VARIABLE PARAMETER ESTIMATION OF CONSUMER PRICE EXPECTATIONS FOR THE SOUTH AFRICAN ECONOMY

RENEE KOEKEMOER

INDIVIDUALS FREQUENTLY FORM EXPECTATIONS about the future level of prices *e.g.* when making consumption expenditure decisions and during wage bargaining. Expectations are formed conditional on economic agents' perceptions of the current economic environment or regime as well as possible time-related changes in that environment.

Expectations, *i.e.* anticipations or views of the future, have featured prominently in macroeconomic literature from the inception of the concept. Since 1930, when Irving Fisher introduced the 'anticipated rate of inflation' as the difference between the nominal and real interest rates, expectations have played an important role in economic theory. Formal analytical treatment of expectations formation, however, only emerged over the last quarter of this century. As a result, important advances in this area occurred.

The development over the past thirty years of macroeconomic models has forced economists to recognise that expectations are not to be treated as exogenous — or to be ignored at will — but instead are central to our understanding of the functioning of the economy (Holden *et al.* 1985:1).

Historically, there have been two distinct methods of dealing with expectations in economic analysis — one is the direct measuring of expectations by conducting surveys to determine expectations, the other is to provide a simple model of expectations formation.

Direct measures are, for obvious reasons, not a very plausible method of obtaining expected values for future outcomes of economic variables. Although undoubtedly useful in preparing economic forecasts, gathering direct measures of expectations are costly and time consuming; in addition the data becomes outdated rather quickly. Furthermore, and perhaps more importantly, direct measures of agents' expectations provide little insight into how expectations would change as policy changes. The breakthrough that led to a more general approach to expectations modelling came with the realisation that expectations could be treated as an unobservable component. In this study, the latter method is adopted: a simple model of expectations formation is specified and the coefficient vector of the expectations rule is treated as an unobservable component.

1. MODELS OF EXPECTATIONS FORMATION

This section briefly discusses the theoretical development of expectations analysis and also refers to its implementation in empirical models.

(a) Adaptive Expectations Mechanisms

During the twenty years after the Second World War, the adaptive expectations hypothesis (AEH), first proposed by Cagan (1956), enjoyed considerable popularity amongst econometricians as a simple and apparently sensible model of how economic agents form expectations.

In modelling expectations of, say, future price levels, the hypothesis of adaptive expectations states that

$$ {}_{t-1}p_t^e = \Phi p_{t-1} + \Phi(1-\Phi)p_{t-2} + \Phi(1-\Phi)^2 p_{t-3} + \dots \qquad (1) $$

where ${}_{t-1}p_t^e$ represents the expected value of the price level in period t formed in period $t-1$, $0<\Phi<1$.

The individual therefore harbours a series of expectations for future outcomes of the price level and, in each period, the expectation for the future is revised in a proportional way with the most recently observed error.

Unobservable expectations with respect to the price level may thus be measured purely in terms of past observations of the actual price level.

The above seemed intuitively appealing, as it states that expectations of the future are a simple extrapolation of the past. The adaptive expectations rule would, however, cause consistent and growing mistakes in periods when prices grow at a constant rate. The adaptive expectations model would in this instance consistently under-estimate the level of p, and it would do so at an increasing absolute amount over time, implying that an economic agent would make entirely predictable errors, even in the very long run. It is hard to believe that such a feature could be true of an intelligent economic agent.

Fixed parameter rules in general are liable to perform poorly in one circumstance or the other. An example of this is a change in the regime generating the underlying variable. The Lucas critique (Lucas 1976) pointed out that when expectations are modelled by functions of lagged variables, the parameters of these functions may vary as the regime for determining the expectations variable changes. So if we assume that agents are intelligent and that they avoid consistent expectations errors, then any fixed parameter extrapolative expectations model will be unable to accommodate policy or other regime changes (Curne and Hall 1994:93).

(b) The Concept of Rational Expectations

The predominant paradigm for modelling expectations is the rational expectations hypothesis. The concept of rational expectations (RE) was originally formulated by Muth (1961). He suggested that agents form their expectations in the same way that they undertake other activities — that is, they use economic theory to predict the value of the variable and this is their 'rational' expectation. Rational expectations are thus simply predictions based on economic theory, using the information available at the time the predictions are made (Holden *et al.* 1985:18).

Walters (1971) preferred the term 'consistent' to 'rational' since the expectations are consistent with the relevant economic theory - it assumes that agents use information efficiently in such a way that the aggregate of all agents' expectations will not be systematically wrong. The seventies witnessed an explosion in theoretical work incorporating rational expectations, which then became closely linked with the neoclassical approach to macroeconomics, to the extent that the two approaches were widely regarded as synonymous.

In the full or strong form of the rational expectations hypothesis, it is assumed that economic agents have a complete knowledge of the economic system about which they need to form expectations. This knowledge includes the functional form, the parameters of the system, as well as any exogenous process governing the system.

Formally, the rational expectations hypothesis (REH) may be stated as follows:

$$ {}_{t}p^{e}_{t+s} = E_{t}(p_{t+s}) \qquad (2)$$

where

p = the variable being forecasted (i.e. the price level)

${}_{t}p^{e}_{t+s}$ = the prediction of the price level for time t+s, formed at time t and E_{t} = the statistical expectation conditional on information available at time t, when the forecast is made.

The rational expectations hypothesis requires that the prediction made by the forecaster be consistent with the prediction generated by the model, conditional on information available at that time. Setting s = l, equation (2) implies that the price fluctuates about its forecast level with a purely random error ε_{t+1} that has a zero mean.

Several arguments for and against the rational expectations hypothesis are found in the literature. Some authors still maintain that one of the most compelling arguments in favour of the rational expectations hypothesis, is the relative weakness of the alternatives.

The most important objections to the rational expectations hypothesis concern the fact that the application thereof requires not only knowledge of the underlying structure of the model, but also of the relevantparameters. Specialists are unable to agree on a model, but even if they should find agreement, they usually obtain varying estimates for relevant coefficients. Much less will consumers, which are presumably less sophisticated in economic theory, but whose expectations we are attempting to model, have such information. This informational argument clearly has merit.

The rational expectations hypothesis has failed to become general practice in a forecasting context, due to a number of practical problems associated with this approach (see Hall 1995; Currie and Hall 1994; and Hall and Garratt 1998).

One conclusion to be drawn from the above, may be that agents do not have full information and full model-consistent expectations but that they learn about regime changes over time and assimilate new information. It would, therefore, be natural to progress from *strong* rational expectations to a *weaker* model, which allows for the possibility of making errors in the short run, while ruling out long-run systematic or predictable errors. This gave rise to learning models. Learning models represent a shift from the assumption of full rationality on the part of economic agents in forming expectations, towards the idea of bounded rationality which is more consistent with the psychology literature on learning processes.

(c) Learning

Learning models of expectations have increasingly received attention in the theoretical literature over the past decade or more. In response to the limitations of the rational expectations hypothesis imposed by the stringent assumption of full knowledge of the true structural model, a number of economists have introduced processes describing the way in which economic agents learn about the underlying economic structure over time.

