HEFAT2010
7th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics
19-21 July 2010
Antalya, Turkey

THE EFFECT OF DIFFERENT MESH SIZE SCALES IN PREDICTION OF RADIATIVE HEAT TRANSFER

Bordbar M.H.* and Hyppänen T.

Department of Energy,
Lappeenranta University of Technology,
P.O. Box 20, FIN-53851 Lappeenranta
Finland,
E-mail: bordbar@lut.fi

ABSTRACT

A brief review of Multiscale Radiative Exchange Method (MREM) is presented and the method is applied to some 2D benchmarks. Within the computational frame of this method, radiative heat transfer in different mesh structures with wide range of optical thicknesses is predicted. The predictions of radiative heat transfer in finer mesh structures are applied in the calculation of radiation heat transfer in the coarser ones. The results obtained by this method showed a good conformity with previously reported results. The method provides a fast and accurate way for simulation of radiative heat transfer in large-scale industrial combustion equipments.

INTRODUCTION

The radiation heat transfer has an important role in prediction of the effect of dust and carbon dioxide on the global environment [1]. In addition, the radiative heat transfer is the dominant heat transfer mechanism in many industrial systems such as boilers [2]. In most of the applications, the radiative heat transfer is accompanied with other modes of heat transfer, reactive turbulent flow and energy generation source i.e. combustion. The complexity of this problem arises from the governing equation for radiative heat transfer in participating media that is in the form of an integro-differential equation. Generally, the media can absorb, emit and scatter the radiation and the surrounding surfaces can absorb, emit and reflect the incident radiation.

Except of some simple geometries, in most of the cases there is no any analytical solution for radiative transfer equation. In boiler technology, the accurate prediction of radiative source term within the furnace and radiative flux on the walls are necessary for a good design. In the recent decades, the improvements in computational power have led the researchers to develop the fast and accurate numerical methods for simulating radiative transfer. Some of the methods such as Monte Carlo method [3] was based on following the emitted photons until they would be absorbed or exit form the system. Some others such as Hottel Zone Method [4, 5], Discrete Ordinate Method [6] and Control Volume Method [7] are based on solving the radiative transfer equation in discretized geometries. The latter methods are called flux methods.

In most of the flux methods used in simulation of radiative heat transfer in participating media, a high mesh size dependency is observed. They usually show different capability in modelling the optically thick media and optically thin media. Therefore, the accuracy and performance of these methods are highly dependent of the optical thickness of the mesh structure used in their calculation.

The main idea of Multiscale Radiative Exchange Method (MREM) is presented in some of our last articles [1, 2, 8]. In this method the radiative heat transfer is predicted in different mesh structures with wide range of optical thicknesses. The predictions of the method in optically thin structures are used in the radiative prediction of the optically thicker structures. This leads to capability of modelling radiative heat transfer in the systems with wide range of optical thicknesses.

The size of different mesh structures used in computational framework of MREM has a direct effect on its accuracy and computational speed. To study this effect, results of a mesh size analysis are presented to provide an overview of the suitable size for different mesh structures used in this method. The organization of the article is as following. First, a review of theory of MREM will be presented and the result of mesh size analysis is presented in the next sections. The method is applied in some two dimensional cases and the result of MREM is compared with those of other methods and with the reported quasi-exact solutions.

NOMENCLATURE

A	[m^2]	Area of the surface cells
B	[m]	Center-to-center distance in the mesh size analysis

B/L	[-]	Aspect ratio of the two coarse cells in mesh size analysis
I		Radiative Intensity
K	[m^{-1}]	Extinction Coefficient
KL	[-]	The optical thickness of coarse cells in mesh size analysis
$K\Delta L$	[-]	The optical thickness of integration elements in mesh size analysis
k_a	[m^{-1}]	Absorptivity
k_r	[m^{-1}]	Reflectivity
k_S	[m^{-1}]	Scattering Coefficient
L	[m]	Center to center distance of two coarse cells in mesh size analysis
M	[-]	Number of the coarse surface cells in the system
N	[-]	Number of the coarse volume cells in the system
q	[J/m^3]	Radiative Heat Energy
RST	[J/m^3]	Radiative Source Term
$\vec{r}$	[-]	Position Vector
S	[m]	Center to center distance between integration elements
$\vec{S}$	[-]	Center to center vector between two cells
$\vec{s}$		
T	[K]	Temperature
V	[m^3]	Volume of the volume cells

