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NUMERICAL SIMULATION OF HYDRODYNAMICS AND HEAT EXCHANGE IN A MULTIBOND PART OF THE IN-LINE HEAT EXCHANGER

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ABSTRACT

In this paper a process of heat and momentum transfer in the in-line cross flow heat exchanger is investigated on the base of numerical solution of Navier-Stokes equations and energy conservation equation.

Obtaining numerical solutions for apparatus of this type has a number of difficulties related primarily to the construction of orthogonal difference grid. Therefore, the vast number of papers is related to the analysis of fluid flow and heat transfer around a cylindrical tube or its symmetric part rather than the entire tube bank. The present work deals with the investigation of heat and momentum transfer for the entire tube bank of heat exchanger. The numerical method used to solve the problem is based on the difference grid which is orthogonal in the entire region.

The method consists in sharing a general Cartesian coordinate system and a local cylindrical coordinate system near each of the cylindrical channel. An exchange of the information between the coordinate systems is carried out in the region of low velocity and temperature gradients.

It allows, on the one hand, to obtain a strict orthogonality on the surface of cylindrical channels and, on the other hand, to use a smaller spatial step in the field of large velocity and temperature gradients at the same time. The results are discussed in terms of dependences of Nusselt number on Reynolds number.

INTRODUCTION

The research deals with the problem of laminar flow of the incompressible Newtonian liquid in a laminar stream past cylinder shaped tube banks. The simulation of temperature and velocity fields was carried out for the flow past the first three rows of the heat exchanger and also for the so- called "middle" cylinder, i.e. for a cylindrical channel located behind a great number of cylindrical tubes. In fact, the analysis of hydrodynamics and heat exchange was carried out for a multibond area consisting of one, two, three cylinders which are placed one after another in a straight line with the same step.

NOMENCLATURE

p	[Pa]	pressure
x	[m]	Cartesian axis direction
y	[m]	Cartesian axis direction
U_x, U_r, U_φ, U_y	[m/s]	components of velocity
ψ	[m^2/s]	current function
ρ	[kg/m^3]	density
τ		dimensionless the time coordinate
t	[K]	time coordinate
ν	[m^2/s]	kinematic viscosity coefficient
a	[m^2/s]	thermal diffusivity
α	[Vt/K·m^2]	heat transfer coefficient
Ω	[1/s^2]	vorticity
T	[K]	temperature
Pr		Number Prandtl
Re		Number Reynolds
Nu		Nusselt number

NUMERICAL METHOD

To solve the problem numerically Navier-Stokes equations for the impulse transfer and the heat transfer equations are applied. The method of solution involves the use of Cartesian coordinate system and the polar one. For Cartesian coordinate system the non-dimensivnal equations are given in the form:

$$\begin{cases} \frac{\partial U_x}{\partial \tau} + U_x \frac{\partial U_x}{\partial x} + U_y \frac{\partial U_x}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{\mathrm{Re}} \nabla^2 U_x, \\ \frac{\partial U_y}{\partial \tau} + U_x \frac{\partial U_y}{\partial x} + U_y \frac{\partial U_y}{\partial y} = -\frac{\partial P}{\partial y} + \frac{1}{\mathrm{Re}} \nabla^2 U_y, \\ \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} = 0, \\ \frac{\partial \theta}{\partial \tau} + U_x \frac{\partial \theta}{\partial x} + U_y \frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Re\,Pr}} \nabla^2 \theta. \end{cases} \quad (1)$$

