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APPLYING TAGUCHI AND GREY RELATIONAL METHODS TO A HEAT EXCHANGER WITH COIL SPRINGS

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ABSTRACT

The goal of this investigation is to apply Taguchi and Grey relational methods, to a concentric heat exchanger application, in order to optimize the design parameters. Coil spring types of the turbulators are used in the experimental study. The design parameters are; Reynolds number ($3000 \leq Re \leq 10000$), outer diameter of the springs (D_s = 7.2, 9.5 and 12 mm), number of the springs (N_s = 4, 5 and 6) and the incline angle of the springs ($\theta = 0°$, 7° and 10°). Nusselt number and friction factor are the results of the study. As a result, the effects of Re, θ, N_s, and D_s on Nusselt number is found to be 79.9%, 3.11%, 1.70% and 13.8%, respectively. Furthermore effect of all told four parameters on friction factor is found as 56.72%, 2.98%, 13.16% and 22.57%.

INTRODUCTION

A heat exchanger is a piece of equipment built for efficient heat transfer from one medium to another. The media may be separated by a solid wall to prevent mixing or they may be in direct contact [1]. Heat exchangers are widely used in industry, especially in heating, ventilation and air-conditioning processes as evaporator, condenser, heater, refrigerator, etc [2].

Turbulators are the most commonly used way to enhance heat transfer in the heat exchangers. A turbulator is a device that turns a laminar flow into a turbulent flow. The heat transfer coefficient for liquids and gases flowing through pipes in heat exchangers tends to be limited due to a fluid boundary layer close to the pipe wall that is stagnant or moves at slow speed, thus acting as an insulating layer. This boundary layer can be broken or reduced in thickness if turbulators are placed in the pipe, which create a turbulent flow that reduces the boundary-layer thickness and thereby increase the heat-transfer coefficient. Examples of turbulators for pipe flow are: twisted-tape, brock, wire, springs, cones, etc. [1].

The co-author of this study has already worked on coil-springs inserted in a concentric tube. The experimentally found results was evaluated by means of the 1st [2] and 2nd [3] Law of Thermodynamics. The pitch of the coil-wire [3-11], the diameter of the coil-wire [5, 6, 8, 9, 10], and the length and the segmentation of the coil-spring [12] are mostly common parameters that are investigated by researchers.

As seen from the literature the design parameters may vary, and all of these variations are done to enhance the heat transfer of course. However, enhancing the heat transfer causes with an enhancing pressure drop meanwhile. Furthermore, to estimate the optimum number of design parameter is a great important problem.

El Sayed et al. [13] investigated the effects of height, thickness, inter-fin spaces, number and tip-shroud clearance of fins on the heat transfer, fluid flow and pressure drop. Naik et al. [14] proposed a design correlation which shows the distribution of optimal rib spacing for a wide range of rib geometries and operational conditions. Sahin et.al [15] investigated the effects of the longitudinal and lateral separations of consecutively enlarged-contracted arranged fin pairs, widths of the fins, angle of attack, heights of fins and flow velocity on the heat and pressure drop characteristics by using the Taguchi method.

The Taguchi Method is new-developed method which involves design of experiments [16]. It is developed by Genichi Taguchi in order to improve the quality of manufactured goods, and more recently also applied to engineering biotechnology marketing and advertising [17].

A second way of parameter optimization is the Grey method. The Grey prediction has also been widely used in studies of social sciences, agriculture, procreation, power consumption and management. In contrast to the other approaches, the Grey prediction needs as few as four data items without presuming sequential distribution of the lagged data. [18].

TAGUCHI AND GREY METHODS FOR OPTIMIZATION

The Taguchi method defines the process objective, or more specifically, a target value for a performance measure of the process. This may be a flow rate, temperature, etc. The target of a process may also be a minimum or maximum; for example, the goal may be to maximize the output flow rate. The deviation in the performance characteristic from the target value is used to define the loss function for the process. The method, determines the design parameters affecting the process. Parameters are variables within the process that affect the performance measure such as temperatures, pressures, etc. that can be easily controlled. The number of levels that the parameters should be varied at must be specified.