Greek Letters

α	[J]	Attenuated radiative energy
ΔL	[m]	Characteristic length of the integration elements
Υ_{IJ}	[-]	Absorption Exchange Factor
Υ_{II}	[-]	Self-absorption of the coarse volume cells
δ_{IJ}	[-]	Scattering Exchange Factor
δ_{II}	[-]	Self-scattering of the coarse volume cells
Ψ_{IJ}	[-]	Extinction exchange factor
Ψ_{II}	[-]	Self-extinction of the coarse volume cells
θ	[Stradian]	Solid angle
σ	[W/m^2K^4]	Stefan-Boltzmann constant ($5.672\times10^{-8} W/m^2.K^4$)
τ_I	[-]	Share of radiative energy of the volume cells that goes out from the faces of the cells $\frac{(1-\Psi_{II})}{(1-\delta_{II})}$
η_I	[-]	Share of radiative energy of the volume cells absorbed in the cell itself and is defined as $\frac{\Upsilon_{II}}{(1-\delta_{II})}$
Ω	[-]	Scattering Phase Function
ω	[-]	Albedo; k_s/K
ϕ	[stradian]	Solid angle in Eq. (1)

a	[-]	Absorption
c	[-]	cold walls
e	[-]	emssion
h	[-]	hot wall
I,J	[-]	Coarse cells
i,j	[-]	Integration elements
in	[-]	Incoming radiative energy to the coarse cells
out	[-]	Outgoing radiative energy from the coarse cells
r	[-]	Reflection
S	[-]	Surface Cell
s	[-]	Scattering
V	[-]	Volume cell
λ	[-]	Wavelength properties[-]

Superscript

*	[-]	Dimensionless properties

THEORY

The radiative balance equation in participating media where the media can absorb, emit and scatter the incident radiation is presented as an integro-differential equation

$$\frac{dI(\vec{r},\vec{s})}{ds}+(k_a+k_s)I(\vec{r},\vec{s})=k_a n^2\frac{\sigma T^4}{\pi}+\frac{k_s}{4\pi}\int_0^{4\pi} I(\vec{r},\vec{s}')\Omega(\vec{s}.\vec{s}')d\phi \quad (1)$$

where k_a and k_s are the spectral absorption and scattering coefficients, respectively. s is the path length in direction $\vec{s}$, $\Omega(\vec{s}.\vec{s}')$ is the phase function from direction $\vec{s}$ to $\vec{s}'$, and ϕ is the solid angle.

Figure 1 shows how the change of radiative intensity in distance ds, i.e. the path length of a volume cell, is the sum of the emission power of the gas molecules and additional scattering minus the amount of loss due to absorption and scattering.

The scattering in the volume cell is assumed to be isotropic and uniformly distributed within the volume cell. Thus the scattering phase function is equal to unity, and therefore

$$\frac{dI(\vec{r},\vec{s})}{ds}=-(k_a+k_s)I(\vec{r},\vec{s})+k_a n^2\frac{\sigma T^4}{\pi}+\frac{k_s}{4\pi}\int_0^{4\pi} I(\vec{r},\vec{s}')d\Omega, \quad (2)$$

Thus, the radiative heat transfer in the participating media, consists of four different terms as follows:

1. $\frac{dI(\vec{r},\vec{s})}{ds}$ represents the source term of radiation in the volume cell.
2. $-(k_a+k_s)I(\vec{r},\vec{s})$ represents the attenuation of radiation intensity for a single ray.
3. $k_a n^2\frac{\sigma T^4}{\pi}$ represents the emission power of the gas molecules in the volume cell.
4. $\frac{k_s}{4\pi}\int_0^{4\pi} I(\vec{r},\vec{s}')d\Omega$ represents the scattering power of the volume cell and depends directly on the amount of radiative intensity coming from all of the 4π steradian . In other words, this term depends on the amount of intensity coming from all of the other cells in the system.

The radiative transfer between two infinitesimal radiative elements can be approximated by a single ray. For this case, the attenuation of radiation intensity between two elements is calculated as follows:

$$\frac{dI(\vec{r},\vec{s})}{ds}=-(k_a+k_s)I(\vec{r},\vec{s}) \quad (3)$$

There is no analytical solution for eq. 1 except for some simple geometries with considering some simplification assumptions. This shows the clear need of numerical solution methods in this field of research.

