

MIXED CONVECTION FLOW OF COUPLE STRESS FLUID BETWEEN VERTICAL PARALLEL PLATES WITH RADIATION AND CHEMICAL REACTION EFFECTS

Kaladhar K.*¹ and Srinivasacharya D.²

*¹Department of Mathematics, National Institute of Technology Puducherry, Karaikal, India

¹E-mail: kk_r_nitw@yahoo.co.in

²Department of Mathematics, National Institute of Technology Warangal-506 004, India

²E-mail: dsc@nitw.ac.in

ABSTRACT

This paper studies the effects of thermal radiation and chemical reaction on two dimensional incompressible couple stress fluid flow with mixed convective heat and mass transfer between two vertical parallel plates in a porous space. The plates are kept at different but constant temperature and concentrations. The governing non-linear partial differential equations are transformed into a system of ordinary differential equations using similarity transformations. The resulting equations are then solved using the homotopy analysis method. The effects of the radiation parameter, chemical reaction parameter and couple stress fluid parameter on velocity, temperature and concentrations are discussed and shown graphically. Also the effects of the pertinent parameters on the rates of heat and mass transfer are tabulated.

INTRODUCTION

In space technology applications and at higher operating temperatures, radiation effects can be quite significant. Since radiation is quite complicated, many aspects of its effect on free convection or combined convection have not been studied in recent years. The combined radiation and mixed convection from a vertical wall with suction/injection in a non- Darcy porous medium was studied by Murthy et al. [1]. Grosan and Pop [2] considered the effect of thermal radiation on fully developed mixed convection flow in a vertical channel. Raptis [3] studied the influence of radiation on free convection flow through a porous medium. Most recently, radiation effects on mixed convection about a cone embedded in a porous medium filled with a nanofluid have been presented numerically by Chamkha et al. [4].

Diffusion rates can be tremendously altered by chemical reactions. Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. Studies on chemical reaction in channel/near plate date back to 1958, when Chambre and Young [5] have analyzed a first- order chemical reaction near a stationary horizontal plate using a similarity transformation. Das et al. [6] have studied the effect of a homogeneous first-order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer using Laplace

NOMENCLATURE

| | |
|--------|--|
| A | Constant pressure gradient |
| Br | Brinkman number |
| C | Concentration |
| C_p | Specific heat at constant pressure |
| C_S | Concentration susceptibility |
| C_T | Temperature ratio |
| D | Solutal diffusivity |
| f | Reduced stream function |
| g | Acceleration due to gravity |
| Gr_C | Mass Grashof number |
| Gr_T | Temperature Grashof number |
| K_f | Coefficient of thermal conductivity |
| K_T | Thermal diffusion ratio |
| Nu | Nusselt number |
| p | Pressure |
| Pr | Prandtl number |
| q'' | Radiation heat flux |
| R | Suction induction parameter |
| Ra | Radiation parameter |
| Re | Reynolds number |
| S | Couple stress parameter |
| Sc | Schmidt number |
| Sh | Sherwood number |
| T | Temperature |
| T_m | Mean fluid temperature |
| u, v | Velocity components in the x and y directions respectively |
| x, y | Cartesian coordinates along the plate and normal to it |

Special characters

| | |
|--------------------|---|
| α | Thermal diffusivity |
| β_T, β_c | Coefficients of thermal and solutal expansion |
| \mathcal{X} | Mean absorption coefficient |
| η | Similarity variable |
| η_1 | Coupling material constant |
| σ | Stefan-Boltzman constant |
| θ | Dimensionless temperature |
| ϕ | Dimensionless concentration |
| μ | Dynamic viscosity |
| ν | Kinematic viscosity |
| ρ | Density of the fluid |

transform technique. Recently, Prathap Kumar et al. [7] presented the effect of homogeneous and heterogeneous reactions on the solute dispersion in composite porous medium analytically. Most recently, Saleh et al. [8] discussed the reversal flow of fully developed mixed convection in a vertical channel with chemical reaction.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. The theory of polar fluids and related theories are models for fluids whose microstructure is mechanically significant. The micro-continuum theory of couple stress fluid proposed by Stokes [9], defines the rotational field in terms of the velocity field for setting up the constitutive relationship between the stress and strain rate. In view of applications, Muthuraj et al. [10] have studied the heat and mass transfer effects on MHD flow of a couple-stress fluid in a horizontal wavy channel with viscous dissipation and porous medium. Most recently Hayat et al. [11] analyzed the stagnation point flow of couple stress fluid with melting heat transfer and the analytical study of Hall and Ion-slip effects on mixed convection flow of couple stress fluid between parallel disks have been presented by Srinivasacharya and Kaladhar [12].

