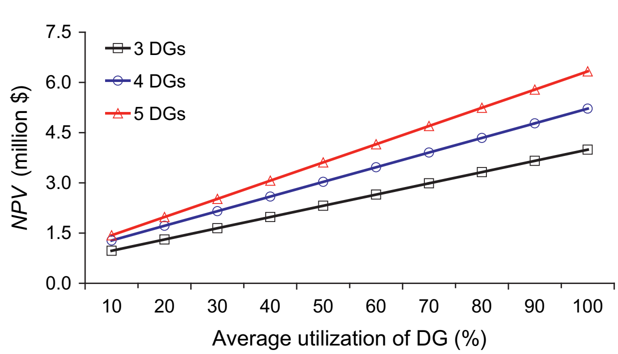

Fig. 8. *NPV* at various average DG utilization factors for different scenerios over planning horizon.

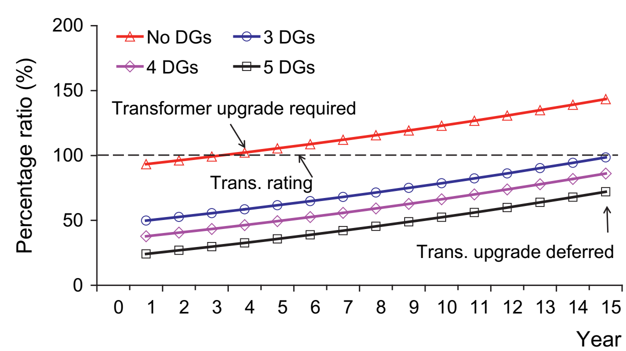

Fig. 9. Percentage ratio of the total demand plus loss to the thermal transformer limit over planning horizon.

a higher *BCR*, a higher *IRR* and a shorter payback period when compared to exclusion of the additional benefits. In addition, it is shown from Table 5 that the *NPV* value will increase further with increasing DG units installed. However, the system cannot accommodate more than five DG units due to the violation of the maximum DG penetration constraint as defined by Eq. (13).

Fig. 8 shows an increase in the *NPV* with the corresponding average DG utilization factors over the planning horizon of 15 years for three scenarios (i.e., 3, 4 and 5 units) with the addi-tional benefits. It is observed from the figure that given a certain average DG utilization factor in the range of 0–100%, the *NPV* is highest for the scenario with five DG units, while this figure is lowest for the scenario with three DG units. In addition, for each scenario, the *NPV* is maximum when the utilization factor is 100% as estimated in Table 5. However, in practice, this factor may be less than 100% due to interruption for maintenance and others. Consequently, the respective *NPV* will be reduced as shown in Fig. 8.

Fig. 9 shows the percentage ratio of the total demand plus total loss to the thermal limit (6 MVA) of the transformer over the planning horizon with a demand growth of 3%. For the scenarios with DG units, the curves are plotted at the average DG utilization factor of 100%. Obviously, without DG connection, an investment would be needed to add a new transformer before year 4. However, the selected three-DG scenario can defer in upgrading this current transformer (6 MVA) to 11 years. A higher deferral is achieved for the scenarios with four or five DG units.

6. Conclusions

This paper has developed an investment planning framework for integrating multiple Distributed Generation (DG) units in industrial distribution systems where the DG units are assumed to be owned and operated by utilities. In this framework, analytical expressions are proposed to efficiently identify the optimal power factor of DG units for minimizing energy losses and enhancing voltage stability. The decision for the optimal location, size and number of DG units is achieved through a benefit–cost analysis. The total benefit includes energy sales and three additional bene-fits including loss reduction, network upgrade deferral and emis-sion reduction. The total cost is a sum of capital, operation and maintenance costs. The results obtained on a 69-bus test distribu-tion system indicated that the additional benefits, particularly the loss incentive have a significant impact on decision-making about the total number of DG units or the amount of DG capacity in-stalled. The additional benefits together accounted for 10–11% of the total benefit when compared to the energy sales of 89–90%. Inclusion of these benefits in the study can lead to faster invest-ment recovery with a high benefit–cost ratio, a high internal rate of return and a short payback period.

When DG units are owned by DG developers, the additional benefits should be shared between the distribution utility and DG developer to encourage DG connection. In this situation, the proposed methodology could be used as guidance for the utility on how to plan and operate DG units to obtain the additional benefits.

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