

# Regime Switching Model of US Crude Oil and Stock Market Prices: 1859 to 2013\*

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## Abstract:

This paper examines the relationship between US crude oil and stock market prices, using a Markov-Switching vector error-correction model and a monthly data set from 1859 to 2013. The sample covers the entire modern era of the petroleum industry, which typically begins with the first drilled oil well in Titusville, Pennsylvania in 1858. We estimate a two-regime model that divides the sample into high- and low-volatility regimes based on the variance-covariance matrix of the oil and stock prices. We find that the high-volatility regime more frequently exists prior to the Great Depression and after the 1973 oil price shock caused by the Organization of Petroleum Exporting Countries. The low-volatility regime occurs more frequently when the oil markets fell largely under the control of the major international oil companies from the end of the Great Depression to the first oil price shock in 1973. Using the National Bureau of Economic research business cycle dates, we also find that the high-volatility regime more likely occurs when the economy experiences a recession.

**Keywords:** Markov switching, vector error correction, oil and stock prices

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## 1. Introduction

Macroeconomic policymakers consider wealth as an important driver of the economy and view asset prices as important predictors of the business cycle. For example, researchers have long followed the stock market in the US as an important predictor of the business cycle. (Moore 1983; Siegel 1991; Chauvet 1998-1999). For example, Siegel (1991) determines that for the 41 recessions observed since 1802, at least an 8-percent loss in stock returns preceded 38. On the negative side, 12 false signals occurred using this criterion, where no recession followed. Seven of these false signals happened after WWII. Other researchers consider the predictive ability of crude oil prices. Hamilton (2003, 2009) argues that oil price shocks proximately cause post-WWII recessions in the US.<sup>1</sup>

The existing literature contains several attempts to identify the effects of changes in crude oil prices on certain macroeconomic variables, such as, real GDP growth rates, inflation, employment, and exchange rates (Hamilton, 1983; Gisser and Goodwin, 1986; Mork, 1989; Hooker, 1996; Davis and Haltiwanger, 2001, Hamilton and Herrera, 2002, Lee and Ni, 2002; Hooker, 2002, and so on). These research papers differ from each other and do not produce a general consensus.

Fewer research papers examine the relationship between crude oil prices and other asset prices, such as stock prices or stock returns. Market participants want a framework that identifies how oil-price changes affect stock prices or stock market returns. On theoretical grounds, oil-price shocks affect stock market returns or prices through their effect on expected earnings (Jones *et al.*, 2004). The relevant literature includes the following studies. Kaul and Seyhun (1990), Sadorsky (1999), Hong et al. (2002), O'Neil *et al.* (2008) and Park

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<sup>1</sup> Even other views exist, however. For example, Leamer (2007) argues that “housing is the business cycle,” showing that significant declines in housing construction preceded 8 of the last 10 recessions. Iacoviello and Neri (2010) estimate a DSGE model and consider the role of the housing market in the propagation of the business cycle. They conclude that “... spillovers from the housing market to the broader economy are non-negligible, concentrated on consumption rather than business investment, and have become more important over time, ...” (p. 57).

and Ratti (2008) report a negative effect of oil-price volatility on stock prices. Sadorsky (2001) finds, on the contrary, a positive relationship between oil prices and stock returns. Jones and Kaul (1996) show that international stock prices do react to oil price shocks. Huang *et al.* (1996) provide evidence in favor of causality effects from oil futures prices to stock prices. More recently, Faff and Brailsford (2000) report that oil-price risk proved equally important to market risk, in the Australian stock market. Pollet (2005) and Driesprong *et al.* (2008) find that oil-price changes predict stock market returns on a global basis, while Hammoudeh and Li (2004), Hammoudeh and Eleisa (2004), and Malik and Hammoudeh (2013) also discover the importance of the oil factor for stock prices in certain oil-exporting economies. Bittlingmayer (2005) documents that oil-price changes associate with war risk and those associated with other causes exhibit an asymmetric effect on the behavior of stock prices. Sawyer and Nandha (2006), however, using a hierarchical model of stock returns, report results against the importance of oil prices on aggregate stock returns, while they retain their explanatory power only on an industrial (sectoral) level. Finally, Gogineni (2008) also provides statistical support for a number of hypotheses, such as oil prices positively associate with stock prices, if oil price shocks reflect changes in aggregate demand, but negatively associate with stock prices, if they reflect changes in supply. Moreover, stock prices respond asymmetrically to changes in oil prices.

Recently, however, researchers began asking whether changes in macroeconomic variables cause oil price changes, leading to the decomposition of those oil price changes into the structural shocks hidden behind such changes (Kilian, 2008a; Kilian and Park, 2009). That is, different sources of oil price changes may imply non-uniform effects on certain macroeconomic variables. More specifically, the relevant literature generates mixed views regarding the effect of such oil-price shocks on asset prices, such as stock prices. Chen *et al.* (1986) argue that oil prices do not affect the trend of stock prices, while Jones and Kaul

(1996) present evidence that favors a negative association. This negative relationship, however, does not receive support by Huang *et al.* (1996) and Wei (2003).

Kilian (2008a) criticizes all these analyses, because researchers treat oil-price shocks as exogenous. Certain work, however, argues that oil prices respond to factors that also affect stock prices (Barsky and Kilian 2002, 2004; Hamilton 2005; Kilian 2008b, Apergis and Miller 2009). Thus, researchers must decompose aggregate oil price shocks into the structural factors that reflect the endogenous character of such shocks.

Thus, this paper investigates the relationship between the Standard and Poor's S&P 500 (SP500) stock market index and the West Texas Intermediate (WTI) spot crude oil price from September 1859 through December 2013, using a Markov-switching vector error-correction (MS-VEC) model. The MS-VEC model includes two regimes -- high- and low-volatility regimes. One unique feature of our analysis is that the sample period runs from the beginning of the modern era of the petroleum industry with the drilling of the first oil well in the US at Titusville, Pennsylvania in 1859.<sup>2</sup> The world economy experienced the Great Depression, leading to an 86-percent decline in the SP500 index from October 1929 through June 1932 and a 68-percent decline in the WTI price from January 1929 through May 1933. Figure 1 documents that the WTI price remained relatively stable from World War II until the effective emergence of OPEC in 1973. That is, although OPEC formed in 1960, they did not exert significant control over world crude oil prices until the countries in OPEC nationalized

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<sup>2</sup> The modern era began in the middle of the 19th Century with the discovery of how to refine kerosene (and paraffin) from crude oil and the invention of the kerosene lamp. Kerosene replaced whale oil as the fuel of choice for lamps. The discovery by Edison of the first commercially practical incandescent light bulb in 1879 led to a phasing out of kerosene lamps. The invention of a commercially successful internal combustion engine in 1879 boosted the demand for gasoline and other petroleum distillates for transportation purposes. As we see from Figure 1, the early part of our sample sees boom and bust cycles in the WTI series. That is, crude oil production and demand experienced significant swings after the birth of the modern era. As the petroleum industry matured, price wars amongst the major international companies in the 1920s led to the 1928 Achnacarry Agreement, which established the international petroleum cartel (Federal Trade Commission 1952). The Agreement divided markets, fixed prices, restricted production, and limited competition. The fixed prices supported the profitability of "high-cost" producers of petroleum and the market divisions and production restrictions attempted to prevent "low-cost" producers from lowering prices to expand their market shares.

their domestic oil industry. Figure 1 also shows that the WTI price experiences much more volatility post-1973.

Our findings imply that the high-volatility regime more frequently existed prior to the Great Depression and after the 1973 oil price shock caused by the Organization of Petroleum Exporting Countries (OPEC). The low-volatility regime occurred more frequently during the period of time from the end of the Great Depression to the first OPEC oil price shock, where the oil markets fell largely under the control of the major international oil companies. We also find a proclivity for the high-volatility regime to occur when the economy experiences a recession.

The paper unfolds as follows. Section 2 outlines the methodology used in the analysis. Section 3 implements the several steps in estimating the MS-VEC – data description, unit-root tests, cointegration tests, MS-VEC estimation, calculation of smoothed probabilities of the high-volatility regime, and impulse response analysis -- and discusses the findings. Section 4 concludes.

## **2. Methodology**

Conventional wisdom suggests that macroeconometric time-series models must address structural change and/or regime shift (see Granger, 1996). Indeed, the survey paper by Hansen (2001) or Perron (2006) affirm that econometric applications should distinctly consider regime shifts.

Econometricians recently developed new models that can tackle sufficiently certain types of structural changes. One appealing, the Markov switching (MS) approach proposed by Hamilton (1990) and later extended to multivariate time-series models by Krolzig (1997, 1999), can address structural breaks. Hamilton (1990) introduced univariate Markov switching autoregressive (MS-AR) model while Krolzig (1997, 1999) developed multivariate extensions to Markov switching vector autoregressive (MS-VAR) and Markov switching

vector error correction (MS-VEC) models. The MS models fall within the category of nonlinear time-series models that emerge from nonlinear dynamic processes, such as high-moment structures, time-varying parameter, asymmetric cycles, and jumps or breaks in a time series (Fan and Yao, 2003). The long sample period includes several influential prior events, such as the Panic of 1907, the Great Depression, World War I and II, first and second OPEC oil-price shocks, and more recently the global financial crisis and Great Recession of 2008. The data also includes a significant number of influential business cycles. MS models can provide a good fit to such time series data with business cycles features and regime shifts.