The Homotopy analysis method (HAM) was first proposed by Liao in 1992, is one of the most efficient methods in solving different types of nonlinear equations such as coupled, decoupled, homogeneous and non-homogeneous. Also, HAM provides us a great freedom to choose different base functions to express solutions of a nonlinear problem [13]. The application of the HAM in engineering problems is highly considered by scientists, because HAM provides us with a convenient way to control the convergence of approximation series, which is a fundamental qualitative difference in analysis between HAM and other methods. Later, Liao [14] presented an optimal Homotopy Analysis approach for strongly nonlinear differential equations. HAM is used to get analytic approximate solutions for heat transfer of a micropolar fluid through a porous medium with radiation by Rashidi et al. [15]. Recent developments of HAM, like convergence of HAM solution, Optimality of convergence control parameter discussed by Srinivasacharya and Kaladhar [16] for the couple stress fluid.

In this paper, we have investigated the Radiation effect on steady mixed convective heat and mass transfer flow between two vertical parallel plates in couple stress fluid with chemical reaction. The Homotopy Analysis method is employed to solve the governing nonlinear equations. The behavior of emerging flow parameters on the velocity, temperature and concentration are discussed.

MATHEMATICAL FORMULATION

The Consider a steady fully developed laminar mixed convection flow of a couple stress fluid between two vertical parallel plates distance $2d$ apart. Choose the coordinate system such that x - axis be taken along vertically upward direction through the central line of the channel, y is perpendicular to the

plates and the two plates are infinitely extended in the direction of x . The plate $y = -d$ has given the uniform temperature T_1 and concentration C_1 , while the plate $y = d$ is subjected to a uniform temperature T_2 and concentration C_2 . Since the boundaries in the x direction are of infinite dimensions, without loss of generality, we assume that the physical quantities depend on y only. The fluid properties are assumed to be constant except for density variations in the buoyancy force term. The fluid is considered to be a gray, absorbing/emitting radiation, but non-scattering, medium and the Rosseland approximation [17] is used to describe the radiative heat flux in the energy equation. In addition, the thermo diffusion effects considered. The flow is a mixed convection flow taking place under thermal buoyancy and uniform pressure gradient in the flow direction. The flow configuration and the coordinates system are shown in Figure 1. The fluid velocity u is assumed to be parallel to the x -axis, so that only the x -component u of the velocity vector does not vanish but the transpiration cross-flow velocity v_0 remains constant, where $v_0 < 0$ is the velocity of suction and $v_0 > 0$ is the velocity of injection.

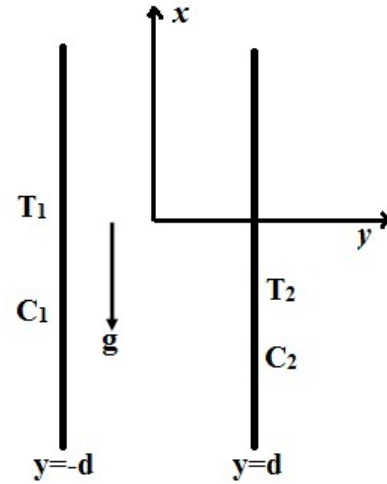


Figure 1: Physical model and coordinate system

With the above assumptions and Boussinesq approximations with energy and concentration, the equations governing the steady flow of an incompressible couple stress fluid are

$$\frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\rho v_0 \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} - \eta_1 \frac{\partial^4 u}{\partial y^4} - \frac{\partial p}{\partial x} + \rho g \beta_T (T - T_1) + \rho g \beta_C (C - C_1) \quad (2)$$

$$v_0 \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q^r}{\partial y} + 2 \frac{v}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\eta_1}{\rho C_p} \left(\frac{\partial^2 u}{\partial y^2} \right)^2 \quad (3)$$

$$v_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 (C - C_1) \quad (4)$$

where u is the velocity component along x direction, ρ is the density, g is the acceleration due to gravity, p is the pressure, μ is the coefficient of viscosity, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansion, α is the

thermal diffusivity, D is the mass diffusivity, C_p is the specific heat capacity, C_s is the concentration susceptibility, T_m is the mean fluid temperature, K_T is the thermal diffusion ratio, K_f is the coefficient of thermal conductivity, η_l is the additional viscosity coefficient which specifies the character of couple-stresses in the fluid and q^r is the radiation heat flux.