By Taguchi method, orthogonal arrays can be created for the parameter design indicating the number of and conditions for each experiment. The selection of orthogonal arrays is based on the number of parameters and the levels of variation for each parameter, and will be expounded below. Finally, the method conducts the experiments indicated in the completed array to collect data on the effect on the performance measure, and it completes data analysis to determine the effect of the different parameters on the performance measure.

In Taguchi method, a loss function is used to calculate the deviation between the experimental value and the desired value. This loss function is further transformed into a signal-to-noise (*S/N*) ratio [19]. Several *S/N* ratios are available depending on the type of characteristics; lower is better (*LB*), nominal is best (*NB*), or higher is better (*HB*). The *S/N* ratios which condenses the multiple data points within a trial, depends on the type of characteristics being evaluated. In this study higher Nusselt number with lower friction factor is the indication of better performance. Therefore, *HB* for the *Nu* number and *LB* for the friction factor are selected for obtaining optimum performance characteristics:

$$L_{HB} = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i^2} \tag{1}$$

$$L_{LB} = \frac{1}{n} \sum_{i=1}^{n} y_i^2 \tag{2}$$

where y expresses the resulting value and n indicates the number of experiments run under such experimental conditions.

The *S/N* ratio η_{ij} for the i_{th} performance characteristic in the j_{th} experiment can be expressed as:

$$\eta_{ij} = -10\log(L_{ij}) \tag{3}$$

An analysis of variance ANOVA is performed to determine which parameters are statistically significant. The *S/N* ratio and ANOVA analyses allow the prediction of the optimal combination of process parameters [20]. A confirmation experiment is then conducted to verify the optimal process parameters determined from the parameter design. ANOVA and F_{test} are used to analyse the experimental data as [16]:

$$S_m = \frac{\left(\sum n_i\right)^2}{J}, \; S_T = \sum \eta_i^2 - S_m \tag{4}$$

$$S_A = \frac{\left(\sum n_{Ai}^2\right)^2}{N} - S_m, \; S_E = S_T - \sum S_A \tag{5}$$

$$V_A = \frac{S_A}{f_A}, \; F_{Ao} = \frac{V_A}{V_E} \tag{6}$$

where S_T is the sum of squares due to the total variation, S_m is the sum of squares due to the means, S_A is the sum of squares due to parameter A (in present study A represents Reynolds number), S_E is the sum of squares due to error, η_{ij} is the η value of each experiment, J is the number of experiments in orthogonal array, η_{Ai} is the sum of the i^{th} level of parameter A, N is repeating number of each level of parameter A, f_A is the degree of freedom of parameter A, and V_A is the variance of parameter A.

Grey relational analysis, which is based on the grey system theory, can be used to determine the complicated inter-relationships between multiple performance characteristics. The grey relational coefficient is found as follows [21]:

$$r\left(x_0(k), x_i(k)\right) = \frac{\min\limits_{i}\min\limits_{k}\left|x_0(k) - x_i(k)\right| + \xi \max\limits_{i}\max\limits_{k}\left|x_0(k) - x_i(k)\right|}{\left|x_0(k) - x_i(k)\right| + \xi \max\limits_{i}\max\limits_{k}\left|x_0(k) - x_i(k)\right|} \tag{7}$$

where $x_i(k)$ is the normalized value of the k^{th} performance characteristic in the i^{th} experiment and ξ is the distinguishing coefficient ($\xi \in |0,\ 1|$). The value of ξ can be adjusted in accordance with actual system requirements. The grey relational grade is a weighting-sum of the grey relational coefficient. It is defined as follows [21]:

$$r\left(x_0, x_i\right) = \frac{1}{p} \sum_{k=1}^{p} r\left(r_0(k), x_i(k)\right) \tag{8}$$

where p is the number of performance characteristics.

