Optimal Maintenance Planning for Building Energy Efficiency Retrofitting From Optimization and Control System Perspectives

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Abstract

This paper discusses the maintenance plan optimization problem for the energy efficiency purpose in the building energy efficiency retrofitting context. A subproblem namely the Building Retrofitted Facilities Corrective Maintenance Planning (BRFCMP) problem is proposed, where the corrective maintenance for malfunctioning retrofitted items are involved. The aggregate performances of the homogeneous retrofitted item groups, instead of the individual item performances, are the main consideration of the optimization issue. An aggregate population level optimization model is proposed to address the BRFCMP problem. When further taking into account the uncertainties, the optimization problem is cast into an optimal control problem to reduce the consequent adverse impact, given the dynamic nature of the aggregate performances of the item groups during operation. Both the optimization and control system approaches are applied to solve the BRFCMP problem without or considering uncertainties. An actual building retrofitting project is used as the case study to investigate the important role of maintenance to the building energy efficiency. The effectiveness of the proposed approaches is verified by simulation results.

Keywords: Energy efficiency, building retrofitting, facilities maintenance, control system framework, optimal control, model predictive control

1. Introduction

Building energy efficiency is one of the most popular research areas today. One major topic in this area is the energy efficiency retrofitting in existing buildings. The focuses of energy efficiency retrofitting research at the current stage are the implementation of energy conservation technologies [1, 2, 3, 4, 5, 6] and the optimization of retrofitting plans [7, 8, 9, 10, 11, 12]. However, only a few studies have been conducted on the maintenance part of an energy efficiency project [13, 14, 15]. The asset maintenance for reliability purpose is a widely studied topic in the reliability engineering area, whereas the maintenance for energy efficiency purpose lacks exploration.

For a building energy efficiency retrofitting project, maintenance is required for the sustainability of the energy performances. In practice, the performances of the energy efficiency retrofits can deteriorate subject to usage and failures of the retrofitted items [16]. According to the Measurement and Verification (M&V) principles [17, 18], the energy efficiency of a retrofitting project can be evaluated by its aggregate energy saving. The energy saving cannot be directly measured, since they represent the absence of energy use. Instead, savings are determined by comparing measured use before and after implementation of a project, making appropriate adjustments for changes in conditions. Taking advantage of the corresponding energy audit results, the retrofitting plan can be optimized to maximize

the aggregate energy saving of the project, subject to a series of constraints. However, most existing studies ignore the possible dynamics of the retrofits' energy performances during operation. From the energy efficiency point of view, a malfunctioning retrofitted item contributes less or even zero energy saving. Wang et al. [11] takes into account such impact when optimizing the long-term energy saving of a retrofitting project. From the management perspective, the increase of malfunctioning items, i.e., the decrease of the population of available items, implies the inconsistency of the aggregate energy performances of the project. Given the absence of the maintenance actions, such energy efficiency deterioration cannot be reversed, resulting in the inefficiency of the retrofitting project and even worse, the violation of the energy performance contract. In conclusion, it is necessary to take into account maintenance in building energy efficiency retrofitting projects.

The scope of maintenance actions includes the activities required to operate and maintain the facilities and their supporting infrastructures in a condition to be used to meet their intended function over the operating period ¹. In the reliability engineering area, maintenance and replacement problems of deteriorating systems have been studied for many years [19]. Maintenance actions are generally classified into two categories: Corrective Mainte-

¹Comprehensive Facility Operation & Maintenance Manual, 2013, http://www.wbdg.org/om/om_manual.php

nance (CM) and Preventive Maintenance (PM). CM involves the repairs and replacements against failures and PM refers to all actions performed in an attempt to retain an item in a specified condition, according to MIL-STD-721C². Usually, CM occurs when the need arises, while PM is a planned program based on periodic inspections, adjustments, and replacements. Besides the usual CM and PM, emergency maintenance, such as restoring lost electrical power, can be taken into account in some projects.

From the perspective of building energy efficiency, the implementation of CM and PM actions adjusts the aggregate population and condition of the available items to preserve the energy performances of the retrofitting project. Moreover, the operation of the retrofits can influence the fatigue and energy performance of an item over the long term, and consequently the available item population dynamics and aggregate energy performances. Given a retrofitting project, both the maintenance plan and the operating schedules of the retrofitted items can be optimized to support the sustainability of the building energy efficiency. However, a retrofitting plan is usually optimized based on the performance characteristics that are estimated over a long time period, e.g., 10 years, the population of the retrofitted item group manifests a dynamic change in one or two years due to the maintenance actions, and the operation usually concerns issues of short intervals such as days or hours. In addition, budget limits are introduced in many practical cases, which also restrict the implementation of the maintenance actions. Therefore, building energy efficiency retrofitting optimization problems with multiple time scales and substantial magnitude could become complex if the maintenance and operation of the retrofitted facilities are taken into account, due to the complexity and interplay of the retrofitting planning, maintenance and operation.

The main purpose of this paper is to propose a method to incorporate the maintenance planning optimization into the building energy efficiency retrofitting project. At the current stage, a subproblem of the maintenance planning optimization problem that is adopted from [13], namely the Building Retrofitted Facilities Corrective Maintenance Planning (BRFCMP), is investigated for this purpose. The retrofitted facilities refer to the totality of the retrofitted items involved in the retrofitting project. The BRFCMP problem considers only the planning of CM actions that restore the malfunctioning items from failures and breakdowns to normal working conditions. For simplicity, it is assumed that the BRFCMP issue addressed here is planning the repair and replacements of the malfunctioning items according to predefined maintenance time schedule. The optimization of maintenance periodicity is therefore excluded at the current stage. The maintenance actions are planned at the aggregate population level, rather than at the level of individual items. In the BRFCMP problem, items of the homogeneous classes corresponding to to different retrofits are aggregated to obtain the managed retrofitted item groups. A hypothesis is made to obtain the classification of homogeneous retrofitted items. There are three kinds of related characteristics: the inherent energy and reliability performance, the operational environment of the items and the corresponding operating schedules. For example, a group of retrofitted items are considered as one class when they are the same model from the same producer, and they all work in a cool and dry climate, with similar workloads. Items from the same homogeneous class are assumed to manifest same energy and financial performances. The robustness of this hypothesis in practice yet remains an open problem that requires further exploration. Emergency maintenance is not an option that can be planned, therefore the equipment concerning emergency maintenance are not included in any of the homogeneous classes. The energy and financial performances of these classes are thus the main concerns of the discussed maintenance planning.

