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THE COMBINED EFFECT OF IRREGULAR SHAPE PARTICLES AND FLUID RHEOLOGY ON SETTLING VELOCITY MEASUREMENT

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ABSTRACT

The drag coefficient of a solid particle depends mainly on the particle Reynolds number, particle sphericity, and fluid rheology. The sphericity of a solid particle is the degree to which the shape of a solid particle approaches that of a sphere. Non-Newtonian fluids are those fluids which do not show linear relationship between shear stress and shear rate. Practically, the apparent viscosity for shear-thinning fluids is decreasing with increasing shear rate. Settlement of solid particles in shear-thinning fluids is of great importance and has many applications in drilling operations, chemical industry, and mining processes.

In this study, the combined effects of particle sphericity and fluid rheology on settling velocity measurement have been studied experimentally. Fifty irregular-shape solid particles with different sphericities (ranged from 0.575 to 0.875), and four shear-thinning fluids with flow behavior indices (ranged from 0.60 to 0.92) were used.

A new drag coefficient charts have been developed for irregular-shape solid particles when they settled down through various shear-thinning fluids, which cover laminar to transient flow regimes around the particles. These charts show linear relationships between the drag coefficient and particle Reynolds number for all fluids, which have the same slope but with different intercepts.

These charts show that the drag coefficient at a given particle Reynolds number is increased as the flow behavior index is decreased (i.e. as the fluid becomes more non-Newtonian), which means higher resistance to particle movement. And, for a given fluid rheology, the drag coefficient is decreased as the particle Reynolds number is increased, which means less resistance to particle movement (i.e. faster slip velocity).

Finally a general equation has been developed for irregular-shape particles when they settle down in various shear-thinning fluids, which can be used to calculate easily and directly the settling velocity and the particle Reynolds number of these particles. This equation can also be used for spherical and disk particles.

INTRODUCTION

The movement of irregular solid particles in shear-thinning fluids has many applications in petroleum, mining, and chemical engineering. In drilling operation, it is required to clean the hole continuously from the cuttings generated at the bottom of the hole in order to have efficient lifting capacity and to prevent many drilling problems. These cuttings have irregular shape and can be lifted to the surface through non-Newtonian drilling fluids, which are mostly shear-thinning fluids.

Previously, different correlations have been developed between drag coefficient and particle Reynolds number, which are basically used for Newtonian fluids only. But incorrectly, most engineers used these correlations to predict the drag coefficient of solid particles moving in non-Newtonian fluids, which gives inaccurate results for all flow regimes around the particle.

The purpose of this study is to develop new drag coefficient charts for irregular shape particles when they settle down through various shear-thinning fluids. Numerous experiments have been done to study the combined effects of the rheological properties and the particles sphericity on the settling velocity measurements.

PREVIOUS WORK

Many authors studied the movement of solid particles in shear-thinning non-Newtonian fluids. Some authors [1-7] stated that the equations developed for Newtonian fluids could be used for non-Newtonian fluids by simply replacing the viscosity term of Newtonian fluid with an apparent viscosity of non-Newtonian fluid.

However, Shah [9] in 1982 clearly stated that this approach is not accurate, and he presented a method whereby particle settling velocity in non-Newtonian fluids can be properly estimated. He proposed plotting C_d^{2-n} versus R_{ep} to show the dependency of C_d on the fluid flow behavior index n. He clearly concluded that the drag coefficient depends on both n and R_{ep}.

Chhabra [1, 3] in 1990 attempted to obtain a unified model to predict the drag coefficient of a falling sphere in power-law fluids. He conclude that the standard curve available for Newtonian fluids provides adequate representation for power-law fluids without any dependence on flow behavior index, within the following ranges of $1.0 < R_{ep} < 1000$ and $0.535 < n < 1.0$.

Vassilios [7] in 2003, proposed an equation to predict the terminal settling velocity of solid spheres falling in non-Newtonian shear thinning fluids. He reached to his equation by comparing used equations with his experimental data. He concluded that the terminal velocity of solid sphere through stagnant shear thinning fluids can be predicted with acceptable accuracy from the correlations used for Newtonian fluids. His conclusion was based on the fact that his experimental data fell along the standard Newtonian drag curve.

