

## A MAGNETOACOUSTIC AUTOWAVE PULSE IN ACOUSTICALLY UNSTABLE REGIONS OF SOLAR ATMOSPHERE

Molevich N. E.\*, Zavershinskiy D. I. and Zavershinskiy I. P.

\*Author for correspondence

Department of Theoretical Physics, Samara branch,  
P.N. Lebedev Physical Institute RAS, Samara State Aerospace University,  
Samara, Russia,  
E-mail: [molevich@fian.smr.ru](mailto:molevich@fian.smr.ru)

### ABSTRACT

Taking into account the most recent net radiation function model and five different kinds of heating scenarios for the upper solar atmosphere, we have defined some regions where the magnetoacoustical amplification may take place. The nonlinear equation describing the evolution of fast and slow magnetoacoustic waves in a heat-releasing completely ionized plasma has been obtained. The shape and parameters of the magnetoacoustic pulse that is an automodel solution of this equation under conditions of magnetoacoustic instability have been determined

### INTRODUCTION

Recently, propagating slow quasi-periodic magnetoacoustic waves with 5-15 minute period have been detected in the solar corona from ground and space-based instruments. Amplified flows of magnetoacoustic waves are formed in the upper regions of solar spots. These zones are filled with so-called shockwave solitons. Such structures have been registered in coronal loops, in coronal holes, in coronal plumes and in prominences. These waves look like relatively short trains. The sharp steep outside the detection region remains an open question, as well as the question about wave appearance. In this paper, we will show that in a thermally unstable medium the fast nonlinear steepness of acoustic waves or magnetoacoustic waves do not lead to strong wave damping. Moreover, we will demonstrate that the nonlinear stage of the instability results in the self-formation of a series of strongly asymmetric autowave pulses with a sharp front and an exponential trailing edge.

### NOMENCLATURE

$\rho$	[g/cm <sup>3</sup> ]	Density
$T$	[K]	Temperature
$P$	[dyn/cm <sup>2</sup> ]	Pressure
$k_B$	[erg/K]	Boltzmann constant
$V$	[cm/s]	Velocity
$B$	[G]	Magnetic induction

$T$	[K]	Temperature
$x$	[cm]	Cartesian axis direction
$y$	[cm]	Cartesian axis direction
$z$	[cm]	Cartesian axis direction
$C$	[erg/g K]	Specific heat
$L$	[erg/g s]	Cooling rate
$Q$	[erg/g s]	Heating rate
$\mathfrak{Z}$	[erg/g s]	Heat-loss function
$c$	[cm/s]	speed of sound, magnetoacoustic, Alfvén wave
$\tau$	[s]	Characteristic time of heating
$\alpha$		Angle between magnetic field vector and z-axis
Subscripts		
$0$		Stationary quantities or low-frequency sound
$\infty$		High-frequency sound
$f$		Fast magnetoacoustic wave
$s$		Slow magnetoacoustic wave
$V$		Constant volume
$P$		Constant pressure
$a$		Alfvén wave

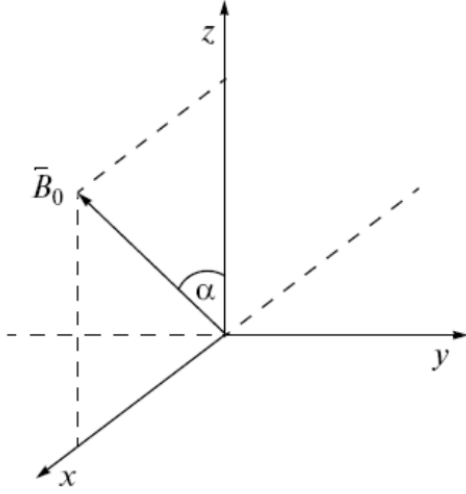
### MATHEMATICAL FORMULATION

The system of the ideal magneto hydrodynamic (MHD) equations describing processes in a heat-releasing fully-ionized medium can be written as