For grey relational analysis, the experimental results for *Nu* number and friction factor are first normalized in the range between 0 and 1, which is termed, grey relational generation. Using the grey relational analysis and the statistical analysis of variance, the optimal combination of parameters can be predicted.

EXPERIMENTAL PROCEDURE

Figure 1 presents the schematic view of the whole set up. Detailed information about the experimental procedure is already given in an article of co-authors [2]. Reynolds number ($\sim 3000 < Re < \sim 10000$) base on the flowrate of the air flow and diameter of inner pipe, incline angle (position of the spring in the inner tube) of the springs (θ = 0°, 7° and 10°), number of the springs (N_s = 4, 5 and 6), and outer diameter of the springs (D_s = 7.2, 9.5, 12 mm) are the independent design parameters that affect the heat transfer and pressure drop. Nusselt numbers (*Nu*) and friction factor (*f*) are the non-dimensional dependent parameters that are aimed to be obtained as the results.

Table 1 Factors and levels used in the experiments

Parameter	***Re***	**θ**	***N***	***D***
Par. Code	**A**	**B**	**C**	**D**
Level 1	3399	0	4	7.2
Level 2	4676	7	5	9.5
Level 3	5972	10	6	12
Level 4	7277	--	--	--
Level 5	8564	--	--	--
Level 6	9890	--	--	--

Varying Reynolds number from ~3000 to ~10000 means turbulent flow is considered as flow region. As previously mentioned, the independent parameters are coded as A, B, C, D for reading facility. The control factors and their levels are listed in Table 1. According to Taguchi quality design concept, a L_{18} orthogonal array with 18 rows (corresponding to the number of experiments) is set and presented in Table 2.

Regardless of category of the performance characteristics, a greater η value corresponds to a better performance. Therefore, the optimal level of the design parameters is the level with the greatest η value. By applying Eqs. (1) (2) and (3), the η values for each experiment of L_{18} array (Table 2) are calculated. The results for Nusselt number are exhibited in Table 3 and the results for friction factor are given in Table 4.