Several studies successfully use MS models to analyze aggregate output and business cycles (e.g., Hamilton 1989; Diebold, *et al.* 1994; Durland and McCurdy 1994; Filardo 1994, Ghysels 1994; Kim and Yoo 1995; and Filardo and Gordon 1998). Numerous studies also utilize MS models in the context of stock market returns (e.g. Tyssedal and Tjostheim 1988; Schwert 1989; Pagan and Schwert 1990; Kim, *et al.* 1998; and Kim and Nelson, 1998). Following these studies, we consider the MS-VEC model, which, with its rich structure, can accommodate the dynamic features of oil and stock price data in our sample. The model choice unlike other traditional models not only efficiently captures the dynamics of the process in a co-integration space, but also possesses an appealing structural form and provides economically intuitive results.

The method chosen uses a vector-error correction (VEC) model with time-varying parameters where, given our objectives, the parameter time-variation directly reflects regime switching. In this approach, we treat changes in the regimes as random events governed by an exogenous Markov process, leading to the MS-VEC model. A latent Markov process determines the state of the economy, where the probability of the latent state process takes a certain value based on the sample information. In this model, inferences about the regimes reflects the estimated probability, which measures the probability of each observation in the

sample coming from a particular regime. The MS-VEC model analyzes the time-varying dynamic relationship between the monthly spot crude oil and stock prices and extends the class of autoregressive models studied in Hamilton (1990) and Krishnamurthy and Rydén (1998). It also permits asymmetric (regime-dependent) inference for impulse response analysis. The structure of the MS-VEC model comes from the model studied in Krolzig (1997, 1999). Examples of these models, among others, include Psaradakis et al. (2004), Krolzig et al. (2002), and Francis and Owyang (2003). Our estimation approach uses the Bayesian Markov-chain Monte Carlo (MCMC) integration method of Gibbs sampling, from which we can calculate confidence intervals for the impulse response function of the MS-VEC model.

More specifically, let  $F_t$  and  $R_t$  denote the monthly spot crude oil and stock prices, respectively.<sup>3</sup> Define the time-series vector  $X_t$  up to and including period  $t$  as  $X_t = [F_t, R_t]'$  and let  $\mathfrak{F}_t = \{X_\tau | \tau = t, t-1, \dots, 1-p\}$ , where  $p$  is a nonnegative integer. For the vector valued time series  $X_t$  of random variables, assume that a (probability) density function  $f(X_t | \mathfrak{F}_{t-1}, \theta)$  exists for each  $t \in \{1, 2, \dots, T\}$ . We denote the parameters and the parameter space by  $\theta$  and  $\Theta$ , respectively and denote the true value of  $\theta$  by  $\theta_0 \in \Theta$ . Let the stochastic variable  $S_t \in \{1, 2, \dots, q\}$  follow a Markov (chain) process with  $q$  states. In the MS-VEC model, the latent state variable  $S_t$  determines the probability of a given state in the economy at any point in time. Noting that oil and stock prices are potentially cointegrated, but their dynamic interactions may exhibit time-varying parameters,<sup>4</sup> our analysis uses the following MS-VEC model<sup>5</sup>:

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<sup>3</sup> The oil and stock prices we analyze are all nonstationary time series as shown by the unit root tests given in Section 3. However, these oil and stock prices maintain a cointegration relationship, leading to the MS-VEC model.

<sup>4</sup> Several studies found that the dynamic links between the oil and stock prices are sensitive to sample period. Ciner (2001) finds strong linkage between oil prices and the stock market in the 1990s, but not in the 1970s and 1980s. Silvapulle and Moosa (1999) using daily data covering the period of 1985 – 1996 reported that their

$$DX_t = m_{S_t} + \sum_{k=1}^{p-1} G_{S_t}^{(k)} DX_{t-k} + P_{S_t} X_{t-1} + e_t, \quad t = 1, 2, \dots, T \quad (1)$$

where  $p$  is the order of the MS-VAR model,  $[e_t | S_t \sim N(0, \Omega_{S_t})]$ , and  $\Omega_{S_t}$  is a  $(2 \times 2)$  positive definite covariance matrix. The random state or regime variable  $S_t$ , conditional on  $S_{t-1}$ , is unobserved, independent of past  $X$ s, and assumed to follow a  $q$ -state Markov process. In other words,  $\Pr[S_t = j | S_{t-1} = i, S_{t-2} = k_2, \dots, \mathfrak{S}_{t-1}] = \Pr[S_t = j | S_{t-1} = i, \mathfrak{S}_{t-1}] = p_{ij}$ , for all  $t$  and  $k_t$ , regimes  $i, j = 1, 2, \dots, q$ , and  $l \geq 2$ . More precisely,  $S_t$  follows a  $q$ -state Markov process with transition probability matrix given by

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1q} \\ \vdots & \vdots & & \vdots \\ p_{q1} & p_{q2} & \cdots & p_{qq} \end{bmatrix}, \quad \sum_{j=1}^q p_{ij} = 1. \quad (2)$$

Thus,  $p_{ij}$  equals the probability of being in regime  $j$  at time  $t$ , given that the economy was in regime  $i$  at time  $(t-1)$ , where  $i$  and  $j$  take possible values in  $\{1, 2, \dots, q\}$ . This MS-VEC model allows all parameters, including the variance matrix  $\Omega_{S_t}$ , to depend on the latent regime or state variable  $S_t$ .

The  $\Pi_{S_t}$  matrix contains the long-run relationships between the oil and stock prices in the MS-VEC model specified in equation (1). We can interpret switching  $\Pi_{S_t}$  in three ways: switching in the cointegrating vectors ( $\beta'$ ), the weighting matrix ( $\alpha$ ), or both. Although these approaches are de facto equivalent, our specification in the error-correction term implies a single set of long-run relationships and preserves the Engle-Granger notion of cointegration. We can write the long-run impact matrix  $\Pi_{S_t}$  as follows:

$$\Pi_{S_t} = \alpha_{S_t} \beta', \quad (3)$$

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findings supports oil futures prices leading spot prices but more importantly there may be a changing pattern of leads and lags over the time period under considered.

<sup>5</sup> Camacho (2005) shows that the asymmetric dynamics of the equilibrium errors lead to the MS-VEC model.

where  $\Pi_{s_t}$  are the state-dependent, long-run impact matrices defined by the  $(n \times r)$  matrix of the state-independent cointegrating vectors  $\beta$  and the  $(n \times r)$  state-dependent weighting matrix  $\alpha_{s_t}$ .<sup>6</sup> In other words,  $\beta$  represents the coefficients of the long-run effects that do not change over the entire sample period and  $\alpha_{s_t}$  stands for the regime-dependent adjustment coefficient that controls how the endogenous variables respond to the disequilibria represented by the  $r$ -dimensional vector  $\beta'X_{t-1}$ . As such, the speed at which the variables adjust to the long-run equilibrium varies across regimes, which is a key distinction of the MS-VEC model in equations (1) to (3). For example, a shock in the oil price will exert a different effect on the stock price depending on whether the economy experiences a low or high volatility regime. In this model, due to the nonlinear dynamics of the equilibrium errors,<sup>7</sup> denoted by  $z_t = \beta'X_{t-1}$ , both the strength with which the equilibrium errors correct (i.e.,  $\alpha_{s_t}$ ) and the short-run dynamics of the endogenous variables (i.e.,  $\Gamma_{s_t}$ ) vary over time. In our specification, the switches capture differences across regimes in the rate of long-run adjustment.

In our paper, we assume that two regimes (i.e.,  $q = 2$ ) will sufficiently describe the dynamic interactions between the oil and stock prices. That is, the two-regime model the crises-recovery (recession-expansion) cycles observed in many macroeconomic time series. A large number of studies show that a two-regime MS model proves rich enough to capture the regime switching behavior in macroeconomic time series (e.g., Hamilton 1988, 1989;

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<sup>6</sup> Following Krolzig (1997, 1999), we estimate the parameters in the cointegration vector  $\beta$  using the Johansen (1988, 1991) method and imposing one cointegration relationship. These estimates enter the MS-VEC model as predetermined. Our specification assumes constant and regime independent cointegration vectors, while allowing for the presence of the regime-dependent adjustment to the equilibrium. This specification conforms with the nonlinear adjustment to equilibrium examined in Savit (1988).

<sup>7</sup> Although the long-run parameters (i.e.,  $\beta$ ) are state-independent, Camacho (2005) shows that the equilibrium errors follow an MS-VAR model under specifications in equations (1) and (3). Indeed, equation (1) emerges from a model where the equilibrium errors follow an MS-VAR process.

Diebold, *et al.* 1994; Durland and McCurdy 1994; Filardo 1994, Ghysels 1994; Kim and Yoo 1995; and Filardo and Gordon 1998).