Cuthbertson (1988:224) reports on an early attempt by Friedman (1979) to provide an alternative optimising framework to rational expectations. Friedman advocates that given the true model $y_t = x_t\beta + u_t$ (where u_t is a white noise error term), agents may sequentially update their estimate of the fixed true parameter vector β as more information on (y_t, x_t) becomes available - a process of recursive least squares. Cuthbertson extends Friedman's framework to include the case where (i) agents have some prior information about β (at time t=0) and (ii) β is allowed to vary stochastically. Friedman (1979:33-34) alludes to the latter outcome when he discusses the possibility that agents may perceive a simple linear model with time varying parameters as a good approximation to the complex 'true' model. Such a model would then represent a boundedly rational learning model of expectations.

The concept of boundedly rational learning is the result of a slightly weaker informational assumption. It implies that agents use some 'reasonable' rule to form expectations and that the form of this rule remains constant over time while agents 'learn' the parameters of this rule. Agents are therefore not assumed to instantaneously know the 'true' model but they do use information optimally (or efficiently).

Implementing boundedly rational learning in a macro model entails the specification of a relatively simple expectations rule and then allowing the parameters of this rule to vary, thus correcting previous errors made by the rule. Over time, the expectations rule would be expected to adjust to the particular regime under which the model is operating and a rational expectations solution will be established. But in the short term, the learning rule allows for errors and hopefully generates a more plausible path to equilibrium.

A serious issue with regard to the learning approach, and to a certain extent unresolved, is the selection of an appropriate expectations rule. In many instances, the reduced form of the structural equation containing the expectations variable is the best vantage point for formulating the expectations rule *(e.g.* DeCanio 1979; Radner 1982; Bray and Slavin 1986; Hall and Garratt 1992a, 1992b).

The specification of an expectations rule is based on the assumption that some element of the rule - usually the parameters - is not known with certainty. The basic idea is that over time, the economic agent will methodically increases his knowledge about the true values of these parameters. A model containing expectations may be formulated to explicitly include the parameters of the rule:

$$ {}_t y^e_{t+1} = g_k(Y_t, X, \xi_t) \qquad k = 1...m \tag{3} $$

where ${}_t y^e_{t+1}$ represents the expectation of Y for period $t+1$, based upon information available at period t, Y_t is the vector of current and lagged values of the endogenous variables and X the vector of exogenous variables over all periods. ξ_t is the vector of agents' estimates of the parameters at period t. Having specified the learning rule or the expectations rule, a mechanism that governs the evolution of these parameters through time needs to be specified.

The way in which the parameters change can be determined in a number of ways. At the simplest level, agents could simply perform a set of 'rolling' regressions using ordinary least squares, each period adding the latest expectation error to the data set. According to Hall and Garratt (1992a: 52), this simple form of learning does not respond well to structural changes. The more sophisticated mechanism based on the Kalman filter, is therefore preferable, although the steps are conceptually similar.

2. STATE-SPACE MODELS AND THE KALMAN FILTER

State-space models were originally developed by control engineers (Wiener 1949; Kalman 1960, 1963). Kalman filtering, named after the contributions of R.E. Kalman, found applications in, for example, the technology of radars, aircraft stabilisation, the determination of coordinates in nautical or aerospace applications and chemical processes. It was not until the 1980s that state-space models received attention in economics literature (amongst others, Laws on 1980; Harvey *et al.* 1986; Cuthbertson 1988; Barrel *et al.* 1994; Currie and Hall 1994).

Extensive surveys of applications of state-space models in econometrics can be found in Hamilton (1994:372-408) and Cuthbertson *et al.* (1992:191-225). Cuthbertson *et al. {op at:* 191) distinguish between two types of models especially amenable to representation via the Kalman filter, namely *unobservable components* models and *time-varying parameter* models. In this study, the state-space model with stochastically time-varying parameters has been applied to a linear regression, *i.e.* the price expectations rule, in which the coefficient vector changes over time.

The next section describes how a dynamic system can be written in state-space form, which form is suitable for the application of the Kalman filter.

(a) The State-space Representation of a Dynamic System

The state-space representation of the dynamics of an $(n\times 1)$ vector, y_t, is given by the following system of equations:

$$y_t = A'x_t + H'\xi_t + w_t \tag{4}$$

$$\xi_{t+1} = F\xi_t + v_{t+1} \tag{5}$$

where A', H' and F are matrices of parameters of dimension (nxk), (nxr) and (rxr), respectively, and x_t is a (kxl) vector of exogenous or predetermined variables, ξ_t is a (rxl) vector of possibly unobserved state variables, known as the *state vector.* The first equation is known as the *observation* (or measurement) equation and the second is known as the *state* (or transition) equation. The (nxl) and (rxl) disturbance vectors w_t and v_t are assumed to be independent white noise with

$$E(v_t v'_\tau) = \begin{cases} Q & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$$E(w_t w'_\tau) = \begin{cases} R & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where Q and R are (rxr) and (nxn) matrices, respectively. The disturbances v_t and w_t are assumed to be uncorrelated at all lags:

$$E(v_t w'_\tau) = 0 \qquad \text{for all t and } \tau. \tag{8}$$

The statement that x_t is predetermined or exogenous means that x_t provides no information about ξ_{t+s} or w_{t+s} for s=0,1,2,... beyond what is contained in $y_{t-1}, y_{t-2}, \ldots, y_1$. Thus, x_t could include lagged values of y or variables which are uncorrelated with ξ_τ and w_τ for all τ.

The system of equations (4) through (8) is typically used to describe a finite series of observations $\{y_1, y_2, \ldots, y_T\}$ for which assumptions about the initial value of the state vector ξ_t are needed.

The various parameter matrices (F, Q, A, H or R) could be functions of time, in which case equations (4) and (5), *i.e.* the state-space representation may be altered to:

$$y_t = a(x_t) + [H(x_t)]' \xi_t + w_t \tag{9}$$

$$\xi_{t+1} = F(x_t)\xi_t + v_{t+1}. \tag{10}$$

Here $F(x_t)$ denotes an $(r \times r)$ matrix whose elements are functions of x_t; $a(x_t)$ similarly describes an (nxl) vector-valued function and $H(x_t)$ an (rxn) matrix-valued function. It is assumed that conditional on x_t and on the data observed through date t-1, denoted

$$\vartheta_{t-1} \equiv (y'_{t-1}, y'_{t-2}, \ldots, y'_1, x'_{t-1}, x'_{t-2}, \ldots, x'_1)',$$

the vector $(v'_{t+1}, w'_t)'$ has the Gaussian distribution

$$\begin{bmatrix} v_{t+1} \\ w_t \end{bmatrix} | x_t, \vartheta_{t-1} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Q(x_t) & 0 \\ 0 & R(x_t) \end{bmatrix}\right). \tag{11}$$

Equations (9) to (10) allows for stochastically varying parameters, but it is more restrictive in the sense that a Gaussian distribution is assumed.

(b) Implementation of the Learning Process in a Macroeconomic Model

The learning process is generally implemented in model context as follows. The model essentially contains three blocks of equations, namely:

$$y_{it} = f_i(Y_t, X, {}_tY^e_{t+1}) \qquad i = 1...n,\ t = 1...T \qquad (12)$$

$$ {}_ty^e_{t+1} = g_k(Y_t, X, \xi_t) \qquad k = 1...m \qquad (13)$$

$$\xi_t = \xi_{t-1} + v_t \qquad v_t \sim N(0,Q). \qquad (14)$$

Equation (13) represents the measurement (or observation) equation(s) of the state-space model in Kalman filter terms, while equation (14) represents the state (or transition) equation(s).