Using the Rosseland approximation for radiation [17] the radiative heat flux q^r is simplified as

$$q^r = -\frac{4\sigma}{3\chi} \frac{\partial T^4}{\partial y} \quad (5)$$

where σ is the Stefan - Boltzman constant, χ is the mean absorption coefficient

The boundary conditions are

$$u=0 \text{ at } y=\pm d \quad (6a)$$

$$u_{yy}=0 \text{ at } y=\pm d \quad (6b)$$

$$T=T_1, C=C_1 \text{ at } y=-d \text{ and } T=T_2, C=C_2 \text{ at } y=d \quad (6c)$$

The boundary condition (6a) corresponds to the classical no-slip condition from viscous fluid dynamics. The boundary condition (6b) implies that the couple stresses are zero at the plate surfaces.

Introducing the following similarity transformations

$$y = \eta d, u = u_0 f, T - T_1 = (T_2 - T_1) \theta$$

$$C - C_1 = (C_2 - C_1) \phi, p = \frac{\mu u_0}{d^2} P \quad (7)$$

in equations (2) - (4), we get the following nonlinear system of differential equations

$$S^2 f^{(iv)} - f'' + Rf' - \frac{Gr_T}{Re} \theta - \frac{Gr_C}{Re} \phi + A = 0 \quad (8)$$

$$\theta'' - RPr \theta' + \frac{4}{3} R_d [(C_T + \theta)] + 2Br(f')^2 + S^2 Br(f'')^2 = 0 \quad (9)$$

$$\phi'' - RSc \phi' - KSc \phi = 0 \quad (10)$$

where primes denote differentiation with respect to η alone,

$Re = \frac{u_0 d}{\nu}$ is the Reynolds number, $Sc = \frac{\nu}{D}$ is the Schmidt

number, $Pr = \frac{\mu C_p}{K_T}$ is the Prandtl number, $R = \frac{\nu_0 d}{\nu}$ is the

suction/induction parameter, $A = \frac{dp}{dx}$ is the constant pressure

gradient, $Gr_T = \frac{g\beta_T(T_2 - T_1)d^3}{\nu^2}$ is the temperature Grashof

number, $Gr_C = \frac{g\beta_T(C_2 - C_1)d^3}{\nu^2}$ is the mass Grashof number,

$C_T = \frac{T_1}{T_2 - T_1}$ is the temperature ratio, $R_d = \frac{4\sigma(T_2 - T_1)^3}{K_T \chi}$ is the

radiation parameter, $Br = \frac{\mu \nu^2}{K_T d^2 (T_2 - T_1)}$ is the Brinkman

number, $S_r = \frac{DK_T(T_2 - T_1)}{\nu T_m(C_2 - C_1)}$ is the Soret number, $S = \frac{1}{d} \sqrt{\frac{\eta_1}{\mu}}$ is the couple stress parameter.

Boundary conditions (6) in terms of f, θ, ϕ become

$$\begin{aligned} f = 0, f'' = 0, \theta = 0, \phi = 0 \text{ at } \eta = -1 \\ f = 0, f'' = 0, \theta = 1, \phi = 1 \text{ at } \eta = 1 \end{aligned} \quad (11)$$

The physical quantities of interest in this problem the Nusselt number and the Sherwood number. The heat and mass fluxes at the vertical plate surfaces can be obtained from

$$q_m = \left[-D \frac{\partial C}{\partial y} \right]_{y=\pm d}, q_w = \left[-K_f \frac{\partial T}{\partial y} + q^r \right]_{y=\pm d} \quad (12)$$

The Nusselt number $Nu = \frac{q_w d}{K_f(T_2 - T_1)}$ and the Sherwood

number $Sh = \frac{q_m d}{D(C_2 - C_1)}$ are given by

$$Nu_{1,2} = - \left[1 + \frac{4}{3} (C_T + \theta)^3 \right] \theta' \Big|_{\eta=-1,1}, Sh_{1,2} = - \phi' \Big|_{\eta=-1,1} \quad (13)$$

Effect of the various parameters involved in the investigation on these coefficients is discussed in the following section.

