TRENDS AND CYCLES IN HISTORICAL GOLD AND SILVER PRICES[#]

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Abstract

The study proposes an alternative modelling specification for the real prices of gold and silver that allows the long run trend and cyclical behaviour to be modelled simultaneously by incorporating two differencing parameter in a fractional integration framework. However, we also consider the separate cases of a standard I(d) process, with a pole or singularity at the zero frequency and a cyclical I(d) model that incorporates a single pole in the spectrum at a non-zero frequency. We use annual data spanning from 1833 to 2013 for gold and 1792 to 2013 for silver. Based on the more flexible model that permits a pole at both zero (trend) and non-zero (cycle) frequencies, we find that in general the estimates associated to the long run or zero frequency appear to be above 1 in case of gold and below 1 for silver, while the order of integration associated with the cyclical frequency is slightly above 0 in the majority of the cases in the two series. Further, higher orders of integration are associated to the long run component compared with the cyclical one. The implications of these findings are highlighted.

Keywords: Gold and Silver Prices, Cycles, Persistence, Long memory

JEL Classification: C22, Q02

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1. Introduction

Commodity prices are generally known to be very volatile leading to uncertainty over future revenue and cost streams. This consequently inhibits planning, deters investment and hence acts as a drag on future growth and poverty reduction prospects. The recent global economic and financial crisis which led to significant increase in commodity prices between 2003 and 2008 has renewed interest in modelling commodity price behaviour. The real prices of energy and metals more than doubled in five years from 2003 to 2008, while the real price of food commodities increased by 75% (Erten and Ocampo, 2013). When the global economic growth slowed down, this led to diminishing demand pressures on commodity prices. However, commodity prices have started to recover very fast, and this has been attributed to the rapid growth of the emerging markets and dramatic increase in worldwide demand for commodities.

Given the swings in prices, the need to model the trend and cycle behaviour of metal prices in general and gold and silver prices in particular cannot be overstressed. They are important for both the producing and consumer countries and for both the private and government. For the producing countries especially in developing countries, export earnings from metals are often the main source of revenue for many governments. The revenues may either come from direct ownership of resource extraction companies or the tax revenues and royalties obtained from private firms. For the consumer countries, they are a key input factor in many industries and hence drastic price increases can affect these industries negatively through higher input costs.

Further, precious metals are considered as leading indicators of inflation or as a variable which can transmit the outlook of monetary policy to the economy. In other words, the pro-cyclical character of the demand for precious metals has underlined their roles as safe-havens against inflation and stores of value and may provide important information as to where the real economy is heading (Baur and Lucey, 2010; Gil-Alana et al., 2015). Thus, precious metals offer valuable diversification opportunities to investors and serve as monetary medium when the market is uncertain (Batten et al., 2010; Arouri et al., 2012; Harvey et al., 2012). Precious metals in general but gold and silver in particular have multiple industrial and investment uses. They can be used as storage of value, reserve for money issuance, anti-inflation shelter and financial instrument among others (Baur and Mcdermott, 2010; Shafiee and Topal, 2010).

Overall, the dynamics of gold and silver prices have implications for investment decisions, profitable capacity expansion and economic planning. Fluctuations in these prices may have a major impact on overall macroeconomic performance and living standards in these countries, hence justifying the need to understand their trend and cycles. Thus, the current work displays a new modelling framework for trends and cycles incorporating different degrees of persistence at each component by means of fractional differentiation. In other words, the main objective of the paper is to present a new time series modeling for gold and silver prices taking simultaneously into account the main two features of the data which are its dependence and its cyclicity.

A number of studies have modelled the trend and cycle features of commodity prices using methods which range from informal graphical inspection of the data to rigorous statistical decomposition techniques and recently to fractional integration. Focussing on this latter approach, the results are mixed. For instance Arouri et al (2012) used several parametric and semiparametric methods including ARFIMA- FIGARCH model and found strong evidence of long memory in the daily conditional return and volatility processes of four precious metals: gold, silver, platinum and palladium. Uludag and Lkhamazhapov (2014) used a similar approach as Arouri et al (2012), and found evidence of anti-persistence in spot returns and a lack of long memory property in gold futures returns. They concluded that long memory is a true feature of the data and not due to structural breaks. Gil-Alana et al. (2015) analysed the persistence properties of five metal prices- gold, silver, platinum, palladium, and rhodium- within a fractional integration framework while accounting for structural breaks. In general they find evidence of long memory behaviour and hence strong dependence across time in the precious metals examined.

From a cyclical viewpoint, Cuddington and Jerrett (2008) use band-pass (BP) filters to search for evidence of super cycles in price of six base metals traded in London Metal Exchange (LME) - aluminum, copper, lead, nickel, tin, and zinc. They find considerable evidence of three past super cycles in real metal prices, defined as cyclical components with expansion phases from 10 to 35 years. They also show that the amplitude of the super cycles is large with variations of 20 to 40% above and below the long-run trends.

Harvey et al. (2012) disentangle trend and cyclical components for relative commodity prices using new and ultra-long aggregate commodity prices, covering the period 1650- 2010. Employing quasi-feasible GLS-based testing approach, it is shown that the trend path of the commodity series can be split into four regimes (i.e. 1650 to the early 1820s, the early 1820s to the early 1870s, the early 1870s to the mid-1940s, and the mid-1940s to 2010). Moreover, using BP filter, they find that long-run cycles (often called super-cycles) last for approximately twenty seven years, but appear to be increasing in periodicity over the 20th century, whereas shorter-run cycles last for approximately four years but appear to be decreasing in periodicity.