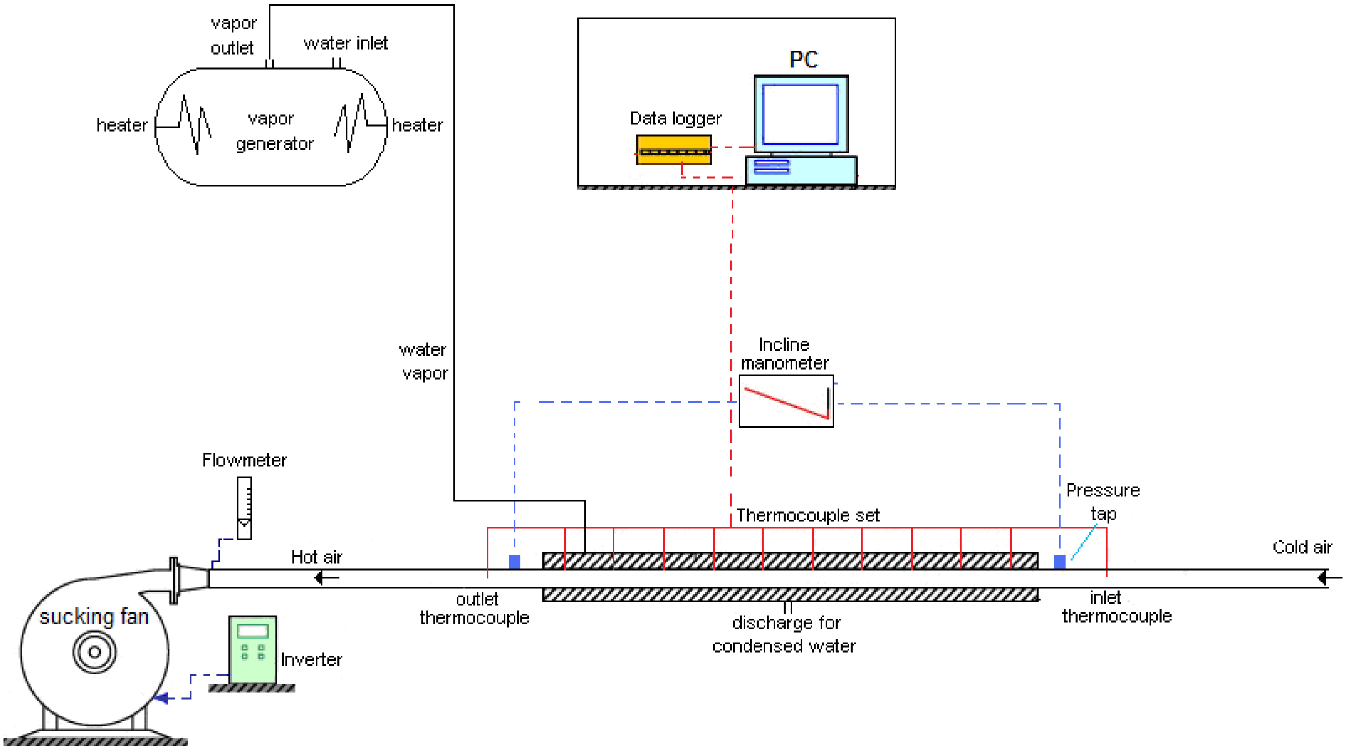

Figure 1 Schematic view of experimental setup [2]

Table 2 *S/N* ratios for Nusselt number and friction factor

	Control factors				*S/N* ratio (η)	*S/N* ratio (η)
Exp.no	**A**	**B**	**C**	**D**	**Nu**	**f**
1	1	1	1	1	25.8019438	1.771767
2	1	2	2	2	28.33638361	3.960798
3	1	3	3	3	30.19963341	7.976429
4	2	1	1	2	28.35773713	1.880798
5	2	2	2	3	32.56714502	3.251545
6	2	3	3	1	30.57218232	3.211967
7	3	1	2	1	30.89207267	1.158993
8	3	2	3	2	33.04241161	1.988726
9	3	3	1	3	34.19201291	2.240665
10	4	1	3	3	34.9953445	2.228327
1	4	2	1	1	32.25906609	0.942919
12	4	3	2	2	33.83647692	1.507140
13	5	1	2	3	35.74906779	1.609814
14	5	2	3	1	34.24474189	1.203786
15	5	3	1	2	34.70622808	1.032122
16	6	1	3	2	35.66878072	1.463796
17	6	2	1	3	37.470597	1.525015
18	6	3	2	1	35.25912007	0.992112

According to Table 3, it is seen that, based on the analysis of *S/N* ratio, the effects of the optimal Reynolds number, incline angle, spring numbers, springs' diameter, and are found to be 36.13 (Level 6-A6), 33.13 (Level 3-B3), 33.12 (Level 3–C3) and 34.2 (Level 3-D3), respectively. It is to say that; if we consider the levels A6B3C3D3, we can obtain the optimal Nusselt number.

Table 3 Average *S/N* ratios for Nusselt number

Control factors	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
A	28.11	30.50	32.71	33.70	34.90	36.13[*]
B	31.91	32.99	33.13[*]	--	--	--
C	32.13	32.77	33.12[*]	--	--	--
D	31.50	32.32	34.20[*]	--	--	--

[*] Optimum level, Overall mean = 32.68 dB

Table 4 Average *S/N* ratios for friction factor

Control factors	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
A	-11.65	-8.62	-4.75	-3.34	-2.01[*]	-2.30
B	-4.36[*]	-5.49	-6.49	--	--	--
C	-3.48[*]	-5.18	-7.67	--	--	--
D	-2.90[*]	-5.09	-8.34	--	--	--

[*] Optimum level, Overall mean = -5.45 dB

A similar conclusion can be made about the friction factor by considering Table 4. From the table we see that, the effects of the optimal Reynolds number, incline angle, spring numbers, springs' diameter are found to be -2.01 (Level 5-A5), -4.36 (Level 1-B1), -3.48 (Level 1–C1) and -2.9 (Level 1-D1), respectively. It means; if we consider the levels A5B1C1D1, we can obtain the optimal friction factor. Fig 2 represents these results in graph form for Nusselt number, and Fig 3 does it for friction factor.

