HEFAT2014
10th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics
14 – 16 July 2014
Orlando, Florida

ON THE OPTIMUM METAL PARTICLE DISPOSITION IN THE PROPELLANT GRAIN

Minkov L.*, Shrager E, Pikushchak E.
*Author for correspondence
Department of Mathematical Physics,
Tomsk State University,
Tomsk, 634050,
Russian Federation,
E-mail: lminkov@ftf.tsu.ru

ABSTRACT

Using the dispersed metal in solid propellants to increase the temperature of combustion products leads to such a problem as the specific impulse loss due to the incomplete combustion of metal particles in the exhaust products. A redistribution of metal loaded into the propellant grain is one of the methods to decrease the specific impulse loss. This paper reports on the ways to obtain the optimum metal particle disposition for the case-bounded propellant grain of tube cross-sectional type. Three different approaches to analyze the metal combustion efficiency are discussed. The influence of the dynamic non-equilibrium of two phase flow on the optimum metal particles disposition in the propellant grain of tube cross-sectional type is investigated.

INTRODUCTION

Increasing the energy efficiency of propulsion is an important task of designing solid rocket motors (SRM), one of the component parts of which is the creation of a propellant with the high specific impulse. Composite solid propellants are currently widely used in SRM, which contain aluminium, magnesium and other metals as fuel additives to increase the specific impulse and combustion stability.

It is known the metal additives, on the one hand, lead to improving the energy performance of composite solid propellant due to high temperature of combustion, and on the other hand, they are the sources of specific impulse losses due to incomplete combustion of metal particles (especially for small-scale SRM and nozzleless SRM). The latter can be decreased by the redistribution of metal particles inside a solid propellant in accordance with particularities of a propellant grain and SRM.

The problem of increasing the specific impulse due to the redistribution of metal inside the grain has been dealt with in the last 40 years. For example, in Japan in 1976 it was patented a composition of the solid propellant in which the metal redistribution is carried out by varying the particle size [1]. It was noted that for the grain of SRM with the channel length to channel diameter ratio equals to 5–30 a redistribution of particle by sizes (44% of channel length is occupied by 50–150 micron particles, 28% – by 20–50 micron particles and 28% – by less than 20 micron particles) can give a 4–7% increase in the specific impulse as compared with uniform distribution of the metal powder inside of the propellant. In addition, the metal redistribution accomplished by changing its mass fraction is another way to increase the specific impulse.

A numerical investigation of the influence of the metal particle redistribution in accordance with a metal mass fraction on the combustion efficiency was carried out in papers [2] by assuming that the mass fraction of metal particles $z(X)$ varies linearly along the channel. It was shown there exists an optimal slope of the line $z(X)$ providing maximum combustion efficiency for a given mass of metal in the propellant grain. The dependence of the coordinate of the point at which the metal particle leaves the grain surface on the initial size of the particle burning out at the channel outlet was given in the paper [3]. The solution was obtained for the flow in one-dimensional approximation and expressed in terms of modified Bessel functions.

It is of interest to reveal the ways of an optimal disposition of metal particles in the propellant grain with a cylindrical channel from the maximum combustion efficiency point of view and to identify main parameters affecting the combustion efficiency.

NOMENCLATURE

A	[m^2]	Nozzle throat area
D	[μm]	Particle diameter
E	[-]	Metal combustion efficiency
$\dot{G}$	[kg/sec]	Mass rate
k	[sec/m$^{1.5}$]	Coefficient in the particle burning rate law
L	[m]	Channel length
M	[kg/mole]	Molar mass of species

p	[-]	Exponent in the particle burning rate law
R	[J/kgK]	Gas constant
R_k	[m]	Radius of channel
Re	[-]	Reynolds number
S	[m^2]	Lateral surface area of grain
t	[sec]	Time
T	[K]	Temperature
U,V	[m/sec]	Axial and radial components of velocity
u,v	[-]	Dimensionless components of velocity
w	[m/sec]	Velocity at the burning surface
W	[m^3]	Volume
X,Y	[m]	Axial and radial coordinates
x,y	[-]	Dimensionless coordinates
z	[-]	Mass fraction of metal

Special characters

α	[-]	Molar fraction of oxidizer
β	[-]	Volume fraction
δ	[-]	Dimensionless particle size
Γ	[-]	Gas dynamic complex
γ	[-]	Specific heat capacity ratio of gas phase
λ	[-]	Drag function
ρ	[kg/m^3]	Bulk density
τ	[-]	Dimensionless time
μ	[Pa·sec]	Viscosity
ν	[-]	Stoichiometric coefficient
θ	[-]	Burning-to-residence time ratio
Φ	[-]	Mass flux ratio
ψ	[-]	Relaxation-to-residence time ratio

Subscripts

c	Oxide
cc	Combustion chamber
b	Binder
bs	Burning surface
i	Inert gas
g	Gas phase
lim	Limit
m	Metal
mb	Metal in binder
ox	Oxidizer
out	Channel outlet
p	Combustion products
r	Residence
0	Initial value