Rossen (2014) explores the dynamics (co-movement, price cycles and long-run trends) of monthly twenty metal prices including gold and silver during January 1910 and December 2011. Results based on the asymmetric band-pass (BP) filter show that price cycles are asymmetric; the number of cycles varies significantly depending on the specific metal under consideration, the majority of the metal price series can be characterized by four super cycles consistent with previous studies. However, there is no evidence of duration dependence and the long-run component of metal prices considerably varies over the set of mineral commodities.

In this paper we propose an alternative modelling specification for the real prices of gold and silver. Classical methods include the trend stationary I(0) and the nostationary unit roots (I(1)) models. However, during the last twenty years fractional integration has become an alternative plausible way of modelling many economic time series. The I(d, d> 0) specification imposes the existence of a pole or singularity in the spectrum at the zero frequency, which is usually associated with the long run trend behaviour. However, as earlier mentioned, another inherent feature observed in many series, including gold and silver prices, is the existence of a cyclical pattern that in many times is reduced to the short run dynamics and incorporated throughout a simple AR(2) process with complex roots. In this paper, we incorporate cycles in the long run dynamics by means of allowing for the existence of another pole in the spectrum at a non-zero frequency. Thus, we consider three different specifications: a) a standard I(d) process, with a pole or singularity at the zero frequency; b) a cyclical I(d) model that incorporates a single pole in the spectrum at a non-zero frequency, and c) a general model that incorporates the two features in a single framework and that includes two poles in the spectrum of the series. Therefore, this latter model incorporates two fractional differencing parameters, one at the long run or zero frequency and another one at a non-zero (cyclical) frequency. To the best of our knowledge, this is the first attempt to model trend and cyclical properties of long-span gold and silver prices simultaneously in a fractional integration framework.

The outline of the paper is as follows: Section 2 describes the models employed in the paper and the methodology used. Section 3 presents the data and the main empirical results, while Section 4 concludes the paper.

2. The models

Assuming that $\{u_t, t = 0, \pm 1, ...\}$ is a covariance stationary I(0) process, three models are examined in this work.

a) The standard I(d) model

It is specified as:

$$(1 - L)^d x_t = u_t, \quad t = 0, \pm 1, ..., \tag{1}$$

where d can be any real value, L is the lag-operator ($Lx_t = x_{t-1}$) and u_t is I(0), defined for our purposes as a covariance stationary process with a spectral density function that is positive and finite at any frequency in the spectrum.

The I(d) model of the form given by equation (1) was introduced by Granger (1980, 1981), Granger and Joyeux (1980) and Hosking (1981) and since then it has been widely employed to describe the behaviour of many economic time series. Note that the parameter d plays a crucial role in describing the degree of dependence (long run persistence) of the series. Specifically, if d = 0 in (1), $x_t = u_t$, and the series is I(0), potentially including ARMA structures with the autocorrelations decaying at an exponential rate. If d belongs to the interval (0, 0.5), the series is still covariance stationary but the autocorrelations take longer to disappear than in the I(0) case. If d is in the interval [0.5, 1), the series is no longer covariance stationary; however, it is still mean-reverting with shocks affecting it disappearing in the long run. Finally, if $d \geq 1$ the series is nonstationary and non-mean-reverting.

There exist many methods for estimating and testing d in equation (1). Some of them are parametric while others are semiparametric and they can be specified in the time or in the frequency domain. In this paper we use a parametric method that uses the Whittle function in the frequency domain (Dahlhaus, 1989) along with a Lagrange Multiplier (LM) test proposed by Robinson (1994) and that it has been used in numerous empirical applications (Gil-Alana and Robinson, 1997; Gil-Alana, 2000; Gil-Alana and Henry, 2003; etc.) .

b) The cyclical I(d) model

This specification is based on the Gegenbauer process, and it is defined as:

$$(1 - 2\cos w_r L + L^2)^d x_t = u_t, \qquad t = 1, 2, \ldots, \tag{2}$$

where w_r and d are real values, and u_t is I(0). For practical purposes we define $w_r = 2\pi r/T$, with r = T/s, and thus s will indicate the number of time periods per cycle, while r refers to the frequency that has a pole or singularity in the spectrum of x_t. Note that if r = 0 (or s = 1), the fractional polynomial in (2) becomes $(1 - L)^{2d}$, which is the polynomial associated with the previous case of fractional integration at the long-run or zero frequency. This type of process was introduced by Andel (1986) and subsequently analysed by Gray, Zhang and Woodward (1989, 1994), Giraitis and Leipus (1995), Chung (1996a,b), Gil-Alana (2001) and Dalla and Hidalgo (2005) among others. It can be shown that denoting $\mu = \cos w_r$, for all $d \neq 0$,

$$(1 - 2\mu L + L^2)^{-d} = \sum_{j=0}^{\infty} C_{j,d}(\mu) L^j,$$

where $C_{j,d}(\mu)$ are orthogonal Gegenbauer polynomial coefficients recursively defined as:

$$C_{0,d}(\mu) = 1, \qquad C_{1,d}(\mu) = 2\mu d,$$

$$C_{j,d}(\mu) = 2\mu\left(\frac{d-1}{j}+1\right)C_{j-1,d}(\mu) - \left(2\frac{d-1}{j}+1\right)C_{j-2,d}(\mu), \quad j = 2, 3, \ldots.,$$

Gray et al. (1989) showed that x_t in (2) is (covariance) stationary if d < 0.5 for $|\mu = \cos w_r| < 1$ and if d <0.25 for $|\mu| = 1$.The existence of a pole at a non-zero frequency indicates that the series displays a cyclical pattern.

