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INFLUENCE OF PROPERTY VARIATION ON NATURAL CONVECTION IN A CYLINDRICAL POROUS ANNULUS

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ABSTRACT
Natural convection in cylindrical annulus filled with gas saturated porous medium has been studied numerically with property variation. It has been found that effect of variable properties is negligible for temperature difference ratio, $\theta^* \le 0.1$. There exists a reference temperature, $T_q = T_c + 0.4(T_h - T_c)$ at which if thermo physical properties are evaluated, the average Nusselt number values for variable properties and constant properties are nearly the same.

INTRODUCTION
Studies on fluid flow and heat transfer in porous media are being extensively reported in the literature owing to relevance in several physical systems of present day interest. Some of the physical systems that are presently under active consideration are pebble bed nuclear reactors, enhanced recovery of oil by thermal methods, high performance insulation for cryogenic containers and buildings, matrix heat exchangers, nuclear waste disposal systems and underground spread of pollutants. Studies on natural convection in vertical cylindrical annulus have been reported in the literature [1-4]. These studies are within the framework of Boussinesq approximation. No work has been reported in the literature on natural convection in cylindrical porous annuli with property variation. However, studies on natural convection with variable properties in rectangular porous cavity are available in the literature [5-10].

In the present study, Boussinesq approximation has been relaxed considering variation of density, viscosity and thermal conductivity of the gas with temperature. Power law variation for viscosity and conductivity of gas with temperature has been employed. Density variation follows perfect gas law. Successive Accelerated Replacement (SAR) scheme [9-10] has been employed to obtain numerical solutions for wide range of parameters.

NOMENCLATURE

A	[-]	Aspect Ratio, H/D
c	[J/kgK]	Specific heat of fluid
D	[m]	Width of porous annuli
g	[m/s^2]	Acceleration due to gravity
H	[m]	Height of porous annuli
K	[m^2]	Permeability of porous medium
k	[W/mK]	Thermal conductivity of porous medium
Nu	[-]	Average Nusselt number, hD/k
p	[Pa]	Fluid pressure
P	[-]	Dimensionless pressure
q	[-]	Reference parameter
r	[m]	Radius of porous annuli
R	[-]	Dimensionless radius of annuli
R^*	[-]	Radius ratio, r_0/r_i
Ra	[-]	Modified Rayleigh number
T	[K]	Temperature of porous medium
u, v	[m/s]	Fluid velocity in r and z directions
U, V	[-]	Dimensionless fluid velocity
z	[m]	Cartesian axis
Z	[-]	Dimensionless Cartesian axis
Special Characters		
α	[m^2/s]	Thermal diffusivity of porous medium
β	[K^{-1}]	Thermal expansion coefficient of fluid
ε	[-]	Error tolerance limit
φ	[-]	Porosity of porous medium
μ	[N-s/m]	Dynamic viscosity of fluid
ν	[m^2/s]	Kinematic viscosity of fluid
ρ	[kg/m^3]	Density of fluid
ψ	[-]	Dimensionless stream function
θ	[-]	Dimensionless temperature
θ^*	[-]	Temperature difference ratio
Subscripts		
c	cold wall	
f	fluid	
h	hot wall	
i	inner wall	
o	outer wall	

q	reference condition
s	solid

MATHEMATICAL FORMULATION

The physical model shown in Fig.1 is a cylindrical annuli of height H, inner radius r_i and outer radius r_o filled with fluid saturated porous medium. The inner face is at temperature, T_h and outer face is at temperature, T_c. The top and bottom faces are insulated. The problem has been studied assuming that the flow is steady, laminar and governed by Darcy law. The porous medium is isotropic and homogenous. The fluid and the solid matrix are in thermal equilibrium.