El-fadili [12] in 2005 plotted his experimental data as $\sqrt{C_d^{2-n} \cdot R_{ep}^2}$ vs. R_{ep} on a log-log chart, rather than the conventional plot of C_d vs. R_{ep}. He proposed a unified model to predict the drag coefficient of a single particle settling in Newtonian and shear thinning fluids. He concluded that his model is good for all shear thinning fluids, where the particle Reynolds number range is $0.001 < R_{ep} < 1000$.

Aswad [13] in 2008 presented a new and simple approach for estimating the settling velocity of solid particles in shear thinning fluids. His approach includes three steps; the first one is to measure the rheological properties of the given non-Newtonian fluid (i.e. to measure shear stress at different shear rates). The second step is to select the best rheological model that fits this fluid. The third step is to calculate the settling velocity using the appropriate equation, which includes the effect of fluid rheology.

Aswad and Mohammed [14] in 2010 developed new drag coefficient charts for spherical and disk particles settling in various shear-thinning fluids. Their charts included the effects of fluid rheology, particle shape and size, and particle settling velocity, and covered flow behavior indices from 0.60 to 0.92 for laminar to transient flow regimes around the particles. From their charts, they developed empirical correlations between C_d and R_{ep} for spherical and disk particles.

SPHERICITY OF IRREGULAR-SHAPE PARTICLES

Sphericity [15] is defined as the degree to which the shape of a solid particle approaches that of a sphere (or how far a solid particle is from the spherical shape). Sphericity could be thought of as the degree of equality of the three axes of a solid particle, where in a perfect sphere the length, width and thickness are all equal. Thus, the sphericity of a complete spherical particle equals one.

For settling purposes, the sphericity of a solid particle can be defined as [15];

$$\Psi_P = \sqrt[3]{\frac{S^2}{L \bullet I}} \qquad (1)$$

Where;

ψ_P = projection sphericity of a solid particle, dimensionless

S = short dimension of the particle, cm

I = intermediate dimension of the particle, cm

L= long dimension of the particle, cm

SHEAR-THINNING FLUIDS

Non-Newtonian fluids (such as polymers, paints, drilling mud, cement slurries, surfactants, blood, jelly, and starch) are those fluids which do not show linear relationship between shear stress and shear rate (i.e. their shear stress or viscosity is changing with shear rate). Practically, non-Newtonian fluids are classified into three major groups: time-independent fluids, time-dependent fluids, and visco-elastic fluids. The time–independent fluids include three types: Pseudo-plastic fluids (also called shear-thinning fluids), Dilatant fluids (also called shear-thickening fluids), and Bingham plastic fluids. On other hand, the time-dependent fluids include only thixotropic and rheopectic fluids.

Fluid viscosity is defined as the internal resistance of fluid to flow, which is equal to the ratio of shear stress to shear rate. For non-Newtonian fluids, the measured viscosity is the “effective or apparent” viscosity. For shear-thinning fluids, the apparent viscosity decreases as the shear rate is increased, while for shear-thickening fluids the apparent viscosity increases as the shear rate is increased.

SETTLING VELOCITY

The settling velocity of a solid particle is defined as the rate at which the particle settles in still fluid. It is a function of particle size and shape (or particle sphericity), particle density, fluid viscosity (or fluid rheology), and fluid density. Since most solid particles have irregular shapes, all particles will be related to a sphere by using the concept of “equivalent diameter” and the concept of “sphericity”.

