Heat and mass transfer

HEFAT2010
7th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics
19-21 July 2010
Antalya, Turkey

ASYMPTOTIC LAW FOR TURBULENT FORCED CONVECTION FROM WALL SURFACE IN PACKED BEDS

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ABSTRACT
The available quantitative measurements of structural characteristics and visual observations of various authors demonstrate that the velocity pulsation spectra and the near-wall zone velocity and temperature profiles fully correspond to the similarity laws that were obtained earlier for turbulent separated flows. With this analogy in mind a physical model of convective transfer processes in near-wall zone of duct filled with packed bed (or surfaces embedded in grainy layers) at high Reynolds numbers is proposed and the asymptotic universal heat transfer law is developed. The universal character of heat and mass transfer law for the surfaces embedded in grainy layer is confirmed by the author’s data and data of various authors.

NOMENCLATURE

\( \delta \) [m] visous sublayer thickness
\( e \) [-] bed porosity
\( \lambda \) [W/mK] Thermal conductivity
\( \nu \) [m^2/s] kinematic viscosity
\( \rho \) [kg/m^3] liquid density

Subscripts
\( d \) refer to the particle diameter
\( e \) effective, refer to equivalent diameter
\( L \) liquid
\( rms \) root-mean-square
\( W \) near-wall zone

INTRODUCTION
 Forced convection in packed beds is a complex problem of modern thermophysics in consequence of many length scales and package structure distinctions between the near-wall zone and the bulk flow, the necessity of adoption the concept of effective transport coefficients, the choice of an averaging procedure, etc.

Interest in the problem is dictated by the needs of a variety of branches of industry such as chemical technology, metallurgy, gas and oil production, and nuclear reactors with spherical-fuel-elements. To the above-mentioned industrial areas of application must be added those related to transport phenomena in the Earth crust. Packed bed is often used as a model of porous media with a pre-dominant structure characterised by the unique dimension of balls.

Due to large number of generalizing monographs and proceedings of specialized conferences we will point out only the most important tendencies and conclusions associated with our goal.

It is a common practice (see, for example, [1] and [2]) to recognize three basic fluid flow regimes on the basis of the measurements of hydraulic resistance:
- a linear one \((\Delta p \propto U)\) realized at Reynolds numbers \(Re_d = Ud/\nu < 10\) and named the Darcy regime;
- a quadratic-law one \((\Delta p \propto U^2)\) observable, according to data by several authors, at \(Re_d > 10^5 \sim 10^6\);
• an intermediate nonlinear/inertial one, in which the power exponent of the $\Delta p \propto U^n$ dependence varies from 1 to 2. Theoretical investigations are based on one or another averaging procedure, the introduction of effective transport coefficients and a gradient hypothesis, empirical relationships for the velocity filtration dependence of the pressure drop, and semi-empirical models for effective thermal conductivity.

In the simplest model [3], [4], filtration velocity and effective thermal conductivity are taken to be constant over the duct cross-section (slug-flow model). With linear relationship $\lambda \sim Re$ at high Reynolds numbers the model leads to $Nu \propto Re$.

At the same time, close to surfaces there are sharp changes of the velocity and temperature fields at distances of the order of grain diameter or less, stemming from the elevated porosity approaching the cubic-package value, [5, 6]. This causes what is known as channeling with the filtration velocity standing out above the level of the core (bulk) flow velocity.

The most commonly data handling and interpretation are oriented to a simple two-layer model where a grainy layer in its heat-transferability is divided into two regions, namely, a core with constant over the duct cross-section velocity and effective thermal conductivity and a near-wall zone with thermal resistance involving all distinguishing features of package and flow in this region.

The key difference between the various two-layered approaches lies in models of the near-wall thermal resistance.

In the first two-layer model [7] laminar boundary layer is assumed to develop on the wall surface downstream of each contact point of a ball with the wall and to be destroyed in the void space by mixing with other streams. Clearly, this model admits only the laminar flow scenario with $Nu_\text{w} \propto Re^{1/2}$.