Assuming that the value of ξ_{t-1} is known, the last block of equations for the expected value of ξ_t can be solved, which is simply the Kalman filter prediction equations for ξ, called the state vector. Given ξ, the second block of equations for the expected value of ${}_tY^e_{t+1}$ can be solved, and given this, the first block of equations for Y_t can finally be solved. For a technical exposition of the derivation of the updating equations and the recursive algorithm of the Kalman filter, refer to Hamilton (1994:372-408) and Cuthbertson (1992:191-225).

The underlying assumptions of the learning process are still quite strong, as agents are still assumed to process all available information in an optimal fashion and a substantial degree of sophistication on the part of the economic agent is still assumed. The learning model may, however, fulfil the criteria for weak rational expectations, since agents are not assumed to have full information. They will most likely make mistakes in the short run, but systematic errors over an extended period of time are ruled out.

3. A LEARNING MODEL OF PRIVATE CONSUMPTION EXPENDITURE IN SOUTH-AFRICA

In this section, the hypothesis that South African consumers are forward-looking with respect to prices when making consumption expenditure decisions, is tested. It is assumed that consumers learn using a Kalman filter-based (boundedly rational learning) process for updating their expectations conditional on prior errors made when forecasting the price level.

The first stage of implementing the boundedly rational learning approach involves the estimation of the time-varying mechanism, which represents economic agents using incomplete historical information to form expectations. In the second stage, the expectations formation mechanism is incorporated into the behavioural equation. The theoretical specification of the private consumption expenditure function would therefore include a price expectations variable, namely the expected one-period-ahead consumer price level.

(a) A Model for Price Expectations Formation

A boundedly rational learning approach towards the formation of price expectations is intuitively attractive. It is also consistent with psychology literature on learning processes, unlike the rational expectations hypothesis, which demands full information from economic agents regarding the model as well as its parameters.

The boundedly rational learning approach is based on the assumption that expectations are formed by intelligent agents who are not fully informed, but learn about their environment as time progresses. The learning model requires the specification of a time-varying parameter rule of expectations formation. In many applications, the expectations rule is derived from the reduced form of the structural equation. However, citing Hall and Garratt (1992b: 11), "How one derives an expectations equation is by no means exclusive. In principle, formation of expectations could be the result of information anywhere in the model, if it is thought to be relevant".

In setting up the expectations rule in this case, the application of price expectations to wage behaviour of countries in the global econometric model (GEM) (Barrel *et al.* 1994:174) is followed. The criteria used to decide on the inclusion of variables in the time-varying expectations rule, according to Barrel, do not follow from a tightly formulated theory, but inclusion of variables are also not completely *ad hoc.* Variables are selected to capture important endogenous linkages in the model and also enable the price expectations equation to adjust the coefficients on existing variables in accordance with movements in variables not included in the behavioural equation.

The expectations rule may therefore be represented by the following theoretical specification:

$$\Delta cpi_{t+1} = e^{\xi_{1t}} \Delta cpi_{t-1}^{\xi_{2t}} \Delta rl_{t-1}^{\xi_{3t}} \Delta exch_{t-1}^{\xi_{4t}} e^{w_t} . \qquad (15)$$

The dependent variable for the price expectations equation is the change in the consumer price index one period ahead (Δcpi_{t+1}). The information set or the independent variables include the change in the representative long-term interest rate (Δrl_{t-1}) and the change in the rand/US dollar spot nominal exchange rate $(\Delta exch_{t-1})$. The above specification implies that information of period t—1 is utilised to form price expectations in period twith respect to period t+1.

Alternative specifications that deserve consideration and empirical testing include the 3-month bankers' acceptance rate, or the prime interest rate or even the M3 money supply, instead of the representative long-term interest rate. The spot rand/US dollar exchange rate may possibly be replaced in the specification by *e.g.* import prices, the terms of trade, or the effective exchange rate. In addition, the wage rate and capacity utilisation in the economy may also be considered as possible explanations of the expected price level. The GEM specification *(op. at.:* 174) mentioned earlier, does include a capacity utilisation variable in the information set, in addition to lagged prices, interest rates and the relevant exchange rate.

The above theoretical specification in a South African context may be motivated as follows. The specification, including lagged prices, interest rates and the exchange rate is an attempt to model the psychological expectations formation process of the (often unsophisticated) consumer. Given that 19 per cent of the adult South African population is illiterate (has not completed primary school) (Stats SA1998), the adjustment of parameters of an expectations rule based on variables like the terms of trade, capacity utilisation or the money supply probably implies an unrealistically sophisticated consumer. Information about changes in prices, interest rate levels and the exchange rate is perhaps more accessible to the average consumer than any other economic variables influencing price changes. Price expectations are expected *a priori* to be autoregressive for the most part, with a less significant contribution by other variables included in the information set.

The first step in implementing the learning model of price expectations would be to formulate the expectations rule in state-space form. In terms of the specification of equation (15), y_t and x, in equation (9) will be given by:

$$\begin{aligned} y_t &= \Delta cpi_{t+1} \\ x'_t &= [1, \Delta cpi_{t-1}, \Delta rl_{t-1}, \Delta exch_{t-1}]'. \end{aligned} \tag{16}$$

The unknown parameters of the system will be estimated along with the (4x1) state vector, ξ_t. The state vector will be assumed to evolve through time according to a random walk with drift process, that is $\xi_{t+1} = \xi_t + v_{t+1}$.

Since all variables are integrated of order 1 (see Appendix 1), first differences are taken and all variables are utilised in natural logarithm form. The empirical result of the expectations rule will be reported in the next section.

(b) Time-varying Parameter Estimation of the Expectations Rule

The first step in the estimation process would be to estimate the time-varying parameters of the expectations rule (the observation equation), $\Delta cpi_{t+1} = f(\Delta cpi_{t-1}, \Delta rl_{t-1}, \Delta exch_{t-1})$. Table 1 reports the hyperparameters and residual diagnostics for the price expectations equation when estimated using the

Kalman filter.

The variances of the error term of the observation equation and the covariance matrix of the state equation error terms, which is assumed to be diagonal, are reported in the top part of the table. $R(1,1)=\sigma^2$ represents the variance of the error term of the observation equation and Q represent the covariance matrix of the error terms or adjustment factors of the state equations. The latter, which are often called hyperparameters, determine the speed of learning and reflect the signal to noise ratio for each variable; hence they also reflect the rate of convergence of the model. Note that the hyperparameters are not necessarily expected to be statistically significantly different from zero as measured by the t-statistic. Both the variances of the error term of the observation equation as well as the covariance matrix of the state equation error terms may be very small and not statistically different from zero. (If they were actually zero, this would mean that the true coefficients were fixed. Under this condition, the Kalman filter gives recursive OLS and the estimated coefficients would still change as learning with respect to the true coefficients take place.)

The hyperparameters further only make sense when compared to the variance of the error term of the observation equation, which in this case is for all practical purposes equal to zero. In this comparison, the magnitude of the hyperparameters is indicative of a fairly rapid learning process with respect to lagged prices, and to a lesser extent with respect to interest rates and the exchange rate. Put differently, the covariance matrix of the state equation error terms represents the 'forgetting factor' (Hall and Garratt 1992b: 7). When the matrix contains only zeros, the Kalman filter will generate OLS estimations of the state vector, ξ. As the diagonal elements of the covariance matrix rise, the parameters are allowed to change more rapidly, and in effect 'forget' the past.