Based on the homogeneous item groups classification, an aggregated population level optimization model for the BRFCMP problem is proposed when uncertainties are not taken into account. Given the BRFCMP a problem succeeding the retrofitting planning optimization that is often multi-objective [20], two objectives are introduced: maximizing the long-term aggregate energy saving and maximizing the economy of the project over a finite period of time. The objective functions are formulated according to a series of performance measures, and a weighted sum method is employed as the solution to the multi-objective optimization problem, as the weighted sum method provides a basic and easy-to-use approach that gives an acceptable approximation of one's preference function when the preference information is not too complex [21]. One of the key issues of applying maintenance planning optimization method is the characterization of the population deterioration of the homogeneous item groups, where the building energy optimization brings in the reliability engineering studies, where deterministic or stochastic models of facilities reliability can be found. A series of common failure distributions, reliability and hazard rate functions for facilities with various reliability characteristics is provided by [22], according to which the population degradation of various types of retrofitted items, e.g., the nonrepairable products and the repairable products, can be characterized. It is expected that the research progress in the reliability engineering area will facilitate the advance of building energy optimization studies, and vice versa. The characterization of item group population degradations can also be found in some existing studies in the Clean Development Mechanism (CDM) environment, which consider the population degradation either by a simplified linear assumption [23] or an experimental data fitting [24]. The optimization model formulation is the first part of the main work in this paper.

²MILITARY STANDARD: DEFINITIONS OF TERMS FOR RELIABILITY AND MAINTAINABILITY, 1981, http://www.everyspec.com/MIL-STD/MIL-STD-0700-0799/MIL-STD-721C_1040/

Another issue of the BRFCMP problem is the adverse impact of the uncertainties during operation. In practice, the measurements of the retrofitted building, namely the retrofitted plant, are usually done by the sampling and estimation methods. As a result, sampling errors can be introduced into the optimization model. The accuracy of the population degradation models is also limited. Furthermore, the influence of human behaviors, environmental factors and the stochastic reliability performances of the products are involved in practical cases, which all result in performance uncertainties. The existing optimization models lack the ability to address the problem of the uncertainties. Given the dynamic nature of the aggregate performances of the retrofitted plant, the control system approach, which is an almost unexplored perspective to solve the building energy efficiency retrofitting optimization problems with uncertainties, can be brought in. Several studies have been conducted to attempt to employ the control system approaches to solve the maintenance optimization problem for reliability purposes [25, 26, 27, whereas most of which focused on single-unit or two-component machines. The main concern of these existing studies is the control of single machine or unit. Recently, Ye et al. [15] investigates employing the control system approach to the optimal maintenance plan for energy efficiency lighting projects. This research focuses on the replacements of the malfunctioning lamps, where the population deterioration of the lamp groups, rather than the performances of individual lamps, are taken as the plant of the control system. Wang et al. [13] also employ the control system perspective to investigate the maintenance plan optimization in a building retrofitting context. Similarly, for the BRFCMP problem, the totality of the retrofitted item groups are taken to be our control plant, where the population of available items from each homogeneous item group as the state variables of the system. The corrective maintenance actions, i.e., the respective numbers of maintained items from the homogeneous item groups, are taken as the control inputs. The measured output of the system can be the aggregate energy savings, capital investment or other performance measures in different cases. Accordingly, the population deterioration can represent the internal dynamics of the state variables. The aforementioned uncertainty factors can be described as disturbances on the state variables or measured output. For simplicity, two further assumptions are made, namely: the disturbances of the system are generally considered as a random noise on state variables; the sampling errors are simplified as a random noise on the measured output. In this way, the BRFCMP optimization problem considering uncertainties is cast into an optimal control problem, where the optimization objectives are transformed into the control objectives. A Model Predictive Control (MPC) based approach is employed to solve the BRFCMP optimal control problem. The MPC approach finds the optimal control inputs by predicting the future based on the present state of the system, and is inherently robust against disturbances. It has become one of the most widely used control algorithms to solve many industrial control problems in the fields of engineering, food processing, automotive applications, and aerospace applications [28], demand-side management [29] and dispatch of power generation [30]. The control system approach is the second part of our main work. As a case study, a practical building retrofitting project is used to test and verify the feasibility of the presented optimization and control approaches.

The remainder of the paper consists of five sections. Section 2 gives the formulation of the multi-objective BR-FCMP optimization problem. Section 3 introduces the weighted sum method as a solution to the BRFCMP problem. Section 4 introduces the control system approach to the BRFCMP optimal control problem when considering uncertainties, and the MPC approach as a solution. Section 5 provides the details of the case study and the simulation results and analysis. Section 6 draws conclusion and discusses future research.