In MREM, by using the same idea of zone method, the participating media and its surrounding walls are decomposed to some volume and surface cells which are optically thick. In the next step, the exchange factors between each of those coarse cells are numerically calculated. Two different exchange factors are defined in this method; the absorption exchange factor and the scattering exchange factor. These two different exchange factors are defined as the ratio of two radiative energies and therefore are dimensionless. In calculation of the exchange factors, the finer mesh structures are used. The optimal size for integration elements are proposed by mesh size analysis. By using the suggested optical thicknesses for integration elements, the exchange factors are calculated in an accurate and fast manner.

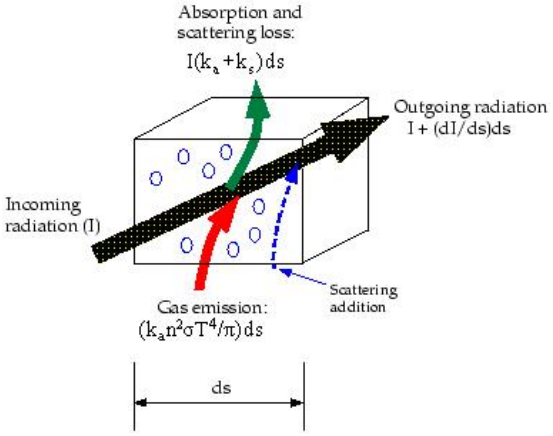

Figure 1 The radiative heat transfer in participating media.

In the next sections, a new analysis for radiative transfer balance in coarse surface and volume cells is presented. To write the radiative balance in coarse cells, the pre-calculated exchange factors are used. By solving the system of equations consists of radiative balance in coarse cells, one can obtain the radiative source terms in participating media and the radiative heat flux on the walls.

DEFINITION OF EXCHANGE FACTORS

There are two different exchange factors in this method; Absorption exchange factor and scattering exchange factor. The scattering exchange factor is defined only when a volume cell is the destination of the radiation while the absorption exchange factors is defined for all the cases.

In analogy with classical view factors which are used in radiative analysis of the non participating media, the exchange factors in MREM are defined as the ratio of two different radiative energies.

The absorption exchange factor is the ratio between the share of the outgoing energy from the source cell that is absorbed in the destination volume cell or reaches to the destination surface cell and the radiative outgoing energy from source cell. Please note that for this case the destination cell can be a volume or a surface cell.

The scattering exchange factor is the ratio between the share of the outgoing energy from the source cell that is scattered in the destination volume cell and the radiative outgoing energy from a source cell. Please note that for this case the destination cell can be only a volume cell.

When the source and destination of radiation is a same volume cell, one can obtain the following definition for self-absorption and self-scattering in coarse volume cells. The self-absorption in the coarse volume cell is the ratio between the share of the emission energy of the cell that is absorbed within the cell itself and the radiative emission of the cell. In the same way, the self-scattering in the coarse volume cell is the ratio between the share of the emission energy of the cell that is scattered within the cell itself and the radiative emission of the cell. Note that the self-absorption and self-scattering is defined only for coarse cells i.e. optically thick.

For infinitesimal cells which are used as the integration elements in calculation of exchange factors, self-absorption and self-scattering are assumed negligible. In most of the numerical methods for simulation of heat and mass transfer, there are some simplifying assumptions. Although, these assumptions make the simulation possible they might induce some error in predictions of the methods. In MREM, we assume that there is no self extinction in the infinitesimal volume elements which are used in calculation of exchange factors between the coarse cells. Although this assumption may insert some errors in the prediction of the method, it has some benefits. By ignoring the self extinction within the integration elements, the singularity problem in calculation of exchange factors between coarse cells is solved. Moreover, we carried out the mesh size analysis to study the effect of this assumption on the overall accuracy of the method. The mesh size analysis present some criteria for the size of the integration elements used in calculation of exchange factors between coarse cells in different situations, i.e. volume-Volume, Volume-Surface, Surface-Volume and Surface-Surface. These criteria may help users of the method to control the error induced by this assumption.

THE FORMULATION OF EXCHANGE FACTORS[1]

The average of radiation emission energy of a coarse volume cell can be written as $\int_{V_I} 4K\sigma T^4 dV$ [9] where K represents the extinction coefficient. Thus, we can simply write the following equation for the self-absorption of the coarse volume cells:

$$\Upsilon_{II} = \frac{\int_{V_I}\int_{V_J=V_I} \alpha_{ij} \frac{K_i k_{a,j}}{\pi S_{ij}^2} (\sigma T_i^4) dV_i dV_j}{\int_{V_I} 4K_i \sigma T_i^4 dV_i} \qquad (4)$$

α_{ij} represents the attenuation of radiant power between centers of two infinitesimal volume elements. The amount of attenuated radiant power is obtained by solving Eq. (3) numerically and is equal to:

$$\alpha_{ij} = e^{-K_{ij}S} \qquad (5)$$

[1] In this section and whole article the lower letters subscript are used to show the infinitesimal cells while the capital letters subscripts represent the coarse cells.