The final values of the state vector, ξ, with associated standard errors, are also reported in the top part of the table. ξ_1, ξ_2, and ξ_3 represent the time-varying coefficients of the variables in the expectations rule, Δcpi_{t-1}, Δrl_{t-1} and $\Delta exch_{t-1}$ respectively. The intercept coefficient of the price expectations rule was defined as time-independent (constant).

Residual diagnostics (of the observation equation) reported in Table 1 indicate adherence to the normality assumption. There is also no serial correlation present. The regression statistics *{e.g.* the R-squared) for time-varying coefficient models are computed by EViews, using the smoothed residuals $y_t - A'x_t - H'\xi_{t|T}$ from the observation equation. The R^2 and adjusted R^2 are equal to unity in this instance.

Each of the estimated time-varying slope coefficients, assumed to evolve as a random walk with drift, displays a reasonable degree of variation over the period. The intercept coefficient of the expectations rule was not allowed to vary. Fig. 1 to 3 illustrate the evolution of the time-varying coefficients.

The time-varying coefficient of lagged prices clearly displays the largest degree of variation. This coefficient varies roughly between 1.0 and 0.0, with an upward trend between 1974 and 1986, coinciding with a period of rapid increase in the general domestic price level. Double-digit inflation figures were first recorded in 1974 and ascribed mainly to the oil price shock experienced in 1973. This was repeated in 1979.

The high inflation figures during this period reflected the world-wide upward trend in general price levels. Although the main trading partners of South Africa managed to curb their domestic inflation during the 1980s, the upward trend

Table 1. Hyperparameters, Final Value of the State Vector and Equation Diagnostics from the Time-varying Kalman Filter Estimates

Dependent variable: Δcpi_{t+1}

Variables	**Coefficient**	**Std.Error**
R(1,1)	6.9E-65	0.0042
Q(1,1)	0.01584	0.0270
Q(2,2)	0.00066	0.6918
Q(3,3)	0.00044	0.7576
constant	0.06640	0.0042
Final ξ_1	0.11123	0.1624
Final ξ_2	-0.02918	0.0428
Final ξ_3	-0.02843	0.0526

Sample period (adjusted): 1969 to 1997
Adjusted $R^2 = 1.000$
s.e. = 7.6E-15

BJ (normality) = 0.0181 [0.991]

LB(1)=0.0496 [0.824]
LB(2)=5.4780 [0.065]
LB(3)=6.1115 [0.106]
LB(4)=6.1125 [0.191]

Note: ξ_1, ξ_2, and ξ_3 are the final values of the time-varying coefficients of Δcpi_{t-1}, Δrl_{t-1} and $\Delta exch_{t-1}$, respectively.

in South African inflation continued. Inflation peaked in 1986 at 18.4 per cent, a period in South African history dominated by international diplomatic and economic isolation. Since 1986 onwards, there has been a general downward tendency in inflation in South Africa, slowing down to a single digit from 1993 onwards. This process was aided by consistently strict monetary policy.

Each of the estimated time-varying slope coefficients, assumed to evolve as a random walk with drift, displays a reasonable degree of variation over the period. The intercept coefficient of the expectations rule was not allowed to vary. Fig. 1 to 3 illustrate the evolution of the time-varying coefficients.

The time-varying coefficient of lagged prices clearly displays the largest degree of variation. This coefficient varies roughly between 1.0 and 0.0, with an upward trend between 1974 and 1986, coinciding with a period of rapid increase in the general domestic price level. Double-digit inflation figures were first recorded in 1974 and ascribed mainly to the oil price shock experienced in 1973. This was repeated in 1979. The high inflation figures during this period reflected the world-wide upward trend in general price levels. Although the main trading partners of South Africa managed to curb their domestic inflation during the 1980s, the upward trend in South African inflation continued. Inflation peaked in 1986 at 18.4 per cent, a period in South African history dominated by international diplomatic and economic isolation. Since 1986 onwards, there has been a general downward tendency in inflation in South Africa, slowing down to a single digit from 1993 onwards. This process was aided by consistently strict monetary policy.

It is interesting to observe that the evolution of the time-varying coefficient of the lagged price level in the price expectations rule to a certain extent mimicked the trend of the actual change in price levels. First, an overall increasing trend emerged from 1974 to 1986 (after an initial downward adjustment of the coefficient), followed by a descent from a high of 0.8 in 1986 to a final value of 0.11 in 1998 when the lowest inflation rate in 28 years was recorded in South Africa, namely that of 6.8 per cent.

What can be gathered from the above may be that consumers are indeed learning from new information as time progresses. In periods of high and rapidly increasing price levels, consumers continuously adjust the parameters of the rule upwards and, as soon as they realise that price levels are declining, they start adjusting their parameters downward, leading to lower expected price levels.

The coefficient of the second variable in the information set of the price expectations rule displays a considerably smaller degree of variation, especially when taking into account that the scale in Fig. 2 is only 60 per cent of that of Fig. 1. The coefficient evolves around 0.04 within the band (-0.02, 0.12). The general trend is slightly upward for the period 1974 to 1985, when interest rates peaked at 18.62 per cent, followed by a movement sideways, in line with interest rate movements.

The variation in the coefficient of the lagged spot nominal rand/US dollar exchange rate is also significantly less than that of lagged prices. The coefficient varies around -0.016, between -0.089 and 0.017. The coefficient evolves rather smoothly, with only one significant jump in 1985, a year marked by the largest depreciation in the rand/US dollar exchange rate in the history, namely a depreciation of 50 per cent from R1,48/US dollar to R2,23/US dollar. This pronounced depreciation may clearly be attributed to political and economic isolation, the debt standstill and

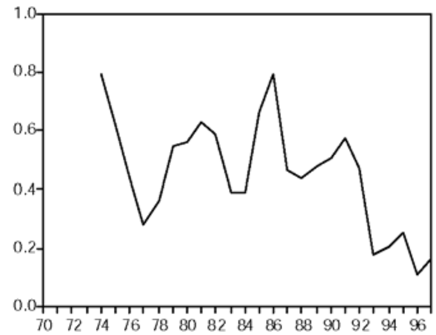

Figure 1. Coefficient on Lagged Consumer Price Inflation

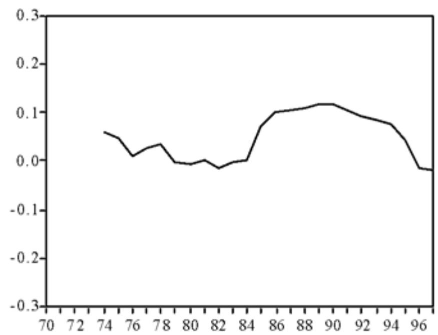

Figure 2. Coefficient on Lagged Long-term Interest Rate

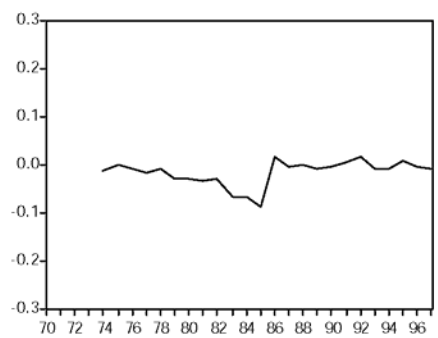

Figure 3. Coefficient on Lagged Rand/US Dollar Exchange Rate

disruptive balance of payments adjustments.