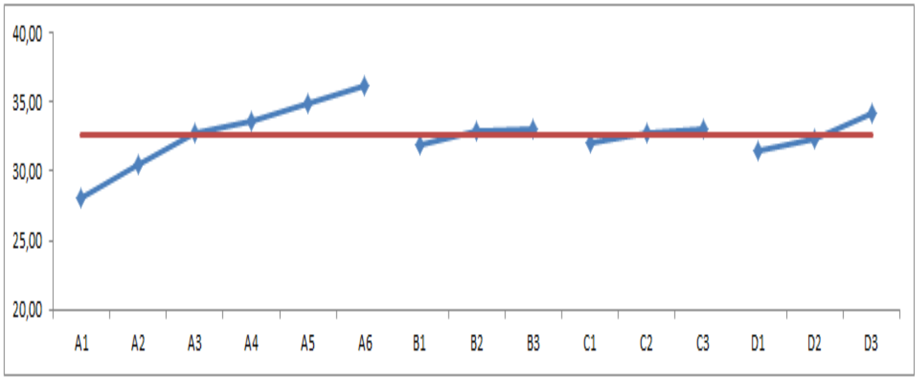

Figure 2 *S/N* ratios for Nusselt number

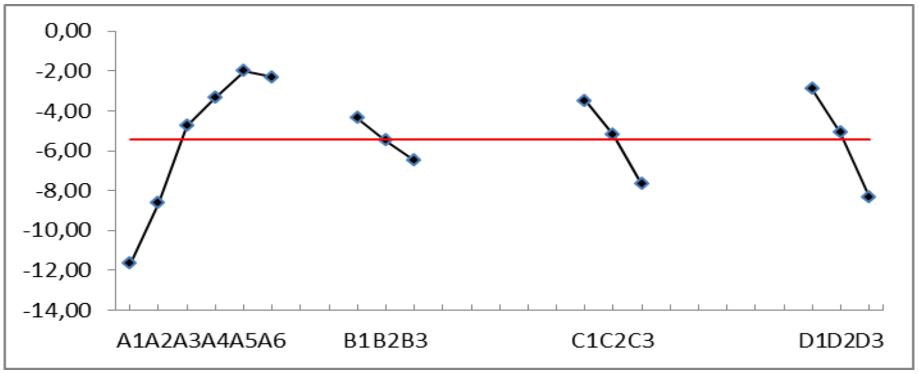

Figure 3 *S/N* ratios for friction factor

A better feel for the relative effect of the different experimental parameters on Nusselt number and friction factor are obtained by decomposition of variance, which is called analysis of variance. The magnitude of the effects of the experimental parameters on Nusselt number and friction factor are aimed to be determined by means of variance analysis method ANOVA [22]. The ANOVA evaluates the experimental errors and test of significance to understand the effect of various factors. The method and equation of the ANOVA are in Table 5 and Table 6, respectively for Nusselt number and friction factor.

Table 5 Results of the analysis of variance for Nusselt number

	degree of freedom (df)	sum of square (SS)	variance (V)	F_{test}	$F_{0.05}$	contri-bution %
A	5	130.50	26.1	198.5*	4.38	79.9
B	2	5.316	2.65	20.22*	5.14	3.11
C	2	3.023	1.51	11.49*	5.14	1.70
D	2	22.826	11.4	86.83*	5.14	13.88
error	6	0.788	0.13	--	--	1.38
sum	--	162.46	--	--	--	100

