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TOPOLOGY OPTIMIZATION FOR AN INTERNAL HEAT CONDUCTING COOLING SCHEME IN A SQUARE DOMAIN

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ABSTRACT

Solid state conductive heat transfer is of importance for the cooling of electronic equipment. At smaller length scales internal cooling via conductive heat transfer has advantages above internal convective heat transfer, especially because it does not rely on support system such as pumps. However, in order for conductive cooling to become effective, the use of high conducting materials and the correct distribution thereof is essential, especially when the volume which needs to be cooled has a low thermal conductivity.

In this two-dimensional numerical study the optimum distribution of high conductive material within a square-shaped uniform heat generating medium is investigated by using a topology optimization algorithm. The use of such a method is considered for two different cost (driving) functions and the resulting cooling material distribution or conducting trees are compared.

INTRODUCTION

Conductive cooling of heat-generating volumes has been approached by a number of researchers employing a number of methods. Of interest in many such investigations is the optimum distribution of a restricted amount of material to aid heat transfer from a volume to a point or to a surface.

It has been observed that in many cases optimum distributions of material resembles that which is observed in nature. Fractal theory has been identified to be able to describe a number of such occurrences and has been employed to obtain or describe general optimum transport lay-outs for both living and non-living structures [1,2].

In later years constructal theory gained momentum and attempted to describe higher level transport system lay-outs, for amongst others, heat transfer using constructal levels. Such optimization processes are driven by predefined or deterministic constructal shapes and geometric inter-dependences. Some of these inter-dependences are carried over to the next construct.

In many ways this approach is opposite to fractal theory approach, where the lowest or smallest element is determined first and more complex schemes are constructed from it upwards. For conductive heat transfer such an approach entails solving a volume- or area-to-point heat transfer problem [3]. As a result thermal tree theories have been developed to describe and optimise the distribution of low thermal resistant paths for different thermal tree constructs [4-9].

NOMENCLATURE

A	m	x -directional length of half the domain
B	m	y -directional length of full domain
C	m	x -directional length of half the exposed boundary
f		Function used in MMA optimization
\hat{f}		Upper limit of constraint function
k	W/mK	Thermal conductivity
M	-	Number of elements in x direction
N	-	Number of elements in y direction
p	-	Penalization factor
\dot{q}	W/m ³	Thermal heat generation rate
R	K/W	Thermal one-dimensional resistance
T	K or °C	Temperature
\sim	-	Vector or single value of design variables
$\tilde{\sim}$	-	Adjusted design variables vector or single value
x	-	Cartesian coordinate
y	-	Cartesian coordinate
z	-	Cartesian coordinate
Δx	m	Element size in x direction
Δy	m	Element size in y direction
Special characters:		
\mathcal{E}	-	Definiteness measure of tree structure
γ	-	Thermal conductivity ratio
ϕ	-	Volume fraction occupied by the high conductive insert
\mathcal{K}	-	Iteration number
Subscripts:		
0		Base or heat producing solid
1		High conductive or cooling solid
j		Nodal or element index
N		Northern direction
S		Southern direction
E		Eastern direction
W		Western direction

Constructal theory has the very important ability of simplifying the building block of a thermal network, but it also has the drawback of not allowing natural or free-morphing of the constructal shape. In certain cases the imposed boundary conditions of adjacent constructs or building blocks are oversimplified and the system level thermal communication across construct boundaries might be neglected.

Another approach in finding the optimum distribution of material in heat transfer problems involves shape and topology optimization algorithms. One such method, entitles the “Evolutionary Structural Algorithm” which employs a rejection criteria to replace or alter under-performing regions within a heat transfer structure [10-13]. The relative performance of a region is, however, dependant on the rest of the structural lay-out. This result in evolutionary path being history dependent which possibly causes non-optimum final solutions.

Another approach is topology optimization which have been employed originally for the optimization of physical force and stress carrying structures [14,15]. In recent times this method have however been extended to heat transfer and fluid problems [16]. These algorithms have the intrinsic ability to allow for free-morphing of the obtained material lay-out scheme and is based on a numerical model which is used as a work-horse engine. In this paper the “Method of Moving Asymptotes” [14] topology optimization algorithm is employed directly and resulting lay-outs are investigated.