Generally, there are three forces acting on a solid particle falling through a given fluid, which are gravity force, buoyancy force due to particle being submerged in fluid, and viscous drag force resulting from fluid resistance to particle movement. When the particle is moving at constant speed, the upward forces are equal to the downward forces. Thus we have;

Drag force + Buoyancy force = Gravity force

$$C_d \left(\frac{1}{2} \rho_f V_s^2 \right) \frac{\pi}{4} d_p^2 + \frac{\pi}{6} d_p^3 \rho_f g = \frac{\pi}{6} d_p^3 \rho_p g \qquad (2)$$

From which, the general slip velocity is defined by equation 3;

$$V_s = \left[\frac{4(\rho_p - \rho_f) \cdot d_p g}{3 \rho_f C_d} \right]^{1/2} \qquad (3)$$

Where;
V_s = slip velocity, cm/sec.
ρ_p = density of the solid particle, g/cm^3.
ρ_f = fluid density, g/cm^3.
C_d = drag coefficient, dimensionless.
d_p = diameter of sphere, or equivalent diameter of a particle having the same volume as a sphere, cm.
g = gravity constant, cm/sec^2.
To calculate the slip velocity of a given particle, the equivalent diameter and the drag coefficient should be determined first.

DRAG COEFFICIENT

The drag coefficient of a solid particle is a dimensionless quantity that is used to quantify the drag or resistance of the particle when it is moving down through a given fluid. It is used in the drag force term, where a lower drag coefficient indicates that the particle will have less hydrodynamic drag. The drag coefficient can be determined from equation 4:

$$C_d = \left[\frac{4(\rho_p - \rho_f) \cdot d_p g}{3\rho_f v_s^2} \right] \quad (4)$$

To calculate the drag coefficient of a given particle, the equivalent diameter and the slip velocity should be determined first.
The drag coefficient is always a function of particle Reynolds number, which is a function of slip velocity, fluid rheology, and particle shape and size. It has been shown from the literature that the drag coefficient is inversely related to the particle Reynolds number, and depending on their range, different relationships between them are developed which are limited only to Newtonian fluids. Inadequately, most engineers used these relations to predict the drag coefficient (or the slip velocity) of solid particles in non-Newtonian fluids, which give inaccurate results for all flow regimes around the particle.

PARTICLE REYNOLDS NUMBER

The particle Reynolds number (or the Reynolds number around the particle), is a dimensionless group which reflects the type of flow regime around a solid particle when settles in a given fluid, and is given by:

$$R_{ep} = \frac{\rho_f \bullet d_p \bullet v_s}{\mu_{app}} \quad (5)$$

Where
R_{ep} = particle Reynolds number, dimensionless.
μ_{app} = apparent fluid viscosity, (dyne-sec)/cm^2

Practically, the values for Reynolds number ranged from 0.001 to 10000, which cover three distinct flow regimes around the particles: laminar, transition, and turbulent.

EXPERIMENTAL DATA

To investigate experimentally the combined effects of particles sphericity and fluids rheology on the drag coefficient chart, fifty irregular-shape solid particles and four non-Newtonian shear-thinning fluids have been used.

The fifty solid particles were carefully selected which then dropped in the four shear-thinning fluids and the settling time was measured. The three dimensions (S, I, and L) of these particles have been measured in the lab using Vernier caliper. Then the sphericities of these solid particles were determined using equation 1. The fifty solid particles were grouped into seven groups according to their sphericity values. The number of irregular particles used and their sphericity value are given in table 1.

The rheological properties of the four shear-thinning fluids (designated as F1, F2, F3, and F4) have been measured in the lab using Chandler-3500 rotational viscometer, which is a direct reading 12-speeds (i.e. 1, 2, 3, 6, 10, 20, 30, 60, 100, 200, 300, and 600 rpm) viscometer. The flow behavior index (n) for fluids F1, F2, F3, and F4 are equal to 0.60, 0.74, 0.86, and 0.92, respectively. The apparent viscosities of these fluids were determined using equation 6.

$$\mu_{app} = \frac{K_1 \bullet K_2 \bullet \theta}{\gamma} \quad (6)$$

Where
K_1 = viscometer constant = 386.0
K_2 = constant = 0.01323
θ = viscometer dial reading, dyne/ cm^2
γ = shear rate, sec^{-1}