The MS-VEC model in equations (1) to (3) contains several satisfying properties for analyzing the short-run dynamic interactions of the variables and their responses to disequilibria. First, we can classify regimes as depending on the parameter switches in the full sample and, therefore, we can detect changes in dynamic interactions between the variables. Second, this model allows for many possible changes in the dynamic interactions between the variables at unknown periods. Third, we can make probabilistic inferences about the dates at which changes in regimes occur. We can evaluate whether a regime change actually occurs, and also identify the dates of such regime changes. Finally, we can also use this model to drive regime dependent impulse response functions and to determine whether the effect of an oil price shock on the stock price varies with regimes.

The empirical procedure for building a suitable MS-VEC models starts with identifying a possible set of models to consider. We determine the order  $p$  of the MS-VEC model using the Bayesian Information Criterion (BIC) in a linear VAR( $p$ ) model. The MS-VEC model specifications may differ in terms of regime numbers ( $q$ ) and the variance matrix specification. We only consider regime-dependent (heteroskedastic) variance models, because both the oil and stock price series span a number of periods where volatilities vary significantly. Moreover, the actual variances of both oil and stock price series in the high-volatility regime exceed their respective variances in the low-volatility regime.

Once we identify a specific MS-VEC model, we next test for the presence of nonlinearities in the data. When testing the MS-VEC model against the linear VEC alternative, we follow Ang and Bekaert (2002) and use the likelihood-ratio statistic (LR), which is approximately  $\chi^2(q)$  distributed with  $q$  restrictions plus the nuisance parameters (i.e., free transition probabilities) that are not identified under the null. We use  $p$ -values based on

the conventional  $\chi^2$  distribution with  $q$  degrees of freedom, while for the approximate upper bound for the significance level of the LR statistic, we use Davies (1987). If we establish nonlinearity, then we can choose the number of regimes and the type of the MS model based on both the likelihood-ratio statistic and the Akaike information Criterion (AIC).<sup>8</sup>

To estimate the MS-VEC model, we adopt a two-step procedure due to Krolzig (1997), Saikkonen (1992), Saikkonen and Luukkonen (1997), and Krolzig, et al. (2002). Since all variables in the MS-VEC model are stationary, the estimators are asymptotically normally distributed and the usual statistical inference applies (Krolzig, 1997; Saikkonen, 1992; Saikkonen and Luukkonen, 1997; Krolzig, et al., 2002). First, we determine the number of cointegrating relationships using the Johansen (1988, 1991) procedure. We obtain the equilibrium errors  $z_t = \beta' X_{t-1}$  in this first step. Second, we estimate the MS-VEC model using the  $z_t$  determined in the first step. Saikkonen (1992) and Saikkonen and Luukkonen (1997) show that the Johansen procedure estimates the cointegrating vectors consistently, even in the presence of regime switching.

Three commonly used methods can estimate the parameters of the MS models. First, the maximum likelihood (ML), although the simplest method of estimation, proves computationally demanding and may exhibit slow convergence.<sup>9</sup> The ML method faces two important practical difficulties. One, finding the global maximum of the likelihood function may prove difficult to locate. Two, the likelihood function for the important class of mixtures of normal distributions is not bounded and the ML estimator does not exist for the global maximum.

Second, the more commonly used method of estimation for the MS models is the expectation maximization (EM) algorithm (Dempster *et al.* 1977; Lindgren 1978; Hamilton

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<sup>8</sup> Krolzig (1997) and Psaradakis and Spagnolo (2003) suggest selecting the number of regimes and the MS model using the AIC, and Psaradakis and Spagnolo (2003) show that using Monte Carlo experiment, the AIC generally yields better results in selecting the correct model.

<sup>9</sup> Redner and Walker (1984) provide an excellent review of the ML estimation of the MS models.

1990, 1994). Assuming a normal conditional distribution of  $X_t$  given  $\{\mathfrak{S}_t, S_t, S_{t-1}, \dots, S_0; \theta'\}$ , the likelihood function is numerically approximated using the EM algorithm in two steps. One, given the current parameter estimates and the data, we compute the conditional expectation of log likelihood (E-step), and two, we compute the parameters that maximize the complete-data log likelihood function (M-step). The EM algorithm may experience slow convergence and also we cannot directly compute the standard errors of the parameters from the EM algorithm.

Third, we can use the Bayesian MCMC parameter estimation based on the Gibbs sampling. The ML and EM methods usually fail for certain types of models, where we cannot compute the full vector of likelihoods for each regime for each period. The MCMC works only with one sample path for the regimes rather than a weighted average of sample paths over all regimes and, therefore, avoids the problem faced by the ML and EM methods.

The MCMC indeed treats the regimes as distinct set of parameter. Our MCMC implementation uses the following steps:<sup>10</sup> (i) Draw the model parameters given the regimes. In our case, transition probabilities do not enter this step. (ii) Draw the regimes given the transition probabilities and model parameters. (iii) Draw the transition probabilities given the regimes. In our case, model parameters do not enter this step.

First, we draw  $W_{S_t}$  given regimes,  $P$ , and  $h_{S_t} = (b, m_{S_t}, a_{S_t}, G_{S_t})'$  using a hierarchical prior. Our implementation draws a common covariance matrix from the Wishart distribution, given the inverse of the regime specific covariances. Then, we draw the regime specific covariances from the inverse Wishart distribution, given the common covariance. The degrees of freedom priors for Wishart and inverse Wishart distributions both equal 4. We use a flat prior and draw  $h_{S_t} = (b, m_{S_t}, a_{S_t}, G_{S_t})'$ , given regimes,  $P$ , and  $W_{S_t}$  from a multivariate Normal distribution with 0 mean.

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<sup>10</sup> See Fruehwirth-Schnatter (2006) for the details of the MCMC estimation of the MS models.

Second, we draw regimes  $S_t$  given  $h_{S_t} = (b, m_{S_t}, a_{S_t}, G_{S_t}) \mathcal{L}$ ,  $P$ , and  $W_{S_t}$ . This comes from the Bayes formula, where relative probability of regime  $i$  at time  $t$  equals the product of the unconditional regime probability times the likelihood of regime  $i$  at time  $t$ . We draw regimes as a random index from  $\{1, \dots, q\}$ , given relative probability weights. That is, we use the Forward Filter-Backwards Sampling (FFBS) (also called Multi Move Sampling) algorithm described in Chib (1996) to draw the regimes. Then, we reject any draw, if less than 5 percent of the observations fall in any of the regimes.

Finally, we draw unconditional probabilities  $P$ , given the regimes, from a Dirichlet distribution. We set the priors for the Dirichlet distribution as 80% probability of staying in the same regime and 20-percent probability of switching to the other regime. We perform the MCMC integration with 50,000 posterior draws with 20,000 burn-in draws.

We use the impulse response function (IRF) to analyze the dynamic interaction between the oil and stock price. The IRF analysis studies how a given shock in one variable propagates to all variables in the system over time (e.g.,  $h = 1, 2, \dots, H$  steps after the shock hits the system). Computing multi-step IRFs from MS-VEC models as well as from all nonlinear time-series models prove complicated because no ordinary method of computing the future path of the regime process exists. The IRF analysis requires that we know the future path of the regime process, since the impulses depend on the system's regime in every time period.

Ideally, the IRFs of the MS-VEC model should integrate the regime history into the propagation period, which proves difficult to resolve. Two approaches work-around the history dependence of the IRFs in the MS models. Ehrmann et al. (2003) suggest assuming that regimes do not switch beyond the shock horizon, leading to regime-dependent IRFs (RDIRF). On the other hand, Krolzig (2006) acknowledges the historical dependence and allows the regime process to influence the propagation of the shocks for the period of

interest,  $h=1, 2, \dots, H$ . In Krolzig's approach, we compute the conditional probabilities of future regimes,  $S_{t+h}$ , given the regime  $S_t$  and the transition probabilities,  $P$ .

One major attraction of the RDIRF analysis is the possibility of determining the time variation in the responses of variables to a particular shock. The RDIRF traces the expected path of the endogenous variable at time  $t+h$  after a shock of given size to the  $k$ -th initial disturbance at time  $t$ , conditioned on regime  $i$ . The  $k$ -dimensional response vectors  $\psi_{ki,1}, \dots, \psi_{ki,h}$  represents a prediction of the response of the endogenous variables. (Ehrmann *et al.* 2003). We can define the RDIRFs as follows:<sup>11</sup>

$$y_{ki,h} = \frac{\mathbb{E}_t X_{t+h}}{\mathbb{E}_t u_{k,t}} \Bigg|_{S_t = \dots = S_{t+h} = i} \quad \text{for } h \geq 0, \quad (4)$$

where  $u_{k,t}$  denotes the structural shock to the  $k$ -th variable. In general, the reduced-form shocks  $e_t$  will correlate across the equations and  $e_{k,t}$  will not correspond to  $u_{k,t}$ . This leads to the famous identification problem for which several solutions exist. We identify the structural shocks as  $e_t = F_{S_t} u_t$ . To make structural inferences from the data, we must identify the structural disturbances and, hence,  $F$ . In other words, we impose sufficient restrictions on the parameter estimates to derive a separate structural form for each regime, from which we compute the RDIRFs. As standard practice to measure the effect of the oil price on the stock price, we order the stock price last and use the recursive identification scheme (Sims 1980). The recursive identification scheme uses the Cholesky decomposition of the covariance matrix as  $W_{S_t} = L_{S_t} L'_{S_t}$  and identifies structural shocks from  $u_t = F_{S_t}^{-1} e_t$  with  $F_{S_t} = L_{S_t}$ .