DOMAIN EXTERNAL BOUNDARY CONDITIONS

Figure 1 shows a representation of the two-dimensional model used in this study. The rectangular heat generating domain has lengths of $2A$ [m] and B [m] in the x and y directions respectively. A portion of the lower boundary (length $2C$) is exposed to an external temperature of T_0 while all other boundaries are adiabatic. A pure heat generating material has a thermal conductivity of k_0 [W/mK] and produces heat at a rate of \dot{q}_0 [W/m³]. Due to the symmetry nature of the domain only the one half is modelled.

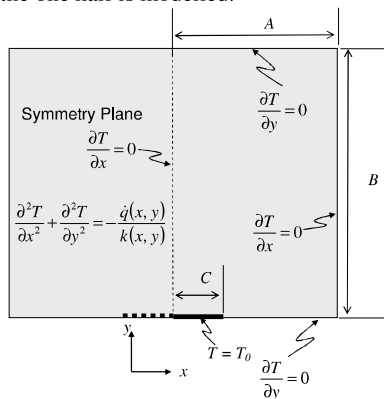


Figure 1 Two-dimensional domain under consideration

In order to reduce the peak temperature in the domain, high thermal conductivity material is introduced. This material does not contribute to heat generation and has a thermal conductivity

higher than that of the heat producing material. Such a pure high heat conduction material has a thermal conductivity of k_1 .

A thermal conductivity ratio can be defined as follows:

$$\gamma = \frac{k_1}{k_0}$$

The amount of high thermal conductive material is however limited by restricting the ratio of its effective volume to the total domain volume. The limiting ratio is given by ϕ :

$$\phi = \frac{\text{maximum allowed volume of heat extracting solid}}{\text{total volume of domain}}$$

A finite difference numerical scheme is used to solve the two-dimensional temperature field within the rectangular domain:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -\frac{\dot{q}(x,y)}{k(x,y)}$$

It must be emphasised that both the heat generation rate and thermal conductivity are allowed to be spatially non-uniform and are dependent on both the x and y coordinates.

NUMERICAL METHOD

In order to make use of a topology optimization scheme, a user-defined code was developed. The domain is uniformly divided into rectangular elements such that that the x and y directional dimensions (Δx and Δy) of all elements are equal:

$$\Delta x = \frac{A}{M} = \Delta y = \frac{B}{N}$$

Here M and N refer to the number of elements in the x and y directions respectively. It must be noted that some restrictions on the choice of dimensions A and B are imposed by this in order to maintain equality between Δx and Δy .

For simplicity purposes and to prevent temperature discontinuity at element interfaces, element-centred temperatures are used in the numerical model. No nodes are thus defined on the boundary of the domain. Each element has uniform thermal conductivity as well as heat generation densities according to a design variable, $\chi_j \in [0;1]$ allocated to each element:

$$k = k_0 + (\chi_j)(k_1 - k_0)$$

$$\dot{q} = \dot{q}_0(1 - \chi_j)$$

Here the subscript j refers to the index number of the element. A pure heat generating element is represented by an elemental design variable $\chi_j = 0$, while a pure heat conducting (or cooling) element is represented by $\chi_j = 1$. Composite elements which have both heat generating and conductive cooling functions are obtained for design variable between 0 and 1.

Thermal interaction between a particular element and its neighbours are represented in Figure 2 following the notation of Patankar [17]. Here T refers to the temperature value at the node at the centre of a cell and subscripts N , E , S , and W refer

to the relevant “northern”, “eastern”, “southern” and “western” directions relative to the central element.