In the polar coordinate system we have

$$\begin{cases} \frac{\partial U_r}{\partial \tau} + U_r \frac{\partial U_r}{\partial r} + \frac{U_\varphi}{r} \frac{\partial U_r}{\partial \varphi} - \frac{U_\varphi^2}{r} = -\frac{\partial P}{\partial r} + \\ + \frac{1}{\mathrm{Re}} \left(\nabla_c^2 U_r - \frac{U_r}{r^2} - \frac{2}{r^2} \frac{\partial U_\varphi}{\partial \varphi} \right), \\ \frac{\partial U_\varphi}{\partial \tau} + U_r \frac{\partial U_\varphi}{\partial r} + \frac{U_\varphi}{r} \frac{\partial U_\varphi}{\partial \varphi} + \frac{U_\varphi U_r}{r} = -\frac{1}{r} \frac{\partial P}{\partial \varphi} + \\ + \frac{1}{\mathrm{Re}} \left(\nabla_c^2 U_\varphi - \frac{U_\varphi}{r^2} + \frac{2}{r^2} \frac{\partial U_r}{\partial \varphi} \right), \\ \frac{\partial U_r}{\partial r} + \frac{U_r}{r} + \frac{1}{r} \frac{\partial U_\varphi}{\partial \varphi} - = 0, \\ \frac{\partial \theta}{\partial \tau} + U_r \frac{\partial \theta}{\partial r} + \frac{U_\varphi}{r} \frac{\partial \theta}{\partial \varphi} = \frac{1}{\mathrm{Re\,Pr}} \nabla_c^2 \theta. \end{cases} \quad (2)$$

To solve the flat and axially symmetric problems the most widely known method of solving Navier-Stokes equations for an incompressible fluid is the way of applying the alternating values of the eddy vortex – the current function. We will use the determination of the vorticity:

$$\Omega = \frac{\partial U_x}{\partial y} - \frac{\partial U_y}{\partial x} \quad (3)$$

We introduce the current function ψ, such that

$$U_x = \frac{\partial \psi}{\partial y}, \quad U_y = -\frac{\partial \psi}{\partial x}. \quad (4)$$

By substituting the velocity components into the formula to determine the vorticity one obtains Poisson equation for the current function:

$$\Omega = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}. \quad (5)$$

Then it is necessary to obtain the equation for the vorticity transfer. For this purpose the equation for the speed component transfer U_x should be differencated with respect to y, and the U_y components - with respect to x. By subtracting the second equation from the first one and converting the obtained expression by means of the continuity equation and by determining the vorticity (3), one obtains the equation for the vorticity transfer. In this way the system of the differential equations (1) is transfered into the vorticity transfer equations, Poisson equation for the function of the current and the equation of heat transfer, as a result it has the following form:

$$\begin{cases} \frac{\partial \Omega}{\partial \tau} + U_x \frac{\partial \Omega}{\partial x} + U_y \frac{\partial \Omega}{\partial y} = \frac{1}{\mathrm{Re}} \nabla^2 \Omega, \\ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \Omega, \\ \frac{\partial \theta}{\partial \tau} + U_x \frac{\partial \theta}{\partial x} + U_y \frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Re\,Pr}} \nabla^2 \theta. \end{cases} \quad (6)$$

In the polar coordinate system the vorticity equation has the following form:

$$\Omega = \frac{\partial U_\varphi}{\partial r} - \frac{1}{r} \frac{\partial U_r}{\partial \varphi} + \frac{U_\varphi}{r}. \quad (7)$$

The current function ψ is determined by the following relations

$$U_r = \frac{1}{r} \frac{\partial \psi}{\partial \varphi}, \quad U_\varphi = -\frac{\partial \psi}{\partial r}. \quad (8)$$

It should be noted that the choice of the sign for the components of velocity U_r, U_φ (in the formula (8)) should correspond to the sign chosen for the velocity components U_x, U_y. One can obtain Poisson equation for the current function in the polar coordinates in the same way.

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \varphi^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} = -\Omega. \quad (9)$$

Thus, by analogy with Cartesian coordinate system, one can obtain the system of equations in terms of the variables the vorticity - the current function

$$\begin{cases} \frac{\partial \Omega}{\partial \tau} + U_r \frac{\partial \Omega}{\partial r} + \frac{U_\varphi}{r} \frac{\partial \Omega}{\partial \varphi} = \frac{1}{\mathrm{Re}} \nabla_c^2 \Omega, \\ \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \varphi^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} = -\Omega, \\ \frac{\partial \theta}{\partial \tau} + U_r \frac{\partial \theta}{\partial r} + \frac{U_\varphi}{r} \frac{\partial \theta}{\partial \varphi} = \frac{1}{\mathrm{Re\,Pr}} \nabla_c^2 \theta. \end{cases} \quad (10)$$