Another asymptotic model assumes the thickness of the near-wall zone to be defined by the critical Reynolds number corresponding to the transition to turbulent flow past a spherical ball [8]. The model leads to linear relationships $Nu \propto Re$ for both the near-wall and overall heat transfer coefficients.

When generalizing experimental data on heat transfer by analogy with forced convection in empty channels these data are often represented in the form

$$Nu = cRePr(\alpha y)$$

But they can be of more complicated form also [9].

It is apparent that the available generalising empirical correlations hold only for experimental conditions under which they were obtained.

By virtue of the fact that it is hardly possible to expect an exact theory will be developed in not too far distant future permitting calculation of an averaged and structural flow characteristics in systems under study, the role of simple pictorial physical models grasping fundamental features of transport phenomena in packed beds based on the most important experimental information on the structure and qualitative flow characteristics to be quite considerable. The objectives to be pursued by these models are a plausible explanation for experimental data and the stage for the elaboration of successful calculation methods.

### PHYSICAL MODEL OF TRANSFER PROCESSES IN THE NEAR-WALL ZONE IN PACKED BEDS

The basis for our model is the qualitative similarity between turbulent separated flows and turbulent filtration in packed beds.

#### Turbulent separated flows

In our papers [10, 11] a general similarity theory was developed for the near-wall zone of turbulent separated flow that is based on the assumption of the governing role of the induced local pressure gradient in transfer processes that take place in the vicinity of the wall in separated flows. This assumption is related to the idea (based on experimental observations) that flow in the near-wall zone is subjected to intense instantaneous accelerations induced by a large-scale vortex flow structure that prevents the formation of a sublayer with constant shear stress. In this connection, the friction velocity is not the governing parameter of the near-wall layer of turbulent separated flows anymore. The statistical regime of turbulent flow in this region is determined by local pressure gradient, or, in other words, local instantaneous accelerations $\alpha$ and distance $y$ from surface. The root-mean-square wall pressure gradient $\alpha_{w}$ is a qualitative characteristic of these accelerations. From dimensional considerations, similarity laws for mean velocity and temperature, the velocity and wall pressure fluctuations spectra were obtained.

These laws have the following form:

$$u \propto (\alpha_{w} y)^{1/2}, \quad T \propto \frac{\rho}{\rho_{f}} (\alpha_{w} y)^{-1/2}$$
$$E_{u} \propto \alpha_{w} k_{s}^{2}, \quad E_{p} \propto \rho \alpha_{w}^{2} k_{s}^{2}$$

For the thickness of the viscous sublayer one can write

$$\delta \propto \nu^{2/3} \alpha_{w}^{-1/3}$$

It is easy to show that the main part of the temperature difference occurs in the viscous sublayer, which has effective thermal conductivity close to the molecular one.

Then for the heat transfer coefficient we have

$$h_{w} \propto \frac{\delta}{\alpha} \propto \alpha_{w}^{2/3} k_{s}^{1/3}$$

The effect of Prandtl number on the heat transfer coefficient is usually taken into account by including the function $Pr^{B}$ in the heat transfer relationship.

Finally, by using the Nusselt and Reynolds numbers definitions, expression (2) takes the form

$$Nu = const C_{a}^{1/3} Re^{2/3} Pr^{B}$$

where $C_{a} = \alpha_{a} L / U^{2} = 2 p_{w} / \langle \rho U^{2} \rangle$ is the pressure pulsations coefficient.

Inasmuch as the coefficient $C_{a}$ is independent of the Reynolds number in turbulent separated flows, expression (3) is the "2/3 power law" ($Nu \propto Re^{2/3}$) for these flows. A large body of experimental research on turbulent separated flows provides support for the similarity laws (1) and the "2/3 power law" (3) for heat transfer.