THE HAM SOLUTION OF THE PROBLEM

For HAM solutions, we choose the initial approximations of $f(\eta), \theta(\eta)$ and $\phi(\eta)$ as follows:

$$f_0(\eta) = 0, \theta_0(\eta) = \frac{1+\eta}{2}, \phi_0(\eta) = \frac{1+\eta}{2} \quad (14)$$

and choose the auxiliary linear operators:

$$L_1 = \frac{\partial^4}{\partial y^4}, L_2 = \frac{\partial^2}{\partial y^2} \quad (15)$$

such that

$$L_1(c_1 + c_2 \eta + c_3 \eta^2 + c_4 \eta^3) = 0, L_2(c_5 + c_6 \eta) = 0 \quad (16)$$

where $c_i (i = 1, 2, \dots, 6)$ are constants. Introducing non-zero auxiliary parameters h_1, h_2 and h_3 , we develop the Zeroth-order deformation problems as follow:

$$(1-p)L_1[f(\eta; p) - f_0(\eta)] = ph_1 N_1[f(\eta; p)] \quad (17)$$

$$(1-p)L_2[\theta(\eta; p) - \theta_0(\eta)] = ph_2 N_2[\theta(\eta; p)] \quad (18)$$

$$(1-p)L_2[\phi(\eta; p) - \phi_0(\eta)] = ph_3 N_3[\phi(\eta; p)] \quad (19)$$

subject to the boundary conditions

$$\begin{aligned} f(-1; p) = 0, f(1; p) = 0, f'''(-1; p) = 0 \\ f''(1; p) = 0, \theta(-1; p) = 0, \theta(1; p) = 1 \\ \phi(-1; p) = 0, \phi(1; p) = 1 \end{aligned} \quad (20)$$

where $p \in [0, 1]$ is the embedding parameter and the non-linear operators N_1, N_2 and N_3 are defined as:

$$\begin{aligned} N_1[f(\eta; p), \theta(\eta; p), \phi(\eta; p)] = S^2 f^{(iv)} - f'' \\ + Re f' - \theta - N\phi + A \end{aligned} \quad (21)$$

$$N_2[f(\eta; p), \theta(\eta; p), \phi(\eta; p)] = \theta'' - RPr \theta' + \frac{4}{3} R_d \left[(C_T + \theta) \right] + 2 Br (f')^2 + S^2 Br (f'')^2 \quad (22)$$

$$N_3[f(\eta; p), \theta(\eta; p), \phi(\eta; p)] = \phi'' - RSc \phi' - KSc \phi \quad (23)$$

For $p = 0$ we have the initial guess approximations

$$f(\eta; 0) = f_0(\eta), \theta(\eta; 0) = \theta_0(\eta), \phi(\eta; 0) = \phi_0(\eta) \quad (24)$$

When $p = 1$, equations (17) - (19) are same as (8) - (10)

respectively, therefore at $p = 1$ we get the final solutions

$$f(\eta; 1) = f(\eta), \theta(\eta; 1) = \theta(\eta), \phi(\eta; 1) = \phi(\eta) \quad (25)$$

Hence the process of giving an increment to p from 0 to 1 is the process of $f(\eta; p)$ varying continuously from the initial guess $f_0(\eta)$ to the final solution $f(\eta)$ (similar for $\theta(\eta; p)$ and $\phi(\eta; p)$). This kind of continuous variation is called deformation in topology so that we call system Eqs. (17) - (19), the zeroth-order deformation equation. Next, the m^{th} -order deformation equations follow as

$$L_1[f_m(\eta) - \chi_m f_{m-1}(\eta)] = h_1 R_m^f(\eta) \quad (26)$$

$$L_2[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = h_2 R_m^\theta(\eta) \quad (27)$$

$$L_2[\phi_m(\eta) - \chi_m \phi_{m-1}(\eta)] = h_3 R_m^\phi(\eta) \quad (28)$$

with the boundary conditions

$$f_m(-1) = 0, f_m(1) = 0, f_m''(-1) = 0, f_m''(1) = 0 \quad (29)$$

$$\theta_m(-1) = 0, \theta_m(1) = 0, \phi_m(-1) = 0, \phi_m(1) = 0$$

Where

$$R_m^f = S^2 f^{(iv)} - f'' + \text{Re } f' - \theta - N\phi + A(1 - \chi) \quad (30)$$

$$\begin{aligned} R_m^\theta &= \theta'' + 2Br \sum_{n=0}^{m-1} f_{m-1-n} f_n' + \frac{4}{3} Rd C_T^3 f'' + \frac{4}{3} Rd \\ &+ S^2 Br \sum_{n=0}^{m-1} f_{m-1-n} f_n'' + 3C_T^2 \sum_{n=0}^{m-1} f_{m-1-n} f_n' \\ &+ \frac{4}{3} Rd \sum_{n=0}^{m-1} f_n'' \sum_{i=0}^{m-1-n} f_i' \sum_{j=0}^{m-1-n-i} f_j f_{m-1-n-i-j} \\ &+ 12Rd C_T \sum_{n=0}^{m-1} f_n'' \sum_{i=0}^{m-1-n} f_i f_{m-1-n-i} \\ &+ 8Rd C_T \sum_{n=0}^{m-1} f_n' \sum_{i=0}^{m-1-n} f_i' f_{m-1-n-i} \\ &+ 4Rd C_T^2 \sum_{n=0}^{m-1} f_n' f_{m-1-n} + \frac{4}{3} Rd \\ &+ 4Rd \sum_{n=0}^{m-1} f_n' \sum_{i=0}^{m-1-n} f_i' \sum_{j=0}^{m-1-n-i} f_j f_{m-1-n-i-j} \end{aligned} \quad (31)$$