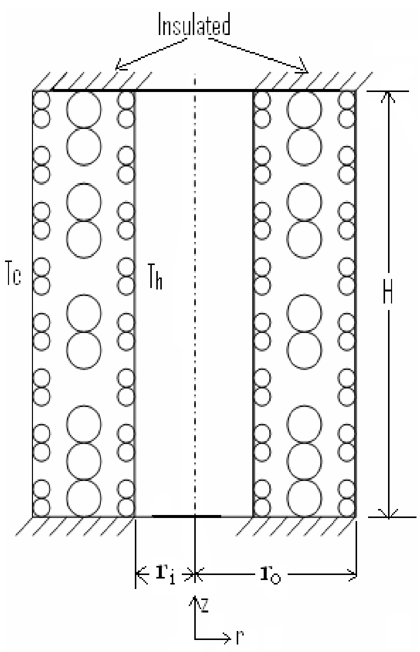

Fig. 1: Physical model and coordinate system

With the foregoing assumptions, governing equations for natural convection in the porous annuli comprising of conservation of mass, momentum and energy are given by,

$$\frac{\partial(\rho u r)}{\partial r}+\frac{\partial(\rho v r)}{\partial z}=0 \tag{1}$$

$$u=-\frac{K}{\mu}\frac{\partial p}{\partial r} \tag{2}$$

$$v=-\frac{K}{\mu}\left[\frac{\partial p}{\partial r}+\rho g\right] \tag{3}$$

$$\rho c\left[u\frac{\partial T}{\partial r}+v\frac{\partial T}{\partial z}\right]=\frac{1}{r}\frac{\partial}{\partial r}\left[rk\frac{\partial T}{\partial r}\right]+\frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right] \tag{4}$$

Solutions to governing equations (1-4) are subject to the following boundary conditions,

$$u=0 \text{ at } r=r_i, r_0 \text{ for } 0\le z\le H$$

$$v=0 \text{ at } z=0, H \text{ for } r_i\le r\le r_0$$

$$T=T_h, T_c \text{ at } r=r_i, r_0 \text{ for } 0\le z\le H$$

$$\frac{\partial T}{\partial z}=0 \text{ at } z=0, H \text{ for } r_i\le r\le r_0 \tag{5}$$

Thermal conductivity of the porous medium can be modeled as [6],

$$k=(k_s)^{1-\phi}(k_f)^{\phi} \tag{6}$$

Property variation of ideal gas can be expressed as,

$$\rho=C_\rho T^{n\rho} \tag{7}$$

$$\mu=C_\mu T^{n\mu} \tag{8}$$

$$k_f=C_k T^{nk} \tag{9}$$

Constants and exponents in equations (7)-(8) vary for different gases.

Stream Function Formulation

Governing equations are rendered dimensionless employing following non-dimensional variables,

$$R=\frac{r}{D},\ Z=\frac{z}{D},\ U=\frac{uD}{\alpha_q},\ V=\frac{vD}{\alpha_q}$$

$$\theta=\frac{T-T_c}{T_h-T_c},\ P=\frac{Kp}{\mu_q\alpha_q},$$

$$\bar\rho=\frac{\rho}{\rho_q},\ \bar\mu=\frac{\mu}{\mu_q},\ \bar k=\frac{k}{k_q} \tag{10}$$

Introducing stream function (ψ) which satisfies the continuity equation (1), U and V are related to ψ as,

$$U=\frac{1}{\bar\rho R}\frac{\partial\psi}{\partial Z},\ V=\frac{1}{\bar\rho R}\frac{\partial\psi}{\partial R} \tag{11}$$

Momentum equations (2) & (3) after elimination of pressure become,

$$\frac{\bar\mu}{\bar\rho^2 R}\left[\frac{\partial\bar\rho}{\partial R}\frac{\partial\psi}{\partial R}+\frac{\partial\bar\rho}{\partial Z}\frac{\partial\psi}{\partial Z}\right]-\frac{\bar\mu}{\bar\rho R}\left[\frac{\partial^2\psi}{\partial R^2}+\frac{\partial^2\psi}{\partial Z^2}\right]-\frac{1}{\bar\rho R}\left[\frac{\partial\bar\mu}{\partial R}\frac{\partial\psi}{\partial R}+\frac{\partial\bar\mu}{\partial Z}\frac{\partial\psi}{\partial Z}\right]+\frac{\bar\mu}{\bar\rho R^2}\frac{\partial\psi}{\partial R}=\bar\rho^2 Ra\frac{\partial\theta}{\partial R} \tag{12}$$