2. Multi-objective BRFCMP Problem

2.1. Variables definitions

Assumed there are I groups of homogeneous retrofitted items involved in a building energy efficiency retrofitting project. Let $t_k = kS, k = 0, 1, 2, ..., T$ denote the sampling instants over the a finite decision horizon [0, TS), namely the sustainability period, where S indicates the sampling interval. The population of the item group i at time t_k is represented by $x_i(t_k)$, and accordingly the system state can be described:

$$\mathbf{x}(t_k) = (x_1(t_k), x_2(t_k), ..., x_I(t_k))^T.$$
 (1)

 $\mathbf{x}(t_0) = \mathbf{x}_0$ indicates the initial state of the retrofits that is decided by the retrofitting plan. In practice, $x_i(t_k)$ with k>0 are obtained by the inspection at t_k . For the energy conservatism, $x_i(t_k)$ is considered the state over interval $[t_{k-1},t_k)$. The maintenance action for item group i is decided based on the inspection result $x_i(t_k)$ and implemented over interval $[t_k,t_{k+1})$. Let $u_i(t_k)$ denote the maintenance action applied to item group i over interval $[t_k,t_{k+1})$. The aggregate population level maintenance plan at t_k can be represented by:

$$\mathbf{u}(t_k) = (u_1(t_k), u_2(t_k), ..., u_I(t_k))^T.$$
(2)

For the convenience of further derivation, \mathbf{x} and \mathbf{u} are employed to represent the system states and maintenance actions over the sustainability period, where \mathbf{u} are the decision variables in the BRFCMP problem. The system state at the next sampling instant can thus be estimated:

$$x_i(t_{k+1}) = D_i(x_i(t_k)) + u_i(t_k).$$
 (3)

where $D_i(\cdot)$ denotes the population degradation of the item group i over $[t_k, t_{k+1})$. Given $D_i(\cdot)$ with i = 1, 2, ..., I

known a priori and taking advantage of equation (3), it is possible to find a series of maintenance actions, i.e., the optimal maintenance plan, that maximizes the selected performance measures. In our model, two types of performance measures, the energy performance indicator and the economy performance indicator are selected. The formulations are given in the next section.

2.2. Performance measures formulation

The performance measures are computed by estimating the system states over the sustainability period. The energy performance indicator in this model is the long-term energy saving i.e., the aggregate energy saving over the sustainability period. The economy performance indicator is the Internal Rate of Return (IRR). To obtain the two performance measures, a series of performance characteristics of the involved retrofitted items are defined and utilized. Given the energy saving of the retrofitted item a performance characteristics known a priori, let $\mathbf{a}(t_k)$ denote the saving amount over interval,

$$\mathbf{a}(t_k) = (a_1(t_k), a_2(t_k), ..., a_I(t_k))^T, \tag{4}$$

where $a_i(t_k)$ denotes the energy saving that an item from item group i contributes over $[t_{k-1}, t_k)$. Similarly, the cost savings can be represented,

$$\mathbf{b}(t_k) = (b_1(t_k), b_2(t_k), ..., b_I(t_k))^T,$$
(5)

and the maintenance costs per item at instant t_k are given:

$$\mathbf{c}(t_k) = (c_1(t_k), c_2(t_k), ..., c_I(t_k))^T.$$
(6)

Taking advantage of these characteristics, the aggregate long-term energy saving can be obtained:

$$ES|_{all} = \sum_{k=1}^{T} ES(\mathbf{x}(t_k), t_k) = \sum_{k=1}^{T} \sum_{i=1}^{I} a_i(t_k) x_i(t_k), \quad (7)$$

and the aggregate cost saving:

$$CS|_{all} = \sum_{k=1}^{T} B(\mathbf{x}(t_k), t_k) = \sum_{k=1}^{T} \sum_{i=1}^{I} b_i(t_k) x_i(t_k),$$
 (8)

where $B(\mathbf{x}(t_k), t_k)$ denotes the aggregate cost saving over interval $[t_{k-1}, t_k)$. The cost saving in our model is considered the main income of the retrofitting project, thereby $B(\mathbf{x}(t_k), t_k)$ also represents the cash inflow over $[t_{k-1}, t_k)$, from the economy point of view. The cash outflow is the expenditure of the maintenance actions given as following,

$$h|_{all} = h_0 + \sum_{k=1}^{T} h(\mathbf{u}(t_k), t_k) = h_0 + \sum_{k=1}^{T} \sum_{i=1}^{I} c_i(t_k) u_i(t_{k-1}),$$
(9)

where h_0 denotes the initial investment for the implementation of the retrofitting plan and $h(\mathbf{u}(t_k), t_k)$ the cash

outflow over $[t_{k-1}, t_k)$. Taking advantage of the time-dependent cash inflow and outflow, the IRR can be obtained. The calculation of IRR is related to the Net Present Value (NPV) which is computed as following,

$$NPV = \sum_{k=1}^{T} \frac{B(\mathbf{x}(t_k), t_k) - h(\mathbf{u}(t_k), t_k)}{(1+d)^{n-1}} - h_0, \qquad (10)$$

where d is the selected discount rate. n=1,2,... represents that the sampling instant t_k lies within the n-th year after the implementation of the retrofitting project. IRR, denoted by $d_R|_T$, refers to the discount rate that makes the NPV over [0,TS) equal to 0. A larger IRR implies the better economy of a project.

As mentioned in the previous section, the nature of the BRFCMP problem is a multi-objective optimization problem. Taking advantage of the aforementioned performance measures, two objective functions are formulated:

$$\begin{cases} f_e(\mathbf{x}, \mathbf{u}) = \frac{ES|_{all}}{\alpha}, \\ f_r(\mathbf{x}, \mathbf{u}) = IRR, \end{cases}$$
 (11)

where ${\bf x}$ and ${\bf u}$ are noted to emphasize the dynamic nature of the aggregated performances. α denotes the targeted energy saving amount that is contracted before the retrofitting. A series of constraints are involved in the BR-FCMP problem, including the energy performance agreement, the maintenance budget limit, the payback period limit, and the pre-decided maintenance time schedule. The constraints are represented by the following equation,

$$\begin{cases}
ES|_{all} \ge \alpha, \\
\sum_{k=1}^{T} h(t_k) \le \beta, \\
NPV|_0^{T_p} \ge 0, \\
u_i(t_k) = 0, \ k \notin Q,
\end{cases} \tag{12}$$

where β denotes the overall maintenance budget limit. $NPV|_0^{T_p}$ denotes the NPV computed over $[0,T_pS)$ where T_p represents the maximum acceptable payback period. Q represents the maintenance time schedule, where $Q = \{k_1, k_2, ...\}$ denotes the set of time instants over [0, TS). After an arbitrary time instant from Q, the maintenance actions is planned and implemented over the following sampling interval. Accordingly, if $k \notin Q$, $u_i(t_k) = 0$.