One can see that the TSF regularities differ crucially from those for isotropic turbulence (the Kolmogorov's similarity laws).
Structural characteristics of turbulent filtration in packed beds and asymptotic heat and mass transfer law

First of all, the results of visual observations [12-14] showed the occurrence of flow separation and attachment, and distinct regions of reverse flows.

It is worth noting that contrary to single-phase turbulent boundary layer or turbulent duct flow where the transition from laminar to turbulent flow in ducts has a quite sharp or "intermittent" character in a certain region of Reynolds number close to critical number, in packed beds velocity pulsations increase gradually.

In [15, 16] high-velocity filtration was studied using the method of optical homogeneity, by which the refractive indices of the glass spheres and filtrating liquid are matched to a high accuracy. The experimental set up is a closed hydrodynamic loop where test section is a rectangular channel with dimensions 3d x 5d, where the diameter of a sphere of 18 mm. Fourteen layers of spheres were placed into a 255 mm high test area. The measurements were performed at the seventh layer.

**Figure 1** Velocity fluctuations spectrum in a cubic cell [15]

The authors described separation phenomena occurring in flow past packing elements. Spectral characteristics of the turbulent pulsations in a cell at the symmetry axis are given in Figure 1 (the figure is represented in its original form following paper [16]) for Reynolds numbers 10^3, 5 x 10^3 and 2 x 10^3 with the local Strouhal number, \( fD/\bar{U}_m \), along the x-axis and the normalized energy \( \overline{u'^2} \) in a broad frequency band, \( \phi_\overline{U}_m/\overline{u'^2} \), where \( \phi = \overline{u'^2} \), / \( \Delta f \) is the energy density in the frequency band, along the y-axis.

These spectra of the longitudinal velocity fluctuations are strictly faithful to the "-2 power law". It is very remarkable that there are no major differences between the flow patterns in near-wall zone and central cells.

The implementation of the similarity variables developed for turbulent separated flows [10, 11] allows the generalization of experimental temperature and velocity profiles in a channel packed with spherical balls.

Experimental velocity profiles in the near-wall zone of a duct filled in with spherical balls [16] are represented in Figure 2 in the turbulent separated flow similarity variables. The figure clearly demonstrates that velocity field has the similarity properties and it shows as well, the velocity profiles measured in the near-wall zone follow the similarity law.

\( u \propto y^{1/3} \) developed for the velocity profile in turbulent separated flows (y here is the distance measured from the wall).

**Figure 2** Velocity profiles of water in the near-wall zone in packed bed at Re = 5000 for different cross-sections (indicated at the upper part of the figure)

Experimental study of temperature profiles in the near-wall zone of the 20 x 40 mm² rectangular duct filled with 16 rows of cubically packed spheres with a diameter of 10 mm each is represented in [17]. The temperature profiles were measured at the seven most typical cross-sections of the duct.

**Figure 3** Temperature profile of water in the near-wall zone in packed bed (Re = 4900)

Implementation of the turbulent separated flows similarity variables makes possible the representation of experimental data in a generalized, universal form. In Figure 3 such a generalization is represented, where \( \theta \) stands for the non-dimensional temperature and y represents the distance measured from the wall (in reference to the legend of Figure 3, the x-coordinate was measured along the flow from the cross-section of maximum blockage in the 13th row of spheres. The 0-mm-section indicates the minimum cross sectional area between two spheres and 7.5 mm refers to the maximum cross-sectional area between 2 spheres). The experimental data support the similarity law, \( T \propto y^{1/3} \) for the temperature profile developed for turbulent separated flows.
So, visually observed flow separations and reattachment in packed beds, the existence of large-scale velocity fluctuations in the void space, the quite high level of the maximum relative turbulence intensity in comparison with "normal" turbulent boundary layer and the character of its change, the turbulence spectrum and behaviour of velocity and temperature profile all give grounds to put forward the following physical model:

In the vicinity of duct walls and packing element surface turbulent flow through packed beds has properties of turbulent separated flows with structural characteristics following the similarity laws (1) of the near-wall turbulence.