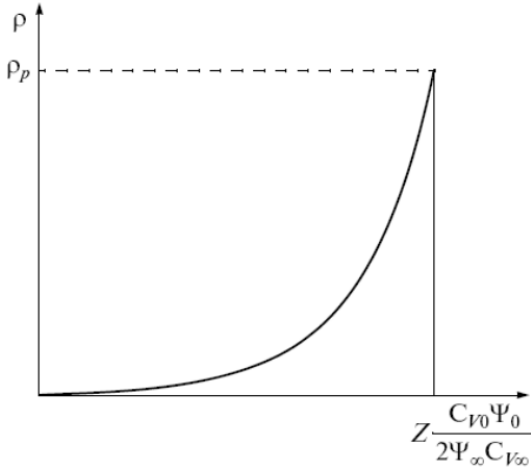
$$\begin{aligned} \frac{\partial \vec{B}}{\partial t} &= \text{rot}[\vec{V} \times \vec{B}], \quad \text{div} \vec{B} = 0, \quad \rho \frac{d\vec{V}}{dt} = -\nabla P + \frac{1}{4\pi} [\text{rot} \vec{B} \times \vec{B}], \\ \frac{\partial \rho}{\partial t} + \text{div} \rho \vec{V} &= 0, \quad C_{V\infty} \rho \frac{dT}{dt} - \frac{k_B \cdot T}{m} \cdot \frac{d\rho}{dt} = -\rho \mathfrak{Z}(\rho, T), \\ P &= \frac{k_B \cdot T \cdot \rho}{m}, \quad \mathfrak{Z}(\rho, T) = L(\rho, T) - Q(\rho, T) \end{aligned} \quad (1)$$

In (1),  $d/dt = \partial/\partial t + \vec{V} \nabla$ ;  $\mathfrak{Z}(\rho, T)$  is the generalized heat-loss function that is widely applied in the study of thermal instabilities. This function phenomenologically takes into account both cooling power in the medium per volume unit (for example, of radiative nature) and power of medium heating as a result of different exothermal processes. In stationary conditions, it equals zero  $\mathfrak{Z}(\rho_0, T_0) = 0$ .

The vector of the stationary magnetic field  $\vec{B}$  lies in the  $x, z$  plane (Figure 1). We consider waves propagating along the  $z$  axis. The dependences on  $x$  and  $y$  are neglected ( $\partial/\partial x = \partial/\partial y = 0$ ).



**Figure 1** Direction of the magnetic induction vector of a nonperturbed magnetic field.



**Figure 2** Autowave pulse

### NONLINEAR EQUATION. AUTOWAVE PULSE

Using the theory of perturbations [1], we have obtained the nonlinear equation describing the propagation of magnetoacoustic waves in a moving coordinate system ( $\varsigma = (z - c_\infty t) / c_\infty \tau, Y = \theta t / \tau, \theta \ll 1$ ) up to quantities of the second order of smallness ( $\sim \theta^2$ ) with respect to the amplitude:

$$\left( \frac{\partial \tilde{\rho}}{\partial Y} + \frac{\Psi_\infty}{2} \frac{\partial \tilde{\rho}^2}{\partial \varsigma} \right)_\varsigma - \quad (2)$$

$$\frac{C_{V0}}{C_{V\infty}} \left( \frac{\partial \tilde{\rho}}{\partial Y} + \frac{\beta \Xi_\infty}{4} \frac{\partial \tilde{\rho}}{\partial \varsigma} + \frac{\Psi_0}{2} \frac{\partial \tilde{\rho}^2}{\partial \varsigma} \right) = 0$$

We use the following notations:

$$\tilde{\rho} = \frac{\rho - \rho_0}{\rho_0}, \quad \tau = \frac{k_B \cdot T_0}{m \cdot Q_0}, \quad \Xi_\infty = \left( 1 \pm \frac{c_\infty^2 - c_a^2 \cos 2\alpha}{\sqrt{c_\infty^4 + c_a^4 - 2c_\infty^2 c_a^2 \cos 2\alpha}} \right),$$

$$|\beta| = \left| \frac{c_{0f,s}^2 - c_{\infty f,s}^2}{c_{\infty f,s}^2} \right| \sim \theta \ll 1, \quad c_{az}^2 = c_a^2 \cos^2 \alpha$$

$$c_0^2 = C_{P0} k_B T_0 / m C_{V0} = \gamma_0 k_B T_0 / m, \quad C_{V0} = \frac{k_B \mathfrak{T}_{0T}}{m},$$

$$c_\infty^2 = C_{P\infty} k_B T_0 / m C_{V\infty} = \gamma_\infty k_B T_0 / m, \quad C_{P0} = \frac{k_B \cdot (\mathfrak{T}_{0T} - \mathfrak{T}_{0\rho})}{m},$$

$$\Psi_\infty = \frac{c_\infty^2 (\gamma_\infty + 1) (c_{\infty f,s}^2 - c_{az}^2)}{2c_{\infty f,s}^2 (2c_{\infty f,s}^2 - c_\infty^2 - c_a^2)} + \frac{3(c_{\infty f,s}^2 - c_\infty^2)}{2(2c_{\infty f,s}^2 - c_\infty^2 - c_a^2)},$$

