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ENTROPY GENERATION DUE TO MIXED CONVECTION BETWEEN VERTICAL PARALLEL PLATES UNDER ISOFLUX/ISOTHERMAL BOUNDARY CONDITIONS

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ABSTRACT

The optimum values of the modified buoyancy parameter $(Gr/Re)_{optimum}$ at which the entropy generation assumes its minimum for fully developed mixed convection in vertical channel between parallel plates have been obtained analytically and presented in this article for thermal boundary condition of the 4th kind. This thermal boundary condition is obtained via subjecting one plate to uniform heat flux (isoflux) while keeping the opposite plate isothermal. The effects of other operating parameters on the entropy generation are also discussed.

INTRODUCTION

Recently, increasing attention has been focused on the parallel-plate ducts since it is encountered in many of energy related applications. This configuration is relevant to solar energy collection, as in the conventional flat plate collector, and in the cooling of modern electronic systems. Electronic components are mounted on circuit cards, an array of which is positioned vertically in a cabinet forming vertical flat channels through which coolants are passed, Otani and Tanaka [1] and EPP [2]. The coolant may be propelled by forced/mixed convection for large applications. Mixed convection takes place when the effect of buoyancy force on forced convection becomes significant. This effect is especially pronounced in situations where the forced-flow velocity is low and the temperature difference is large. In mixed convection flows, the forced-convection effects and the free-convection effects are of comparable magnitudes. Mixed convection flow is called buoyancy-aided flow if the buoyancy forces act in the flow direction while it is called buoyancy-opposed flow if the buoyancy forces oppose the flow direction. Design information for mixed convection should reflect the interacting effects of free and forced convection.

Laminar flow exists in the majority of compact heat exchangers because of their low hydraulic diameters. The huge amount of the research related to flow and heat transfer through parallel plate channels has been well cited by Peterson and Alfonso [3] in their analysis of thermal control of electronic equipment and the more recent work reported by [4-8]. The vertical parallel plate configuration is applicable in the design of cooling systems for electronic equipment and of finned cold plates in general. When the spacing between the plates is small relative to the height of the channel, the fully developed flow approximation can be invoked. Constant-property fully developed mixed convection between vertical parallel plates has been of interest in research for many years [9-14].

It is well known that flow and heat transfer processes are always irreversible. In other words, for all heat transfer processes, there will exist an entropy generation. This thermodynamic irreversibility, or entropy generation, is attributed to two main sources. One of these sources is the heat transfer due to finite temperature difference, which is referred to as the thermal entropy generation. The other source of irreversibility is attributed to the viscous friction due to fluid flow. This source of entropy generation is referred to as the viscous entropy generation. The entropy generation of the process, irreversibility, is directly proportional to the amount of dissipated useful energy in the process. Understanding how entropy is being generated, one can reduce the irreversibility of the heat transfer process and consequently enhances its efficiency. An optimal design can be obtained by compromising the pertinent operating parameters.

Among the huge amount of literatue regarding the flow and heat transfer in vertical channels between parallel plates, only a few literature was devoted to the analysis of entropy generation due to flow through channels between parallel plates. However, entropy generation due to different modes of flow and heat transfer in vertical channels between two vertical parallel plates has recentely received an increasing attention of researchers. In this regard, Bejan [15] investigated the mechanism of entropy production encountered in forced convective heat transfer process in four fundamental flow configurations. It was found that the design features and the geometry selection are very essential to minimize production of entropy and hence minimize the destruction of available work in convective heat transfer processes. Abbasi et al. [16] investigated the entropy production in Poiseuille-Benard channel flow. The study included a numerical solution of Navier-Stokes and energy equations utilizing the finite control volume method. The study showed that the maximum entropy generation is located at the heat transfer area and no significant entropy is generated in the main flow area. Andreozzi et al. [17] have numerically predicted the entropy generation for a natural convective flow between symmetrically heated plates at a uniform heat flux. The study focused on the effect of Rayleigh number and aspect ratio values. It was shown that different behaviors of local entropy generation occur at different Rayleigh numbers. Also, a correlation was derived in the range of $10^3 \leq Ra \leq 10^6$ and $5 \leq L/b \leq 20$ to relate the global entropy generation, Rayleigh number, and the aspect ratio (L/b). On the other hand, Erbay et al. [18] have conducted a two dimensional numerical analysis of entropy generation during transient convective heat transfer for laminar flow between two parallel plates. The plates were kept at constant equal temperatures higher than that of the fluid. The bottom plate moves in either parallel or in inverse direction to the flow.

Mahmud and Fraser [19] gave special focus to the entropy generation characteristics and its dependency on the various dimensionless parameters during their analysis of the mixed convection-radiation interaction in a vertical channel. In this study, a steady-laminar flow of an incompressible-viscous fluid was assumed through the channel with negligible inertia effect. Fluid was further considered as an optically thin gas and electrically conducting. Governing equations in Cartesian coordinate were solved analytically under fully developed conditions. Expressions for velocity, temperature, local and average entropy generation rate are derived and presented graphically. The results of this study showed that the optimum radiation parameters determined based on the concept of entropy generation minimization, increase with $(Gr \times Ri)^{0.5}$ where Ri is the Richaedson number that equals to Gr/Re^2. This means that the value $(Gr \times Ri)^{0.5}$ used in [19] is equivalent to the modified buoyancy parameter (Gr/Re) used in the present work. Boulama et al. [20] investigated the steady-state, laminar, fully-developed mixed convection of a binary non-reacting gas mixture flowing upwards in a vertical parallel-plate channel from the second law of thermodynamics point of view. Analytical expressions were derived for the entropy generation rate for two combinations of boundary conditions: uniform wall temperature with uniform wall concentration and uniform wall heat flux with uniform wall concentration. These expressions include three sources of irreversibility: heat conduction, fluid friction and species diffusion. The results showed that for humid air, the contribution of fluid friction is negligible for both cases while heat conduction and species diffusion effects appear to be of comparable orders of magnitude.