2.3. Population degradation formulation

In the current BRFCMP problem, two types of population degradation models are employed to characterize the population dynamics of different item groups [13]. One corresponding to nonrepairable items such as the lighting facilities and motion sensors, and the other to repairable items such as heat pumps. Equations (13) and (14) describe these degradation models, respectively:

$$D_i(x_i(t_k)) = b_i c_i x_i(t_k)^2 / x_i - b_i x_i(t_k) + x_i(t_k),$$
 (13)

$$D_i(x_i(t_k)) = x_i(t_k)e^{-\zeta_i}, \tag{14}$$

where the coefficients b, c, ζ are estimated by the Mean Time To Failure (MTTF) of the nonrepairable items and the Mean Time Between Failures (MTBF) of the repairable items. The population degradation model for nonrepairable products as described in equation (13) is taken from [24]. Let L_i denote the MTTF, i.e., the rated lifetime of the item from item group x_i . The general form of the time-domain degradation model $P_i(t) = (c_i + e^{b_i t - L_i})^{-1}$ can be found in [24], where $P_i(t)$ is the proportion of surviving items in the whole group. For this degradation model, given L_i is known , b_i and c_i can be obtained by solving out the following equations:

$$\begin{cases}
P_i(0) = 1, \\
P_i(L_i) = 0.5,
\end{cases}$$
(15)

where b_i and c_i can also be identified from the experimental data. Equation (14) describes the degradation model for repairable products. As the rated lifetime is usually several times longer than the MTBF for such items, according to the reliability bathtub curve, the failure rate of such items is an approximately low constant before the end of the lifetime. Therefore an exponential degradation model is adopted from [22] in equation (14). Let θ_i denote the MTBF of the facility, ζ_i is then obtained from the following equation:

$$\zeta_i = (\theta_i)^{-1}. \tag{16}$$

Both equations (13) and (14) are actually statistical models considered as first-order Markov processes in the current BRFCMP problem. Once again for simplicity of discussion, another important assumption is made: the replaced or repaired items and the malfunctioning items are from the same respective homogeneous classes, they thus share the same failure pattern. The MTTF and MTBF are known a priori according to the model. Such information can be obtained from the equipment producers or the historical performances of the items.

3. The Weighted Sum Solution to the BRFCMP Problem

3.1. The weighted sum of objective functions

At the current stage, a weighted sum method that has been utilized in similar studies [10, 11] is employed to solve the multi-objective BRFCMP problem. Taking advantage of equation (11) and (12), the multi-objective optimization problem is translated into the minimization of a utility function, which is the weighted sum of the objective functions associated with stationary penalty functions corresponding to the constraints, given by equation (17):

$$J = -\lambda_1 f_e(\mathbf{x}, \mathbf{u}) - \lambda_2 f_r(\mathbf{x}, \mathbf{u}) + \omega \sum_{n=1}^{3} \max(0, P_n), \quad (17)$$

where λ_1 , λ_2 are positive constants subject to $\lambda_1 + \lambda_2 = 1$. ω is a large positive integer. P_n are the penalty functions defined as following:

$$P_{n} = \begin{cases} \alpha - ES|_{all}, & n = 1, \\ \sum_{k=1}^{T} h(t_{k}) - \beta, & n = 2, \\ -NPV|_{0}^{T_{p}}, & n = 3 \end{cases}$$
 (18)

The BRFCMP optimization problem is actually to find a \mathbf{u} to minimize the utility function in equation (17) and (18). The DE algorithm is employed to solve this minimization problem over the interval [0, TS). The details of the DE algorithm is introduced in the following section.

3.2. A Differential Evolution algorithm solution

In existing studies, DE and other evolutionary algorithms are widely and successfully applied to solve the energy optimization problems. The DE algorithm is easy to implement, and it is verified to be effective for a similar optimization problem in [11]. Given the non-analytic objective function in equation (11), the DE algorithm is more feasible than conventional NLP algorithm. In DE al-

Algorithm 1 Pseudocode of the DE algorithm

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Definition:
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NP: the population size;

D: dimension of the problem;

X: the decision matrix with the size of NP*D;

F: the learning rate;

J: the function value vector with the size of np*1;

Mg: the maximum number of generations for stopping criterion.

```
1: BEGIN
 2: Set CR = 0.5; F = 0.5; A = \emptyset;
 3: Create a random initial population \{X_{i,0}|i=1,2,...,NP\};
 4: while q = 1 to Mq do
         while i = 1 \text{ to } NP \text{ do}
              Randomly select X_{r1,q} \neq X_{i,q} from current population
 6:
    \mathbf{P};
 7:
             Randomly select X_{r2,g} \neq X_{i,g} from current population
     \mathbf{P};
              Randomly select X_{best,q}^p as one of the 10% best vectors
 8:
     from P;
              V_{i,g} = X_{i,g} + F \cdot (X_{best,g}^p - X_{i,g}) + F \cdot (X_{r1,g} - X_{r2,g});
Generate j_{rand} = randint(1, D);
 9:
10:
              while j = 1 \text{ to } D \text{ do}
11:
                  if j = j_{rand} or rand(0, 1) < CR then
12:
                      U_{j,i,g} = U_{j,i,g};
13:
14:
                      U_{j,i,g} = X_{j,i,g};
15:
16:
                  end if
17:
              if J(U_{i,g}) \leq J(X_{i,g}) then
18:
19:
                  X_{i,g+1} = U_{i,g};
20:
                  X_{i,g+1} = X_{i,g}
21:
22:
              end if
23:
         end while
```

gorithm, a set of candidate solutions, namely individuals,

24: end while

are adopted to represent the possible values of the decision variables **u**. These individuals are moved around in the search-space which is regulated by the boundary of the problem. By implementing a series of mathematical operations including *Mutation*, *Corssover* and *Selection* (see [31] for further details), a satisfactory solution can hopefully, although not guaranteed, be discovered. The pseudo-code of the employed DE algorithm is given in Algorithm 1.