Energy equation (4) becomes,

$$\frac{1}{R}\left[\frac{\partial\theta}{\partial R}\frac{\partial\psi}{\partial Z}-\frac{\partial\theta}{\partial Z}\frac{\partial\psi}{\partial R}\right]=\bar k\left[\frac{1}{R}\frac{\partial\theta}{\partial R}+\frac{\partial^2\theta}{\partial R^2}+\frac{\partial^2\theta}{\partial Z^2}\right]+\left[\frac{\partial\bar k}{\partial R}\frac{\partial\theta}{\partial R}+\frac{\partial\bar k}{\partial Z}\frac{\partial\theta}{\partial Z}\right] \tag{13}$$

Dimensionless property variations are given by,

$$\bar\rho=\left[\frac{1+\theta^*\theta}{1+\theta^* q}\right]^{n\rho} \tag{14}$$

$$\bar{\mu} = \left[\frac{1+\theta^*\theta}{1+\theta^* q}\right]^{n\mu} \tag{15}$$

$$\bar{k} = \left[\frac{1+\theta^*\theta}{1+\theta^* q}\right]^{nk\phi} \tag{16}$$

Boundary conditions are given by,

$$\psi = 0, \quad \theta = 1 \qquad at \qquad R = \frac{1}{R^* - 1}$$

$$\psi = 0, \quad \theta = 0 \qquad at \qquad R = \frac{R^*}{R^* - 1}$$

$$\psi = 0, \quad \frac{\partial \theta}{\partial Z} = 0 \qquad at \qquad Z = 0$$

$$\psi = 0, \quad \frac{\partial \theta}{\partial Z} = 0 \qquad at \qquad Z = 0, A \tag{17}$$

The non-dimensional parameters are modified Rayleigh number (Ra), temperature difference ratio (θ^*), aspect ratio (A) and the reference temperature parameter (q) given by,

$$Ra = \frac{Kg\theta^* D}{\nu_q \alpha_q (1+q\theta^*)} \tag{18}$$

$$\theta^* = \frac{T_h - T_c}{T_c} \tag{19}$$

$$R^* = \frac{r_0}{r_i} \tag{20}$$

$$A = \frac{H}{D} \tag{21}$$

$$q = \frac{T_q - T_c}{T_h - T_c} \tag{22}$$

NUMERICAL SCHEME

Solutions to governing equations are obtained employing Successive Accelerated Replacement (SAR) scheme [9-10]. The basic philosophy of this scheme is to guess the profile for each variable that satisfies the boundary conditions. The equations are transformed into finite difference form employing central differencing scheme. Let governing equation of variable ψ is given by $\tilde{\psi}_{i,j} = 0$ at any mesh point i, j corresponding to R and Z position. The error arising out of the guessed profile is evaluated. Let the error arising in the equation at (i, j) and the n iteration be $\tilde{\psi}_{i,j}{}^n$. The (n+1) approximation to the variable ψ is given as,

$$\psi_{i,j}{}^{n+1} = \psi_{i,j}{}^n - \varpi \frac{\tilde{\psi}_{i,j}{}^n}{\partial \tilde{\psi}_{i,j}{}^n / \partial \psi_{i,j}} \tag{23}$$

ϖ is the acceleration factor which varies from 0 to 2.The procedure of correcting the variable at every mesh point is repeated until a set of convergence criteria is satisfied .The criterion is given by,

$$\frac{\sum_i \sum_j \left|\psi_{i,j}{}^{n+1} - \psi_{i,j}{}^n\right|}{\sum_i \sum_j \left|\psi_{i,j}{}^{n+1}\right|} \le \varepsilon \tag{24}$$

ε is the error tolerance limit. The feature of using the corrected value of the variable immediately upon becoming available is inherent in this method.

Numerical code based on SAR scheme has been developed in C++ for obtaining numerical solution to the governing equations.