The stated physical model of turbulent filtration allows one to obtain the turbulent forced convective heat and mass transfer law leaning upon the analogy with transfer processes in turbulent separated flow and dimensionality consideration.

The intensity of large-scale pressure gradient in packed beds and local pressure pulsations can be estimated as follows:

\[ p' \approx U^2 / d \]

from which it follows that

\[ C_a \approx \frac{U^2}{d} \frac{d}{U^2} = \text{const} \]

resulting in, bearing in mind a great quantity of pores in a packed bed, the asymptotic universal law

\[ Nu = \text{const} \operatorname{Re}^{2/3} \operatorname{Pr}^{0.6} \]  \hspace{1cm} (4)

The constant is to be determined from experimental data. The grain diameter or equivalent pore dimension can be used as a characteristic length scale in this law.

Following is experimental verification of the law; for the moment we point out that, as it is known, a wall has an ordering effect on a packed bed that can be followed up to \((3 \pm 5)d\) deep into the packing. As a consequence of this effect the first layer of balls forms always-stable structure approaching the cubic one. Average porosity varies over a narrow range and has a weak dependence on the core porosity. In this connection the dependence of various transfer process characteristics on near-wall porosity was expected to be feebly marked in comparison with those for the core of packed bed.

**EXPERIMENTAL SET-UPS AND RESULTS**

The experimental setup used in this study consists of a closed circulation loop. The working fluid (water or 47% - glycerine aqueous solution) was pumped sequentially from a reservoir by a rotary pump into the working section, mixer section, flow-meter section, shell-tube heat exchanger and than it returned into the reservoir.

The working section was a copper tube with inner diameter \( D = 52 \) mm, wall thickness of \( 1.6 \) mm and length of \( 566 \) mm. Twelve nichrome-constantan thermocouples were cased in the tube wall. A thin layer of electrical (mica) insulation was applied to the outer tube wall, and over it a nichrome tape of \( 0.5 \times 10 \) mm cross-section was wound. The outside of the test section was covered by a layer of thermal insulation. Together with the wall temperature, the fluid temperature at both the working section inlet and outlet (past mixer), the heater temperature and the pressure drop across the working section were all measured. As a grainy medium, glass balls with diameter \( d = 0.9, 3.2, \) and \( 8.9 \) mm were used.

Temperature profiles over the tube cross section were measured by a comb-type probe of nine regularly spaced thermocouples. The thermocouples were made from nichrome and constantan wires of 0.1 mm diameter.

The thermocouples next to the tube wall were positioned at distances of 6 mm.

The heat transfer coefficients on the stabilised heat transfer section were measured over a wide range of the flow parameters.

Figure 4 presents the experimental data for mono-dispersed packed bed composed of glass balls with \( d = 0.9, 3.2, \) and \( 8.9 \) mm for the filtration of water and for the beds formed by the balls with \( d = 3.2 \) mm for the filtration of 47% - glycerine aqueous solution.

Of particular interest are the data for the 3.2 mm-balls, which are indicative of the existence of three laws of heat transfer in the flow range studied. For the case of water filtration we have:

\[ \frac{Nu}{Pr^{0.4}} \approx \text{Re}_d^{1/5} \quad \text{Re}_d < 40 \]

\[ \frac{Nu}{Pr^{0.6}} \approx \text{Re}_d \quad 40 < \text{Re}_d < 180 \]

\[ \frac{Nu}{Pr^{0.3}} \approx \text{Re}_d^{2/5} \quad \text{Re}_d > 200 \]

The solid curve presents theoretical asymptotic law, equation (4), with \( \text{const} = 0.4 \). When processing our experimental data the exponent \( \beta \) was found to be \( \beta = 0.4 \).