Here, we will employ first a testing parametric method of Dalla and Hidalgo (2005). It tests the null hypothesis of no cycles against the alternative of strong cycles of form as in (2).

Additionally, we will employ the methods of Giraitis et al. (2001) and Hidalgo (2005), testing both r and d in equation (2) in a parametric and a semiparametric way respectively.

c) A model with two poles at the spectrum

The third model combines the two previous approaches in a single framework. Thus, the model examined is the following:

$$(1 - L)^{d_1}(1 - 2\cos w_r L + L^2)^{d_2} x_t = u_t, \quad t = 1, 2, \ldots, \qquad (3)$$

where d_1 is the order of integration corresponding to the long-run or zero frequency, and d_2 is the order of integration with respect to the non-zero (cyclical) frequency, and u_t is assumed once more to be an I(0) process. As in the previous cases, d_1 and d_2 are allowed to be real values and thus they are not restricted to be integers.

Dealing with the model in equation (3), Robinson (1994) proposes a Lagrange Multiplier (LM) test, testing the null hypothesis:

$$H_o: \; d \equiv (d_1, d_2)^T = (d_{1o}, d_{2o})^T \equiv d_o, \qquad (4)$$

in (3), for real values d_o, where T means transposition, and x_t are the regression errors in a model of form:

$$y_t = \beta^T z_t + x_t, \quad t = 1, 2, \ldots, \qquad (5)$$

wherey_t is the observed time series; β is a (kx1) vector of unknown parameters, and z_t is a (kx1) vector of deterministic terms, that might include, for example, an intercept (i.e. $z_t = 1$) or an intercept with a linear trend ($z_t = (1,t)^T$). The specific form of the test statistic (denoted by $\hat{R}$) is presented in the Appendix. Under very general regularity conditions, Robinson (1994) showed that for this particular version of his tests,

$$\hat{R} \to_d \chi_2^2, \quad as \quad T \to \infty, \qquad (6)$$

where T indicates the sample size, and "$\rightarrow_d$" stands for convergence in distribution. Thus, unlike in other procedures, we are in a classical large-sample testing situation. A test of (5) will reject H_o against the alternative H_a: $d \neq d_o$ if $\hat{R} > \chi^2_{2,\alpha}$, where Prob ($\chi^2_2 > \chi^2_{2,\alpha}$) = α. There are several reasons for using this approach. First, this test is the most efficient in the Pitman sense against local departures from the null, that is, if it is implemented against local departures of the form: H_a: $d = d_o + \delta T^{-1/2}$, for $\delta \neq 0$, the limit distribution is a $\chi^2_2(v)$, with a non-centrality parameter v that is optimal under Gaussianity of u_t. Moreover, Gaussianity is not necessary for the implementation of this procedure, a moment condition of only order 2 being required.

3. Data and empirical results

The data examined correspond to the annual data for real prices of gold (1833 – 2013) and silver (1792 – 2013) in natural logarithmic form, with the start and end points of the samples purely driven by data availability at the time of writing this paper. The data on nominal prices (London PM Fix US dollar per ounce) for gold and silver is obtained from www.kitco.com, while the annual Consumer Price Index (CPI) data used to deflate the nominal prices to obtain the real values of gold and silver prices, are derived from the website of Professor Robert Sahr (http://oregonstate.edu/cla/polisci/sahr/sahr). Figure 1 displays the time series plots (in logs) along with their first differences. It can be observed that gold price appear to have longer price swings than silver price. Silver price rather looks more volatile. Both exhibit two large spikes around the 1980s and 2013 suggesting possibility of structural breaks in the series. The first differenced data display some heteroscedastic behaviour but the methods employed in this paper for estimating and testing the differencing parameters seem to be robust to this feature.

[Insert Figures 1 – 3 about here]

The correlograms and the periodograms of the series and the first differenced data are displayed in Figures 2 and 3 respectively. The correlograms show a cyclical pattern that is clearer observed for the gold real prices. The periodograms of the original data show the highest value at the zero frequency while those of the first differenced data display the highest values at a non-zero frequency.

In order to accommodate deterministic terms, we suppose that z_t in (5) is equal to $(1, t)^T$, such that

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, \ldots, \tag{7}$$

and we start presenting the results for the standard case of I(d) models. Table 1 displays the estimates of d (and the 95% confidence interval) for the three standard cases of no regressors (i.e., $\beta_0 = \beta_1 = 0$ a priori in (7)), an intercept (β_0 unknown and $\beta_1 = 0$ a priori), and an intercept with a linear time trend (β_0 and β_1 unknown), assuming that u_t is a white noise process, an AR(1) and that it follows the model of Bloomfield, which is a non-parametric approach of modelling the I(0) error term.[1]

[Insert Table 1 about here]

Starting with the deterministic terms, the first thing we note is that an intercept seems to be sufficient to describe the deterministic part of the model. Focussing first on the gold prices, we see that if u_t is white noise, the estimated value of d is 1.32, and the unit root null hypothesis is rejected in favour of higher orders of integration, however, allowing for autocorrelated disturbances, the estimated ds are below 1, and the unit root is almost rejected. Looking at the silver log-prices, the three estimates of d are below 1, the unit root cannot be rejected with white noise errors, and this hypothesis is decisively rejected in favour of mean reversion ($d < 1$) with autocorrelated errors.

[1] This nonparametric approach of Bloomfield (1973) accommodates very well in the context of fractional integration (Velasco and Robinson, 2000; Gil-Alana, 2004; etc.)