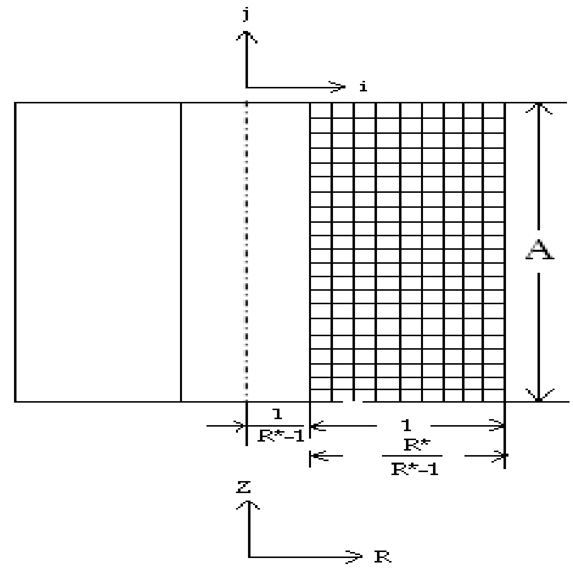

Fig.2 Discretisation of the computational domain

Average Nusselt Number

The average Nusselt number is computed by numerical integration of local Nusselt number employing Simpson's 1/3rd rule.

$$Nu_{h,c} = -\frac{1}{A}\int_0^A \bar{k}\frac{\partial \theta}{\partial R}\bigg|_{R=\frac{1}{R^*-1},\frac{R^*}{R^*-1}} \tag{25}$$

RESULTS AND DISCUSSION

Extensive numerical simulations have been carried out to determine the optimum parameters of the SAR scheme. Grid size of 41x41, error tolerance limit, $\varepsilon = 10^{-4}$ and acceleration factor, $\omega = 1.0$ have been used for all the computations. Numerical results have been validated by comparing average Nusselt number values obtained for constant properties i.e. $n\rho = n\mu = nk = 0$ with values reported in the literature (Table 1). Results of the present work are in excellent agreement with values reported in the literature. This validates the present numerical scheme.

Table 1: Comparison of present results with reported in the literature for constant properties.

R^*	2		5		10	
Ra	50	50	120	80	135	90
A	1	10	6.25	2.5	5.55	2.22
Havstad & Burns [1]	2.92	1.83	4.62	5.14	6.82	7.31
Prasad & Kulacki [2]	2.77	1.72	4.15	4.83	6.05	6.88
Present work	2.86	1.72	4.21	4.87	6.69	5.94

Flow and temperature fields shown in Figs. 3& 4 are similar to that reported in the literature [2]. Adverse temperature gradients are observed at the bottom of hot inner wall and top of the outer cold wall.

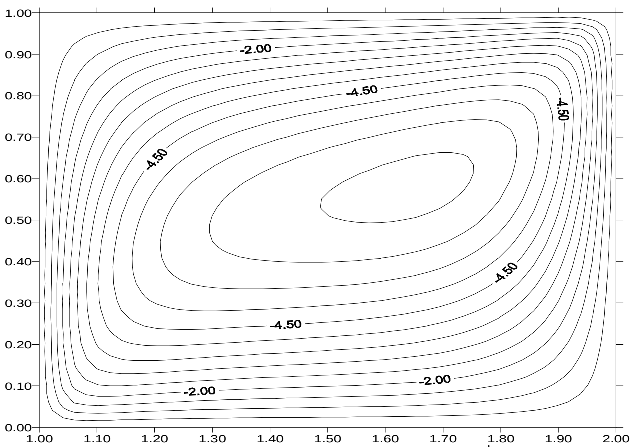

Fig. 3 Streamlines for Ra=100,A=1 and R^*=2 (Constant property)