K_{ij} represents the average of the extinction coefficient of media along the center-to-center vector $\vec{S}$ and is defined as

$$K_{ij} = \frac{1}{S}\int_{\vec{S}} K dS \tag{6}$$

Fig. 2 illustrates a schematic of the numerical procedure which should be carried out for calculating the attenuation between two integration elements during the calculation of exchange factors.

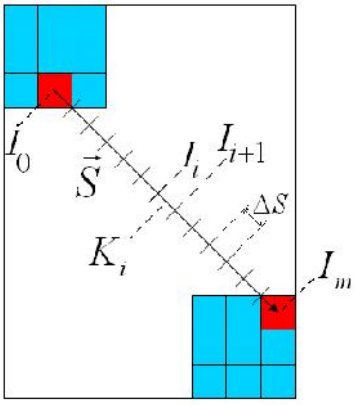

Figure 2 calculation of attenuation between two integration elements.

Similarly, the self-scattering in coarse volume cells is derived as

$$\delta_{II} = \frac{\int_{V_I}\int_{V_J=V_I} \alpha_{ij}\frac{K_i k_{s,j}}{\pi S_{ij}^2}(\sigma T_i^4)dV_i dV_j}{\int_{V_I} 4K_i\sigma T_i^4 dV_i} \tag{7}$$

The self-extinction (Ψ_{II}) of the coarse cells is defined as the sum of the self-absorption and self-scattering of the volume cells.

$$\Psi_{II} = \Upsilon_{II} + \delta_{II} \tag{8}$$

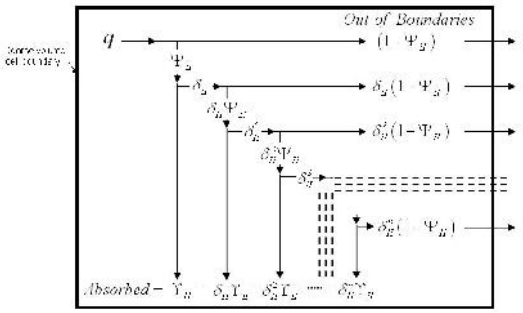

Figure 3 Schematic of a coarse volume cell and physical phenomena related to radiative energy in a coarse volume cell.

From the emission and scattering powers of the coarse volume cells, some share exits from the cell faces, and the rest are absorbed or scattered inside the cell itself. From the scattering share, again some part exits from the faces of the cell and the rest is absorbed or re-scattered within the cell itself. By using the content of self-absorption, self-scattering and self-extinction, Fig. 3 shows the physical phenomenon that occurs for the radiative energy of the coarse volume cell within the cell itself. Based on the schematic illustrated in Fig. 3, the share of radiative powers that exits the boundary of the coarse volume cell is

$$\tau_I = (1-\Psi_{II})\left[1+\delta_{II}+\delta_{II}^2+.....+\delta_{II}^n\right], \tag{9}$$

and the share that is absorbed in the cell is

$$\eta_I = \Upsilon_{II}\left[1+\delta_{II}+\delta_{II}^2+.....+\delta_{II}^n\right]. \tag{10}$$

Since $0 \le \delta_{II} \le 1$ by using the Maclaurin seri, the form of the last two equations is changed to

$$\tau_I = \frac{(1-\Psi_{II})}{(1-\delta_{II})} \tag{11}$$

$$\eta_I = \frac{\Upsilon_{II}}{(1-\delta_{II})} \tag{12}$$

By using the above described analysis in the definition of absorption exchnage factor, one can obtain the following equation for absorption exchange factor between two coarse cells (V-V, S-V, V-S, and S-S):

$$\Upsilon_{V_I-V_J}\Big|_{I\neq J} = \frac{\int_{V_I}\int_{V_J} \alpha_{ij}\frac{K_i k_{a,j}}{\pi S_{ij}^2}(\sigma T_i^4)dV_i dV_j}{\tau_{V_I}\int_{V_I} 4K_i\sigma T_i^4 dV_i} \tag{13}$$

$$\Upsilon_{S_I-V_J} = \frac{\int_{A_I}\int_{V_J} \alpha_{ij}\frac{k_{a,j}\cos\theta_i}{\pi S_{ij}^2}dA_i dV_j}{A_I} \tag{14}$$

