A NEW APPROACH FOR DETERMINING THE PERMEABILITY OF CELLULAR METALS

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ABSTRACT
Open-cell metal foam has enjoyed significant industrial and scientific interest as a viable multi-functional engineering material. Metal foam possesses a number of advantageous properties. Several applications require fluid flow through the open pores of metal foam. A key flow property of metal foam is the permeability, which has been subject to research and debate for a long time. This paper presents a new approach for evaluating this critical transport property by considering the physics of energy dissipation, i.e., the viscous shear and form drag. The current work emphasizes the necessary initial step of clearly distinguishing between different flow regimes (Darcy and Forchheimer), prior to evaluating the permeability. This distinction is sometimes absent or overlooked in several previous studies. The new approach is used to determine two types of permeability for wind-tunnel steady-state unidirectional airflow through open-cell aluminum foam samples having different pore densities.

INTRODUCTION
Lower cost of production has increased the applications of metal foams [1-5]. Flow characteristics and pressure drop in metal foam are crucial in many practical designs. The permeability of a porous medium is a key material property in fluid flow applications. Due to the structure of the foam, the flow field is complex and exact solutions of the flow field is only possible for very simple cases, otherwise they are impossible [6,7]. Researchers use experiments and to assess the pressure drop in metal foam and to obtain the flow properties including the permeability. See for example [8-10].


Other researchers used idealized geometrical shapes to develop theoretical models for the structure of metal foam. See for example Bhattacharya et al. [12], du Plessis et al. [13], Foure and du Plessis [14], Despois and Mortensen [15] and Boomsma et al. [16].

Other experimental studies include those of Boomsma and Poulisakos [17], who measured the hydraulic and thermal performance of open-cell aluminum foam, and Hwang et al. [6] who studied the friction drag for airflow in aluminum foam. In both studies, the permeability and the inertia coefficient were determined from the Forchheimer relation.

Lage et al. [7] and Antohe et al. [18] experimentally determined the permeability and the inertia coefficient for air and poly-alpha-olefin oil flow in aluminum foam. The discrepancy between the results obtained with air and oil were 18% for the permeability and 51% for the inertia coefficient. Paek et al. [11] observed that, in general, the experimental pressure drop data for metal foam in the literature were divergent.

This paper presents results of an experimental study of airflow through a number of aluminum foam samples. It also provides a clear empirical approach for correlating the pressure drop data, and for obtaining representative consistent values for the permeability and the drag coefficient of metal foam, as fixed material properties in different flow regimes (Darcy and Forchheimer). The new approach may help in re-examining and reconciling the divergent permeability data in the literature, and may increase understanding of the pressure drop characteristics in porous metals.
NOMENCLATURE

\( C \) \([\text{m}^2/\text{c}]\) \text{Form drag coefficient}
\( c \) \([-\text{c}]\) \text{Universal drag coefficient}
\( f \) \([-\text{c}]\) \text{Friction factor}
\( K \) \([\text{m}^2] \) \text{Permeability}
\( K_v \) \([\text{m}^2] \) \text{Darcy permeability}
\( K_f \) \([\text{m}^2] \) \text{Forschheimer permeability}
\( L \) \([\text{m}] \) \text{Thickness of foam sample in the flow direction}
\( p \) \([\text{Pa}]\) \text{Static pressure}
\( p_{\text{pp}} \) \([-\text{c}]\) \text{Number of pores per inch}
\( V \) \([\text{m/s}] \) \text{Darcian velocity}

Special characters
\( \delta \) \([\text{c}]\) \text{Uncertainty}
\( \varepsilon \) \([-\text{c}]\) \text{Porosity}
\( \mu \) \([\text{kg/m.s}] \) \text{Kinematic viscosity}
\( \rho \) \([\text{kg/m}^3] \) \text{Density}

FLOW REGIMES IN POROUS MEDIA

For a purely-viscous creeping flow of a fluid through a porous medium, the Darcy equation, relates the pressure drop to the Darcian velocity:

\[
\frac{\Delta p}{L} = \frac{\mu}{K} V
\]

(1)

where \( \Delta p \) is the pressure drop, \( L \) is the length of the porous medium in the flow direction, \( \mu \) is the fluid viscosity and \( K \) is the permeability of the porous medium. The Darcian velocity \( V \) is a superficial quantity calculated by dividing the volumetric flow rate by the cross-sectional area.

du Plessis and Woodberg [19] provided a critical Reynolds number for departure from the Darcy regime

\[
Re_c = \frac{50.8 \varepsilon (1 - \varepsilon)^{1/3}}{1.9[1 - (1 - \varepsilon)^{1/3}]}
\]