Cheng et al. [21] have numerically predicted the entropy generation of developing laminar mixed convection flow in vertical parallel plates with a series of transverses fins placed on the hotter plate. Their study showed that although fins enhance heat transfer, they cause a significant increase in entropy generation. In their study, local entropy generation profiles for different height of fins were obtained and compared with the entropy generation in smooth channels. Demirel [22] have studied the entropy generation in the planar and circular Couette flow with constant properties under asymmetric wall temperature condition. The study showed that entropy generation rate is influenced by the pressure gradient and Brinkman number. The study reported minimum entropy generation at $Y = 0.3$ and at $Y = 0.7$ at a dimensionless pressure gradient $= -3.0$ and 3.0, respectively. Investigating the existence of thermodynamic irreversibility exrtrema, Sekulic et al. [23] have studied fully developed laminar flow through different geometric channels. These channels include parallel pates channel, circular channel, triangular channel, rectangular channel, square channel, sine channel and longitudinal finned circular tube channel. They have reported that the improvement and/or deterioration of the performance of one geometry with respect to the other, is dependent on the channel geometric parameters, Reynolds number, and the inlet to wall temperature ratio. Balaji et al. [24] reported the results of their numerical investigation of turbulent mixed convection from a symmetrically heated vertical channel, bathed by a steady upward flow of cold air. In this regard, they reported that the optimal inlet velocities at which the total entropy generation rate reaches a minimum value were found to exist, for every set of heat flux and aspect ratio. Further, this optimum velocity turns out to be independent of the aspect ratio and increases linearly with the heat flux. Simple and easy to use correlations for the optimum Reynolds number and the dimensionless average wall temperatures corresponding to the optima were developed. For the range of parameters considered in this study, it is seen that for optimum conditions, the ratio of the entropy generation due to fluid friction to total entropy generation rate, known in literature as the Bejan number, varies within a narrow band (0.14–0.22).

Focusing on the applied side of the optimization of the flow and heat transfer in one of the fundamental flow geometries, flow in channels between parallel plates, Ordóñez and Bejan [25] investigated the possibilities of entropy generation minimization in parallel-plates counter flow heat exchangers. The study showed that the entropy generation is directly affected by the spacing ratios between the two channels and the heat transfer area between the two streams which gives the opportunity to minimize the irreversibility by controlling these two factors. Moreover, the study showed that the ratio of the heat capacity rates of the two streams is another strong factor that should be utilize as a design optimization factor. Yang et al. [26] recently presented thermal optimization of a stack of printed circuit boards using entropy generation minimization (EGM) method. In this study, Yang et al. [26] numerically integrated the governing thermal-fluid flow equations in the laminar-flow regime subject to the appropriate boundary conditions. After the flow and temperature fields were solved, the volumetric rate of local entropy generation was integrated to determine the total entropy generation rate in the system which consists of two components, one by heat transfer and the other by viscous friction. The Reynolds number, block geometry and bypass flow area ratio were varied to search for an optimal channel spacing. Having the same interest in optimizing stacked packaging of laminar-convection-cooled printed circuits, Takahiro et al. [27], also used the entropy generation minimization (EGM) method to optimize the fin pitch of heat sink in a free-convection environment. In connection with the electronic printed circuit packaging and electronic cooling devices, Ilis et al. [28] numerically investigated the effect of aspect ratio on entropy generation in a rectangular cavity with differentially heated vertical walls. The vertical walls of the cavity were at different constant temperatures while the horizontal walls were adiabatic. Heat transfer between vertical walls occurs by laminar natural convection. Based on the obtained dimensionless velocity and temperature values, the distributions of local entropy generation due to heat transfer and fluid friction, the local Bejan number and local entropy generation number were determined and related maps were plotted for $Pr = 0.7$. On the same area of interest, Zahmatkesh [29] analyzed the importance of thermal boundary conditions of the heated/cooled walls in heat transfer and entropy generation characteristics inside a porous enclosure, heated from below.

It is clearly noted from the thorough literature cited above that there is little quantitative information available on the analysis of entropy generation due to mixed convection through parallel plate vertical channels. It can be also concluded that there exists no attempt in the literature to estimate the values of the modified buoyancy parameter Gr/Re that would result in a minimum entropy generation due to mixed convection through vertical channels between parallel plates. This lack of information regarding the optimum values of the modified buoyancy parameter as well as the lack of information regarding the effects of other operating parameters on the entropy generation motivated the author to conduct a comprehensive study on the buoyancy effects on entropy generation due to mixed convection in vertical channels with different cross sections under different thermal boundary conditions [30] due to its importance for the scientific and applied research applications in the industry of heat transfer equipment. The present article presents the results and analysis of an analytical solution for entropy generation due to mixed convection in the fully developed region between two vertical parallel plates under isoflux/isothermal boundary conditions.

PROBLEM DESCRIPTION AND FORMULATION

The main objective of the present article is to obtain analytically the values of the modified buoyancy parameter $(Gr/Re)_{optimum}$ at which the entropy generation assumes its minimum values. These optimum values are obtained via solving the problem of fully developed laminar mixed convection in open-ended vertical channel between two parallel plates under thermal boundary condition of 4[th] kind. This type of boundary condition is obtained via having one of the plates subjected to a constant heat flux while having the other plate kept isothermal at a given temperature. In this regard, Figures 1(a & b) depict two-dimensional channel between two vertical parallel plates. The distance between the plates is ‘b’ i.e., the channel width. The Cartesian coordinate system is chosen such that the z-axis is in the vertical direction that is parallel to the flow direction and the gravitational force ‘g’ always acting downwards independent of flow direction. The y-axis is orthogonal to the channel walls, and the origin of the axes is such that the positions of the channel walls are y = 0 and y = b.

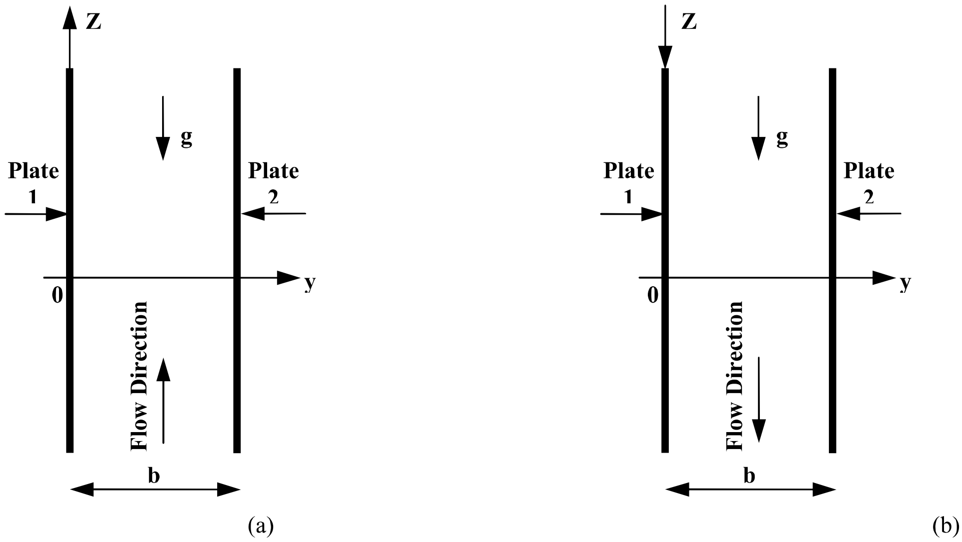


Fig. 1: Schematic view of the system and coordinate axes corresponding to (a) Upflow (b) Downflow

The flow is assumed to be laminar and the fluid is assumed to be a Newtonian fluid with constant properties but obeys the classic Boussinesq approximation according to which the fluid density is treated as constant in all terms of the governing equations except in the buoyancy term of the vertical flow direction momentum equation where it is considered as function of temperature. The flow is assumed to be thermally and hydrodynamically fully developed with no internal heat generation. It is also assumed that the viscous dissipation effect on the temperature distribution is neglected while its effect on the entropy generation is considered [15 – 29].