$$\Upsilon_{V_I-S_J} = \frac{\int_{V_I}\int_{A_J} \alpha_{ij}\frac{K_i\cos\theta_j}{\pi S_{ij}^2}(\sigma T_i^4)dV_i dA_j}{\tau_{V_I}\int_{V_I} 4K_i\sigma T_i^4 dV_i} \tag{15}$$

$$\Upsilon_{S_I-S_J} = \frac{\int_{A_I}\int_{A_J} \alpha_{ij}\frac{\cos\theta_i\cos\theta_j}{\pi S_{ij}^2}dA_i dA_j}{A_I} \tag{16}$$

In the same way, for scattering exchange factor one can derive the following equations:

$$\delta_{V_I-V_J}\Big|_{I\neq J} = \frac{\int_{V_I}\int_{V_J} \alpha_{ij}\frac{K_i k_{s,j}}{\pi S_{ij}^2}(\sigma T_i^4)dV_i dV_j}{\tau_{V_I}\int_{V_I} 4K_i\sigma T_i^4 dV_i} \tag{17}$$

$$\delta_{S_I-V_J}=\frac{\int_{A_I}\int_{V_J}\alpha_{ij}\frac{k_{s,j}\cos\theta_i}{\pi S_{ij}^2}dA_i dV_j}{A_I}. \quad (18)$$

We use the numerical integration for calculating the exchange factors between the coarse cells. Bordbar and Hyppänen presented the formulation of exchange factors for two-dimensional Cartesian coordinates when the geometry has an infinite dimension in the third direction [12].

To make the formulation easier, the extinction exchange factor is defined as the sum of the absorption exchange factor and the scattering exchange factor:

$$\Psi_{IJ}=\Upsilon_{IJ}+\delta_{IJ} \quad (19)$$

To satisfy the radiative energy conservation, all of the outgoing radiative power from each cell in the system should be absorbed or scattered in the volume cells or reach the surface cells in the system. Therefore, for each coarse radiative cell "I" as the source of radiation, it can be written that

$$\begin{cases}\sum_{\substack{J=1\\J\neq I}}^{M+N}\Psi_{IJ}=1 & \text{for Volume Cells}\\ \sum_{J=1}^{M+N}\Upsilon_{IJ}=1 & \text{for Surface Cells}\end{cases} \quad (20)$$

These equations present a good criterion for checking the accuracy of the calculation of exchange factors by satisfying the overall radiative energy balance. Thus, the difference between the summation of calculated exchange factors for each cell and unity presents the total error in calculation of exchange factors. Therefore, the calculated values for exchange factors which are obtained numerically can be modified by being scaled with the scale factor of $1\Big/\sum_{\substack{J=1\\J\neq I}}^{M+N}\Psi_{IJ}$ for the volume cells and $1\Big/\sum_{J=1}^{M+N}\Upsilon_{IJ}$ for the surface cells. By using this scaling procedure, the summation of exchange factors before being used in radiative balance equations for each cell would be equal to one and it will meet the radiation energy balance of the system. Thus, it decreases the effect of inaccuracy of exchange factors calculation on the overall accuracy of the method. This method should not be considered as a solution for inaccuracies of exchange factors. Thus, correction factor obtained at this stage shall be close to one and it can be used as one criterion for checking the accuracy of exchange factors calculation and the whole method.

RADIATIVE BALANCE IN COARSE CELLS

In the MREM method, the participating media and its surrounding surfaces are decomposed into M surface and N volume cells. For each volume cell, the amount of outgoing radiative energy is the sum of emission and scattering energys. Similarly, in the surface cells, the amount of outgoing radiative energy is the sum of emission and reflecting energys. The scattering energy of the volume cells and reflection energy of the surface cells depend on the incoming radiative energy into the cell from other cells in the system. The amount of incoming radiative energy from other cells in the system is calculated by using the concept of the exchange factors. Finally, the radiative source term for each cell is calculated as the difference between incoming and outgoing radiative energys of the cell.

The current formulation of the method is developed for the finite systems and exchange factors are calculated by numerical integration between the coarse radiative elements that can be either surface or volume cells.