$$R_m^\phi = \phi'' - RSc \phi' - KSc \phi \quad (32)$$

and, for m being integer

$$\chi_m = 0 \quad \text{for } m \leq 1 \\ \chi_m = 1 \quad \text{for } m > 1 \quad (33)$$

The initial guess approximations $f_0(\eta)$, $\theta_0(\eta)$ and $\phi_0(\eta)$, the linear operators L_1 , L_2 and the auxiliary parameters h_1 , h_2 and h_3

are assumed to be selected such that equations (17) – (19) have solution at each point $p \in [0, 1]$ and also with the help of Taylor's series and due to Eq. (24); $f(\eta; p)$, $\theta(\eta; p)$ and $\phi(\eta; p)$ can be expressed as

$$f(\eta; p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) p^m \quad (34)$$

$$\theta(\eta; p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) p^m \quad (35)$$

$$\phi(\eta; p) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta) p^m \quad (36)$$

in which h_1 , h_2 and h_3 are chosen in such a way that the series (34) - (36) are convergent at $p = 1$. Therefore we have from (25) that

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \quad (37)$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) \quad (38)$$

$$\phi(\eta) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta) \quad (39)$$

for which we presume that the initial guesses to f , θ and ϕ the auxiliary linear operators L and the non-zero auxiliary parameters h_1 , h_2 and h_3 are so properly selected that the deformation $f(\eta; p)$, $\theta(\eta; p)$ and $\phi(\eta; p)$ are smooth enough and their m^{th} -order derivatives with respect to p in equations (37)-(39) exist and are given respectively by

$$f_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m f(\eta; p)}{\partial p^m} \right|_{p=0}, \quad \theta_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \theta(\eta; p)}{\partial p^m} \right|_{p=0},$$

$$\phi_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \phi(\eta; p)}{\partial p^m} \right|_{p=0}$$

It is clear that the convergence of Taylor series at $p = 1$ is a prior assumption, whose justification is provided via a theorem (Srinivasacharya and Kaladhar, [21]), so that the system in (37)-(39) holds true. The formulae in (37)-(39) provide us with a direct relationship between the initial guesses and the exact solutions. All the effects of interaction of the chemical reaction as well as of the mass transfer, Soret and Dufour effects and couple stress flow field can be studied from the exact formulas (37)-(39). Moreover, a special emphasize should be placed here that the m^{th} -order deformation system (26) - (29) is a linear differential equation system with the auxiliary linear operators L whose fundamental solution is known.

RESULTS AND DISCUSSION

In the absence suction/injection parameter R and Buoyancy ratios Gr_T/Re and Gr_C/Re , Eq. (8) reduces to the equation of motion for the flow between parallel plates given in text book by Stokes ([18], page no. 44). Analytical solution of that

equation with type A conditions and HAM solution at different S are shown in Table 1. The comparisons are found to be in a very good agreement. Therefore, the HAM code can be used with great confidence to study the problem considered in this paper.

The expressions for f , θ and ϕ contain the auxiliary parameters h_1 , h_2 and h_3 . As pointed out by Liao (2003), the convergence and the rate of approximation for the HAM solution strongly depend on the values of auxiliary parameter h . For this purpose, h -curves are plotted by choosing h_1 , h_2 and h_3 in such a manner that the solutions (34) - (36) ensure convergence [9]. Here to see the admissible values of h_1 , h_2 and h_3 , the h -curve is plotted for 15th-order of approximation in Fig. 2 by taking the values of the parameters $Sc = 0.22$, $Pr = 0.7$, $Br = 0.1$, $K = 1$, $S = 1$, $Rd = 0.5$ and $CT = 0.1$. It is clearly noted from Fig. 2 that the range for the admissible values of h_1 , h_2 and h_3 are $-1.15 < h_1 < -0.6$, $-1.2 < h_2 < -0.6$ and $-1.15 < h_3 < -0.4$ respectively. A wide valid zone is evident in these figures ensuring convergence of the series.