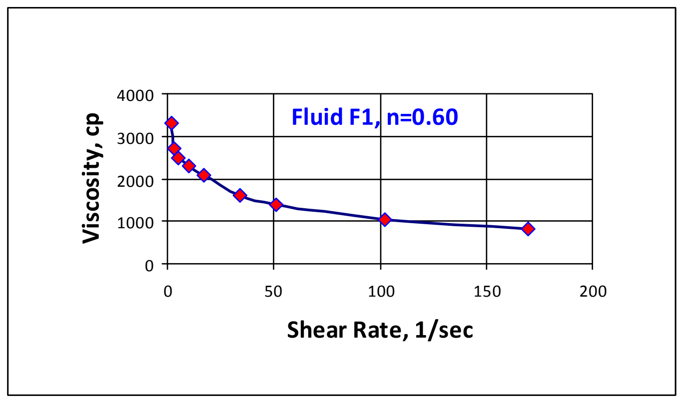

Figure 1 Apparent Viscosity vs. Shear Rate for Fluid F1

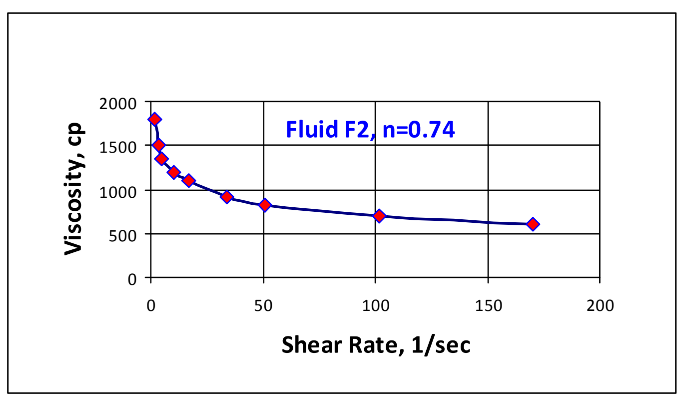

Figure 2 Apparent Viscosity vs. Shear Rate for Fluid F2

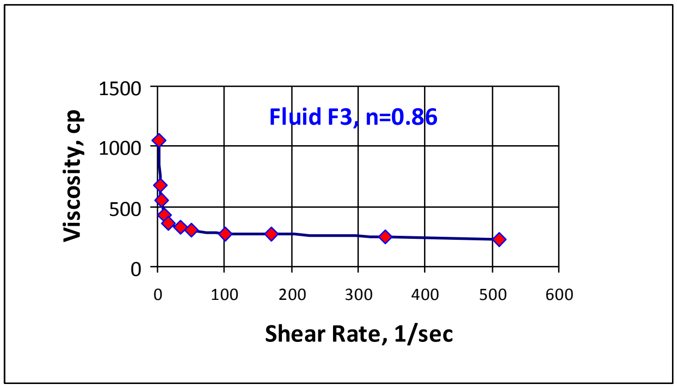

Figure 3 Apparent Viscosity vs. Shear Rate for Fluid F3

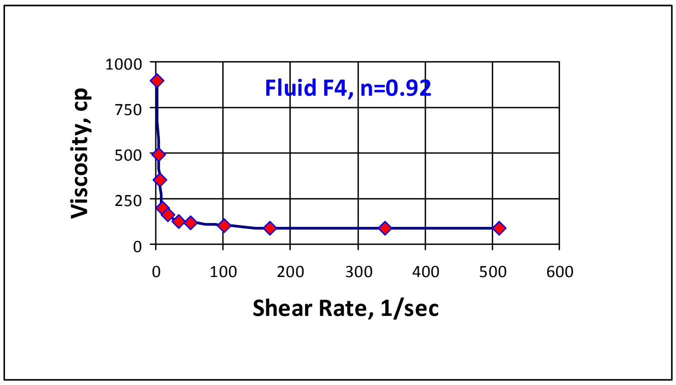

Figure 4 Apparent Viscosity vs. Shear Rate for Fluid F4

RESULTS AND ANALYSIS

1- Figures 1, 2, 3 and 4 show the plots of fluid viscosity vs. shear rate for fluids F1, F2, F3 and F4, respectively. These figures confirmed the shear-thinning behavior of these fluids.