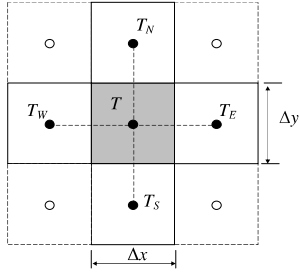


Figure 2 Element centred nodal scheme

The temperature at the central node is dependent on the surrounding temperature values as follows for a unit z -directional depth:

$$\frac{\Delta T_N}{R_N} + \frac{\Delta T_E}{R_E} + \frac{\Delta T_S}{R_S} + \frac{\Delta T_W}{R_W} + \dot{q}\Delta x\Delta y = 0$$

Here $\Delta T_N = T_N - T$ etc., and with R being the thermal resistance between the node and its neighbour. In general the thermal resistance values between a node and its northerly neighbour can be written as:

$$R_N = \frac{\Delta y}{2k\Delta x} + \frac{\Delta y}{2k_N\Delta x}$$

Here k and k_N refers to the thermal conductivity of a particular element and that of its northerly neighbour as defined earlier in terms of the elemental design variables. Similar expressions can be drawn up for the other directional resistance values. Special attention is required for elements on the external boundary of the domain and in particular those elements on the exposed boundary length (length C).

By utilizing an appropriate nodal numbering scheme, a fully implicit matrix approach is used to solve for the steady state temperature field for a given set of design variables. The numerical model was validated against existing commercial software.

TOPOLOGY OPTIMIZATION

The “Method of Moving Asymptotes” (MMA) optimization procedure published by Swanberg in 1987 [14] is used in this study to attempt to determine the optimum distribution of the highly conductive material. This method was originally employed for the optimization of structures, but has since been used in a wide range of topology optimization problems. It is the aim of the optimization scheme to minimize some cost function, $f_0(\cdot)$ subject to one or more behaviour constraints:

$$f_1(\cdot) \leq \hat{f}_1$$

Here \cdot is the vector or matrix of design variables, which has an impact on the cost function.

In this investigation we aim to minimize the cost function in terms of the distribution of the high thermal conductive material within the domain. From initial investigations, it was become clear that the choice of the cost function is an important factor. The following cost functions are used in this paper:

$$\text{Cost function 1: } f_0(\cdot) = T_{\max}$$

$$\text{Cost function 2: } f_0(\cdot) = \frac{1}{MN} \sum_{j=1}^{MN} T_j$$

Index j refers the element numbers of the domain. In case 1 the emphasis is to try and minimize the peak temperature in the domain, while in case 2 the emphasis is to minimize the average temperature in the domain. The only restriction placed on the optimization scheme is to keep the effective volume fraction of the high conductive material to below ϕ_{\max} as was defined earlier. This is implemented as follows:

$$f_1 = \sum_{j=1}^{MN} \phi_j \quad \text{with} \quad \hat{f}_1 = \phi_{\max} MN$$

In order to perform the optimization, a sequence of explicit sub-problems are generated and solved following an iterative scheme as reported by Swanberg (super script with brackets refer to the iteration number):

- STEP 0) A starting point $\cdot^{(0)}$ is chosen for iteration $\mathcal{K} = 0$
- STEP I) For a given iteration, \mathcal{K} , the following is determined:
- i. Constraint function values: $f_1(\cdot^{(\mathcal{K})})$,
 - ii. Gradients of both the cost function and constraint functions: $\nabla f_1(\cdot^{(\mathcal{K})})$
- STEP II) Generate a sub-problem based on the original problem by replacing the original implicit functions with approximating explicit functions based on the results of STEP I.
- STEP III) Find the optimal solution of the sub-problem and let this solution be the next iteration point $\cdot^{(\mathcal{K}+1)}$. Go to STEP I) and repeat until some convergence criteria is met.

Using cost function 1 for a case where the maximum volume fraction is 10% ($\phi_{\max} = 0.1$) and the conductivity ration is $\gamma = 500$, the topology lay-outs as shown in Figure 3 are obtained after different number of iterations. An effective mesh size of 80 by 80 elements were used. $\cdot^{(0)} = 0$ was used and the dimensions of the domain was set to $A = 0.5$ m, $B = 1$ m, and $C = 0.025$. This resulted in the effective domain being square.