It must be emphasized that the problem of the Prandtl number effect is valid one for special investigation (the situation bears a close analogy to that in turbulent flows in ducts with no packing).

The Nusselt number and Reynolds number were calculated with the ball diameter as a characteristic length and the working liquid thermal conductivity \( \lambda_d \), so that the heat transfer law takes the form

\[ Nu = 0.4 \text{Re}^{2/3} \text{Pr}^{0.4} \]  \hspace{1cm} (5)

In our paper [18] we presented data with \( d = 3.2 \) mm on pressure drop variation with Reynolds number. These data showed that heat transfer laws matched the changes of pressure drop and followed filtration regimes.
The turbulent filtration regime is our chief interest. Correlation (5) is structurally identical to heat transfer law (4) developed on the basis of the analogy with turbulent separated flows where the near-wall zone governs transfer processes.

So, the next step is an estimation of the near-wall zone and core region thermal resistance contribution into correlation (5), what can be done by using a two-layer model representing total resistance to heat transfer as a sum of the near-wall thermal resistance and that of the core, $1/h = 1/h_W + D/h_c$.

Since there are no reliable theoretical or experimental correlations for the effective conductivity, $\lambda_e$ averaged over the cross section was determined from the solution of the energy equation for fully developed heat transfer and the measured across the tube temperature profiles by analogy with [19].

Experiments showed the temperature profiles became more complete with increase in the Reynolds number. A sharp drop in fluid temperature in the vicinity of the wall at high Reynolds numbers points to the relative importance of the wall heat transfer resistance. The correlation equation for $\lambda_e/\lambda_A$ derived from the measured radial temperature profiles of working fluid flow and heat fluxes for large Péclet numbers $Pe = Re_D Pr$ is

$$\frac{\lambda_e}{\lambda_A} = 0.083 \frac{Re_D}{Pr}$$

The contribution of the stagnant conductivity to the total effective thermal conductivity was found to be insignificant.

On the basis of the measured overall heat transfer coefficient $h$ (the maximum experimental error in the heat transfer coefficient is estimated to be 6%) and the effective thermal conductivity one can estimate the near-wall heat transfer coefficient $h_W$.

The table below presents estimations of the proportional contribution of the near-wall thermal resistance to the overall thermal resistance for water filtration through the tube packed with the spheres of diameter $d = 3.2$ mm:

<table>
<thead>
<tr>
<th>$Re_D$</th>
<th>534</th>
<th>588</th>
<th>309</th>
<th>207</th>
<th>170</th>
<th>71</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_W/\lambda_A$</td>
<td>0.94</td>
<td>0.90</td>
<td>0.88</td>
<td>0.83</td>
<td>0.77</td>
<td>0.74</td>
</tr>
</tbody>
</table>

This table provides evidence for the dominant contribution of the near-wall zone to overall heat transfer resistance at high Reynolds numbers corresponding to turbulent flow; it can be of order of 90% and more. In the filtration regime preceding the turbulent flow regime further increases of the core thermal resistance increases. This fact supports the correctness of the equations (2) and (3) use for analysis of turbulent forced convective heat transfer in packed beds, culminating in the asymptotic universal law, equation (4).

Of particular interest is the comparison of the above heat transfer trends with those in the annular filled with one layer of spherical balls essentially modelling transfer processes in the near-wall zone. For this purpose inside the test section an air-filled and hermetically sealed thin-glass tube of diameter $D = 34$ mm was centrally located. The experiments with 8.9 mm-diameter balls filling the annular were conducted for two types of packing — cubic and rhombohedral. It makes sense to compare tube and annular heat transfer data processed in terms of equivalent characteristic length scales. For a tube with a large number of balls across the tube cross-section it is $\frac{d}{l} = \frac{2}{3} \frac{e}{l(1-e)}$ and for an annular channel with one-layered ball packing the equivalent hydraulic diameter is calculated from the formula $\frac{d}{l} = \frac{D_2 - D_1}{e} \frac{1.5(1-e)(D_2 - D_1)}{l}$.