Next we focus on strong cycles and consider the model given by equation (2). Here first we perform the Dalla and Hidalgo's (2005) method and the results reject the null hypothesis of no cycles in the two series in favour of cyclical long range dependence. Estimating the location for the pole or singularity at the spectrum with the semiparametric method of Hidalgo (2005), the estimates range in the two series between 4 and 9 periods depending on the choice of the bandwidth number. Tables 2a and 2b displays the estimates of d in (2) for r = 4, 5, 6, 7 and 8, again for the three cases of no regressors, an intercept, and an intercept with a linear trend. Table 2a refers to the case of white noise disturbances, while Table 2b focuses on autocorrelatedAR(1) errors.

[Insert Tables 2a and 2b about here]

As with the previous model, the most significant results are those based on an intercept but with no linear trend. Generally the estimates of d are positive in the two series though more evidence of significant positive values of d is found in the gold real prices data. Employing the exponential spectral model of Bloomfield (1973) for the I(0) error term u_t in (2) produced estimates of d fairly similar to those reported in Table 2b for the case of AR(1) errors.

Finally, we employ model (3), which is a more flexible specification than (1) and (2) in the sense that it allows us to jointly estimated the fractional differencing parameters corresponding to the zero and non-zero frequencies. Table 3a refers to the case of white noise errors, while Tables 3b and 3c displays the estimates for the cases of AR(1) and Bloomfield-type disturbances. Focussing on the case with an intercept and starting with Table 3a (white noise) we see that for gold, d_1 is above 1 in all cases ranging from 1.15 (r = 7, 8) to 1.30 (r = 5), while d_2 is slightly above 0.2 for 4 values of r and close to 0 if r = 4. Looking at the results for silver in the same table, d_1 is slightly below 1 and d_2 is slightly above 0 for the five values presented for r.

[Insert Tables 3a, 3b and 3c about here]

Table 3b refers to AR(1) disturbances. For gold, the estimates of d ranges between 0.53 (r = 5) and 0.89 (r = 6, 7), while d_2 is equal to 0.03 with r = 4 and it ranges between 0.12 and 0.17 in the remaining cases. For silver, d_1 oscillates between 0.55 (r = 8) and 0.68 (r = 4), while d_2 is very close to 0. Using the exponential spectral model of Bloomfield (1973) (in Table 3c) the estimates are very close to those based on white noise disturbances. Thus, for gold d_1 is above 1 and d_2 above 0, and for silver, the estimates of d_1 are slightly below 1 while those of d_2 are slightly above 0. The fact that the orders of integration following this third specification are all significantly positive in case of d_1, but also in the majority of the cases for d_2 suggests that this specification is preferred over (1) and (2) since these two models are particular cases of (3) with $d_1 = 0$ and $d_2 = 0$ respectively. A more difficult task is to determine which form is the most adequate one for the I(0) disturbance term. Using LR tests and several diagnostic tests on the residuals the results were a bit ambiguous, however, we came to the conclusion that the non-parametric approach of Bloomfield (1973) accommodates well for the two series under study. Thus, we can conclude by saying that higher orders of integration are associated to the long run component compared with the cyclical one. Thus, the estimates associated to the long run or zero frequency seem to be above 1 in case of gold and below 1 for silver, while the order of integration associated with the cyclical frequency is slightly above 0 in the majority of the cases in the two series.

4. Conclusions

The study proposes an alternative modelling specification for the real prices of gold and silver. We use annual data covering the period 1883-2013. We consider both the long run trend behaviour and the cycle patterns in these two series within a fractional integration framework. We incorporate cycles in the long run dynamics by means of allowing the existence of an additional pole in the spectrum at a non-zero frequency. In other words, the model incorporates two fractional differencing parameters, one at the long run or zero frequency and another one at a non-zero (cyclical) frequency. This is an extension to the conventional fractional integration model that imposes the existence of a unique pole or singularity in the spectrum at the zero frequency, usually associated with the long run trend behaviour. However, we also consider the separate cases of a standard I(d) process, with a pole or singularity at the zero frequency and a cyclical I(d) model that incorporates a single pole in the spectrum at a non-zero frequency. We consider the estimates of the differencing parameter for the three standard cases of no regressors, an intercept, and an intercept with a linear time trend, and assuming that the residual is a white noise process, an AR(1) and that it follows the model of Bloomfield. The most significant specification is that with intercept only.

Results based on the standard I(d) process shows that real gold price is non-mean reverting while real silver price is mean reverting. In other words we find evidence of long memory behaviour in the gold price inflation rate but not in the silver inflation rate.Based on the cyclical I(d) model, the null hypothesis of no cycles in the two series is rejected in favour of cyclical long range dependence and the location for the pole or singularity at the spectrum in the two series is estimated to be between 4 and 9 periods depending on the number of the bandwidth parameter chosen. Finally, results from the more flexible model indicates that in general the estimates associated to the long run or zero frequency appear to be above 1 in case of gold and below 1 for silver, while the order of integration associated with the cyclical frequency is slightly above 0 in the majority of the cases in the two series. Further, higher orders of integration are associated to the long run component compared with the cyclical one.

These results have important implications for policy. For instance in the event of exogenous shocks, the effects will be permanent in gold and strong policy measures need to be adopted to ensure that gold price returns to its original trend. The persistent property in gold and the cyclical long range dependence property in both gold and silver is important for monetary policy especially with respect to inflation targeting since these properties are likely to affect the persistence nature of the aggregate inflation of an economy. This will have implications for economic variables such as interest rates, consumption, investment, and output growth. More so, the findings have implications for portfolio diversification, forecasting of gold and silver prices and overall economic planning.