* Significant at least 95% confidence level

Table 6 Results of the analysis of variance for friction factor

	degree of freedom (df)	sum of square (SS)	variance (V)	F_{test}	$F_{0.05}$	Cont-ribution %
A	5	225.750	45.1	43.1*	4.38	56.72
B	2	13.685	6.84	6.53*	5.14	2.98
C	2	53.244	26.6	25.4*	5.14	13.16
D	2	89.838	44.9	42.9*	5.14	22.57
error	6	6.278	1.04			4.57
sum		338.796				100

* Significant at least 95% confidence level

The F_{test} is used to make a decision about the quantity of the effects', in other words; the magnitude of the found analyses' results is evaluated by means of F_{test}. The calculated F values are compared to appropriate standard confidence tables which are presented by Ross's textbook [22]. In the case of any F value turns out as a result of the mentioned comparison to be higher than such F value on the table, it is concluded that the analysis is at the assumed confidence level.

According to this analysis, the most effective parameters with respect to Nusselt number is the Reynolds number (43.145) and then diameter of the spring, incline angle and number of the springs in order. On the other hand, the effect of

Re number is the highest on friction factor and the diameter of the springs, number of the springs and inclined angle comes orderly.

Table 7 and Table 8 show the confirmation for the analysis. It is seen here that, the estimated Nusselt number by the analysis is 84.64, and it is found 86.88 by the experiments. When the friction factor is handled it is observed that the predicted *f* is 0.66, but the experimentally found one is 0.68.

Table 7 Results of the confirmation experiment for Nusselt number

	Initial parameters	Optimum parameters	
		Prediction	Experiment
Level	A6B3C2D1	A6B3C3D3	A6B3C3D3
Nu	59.66	84.64	86.88
S/N ratio (*dB*)	35.51	38.55	38.78

* Improvement of S/N ratio for Nusselt number = 3.27 *dB*

Table 8 Results of the confirmation experiment for friction factor

	Initial parameters	Optimum parameters	
		Prediction	Experiment
Level	A6B3C2D1	A5B1C1D1	A5B1C1D1
f	1.065	0.66	0.68
S/N ratio (*dB*)	-0.54	3.59	3.35

* Improvement of *S/N* ratio for Nusselt number = 3.27 *dB*

Now the attention is turned to the grey relational analysis. The results of the analysis are presented in Table 9 and Table 10. According to the performed experiment design, it is clearly observed from Table 9 and Fig 4 that, experiment 17 has the highest grey relational grade. It means the highest *Nu* number and lowest friction factor that is obtained at the same time can be reached at experiment 17. So it is the optimal value.

Table 9 Grey relation coefficient and grey relational grade values

Exp. no	Control factor				Grey Relational coeff.		Grey relational grade	Order
	A	B	C	D	*Nu*	*f*		
1	1	1	1	1	0.333	0.809	0.571	14
2	1	2	2	2	0.362	0.538	0.450	17
3	1	3	3	3	0.395	0.333	0.364	18
4	2	1	1	2	0.363	0.789	0.576	13
5	2	2	2	3	0.461	0.604	0.533	15
6	2	3	3	1	0.403	0.608	0.505	16
7	3	1	2	1	0.410	0.942	0.676	9
8	3	2	3	2	0.481	0.771	0.626	12
9	3	3	1	3	0.540	0.730	0.635	11
10	4	1	3	3	0.598	0.732	0.665	10
11	4	2	1	1	0.450	1.00	0.725	7
12	4	3	2	2	0.519	0.862	0.691	8
13	5	1	2	3	0.673	0.841	0.757	5
14	5	2	3	1	0.544	0.931	0.737	6
15	5	3	1	2	0.575	0.975	0.775	3
16	6	1	3	2	0.664	0.871	0.767	4
17	6	2	1	3	1.00	0.858	0.929	1
18	6	3	2	1	0.622	0.986	0.804	2

The grey relational grade values for each level of the turbulator parameters are calculated by using the same method. The grey relational grade values are shown in Table 10. Since the grey relational grade represents the level of correlation between the reference sequence and the comparability of sequence, the greater value of the grey relational grade means that the compatibility sequence has a stronger correlation, to the reference sequence.