The RDIRF analysis, although simple to derive and to construct confidence intervals via bootstrap, is not appropriate, if regime switching likely occurs during the propagation of

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<sup>11</sup> Refer to Ehrmann *et al.* (2003) for details on characteristics and computation of the regime-dependent impulse responses.

shocks. The Krolzig (2006) solution possesses appeal, but it cannot construct confidence intervals. In our study, we combine RDIRF analysis with MCMC integration. We examine whether the dynamic response of the stock price to oil price shocks depends on the state of the economy, such as crash or recovery periods, assuming a given regime (i.e., regime switching does occur during the shock propagation periods) and studying the propagation of the oil price shock in the future, which proves appropriate for our purposes. Building on the Bayesian impulse responses for the linear VAR models (Ni *et al.* 2007), we derive the posterior density of the RDIRFs from the Gibbs sampling. The simulations of the posteriors of the parameters jointly with the identification of the structural shocks via the Gibbs sampler directly yield the posterior densities of the RDIRFs. We compute the confidence bands by the MCMC integration with Gibbs sampling of 50,000 posterior draws with a burn-in of 20,000.

### **3. Data and Empirical Findings**

#### *3.1. Data*

We collected data on the Standard and Poor's S&P 500 (SP500) stock market index, and West Texas Intermediate (WTI) spot crude oil price from September 1859 through December 2013 for 1,852 observations. Data come from the Global Financial Database. We seasonally adjust the data using Census X13. Figure 1 plots the natural logarithm of the two series. The shaded (grey) bars identify the National Bureau of Economic Research (NBER) recessions in the US economy. Frequently, the SP500 and the WTI fall during recessions, although not in every recession.

Table 1 reports the descriptive statistics for the natural logarithms of the SP500 and WTI series in Panel A. Panel B gives the descriptive statistics for SP500 and WTI returns, measured as the logarithmic difference in the price series. In addition to the mean, standard deviation (SD), coefficient of variation (CV), minimum (min), maximum (max), skewness, and kurtosis statistics, Table 1 also reports the Jarque-Berra normality test (JB), the Ljung-

Box first [Q(1)] and fourth [Q(4)] autocorrelation tests, and the first [ARCH(1)] and fourth [ARCH(4)] order Lagrange multiplier (LM) tests for autoregressive conditional heteroskedasticity (ARCH).

For both series in logarithmic levels and logarithmic differences, we reject normality and find evidence of first- and fourth-order autocorrelation and autoregressive conditional heteroskedasticity. Using the coefficient of variation to measure relative volatility, we see that the WTI series exhibits more volatility than the SP500 in both logarithmic levels and logarithmic differences.<sup>12</sup>

### 3.2. *Unit-root tests*

Table 2 Panel A reports unit-root tests for the log levels of the series with a constant and a linear trend in the test equation. Panel B reports unit-root tests for the first differences of the log series with only a constant in the test equation. ADF denotes the augmented Dickey-Fuller (Dickey and Fuller, 1979) test,  $Z_\alpha$ , the Phillips-Perron  $Z_\alpha$  unit root test (Phillips and Perron, 1988),  $MZ_\alpha$  and  $MZ_t$ , the modified Phillips-Perron tests of Perron and Ng (1996), DF-GLS, the augmented Dickey Fuller test of Elliot et al. (1996) with generalized least squares (GLS) detrending, KPSS, the Kwiatkowski et al. (1992) stationarity test, and Zivot-Andrews, the endogenous structural break unit root test of Zivot and Andrews (1992) with breaks in both the intercept and linear trend.  $Z_\alpha$ ,  $MZ_\alpha$ , and  $MZ_t$  tests depend on GLS detrending. For the ADF unit root statistic, we select the lag order by sequentially testing the

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<sup>12</sup> Although we implement the MS-VEC methodology to address regime changes and structural breaks, one referee suggested that we also try an explicit break-test method. We performed the Bai-Perron (2003) procedure for identifying break points on each of the two equations of the VEC model. The results appear in Tables A1 and A2 of the Appendix. We find one break in each equation – September 1884 for the oil equation and July 1933 for the stock price equation. The break dates provide interesting links to Figures 1 and 2. To wit, September 1884 nearly coincides with the switch from Kerosene to electric lamps as well as the introduction of the automobile and its needs for gasoline. Of course, July 1933 lies in the early stages of the Great Depression. Further, since the Bai and Perron (2003) test requires trimming of observations from both ends of the sample (we choose 15-percent trimming), we could miss some possible break points. Thus, we also applied the Brock, Dechert, Scheinkman, and LeBaron (1996) (BDS) test of nonlinearity on the residuals recovered from the individual equations of the VEC model. Tables A3 and A4 show that the test overwhelmingly rejects the null of iid residuals for both equations at all dimensions, thus indicating nonlinearity.

significance of the last lag at the 10-percent significance level. We select the bandwidth or the lag order for the  $MZ_\alpha$ ,  $MZ_t$ , DF-GLS, and KPSS tests using the modified Bayesian Information Criterion (BIC)-based data dependent method of Ng and Perron (2001).

In Panel A of Table 2, the KPSS test rejects the null hypothesis of stationary series. All the other tests cannot reject the null hypothesis of nonstationary series. In Panel B, the KPSS test cannot reject the null hypothesis of stationary series whereas all other tests reject the null hypothesis of nonstationary series. In sum, all series test as first-difference stationary series. In sum, we conclude that both the SP500 and WTI series are nonstationary in their logarithmic differences.

### 3.3. *Multiple cointegration tests*

Table 3 reports selection criteria and multivariate cointegration tests for the VAR(p) model of the natural logarithms of SP500 and WTC. Panel A reports the AIC, BIC, and Hannan-Quinn (HQ) information criteria. The VAR order of 2 comes from the minimum BIC and HQ values. Moreover, we prefer a more parsimonious specification for our analysis. Panel B reports maximum eigenvalue ( $\lambda_{\max}$ ) and trace ( $\lambda_{\text{trace}}$ ) cointegration order tests of Johansen (1988, 1991). Both the maximum eigenvalue and trace tests support cointegration between SP500 and WTI. Panel C reports the multivariate cointegration test of Stock and Watson (1988). Under the null  $q(k, k-r)$  of Stock-Watson cointegration test, we test  $k$  common stochastic trends against  $k-r$  common stochastic trends (or  $r$  cointegration relationships). We find support for cointegration between SP500 and WTI, using the Stock-Watson test.

### 3.4. *Estimation of MS-VEC model*

Table 4 reports estimation results and model selection criteria for the MS-VEC model given in equations (1) to (3). We select the lag order by the minimum BIC value in a VAR in levels as 2 for both linear VEC and MS-VEC models. We estimate the MS-VEC model using Bayesian Monte Carlo Markov Chain (MCMC) method with Gibbs sampling. The MCMC

estimates employ 20,000 burn-in and 50,000 posterior draws. All reported estimates in Table 4 for the MS-VEC model come from the Bayesian estimation.

The likelihood ratio statistic tests the linear VEC model under the null against the alternative MS-VEC model. The LR test is nonstandard, since unidentified parameters exist under the null. The  $\chi^2$  p-values (in square brackets) with degrees of freedom equal to the number of restrictions as well as the number of restrictions plus the numbers of parameters unidentified under the null reject the null of linearity. The p-value of the Davies (1987) test also rejects linearity. We estimate these models over the full sample period 1959:12-2012:12 with 1849 observations.

The long-run average probabilities of a low and high-volatility regime equal 0.72 and 0.28, respectively. That is, for our 1849 observations, we expect the low- (high-) volatility regime to occur on 1,334 (515) occasions. These numbers imply an average duration in the low (high-) volatility regime of 20.7 (8.0) months. The actual outcomes over our sample in the low- (high-) volatility regime equal 1,279 (570).

### 3.5. *Smoothed probabilities*

Figures 2 and 3 plot the estimates of the smoothed probabilities of high volatility regime (regime 2) of the MS-VEC model given in equations (1) to (3). The smoothed probabilities equal the means of the 50,000 posterior draws for each time period based on the FFBS algorithm.

In Figure 2, the shaded (grey) bars correspond the periods where smoothed probability of the high volatility regime equals or exceeds 0.5. We note that the high volatility region occurs much more frequently in the pre-1990 era, during the Great Depression, and in the post 1973 period. Since recessions occur more frequently during the pre-1990 period, Figure 3 also plots the smoothed probability in regime 2 along with shaded (grey) bars that correspond to NBER business cycle recession. Visually, Figure 3 seems to

show that a high smoothed probability of a high volatility region associates with a recession, although not all recessions associate with a high smoothed probability of a high volatility region. We calculated the average probability of the high volatility region during recessions and expansions, finding values of 0.49 and 0.28.