In terms of an equivalent characteristic scale equation (5) takes the form

$$Nu_e = 0.15 Re_D^{2/3} Pr^{0.4}$$

Figure 5 Heat transfer coefficient in tube and annular channel. 1, 2 — random packing of 3.2 mm glass balls in tube for the water and aqueous glycerol solution, respectively. 3 — d=0.9 mm; tube; water. 4, 5 — cubic and rhombohedral packing of 8.9 mm glass balls in annular channel, respectively. 6— eq (6), 7— Nu ~ Re^{0.5}

Figure 6 Summary plot on heat transfer upon the filtration of liquids and air. 1— d/D=0.072 mm, 2 — 0.167 mm [19]; 3— d=4 mm, nitrobenzene, Pr=9.5, 4—17.1 mm, toluol, Pr=9.5, 5—5.5 mm, glycerine solution, Pr=10.9 [20]; 6— d/D=0.1, 7—0.06 [21]; 8— d/D=0.272, 9— 0.299, 10— 0.254, 11— 0.15, 15— 0.194, 16— 0.283 [22]; 12— d/D=0.2, 13— 0.3 [23]. 1, 2, 6 — 13 — data for air.

From Figure 5 it is seen that the results for the cubic and rhombohedral one-layered packing in the annular channel and for random packing in the tube are in excellent agreement with one another and with the asymptotic law written in the form of equation (6).

In Figure 6 a large body of experimental data by various authors on forced convective heat transfer of air and different liquids in tubes filled with spherical balls at high Reynolds numbers are shown.

It is worth noting that these data agree wholly satisfactorily
with the asymptotic law in form of equation (5). Unfortunately, it does not always happen that authors provide complete information on their experiments.

Recent generalized empirical correlations recommended in [9] for forced convection heat transfer in pipes filled with spherical beds (notice that they are rather cumbersome with many empirical constant because the correlations cover the Darcy, inertial and turbulent regimes of flow) and an experimental investigation [24] of air forced convection in large rectangular packed ducts with asymmetric heating confirm very well the “2/3 power law”.

Experimental data [25] on air convection heat transfer from a cylinder embedded in a packed bed of spherical particles provide evidence in support of this law also.

CONCLUDING REMARKS

Experimental data, both qualitative and quantitative, of a variety of authors show that the main properties of forced convective transfer processes in the near-wall region in channels with packed beds or surfaces embedded in a granular medium correspond to those of turbulent separated flows. This enables a physical model of transfer processes in the near-wall region in packed beds to be proposed. The similarity laws of transfer processes in the near-wall zone of turbulent separated flows obtained previously are valid for packed beds and may be used as an algorithm, a basis, for a purposeful study of structural flow characteristics in packed beds and to reveal the effect of a variety of factors.

The application of these similarity laws and dimensionality analysis to heat and mass transfer of surfaces embedded in packed beds at high Reynolds numbers corresponding to the turbulent regime of filtration has for the first time led to the fundamental relation of the Nusselt and Reynolds numbers, asymptotic universal “two-thirds power law”.

Careful experiments on heat transfer in channels with packed beds support the heat transfer universal law at turbulent filtration. Experimental effective thermal conductivity provide quantitative evidence of the prominent role of the near-wall thermal resistance at high Reynolds numbers. The comparison with one-layered packing in an annulus serves an important argument in favour of the heat transfer law universality and the physical model of the heat transfer mechanism in the near-wall zone at high Reynolds numbers.

This new approach provides the possibility of constructing physically correct two-layer models of heat transfer matching the condition of limiting transition to the “two-third power law”. The issues of Prandtl number and thermal conductivities of liquid and material consisting of the packed bed ratio effects are still open questions.

REFERENCES


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