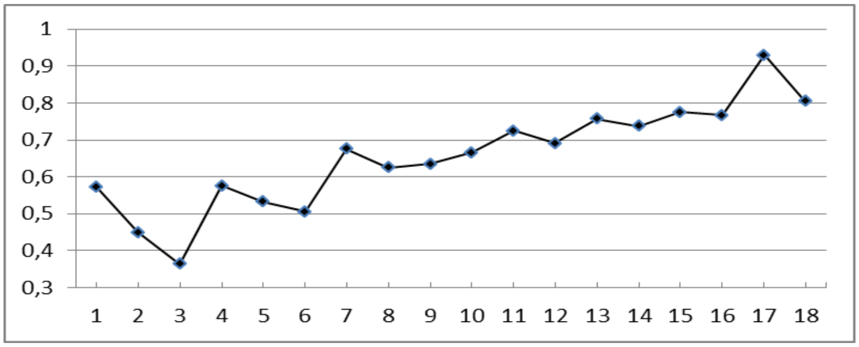

Figure 4 Grey relational grades

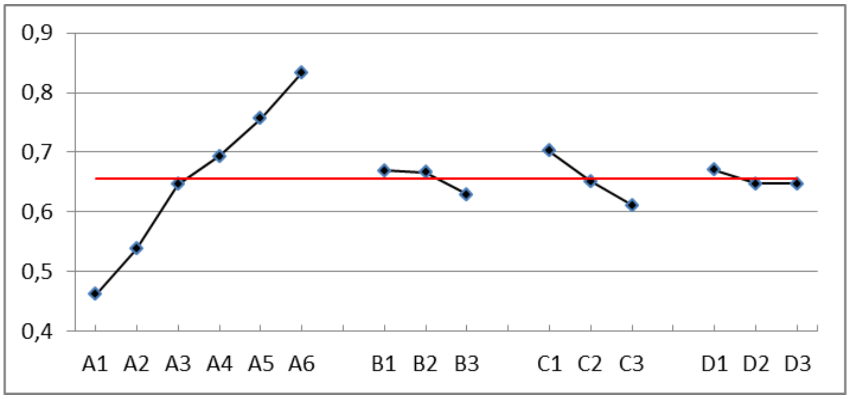

Figure 5 Effect of design parameters on multi performance characteristics

Table 10 The response table for grey relational grade

Control factor	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6	Max-Min
A	0.462	0.538	0.646	0.694	0.756	0.833*	0.372
B	0.669*	0.667	0.629	--	--	--	0.040
C	0.702*	0.652	0.611	--	--	--	0.091
D	0.670*	0.648	0.647	--	--	--	0.023

* overal mean =0 65

Based on the grey relational grade values given in Table 10, the optimal design parameter was obtained for *Re* number at Level 6, (A6), incline angle Level 1 (B1), number of the springs Level 1, (C1), and diameter of the springs Level 1 (D1). Effect of design parameters on the multi-performance is also given in Fig 5 for better understanding. These results improve that the for maximum Nusselt number, minimum friction factor the optimal case can be reached with the experiment A6B1C1D1.

CONCLUSIONS

In this study, an experimental work on the energy analysis of a concentric tube heat exchanger with coil spring turbulators was handled. The optimization of the design parameters were performed by use of two methods, namely Taguchi and Grey methods. The effect of design parameters, such as Re number, incline angle of the springs in the tube, diameter of the spring insert the tube and number of the springs, on the Nusselt number and friction factor were aimed to be determined.

As a result, the effects of design parameters Re, θ, N_s, and D_s on Nusselt number was found as 79.9%, 3.11%, 1.70% and 13.8%, respectively. As it is expected, increasing heat transfer causes increasing friction factor. By these methods, the effect of four design parameters on friction factor was found as 56.72%, 2.98%, 13.16% and 22.57%.

It is concluded that the optimal case was obtained with the experiment A6B1C1D1, which indicates maximum Nusselt number, minimum friction factor.

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