Darker shades of grey indicate the recommended position of the high-conductive material within the domain. For the current case it may be seen that the lay-out started to stabilize after about 35 iterations. Figure 4 gives the maximum temperature and ϕ histories of this optimization run. It may be seen that the optimization algorithm is able to adhere to the restriction placed on ϕ .

Considering the temperature history, an initial sharp drop in peak temperatures is evident where-after the temperature picks up and stabilizes. The initial drop in temperature is accompanied by a wide-spread grey distribution within the domain. Such a distribution appears to be thermally efficient, but topologically the cooling material is not coalesced in exact or certain positions. Such “grey” distributions are undesired.

The temperature distribution at iteration 40 is shown in Figure 5. As expected the lowest temperature is at the lower boundary exposed to temperature T_0 , while the maximum temperatures are close to the domain boundaries.

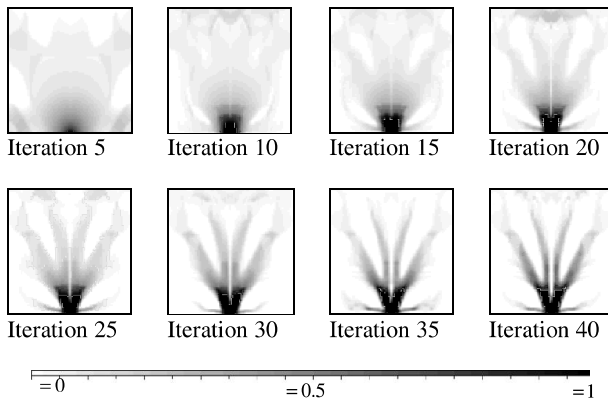


Figure 3 Obtained topology lay-outs after different number of iterations [effective 80x80 mesh size].

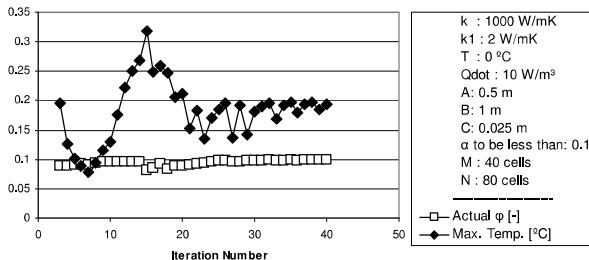


Figure 4 Temperature and volume fraction histories for a particular case

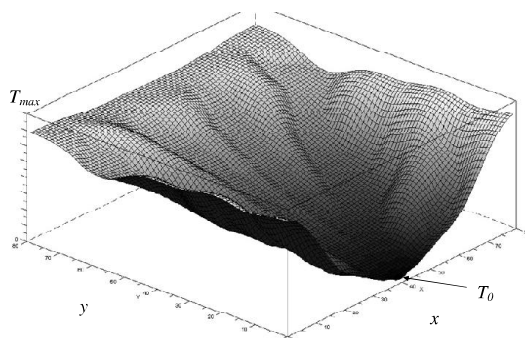


Figure 5 Temperature distribution of the case shown in Figure 3 at iteration 40.

DEFINITENESS MEASURE

The topology optimization algorithm has the freedom to create composite elements (grey shade) as required where applicable to minimize the cost function. Such elements are neither heat producing nor conductive cooling elements and is in essence impossible to manufacture. The following measure is defined to be used to express the definiteness of the obtained distribution of the high conducting material:

$$\epsilon = 1 - \frac{\sum_{j=1}^{MN} (\phi_j^2 - \phi_j)}{(\phi^2 - \phi)MN}$$

For cases where all elements are either purely defined as being either heating elements or conductive cooling elements, ϵ will be equal to 1. For such a case a well defined boundary between the heating and cooling segments of the domain will be present. On the other hand the worst case will be where all elements are equally ill-defined or equally composite in nature. For such a case ϵ will be equal to zero and no observable distribution of the high heat conducting material will be visible. In stead, the high conductive material will be evenly dispersed through-out the domain.