The exchange factors are calculated between all pairs of the cells in the system and the radiative balance for each volume and surface cell is written by using these pre-calculated exchange factors. For the volume cells, the radiative balance is written as:

$$q_{out,I}=\tau_I\left[\int_{V_I}4k_e\sigma T^4dV+\sum_{\substack{J=1\\J\neq I}}^{M+N}\delta_{JI}q_{out,J}\right] \quad (21)$$

The incoming radiative power to the volume cell can be written as:

$$q_{in,I}=\sum_{\substack{J=1\\J\neq I}}^{M+N}\Psi_{JI}q_{out,J} \quad (22)$$

Where Ψ_{JI} is defined as $\Psi_{JI}=\Upsilon_{JI}+\delta_{JI}$.

For the surface cells, the radiative balance is written as:

$$q_{out,I}=\int_{A_I}k_e\sigma T^4dA+k_{r,I}\sum_{J=1}^{N+M}\left(\Upsilon_{JI}\,q_{out,J}\right) \quad (23)$$

By using the definition of absorption exchange factors, the incoming radiative power from other coarse cells to a coarse surface cell is found as:

$$q_{in,I}=\sum_{J=1}^{N+M}\left(\Upsilon_{JI}\,q_{out,J}\right) \quad (24)$$

the radiative source term for coarse cells is calculated by :

$$RST=q_{out,I}-q_{in,I} \quad (25)$$

therefore after calculating exchange factors, by writing eq. 21 for coarse volume cells and eq.23 for coarse surface cells, a system of equations will be obtained. By solving this system of equation, one can calculates the amount of outgoing radiative energy from coarse cells. These values are in use to obtain the amount of incoming radiative energy to the coarse cells (eq.22 and eq. 24). Finally by using eq.25, the radiative source term in coarse volume cells is calculated as well as radiative heat flux in the coarse surface cells.

MESH SIZE ANALYSIS

To achieve a reasonable accuracy in the calculation of exchange factors, the optical thickness of integration elements plays a key role. The size of the integration elements should be small enough so that they see each other as the differential elements and the radiation transfer between two integration elements can be approximated by a single ray and the amount of self-absorption and scattering can be ignored. Therefore, a

mesh size analysis is done to find a suitable optical thickness for integration elements in order to achieve a high level of accuracy in the calculation of exchange factors. We used different integration elements in numerical calculation of exchange factors. Using the finer integration elements provide more accurate exchange factors. However, from a certain point, the accuracy shows independency from the size of the integration elements. Figure 4 shows one of the typical results of the mesh size analysis for exchange factors between a coarse volume and surface cells.

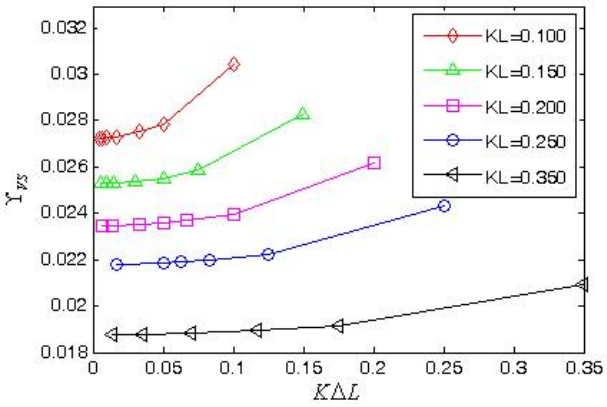

Figure 4 Absorption Exchange factor vs. the optical thickness of integration elements for the volume to surface state where aspect ratio is 1.5 and for different coarse cell sizes.

Based on the result of the mesh size analysis there are two dimensionless parameters affect on the accuracy of the exchange factors; The optical thickness of the integration elements shown by $K\Delta L$ and an aspect ratio B/L where B is the center to center distance of two coarse cell and L is the optical depth of the coarse cells.

The mesh size analysis shows that when the aspect ratio is small, the finer integration elements are needed to achieve a good level of accuracy in a reasonable computational time. When two coarse cells are far enough from each other (typically $B/L \geq 2.5$ for V-S state and $B/L \geq 2.0$ for V-V state), even using the relatively coarse integration elements leads to a good level of accuracy. Figure 5 shows this for exchange factors between a coarse volume and surface cells.

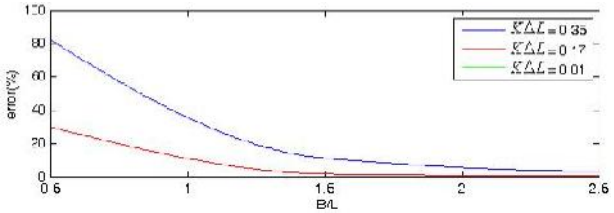

Figure 5 Dependency of error in calculation of absorption exchange factor between a coarse volume cell and a coarse surface cell to aspect ratio and optical thickness of integration elements for a certain coarse cell size ($KL = 0.35$).The error versus the aspect ratio for different integration elements sizes.