The general governing equations of flow and heat transfer are the conservation equations of mass, momentum, and energy along with the entropy production equation that is used to predict the irreversibility associated with the heat and fluid flow. Assuming constant physical properties, one can write the full conservation and entropy production equations in a vectorial form as follows.

Continuity Equation

$$\nabla . V = 0 \tag{1}$$

Momentum Equation

$$\rho \frac{DV}{Dt} = F - \nabla p + \mu \nabla^2 V \tag{2}$$

Energy Equation

$$\rho c_p \frac{DT}{Dt} = k \nabla^2 T \tag{3}$$

Local Entropy Equation

$$s'''_{gen} = s'''_{Thermal} + s'''_{Viscous}$$

$$s'''_{gen} = \frac{k}{T^2} (\nabla T)^2 + \frac{\mu}{T} \Phi \tag{4}$$

Along with the Boussinesq approximation

$$\rho = \rho_0 \left(1 \mp \beta |T - T_0|\right)$$

Under the thermal and hydrodynamic fully developed conditions and assumptions listed above, the differential form of the continuity equation, eq. (1) is readily satisfied and the general governing equations (1- 3) are reduced to:

z – Momentum Equation

$$-\frac{d(p - p_o)}{dz} \pm \rho_o g \beta |T - T_o| + \mu \left(\frac{d^2 u}{dy^2}\right) = 0 \tag{5}$$

where **plus** and **minus** signs indicate **buoyancy aiding flow** and **buoyancy opposing flow** respectively.

Energy Equation:

$$\frac{d^2 T}{dy^2} = 0 \tag{6}$$

To complete the mathematical model to solve for the three unknowns, u, T and p, one more equation is required. This equation is the integral form of the continuity equation.

Integral Continuity Equation:

$$\bar{u} = \frac{1}{b} \int_0^b u dy \tag{7}$$

The volumetric rate of local total entropy generation can be expressed as the sum of thermal entropy generation and viscous entropy generation as given by equation (4). The thermal entropy generation is attributed to the heat transfer across the fluid due to temperature difference while the viscous entropy generation is attributed to the viscous dissipation associated with the flow of viscous fluids. The total local entropy production equation under the thermal and hydrodynamic fully developed conditions can be written as:

$$s'''_{gen} = \frac{k}{T^2} \left(\frac{dT}{dy}\right)^2 + \frac{\mu}{T} \left(\frac{du}{dy}\right)^2 \tag{8}$$

Obtaining the local entropy generation via the direct substitution of the temperature and velocity gradients in the above equation, one can obtain the total entropy by integrating the local entropy over the volume under consideration. This integration can be expressed in the following form:

$$s_{total} = \int_V s'''_{gen}(y, z, x) dV = \int_0^w \int_0^l \int_0^b s'''_{gen}(y, z, x) dy dz dx \tag{9}$$

Considering vertical channel between parallel plates whose width (*w*) is much larger than the distance between the two plates, one can reduce the problem into a two dimensional problem. Moreover, invoking the fully developed conditions, one can reduce the above integration to:

$$s_{total} / (w . l) = s''_{total} = \int_0^b s'''_{gen}(y) dy \tag{10}$$

where s''_{total} is the entropy generation per unit channel width and height.

Thermal Boundary Conditions

One fundamental kind of isothermal boundary conditions is to be presently investigated. This boundary condition can be obtained via having one wall subjected to a constant uniform heat flux (isoflux) while the other wall is kept at constant temperature (isothermal). This boundary condition is referred to in the literature as fundamental thermal boundary condition of the fourth kind according to Reynolds et al. [31]. This boundary condition is presented mathematically as:

at $y = 0 : q'' = \mp k \left(\frac{dT}{dy}\right)_{y=0}$,

$y = b : T = T_0$ (11)

No slip conditions

At $y = 0: u = 0$, at $y = b: u = 0$ (12)

Using the dimensionless parameters given in the nomenclature, the dimensionless form of the governing equation can be written as:

<u>**Z – Momentum Equation**</u>

$$\frac{d^2U}{dY^2} = -\frac{dP}{dZ} \pm \frac{Gr}{\mathrm{Re}}\theta \quad (13)$$

<u>Energy Equation:</u>

$$\frac{d^2\theta}{dY^2} = 0 \quad (14)$$

<u>Integral Continuity Equation:</u>

$$\int_0^1 U dY = 1 \quad (15)$$

<u>Local Entropy Equation:</u>

$$S_{gen} = \frac{s'''_{gen} D_h^2}{k} = \frac{1}{(\theta+\tau)^2}\left(\frac{d\theta}{dY}\right)^2 + \frac{Ec\,\mathrm{Pr}}{(\theta+\tau)}\left(\frac{dU}{dY}\right)^2 \quad (16\text{-a})$$

or : $S_{gen} = S_{Thermal} + S_{Viscous}$

where:

$$S_{Thermal} = \frac{1}{(\theta+\tau)^2}\left(\frac{d\theta}{dY}\right)^2 \quad (16\text{-b})$$

$$S_{Viscous} = \frac{Ec\,\mathrm{Pr}}{(\theta+\tau)}\left(\frac{dU}{dY}\right)^2 \quad (16\text{-c})$$

<u>Total Entropy Equation:</u>

Substituting for the dimensional local volumetric entropy generation in eq. (10) in terms of its dimensionless value as shown in the nomenclature, one can write:

$$s_{Total}/(w\,.l) = s''_{total} = \int_0^b s'''_{gen}(y)dy$$

$$= b\int_0^1 \frac{S_{gen}k}{D_h^2}(Y)dY$$

$$S_{Total} = \frac{s_{Total}}{(w\,.l)}\frac{D_h^2}{k} = \int_0^1 S_{gen}(Y)dY \quad (17)$$