The equation system (6) describing the hydrodynamics and the heat transfer in Cartesian coordinate system is applied to solve the problem under consideration within the whole range in the heat exchanger excluding the local areas around the cylindrical tubes for which the equation system (10) was used. The calculated domain with two rows of tubes is presented in figure 1 to characterise the boundary conditions and those of the information exchange between the coordinate system. The equation system (10) is solved in the vicinity of every cylinder in the polar coordinate system (in figure 1 it is given for one of the cylinders in the area between C_1C_2 D_1D_2 circles). Cartesian coordinate system is a principal one where all the local coordinate systems are involved. Moreover, for convenience of calculations the coordinate lines of Cartesian coordinate system in the vicinity of the cylindrical tubes are chosen in the form as rectangles and as a result the calculation region in Cartesian coordinate system is turned into a rectangle $A_1B_1B_2B_3B_4A_4A_3A_2$ in figure 1.

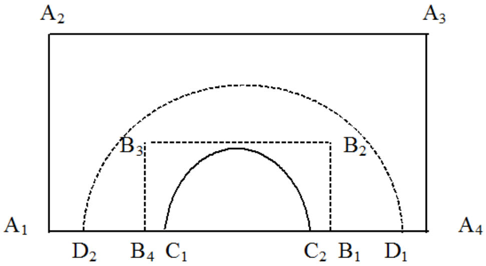

Figure 1 The calculated geometrical region.

The peculiarity of the proposed method is the information transmission from one coordinate system to another. The exchange by the computed information occurs by means of a linear interpolation in a certain geometric area where there exists a solution both in Cartesian system as well as in the polar one. The way to exchange is shown in detail in figure 1. In this way the transmission of the data from Cartesian coordinate system to polar one is realized along the circumference C_1C_2, which is disposed in the field of solving the system (6) in Cartesian coordinate system (the domain $A_1A_2A_3A_4B_4B_3B_2B_1A_1$). Respectively, the information from the polar coordinate system to Cartesian one is transferred along the $B_1B_2B_3B_4$ line to the domains of computation of the polar coordinate system which is disposed between the circles C_1C_2 and D_1D_2. On the boundaries the values of the Ω, ψ, θ variables are transmitted from one coordinate system to another. As the numerical computations of the equation systems (6) and (10) with the second order of precision show that the information transmission by means of the linear interpolation is quite sufficient. One should also point out that the proposed method of solving makes it possible to dispose the boundaries of transition from one coordinate system to another in the regions of relatively small temperature gradients and speeds, which, in turn, raises the accuracy of determining magnitudes and essentially simplifies the formulation of the boundary conditions. The choice of a denser difference grid in the polar coordinate system makes it possible to obtain a more accurate solutions in the boundary layer immediately near the streamlined body. In this case the grid steps in Cartesian coordinate system should be taken considerably longer and, on the whole, this does not influence the accuracy of solution and decreases the time for the solution the problem.

The equations of the vorticity and heat transfer are solved by the implicit method of the alternating directions [2] both for Cartesian and polar coordinate systems. The scheme of this method to solve the equations of vorticity and energy transfer are written in the form of two semi-steps along the time

$$\frac{f^{n+\frac{1}{2}} - f^n}{\Delta t / 2} = \Lambda_x f^{n+\frac{1}{2}} + \Lambda_y f^n + q^n, \qquad , \qquad (11)$$
$$\frac{f^{n+1} - f^{n+\frac{1}{2}}}{\Delta t / 2} = \Lambda_x f^{n+\frac{1}{2}} + \Lambda_y f^{n+1} + q^{n+\frac{1}{2}},$$

here Λ_x and Λ_y are accordingly the operators of the first and second derivatives along the coordinates x and y. A non-conservative central-difference scheme was used for the convective terms in the transfer equations at small Reynolds numbers (a network number Re < 1) and the conservative exponential scheme [3] was used at large Reynolds numbers, where for this case the transfer equations (6) and (10) were written in a conservative form. Poisson equation for the current function was solved by an iterative method of the sequential upper relaxation [1] which essential accelerates the process of solving the problem, the latter consisting in recalculation of the new values ψ, using the formula

$$\Psi_{i,j}^{n+1} = \alpha_1 \Psi_{i,j}^{n+1} + \left(1 - \alpha_1\right) \Psi_{i,j}^{n}. \qquad (12)$$

In the calculations the relaxation coefficient α_1 varied in value from 1.6 to 1.8.