In Algorithm 1, D denotes the dimension of the problem, G refers to the maximum step of iteration and NP represents the population size. The CR and F are the crossover probability and the learning rate respectively. $X_{best,g}^p$ is selected from the 10% best vectors of the current population. $X_{r1.g} - X_{r2.g}$ is a differential vector for the mutation operation, where $X_{r1.g}, X_{r2.g}$ are randomly selected from the current population. The mutation vector $V_{i,g}$ and the child vector $U_{i,g}$ are accordingly generated. $J(\cdot)$ denotes the utility function in equation (17). After Mg steps, the individual with smallest utility function value is the output of the DE algorithm.

4. The Control System Approach to the BRFCMP Problem Considering Uncertainties

The aforementioned optimization model excludes the uncertainties in the system, e.g., the model uncertainties and the sampling uncertainties. As a result, such uncertainties can deliver adverse impact to the maintenance plan, causing further deterioration of the performances. Therefore, we cast the BRFCMP problem into an optimal control problem, based on which the control system approach is introduced for the robustness of the performances against uncertainties. The formulation of the optimal control problem is given in the following section.

4.1. The control system framework formulation

Taking advantage of the system states and decision variable in equation (1)-(2), the control system formulation that describes the system dynamics can be obtained via rewriting equation (3):

$$\begin{cases} \mathbf{x}(t_{k+1}) = \mathbf{D}(\mathbf{x}(t_k)) + \mathbf{u}(t_k) + \mathbf{w}(t_k), \\ \mathbf{y}(t_k) = ES(\mathbf{x}(t_k), t_k) + \mathbf{d}(t_k). \end{cases}$$
(19)

The decision variable $\mathbf{u}(t_k)$ hereby represents the control input of the control system. The measured output $\mathbf{y}(t_k)$ is the measurement of the aggregate energy saving during the sampling period $[t_{k-1}, t_k)$. In practice, $\mathbf{y}(t_k)$ is measured according to the sampling of the item group populations at instant t_k and equation (7). The accuracy and confidential level of the measurement is determined by the sampling size [32]. $\mathbf{w}(t_k)$ and $\mathbf{d}(t_k)$ denote the disturbances to the system states and outputs. Such disturbances are employed to indicate the impact of the uncertainties. The objective of the BRFCMP optimal control problem is thus to find an optimal control law \mathbf{u} that minimizes the performance index in equation (17) for the control system in

equation (19), subject to the constraints in equation (12). An MPC approach is employed to solve the BRFCMP optimal control problem. The details of the MPC algorithm are given in the following section.

4.2. The MPC approach to the BRFCMP optimal control problem

In MPC approaches, an open-loop optimal control problem is repeatedly solved over a finite horizon, namely the control horizon, according to the plant performance prediction. The obtained optimal open-loop control is then used to generate the optimal control input for the problem to be solved, with which the state variables executed over the next finite horizon are estimated. As the optimal controller over the next finite horizon is actually a function of the system state from the previous control step, a closedloop feedback is thus obtained. Given the finite decision horizon in our model, such MPC algorithm is employed: let t_m denote the current instant, consider a control horizon that covers $[t_m, TS)$, i.e., the rest of the sustainability period. A mathematical transformation of the BRFCMP optimal control problem is applied, and the open-loop optimal control problem over the control horizon $[t_m, TS)$ is accordingly defined as the following minimization problem:

$$min J' = -\lambda_1 f'_e|_m(\mathbf{x}, \mathbf{u}) - \lambda_2 f'_r|_m(\mathbf{x}, \mathbf{u}), \qquad (20)$$

where $f'_r|_m(\mathbf{x}, \mathbf{u})$ indicates the discount rate that makes $NPV'|_m = 0$, with

$$\begin{cases}
f'_{e}|_{m}(\mathbf{x}, \mathbf{u}) = \sum_{k=1}^{m} ES(\mathbf{x}(t_{k}), t_{k}) + \sum_{k=m+1}^{T} ES(\mathbf{x}|_{m}(t_{k}), t_{k}), \\
NPV'|_{m} = \sum_{k=1}^{m} \frac{B(\mathbf{x}(t_{k}), t_{k}) - h(\mathbf{u}(t_{k}), t_{k})}{(1+d)^{n-1}} \\
+ \sum_{k=m+1}^{T} \frac{B(\mathbf{x}|_{m}(t_{k}), t_{k}) - h(\mathbf{u}|_{m}(t_{k}), t_{k})}{(1+d)^{n-1}} - h_{0},
\end{cases}$$
(21)

subject to

$$\begin{cases}
\sum_{k=1}^{m} ES(\mathbf{x}(t_k), t_k) + \sum_{k=m+1}^{T} ES(\mathbf{x}|_m(t_k), t_k) \ge \alpha, \\
\sum_{k=1}^{m} h(\mathbf{u}(t_k), t_k) + \sum_{k=m+1}^{T} h(\mathbf{u}|_m(t_k), t_k) \le \beta, \\
NPV|_0^{T_p} \ge 0, \\
u_i(t_k) = 0, \ k \notin Q,
\end{cases} (22)$$

where $\mathbf{x}|_m(t_k)$ denotes the predictive system states and $\mathbf{u}|_m(t_k)$ the scheduled control inputs after t_m . The employed MPC approach takes the existing performances before t_m into account to estimate the long-term performances of the project, based on which the open-loop control problem over the control horizon is solved. Given the non-analytic component in equation (20), we continue

to employ the aforementioned DE algorithm to solve the open-loop control problem.