It is obvious from the above that the most important variable in the information set of the price expectations rule is the lagged price level. The other two variables display a lesser degree of variation and are also statistically less significant in explaining price expectations. Furthermore, the larger variance of the one-period-ahead forecast of the first element of the state vector, namely that of the lagged price level, is also indicative of a faster rate of learning with respect to observed changes in the actual price level than that of the other two independent variables.

The Kalman filter prediction of the actual one-period-ahead price level, cpi^*_{t+1}, is therefore the variable to be implemented in the consumption function as the variable reflecting the price expectations of the consumer. Citing Cuthbertson (1988:226) in this regard: "It is worth noting at the outset that an expectations series generated using the Kalman filter with time-varying parameters may be directly used in behavioural equations where the structural parameters are assumed constant."

(c) Si Forward-looking Consumption Function

Finally, the behavioural equation can be estimated, containing the one-period-ahead forecast of the expectations variable, *i.e.* the actual result obtained through the Kalman filter estimation. The theoretical specification of the consumption function, based on the forward-looking theories of consumption *(i.e.* the life-cycle hypothesis of Modigliani and Brumberg (1954), and Ando and Modigliani (1963), and the permanent-income hypothesis of Friedman (1957)), will include an income variable, some wealth variable, and possibly long-term or short-term interest rates representing monetary conditions. These are, according to theory, the variables to consider for the long-run equilibrium or steady-state relationship:

$$c_t = Ay_t^{\alpha} w_t^{\beta} r_t^{\gamma} e^{\varepsilon_t} \tag{17}$$
$$0<\alpha<1;\ 0<\beta<1;\ \alpha+\beta=1;\ \gamma<0$$

with

c_t = private consumption expenditure in period t
y_t = personal disposable income in period t
w_t = financial wealth stock in period t
r_t = a representative interest rate in period t and
ε_t = the stochastic disturbance term.

The explicit inclusion of both the income and the wealth variables is justified by the forward-looking theories of consumption, and further motivated by the Ball-Drake (1964) hypothesis regarding the derivation of utility from the accumulation of wealth. Both α and β are expected to be positive with $\alpha+\beta=1$.

South Africa is characterised by absolute poverty and an unequal distribution of both income and wealth. A substantial portion of the population possesses virtually no wealth and earns a small income, if any. More specifically, 90 per cent of the population earns less than 50 per cent of total income (Whiteford and van Seventer 1999:ii), while the unemployment rate ranges in the region of 36 per cent (Stats SA 1998). The large portion of the population constrained by very low income levels, spends virtually all of its income on consumption, with very little left to be utilised for wealth accumulation. Wealth therefore plays an insignificant role in the consumption expenditure decisions of these consumers. For South Africa as a whole, one would expect an income elasticity close to unity for total consumption expenditure, and a low wealth elasticity.

Within an intertemporal consumption optimisation framework, the interest rate constitutes a trade-off between current and future consumption. Other variables that were considered as explanatory of consumer behaviour include lagged consumption expenditure *(i.e.* expenditure patterns of the past), variables reflecting labour market sentiment, for example the employment rate, lagged personal savings, credit leasing finance, instalment sale credit, stock market prices, *etc.* The expected price level, that is the one-period-ahead consumer price level forecasted by the Kalman filter, can be used to test for the effect of price expectations on consumption expenditure behaviour within a boundedly rational learning framework. For a list of variables, refer to Appendix 1.

The long-run relationship between total private consumption expenditure, wealth and personal disposable income was estimated in the form c-yd = f(w-yd), thus constraining the sum of wealth and income elasticities to unity (all variables in natural logarithm form), that is

$$c_t = Ayd_t^{\alpha} w_t^{\beta} e^{\varepsilon_t} \text{ with } 0<\beta<1;\ 0<\alpha<1;\ \alpha+\beta=1. \qquad (18)$$

Interest rates were not included in the long-run equilibrium equation, but rather considered as contributing towards the short-run dynamic structure of the system.

A vector autoregressive (VAR) model with a lag length of 1 was used to test for the number of cointegrating relationships. The validity of the Johansen estimation technique depends on white noise error terms — the minimum lag length that renders residuals of the ECM white noise would thus be acceptable. The error correction model for total private consumption expenditure, as reported in Table 5, passed all the standard tests of serial correlation, normality, etc. where long-run coefficients were obtained with a VAR of order 1. The limited sample of the data from 1970 to 1997 (constrained by the unavailability of the financial wealth variable) and the fact that the Johansen cointegration test is rather data-hungry, contributed towards the decision to use a VAR with lag length 1. Although the Akaike information criterion (AIC) and the Schwarz Bayesian criterion (SBC) suggested that a model with lag length 2 is preferable, the values of the information criteria were very close for the two models, see Table 2. Granger (1991:561) also notes in this regard that the AIC criterion is known to 'overfit' in the sense that if the true value of p for VAR(p) is p, (finite), then the criterion is inclined to choose a value for p that is somewhat greater than p_0.

Table 2. Selection of the Order of the VAR: Total Private Consumption Expenditure

Order of VAR	AIC	SBC
1	57.2617	55.3180
2	53.7687	50.6234

Both wealth variables were tested, namely the stock of wealth, w, and the return on wealth, rw. In this instance, total private consumption expenditure, the return on wealth and personal disposable income constituted a cointegrating relationship.

In order to test for cointegration between (c-yd) and (rw-yd), the Johansen test (1989,1990) was employed. For a

VAR(l), the Johansen likelihood ratio test for the number of cointegrating relationships (denoted by r), based on the maximum eigen value and the trace of the stochastic matrix, suggests one cointegrating relationship at a 5 per cent significance level between the variables in the long-run relationship (the cointegration equation). This result is presented in Table 3.

Table 3. Johansen Test for the Number of Cointegrating Relationships: Total Private Consumption Expenditure

Cointegration LR test based on maximal eigenvalue of the stochastic matrix				
Null	Alternative	Statistic	95% Critical value	90% Critical value
r=0	r=1	16.8664	15.8700	13.8100
r ≤ 1	r=2	7.0921	9.1600	7.5300

Cointegration LR test based on trace of the stochastic matrix				
Null	Alternative	Statistic	95% Critical value	90% Critical value
r=0	r ≥ 1	23.9585	20.1800	17.8800
r ≤ 1	r=2	7.0921	9.1600	7.5300

The long-run relationship was estimated with an unrestricted constant. The long-run cointegration relationship, for rank equal to 1 (r=1), is reported in Table 4 (figures in parentheses denote coefficients normalised on the dependent variable). The result suggests the presence of a long-run relationship of the form

$$ctot_t = (1\text{-}0.1966)yd_t + (0.1966)rw_t + 0.5593. \qquad (19)$$

The income elasticity therefore amounts to 0.8034 and the wealth elasticity to 0.1966 for the period under consideration.

Coefficients are of the correct sign and the magnitudes correspond with *a priori* expectations expressed in the hypothesised long-run relationship.

Table 4. Estimate of Cointegration Equation

Variables	**Cointegrating vector**
$(ctot\text{-}yd)_t$	0.4839 (-1.0)
$(rw\text{-}yd)_t$	-0.8813 (0.1966)
constant	-2.5077 (0.5593)

Sample Period: 1971-1997
Order of VAR = 1; r=1

The residual derived from the above, allows for the specification of the error correction model, representing the short-run dynamic adjustment process. This result is reported in Table 5.