At this stage, it is important to point out that, since we use long span data, gold and and silver prices are likely to have witnessed structural breaks. And as indicated by Diebold and Inoue (2001), presence of structural breaks can lead to spurious evidence in favour of long-memory. In light of this, we carried out a similar analysis to the one performed in Section 3 by allowing for potential breaks and outliers, and modelled through dummy variables incorporated in the regression model (5). Though it produced some small differences in the magnitudes of the differencing parameters, qualitatively the results, available upon request from the authors, were very similar to those reported in the paper, with higher orders of integration at the long run or zero frequency compared with the cyclical one, and also higher degrees of persistence in gold compared with silver. Future research should be aimed at re-estimating these models by allowing for non-linearities, which may be conducted for instance by means of the Chebyshev polynomials in time.

Appendix

The test statistic proposed by Robinson (1994) for testing H_o (4) in the model given by equations (3) and (5) is given by:

$$\hat{R} = \frac{T}{\hat{\sigma}^4}\hat{a}'\hat{A}^{-1}\hat{a},$$

where T is the sample size, and

$$\hat{a} = \frac{-2\pi}{T}\sum_j^* \psi(\lambda_j) g_u(\lambda_j;\hat{\tau})^{-1} I(\lambda_j); \qquad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T}\sum_{j=1}^{T-1} g_u(\lambda_j;\hat{\tau})^{-1} I(\lambda_j),$$

$$\hat{A} = \frac{2}{T}\left(\sum_j^* \psi(\lambda_j)\psi(\lambda_j)' - \sum_j^* \psi(\lambda_j)\hat{\varepsilon}(\lambda_j)'\left(\sum_j^* \hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)'\right)^{-1}\sum_j^* \hat{\varepsilon}(\lambda_j)\psi(\lambda_j)'\right)$$

$$\psi(\lambda_j)' = \left[\psi_1(\lambda_j), \psi_2(\lambda_j)\right]; \qquad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau}\log g_u(\lambda_j;\hat{\tau}); \qquad \psi_1(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right|;$$

$\psi_2(\lambda_j) = \log\left|2\left(\cos\lambda_j - \cos w_r\right)\right|$, with $\lambda_j = 2\pi j/T$, and the summation in * is over all frequencies which are bounded in the spectrum. $I(\lambda_j)$ is the periodogram of $\hat{u}_t = (1-L)^{d_{1o}}(1-2\cos w_r L + L^2)^{d_{2o}} y_t - \hat{\beta}'\bar{z}_t$, with

$$\hat{\beta} = \left(\sum_{t=1}^{T}\bar{z}_t\,\bar{z}_t'\right)^{-1}\sum_{t=1}^{T}\bar{z}_t\,(1-L)^{d_{1o}}(1-2\cos w_r L + L^2)^{d_{2o}} y_t;$$

$\bar{z}_t = (1-L)^{d_{1o}}(1-2\cos w_r L + L^2)^{d_{2o}} z_t$, evaluated at $\lambda_j = 2\pi j/T$ and $\hat{\tau} = \arg\min_{\tau\in T^*}\sigma^2(\tau)$, with T^* as a suitable subset of the R^q Euclidean space. Finally, the function g_u above is a known function coming from the spectral density of u_t:

$$f(\lambda) = \frac{\sigma^2}{2\pi} g_u(\lambda;\tau), \qquad -\pi < \lambda \le \pi.$$

Note that these tests are purely parametric and, therefore, they require specific modelling assumptions about the short-memory specification of u_t. Thus, if u_t is white noise, $g_u \equiv 1$, and if u_t is an AR process of the form $\phi(L)u_t = \varepsilon_t$, $g_u = |\phi(e^{i\lambda})|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are a function of τ.

The point estimates were obtained by choosing the values that minimise Robinson's (1994) test statistic over a grid of values for d_1, d_2 and r. These parameter estimates were practically identical to those obtained by maximising the Whittle function in the frequency domain (Dahlhaus, 1989). The confidence intervals were obtained by choosing the values of the differencing parameters where the null hypotheses tested could not be rejected at the 5% level.

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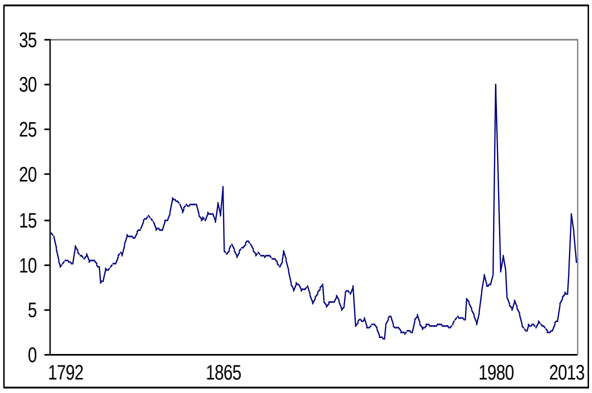
TABLES AND FIGURES

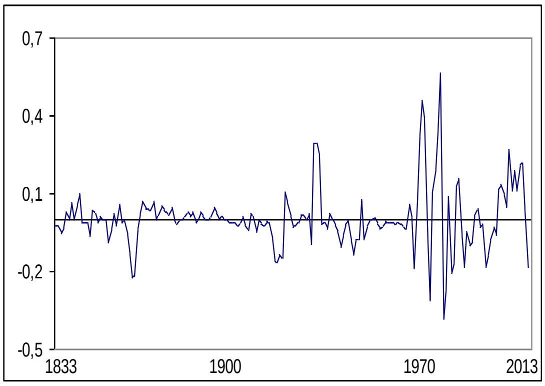
Figure 1: Log of real gold and silver prices and their first differences