By applying DE algorithm, a series of optimal control inputs are obtained, represented by $\mathbf{u}'|_m = {\{\mathbf{u}'|_m(t_k): k = \}}$ m, m+1, ..., T-1. Only the optimal control action over the first sampling period $[t_m, t_{m+1})$ is applied, represented by $\hat{\mathbf{u}}|_m = {\mathbf{u}'|_m(t_m)} = {\hat{\mathbf{u}}|_m(\mathbf{x}(t_m), t_m)}$, where the last equation is to emphasize the functional dependence of the optimal control on the initial state $\mathbf{x}(t_m)$ of the MPC formulation in equations (20)-(22). After $\hat{\mathbf{u}}|_m$ is applied, the predictive state $\mathbf{x}|_{m}(t_{m+1})$ can be obtained. Due to the influences of uncertainties that is represented by disturbance $\mathbf{w}(t_m)$, the actual state $\hat{\mathbf{x}}(t_{m+1}) = \mathbf{x}|_m(t_{m+1}) + \mathbf{w}(t_m)$. In practice, $\hat{\mathbf{x}}(t_{m+1})$ must be obtained by the inspection. $\hat{\mathbf{x}}(t_{m+1})$ then becomes the initial condition of the MPC formulation over the next control horizon $[t_{m+1}, TS)$. When $m \notin Q$, the control input $\mathbf{u}(t_m) = 0$ is implemented as a solution. These take place consecutively over the sustainability period to obtain the optimal control inputs $\hat{\mathbf{u}}$. The measured output $\mathbf{y}(t_k)$ is obtained by equation (19). $\mathbf{x}(t_{k+1})$ is also applied as the initial state for the open-loop optimal control problem over the next control horizon. In summary, the following MPC algorithm can be formulated:

4.3. The MPC Algorithm

Initialization: Let initial state $\mathbf{x}(t_0) = \mathbf{x}_0$ and m = 0.

- (i) Compute the open-loop optimal solution $\{\mathbf{u}'|_m(t_k)\}$ of the problem formulation (20)-(22), where k=m,m+1,...,T-1.
- (ii) The MPC controller $\hat{\mathbf{u}}|_m = \{\mathbf{u}'|_m(t_m)\}$ is applied after the sampling instant t_m . The remains of the open loop optimal solution $\{\mathbf{u}'|_m(t_k): k=m+1,...,T-1\}$ are discarded. The predictive state $\mathbf{x}|_m(t_{m+1})$ is then obtained according to:

$$\mathbf{x}|_m(t_{m+1}) = \mathbf{D}(\mathbf{x}(t_m)) + \mathbf{u}'|_m.$$

Given the impact of disturbance $\mathbf{w}(t_m)$, the actual state $\hat{\mathbf{x}}(t_{m+1}) = \mathbf{x}|_m(t_{m+1}) + \mathbf{w}(t_m)$.

(iii) Let $\hat{\mathbf{x}}(t_{m+1})$ be the initial state for the next predictive horizon, m := m + 1 and go back to step (i).

According to the constraint (22), $\mathbf{u}(t_m) = 0$ when $m \notin Q$, where step (i) is skipped and $\hat{\mathbf{x}}(t_{m+1}) = \mathbf{D}(\mathbf{x}(t_m)) + \mathbf{w}(t_m)$. The above MPC algorithm will go over the sustainability period to solve out the optimal control strategy.

5. Simulation and Verification

5.1. Case study

To verify the effectiveness of the proposed approaches, a small retrofitting project for a government office building is introduced as our case study, where the investor and the stakeholders are the same party. The financial benefit of the project lies in the cost saving from the saved energy as well as from the investor taking into account the maintenance cost as part of the investment. Therefore, the

overall investment is the summation of the initial installation cost and the maintenance cost during operation. The cost saving from the energy saving is considered the main income of the project. The calculation of NPV takes into account these two cash flows. A full maintenance strategy, i.e., all the malfunctioning items are repaired during the maintenance, is introduced as the baseline performance.

The specifications of the retrofits are illustrated in Table 1. There are 5 categories of retrofits involved, including the motion sensors, the 20W retrofit Compact Fluorescent Lamps (CFL), the 23 inch LCD monitors, the 3kW heat-pumps and the 23L microwave ovens. The retrofitted items corresponding to the same homogeneous class are from one item group. The population degradation of the item groups of motion sensors, 20W CFL and 23 inch LCD monitors are corresponding to the population degradation model in equation (13), and item groups of heat-pumps and microwave ovens the degradation model in equation (14). Table 1 indicates different performance characteristics of the retrofits, including the prices to apply the retrofit to one item, the energy savings and cost savings per retrofitted item, the corrective maintenance costs to replace or repair one retrofitted item. To be noticed, the illustrated energy savings and cost savings are the annual average values. Table 1 also indicates the type of the population degradation model that corresponds to the retrofit, and the quantities of the retrofitted item involved in this energy efficiency retrofitting project, i.e., the initial state \mathbf{x}_0 . The parameters of the corresponding population deterioration models are given in Table 2.

In the present retrofitting project, the sustainability period is 10 years, which is an average value in similar An inspection will be performed at the end of every six months, implying that our decision horizon $k = \{0, 1, 2, ..., 24\}$. As above-mentioned, for the sake of the energy conservatism, the inspection result is considered to be the state of the item groups during the prior sampling period [23]. The maintenance actions are scheduled to take place at the end of each year except the last year, i.e., every two sampling periods. Therefore, Q = $\{2,4,6,...,22\}$. The initial investment to implement the retrofitting project is \$176,650. The baseline energy consumption in this year is 4,397,572 kWh, and the targeted energy saving amount that is contracted to be achieved by this retrofitting project is 15% of the baseline consumption in 10 years, i.e., 6,596,358 kWh. The desired payback period is 3 years.