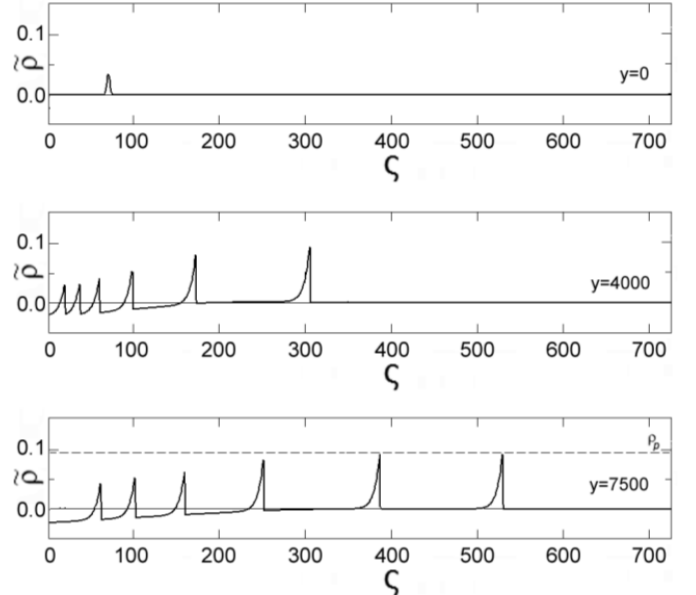
$$\Psi_0 = \frac{c_0^2 (2\gamma_0 - 1) (c_{0f,s}^2 - c_{az}^2)}{c_{0f,s}^2 \gamma_0 (2c_{0f,s}^2 - c_0^2 - c_a^2)} + \frac{3(c_{0f,s}^2 - c_0^2)}{2(2c_{0f,s}^2 - c_0^2 - c_a^2)}$$

$$\frac{c_0^2 (c_{0f,s}^2 - c_{az}^2)}{2c_{0f,s}^2 \gamma_0 \mathfrak{T}_{0T} (2c_{0f,s}^2 - c_0^2 - c_a^2)} \left[ \mathfrak{T}_{0TT} (\gamma_0 - 1)^2 + \mathfrak{T}_{0\rho\rho} \right] + 2\mathfrak{T}_{0\rho T} (\gamma_0 - 1)$$

$$c_{0f,s}^2 = 0.5 \left( c_0^2 + c_a^2 \pm \sqrt{c_0^4 + c_a^4 - 2c_0^2 c_a^2 \cos 2\alpha} \right),$$

$$c_{\infty f,s}^2 = 0.5 \left( c_\infty^2 + c_a^2 \pm \sqrt{c_\infty^4 + c_a^4 - 2c_\infty^2 c_a^2 \cos 2\alpha} \right).$$

Here,  $C_{P\infty}$  is the high-frequency specific heat at constant pressure;  $C_{V0}$  and  $C_{P0}$  are the effective low-frequency specific heats at constant volume and pressure in the heat-releasing medium, respectively;  $\tau$  is the characteristic time of heating;  $c_a$  is the velocity of Alfvén waves; and  $Q_0$  is the steady-state value of the heating power.



**Figure 3** Disintegration of the initial localized perturbation into sequence of pulses obtained by the numerical solution of nonlinear magnetoacoustical equation (2).

The form of equation (2) coincides with the form of the nonlinear acoustic equation of the relaxing and heat-realising media [2, 3].

Under condition (3) of acoustical instability [1, and the references cited therein]

$$[\mathfrak{S}_{0p}/(\gamma_{\infty}-1) + \mathfrak{S}_{0T}] < 0, \quad (3)$$

equation (2) has an automodel solution in the form of a strongly asymmetric pulse (Figure 2)

$$\tilde{\rho}(z) = \begin{cases} \rho_p \exp[(Z-Z_0)C_{V0}\Psi_0/2\Psi_{\infty}C_{V\infty}] & Z \leq Z_0 \\ 0 & Z > Z_0 \end{cases} \quad (4)$$

According to (4), this pulse has a discontinuous leading edge and an exponential trailing edge. It propagates with the following velocity

$$w = w_p = \beta \Xi_{\infty} \Psi_{\infty} / 2(2\Psi_{\infty} - \Psi_0).$$

The pulse is an autowave (self-sustained wave). The velocity and amplitude of this pulse are determined by the parameters of the medium and by the magnitude and angle of the magnetic field. In Figure 3, the numerical simulation of disintegration of the initial small perturbation into sequence of such autopulses is shown.

## ACOUSTICALLY ACTIVE REGIONS OF THE SOLAR ATMOSPHERE

In this section, we will show that the acoustical instability is quite possible for the solar atmosphere.

Usually, for the solar atmosphere the generalized heat-loss function  $\mathfrak{S}(\rho, T)$  represents the difference between an arbitrary heat input  $Q(\rho, T)$  and a radiative loss function  $L(\rho, T)$ .