2- The settling velocities of the fifty irregular-shape solid particles through four shear-thinning fluids have been measured experimentally. Some of these results are given in tables 2 and 3 for fluids F1 and F2, respectively, noting that each fluid has different flow behavior index value *n*.

3- For the fifty irregular-shape solid particles, the drag coefficients and the particle Reynolds numbers have been calculated using equations 4 and 5, respectively, in each of the four shear-thinning fluids. Some of these results are given in tables 2 and 3 for fluids F1 and F2, respectively.

4- The drag coefficients are then plotted vs. particle Reynolds numbers for each shear-thinning fluid, on log-log paper as shown in figures 5, 6, 7 and 8. These figures clearly show linear relationship and symmetry between the plotted points. For each tested fluid, the plotted line through the irregular-shape particles gives the same slope as that for spherical particles but with different intercepts at (*Rep = 1.0*). These figures also confirm that the drag coefficients for irregular particles are much higher (about three times) than that for spherical particles. The data for spherical particles are taken from reference [14].

5- Also figures 5, 6, 7 and 8 show that the effect of particle sphericity on drag-coefficient chart is not clear. It is very hard to distinguish between solid particles of different sphericities,

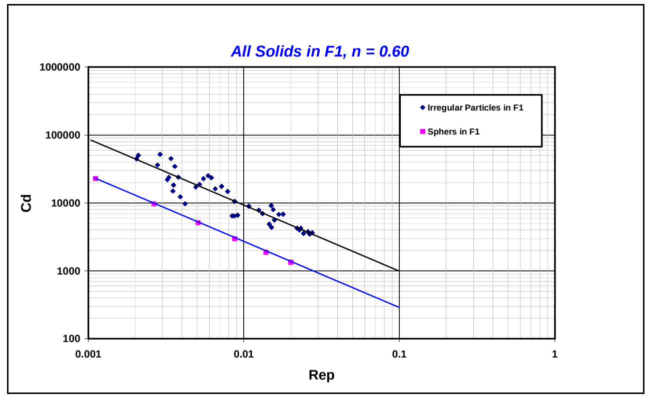

Figure 5 Drag-Coefficient for solid Particles in Fluid F1

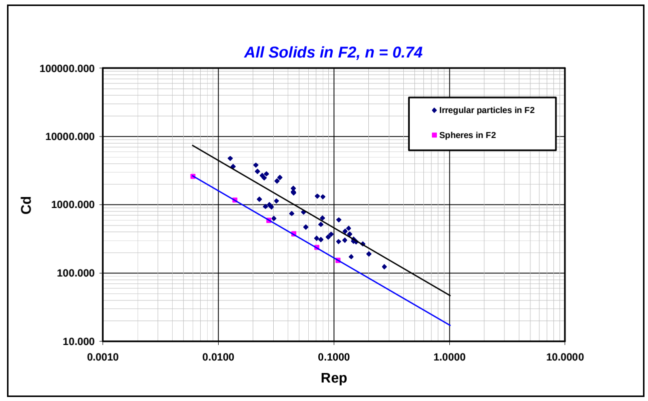

Figure 6 Drag-Coefficient for solid Particles in Fluid F2

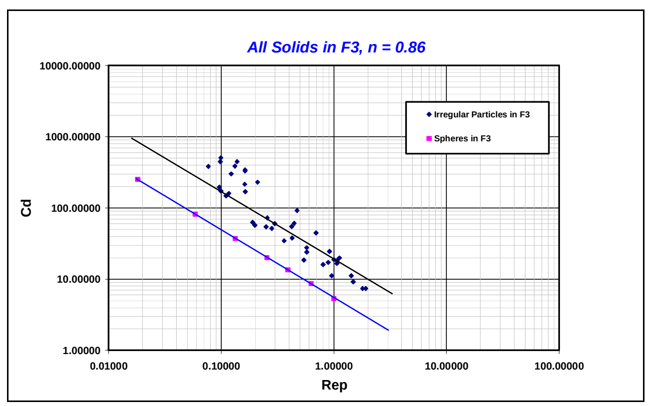

Figure 7 Drag-Coefficient for solid Particles in Fluid F3

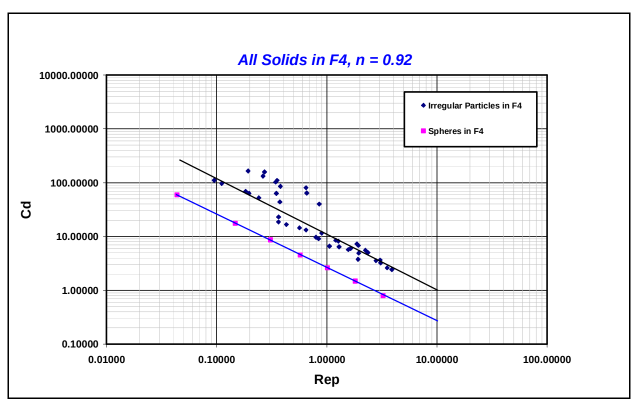

Figure 8 Drag-Coefficient for solid Particles in Fluid F4

which means that there is no clear effect or distinct relationship between them. Therefore, for the seven groups of sphericities, all solid particles are treated together in one linear relationship.