Table 2: Parameters for the corresponding population deteriors and also

rioration models					
Retrofits	type	MTTF	b_i	c_i	ζ_i
		/MTBF			
Motion sensor	I	1.13	1.299	0.895	N/A
20W retrofit CFL	I	1.49	1.2165	0.9494	N/A
23 inch LCD Monitor	I	2.71	1.115	0.996	N/A
3kW Heat-pumps	II	2.08	N/A	N/A	0.24
23L Microwave oven	II	1.98	N/A	N/A	0.25

Table 1: Characteristics of involved retrofits									
Pre-retrofitting	Retrofits	Type	Quantities	Unit Price	Unit Energy	Unit Cost	Corrective		
			(\$)	Saving (kWh)	Saving (\$)	Cost (\$)			
No motion sensor	Motion sensor	I	123	196	1140	121.1	196		
Halogen Classic 75W	20W retrofit CFL	I	408	14	105.6	11.9	14		
Old CRT Monitor	23 inch LCD Monitor	I	250	150	87.8	10.8	150		
Electrical geyser	3kW Heat-pumps	II	85	1250	8640	973.3	201		
Inefficient oven	23L Microwave oven	II	35	88	72	8.3	45		

Our simulation involves five different cases, including the no maintenance case that reveals the adverse impact of the deterioration, the full maintenance case that is considered the baseline performance, and three optimal cases that employ different weights. The three optimal cases are: the *Optimal balance* case with $\lambda_1 = 0.5$ and $\lambda_2 = 0.5$, the Energy prior case with $\lambda_1 = 1.0$ and $\lambda_2 = 0$, and the Economy prior case with $\lambda_1 = 0$ and $\lambda_2 = 1.0$. The $Energy\ prior$ case and $Economy\ prior$ case actually cast the BRFCMP problem into a constrained single objective optimization problem. For the optimal cases, there are two different maintenance budget limits, denoted by tight and sufficient. The tight budget is \$125,000 over the sustainability period and the sufficient budget \$200,000. Performances with different objectives under different budget limits will be given to illustrate the effectiveness of the optimal maintenance strategy and our optimization model. The disturbances are represented by a random noise in our simulation. As system output feedback is employed in the applied MPC approach, this noise is added as a total on the system states. The range of the noise is $\pm 0.1\mathbf{x}(t_k)$.

5.2. Illustrative results and analysis

The performances of the maintenance plan without considering disturbances are illustrated in Table 3. Table 3 illustrates the following information: the aggregate energy saving (kWh) and the percentage saving in comparison with the energy baseline, the IRR, the payback period (years), the NPV (\$), the maintenance cost (\$) and the total investment of the project (\$). First of all, the performances where no maintenances are applied, are unacceptable. The no maintenance performances are illustrated to manifest the important role of maintenance. Then the full maintenance performances are given as the baseline performances. The performances of the Optimal balance case, Energy prior case and Economy prior case with different budget limits are illustrated following the full maintenance performances. An interesting result can be observed from the cases with sufficient budget. In the Energy prior case, the solutions are same with the full maintenance solution. In the Optimal balance case, the IRR is slightly improved with a little lose of the energy saving. Even in the Economy prior case, there are no significant differences. Such performances imply that, when budget is sufficient, the full maintenance strategy is one of the best choices. However, in practice, the maintenance budget can often be insufficient to support full maintenance strategy. In cases with tight budget limits, maintenance plan optimization is

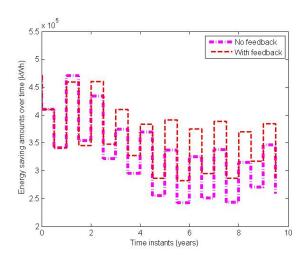


Figure 1: Energy performances of the maintenance plans with and without feedback in *Optimal Balance* case with sufficient budget.

necessary. In our case study, the obtained optimal maintenance plans in the three optimal cases with tight budget are the same one. This reveals that the two objectives employed in our optimization model are not very contradictory. The energy savings are well preserved while the maintenance costs are significantly reduced in comparison with the full maintenance case. The performances of the optimal maintenance plan with tight budget limits verify the effectiveness of our optimization method.

Table 4 illustrates the performances of the maintenance plan under the impact of uncertainties, where the Balance open-loop, Energy open-loop and Economy open-loop represent that, in these cases, the open-loop maintenance plans that are obtained without considering uncertainties are applied when there actually are uncertainties during operation. In contrary to the open-loop cases, Balance with feedback, Energy with feedback and Economy with feedback represent that the applied maintenance plan are obtained via the control system approach with state feedback. The random noises in the corresponding open-loop cases and with feedback cases are the same for a clear comparison of the performances. Generally, from Table 4, the performances in with feedback cases outperform the ones in open-loop cases, showing that the proposed control system approach is robust against uncertainties. The results can prove the effectiveness of the control approach to reduce the adverse impact of uncertainties during operation.

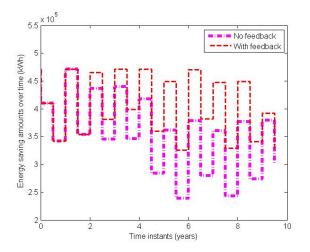
Figs. 1-6 illustrate the energy and economy perfor-

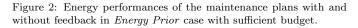
Table 3: Comparison of the performances of cases without disturbances

Cases	Budget	Energy	Percentage	IRR	Payback	NPV	Maintenance	Total
	limit (\$)	savings (kWh)	saved		period (years)	(\$)	cost (\$)	investment (\$)
$No\ maintenace$	N/A	2,065,742	4.69%	0.69%	N/A	-18,755.68	0	176,650
$Full\ maintenace$	N/A	8,830,340	20.08%	40.54%	2.71	$314,\!556.3$	179,223	355,873
$Optimal\ balance$	125,000	8,219,041	18.69%	40.56%	2.7	$310,\!439.4$	124,963	301,613
$Optimal\ balance$	200,000	8,827,172	20.07%	40.65%	2.7	$315,\!168.6$	178,233	354,883
$Energy \ prior$	125,000	8,219,041	18.69%	40.56%	2.7	$310,\!439.4$	124,963	301,613
$Energy \ prior$	200,000	8,830,340	20.08%	40.54%	2.71	$314,\!556.3$	179,223	355,873
$Economy\ prior$	125,000	8,219,041	18.69%	40.56%	2.7	$310,\!439.4$	124,963	301,613
Economy prior	200,000	8,737,285	19.87%	40.79%	2.69	318,225.4	162,328	338,978