Dimensionless form of the Thermal Boundary Conditions:

at $y = 0$: $\left(\frac{d\theta}{dY}\right)_{Y=0} = -1$,

at $Y = 1$: $\theta = 0$ (18)

No slip conditions: at $Y = 0 : U = 0$, at $Y = 1 : U = 0$ (19)

ANALYTICAL SOLUTIONS

In order to obtain a closed form solution for the local and total entropy generation, the temperature and velocity profiles for the fully developed flow regions need to be obtained first. These solutions are obtained via double integrating of the momentum equation, eq. (12) and the energy equation, eq. (13) with respect to *Y* and applying the pertinent boundary conditions. The closed form solutions of the velocity and temperature profiles, and consequently the local entropy generation profiles, depend on the thermal boundary conditions employed. The required closed form solutions under the fourth kind of thermal boundary conditions are derived and presented hereinafter.

The solution of the energy equation (14) under thermal boundary condition of fourth kind is given by the following equation:

$$\theta(Y) = 1 - Y \quad (20)$$

which have the following temperature gradient:

$$\frac{d\theta}{dY} = -1 \quad (21)$$

Substituting for the temperature profile given by eq. (20) into the momentum equation (13) and solving for the velocity profile, one gets the velocity profile:

$$U_{fd,mxd} = -6Y(Y-1) + \frac{1}{12}\left(\pm\frac{Gr}{\mathrm{Re}}\right)(2Y^3 - 3Y^2 + Y) \quad (22)$$

which has the velocity gradient given as:

$$\frac{dU}{dY} = -6(2Y-1) + \frac{1}{12}\frac{Gr}{\mathrm{Re}}(6Y^2 - 6Y + 1) \quad (23)$$

Substituting for the local temperature, local temperature gradient and the local velocity gradients given by eqs. (20 –

23), in equations (16-b and 16-c), the volumetric rates of local thermal and local viscous entropy generations can be written as:

$$S_{Thermal} = \frac{1}{\left(1 - Y + \tau\right)^2} \tag{24}$$

$$S_{Viscous} = \frac{Ec\,\Pr}{\left(1 - Y + \tau\right)} \left(-6(2Y - 1) + \frac{1}{12}\frac{Gr}{\mathrm{Re}}(6Y^2 - 6Y + 1) \right)^2 \tag{25}$$

The above two equations clearly show that the thermal and viscous entropy generation are strong functions of the operating parameters. It is quite clear from equation (24) that the thermal part of the entropy generation is function of the dimensionless reference temperature (τ). However, eq. (24), shows that fully developed thermal entropy generation is not a function of modified buoyancy parameter, Gr/Re. On the other hand, eq. (25) clearly shows the viscous contribution to entropy generation is a strong function of all the operating parameters which are namely, the dimensionless reference temperature(τ), Eckert number (*Ec*), Prandtl number (*Pr*) and the modified buoyancy parameter *Gr/Re*.

Substituting (24) and (25) into (17) the dimensionless total entropy can be calculated as:

$$S_{Tot} = \int_0^1 \left[\frac{1}{\left(1 - Y + \tau\right)^2} + \frac{Ec\,\Pr}{\left(1 - Y + \tau\right)} \left(-6(2Y - 1) + \frac{1}{12}\frac{Gr}{\mathrm{Re}}(6Y^2 - 6Y + 1) \right)^2 \right] dY$$

Now we can integrate for thermal and viscous entropy one at a time:

$$\begin{aligned} \left(S_{Tot}\right)_{Tm} &= \int_0^1 \left[\frac{1}{\left(\theta + \tau\right)^2} \left(\frac{\partial \theta}{\partial Y} \right)^2 \right] dY \\ &= \int_0^1 \frac{1}{\left(1 - Y + \tau\right)^2} dY = \frac{1}{\tau^2 + \tau} \end{aligned} \tag{26}$$

$$\left(S_{Tot}\right)_{Vs} = \int_0^1 \left[\frac{1}{\left(\theta + \tau\right)} \left(\frac{\partial U_{fd,mxd}}{\partial Y} \right)^2 \right] dY$$

$$= \int_0^1 \left(\frac{Ec\,\Pr \left[-6(2Y - 1) + \frac{1}{12}\frac{Gr}{\mathrm{Re}}(6Y^2 - 6Y + 1) \right]^2}{\left(1 - Y + \tau\right)} \right) dY$$

$$\begin{aligned} \Rightarrow \left(S_{Tot}\right)_{Vs} / l = \frac{Ec\,\Pr}{144} \Big\{ &\Big[\left[36\tau^4 + 72\tau^3 + 48\tau^2 + 12\tau + 1 \right] \left[\ln(\tau + 1) - \ln(\tau) \right] \\ &- 3 \cdot \left(12\tau^3 + 18\tau^2 + 8\tau + 1\right) \Big] \left(\frac{Gr}{\mathrm{Re}} \right)^2 - 144 \Big[\left(12\tau^3 + 18\tau^2 + 8\tau + 1\right) \left[\ln(\tau + 1) - \ln(\tau) \right] \\ &- 3 \cdot \left(4\tau^2 + 4\tau + 1\right) \Big] \left(\frac{Gr}{\mathrm{Re}} \right) + 5184 \left[\left(4\tau^2 + 4\tau + 1\right) \left[\ln(\tau + 1) - \ln(\tau) \right] - 2 \cdot (2\tau + 1) \right] \Big\} \end{aligned} \tag{27}$$