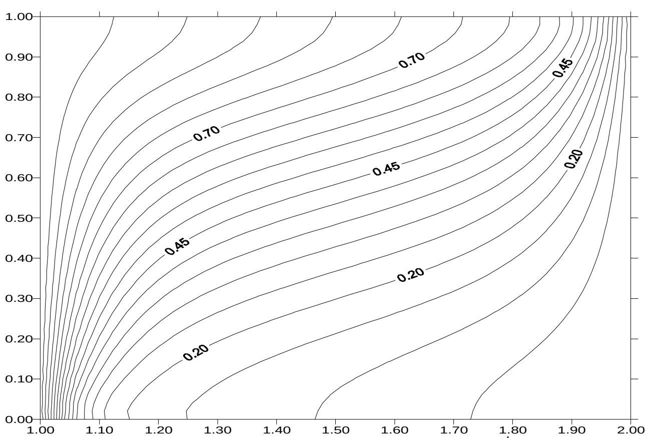

Fig. 4 Isotherms for Ra=100, A=1 and R^*=2 (Constant property)

Fig.5 shows variation of average Nusselt number with aspect ratio for different radius ratios. It can be observed that as aspect ratio increases, average Nusselt number decreases. For a given Rayleigh number and aspect ratio, as radius ratio increases, the average Nusselt number increases. The same trend has been observed by Prasad and Kulacki [2].

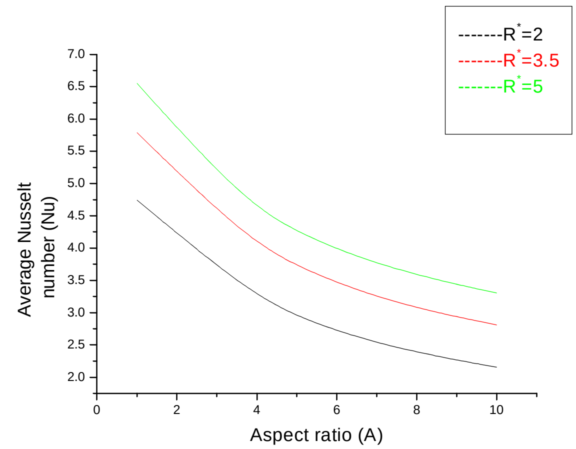

Fig.5: Variation of average Nusselt number with aspect ratio for Ra=100 (constant property)

Results for Property Variation

All computations have been carried out for the porous medium having porosity ϕ =0.4 filled with air. For air, values of indexes have been obtained from best fit curves of data available in [12] as $n\rho = -1$, $n\mu = 0.67$ and $nk = 0.82$.

From results shown in the Tables 2 & 3, it is observed that for temperature difference ratio $\Theta^* < 0.1$, the average Nusselt number for variable and constant properties are nearly same. For $\Theta^* > 0.1$, there is a deviation in average Nusselt number values for variable property over that of constant property. The average Nusselt number values for variable properties and constant properties are nearly same at q=0.4 irrespective of all other parameters.

Table 2: Influence of Temperature difference ratio (θ^*) and reference parameter (q) on average Nusselt number for Ra=100 and R^*=2:

		A=1	5	10
Constant Property		Nu=4.74	2.93	2.16
Variable property q= 0.3	θ^*			
	0.1	4.61	2.90	2.14
	0.5	4.19	2.65	2.09
	0.7	4.06	2.73	2.11
0.4	0.1	4.72	2.95	2.16
	0.5	4.71	2.98	2.19
	0.7	4.76	3.02	2.21
0.5	0.1	4.85	2.99	2.19
	0.5	5.34	3.22	2.30
	0.7	5.64	3.35	2.36

Table 3: Influence of temperature difference ratio (θ^*) and reference parameter (q) on average Nusselt number for Ra=100 and A=1.0.

		R^* = 2	3.5	5.0
Constant Property		Nu=4.74	5.84	6.56
Variable property	Θ^*			
	0.1	4.60	5.75	6.52
q= 0.3	0.5	4.19	5.47	6.37
	0.7	4.04	5.37	6.37
0.4	0.1	4.72	5.88	6.67
	0.5	4.72	6.16	7.12
	0.7	4.76	6.32	7.40
0.5	0.1	4.84	6.02	6.80
	0.5	5.34	6.91	7.84
	0.7	5.64	7.48	8.72

Table 4 shows the effect of property variation on average Nusselt number. For reference parameter q=0.4, average Nusselt number values for constant properties, individual variable property and all variable properties are nearly the same. Flow and temperature fields shown in Figs.6-7 for variable properties at q=0.4 are similar to that shown in Figs.3-4 for constant properties.