Both measures of non-human wealth were introduced in the specification of total private consumption expenditure. While the return on wealth is utilised in the long-run cointegration relationship, the actual real stock of wealth seems to contribute to the short-run dynamics of the system, as do lagged values of wealth stock and current real disposable income, lagged total consumption expenditure and the future expected price level. Lagged values of the dependent variable, *i.e.* total private consumption expenditure, would account for habit persistence or historic expenditure patterns, tastes of the consumer, *etc.* The expected price level, that is the one-period-ahead Kalman filter estimation, is introduced to test the hypothesis that consumers are forward-looking with respect to prices. All variables were used in natural logarithmic form. The interest rate variable did not contribute towards explanation of total private consumption expenditure (although statistically significant in the explanation of the durable component of private consumption expenditure).

Table 5. An Error Correction Model for Total Private Consumption Expenditure

Dependent variable: $\Delta ctot_t$

Variable	Coefficient	Std. Error	t-Statistic
constant	0.0408	0.0116	3.5218
$resid_{t-1}$	-0.2053	0.0710	-2.8924
$\Delta ctot_{t-1}$	-0.2351	0.1054	-2.2316
Δyd_t	0.6704	0.0680	9.8632
Δw_t	-0.3163	0.0404	-7.8219
Δw_{t-1}	0.1208	0.0325	3.7180
Δcpi^*_{t+1}	-0.2167	0.0882	-2.4570

Sample period: 1975 to 1997
R^2 = 0.9097
Adjusted R^2 = 0.8759
s.e.= 0.0094

Normality:	BJ(2)=0.7026	[0.703]
Serial correlation:	LB(8)=3.4096	[0.910]
	LM(2)=2.9052	[0.234]
Heteroscedasticity:	ARCH(1)=0.0415	[0.839]
	White(1)=6.6778	[0.878]
Stability:	RESET(2)=0.7714	[0.680]

Note: cpi^*_{t+1} represents the one-period-ahead Kalman filter prediction of the expected price level.

It might be argued that the fact that the one-period-ahead Kalman filter result appears to be statistically significant is not sufficient to prove that consumers are indeed forward-looking. To prove this fact, the above regression was repeated, this time also incorporating the current period Kalman filter result as well as the result lagged by one period. Only the one-period-ahead variable turned out to be statistically significant while the current period and lagged variables were statistically insignificant (probability values of 0.25 and 0.51 respectively). All other coefficients remained robust and significant. Actual and lagged price levels were also tested with corresponding results.

Diagnostic statistics suggest that the equation is statistically well-specified, with no violations of the Gaussian assumption. The use of a VAR of order 1 for the derivation of the long-run relationship, is therefore justified. Figures in square brackets denote probability values (the probability of falsely rejecting the null hypothesis of a zero restriction on the coefficient or diagnostic). BJ denotes the Bera-Jarque test for the normality assumption; LB and LM denote the Lung-Box Q and Lagrange Multiplier tests for serial correlation; Ramsey's RESET is a test for misspecification and ARCH and White denote tests for the homescedasticity assumption. All test statistics are in χ^2 form, with figures in parentheses denoting degrees of freedom. Recursive estimates also confirmed stability in the parameters.

The result of *ex post* simulation of the private consumption function is presented graphically in Fig. 4.

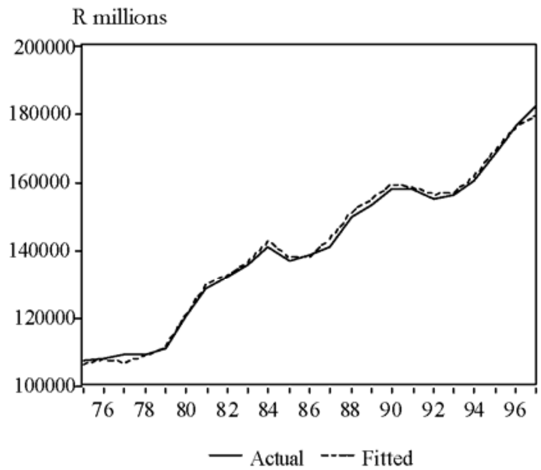

Figure 4. The Overall Dynamic Fit of the Total Private Consumption Expenditure Function

4. CONCLUSION

This paper tested the hypothesis that South African consumers are forward-looking with respect to prices when making consumption expenditure decisions. Furthermore, the assumption that consumers learn through a boundedly rational learning process from expectations errors made in the past and that they find a way of increasing their knowledge about the true values of the parameters in the expectations rule, was tested.

The expectations rule was formulated in an attempt to capture the psychological learning process of intelligent economic agents. Variables influencing price expectations were taken to be the lagged observed price level, lagged interest rates and the lagged exchange rate. The most important variable in the information set proved to be the lagged price level. The time-varying coefficient of this variable also displayed the largest variation (variance) with an upward tendency in periods with high and increasing price levels and *visa versa.*

The price expectations variable was subsequently implemented in a function for total private consumption expenditure. Total private consumption, non-human (financial) wealth and current disposable income constituted a long-run equilibrium relationship. Variables that contributed towards explaining the short-run dynamics of the system, included lagged consumption expenditure, wealth stock and current disposable income. Apart from the above, the one-period-ahead price expectations variable (the result from the Kalman filter estimation), included in the behavioural equation to test for the role of forward-looking inflation in consumption expenditure decisions, proved to be significant. The interest rate variable, although significant in the explanation of durable consumption, did not contribute towards the explanation of total private consumption expenditure.

From the results reported in this paper, it may be concluded that both the time-varying Kalman filter prediction for price expectations and the estimation of the error correction model for private consumption expenditure yielded satisfactory results. The hypothesis that South African consumers consider price expectations when making consumption expenditure decisions has therefore been validated. Furthermore, the fact that consumers may be regarded as intelligent agents who are able to assimilate information and learn from their environment as time progresses has been proven.

APPENDIX 1: Sources of Data and Calculations

The sources and construction of the data series used to empirically estimate the theoretical models in this paper are discussed in this section, as "well as the univariate characteristics of the data.

The income variable is represented by real disposable income. The consumer price index (cpi) is used to deflate this variable. The 3-month bankers' acceptance rate and the eskom rate are considered as the representative short-term and the long-term interest rates respectively.

Non-human "wealth "would ideally include net financial "wealth of households, housing "wealth and possibly an index for stock market prices. The magnitude of the *stock* of "wealth in South Africa is however not available in time series format. An indication of the *flow* of financial "wealth for the household sector is available in the National Financial Account in the form of the financing balance *(i.e.* financial assets — financial liabilities)[2].

In this study the financial "wealth stock variable "was then constructed by accumulating financial flows for the household sector from 1970, assuming a base value in that year. The return on "wealth variable "was constructed by means of the representative long-term interest rate. A constraining factor is that data on financial "wealth is only available from 1970 onwards, "whereas the other variables under consideration date back to 1960 and some to 1946. Wealth variables "were also deflated using the cpi.