If we define high- and low-volatility regimes as 1 and 0, respectively, and recessions and expansions as 1 and 0, respectively, then we can form a contingency table of outcomes. Table 5 reports the findings for the entire sample period of 1,849 observations. We soundly reject the  $\chi^2$  test for independence, which equals 131.45. The odds ratio implies that the economy is three times more likely to experience a recession in the high-volatility regime than in the low-volatility regime. In sum, we find a significant relationship between recessions and a high smoothed probability of the high volatility region

### 3.6. *Impulse response functions*

Figure 4 reports 1 to 20 step impulse responses of the stock price to a 1 standard deviation shock in the oil price. All impulses use the Cholesky factor orthogonalization. Impulses responses appear as solid lines and the 95-percent confidence intervals appear as dotted lines. The confidence intervals for the linear VEC model come from 1,000 bootstrap resampling. The MS-VEC impulse responses are computed using the regime dependent impulse response method suggested by Ehrmann et al. (2003). The confidence intervals for the MS-VEC models come from the 50,000 posterior draws.

The impulse responses for the linear VEC model show a significant positive effect of the oil price shock on the stock price. That is, a positive oil price shock of 1-percent leads to a growing positive movement in the stock price over time. On the other hand, the impulse

responses during the low- and high-volatility regions show no significant effect and a significant negative effect, respectively.<sup>13</sup>

Kilian and Park (2009) argue that the linkage between oil and stock prices depends on the shocks in the oil market. More specifically, shocks to global aggregate demand and shocks to precautionary demand lead to opposite effects on stock prices.<sup>14</sup> That is, an unanticipated increase in global demand will raise both oil and stock prices whereas an increase in precautionary demand will lead to a jump in the oil price and a drop in the stock price, because uncertainty about future oil supply drives precautionary demand.

Our findings point to the importance of conditioning the linkages on the low- and high-volatility environments. When we do not differentiate, we find a positive response of stock prices to a positive shock to oil prices. When we condition on low- and high-volatility regimes, we find no relationship between the oil price shock and stock prices in the low volatility regime, supporting the findings of Huang *et al.* (1996) and Wei (2003), and a negative relationship for the high-volatility regime, supporting the findings of Jones and Kaul (1996).<sup>15</sup>

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<sup>13</sup> A referee inquired about the Granger causal relationship between the two series. We establish a one-way long-run causality from the stock price to the oil price within a linear VEC model, which seems counterintuitive. We cannot formulate the linear VEC model to reflect the overall effects of both regimes, because imposing linear restrictions on the MS-VEC model does not work. In an MS-VEC model, the covariance matrix also depends on the regime and the regimes do indeed propagate to more than one period (Krolzig, 1997). Additionally, regime transition probabilities are nuisance parameters under the linear model. Therefore, the impulse response function of the linear VEC and MS-VEC models are not comparable in that the linear model may not indeed reflect the overall effects of both regimes. The linear model does not incorporate additional dynamic features that an MS-VEC model captures. These features include the asymmetric adjustment to the equilibrium, the asymmetric and history dependent response, and regime switching volatilities. The impulse responses in an MS-VEC model also depend on the volatility of the regime as well as the sign, size, and history (time) of the shock. In a bivariate model, impulse response and Granger causality tests may substitute for each other (Lütkepohl, 2005; p. 44). Therefore, we can interpret the impulse response analysis presented in this subsection as indicating causality links. We include the impulse response function of the linear VEC model to illustrate the likely misleading results that may arise, if we ignore the nonlinear dynamics. The Granger causality links in a MS-VEC model prove complicated and constitute an entirely different paper.

<sup>14</sup> Kilian and Park (2009) also argue that shocks to oil production prove the least important for explaining stock-price movements.

<sup>15</sup> As suggested by one referee, we also analysed the asymmetric effect of positive and negative oil-price shocks on stock returns. For this purpose, we followed Granger and Yoon's (2002) concept of hidden cointegration to disaggregate the oil-price movements into positive and negative components based on the cumulative sum of the

#### 4. Conclusion:

This paper examines the relationship between US crude oil and stock market prices, using a MS-VEC model and a monthly data set from 1859 to 2013. Our sample period begins at the time usually identified as the modern era of the petroleum industry, which links to the drilling of the first oil well in the US at Titusville, Pennsylvania in 1858. The early part of the 20<sup>th</sup> century saw the major international oil companies capturing control of the pricing of crude oil. This control continued until OPEC established its dominance with the nationalization of domestic oil industries in OPEC countries. That effective control by OPEC saw its initial success in the first oil price shock of 1973. Since then, OPEC's power waxed and waned over time.

We find that the natural logarithms of SP500 stock market index and the WTI crude oil price series exhibit non-stationary behavior. Moreover, these two series prove cointegrated, leading to our estimation of the MS-VEC model. Tests reject the null hypothesis of a linear VEC model in favor of the nonlinear MS-VEC model with two regimes. We find that the high-volatility regime more frequently exists prior to the Great Depression and after the 1973 oil price shock caused by OPEC. The low-volatility regime occurs more frequently during the period of time from the end of the Great Depression to the first oil price shock, where the oil markets fell largely under the control of the major international oil companies.

The impulse-response function analysis in our non-linear MS-VEC model determines that an oil price shock delivers a negative effect on stock prices for the high-volatility economy, whereas no relationship exists for the low-volatility economy. Similar analysis in a

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log-returns of the oil price. Upon using the positive and negative components in a linear VEC model with the stock price, we observe that both the shocks (of equal magnitude) positively affect stock returns, with the negative oil price shock exerting a larger effect. So while, our results highlight asymmetry in the effect of oil shocks, the counter-intuitive result of the positive oil shocks not reducing stock price most likely reflects the fact that we use a linear VEC model, even when strong evidence exists of nonlinearity based on both the Bai and Perron (2003) and the BDS (1996) tests. Further details of these results are available upon request from the authors.

linear VEC model produces the opposite finding of a significant positive effect of an oil price shock on the stock price. These findings suggest that ignoring nonlinearity can lead to problematic results. Our finding of no relationship between the oil price shock and the stock price in the low volatility regime supports the results of Huang *et al.* (1996) and Wei (2003), while our finding of a negative relationship for the high-volatility regime supports the results of Jones and Kaul (1996). The analysis of the linkages between oil and stock prices in these paper and the existing literature does not consider the long sample of observations included in our paper, beginning with the modern era of the petroleum industry through the present. Our analysis opens the door for additional detailed analysis of the issues across this longer time horizon.

Regarding long-term investors, our impulse response analysis implies that a positive (negative) oil-price shock leads to a permanent negative (positive) effect on the stock price. But that effect only occurs when the economy experiences the high-volatility regime. Moreover, our data show that the high-volatility regime occurs only 28-percent of the time. The low-volatility regime occurs 72 percent of the time, where the oil price shock exhibits no significant effect on the stock price throughout the impulse response period. Thus, using oil-price shocks to help predict movement in the stock price will usually fail.

Hamilton (2005) claims that positive oil price shocks preceded 9 of the 10 post-WWII recessions in the US. If the high-volatility regime corresponded with the oil price shock, then we would see a decrease in the stock price, which many pundits think forecasts recession. Experience of stock-price movements suggests many false signals of recession. But, using the NBER business cycle dates, we find it more likely that the high-volatility regime occurs when the economy experiences a recession. Thus, a negative stock-price movement that follows a positive oil-price shock may offer a stronger signal to policy makers and others of a recession.

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**Table 1: Descriptive statistics.**

	<b>SP500</b>	<b>WTI</b>
<b><i>Panel A: log levels</i></b>		
<b>Mean</b>	3.337	1.406
<b>SD</b>	1.91	1.317
<b>CV</b>	0.572	0.937
<b>Min</b>	0.352	-2.303
<b>Max</b>	7.499	4.897
<b>Skewness</b>	0.742	0.714
<b>Kurtosis</b>	-0.696	-0.218
<b>JB</b>	207.398 <sup>***</sup>	160.993 <sup>***</sup>
<b>Q(1)</b>	1846.705 <sup>***</sup>	1839.056 <sup>***</sup>
<b>Q(4)</b>	7341.834 <sup>***</sup>	7219.099 <sup>***</sup>
<b>ARCH(1)</b>	1847.788 <sup>***</sup>	1822.354 <sup>***</sup>
<b>ARCH(4)</b>	1844.866 <sup>***</sup>	1824.999 <sup>***</sup>
<b><i>Panel B: log returns</i></b>		
<b>Mean</b>	0.0038	0.0009
<b>SD</b>	0.0477	0.0903
<b>CV</b>	12.553	100.333
<b>Min</b>	-0.3563	-0.6931
<b>Max</b>	0.3524	0.7985
<b>Skewness</b>	-0.5263	-0.2415
<b>Kurtosis</b>	8.5802	13.2378
<b>JB</b>	5780.1050 <sup>***</sup>	13569.2650 <sup>***</sup>
<b>Q(1)</b>	25.6756 <sup>***</sup>	274.7002 <sup>***</sup>
<b>Q(4)</b>	34.8562 <sup>***</sup>	320.4884 <sup>***</sup>
<b>ARCH(1)</b>	92.4441 <sup>***</sup>	198.9482 <sup>***</sup>
<b>ARCH(4)</b>	227.6214 <sup>***</sup>	296.9623 <sup>***</sup>
<b>N</b>	1,852	1,852

**Note:** The table gives the descriptive statistics for Standard and Poor's S&P 500 Stock Market Index (SP500), and West Texas Intermediate spot crude oil price (WTI). All values are in natural logarithms in Panel A. Panel B gives the descriptive statistics for log returns. The sample period covers Sep. 1859-Dec. 2013 with  $n=1852$  observations. S.D. and C.V. denote standard deviation and coefficient of variation, respectively. In addition to the mean, standard deviation (S.D.), minimum (min), maximum (max), skewness, and kurtosis statistics, the table reports the Jarque-Berra normality test (JB), the Ljung-Box first [ $Q(1)$ ] and the fourth [ $Q(4)$ ] autocorrelation tests, and the first [ARCH(1)] and the fourth [ARCH(4)] order Lagrange multiplier (LM) tests for the autoregressive conditional heteroskedasticity (ARCH). <sup>\*\*\*</sup>, <sup>\*\*</sup> and <sup>\*</sup> represent significance at the 1%, 5%, and 10% levels, respectively.