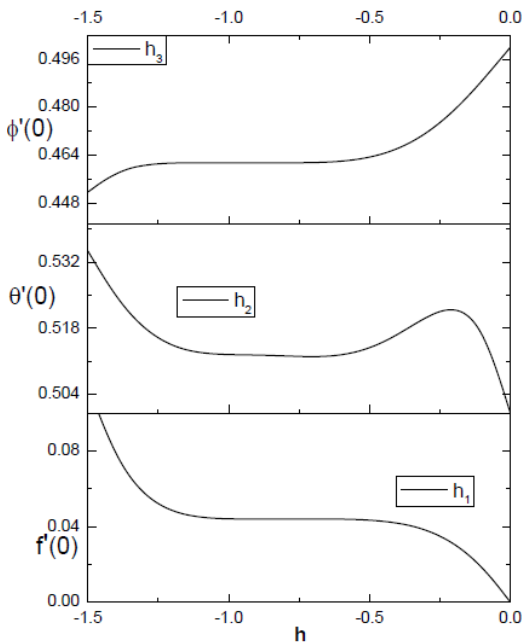


Figure 2: h curve for f , θ and ϕ

The average residual errors are calculated at different order of approximations (m) and found that they are minimum at $h_1 = -0.75$, $h_2 = -0.9$ and $h_3 = -0.9$ respectively. Therefore, the optimum values of convergence control parameters are taken as $h_1 = -0.75$, $h_2 = -0.9$, $h_3 = -0.9$.

The solutions for $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ have been computed and shown graphically in Figs. 3 to 10. The effects of radiation parameter (Rd), chemical reaction parameter (K) and couple stress fluid parameter (S) have been discussed. To study the effect of Rd , K and S , computations were carried out by taking $Pr = 0.7$, $Gr_T = Gr_C = 10$, $Re = 2$, $R = 2$, $Br = 0.1$, $Sc = 0.7$, $C_T = 0.1$ and $A = 1$.

Figures 3 to 4 represent the effect of radiation parameter Rd on $f(\eta)$ and $\theta(\eta)$. It can be seen from these figures that the velocity $f(\eta)$ increase with an increase in the parameter Rd . This implies that the radiation have a retarding influence on the mixed convection flow. The dimensionless temperature increases as Rd increases. The effect of radiation parameter Ra is to increase the temperature significantly in the flow region. The increase in radiation parameter means the release of heat energy from the flow region and so the fluid temperature increases.

Figures 5 to 7 indicate the effect of the couple stress fluid parameter S on $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$. As the couple stress fluid parameter S increases, the velocity $f(\eta)$ decreases. It is also clear that the temperature $\theta(\eta)$ decreases with an increase in S . It can be noted that the velocity in case of couple stress fluid is less than that of a Newtonian fluid case. Thus, the presence of couple stresses in the fluid decreases the velocity and temperature. As the diffusion equation is independent of couple stress fluid parameter, concentration has no significant change with the couple stress parameter.

Figures 8 to 10 represent the effect of chemical reaction K on $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$. It can be seen from these figures that the velocity $f(\eta)$ decrease with an increase in the parameter K . The dimensionless temperature decreases as K increases. The concentration $\phi(\eta)$ decreases with an increase in the parameter K . Higher values of K amount to a fall in the chemical molecular diffusivity, i.e., less diffusion. Therefore, they are obtained by species transfer. An increase in K will suppress species concentration. The concentration distribution decreases at all points of the flow field with the increase in the reaction parameter. This shows that heavier diffusing species have greater retarding effect on the concentration distribution of the flow field.

Variation of chemical reaction parameter (K), Radiation parameter (Rd) together with the Suction/induction parameter (R) is presented in Table 2 with fixed values of other parameters. It can be noted that the heat and mass transfer rates decreases with an increase in K . Further the behavior of the remaining parameters is self evident from the Table 2 and hence is not discussed for brevity.

| | $S=0.5$ | | $S=0.75$ | |
|------|-------------|------------|-------------|-------------|
| | Analytical | HAM | Analytical | HAM |
| -1 | 0 | 0 | 0 | 0 |
| -0.5 | 0.227538568 | 0.22753853 | 0.153714468 | 0.153714468 |
| 0 | 0.316450557 | 0.31645062 | 0.214780383 | 0.214780383 |
| 0.5 | 0.227538568 | 0.22753853 | 0.153714468 | 0.153714468 |
| 1 | 0 | 0 | 0 | 0 |

Table 1: Comparison of flow velocity (f) for $R = Gr_T/Re = Gr_T/Re = 0$.