(2)

where \( Re \) is the Reynolds number based on a particle diameter. Outside the Darcy regime, form drag becomes significant, and the pressure drop becomes a combination of both viscous and form drag. In such case, we have:

\[
\frac{\Delta p}{L} = \frac{\mu}{K} V + \rho C V^2
\]

(3)

where \( \rho \) is the fluid density and \( C \) is a form drag coefficient. \( C \) is sometimes written as \( C_{D,5} \) [20] where \( C \) is a dimensionless coefficient. Equation (3) is known as the Forchheimer equation or the Dupuit-Darcy equation. Both \( K \) and \( C \) are strongly dependent on the structure of the porous medium. Ergun [21] empirically related the permeability and the form drag coefficient of Eq. (3) to the porosity and the particle diameter of porous media. For the random structure of porous metals, the particle diameter is not trivial [22].

EXPERIMENT

The pressure drop was measured in an open-loop wind tunnel shown schematically in Fig. 1. At the outlet of a direct drive 0.5-horsepower blower 1 draws ambient air through the tunnel. As a flow rate control, a piece of Plexiglas 2 is used to partially block the outlet area of the blower. For further adjustment of the flow rate, and the velocity in the test section, a variable speed controller 3. A reducer 4 connects the inlet of the blower to the rest of the tunnel.

![Figure 1 Schematic of Wind Tunnel](image)

The test section 6 is fabricated from 1.27-cm-thick Plexiglas and has a square cross-section of 10.16 cm inner dimension. The test section is placed securely between two square ducts 5 using a set of eight latches 9. Each duct has a length of 152.4 cm, which is greater than ten times the hydraulic diameter of the tunnel. Thus the first upstream duct removes any entrance effects before the flow reaches the test section. A foam sample 7 can be inserted into an opening in the top face of the test section and can fit snugly and be sealed. Two holes 8 are drilled at the center of the top acrylic sheet, one before and one after the test section at a distance of 1.59 cm and 6.67 cm from the test section, respectively. The diameters of these holes are 0.79 cm and they hold pressure differential tubing. The tubing is 0.79 cm outer diameter and 0.48 cm inner diameter. The pressure drop is measured using an Omega differential pressure transmitter. Depending on the air velocity the pressure transmitter may change between model PX653-10D5V, which has a range of 0 to 2,487 Pa, and model PX653-03D5V, which has a range of 0 to 746 Pa. Both transmitters have an output of 1-5 VDC. The velocity meter is an Extech Heavy Duty CPM Metal Vane flow meter (not shown) with power input 15-18 VDC, analog output 4-20 mA, and a range of 0.5 to 35 m/s.

The tunnel entrance section 10 has a square cross-section that gradually decreases from 30.48 cm to 11.43 cm, and is 38.10 cm in length. At the inlet of this section, there is a 6.35-cm long Plexiglas extension that contains roughly 2800 6-mm diameter soda straws 12 that are glued together in a honeycomb pattern. This is followed by a plastic screen having a finer mesh to further filter out any smaller turbulence.

The above set-up was used for experiment in the low velocity range of up to 5 m/s. For higher velocities the blower was replaced with a powerful suction unit (Super Flow 600 Bench, originally designed to test engine components), which produced air flow rates up to 17 m³/min.

Three samples of commercially available aluminum foam (manufactured by BRG Materials and Aerospace [23]) were separately investigated. Each sample had a cross-sectional area of 10.16 cm by 10.16 cm. Other foam parameters are given in Table 1.
2 Topics

The samples were all made from aluminum alloy 6101-T6 and were homogeneous. The samples are marked by an approximate industrial designation referred to as ppi (pores per linear inch). See Fig. 2.

Table 1 Foam Samples

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Pores per inch, ppi</th>
<th>Porosity, e%</th>
<th>Thickness, L (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>92.9</td>
<td>4.80</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>92.5</td>
<td>4.50</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>93.5</td>
<td>3.00</td>
</tr>
</tbody>
</table>

Each sample was placed in the tunnel’s test section, as shown in Fig. 1. The flow rate was varied to realize different velocities at the test section. For each velocity, the static pressure drop was measured using the pressure taps transducers.

Figure 2 Photograph of Foam Samples

The uncertainty in the measured pressure drop had a contribution from the fixed error (e_f = 10.8%) and an error in each reading (e_r = 0.2%), as reported by the pressure transducer’s manufacturer. The root-sum-squares method [24] states that total uncertainty in the pressure values $\delta_p$ is given by

$$\delta_p = \pm \sqrt{e_f^2 + e_r^2}$$

This results in $\delta_p$ of 10.8%.

The average air velocity in the tunnel was measured using an Extech metal vane meter. The reported uncertainties by the manufacturer included a fixed error of $\pm 2\%$ and an error of $\pm 0.2$ m/s of the reading. Using the root-sum-squares method and a conservative representative velocity of 2 m/s, the relative uncertainty in the air velocity is 10.2%.