[2] Financial assets and liabilities as reported in the National Financial Account consist of 24 items including gold and other foreign reserves; cash and demand monetary deposits; short, medium and long-term monetary deposits; deposits "with other institutions and other financial institutions; treasury and other bills; bank loans and advances; trade credit and short-term loans; short and long-term government stock; non-marketable government bonds; securities of local authorities and public enterprises; other loan stock and preference shares; ordinary shares; foreign branch/head office balances; long-term and mortgage loans; interest in retirement and life funds; amounts receivable/payable; other assets/liabilities and a balancing item.

(See Muellbauer and Aron (1999) for a first systematic attempt to construct personal sector "wealth for South Africa. Although their series does include housing "wealth obtained from unpublished constant price housing stock estimates and data on housing prices, they report remaining gaps in the following areas: personal sector ownership of foreign assets, assets of unincorporated businesses included in the personal sector and unfounded pension rights.)

The source of all the data used is the *Quarterly Bulletin* of the South African Reserve Bank. For a list of variables refer to Table Al. All variables are used in natural logarithmic form.

Table A1. List of Variables

Series	Description
$ctot_t$	Real total private consumption expenditure
yd_t	Real personal disposable income
w_t	Real financial wealth
rw_t	Real return on financial wealth
$empl\_na_t$	Employment in the non-agricultural sectors
rs_t	Nominal short-term interest rate (3-month bankers' acceptance rate)
rl_t	Nominal long-term interest rate (eskom rate)
$exch_t$	Rand/US dollar spot nominal exchange rate
cpi_t	Price level (consumer price index)
cpi_e_t	Expected one period ahead price level ($cpi\_e_t = cpi_{t+1}$)
cpi^*_{t+1}	Kalman filter prediction of expected one period ahead price level

Order of Integration

In analysing the univariate characteristics of the data, the Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) tests "were employed to establish the order of integration of the data series, all variables in natural logarithmic form. The testing strategy as suggested by Dolado *et al.* (1990) and as applied by Sturm and De Haan (1995:69) was used (also see Enders 1995:257).

The number of lags used in the estimated equations "was determined in a similar "way as suggested by Perron (1989:1384), namely starting with eight lags and testing downwards, until the last lag is significant or there are no lags left.

In addition, graphing the data series in levels as "well as their first and second differences, looking at autocorrelation functions (correlograms) and spectrum analysis, proved to be helpful when ADF-test results "were inconclusive.

Tables A2, A3 and A4 report the outcomes of the ADF-tests for all relevant data series employed in estimations. The series tested are listed in the first column. The second column reports "whether a trend and a constant (Trend), only a constant (Constant), or neither one (None) is included. In the third column, the number of lags recorded is reported. The next column shows the ADF t-statistic, called X_x "when a trend and a constant are included, T^ "when only a constant is included, and T "when neither is included. The last column reports the F-statistic, O3 ($^{(}$I$^{)}$i), testing "whether the trend (constant) is significant under the null hypothesis of no unit root.

According to Table A2, ADF-tests rendered one of the variables stationary in levels, namely the log of the consumer price index. However, by simply looking at a graphical representation of this variable, it becomes obvious that this series cannot be stationary in levels. Table A3 indicates that all variables are indeed integrated of order 1. From experience and evidence from other tools, prices are known to be integrated of either order 1 or 2. The second differenced form of the consumer price index "was therefore also subjected to a unit root test. In Table A4, it is once again clear that the ADF test is inconclusive in establishing the order of integration of the consumer price index. The increasing rate of the change in the general price level between 1970 and 1986 and the slowdown in the rate of change from then onwards, could be the reason for the ADF-test's failure to be conclusive in this instance. In this study, the consumer price index "was regarded as 1(1) and used in first differenced form.

Table A2. Augmented Dickey-Fuller Tests for Non-Stationarity, Levels, 1970-1997
(All data series in natural logarithmic form)

Series	**Model**	**Lags**	$\tau_\tau, \tau_\mu, \tau$[a]	Φ_3, Φ_1[b]
$ctot_t$	Trend	4	-1.94	4.17
	Constant	2	-2.19	4.99
	None	6	1.00	
yd_t	Trend	2	-3.59	5.01
	Constant	2	-2.71	2.83
	None	0	3.88	
w_t	Trend	8	-3.23	5.19
	Constant	8	-2.87	4.23
	None	5	0.38	
rw_t	Trend	0	-2.83	5.02
	Constant	8	-2.45	1.74
	None	0	1.93	
rl_t	Trend	8	0.14	1.59
	Constant	8	-2.15	1.78
	None	8	1.34	
$exch_t$	Trend	3	-2.34	3.02
	Constant	2	0.73	1.43
	None	0	3.13	
cpi_t	Trend	1	-1.52	17.26**
	Constant	4	-2.46**	12.09**
	None	6	1.02	

*(**)Significant at a 5(1)% level.

a At a 5(1)% significance level the MacKinnon (1990) critical values are -3.63(-4.44) when a trend and a constant are included (τ_τ), and -3.00(-3.77) when only a constant is included (τ_μ) and -1.96 (-2.68) when neither is included (τ). The standard normal critical value is –1.703 (-2.473).

b At a 5(1)% significance level the Dickey-Fuller critical values (for 25 observations) are 7.24(10.61) when a trend and a constant are included (Φ_3) and 5.18(7.88) when only a constant is included (Φ_1).

Table A3. Augmented Dickey-Fuller Tests for Non-Stationarity, First Differenced, 1970-1997
(All data series in natural logarithmic form)

Series	Model	Lags	$\tau_\tau, \tau_\mu, \tau$[a]	Φ_3, Φ_1[b]
$\Delta ctot_t$	Trend	3	-4.21*	4.68
	Constant			
	None			
Δyd_t	Trend	0	-5.83*	16.98**
	Constant			
	None			
Δw_t	Trend	3	-3.95*	4.09
	Constant			
	None			
Δrw_t	Trend	0	-6.20**	19.27**
	Constant			
	None			
Δrl_t	Trend	2	5.03**	13.02**
	Constant			
	None			
$\Delta exch_t$	Trend	2	-4.61**	7.81*
	Constant			
	None			
Δcpi_t	Trend	0	-2.25	4.76
	Constant	0	-2.89**	8.38**
	None			

*(**)Significant at a 5(1)% level.

a At a 5(1)% significance level the MacKinnon (1990) critical values are -3.63(-4.44) when a trend and a constant are included (τ_τ), and -3.00(-3.77) when only a constant is included (τ_μ) and -1.96 (-2.68) when neither is included (τ). The standard normal critical value is –1.703 (-2.473).

b At a 5(1)% significance level the Dickey-Fuller critical values (for 25 observations) are 7.24(10.61) when a trend and a constant are included (Φ_3) and 5.18(7.88) when only a constant is included (Φ_1).

Table A4. Augmented Dickey-Fuller Tests for Non-Stationarity, Second Differenced, 1970-1997 (Data series in natural logarithmic form)

Series	**Model**	**Lags**	$\tau_\tau, \tau_\mu, \tau^a$	$\Phi_3, \Phi_1{}^b$
$\Delta\Delta cpi_t$	Trend	2	-0.59	2.18
	Constant	0	-2.09	4.36
	None	0	-0.35	

*(**)Significant at a 5(1)% level.

a At a 5(1)% significance level the MacKinnon (1990) critical values are -3.63(-4.44) when a trend and a constant are included (τ_τ), and -3.00(-3.77) when only a constant is included (τ_μ) and -1.96 (-2.68) when neither is included (τ). The standard normal critical value is –1.703 (-2.473).

b At a 5(1)% significance level the Dickey-Fuller critical values (for 25 observations) are 7.24(10.61) when a trend and a constant are included (Φ_3) and 5.18(7.88) when only a constant is included (Φ_1).