The comprehensive review of the results of the mesh size analysis is available in [11].

NUMERICAL PROCEDURE

The MREM can be used as a standalone method for calculating radiative source term in volume and radiative heat flux in the walls for the participating media with known temperature field.

When the temperature field is not known, MREM can be used as a part of overall CFD solver to solve the radiation heat transfer. In this case, the MREM uses an initial guess for temperature field and calculates the radiative source term in the domain. The radiative source term is added to the overall energy balance equation in the CFD solver. Then the CFD solver obtains the new temperature field that will be used in the next iteration of MREM. This iterative process will continue until achieving a certain limit of convergence in the radiative source term.

RESULTS AND DISSCUSSION

In this section, the MREM method is applied to several 2D benchmark problems and the results are compared with some of the previously reported quasi-exact solutions and result of other radiative models such as P1 and DO method to study the accuracy of MREM method.

To illustrate the effect of optical thickness of different mesh structures on the accuracy of the MREM method, some of the benchmark problems are solved using different integration elements and coarse mesh structures.

The MREM has been applied to standard 2D square benchmark in which a wall is kept at T=100K and the other walls are cold(T=0K). Three different participating media is considered; (1) Optically thick pure absorbing media $k_a = 10m^{-1}$, (2) Optically thin pure scattering media $k_s = 0.25m^{-1}$ and (3) normal absorbing-isotropically scattering media $k_s = 0.9m^{-1}$ and $k_a = 0.1m^{-1}$.

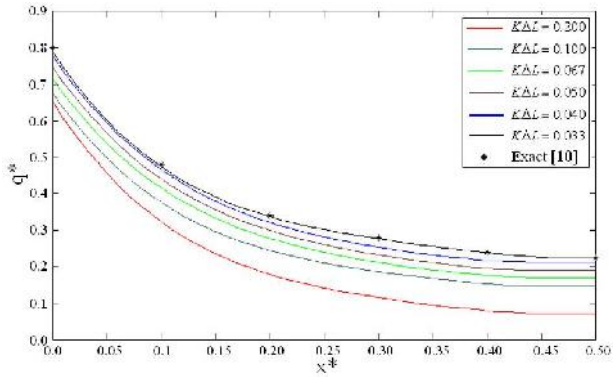

Figure 6 Effect of optical thickness of the integration elements on the accuracy of REM for a fixed size of coarse cell ($KL = 0.2$)

To show the result of the method, the dimensionless parameters are used as

- Dimensionless coordinate: $x^* = x/L_x$.
- Optical thickness of the benchmark: $KL = (k_a + k_s)L_x$
- Dimensionless heat flux on the hot bottom surface $q^* = q/\left(\sigma\left(T_h^4 - T_c^4\right)\right)$ where q is the surface net radiative heat flux on the bottom surface and is defined as $q = q_{in} - q_{out}$ and T_h and T_c are the temperatures of the hot and cold surfaces, respectively.

Figure 6 shows the result of MREM for case 1, pure absorbing optically thick media in comparison with exact solution reported by Modest [10]. To study the effect of integration elements size, several integration element structures are used. By using the finer sizes for integration elements, the predictions of the MREM approach to the exact solution. However, it increases the computational time.

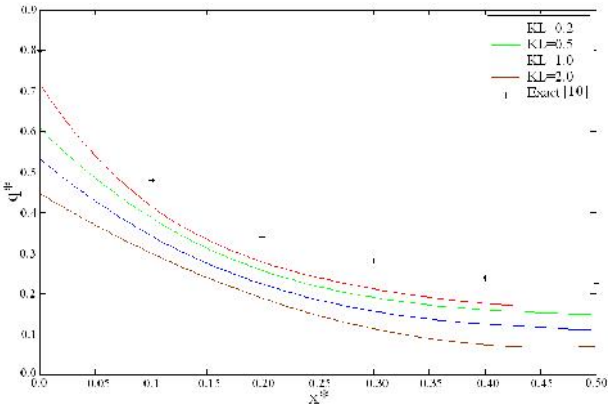

Figure 7: Dimensionless radiative heat flux on the hot bottom wall of the pure absorbing benchmark case 1($\omega = 0$), with optically very thick media ($k_a = 10m^{-1}$); Effect of optical thickness of the integration elements on the accuracy of REM for a fixed size of coarse cell ($KL = 0.2$)