RESULTS

Figure 3 is a plot of the pressure drop per unit length of the foam sample in the flow direction for foam samples versus the Darcian velocity the 10-, 20- and 40-ppi. The pressure drop for the three types of foam increases with increasing velocity in a quadratic fashion, as shown by the lines of the quadratic curve fits Eq. (3). The R value and the quadratic equation are given for each case. The general trends of the pressure drop are in qualitative agreement with previous studies, e.g., [7, 12, 17].

This is one of the most common ways of presenting pressure drop data for porous media. The curve-fit constants are usually used to back-calculate the permeability and the form drag coefficient. The curve fits were obtained by treating the whole span of the velocity (0 to 20 m/s) as one velocity ‘range’. We consider other velocity ranges— one at a time: 0 to 5, 0 to 10 and 0 to 15 m/s. For these cases the permeability and the form drag coefficient are listed in Table 2 for each of the three samples. Both of these properties show significant variations for the

![Figure 3 Pressure Drop vs. Darcian Velocity for the Three Foam Samples](image)

Table 2 Permeability and Form Drag Coefficient for Different Velocity Ranges

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Velocity Range (m/s)</th>
<th>Permeability $KK \times 10^9$ (m²)</th>
<th>% Error in K</th>
<th>Form Drag Coefficient C (m²)</th>
<th>% Error in C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-5</td>
<td>1.14</td>
<td>73.43</td>
<td>68.58</td>
<td>23.37</td>
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<tr>
<td></td>
<td>0-10</td>
<td>1.17</td>
<td>72.73</td>
<td>51.44</td>
<td>42.53</td>
</tr>
<tr>
<td></td>
<td>0-15</td>
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<td>58.97</td>
<td>49.07</td>
<td>45.17</td>
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<tr>
<td></td>
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<td>24.94</td>
<td>88.68</td>
<td>0.92</td>
</tr>
<tr>
<td>2</td>
<td>0-5</td>
<td>0.85</td>
<td>57.71</td>
<td>94.68</td>
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<tr>
<td></td>
<td>0-10</td>
<td>1.33</td>
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<td>87.25</td>
<td>26.32</td>
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<tr>
<td></td>
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<td>32.99</td>
<td>79.05</td>
<td>33.24</td>
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<tr>
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<td>115.63</td>
<td>2.35</td>
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<td>69.04</td>
<td>96.03</td>
<td>26.80</td>
</tr>
<tr>
<td></td>
<td>0-10</td>
<td>1.25</td>
<td>36.55</td>
<td>13.42</td>
<td>89.77</td>
</tr>
<tr>
<td></td>
<td>0-15</td>
<td>1.21</td>
<td>38.58</td>
<td>101.18</td>
<td>22.88</td>
</tr>
<tr>
<td></td>
<td>0-20</td>
<td>1.75</td>
<td>11.17</td>
<td>130.57</td>
<td>0.47</td>
</tr>
</tbody>
</table>

same sample in the different velocity ranges, with no identifiable trend or physical explanation. This is rather surprising since the permeability and the form drag coefficient are supposed to be dependent only on the internal structure of the porous medium.
Reported values of the form drag coefficient and the permeability for samples with similar porosities given in [12, 8, 16] disagree considerably. The discrepancy is not uncommon [11]. Similarly, the data of [17] and [18] for compressed 40-ppi foam show some significant differences for both the permeability and the form drag coefficient. Actually, disagreements existed in the same study [18] in the permeability and the form drag coefficient for the same foam samples when using different working fluids.

Examining the available literature, it becomes clear that what is considered a velocity “range” varies widely from one researcher to another. It also seems that a velocity range sometimes depends on the intended application of the foam and at other times is dictated by the limits of experimental set-ups. The following can serve as examples. Boomsma and Poulikakos [17] used a velocity range extending up to 1.042 m/s; Antohe et al. [18] up to 1.5 m/s; Bonnet et al. [25] up to 20 m/s for air and up to 0.1 m/s for water; and Lage and Antohe [20] up to 5 m/s, approximately.

In light of the above, it is obvious that a consistent approach for assessing the permeability and the form drag coefficient of porous media, along with a resolution of the meaning of a velocity range, are needed. First we observe that the permeability constant was conceived by Darcy for creeping flow of water through sand, as indicated by [20]. This suggests that the ‘Darcy’ permeability of a porous medium should strictly be measured in the low-velocity Darcy regime. In many of the previous works cited above and in recent works, e.g., Bonnet et al. [25] and Reutter et al. [26], the critical distinction between the flow regimes when it comes to evaluating the permeability and form drag coefficient from pressure drop data is overlooked.