Due to the fact that topology optimizations schemes are likely to produce solutions which contains a large proportion of composite cells, penalization methods are commonly employed whereby such solutions are penalized (sometimes referred to as the SIMP rule). A penalisation factor, p is introduced to favour design variables of either $x_j = 0$ or $x_j = 1$. This is accomplished by using \tilde{x}_j in stead of x_j in STEP 1.ii of the the MMA algorithm:

$$\tilde{x}_j = \frac{p}{x_j} \text{ with } p > 1.$$

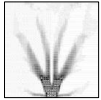

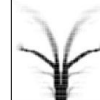
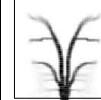
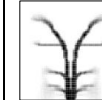
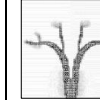
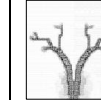
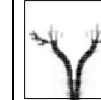
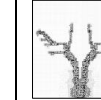
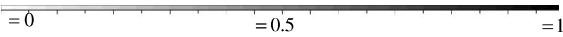
The choice of p is not limited, but may have a significant impact on the solution obtained. The value of p may for instance be kept constant for all MMA iterations, or be increased gradually [18]. In this investigation the value of p was kept constant during an optimization run.

COMPARATIVE RESULTS

Comparative results as obtained using cost function 1 and 2 are shown in Figure 6 for different penalization factors ranging from 1 to 5. The obtained maximum temperature per heat generation rate as well as the final ϕ and definiteness measure, ϵ , are also given. All results are obtained after 40 MMA iterations with a mesh size of effectively 80 by 80 elements using the same dimensions and material properties as before. For all cases optimization convergence was achieved.

It is evident that most of the distribution lay-outs resemble natural trees. These tree-structures are found to be heavily dependent on the choice of the penalization factor as judged by the wide range of obtained lay-outs. It is also seen that for cost function 1 a gradual change in the trees structure with an increase in p is not present. On closer inspection (results not included in this paper) it was found that at certain p values a rapid change in the structure is observed.

Cost Function 1:

	$p = 1$	$p=1.5$	$p = 2$	$p = 2.5$	$p=3$	$p = 3.5$	$p = 4$	$p=4.5$	$p=5$
T_{\max} / \dot{q}_0	0.0073	0.0088	0.0115	0.0139	0.0126	0.0192	0.0203	0.0186	0.0215
ϕ_{final}	0.0987	0.0988	0.0993	0.0990	0.0992	0.0991	0.0970	0.0974	0.0096
$\mathcal{E}_{\text{final}}$	0.5206	0.7294	0.7312	0.7369	0.7888	0.8294	0.8137	0.821	0.0841
$\mathbf{x}_{\text{final}}$									
									

Cost Function 2:

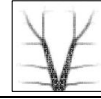

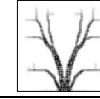
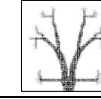
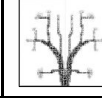
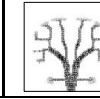
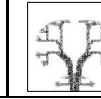
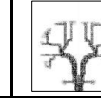
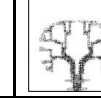
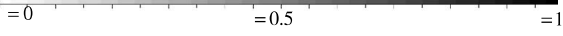
	$p = 1$	$p=1.5$	$p = 2$	$p = 2.5$	$p=3$	$p = 3.5$	$p = 4$	$p=4.5$	$p=5$
T_{\max} / \dot{q}_0	0.00941	0.01116	0.0110	0.0134	0.0149	0.0173	0.0187	0.0209	0.0220
ϕ_{final}	0.0989	0.09905	0.0993	0.0998	0.0997	0.0996	0.0994	0.0993	0.0993
$\mathcal{E}_{\text{final}}$	0.83217	0.85832	0.8827	0.9003	0.8976	0.9117	0.8994	0.899	0.9172
$\mathbf{x}_{\text{final}}$									
									

Figure 6 Obtained distributions of high conductive solids within domain for cost functions 1 and 2. [effective 80x80 mesh size].

For cost function 2 such rapid changes were not observed. At higher p values, the final tree structures appears to become stable, with relative little differences in the obtained trees as p values are changed from 4 to 5. For cost function 2 an increase in p resulted in more heavily branched trees and ultimately a lollipop-shaped structure is achieved.