It is worth mentioning that unlike thermal entropy generation, viscous entropy generation depends on *Gr/Re*, *Ec*, and *Pr* in addition to τ. The thermal entropy generation given by eq.(26) and the viscous entropy generation given by eq.(27) have been plotted for different operating parameter as it is presented in section 4 hereinafter. Plotting the total entropy generation for a given value of Ec, Pr and τ over a wide range of the modified buoyancy parameter that covers the both of the buoyancy-opposed and buoyancy-aided flows, -375 ≤ Gr/Re ≤ 375, it was found that there is a unique value of the modified buoyancy parameter (only for buoyancy-aided flow) at which the entropy generation assumes its minimum. This unique value of the modified buoyancy parameter is referred to as the optimum value of the modified buoyancy parameter $(Gr/Re)_{optimum}$. To obtain the exact values of these optimum modified buoyancy parameters for different values of the operating parameters, one can differentiate the previous lengthy expression for the viscous entropy generation given by eq. (27) (since the thermal entropy generation is not function of the modified buoyancy parameter) with respect to *Gr/Re* in order to find the value of *Gr/Re* at which the total entropy is minimal. Once differentiation is granted, the expression is then equated to zero and solved for the optimum value of *Gr/Re.*

$$\frac{d\left(S_{tot}\right)}{d\left(Gr/\text{Re}\right)} = \frac{d\left(S_{tot}\right)_{Tm}}{d\left(Gr/\text{Re}\right)} + \frac{d\left(S_{tot}\right)_{Vs}}{d\left(Gr/\text{Re}\right)} = \frac{d\left(S_{tot}\right)_{Vs}}{d\left(Gr/\text{Re}\right)} = 0$$

$$\frac{Ec\,\Pr}{144}\left\{ 2\left[\left[36\tau^4 + 72\tau^3 + 48\tau^2 + 12\tau + 1\right]\left[\ln\left(\tau+1\right) - \ln\left(\tau\right)\right]\right.\right.$$

$$\left.\left. -3\cdot\left(12\tau^3 + 18\tau^2 + 8\tau + 1\right)\right]\left(\frac{Gr}{\text{Re}}\right) - 144\left[\left(12\tau^3 + 18\tau^2 + 8\tau + 1\right)\left[\ln\left(\tau+1\right) - \ln\left(\tau\right)\right] - 3\cdot\left(4\tau^2 + 4\tau + 1\right)\right]\right\} = 0$$

The above expression is then reduced to a simple closed form for the optimum value of *Gr/Re*.

$$\left(\frac{Gr}{\text{Re}}\right)_{Optimum} = \frac{72\cdot\left[2\cdot\tau + 1\right]}{6\tau^2 + 6\tau + 1} \qquad (28)$$

RESULTS PRESENTATION AND DISCUSSIONS

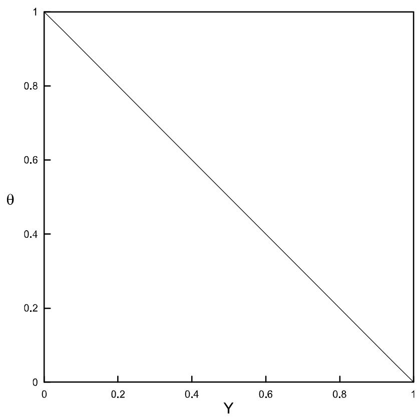


Fig. 2(a): Fully developed temperature profiles

On the other hand, the velocity profile is obtained via the solution of eq. (13) after substituting the pertinent temperature profile given by eq. (20) and applying the no slip conditions at the two plates. The obtained velocity profile for the fourth kind thermal boundary condition is given by eq. (22) and is plotted in Fig. 2(b). Equation (22) shows that the velocity profile under this thermal boundary condition is not function of the operating parameters Ec, Pr nor τ. However, eq. (22) shows clearly that the velocity profile under this kind of thermal boundary conditions is a strong function of the modified buoyancy parameter. Figure 2(b) depicts the effect of the modified buoyancy parameter. This figure along with eq. (22) shows clearly that for Gr/Re = 0, the second term of eq. (22) vanishes and the velocity profile is typically the same as the parabolic isothermal velocity profile for forced flow. Moreover, the figure shows that all of the velocity profiles have the same value at *Y = 0.5* for any value of Gr/Re. This can be clearly explained from eq. (22) in which the second term includes the effect of *Gr/Re*. This term includes the expression ($2Y^3 - 3Y^2 + Y$) that vanishes at *Y = 0.5*. On the other hand, the velocity gradient which plays an important role in the entropy generation is function of the modified buoyancy parameter Gr/Re. The variation of the velocity gradient profiles for different values of the modified buoyancy parameter is shown graphically in Fig. 2(c) and is given mathematically by eq. (23). Figure 2(c) shows that the velocity gradient profiles for all values of the modified buoyancy parameter intersect (have the same values) at two locations between the two plates. These two locations are the values of *Y* at which the second term of eq. (23) that includes the effect of the modified buoyancy parameter Gr/Re vanishes. These values of *Y* are nothing but the values that make the expression ($6Y^2 - 6Y + 1$) of eq. (23) equals to zero which are namely *Y = 0.211* and *Y = 0.78*. Flow reversal takes place when the velocity gradient in the transverse direction becomes less than or equal to zero; $dU/dY \le 0$. This condition is usually encountered at the walls where the flow suffers the highest retarding resistance due the viscous effects. Applying the above conditions for buoyancy-aided and buoyancy-opposed flows revealed that the values of the buoyancy parameter (Gr/Re) that makes flow reversal commence at one of the channel walls is given as $\left(Gr/\text{Re}\right) \ge 72$. It is worth emphasizing here that the flow reversal starts at the first wall

The solution of the fully developed energy equation, eq. (14), under isothermal boundary conditions of fourth kind is given by eq. (20) and is shown in Fig. 2(a). Equation (20) and Fig. 2(a) show that the temperature profiles for this kind of isothermal boundary conditions are neither function of the modified buoyancy parameter (Gr/Re) nor the other operating parameters Ec, Pr and the dimensionless reference temperature τ.

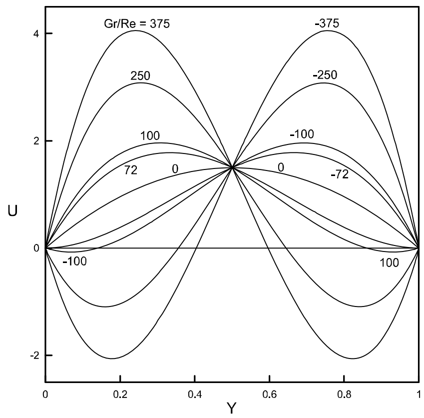


Fig. 2(b): Fully developed velocity profiles at different Gr/Re

(Y = 0) for buoyancy-opposed flow and at the second wall (Y = 1) for buoyancy-aided flow.

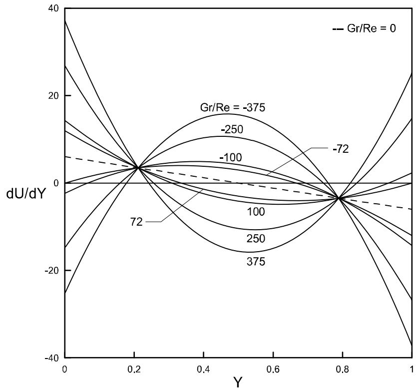


Fig.2(c): Fully developed velocity gradient profiles at different Gr/Re

As discussed earlier the entropy generation is a result of two irreversibilities associated with the heat and fluid flow. The entropy generation associated with the heat transfer due the finite temperature gradient is referred to as the thermal entropy generation. The viscous entropy generation is used to refer to the entropy generation associated with the fluid flow.