**Table 2: Unit root tests.**

	LnSP500	LnWTI
<i>Panel A: Unit-root tests in levels</i>		
<b>ADF</b>	-1.4167 [20]	-1.8971 [24]
<b><math>Z_\alpha</math></b>	-15.1120 [1]	-15.8780 [1]
<b><math>MZ_\alpha</math></b>	-5.4095 [1]	-4.0321 [1]
<b><math>MZ_t</math></b>	-1.5121 [1]	-1.2883 [1]
<b>DF-GLS</b>	-1.5097 [1]	-1.2811 [1]
<b>KPSS</b>	5071.3870 <sup>***</sup> [1]	816.0628 <sup>***</sup> [1]
<b>Zivot-Andrews</b>	-1.4167 [1]	-1.8971 [1]
<i>Panel B: Unit-root test in first differences</i>		
<b>ADF</b>	-10.0940 <sup>***</sup> [19]	-12.8508 <sup>***</sup> [23]
<b><math>Z_\alpha</math></b>	-534.3900 <sup>***</sup> [15]	-700.0900 <sup>***</sup> [21]
<b><math>MZ_\alpha</math></b>	-43.5732 <sup>***</sup> [9]	-106118.0000 <sup>***</sup> [14]
<b><math>MZ_t</math></b>	-4.6579 <sup>***</sup> [9]	-230.3450 <sup>***</sup> [14]
<b>DF-GLS</b>	-3.8396 <sup>***</sup> [9]	-12.7764 <sup>***</sup> [21]
<b>KPSS</b>	0.1461 [9]	0.0039 [20]

**Note:** Panel A reports unit roots tests for the log levels of the series with a constant and a linear trend in the test equation. Panel B report unit root test for the first differences of the log series with only a constant in the test equation. ADF is the augmented Dickey-Fuller (Dickey and Fuller, 1979) test,  $Z_\alpha$  is the Phillips-Perron  $Z_\alpha$  unit root test (Phillips and Perron, 1988),  $MZ_\alpha$  and  $MZ_t$  are the modified Phillips-Perron tests of Perron and Ng (1996), DF-GLS is the augmented Dickey Fuller test of Elliot *et al.* (1996) with generalized least squares (GLS) detrending, KPSS is the Kwiatkowski *et al.* (1992) stationarity test, and Zivot-Andrews is the endogenous structural break unit root test of Zivot and Andrews (1992) with breaks in both the intercept and linear trend.  $Z_\alpha$ ,  $MZ_\alpha$ , and  $MZ_t$  tests are based on GLS detrending. For the ADF unit root statistic the lag order is selected by sequentially testing the significance of the last lag at 10% significance level. The bandwidth or the lag order for the  $MZ_\alpha$ ,  $MZ_t$ , DF-GLS, and KPSS tests are select using the modified Bayesian Information Criterion (BIC)-based data dependent method of Ng and Perron (2001). <sup>\*\*\*</sup>, <sup>\*\*</sup> and <sup>\*</sup> represent significance at the 1%, 5%, and 10% levels, respectively.

**Table 3: Multivariate cointegration tests.**

<i>Panel A: VAR order-selection criteria</i>							
Lag ( $p$ )	1	2	3	4	6	8	10
AIC	-10.9114	-11.0840	-11.0816	<b>-11.0843</b>	-11.0833	-11.0874	-11.0854
HQ	-10.9048	<b>-11.0730</b>	-11.0661	-11.0644	-11.0590	-11.0587	-11.0523
BIC	-10.8934	<b>-11.0541</b>	-11.0396	-11.0304	-11.0174	-11.0095	-10.9956

<i>Panel B: Johansen cointegration tests</i>						
Eigenvalues	0.0174	0.0001				
$H_0$	$\lambda_{\max}$	Critical values			Cointegration vector	
		10%	5%	1%	LSP	LOIL
$r = 1$	0.2600	6.5000	8.1800	11.6500	1.0000	1.0000
$r = 0$	32.5100 <sup>***</sup>	12.9100	14.9000	19.1900	-1.4970	0.9614
$H_0$	$\lambda_{\text{trace}}$	Critical values			Loadings	
		10%	5%	1%	LSP	LOIL
$r \leq 1$	0.2600	6.5000	8.1800	11.6500	-0.0007	0.0002
$r = 0$	32.7800 <sup>***</sup>	15.6600	17.9500	23.5200	0.0102	0.0001

<i>Panel C: Stock-Watson cointegration test</i>			
$H_0: q(k,k-r)$	Statistic	Critical values for $q(4,3)$	
$q(2,0)$	-2.2493	1%	-24.1694
$q(2,1)$	-17.4298 <sup>**</sup>	5%	-17.4041
		10%	-14.0011

**Note:** Table reports selection criteria and multivariate cointegration tests for the VAR( $p$ ) model of variables LSP, and LOIL. Panel A reports the AIC, BIC, and Hannan-Quinn (HQ) information criteria. The VAR order is selected based on minimum BIC and is 2. Panel B reports maximal eigenvalue ( $\lambda_{\max}$ ) and trace ( $\lambda_{\text{trace}}$ ) cointegration order tests of Johansen (1988, 1991). Non-rejection of  $r=0$  for the Johansen tests implies no cointegration. Panel C reports the multivariate cointegration test of Stock and Watson (1988). Under the null  $q(k,k-r)$  of Stock-Watson cointegration test,  $k$  common stochastic trend is tested against  $k-r$  common stochastic trend (or  $r$  cointegration relationship). Rejection of  $q(2,1)$  for the Stock-Watson test implies cointegration. <sup>\*\*\*</sup>, <sup>\*\*</sup> and <sup>\*</sup> represent significance at the 1%, 5%, and 10% levels, respectively.

**Table 4: Estimation results for the MS-VEC model.**

<i>Model selection criteria</i>		
	MS(2)-VEC	Linear VEC(2)
Log likelihood	12267.1548	4996.9223
AIC criterion	-13.2430	-5.3974
HQ criterion	-13.2428	-5.3974
BIC criterion	-13.1713	-5.3765

<i>LR linearity test</i>	Statistic	<i>p-value</i>
	4207.5566	$\chi^2(9)=[0.0000]$ ***
		$\chi^2(11)=[0.0000]$ ***
		Davies=[0.0000]***

*Transition probability matrix*

$$P = \begin{bmatrix} 0.9518 & 0.1250 \\ 0.0482 & 0.8750 \end{bmatrix}$$

**Regime properties**

	Probability	Observations	Duration (months)
Regime 1	0.7217	1334	20.7468
Regime 2	0.2783	515	8.0000