Due to the relatively small C length in terms of length A , a single tree was created in all cases with the main branch (or trunk) at the exposed section of the lower boundary being at the lowest temperature of T_o . It also appears that all trees have two main branches forming a “V” shape, except for cases where $p = 1.5$. In these cases 4 branches of approximately similar thicknesses were created.

It was also found that the overall trees shape is already determined relatively early in the MMA iterations. During later iteration steps minor detail is resolved with relatively little change in the overall lay-out.

In Figure 7 the converged maximum temperatures in the domain is plotted against the employed p -value. The presence of such trees reduced the peak temperatures by between 97% and 99% or about 30 to 100 fold compared to the same domain without any cooling inserts. It can be seen that an increase in the converged peak temperatures is observed with an increase in the penalization factor. It is found that similar peak temperatures are achieved using either of the cost functions under considerations. However, when referring to Figure 8, which plots the same data points as in Figure 7, but for peak temperature against the definiteness measure, it is seen that the

use of cost function 2 resulted in better defined trees structures (having higher ϵ values).

Increases in p values also resulted in an increase in the definiteness of the obtained structures. However, at higher p values an asymptote in terms of ϵ is reached. For cost function 2 this maximum attainable ϵ is found to be approximately 0.93, while for cost function 1 it appears to 0.83. Once this maximum value of ϵ is reached, any additional increases in p causes an increased peak temperature associated with the resultant tree structure, but with no added benefit of better defined structures. Cost function 2 thus performed better.

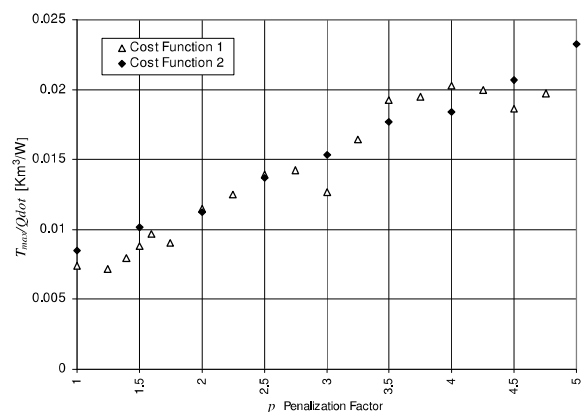


Figure 7 Converged maximum temperatures for different p values.

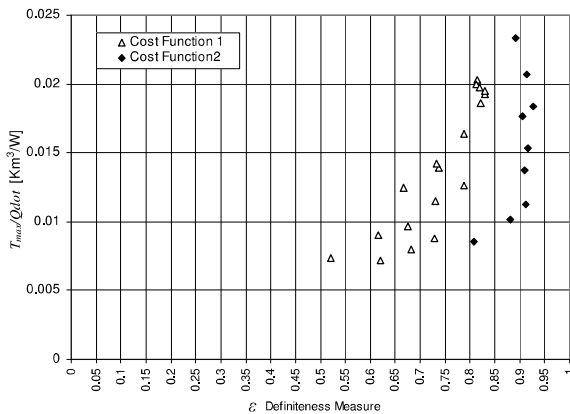


Figure 8 Converged maximum temperatures in terms of the converged tree definiteness measure.

CONCLUSION

The presence of the high conductive material is able to dramatically reduce the peak temperatures within a heat generating domain. The use of the “Method of Moving Asymptotes” was considered for determining the efficient distribution of the conductive material. Two cost functions were considered for a range of penalization factors. Various conducting tree geometries were obtained, most of which have two main branches in a “V” configuration. Increases in the penalization factor resulted in better defined tree structures, but at the cost of slightly higher temperatures. Cost function 2 resulted in better defined trees and should be used in stead of cost function 1. The use of topology optimization algorithms to find effective material distributions can be a valuable tool when design the architecture of internal solid state cooling schemes for possible application to electronics cooling. Further investigation is required to fully characterise the use of such algorithms in terms of different domain shapes and sizes as well as material properties.

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