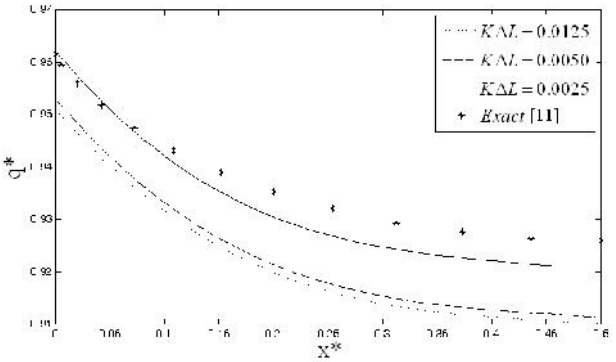

Figure 8: Dimensionless radiative heat flux on the hot bottom wall of the pure scattering benchmark case 2 $(\omega = 1, \Phi = 1)$, with optically very thin media ($k_s = 0.25m^{-1}$); Effect of the optical thickness of the integration elements on the accuracy of REM for a fixed size of coarse cell ($KL = 0.0167$).

Figure 7 shows the effect of coarse cell size on the accuracy of MREM results. Using the smaller size for coarse cell structure may increase the accuracy of the method. However, the size of the integration element has a more highlighted role.

Figure 8 shows result of the MREM method for case 2, optically thin pure scattering media in comparison with quasi-exact solution reported by Crosbie and schrenker [11]. Again, implementing finer integration element improves the accuracy of the method and increase the computational cost of the simulation.

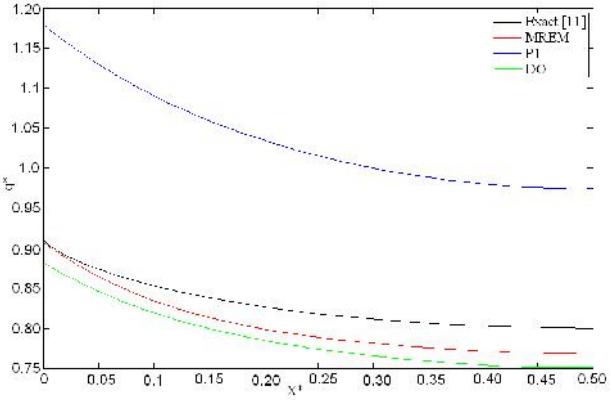

Figure 9: Dimensionless radiative heat flux on the hot bottom wall of the absorbing-isotropically scattering benchmark case 3 $(\Phi = 1,\ \omega = 0.9,\ K = 1.0m^{-1})$ in comparison with result of DO and P1 methods. The optical thickness of the coarse cell is set to 0.05. The optical thickness of integration elements in REM calculation is 0.03 and the same structure is used in P1 and DO methods. The solid angle discretization for DO method is $N_\theta \times N_\phi = 2 \times 2$.

In figure 9 the result of the MREM for case 3 is compared with quasi exact solution reported by Crosbie and Schrenker [11] as well as the results of DO and P1 methods. The integration element structure that is used in MREM is the same as mesh structure used in Do and P1 calculation. For this case, the predictions of MREM are more accurate than those of other methods.

CONCLUSIONS AND REMARKS

The accuracy and computational time of the numerical radiation heat transfer models highly depends on the optical thickness of the discretized elements used in these models. In contrary with the previous proposed methods, in the computational framework of MREM several mesh structures with wide range of the optical thicknesses are used. These different mesh structures allow MREM to model the radiation heat transfer in a fast and accurate manner. The inhomogeneous radiative behavior of the media is modeled by optically thin integration elements used in numerical calculation of exchange factors. This information is used in writing the radiative balance in coarse cells with thick optical thicknesses.

Implementing coarse cell structure for writing the radiative balance allows taking the radiative interactions between all points of the space into account.
Comparing the results of MREM with those of other methods and the available exact solutions in some benchmarks showed the performance of MREM. The accuracy of MREM is highly dependent on the optical thicknesses of the integration element structures. Using very fine integration element structure leads to very accurate result. However, it increases the computational time significantly.
To achieve some optimal values for integration elements, we have carried out a mesh size analysis. The results showed that only for the coarse cells which are located close to each other very fine integration elements are needed to provide accurate exchange factors. For other coarse cells, using the coarser integration elements or even a single calculation instead of numerical integration may also lead to a good level of accuracy in exchange factors.

AKNOWLODGEMENTS

The authors would like to acknowledge the financial support of Academy of Finland for this research which was provided under grant Number 124368.

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