The volumetric rate of the local thermal entropy generation is given by eq. (24) that indicates clearly that the thermal entropy generation is not function of the operating parameters Gr/Re, Ec nor Pr. However, eq. (24) show that the thermal entropy generation is function of the dimensionless reference temperature, τ only. The variation of the local thermal entropy generation with τ is given graphically in Fig. 3(a). Figure 3(a) depicts the volumetric thermal entropy generation rate profiles for different values of τ. The figure shows a higher thermal entropy generation rate for lower τ values.

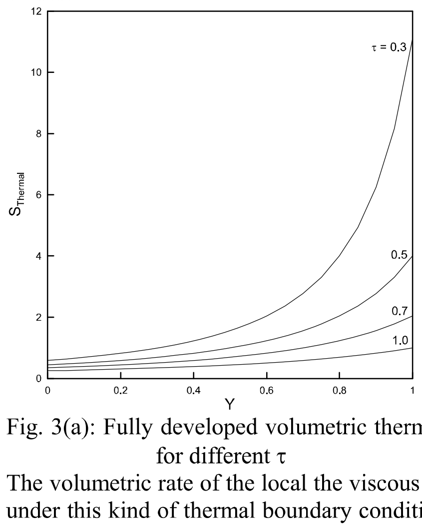


Fig. 3(a): Fully developed volumetric thermal entropy profiles for different τ

The volumetric rate of the local the viscous entropy generation under this kind of thermal boundary conditions is given by eq. (25). The volumetric viscous entropy generation rate under thermal boundary condition of fourth kind is a function of the Eckert number Ec, Prandtl number Pr, modified buoyancy parameter Gr/Re and the dimensionless reference temperature τ. Figures 3(b) and 3(c) depict the profiles of the volumetric rates of viscous and total entropy generation for different values of τ while Figs. 3(d) and 3(e) show the profiles of the volumetric rates of viscous and total entropy generation for different values of Gr/Re. The figures show that the highest viscous entropy is generally anticipated at the walls, and lower values are expected at the core of the flow. However, the flow field gets distorted for high values of modified buoyancy parameter Gr/Re which increases the volumetric viscous entropy, consequently increasing the volumetric rate of total entropy generation at the core of the flow.

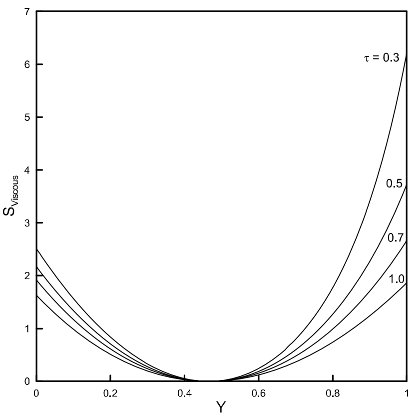


Fig. 3(b): Fully developed volumetric viscous entropy profiles for different τ at: Ec = 0.1, Pr = 0.7, Gr/Re = 10

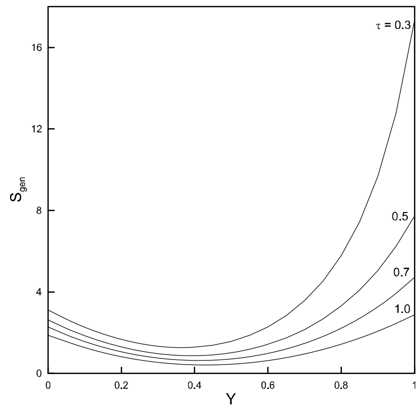


Fig. 3(c): Fully developed total volumetric entropy profiles for different τ at: Ec = 0.1, Pr = 0.7, Gr/Re = 10

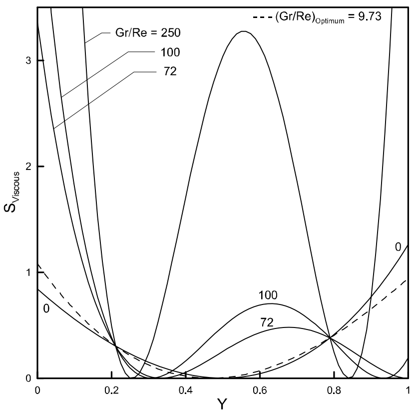


Fig. 3(d): Fully developed volumetric viscous entropy profile for different Gr/Re at: Ec = 0.1, Pr = 0.7, τ = 2

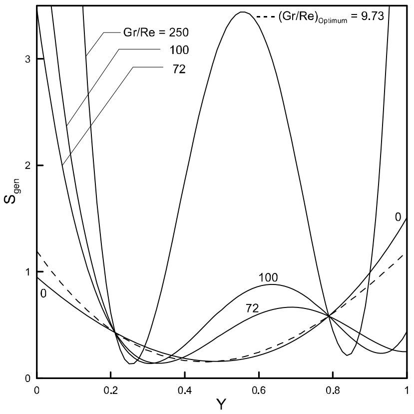
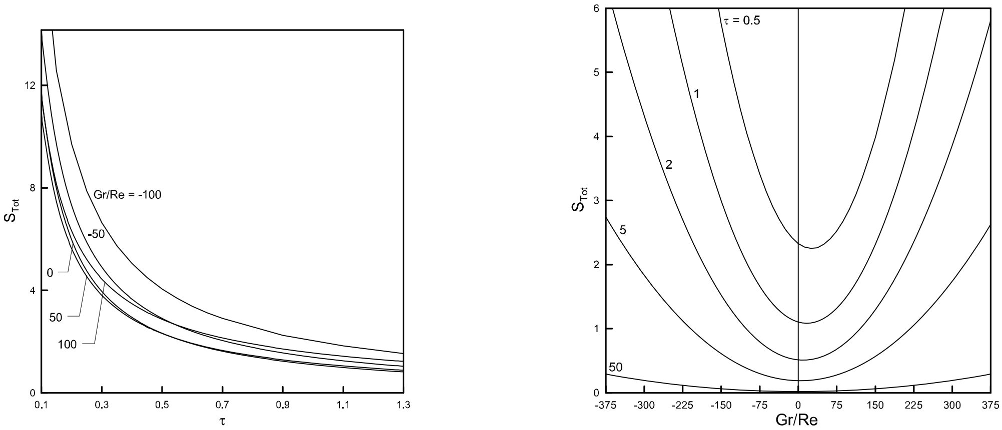