The exact mechanism of outer solar layers heating remains an open question. It is usually assumed that the heating function for these regions can be written as [4 and references therein]:

$$Q(\rho, T) = \text{constant } \rho^a T^b,$$

where  $a$  and  $b$  are given constants.

For five different heating scenarios these constants are as follows:

- case A, constant heating per unit volume ( $a = b = 0$ );
- case B, constant heating per unit mass ( $a = 1, b = 0$ );
- case C, heating by coronal current disipation ( $a = 1, b = 1$ );
- case D, heating by Alfvén mode/mode conversion ( $a = b = 7/6$ );
- case E, heating by Alfvén mode/anomalous conduction damping ( $a = 1/2, b = -1/2$ ).

The radiative loss function in limit of optically thin plasma can be written in the simple form:

$$L(\rho, T) = \rho \cdot f(T), \quad f(T) = \chi^* T^{\varepsilon},$$

where  $\chi^*$  and  $\varepsilon$  are the piecewise functions depending on the temperature.

Using the parametrization model based on the analytical representation of the cooling rate, see Ibanez [5], we have obtained the temperature regions where the solar atmosphere is acoustically unstable. Taking into account the current level of knowledge, we have used another parametrization of the radiative loss function computed from the CHIANTI v7 atomic

database [6] assuming coronal abundances, ionization equilibrium, and a constant pressure of 6.64mPa [7].

As a result of our investigation, the temperature ranges of acoustical instability (3) have been obtained on the base of five models of heating A-E.

For heating scenarios A and E, the acoustic waves always are damped in the range of temperature  $10^4 \leq T \leq 10^8 K$  under consideration. For case B, the magnetoacoustic waves can be amplified in the range of temperature  $2 \cdot 10^6 \leq T \leq 3.98 \cdot 10^6 K$ . For case C, amplification occurs for temperatures between  $1.58 \cdot 10^4 \leq T \leq 2.51 \cdot 10^4 K$ ,  $2.51 \cdot 10^5 \leq T \leq 3.98 \cdot 10^5 K$ ,  $10^6 \leq T \leq 3.98 \cdot 10^6 K$ , and  $10^7 \leq T \leq 3.16 \cdot 10^7 K$ . For case D, one can find the wave amplification in the ranges of temperature  $1.58 \cdot 10^4 \leq T \leq 2.51 \cdot 10^4 K$ ,  $10^5 \leq T \leq 1.25 \cdot 10^5 K$ ,  $2.51 \cdot 10^5 \leq T \leq 3.98 \cdot 10^5 K$ ,  $10^6 \leq T \leq 3.98 \cdot 10^6 K$ , and  $10^7 \leq T \leq 3.16 \cdot 10^7 K$ .

## CONCLUSION

It has been shown that magnetoacoustical instability of solar atmosphere can result in the formation of trains of strongly asymmetric autowave pulses with a discontinuous leading edge and an exponential trailing edge.

The study was supported by the Ministry of education and science of Russia under Competitiveness Enhancement Program of SSAU for 2013-2020 years and by the state assignment to educational and research institutions under projects N 102, 608, GR 114091840046, by RFBR under grants 13-01-97001, 14-02-97030 r\_povolzh'e\_a, by the Grant of RF President for young researchers and post graduate students.

## REFERENCES

- [1] Zavershinsky, D. I., Molevich, N.E., A magnetoacoustic autowave pulse in a heat-releasing ionized gaseous medium. *Technical Physics Letters*, Vol 36, 2013 pp. 676-679
- [2] Makaryan V.G., Molevich N.E., Stationary shock waves in nonequilibrium media, *J. Plasma Sources Sci. Techn.* Vol. 16, 2007, pp. 124-131
- [3] Molevich N.E., Zavershinsky D.I., Galimov R.N., and Makaryan V.G., Traveling self-sustained structures in interstellar clouds with the isentropic instability, *Astrophys. Space Sci.* Vol. 334, 2011, pp. 35-44
- [4] Carbonell M., Oliver R. and Ballester J. L., Time damping of linear non-adiabatic magnetohydrodynamic waves in an unbounded plasma with solar coronal properties, *Astron. Astrophys.* Vol. 415, 2004, pp. 739-750
- [5] Ibanez S. M. H., Sanchez D. N. M., Propagation of sound and thermal waves in a plasma with solar abundances, *Astrophys. J.* Vol. 396, 1992, pp. 717-724
- [6] Landi, E., Del Zanna, G., Young, P. R., Dere, K. P., Mason, H. E. CHIANTI An Atomic Database for Emission Lines. XII. Version 7 of the Database., *Astron. Astrophys.* Vol. 744, 2012 pp. 9

[7] Soler, R., Ballester, J. L., Parenti, S CHIANTI Stability of thermal modes in cool prominence plasmas, *Astron. Astrophys.* Vol. 540, 2012 pp. 6