Real gold prices

Real silver prices



First differences of real gold prices



First differences of real silver prices

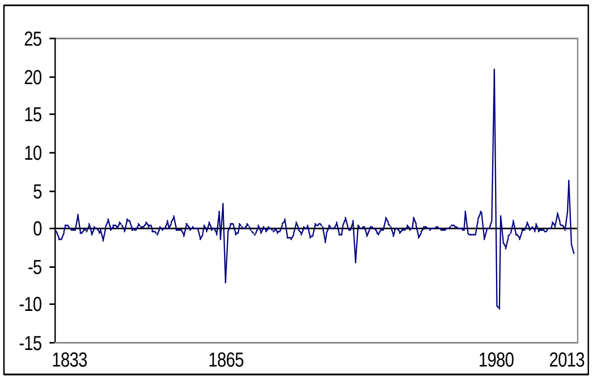
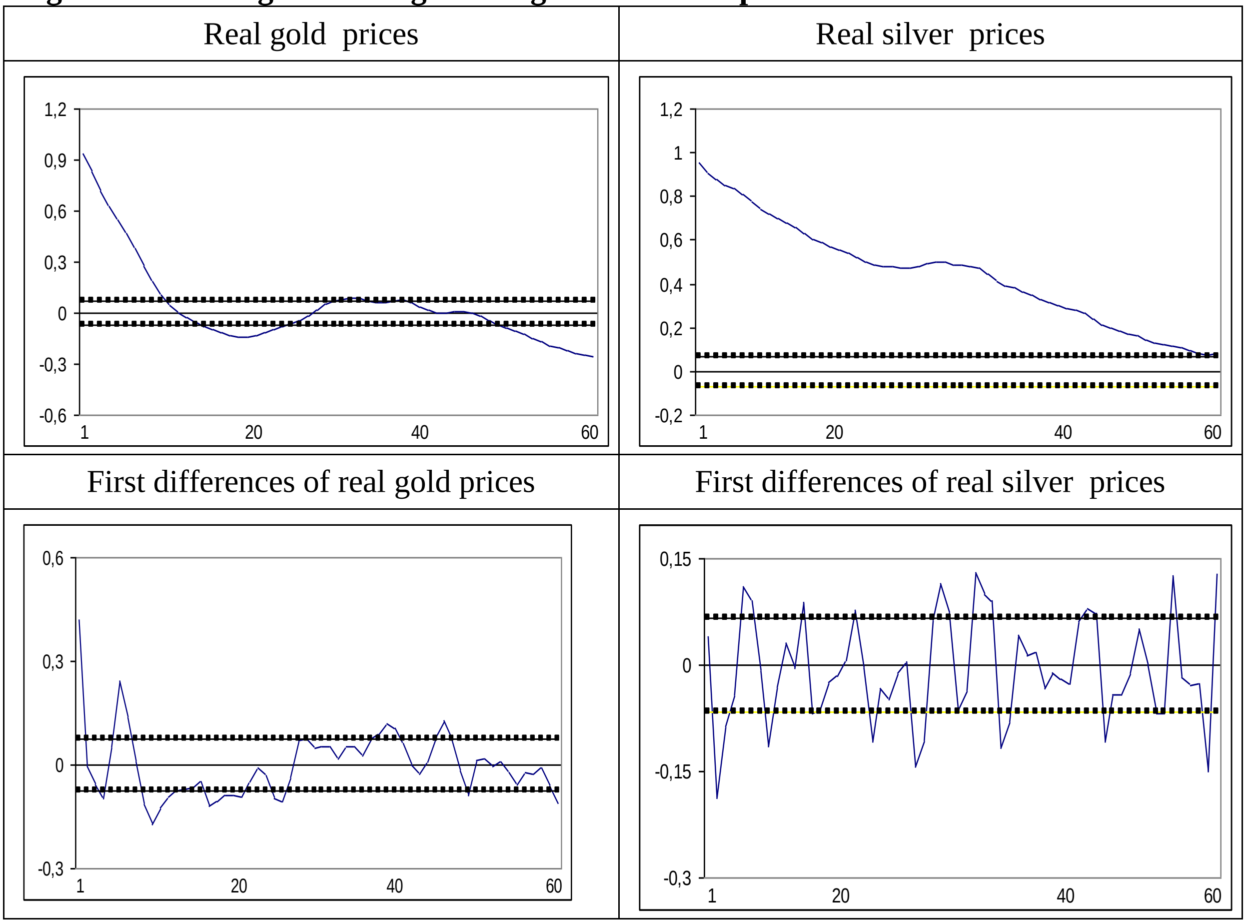
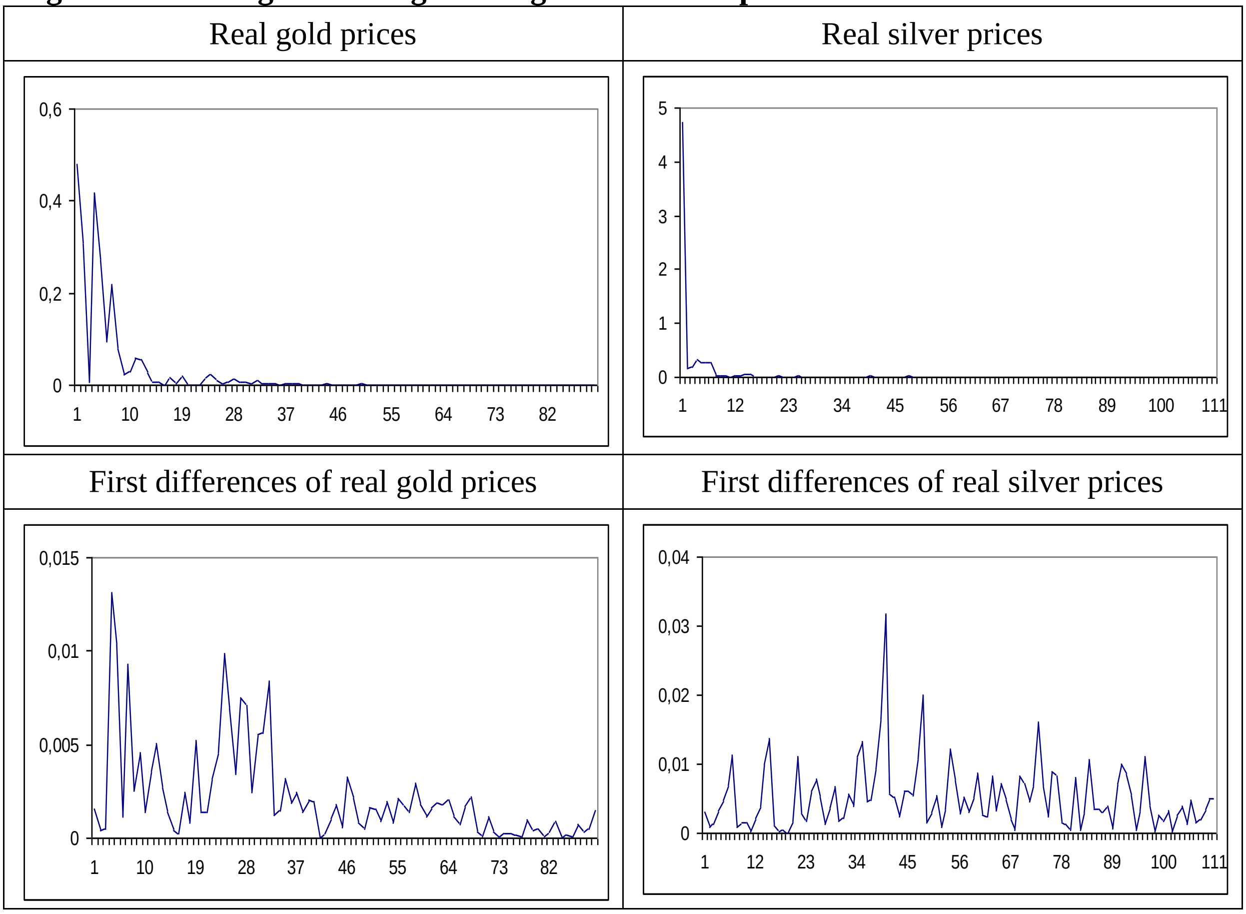