Table 4: Comparison of the performances of cases including disturbances

Cases	Budget	Energy	Percentage	IRR	Payback	NPV	Maintenance	Total
	limit (\$)	savings (kWh)	saved		period (years)	(\$)	cost (\$)	investment (\$)
Balance open-loop	125,000	7,685,571	17.47%	38.43%	2.65	274,193.7	125,361	302,011
$Balance\ with\ feedback$	125,000	7,878,323	17.92%	38.78%	2.81	287,191.1	$125,\!179$	301,829
Balance open-loop	200,000	8,051,623	18.31%	39.24%	2.64	272,539.8	180,795	357,445
$Balance\ with\ feedback$	200,000	8,810,725	20.04%	40.97%	2.59	307,000.8	196,249	372,899
Energy open-loop	125,000	7,167,616	16.30%	33.54%	3.16	232,309.7	125,585	176,650
Energy with feedback	125,000	7,467,160	16.98%	35.29%	3.05	254,924	124,999	301,649
Energy open-loop	200,000	7,719,358	17.55%	37.33%	2.68	249,480	180,144	356,794
Energy with feedback	200,000	8,674,394	19.72%	39.63%	2.58	293,014.1	199,996	376,646
Economy open-loop	125,000	7,063,163	16.06%	36.69%	2.77	258,692	80,259	256,909
Economy with feedback	125,000	7,512,802	17.08%	37.60%	2.81	277,871.4	$94,\!552$	271,202
Economy open-loop	200,000	7,368,024	16.75%	36.04%	2.73	233,466.1	165,144	341,794
Economy with feedback	200,000	7,640,285	17.37%	39.55%	2.71	282,598.7	112,158	288,808





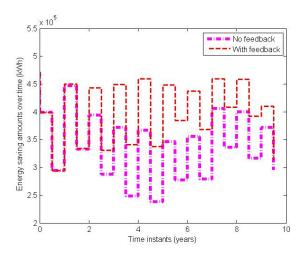


Figure 3: Energy performances of the maintenance plans with and without feedback in $Economy\ Prior$ case with sufficient budget.

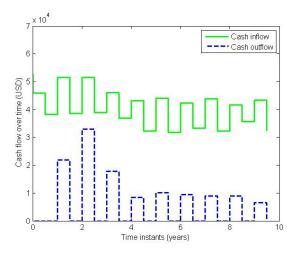


Figure 4: Cash flows of the maintenance plans with feedback in *Optimal Balance* case with sufficient budget.

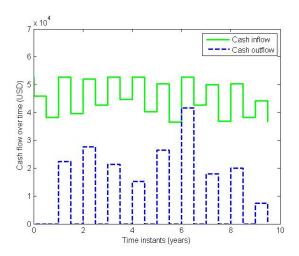


Figure 5: Cash flows of the maintenance plans with feedback in $Economy\ Prior$ case with sufficient budget.

mances of the Balance with feedback, Balance with feedback and Economy with feedback cases, where the performances are influenced by uncertainties. The energy savings and cash flows during each sampling interval in the three cases are illustrated. In Figs 1-3, the thin dashed lines represent the performances in the with feedback cases. In contrary to the maintenance plan with feedback, the thick dash-dot lines represent the energy savings from the open-loop cases, showing that the open-loop maintenance plan lose more energy savings with uncertainties during operation. In Figs. 4-6, the cash flows from the with feedback cases are chosen to be illustrated. The solid lines represent the cash inflows and the dashed line the cash outflows which indicate the level of the corrective maintenance efforts during operation.

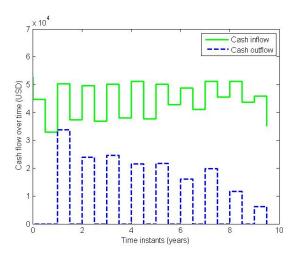


Figure 6: Cash flows of the maintenance plans with feedback in $Optimal\ Balance$ case with sufficient budget.

6. Conclusion

The main work of this paper is to investigate the important role of maintenance to the building energy efficiency in the building energy efficiency retrofitting context, and the potential of improving the performances of an energy efficiency retrofitting project by the maintenance plan optimization. A subproblem namely the Building Retrofitted Facilities Corrective Maintenance Planning (BRFCMP) is adopted as our subject investigated at the current stage. An aggregate population level optimization model is proposed to address the BRFCMP problem without taking into account the uncertainties. Given the BRFCMP a problem succeeding the retrofitting planning optimization that is often multi-objective, two objective are introduced in the optimization model: maximizing the long-term aggregate energy saving over the sustainability period and maximizing the Internal Rate of Return of the project. The objective functions are formulated according to a series of performance measures, and a weighted sum method is employed as the solution to the multi-objective optimization problem. Moreover, when taking into account the uncertainties that can deliver adverse impact to the actual performances of the project, the control system approach as an unexplored perspective is introduced, where the optimization objectives are transformed into the control objectives. A Model Predictive Control (MPC) based approach is employed to solve the BRFCMP optimal control problem.

A practical building retrofitting project is used to test and verify the feasibility of the presented optimization and control approaches, and the simulation results reveal that, when the budget is sufficient, the full maintenance strategy that have all the malfunctioning items repaired becomes the best option. However, the proposed optimization method can preserve the performances while the cost is significantly reduced. Taking advantage of the proposed method, the objectives can be well achieved with tight maintenance budget limit. Furthermore, the performances of the maintenance plans with feedback outperform the open-loop strategy performances, showing robustness against uncertainties. The effectiveness of the optimization and control approaches is thus verified.

Several topics call for further studies on the investigated topic: the retrofitting planning optimization can be combined with the maintenance planning optimization; the robustness of the classification of the retrofitted facilities remains an open problem; the uncertainties factors are possible to be recognized and taken into account in the optimization; the operating schedule optimization can be introduced to further improve the building energy performance. Relaxing the assumption that maintenance actions are kept within the homogeneous class of facilities may call for new modelling and control of systems of varying dimensions.

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