REFERENCES

ARON, J. and MUELLBAUER, J. (2000), Personal and corporate saving in South Africa, *Working Paper Series 2000-21*, Centre for the Study of African Economies, Oxford University.

ANDO, A. and MODIGLIANI, F. (1963), The life-cycle hypothesis of savings, *American Economic Review,* 53, pp. 55-84.

BALL, R.J. and DRAKE, P.S. (1964), The relationship between aggregate consumption and wealth, *International Economic Review,* 5, pp.63-81.

BARREL, R., CAPARALE, G.M., HALL, S.G. and GARRATT, A. (1994), Learning about monetary union. An analysis of boundedly rational learning in European labour markets, in *Applied Economic Forecasting techniques*. Hall, S.G. (ed.) Harvester Wheatsheaf, New York.

BRAY, M.M. and SAVIN, N.E. (1986), Rational expectations equilibria, learning and model specification, *Econometrica* 54 (5), September, pp. 1129-60.

CAGAN, P., (1956), The monetary dynamics of hyperinflation, in M. Friedman (ed.), *Studies in the quality theory of money,* Chicago University Press, Chicago.

CUTHBERTSON, K. (1988), Expectations, learning and the Kalman filter, *Manchester School,* LVI (3), September, pp. 223-46.

CUTHBERTSON, K., HALL, S.G. and TAYLOR, M.P. (1992), *Applied Econometric*

Techniques. Harvester Wheatsheaf, Hemel Hempstead.
CURRIE, D. and HALL, S.G. (1994), Expectations, learning and empirical macro-economic models, in *Applied Economic Forecasting techniques*. Hall, S.G. (ed.) Harvester Wheatsheaf, New York.
DE CARIO, S.J. (1979), Rational expectations and learning from experience, *Quarterly Journal of Economics* 93, pp. 47-57.
DICKEY, D.A. and FULLER, W.A. (1979), Distribution of the estimators for autoregressive time series with a unit root, *Journal of the American Statistical Association,* 74 (366), pp. 427-31.
_____ and _____ (1981), Likelihood ratio statistics for autoregressive time series with a unit root, *Econometrica,* 49 (4), July, pp. 1057-72.
DOLADO, J.J., JENKINSON, T., and SOSVILLA-RIVERO, S. (1990), Cointegration and unit roots: a survey, *Journal of Economic Surveys,* 4(3), pp. 249-76.
ENDERS, W. (1995), *Applied Econometric Time Series,* John Wiley & Sons, Inc, New York.
FRIEDMAN, B.M. (1979), Optimal expectations and the extreme information assumption of 'rational expectations' macro-models, *Journal of Monetary Economics,* 5 (1), pp. 23-41.
FRIEDMAN, M. (1957), *A Theory of the Consumption Function*, Princeton University Press.
GRANGER, C.W.J. (1991), Time Series Econometrics, in Greeenaway, D., Bleany, M. and Stewart, I., *Companion to centempory economic thought*, Routledge, London, pp. 559-73.
HALL, S.G. (1995), Macroeconomics and a bit more reality. *The Economic Journal,* 105 (July). Blackwell Publishers.
_____ and GARRATT, A. (1992a), Expectations and Learning in Economic Models, *Economic Outlook,* Vol. 16, No 5, pp. 52-53.
_____ and _____. (1992b), Model consistent learning and regime switching, *CEF Discussion Paper No 02-92*. London Business School, London.
_____ and _____ (1998), The treatment of expectations in large-scale models, in Klein, L. and Hickman, B.G. (eds.), *Studies in Applied International Economics, Vol. 1, Link proceedings 1991,1992,* World Scientific, Singapore.
HAMILTON, J.D. (1994), *Time Series Analysis*. Princeton University Press, New Jersey.
HARVEY, A.C., HENRY, S.G.B., PETERS, S. and WREN-LEWIS, S. (1986), Stochastic trends in dynamic regression models: an application to the employment-output equations, *Economic Journal,* 96 (384), pp. 975-85.
HOLDEN, K., PEEL D.A., THOMPSON, J.L. (1985), *Expectations: Theory and Evidence*. MacMillan Publishers Ltd., London.
JOHANSEN, S. (1989), Likelihood based inference on cointegration: theory and applications, *Mimeo,* Institute of Mathematical Statistics, University of Copenhagen.
_____ (1991), Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive models, *Econometrica,* 59 (6),pp.1551-80.

_____ and JUSELIUS, K. (1990), Maximum likelihood estimation and inference on cointegration with applications to the demand for money, *Oxford Bulletin of Economics and Statistics,* 52, pp. 169-209.

KALMAN, R.E. (1960), A new approach to linear filtering and prediction problems. *Journal of Basic Engineering, Transition of the ASME Series D,* 82, pp. 35-45.

_____ (1963), New methods in the Wiener Filtering Theory, in John Bogdanoff and Frank Kozin, eds., *Proceedings of the First Symposium of Engineering Applications of Random Function Theory and Probability,* pp.270-388. New York: Wiley.

LAWSON, T. (1980), Adaptive expectations and uncertainty, *Review of Economic Studies,* XLVII (2), pp. 305-20.

LUCAS, R.E. Jnr (1976), Economic policy evaluation: A critique, in *The Phillips Curve and Labour Markets,* vol.1, Carnegie-Rochester Conference series on Public Policy, North-Holland, Amsterdam, pp. 19-46.

MACKINNON, J.G. (1990), Critical values for cointegration tests, *Department of Economics Discussion Paper No. 90-4*, University of California, San Diego.

MODIGLIANI, F. and BRUMBERG, R. (1954), Utility analysis and the consumption function: an interpretation of cross sectional data, in Kutihara, K. (ed.), *Post-Keynesian Economics,* Rutgers, University Press.

MUELLBAUER, J. and ARON, J. (1999), Estimates of personal sector wealth for South Africa, *Working Paper Series 99-17,* Centre for the Study of African Economies, University of Oxford.

MUTH, J.F. (1961), Rational expectations and the theory of price movements, *Econometrica, 29 (3),* pp. 315-35.

PERRON, P. (1989), The great crash, the oil price shock, and the unit root hypothesis, *Econometrica,* 57 (6), pp. 1361-401

RADNER, R. (1982), Equilibrium under uncertainty, in Arrow, K.J. and Intriligator, M.D. (eds), *Handbook of Mathematical Economics* Vol. 2, North-Holland, Amsterdam.

SOUTH AFRICAN RESERVE BANK, *Quarterly Bulletin*. Various issues, Pretoria.

STURM, J.E. and DE HAAN, J. (1995), Is public expenditure really productive?, *Economic Modelling,* 12 (1), pp. 60-72.

STATISTICS SOUTH AFRICA (1998), *The people of South Africa, population census, 1996,* Report No. 1:03-01-11, Pretoria.

WALTERS, A.A. (1971), Consistent expectations, distributed lags and the quantity theory, *Economic Journal,* vol. 81, pp. 273-81.

WHITEFORD, A. and van SEVENTER, D.E. (1999), *Winners and losers, South Africa's changing income distribution in the 1990s,* WEFA Southern Africa, Pretoria.