Figure 2: Correlograms of log of real gold and silver prices and their first differences



The thick lines give the 95% confidence band for the null hypothesis of no autocorrelation.

Figure 3: Periodograms of log of real gold and silver prices and their first differences



The horizontal axis refers to the discrete Fourier frequencies $\lambda_j = 2\pi j/T$, j = 1, …, T/2.

Table 1: Estimates of d in an I(d) model at the zero frequency (Model (1))

i) Log of real gold prices			
u_t (disturbances)	No regressors	An intercept	A linear time trend
White noise	1.00 (0.92, 1.10)	**1.32 (1.17, 1.56)**	1.33 (1.17, 1.56)
AR (1)	1.29 (1.13, 1.50)	**0.78 (0.57, 0.99)**	0.42 (0.16, 0.91)
Bloomfield (1)	1.01 (0.86, 1.21)	**0.77 (0.59, 1.00)**	0.76 (0.59,
ii) Log of real silver prices			
u_t (disturbances)	No regressors	An intercept	A linear time trend
White noise	0.94 (0.87, 1.04)	**0.92 (0.82, 1.05)**	0.92 (0.82, 1.05)
AR (1)	xxx	**0.71 (0.56, 0.88)**	0.70 (0.49, 0.88)
Bloomfield (1)	0.93 (0.81, 1.07)	**0.72 (0.60, 0.88)**	0.71 (0.57, 0.88)

In bold, the significant models according to the deterministic terms.

Table 2a: Estimates of d in a cyclical I(d) model (Model (2))

ii) Log of real gold prices			
White noise u_t	No regressors	An intercept	A linear time trend
r = 4	-0.098 (-0.198, 0.040)	**-0.097 (-0.197, 0.041)**	-0.092 (-0.193, 0.050)
r = 5	0.233 (0.063, 0.460)	**0.233 (0.063, 0.460)**	0.239 (0.069, 0.466)
r = 6	0.318 (0.195, 0.467)	**0.318 (0.195, 0.466)**	0.318 (0.196, 0.465)
r = 7	0.314 (0.207, 0.438)	**0.313 (0.207, 0.438)**	0.312 (0.206, 0.435)
r = 8	0.296 (0.191, 0.413)	**0.296 (0.191, 0.412)**	0.293 (0.189, 0.409)
ii) Log of real silver prices			
White noise u_t	No regressors	An intercept	A linear time trend
r = 4	0.092 (-0.018, 0.243)	**0.091 (-0.019, 0.243)**	0.093 (-0.018, 0.245)
r = 5	0.125 (0.035, 0.233)	**0.125 (0.035, 0.233)**	0.126 (0.036, 0.234)
r = 6	0.099 (0.028, 0.182)	**0.099 (0.028, 0.182)**	0.099 (0.028, 0.182)
r = 7	0.040 (-0.041, 0.131)	**0.040 (-0.041, 0.132)**	0.040 (-0.041, 0.132)
r = 8	0.007 (-0.073, 0.098)	**0.007 (-0.074, 0.098)**	0.006 (-0.074, 0.097)

In bold, the significant models according to the deterministic terms.