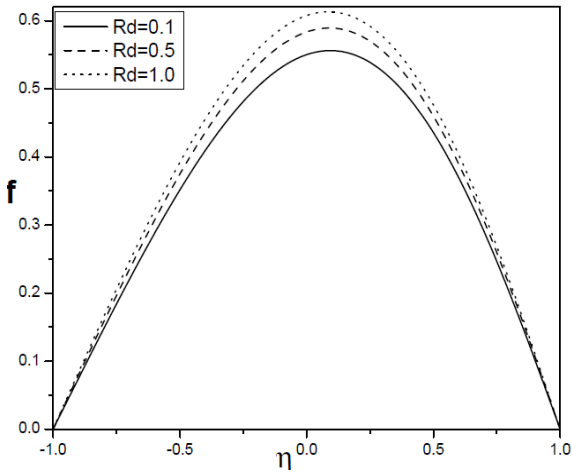


Figure 3: Effect of Rd on f at $S = 1.0, K = 1$

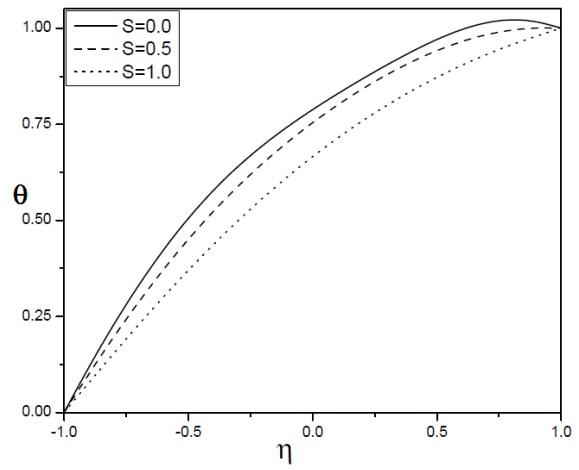


Figure 6: Couple stress parameter S effect on θ at $K = 1, Rd = 0.5$

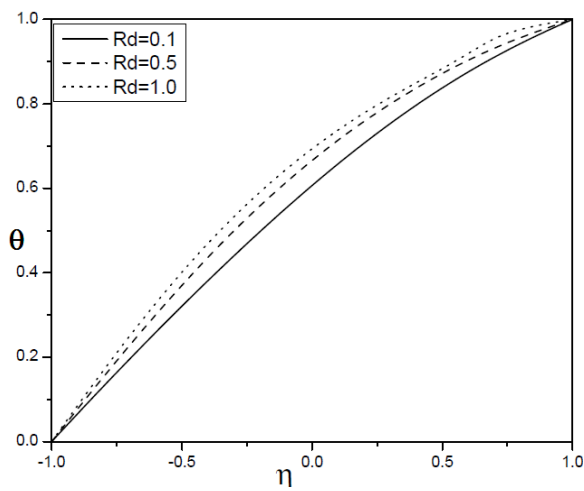


Figure 4: Effect of Rd on θ at $S = 1.0, K = 1$

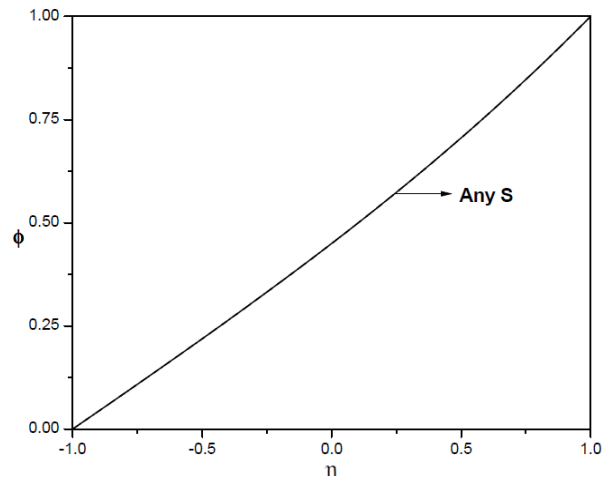


Figure 7: Couple stress parameter effect S on $\phi(\eta)$ at $K = 1, Rd = 0.5$

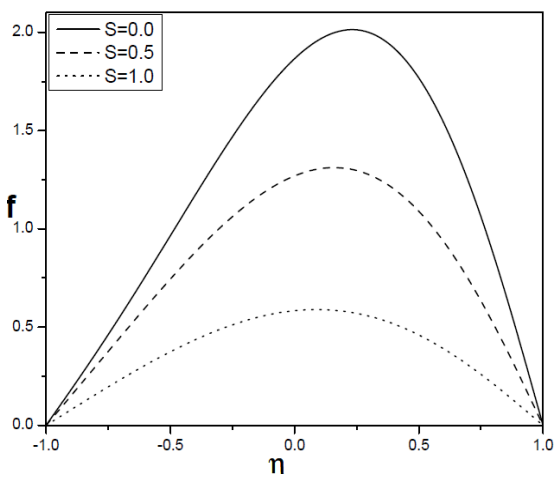


Figure 5: Couple stress parameter effect S on f at $K = 1, Rd = 0.5$

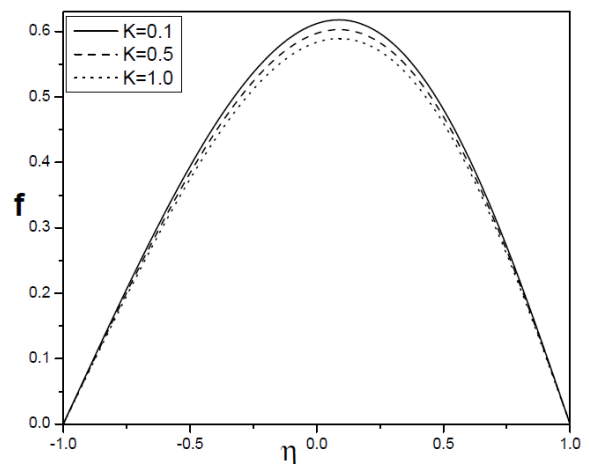


Figure 8: Effect of K on f at $S = 1.0, Rd = 0.5$

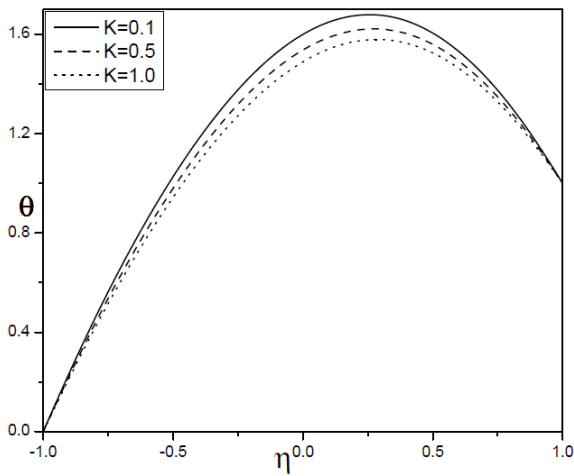


Figure 9: Effect of K on θ at $S = 1.0$, $Rd = 0.5$

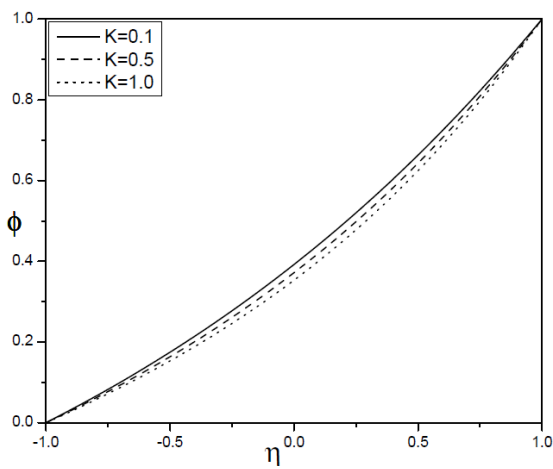


Figure 10: Effect of K on $\phi(\eta)$ at $S = 1.0$, $Rd = 0.5$

| K | Rd | R | Nu_1 | Nu_2 | Sh_1 | Sh_2 |
|-----|-----|---|----------|----------|----------|----------|
| 0.2 | 1 | 2 | -1.07087 | 0.074101 | -0.30301 | -0.78013 |
| 0.4 | 1 | 2 | -1.06213 | 0.067131 | -0.2945 | -0.80779 |
| 0.6 | 1 | 2 | -1.05376 | 0.060443 | -0.28632 | -0.83485 |
| 1 | 0.2 | 2 | -0.9296 | 0.07258 | -0.27088 | -0.88731 |
| 1 | 0.4 | 2 | -1.00293 | 0.052767 | -0.27088 | -0.88731 |
| 1 | 0.6 | 2 | -1.07219 | 0.076515 | -0.27088 | -0.88731 |
| 1 | 0.5 | 0 | -1.26799 | 0.117392 | -0.43356 | -0.63873 |
| 1 | 0.5 | 1 | -1.14956 | 0.087198 | -0.3453 | -0.75593 |
| 1 | 0.5 | 2 | -1.03803 | 0.047852 | -0.27088 | -0.88731 |

Table 2: Variation of heat mass transfer rates at different values of K , Rd , R

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