6- Figure 9 shows the general drag coefficient chart for irregular-shape particles using the four shear-thinning fluids on a log-log plot. The four plotted lines have the same slope but with different intercepts, noting that each fluid has different flow behavior index (*n*) value.

7- Also, it is clear from figure 9 that the drag coefficient decreases as the particle Reynolds number increases and as the flow behavior index increases. On the other hand, for a given flow regime around the particle, the drag coefficient increases as the flow behavior index decreases (or as the fluid becomes more non-Newtonian).

Table 1 Sphericity Values for the Irregular Solid Particles

Group Number	Particles Numbers	Sphericity Value, ψ_P
1	41,42,46,49	0.575
2	7,10,15,26,43,45	0.625
3	3,9,13,20,25,28,32,33,34,50	0.675
4	4,12,16,19,22,29,31,37,39,40	0.725
5	1,14,27,35,44,47,48	0.775
6	2,5,6,8,18,21,30,36,38	0.825
7	11,17,23,24	0.875

GENERAL EQUATION

Since figure 9 shows linear relationships between drag coefficient and particle Reynolds number for all fluids, therefore, the following general equation has been adopted, which represents a straight line equation for log-log plots:

$$C_d = \frac{a}{R_{ep}^{b}} \qquad (7)$$

Where:
a = the intercept of the straight line at (*Rep = 1.0*)
b = the slope of the straight line (it is negative slope)

Equation 7 can be used for general irregular-shape particles and for spherical particles.

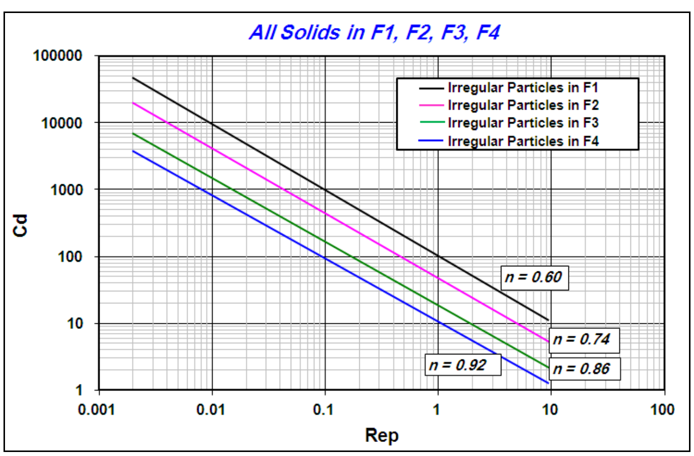

Figure 9 General Drag-Coefficient Chart for Irregular-Shape Particles in Shear-Thinning Fluids

DETERMINATION OF PARAMETERS

The values of the parameters V_S, *Rep*, and C_d can be determined easily and directly by using equation 7 together with figure 9, through the following steps:

1- For any fluid rheology, it is possible to generate the proper straight line that represents the corresponding (*n)* value. Then the value of the intercept (a) can be determined easily at (*Rep = 1.0*).