Fig. 3(e): Fully developed total volumetric entropy profile different Gr/Re at: Ec = 0.1, Pr = 0.7, τ = 2

Figure 3(b) and 3(c) along with a similar set of figures [30] (not presented here due to space limitations) reveal similar trends for low range of modified buoyancy parameter, Gr/Re < 72. However, for values of the modified buoyancy parameter Gr/Re ≥ 72 where the flow reversal commences at the cooler wall for the case of upward heated flow or at the hotter wall for the case of downward cooled flow (buoyancy-aided flow) and vice versa for the buoyancy-opposed flow where the flow reversal commences at Gr/Re ≤ -72, this trend is distorted. It can be noted from Fig. 3(d), and all other Figures 3 (b –e), that the highest local viscous entropy generation takes place at the walls where the highest velocity gradients exist due to the presence of the walls and the no slip conditions. For the case of forced flow (Gr/Re = 0), the minimum local viscous entropy generation occurs in the core of the flow and equals exactly zero at the center of the channel (Y = 0.5) where the velocity gradient is exactly zero. However, for different values of the modified buoyancy parameter other than zero, the velocity profile becomes distorted and deviates much from its parabolic shape (for the case of Gr/Re =0). The distortion of the velocity profile and its deviation from its parabolic profile depends on the value of the modified buoyancy parameter and depends on whether the buoyancy is aiding or opposing the main flow as shown in Fig. 2(b). For the situations in which the flow reversal commences at one of the walls (at Gr/Re = ± 72, where $dU/dY\big|_{wall}=0$) a zero viscous entropy generation occurs at this wall where the velocity gradient is exactly zero, as shown in Fig. 2(c). On the other hand, for the situations in which the flow reversal extended to the core of the flow (Gr/Re >> 72 , or Gr/Re << - 72) the local viscous entropy generation becomes significant at the core of the flow as shown in Fig. 3(d). It is worth noticing here that the volumetric rate of local viscous entropy generation will have the value of zero at two spots between the two plates depending on the value of the modified buoyancy parameter as shown in Fig. 3(d). It is worth mentioning here that the profiles of the volumetric rate of local total entropy generation shown in Fig. 3(e) have similar trends to those presented in Fig. 3(d) for the local viscous entropy generation with expected differences due to the addition of the local thermal entropy generation to the local viscous entropy generation. This indicates that the viscous entropy generation is dominant. The variation of overall rate of total entropy generation in the fully developed region with τ is shown in Fig. 4(a) for different values of Gr/Re. The figure clearly shows that the overall rate of total entropy generation for buoyancy-opposed flow is usually greater than that for pure forced flow (Gr/Re = 0) and buoyancy-aided flow for the same value of Gr/Re. The figure shows also that for some values of the modified buoyancy parameter, Gr/Re, for buoyancy-aided flow situations the entropy generation is less than that for pure forced flow. This indicates the existence of an optimum value of the modified buoyancy parameter $(Gr/Re)_{Optimum}$ at which the entropy generation assumes its minimum as obtained analytically by eq. (28). To show the existence of these values clearly, the variation of the overall rate of total entropy generation with the modified buoyancy parameter (Gr/Re) has calculated and presented hereunder.



(a) (b)

Fig. 4: Variation of fully developed overall rate of total entropy generation: (a) with τ for different Gr/Re at: Ec = 0.1, Pr = 0.7, (b) with Gr/Re for different τ at: Ec = 0.1, Pr = 0.7

The variation of overall rate of total entropy generation with Gr/Re for different values of τ is presented in Fig. 4(b). On the other hand, the variation of Bejan number with Gr/Re is given in Fig. 4(c) for different values of τ. The two figures show the existence of an optimum value of modified buoyancy parameter at which the total entropy generation rate is minimum. This value of buoyancy corresponds to the same value at which Bejan number assumes its maximum. The optimum values of the modified buoyancy parameter Gr/Re is given by eq. (28). Some values of optimum modified buoyancy parameter are tabulated below in Table 1 for different possible values of τ.

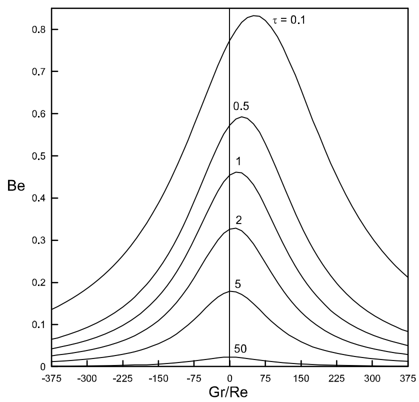


Fig. 4(c): Variation of Bejan number with Gr/Re for different τ at: Ec = 0.1, Pr = 0.7

Table 1: Optimum values of the modified buoyancy parameter for fully developed flow under thermal boundary conditions of fourth kind at different values of τ

τ	**0.1**	**0.25**	**0.5**	**0.75**	**1**	**1.25**	**1.5**	**1.75**	**2**
$(Gr/Re)_{optimum}$	52.05	37.57	26.18	20.28	16.62	14.10	12.26	10.85	9.73

It is clear from eq. (25) that the Eckert number and the Prandtl number are going to scale up or down the local viscous and consequently the local total entropy generation. In other words, the larger the value of the product (Ec Pr), the larger is the associated viscous entropy generation and vice versa. It is worth reminding here that the Prandtl number is a fluid property with Prandtl of order 10^{-3} represents liquid metals, Prandtl of order 1 represent gases, order 10 for liquids (such as water) and order 10^3 for oils. On the other hand, (Ec Pr) is a measure of the importance of the viscous dissipation relative to the change of the energy flux [32]. It is also worth reporting here that the dimensionless reference temperature τ can take any arbitrary value. However, the reference temperature is used to be taken as the ambient temperature. The effect of Eckert and Prandtl numbers on the overall rate of total entropy generation and Bejan number are shown hereunder in Figs. 5 and 6 respectively.