To mark regime change for flow in porous media, some authors [7, 17, 18] use the Reynolds number based on the square root of permeability as a length scale. This is problematic because one must know the permeability of the porous medium a priori. This problem is further complicated by the fact that the transition from one regime to another is not universal, but differs from one medium to another, as they rightly indicated.

A demarcation of different regimes in the pressure drop data for a given porous medium can be obtained by dividing Eq. (3) by the velocity to obtain

$$\frac{\Delta p}{L V} = \frac{\mu}{K} + \rho CV$$  \hspace{1cm} (5)

In a purely Darcian regime, the second term (form drag) on the right hand side of Eq. (5) approaches zero, i.e., the only contribution to the pressure drop would be the viscous drag represented by the first term on the right-hand side. The right-hand side of Eq. (5) is plotted against the velocity in Fig. 4. It is clear that there is a regime change occurring at around a velocity of about 1 m/s, after which the form drag becomes important. Between about 1 and about 5 m/s, there seems to be a transitional regime. The permeability determined from the Darcy regime shall be called the Darcy permeability (K_D) to distinguish it from others. In the Forchheimer regime, the porous medium has a markedly different behavior. Both the form and viscous drag are significant. In this regime, we can fit a line to the reduced pressure drop versus velocity of Fig. 5. We must exclude both the Darcy and transitional regimes in the linear curve fit. Figure 5 shows the linear curve-fits for the three foam samples in the purely Forchheimer regime only. From the curve-fit equations, we can compute a permeability and a form drag coefficient. The permeability obtained here is, as expected, significantly different from the Darcy permeability, and shall be called the Forchheimer permeability (K_F). It is interesting to note that the Darcy permeability is two orders of magnitude greater than the Forchheimer counterpart; and markedly higher than the permeability commonly obtained by fitting a quadratic to the linear pressure drop data as a function of velocity over the entire range, i.e. Fig. 3.

Figure 4 Reduced Pressure Drop vs. Velocity: Full Data Range

Figure 5 Reduced Pressure Drop vs. Velocity: Forchheimer Regime

Errors in permeability caused by not separating regimes can reach as high as 70%, and in the form drag coefficient as high...
as 26%, as reported in Table 2. It is interesting to note that the Forchheimer permeability and form drag coefficient are the closest to the values obtained by curve-fitting the linear pressure drop versus the Darcian velocity over the entire range (0 to 20 m/s). This is true because for such velocity range the error introduced by including any Darcy or transitional regime data in the curve fit is rather small. The opposite is true when the velocity range is relatively small, i.e., 0 to 5 m/s. For this case the error is large since the Darcy and transitional regimes can be present over a significant span of the small velocity range.

<table>
<thead>
<tr>
<th>Sam. No.</th>
<th>Pores per inch</th>
<th>Darcian Permeability $K_0$ (m²)</th>
<th>Forchheimer Permeability $K_f$ (m²)</th>
<th>Form Drag Coefficient $C$ (m⁻¹)</th>
<th>Uni. Drag Coeff. $c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>1.01X10⁻⁶</td>
<td>4.29X10⁻⁶</td>
<td>89.50</td>
<td>0.058</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>6.45X10⁻⁶</td>
<td>2.01X10⁻⁶</td>
<td>118.41</td>
<td>0.053</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>4.40X10⁻⁶</td>
<td>1.97X10⁻⁶</td>
<td>131.19</td>
<td>0.058</td>
</tr>
</tbody>
</table>

CONCLUSION

Experimental data sets were presented for the pressure drop of airflow through samples of open-cell aluminum foam having different pore densities. Three flow regimes were identified: Darcy, transitional and Forchheimer regimes. For each foam sample, two distinct values of the permeability were obtained—one for the Darcy regime and one for the Forchheimer regime. The following observations can be made to serve as notes of caution when treating pressure drop data in porous media:

- Flow regimes must be identified prior to evaluating flow properties.
  - If the flow regime is Darcy, the evaluated permeability is called the Darcy permeability.
  - For the Forchheimer regime, we plot $Dp/LV$ versus $V$ for data points in this region only. We fit a straight line and determine the form drag coefficient from the slope and a different (Forchheimer) permeability from the y-intercept. The two permeabilities and the form drag coefficient obtained by the current approach should be independent of the fluid and should be constant for a given porous medium.
  - Plotting in this format is not a matter of preference or convenience, but it is rather crucial in clearly showing regime changes.
  - Fitting a quadratic curve for linear pressure drop $Dp/LV$ versus velocity $V$ data to determine $C$ and $K$ can be misleading, as no flow regime change can be shown. In this manner we can encounter both the Darcy, transitional and Forchheimer regimes and mix the three.

REFERENCES