2- By knowing the values of the constants (a) and (b), one can use equation 7 to determine the slip velocity of any solid particle that settles through a given shear-thinning fluid.

3- Using the slip velocity value, it is possible to determine the particle Reynolds number and the drag coefficient, hence estimating the type of flow regime around the particle and the drag force affecting the falling particle.

Table 2 Some Experimental Results for Fluid F1, *n* = 0.60

Particle No.	dp,(cm)	Ψp	t,(sec)	Vs	Rep	Cd
1	0.737	0.76	39.97	0.378	0.0087	6428.133
2	0.914	0.80	59.34	0.254	0.0072	17562.72
3	0.874	0.67	46.89	0.322	0.0087	10488.82
4	0.951	0.73	86.38	0.175	0.0052	38705.62
5	0.899	0.85	71.72	0.211	0.0059	25220.4
6	0.906	0.82	68.86	0.219	0.0062	23451.23
7	0.951	0.64	30.60	0.493	0.0146	4857.25
8	0.830	0.84	70.91	0.213	0.0055	22785.87
9	0.725	0.68	166.40	0.091	0.0020	109612.5
10	0.914	0.65	54.35	0.278	0.0079	14733.16

L= 15.10 cm, μ = 32.9992 ρ = 1.025

Table 3 Some Experimental Results for Fluid F2, *n* = 0.74

Particle No.	dp,(cm)	Ψp	t,(sec)	Vs	Rep	Cd
1	0.737	0.76	8.95	1.687	0.0708	322.9474
2	0.914	0.80	17.60	0.858	0.0446	1548.072
3	0.874	0.67	8.39	1.800	0.0895	336.4799
4	0.951	0.73	18.29	0.826	0.0446	1738.778
5	0.899	0.85	22.66	0.666	0.0341	2522.669
6	0.906	0.82	17.33	0.871	0.0449	1488.325
7	0.951	0.64	5.79	2.610	0.1412	173.9496
8	0.830	0.84	22.15	0.682	0.0322	2227.753
9	0.725	0.68	24.96	0.605	0.0250	2471.222
10	0.914	0.65	10.16	1.486	0.0773	515.8855

L = 15.10 cm,μ = 17.9996 ρ = 1.024

CONCLUSIONS

1- New drag coefficient charts have been developed for irregular-shape solid particles when they settle down through various shear-thinning fluids, which include the combined effects of fluid rheology, particle sphericity, and particle settling velocity. These chart cover flow behavior indices from 0.60 to 0.92 and particle sphericities from 0.575 to 0.875 for laminar to transient flow regimes around the particles.

2- The general drag coefficient chart shows linear relationships between C_d and R_{ep} for all shear-thinning fluids, which have the same slope but with different intercepts at ($R_{ep} = 1.0$).

3- The drag coefficient at a given particle Reynolds number increases as the flow behavior index decreases (i.e. as the fluid becomes more non-Newtonian), which means higher resistance to particle movement. And, for a given fluid rheology, the drag coefficient decreases as the particle Reynolds number increases, which means less resistance to particle movement (i.e. faster slip velocity).

4- For each fluid rheology, the linear relationship for the irregular-shape particles has the same slope as that for spherical particles but with different intercepts, and the drag coefficients for irregular particles are about three times higher than that for spherical particles.

5- The effect of particle sphericity on drag-coefficient chart is not clear. It is very hard to distinguish between solid particles of different sphericities, which means that having clear effect or building distinct relationship between them is not possible.

6- A general equation is presented for irregular-shape particles when they settle down in shear-thinning fluids, which can be used to determine easily and directly the settling velocity, the particle Reynolds number, and the drag coefficient for these particles. This equation can also be used for spherical and disk particles.

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