BOUNDARY CONDITIONS

A special feature of defining the boundary conditions is their definition of the polar coordinate system on the surface of cylindrical channels as well as the definition of the other boundary conditions in Cartesian coordinate system. In particular, the current function has a constant value on the symmetry axis A_2A_3 and A_1A_4 Ω=0, and for the temperature the condition is $\partial\theta / \partial y = 0$. The value of speed and temperature was taken to be a consistent at the entrance to the heat exchanger A_1A_2 (figure 1). On the exit boundary from the heat exchanger A_3A_4 (figure 1) the “soft” boundary conditions were used for all alternating values. On the surface of the cylinder circumference (the circle D_1D_2 figure 1), a physical condition of adhesion to the wall and non-penetration of a liquid on the wall were applied, the latter is transformed in the polar coordinate system into the condition

$$\Omega_w = \left(\partial^2 \psi / \partial r^2\right)_w. \qquad (13)$$

By factorization of the current function into Taylor series this condition may be brought to a famous difference formula of Thom first order or Woods second order accuracy [1] which are of the form:

$$\Omega_w = \frac{2\left(\psi_{w+1} - \psi_w\right)}{\Delta r^2}, \Omega_w = \frac{3\left(\psi_{w+1} - \psi_w\right)}{\Delta r^2} - \frac{1}{2}\Omega_{w+1}. \qquad (14)$$

The temperature on the surface of the cylinders was supposed be constant and to be equal to unity in a dimensionless form. While calculating the input conditions for the "middle" cylinder the speed and temperature profiles obtained from the “soft” conditions at the exit were used for the speed and temperature profiles.

PROCESSING OF RESULTS

To check the reliability of the obtained results the operations were performed, they dealt with the stream line flow of a single heated cylinder having a constant temperature by the endless stream. Figure 2 shows the temperature distribution while the stream of a single cylinder for numbers Re = 10, 100.

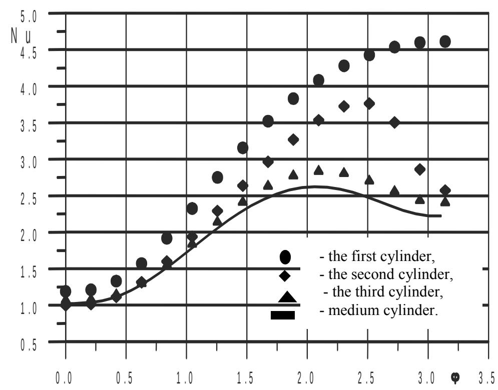

Figure 1 The distribution of the local Nu distribution along the surface of cylinders in an in-line heat exchanger.

Figure 3 and 4 show the temperature distribution for the first pair cylinders of and the three following ones for different Re numbers. The calculated values of the average number Nu were compared in accordance with the criterion Re with the data taken from paper [4] (figure 5). One can see from the figures that a good agreement of the results has been achieved

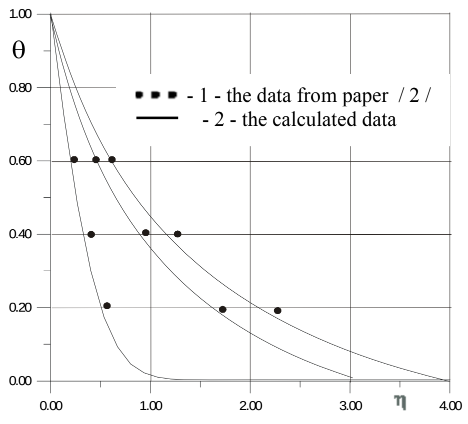

Figure 3 The temperature distribution in a laminar boundary layer on longitudinally streamlined plate