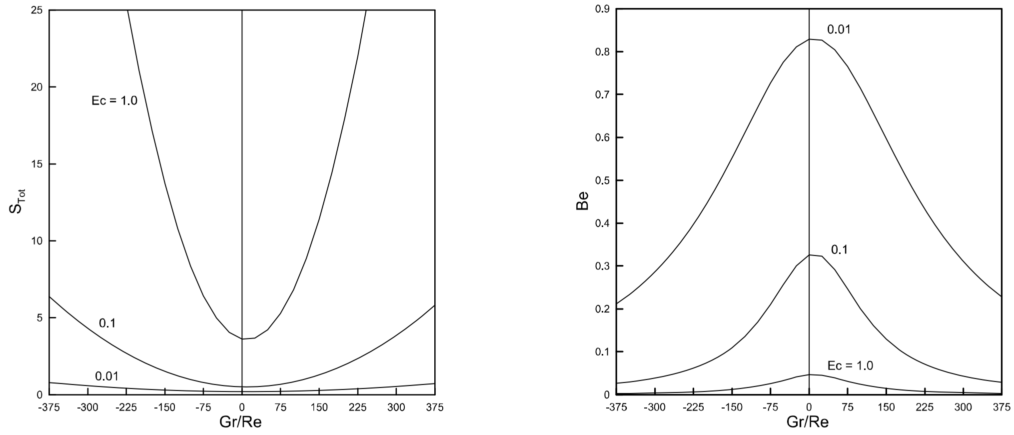


Fig. 5: Variation of (a) the overall rate of total entropy generation, (b) Bejan number with Gr/Re for different □c at: τ = 2, Pr = 0.7.

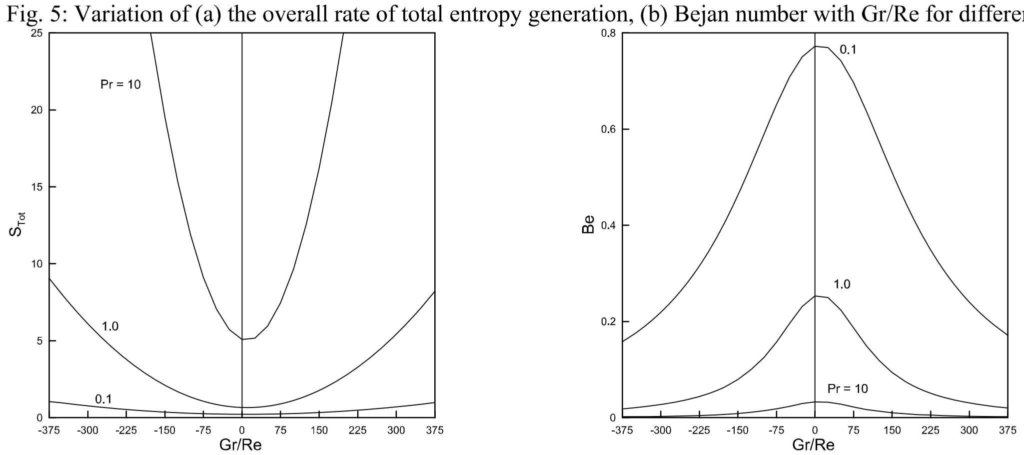


Fig. 6: Variation of (a) the overall rate of total entropy generation, (b) Bejan number with Gr/Re for different Pr at: τ = 2, Ec = 0.1.

These two figures show that both of Eckert and Prandtl numbers have no effect on the optimum value of the modified buoyancy parameter. However, both has the effect of scaling up or down the levels of entropy generations.

CONCLUSIONS

The entropy generation due to fully developed mixed convection between two vertical parallel plates under fourth kind of isothermal boundary condition has been obtained analytically. This kind of thermal boundary condition is obtained via having one plate subjected to constant heat flux while the other plate is kept isothermal. The effects of the operating parameters on the local and total entropy generation have been investigated and discussed. The analysis of the total entropy generation revealed the existence of optimum values of the modified buoyancy parameter Gr/Re at which the total entropy generation assumes its minimum value. These values have been obtained analytically for different values of the dimensionless reference temperature. The effects of Eckert and Prandtl number on the entropy generation for this particular case has been also investigated, presented and discussed.

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NOMENCLATURE

B Gap width between the parallel plates

c_p Specific heat of fluid at constant pressure

D_h Hydraulic or equivalent diameter of the vertical channel; *b* *dp/ dz* Pressure gradient

dP/dZ Dimensionless pressure gradient

Ec Eckert number, $\frac{u_o^2}{C_p(T_w - T_o)}$

Gr Grashof number, $\frac{g\beta q'' D_h^4}{\upsilon^2 k}$

Gr/Re Modified buoyancy parameter

g Gravitational body force per unit mass (acceleration)

h Convective heat transfer coefficient

l Channel height

p Local pressure at any cross section of the vertical channel

p_o Hydrostatic pressure, $\rho_0 gz$ at channel entrance

P Dimensionless pressure inside the channel at any cross section, $\frac{(p - p_o)}{\rho u_o^2}$

Pr Prandtl number, $\frac{\mu C_p}{k}$

q'' Constant heat flux at the isoflux plate

Re Reynolds number, $\frac{\rho u_o D_h}{\mu}$

$s'''_{Thermal}$ Local thermal entropy generation per unit volume

s'''_{Visous} Local Viscous entropy generation per unit volume

s'''_{gen} Total local entropy generation per unit volume

s_{Total} Overall rate of total entropy generation

S_{gen} Dimensionless local total entropy generation, $\frac{s'''_{gen} D_h^2}{k}$

S_{Total} Dimensionless overall total entropy generation,

$$\frac{s_{Total}}{(w.l)}\frac{D_h^2}{k} = \int_0^1 S_{gen}(Y)\,dY$$

$(S_{Tot})_{Tm}$ Dimensionless overall rate of thermal entropy generation

$(S_{Tot})_{Vs}$ Dimensionless overall rate of viscous entropy generation

T Dimensional temperature at any point in the channel

T_o Ambient (reference) temperature and temperature of the isothermal plate

U Axial velocity component

$\overline{u}$ Average axial velocity

U_o Uniform entrance axial velocity

U Dimensionless axial velocity at any point, $\frac{u}{u_o}$

v Transverse velocity component

V Dimensionless transverse velocity, $\frac{v\,\mathrm{Re}}{u_o}$

y Transverse coordinate of the vertical channel between parallel plates

Y Dimensionless transverse coordinate, $\frac{y}{D_h}$

z Axial coordinate (measured from the channel entrance)

Z Dimensionless axial

coordinate, $\frac{z}{D_h \mathrm{Re}}$

Greek Letters

μ Dynamic viscosity of the fluid

υ Kinematic viscosity of the fluid, μ/ρ_o

θ Dimensionless temperature, $(T-T_0)/q'' D_h/2k$

ρ Density of the fluid

ρ_o Density of the fluid at the channel entrance

τ Dimensionless entrance temperature, $\frac{T_o}{(T_w - T_o)}$

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