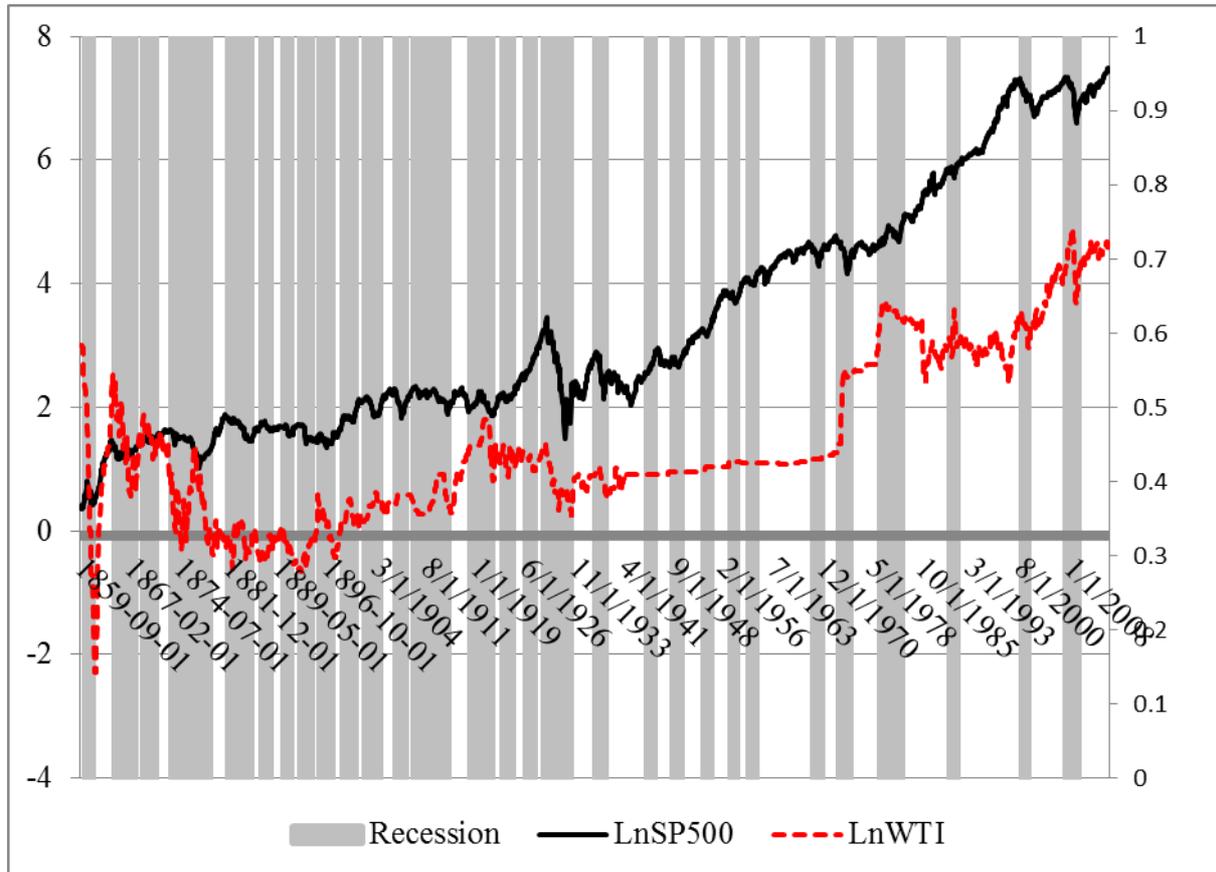
**Note:** Table reports estimation results and model selection criteria for the MS-VEC model given in equations (1) to (3). The lag order is selected by the BIC in a VAR in levels as 2 for both linear VEC and MS-VEC models. The MS-VEC model is estimated using Bayesian Monte Carlo Markov Chain (MCMC) method where we utilize Gibbs sampling. The MCMC estimates are based on 20,000 burn-in and 50,000 posterior draws. All reported estimates in the Table for the MS-VEC model are obtained from the Bayesian estimation. The likelihood ratio statistic tests the linear VEC model under the null against the alternative MS-VEC model. The test statistic is computed as the likelihood ratio (LR) test. The LR test is nonstandard since there are unidentified parameters under the null. The  $\chi^2$  *p*-values (in square brackets) with degrees of freedom equal to the number of restrictions as well as the number of restrictions plus the numbers of parameters unidentified under the null are given. Regime properties include ergodic probability of a regime (long-run average probabilities of the Markov process), observations falling in a regime based on regime probabilities, and average duration of a regime. The *p*-value of the Davies (1987) test is also given in square brackets. The models are estimated over the full sample period 1959:12-2012:12 with 1849 observations. \*\*\*, \*\* and \* represent significance at the 1%, 5%, and 10% levels, respectively.

**Table 5: High- and low-volatility versus recession and expansion outcomes.**

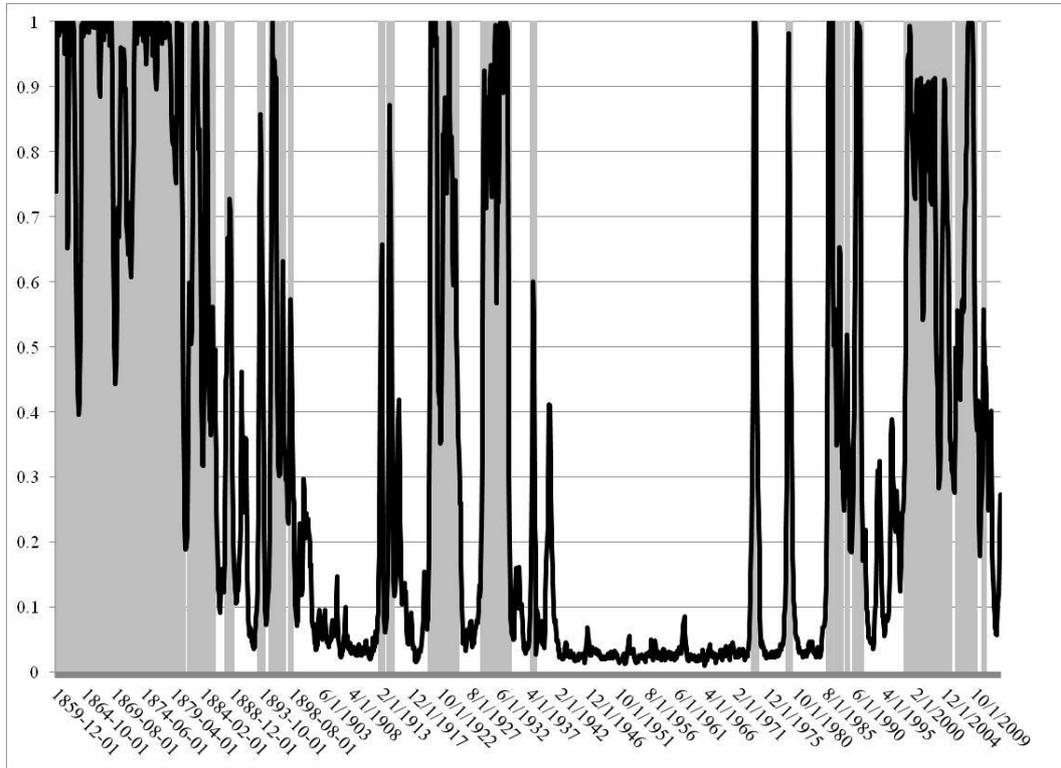
	High Volatility	Low Volatility	Count
<b>Recession</b>	288	302	590
<b>Expansion</b>	282	977	1259
<b>Count</b>	570	1279	1849

**Note:** The high- (low-) volatility regime occurs when the smoothed probability of the high-volatility regime is greater than or equal to (less than) 0.5. Recessions and expansions depend on the NBER business cycle dates.

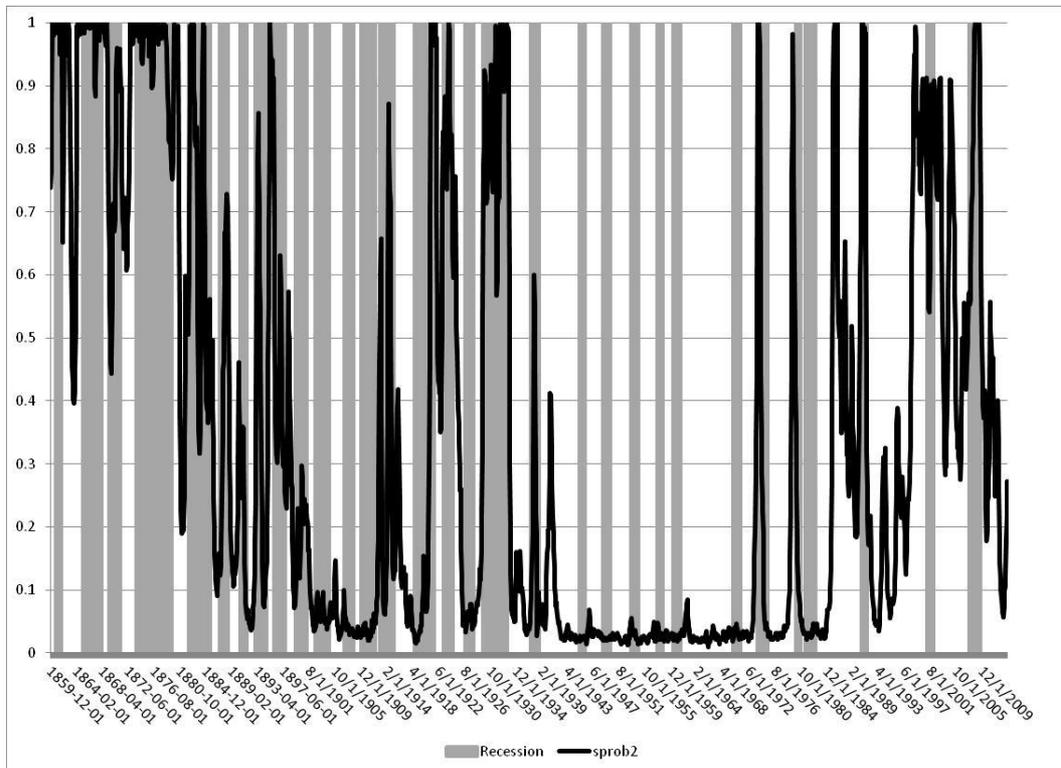
Figure 1: Natural Logarithms of SP500 and WTI: Sept.1859-Dec. 2013