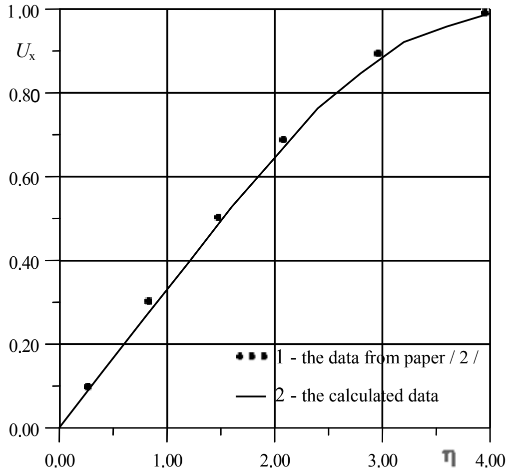

Figure 4 The speed profile in a laminar boundary layer on longitudinally stream lined plate

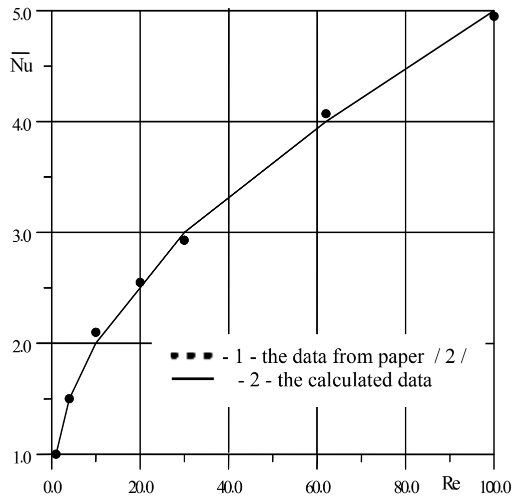

Figure 5 The dependence of the average Nu on Re

While analyzing the first three cylinders and the "middle" ones of the in-line heat exchanger for a fixed number Re (Re=10) (figure 6) the difference in the Nu distribution was found out along the surface. The second and the following cylinders get into the vortex region formed behind the front of standing bodies. It is natural that the conditions of flowing over this region are worse than in the front part of a single cylinder, and, therefore, the maximum value of the local heat removal coefficient is removed into the depth along the flow current. One can see on the figure that difference between the second and the third cylinder begins to decrease, therefore the differences between the third cylinder and the next one will be minimum. This fact is confirmed by the curve for the “middle” cylinder.

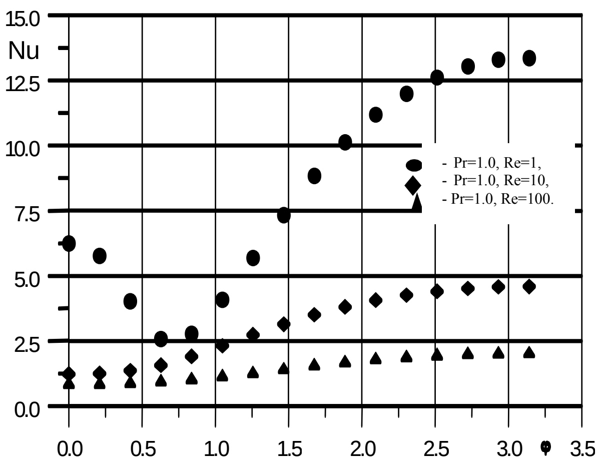

Figure 6 The distribution of the local number Nu the surface of the first cylinder in the in-line heat exchanger at

CONCLUSION

This paper presents the calculation method for the heat exchanger of the in-line type for the laminar flow regime. The method allows one to compute the heat exchangers with a different profile type of the cross-sections of the canals in a strictly orthogonal coordinate system. On the basis of this method the algorithm of the numerical solution was developed and the analysis of the results concerning the influence of different factors on the heat exchange process was carried out. Further, the authors intend to give a generalization of the proposed method on the basis of the turbulent flow regime in the heat exchangers of the in-line and staggered type.

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