Table 2b: Estimates of d in a cyclical I(d) model (Model (2))

i) Log of real gold prices			
AR(1)-type u_t	No regressors	An intercept	A linear time trend
r = 4	0.065 (-0.067, 0.254)	**0.066 (-0.066, 0.256)**	0.066 (-0.066, 0.257)
r = 5	0.117 (-0.002, 0.274)	**0.117 (-0.001, 0.275)**	0.120 (0.001, 0.280)
r = 6	0.142 (0.048, 0.262)	**0.142 (0.049, 0.263)**	0.146 (0.053, 0.268)
r = 7	0.146 (0.054, 0.271)	**0.146 (0.053, 0.270)**	0.151 (0.057, 0.277)
r = 8	0.111 (0.012, 0.239)	**0.110 (0.012, 0.238)**	0.115 (0.017, 0.245)
ii) Log of real silver prices			
AR(1)-type u_t	No regressors	An intercept	A linear time trend
r = 4	0.091 (-0.019, 0.242)	**0.091 (-0.018, 0.242)**	0.093 (-0.018, 0.244)
r = 5	0.155 (0.042, 0.303)	**0.155 (0.042, 0.303)**	0.158 (0.044, 0.307)
r = 6	0.159 (0.057, 0.289)	**0.159 (0.057, 0.289)**	0.162 (0.059, 0.291)
r = 7	0.047 (-0.066, 0.195)	**0.048 (-0.065, 0.195)**	0.049 (-0.063, 0.197)
r = 8	-0.039 (-0.142, 0.088)	**-0.039 (-0.142, 0.089)**	-0.038 (-0.142, 0.089)

In bold, the significant models according to the deterministic terms.

Table 3a: Estimates of d_1 and d_2 in a longrun + cyclical I(d) model (Model (3))

i) Log of real gold prices						
White noise u_t	No regressors		An intercept		A linear time trend	
	d_1	d_2	d_1	d_2	d_1	d_2
r = 4	1.04	0.74	**1.27**	**-0.06**	1.27	-0.06
r = 5	1.02	0.91	**1.30**	**0.23**	1.30	0.23
r = 6	1.00	0.91	**1.21**	**0.25**	1.21	0.25
r = 7	0.99	0.90	**1.15**	**0.25**	1.15	0.25
r = 8	1.00	0.88	**1.15**	**0.22**	1.15	0.22
ii) Log of realsilver prices						
White noise u_t	No regressors		An intercept		A linear time trend	
	d_1	d_2	d_1	d_2	d_1	d_2
r = 4	0.93	0.07	**0.93**	**0.06**	0.93	0.06
r = 5	0.90	0.17	**0.90**	**0.14**	0.90	0.14
r = 6	0.87	0.16	**0.86**	**0.14**	0.86	0.14
r = 7	0.85	0.13	**0.85**	**0.10**	0.85	0.10
r = 8	0.86	0.08	**0.84**	**0.08**	0.84	0.08

In bold, the significant models according to the deterministic terms.

Table 3b: Estimates of d_1 and d_2 in a longrun + cyclical I(d) model (Model (3))

ii) Log of real gold prices						
AR-type u_t	No regressors		An intercept		A linear time trend	
	d_1	d_2	d_1	d_2	d_1	d_2
r = 4	0.86	-0.02	**0.72**	**0.03**	0.71	0.03
r = 5	1.25	0.05	**0.53**	**0.17**	0.52	0.18
r = 6	1.25	0.05	**0.89**	**0.15**	0.89	0.15
r = 7	1.25	0.08	**0.89**	**0.15**	0.86	0.14
r = 8	0.90	0.19	**0.76**	**0.12**	0.76	0.12
ii) Log of real silver prices						
AR.-type u_t	No regressors		An intercept		A linear time trend	
	d_1	d_2	d_1	d_2	d_1	d_2
r = 4	0.68	0.01	**0.68**	**0.01**	0.57	0.01
r = 5	0.73	0.06	**0.61**	**0.05**	0.61	0.05
r = 6	0.72	0.06	**0.62**	**0.05**	0.62	0.05
r = 7	0.69	-0.02	**0.57**	**-0.02**	0.54	-0.02
r = 8	0.69	-0.07	**0.55**	**-0.06**	0.54	-0.06

In bold, the significant models according to the deterministic terms.

Table 3c: Estimates of d_1 and d_2 in a longrun + cyclical I(d) model (Model (3))

iii) Log of real gold prices						
Bloomfield-type u_t	No regressors		An intercept		A linear time trend	
	d_1	d_2	d_1	d_2	d_1	d_2
r = 4	1.34	0.05	**1.34**	**0.05**	1.34	0.05
r = 5	1.29	0.03	**1.30**	**0.03**	1.30	0.03
r = 6	1.21	0.25	**1.21**	**0.25**	1.21	0.25
r = 7	1.14	0.25	**1.15**	**0.25**	1.15	0.25
r = 8	1.11	0.24	**1.11**	**0.24**	1.11	0.24
ii) Log of real silver prices						
Bloomfield.-type u_t	No regressors		An intercept		A linear time trend	
	d_1	d_2	d_1	d_2	d_1	d_2
r = 4	0.95	0.05	**0.92**	**0.05**	0.92	0.05
r = 5	0.90	0.14	**0.90**	**0.14**	0.90	0.14
r = 6	0.86	0.14	**0.86**	**0.14**	0.86	0.14
r = 7	0.85	0.10	**0.85**	**0.10**	0.85	0.10
r = 8	0.85	0.08	**0.84**	**0.08**	0.84	0.08

In bold, the significant models according to the deterministic terms.