**Figure 2: Smoothed probability estimates of high volatility (regime 2).**

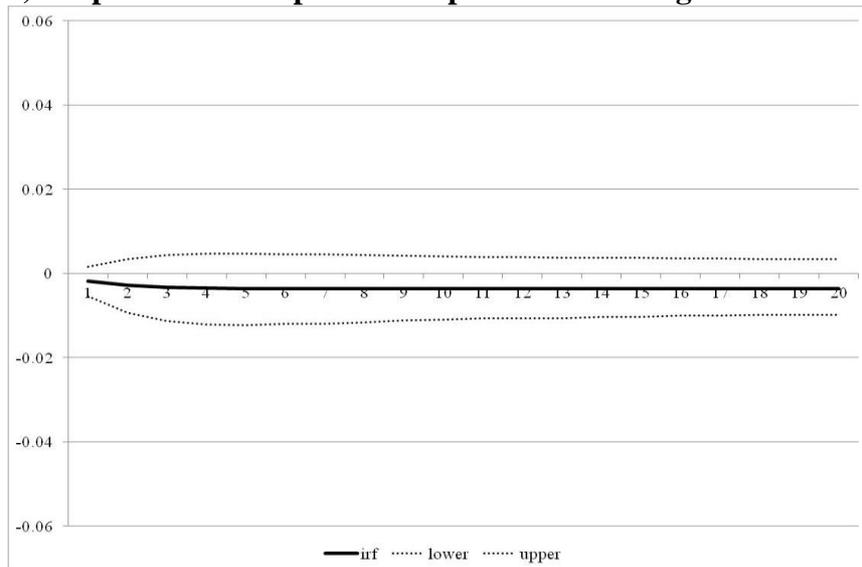


**Figure 3: Smoothed probability estimates of high volatility (regime 2) and recessions.**

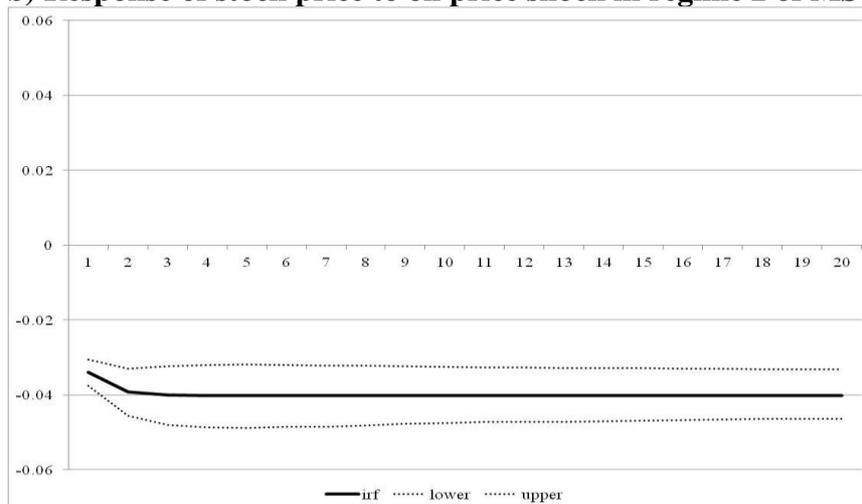


**Figure 4: Impulse response of stock price to oil price in linear VEC and MS-VEC models.**

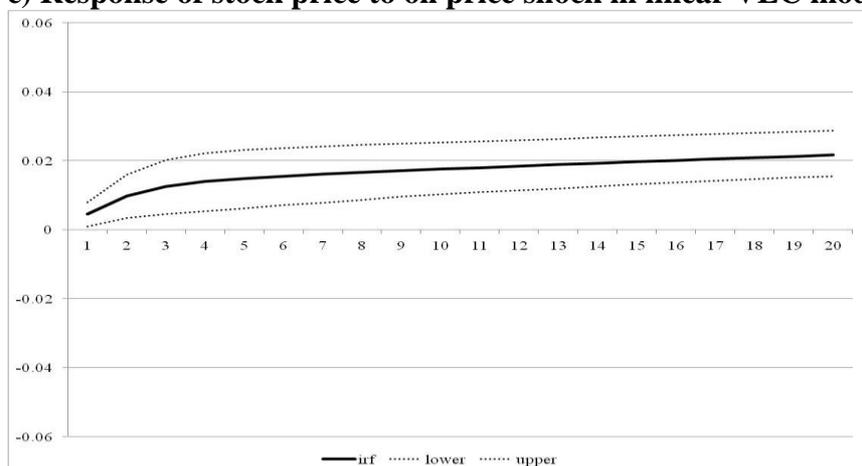
**a) Response of stock price to oil price shock in regime 1 of MS-VEC model**



**b) Response of stock price to oil price shock in regime 2 of MS-VEC model**



**c) Response of stock price to oil price shock in linear VEC model**



## Appendix:

**Table A1: DLOIL Equation: Bai-Perron Break Test**

Multiple breakpoint tests  
 Bai-Perron tests of L+1 vs. L sequentially determined breaks

Sample: 1859M09 2013M12  
 Included observations: 1850  
 Breakpoint variables: C DLSP(-1) DLOIL(-1) ECT(-1)  
 Break test options: Trimming 0.15, Max. breaks 5, Sig. level 0.05

Sequential F-statistic determined breaks:			1
Break Test	F-statistic	Scaled F-statistic	Critical Value**
0 vs. 1 *	7.802926	31.21170	16.19
1 vs. 2	2.030751	8.123006	18.11

\* Significant at the 0.05 level.

Break dates:

	Sequential	Repartition
1	1884M09	1884M09

Notes: DLOIL and DLSP stand for the log-returns of the oil and stock prices; C is the constant and ECT is the error-correction term.

**Table A2: DLSP Equation: Bai-Perron Break Test**

Multiple breakpoint tests  
 Bai-Perron tests of L+1 vs. L sequentially determined breaks

Sample: 1859M09 2013M12  
 Included observations: 1850  
 Breakpoint variables: C DLSP(-1) DLOIL(-1) ECT(-1)  
 Break test options: Trimming 0.15, Max. breaks 5, Sig. level 0.05

Sequential F-statistic determined breaks:			1
Break Test	F-statistic	Scaled F-statistic	Critical Value**
0 vs. 1 *	8.755473	35.02189	16.19
1 vs. 2	3.291364	13.16546	18.11

\* Significant at the 0.05 level.

Break dates:

	Sequential	Repartition
1	1933M07	1933M07

Notes: See Notes to Table A1.

**Table A3: BDS Test: DLOIL residuals:**

Sample: 1859M09 2013M12

Included observations: 1852

<u>Dimension</u>	<u>BDS Statistic</u>	<u>Std. Error</u>	<u>z-Statistic</u>	<u>Prob.</u>	
2	0.043596	0.002767	15.75624	0.0000	
3	0.090362	0.004404	20.51657	0.0000	
4	0.123945	0.005256	23.58112	0.0000	
5	0.145502	0.005492	26.49462	0.0000	
6	0.158487	0.005310	29.84635	0.0000	
Raw epsilon		0.089605			
Pairs within epsilon		2406942.	V-Statistic	0.703270	
Triples within epsilon		3.51E+09	V-Statistic	0.554077	
<u>Dimension</u>	<u>C(m,n)</u>	<u>c(m,n)</u>	<u>C(1,n-(m-1))</u>	<u>c(1,n-(m-1))</u>	<u>c(1,n-(m-1))^k</u>
2	919013.0	0.537914	1201191.	0.703077	0.494318
3	747080.0	0.437752	1199715.	0.702974	0.347390
4	627418.0	0.368034	1198273.	0.702890	0.244089
5	539638.0	0.316887	1196720.	0.702740	0.171385
6	474222.0	0.278775	1195174.	0.702593	0.120288

Notes: See notes to Table A1.

**Table A4: BDS Test: DLSP residuals:**

Sample: 1859M09 2013M12

Included observations: 1852

<u>Dimension</u>	<u>BDS Statistic</u>	<u>Std. Error</u>	<u>z-Statistic</u>	<u>Prob.</u>	
2	0.022906	0.002005	11.42369	0.0000	
3	0.042873	0.003179	13.48459	0.0000	
4	0.057272	0.003778	15.16085	0.0000	
5	0.064286	0.003929	16.36346	0.0000	
6	0.066350	0.003780	17.55155	0.0000	
Raw epsilon		0.059133			
Pairs within epsilon		2405422.	V-Statistic	0.702826	
Triples within epsilon		3.40E+09	V-Statistic	0.537074	
<u>Dimension</u>	<u>C(m,n)</u>	<u>c(m,n)</u>	<u>C(1,n-(m-1))</u>	<u>c(1,n-(m-1))</u>	<u>c(1,n-(m-1))^k</u>
2	882264.0	0.516404	1200195.	0.702495	0.493499
3	664616.0	0.389432	1198758.	0.702413	0.346560
4	512273.0	0.300492	1197205.	0.702263	0.243220
5	399993.0	0.234884	1195620.	0.702094	0.170599
6	316343.0	0.185965	1194056.	0.701936	0.119615

Notes: See notes to Table A1.