Table 4: Influence of reference parameter on average Nusselt number for property variations for Ra=100, A=1
R^*=2 and Θ^*=0.5

Property variation	Average Nusselt number (Nu)		
	q=0.3	0.4	0.5
Constant property $n\rho=n\mu=nk=0$	4.74	4.74	4.74
Variable density $n\rho$=-1, $n\mu=nk=0$	4.19	4.62	5.08
Variable viscosity $n\mu$=0.66, $n\rho=nk=0$	4.54	4.64	4.73
Variable conductivity nk=0.82, $n\rho=n\mu=0$	5.01	4.97	4.94
Variable all properties $N\rho$=-1, $n\mu$= 0.66 nk=0.82	4.19	4.72	5.34

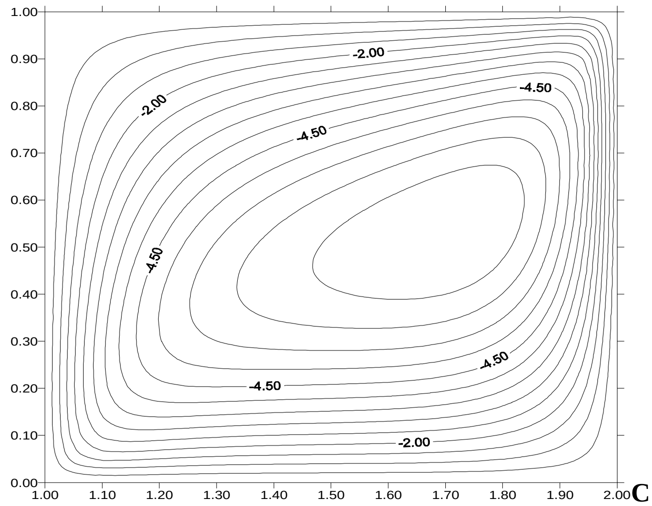

Fig 6: Streamlines for Ra=100,A=1, R^*=2, Θ^* =0.5 and q=0.4 (variable property)

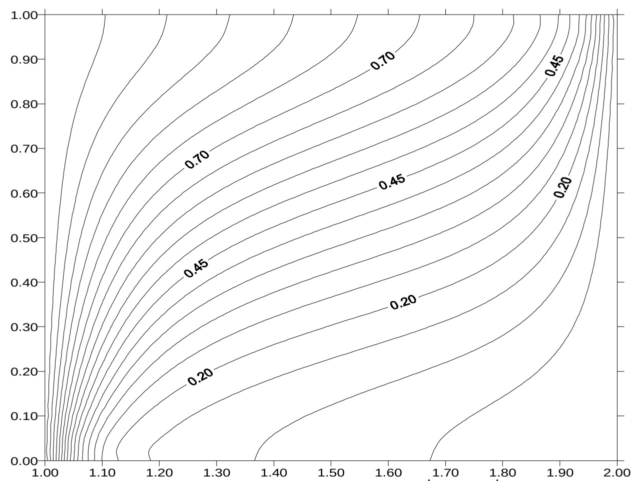

Fig 7: Isotherms for Ra=100,A=1, R^*=2 Θ^* =0.5 and q=0.4 (variable property)

CONCLUSIONS

The following conclusions are drawn from the numerical analysis of natural convection in cylindrical porous annuli:

- For temperature difference ratio, Θ^*< 0.1, the average Nusselt number for variable and constant properties are nearly same. For Θ^*> 0.1, there is a deviation in average Nusselt number values for constant and variable properties.
- The flow fields, temperature fields and average Nusselt number values are nearly same for variable and constant properties if thermo physical properties of fluid are evaluated at a reference temperature, $T_q = T_c + 0.4(T_h - T_c)$.

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