ZERO-DIMENSIONAL APPROACH

Combustion products of the metallized composite solid propellant (MCSP) leaving the burning surface is supposed to consist of metal (Al), oxidizing gases (CO_2, H_2O, O) and inert gases (I). A chemical reaction takes place between the metal and the oxidizing gas to form a metal oxide and inert gases

$$\nu_m Al + \nu_O O = \nu_c Al_2O_3 + \nu_i I \qquad (1)$$

Here O and I are the oxidizing gas and the inert gas, respectively. Equation (1) shows that for complete combustion of one mole of metal requires ν_O/ν_m moles of oxidizing gas. This stoichiometric ratio imposes certain restrictions on the permissible content of metal in the MCSP. We define the limit of the metal mass fraction in the MCSP z_{lim}, at which metal can burn completely. Number of gas moles generated at burning of the MCSP equals to $\dot{G}_{bs}(1-z)/M_g$. Number of metal moles admitted to the flow equals to $\dot{G}_{bs}z/M_m$. Therefore in accordance with equation (1) for the complete combustion of metal the following equality must be hold:

$$\frac{\alpha \dot{G}_{bs}(1-z)/M_g}{\dot{G}_{bs}z/M_m} = \frac{\nu_o}{\nu_m}$$

Hence the limit value of the mass fraction of metal can be found from

$$z_{\lim} = \frac{1}{1+\omega/\alpha} \qquad (2)$$

Here $\omega = M_g\nu_o/M_m\nu_m$. Equation (2) shows that z_{lim} depends on α and the complex ω. For values $z(\alpha) > z_{\lim}(\alpha)$ the metal cannot completely burn up, and for $z(\alpha) < z_{\lim}(\alpha)$ oxidizing gases will be sufficient to do it. For different propellants, the molar mass varies from 15 to 30 kg/Kmole, so that the complex ω varies from 0.8 to 1.6. It follows from equation (2) that decreasing the molecular weight of gas phase and increasing the mole fraction of oxidizing gas α lead to increasing the limit value of $z_{\lim}(\alpha)$.

There are two main time parameters affecting the metal combustion completeness in the SRM, namely the characteristic burning time of metal particle t_b, the characteristic residence time of metal particle in the combustion chamber of the SRM, t_r.

In the case of zero-dimensional approach the residence time can be defined as $t_r = \rho_p W_{cc}/\dot{G}_{out}$. Assuming the nozzle discharge coefficient and the coefficient of thermal losses equal to unity, we find that

$$t_r = \frac{W_{cc}}{\Gamma(\gamma) A\sqrt{RT_p}} \qquad (3)$$

here $\Gamma(\gamma) = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$

The characteristic burning time of metal particle t_b is taken from the experimental dependence [4]:

$$t_b = k\frac{D^p}{\alpha^{0.9}}$$

where $k = 1.062\cdot10^4$ sec/m$^{1.5}$ for p=1.5. The condition of the complete combustion of metal one can be written in the form $\theta < 1$, where $\theta = t_b/t_r$. When the ratio W_{cc}/A is 20 m and α is 0.1 then the diameter of particles to be completely burned at the chamber outlet is less than 50μm, and when α equals to 0.4 the diameter reaches 100μm. Decreasing the volume of the combustion chamber leads to decreasing the diameter of particles which can completely burn. The estimate shows when W_{cc}/A = 2 m and α=0.1, the particle diameter reaches 10 μm, and when α = 0.4, the particle diameter D is 25 μm.

Thus, the zero-dimensional approach allows us to derive conditions under which the metal particles in the combustion chamber of the SRM will be completely burned: 1) the mass fraction of metal particles in the MCSP has to be less than z_{lim}, determined from equation (2); 2) the ratio of the burning time of metal particle to the residence time of metal particle in the combustion chamber has to be less than unity, $\theta<1$.

ONE- AND TWO-DIMENSIONAL APPROACHES

The zero-dimensional approach does not allow to detect the influence of particle distribution along the channel on the combustion completeness of particles. It can be made on the base of one- or two-dimensional approaches.

According to Larson [5] the metal combustion efficiency is defined in terms of mass flow rates as $E = 1 - \dot{G}_{m,out} / \dot{G}_{m,bs}$ and for the equilibrium flow (equality of gas and particles velocities) the main parameters affecting the efficiency are the ratio of the burning time of metal particle to the mean residence time of metal particle in the combustion chamber of the SRM – θ, and the ratio of the mole fluxes of metal and oxidizer on the burning surface of propellant – Φ, taking into account the stoichiometric ratio ν_o/ν_m.

One can be written:

$$\Phi = \frac{\nu_o}{\nu_m}\frac{\rho_m w_m / M_m}{\alpha_0 \rho_g w_g / M_g}, \qquad \theta = \frac{2w_m}{R_k}\frac{kD_0^p}{\alpha_0^{0.9}}$$

At $w_g = w_s = w$, we have:

$$\Phi = \frac{z}{1-z}\frac{\omega}{\alpha_0} \qquad \theta = \frac{2w}{R_k}\frac{kD_0^p}{\alpha_0^{0.9}} \tag{4}$$

The parameter Φ is greater unity in the case of the oxidizer deficiency and Φ is less unity in the case of the oxidizer excess.

As a rule, the MCSP consists of three main components: a binder, an oxidizer and powdered metal. A mass fraction of metal distributed in the propellant can be expressed through mass densities and volume fractions of the components from equation (4'):

$$\frac{z}{1-z} = \frac{\beta_{mb}\rho_m}{(1-\beta_{mb})\rho_b + \dfrac{\beta_{ox}}{1-\beta_{ox}}\rho_{ox}} \tag{4'}$$

where β_{mb} is the volume fraction of metal distributed in the binder, equals to $\beta_m/(\beta_{bi} + \beta_m)$. Since the velocity of combustion products leaving the burning surface of the SRM is expressed in terms of the temperature through the correlation

$$w = \frac{A}{S_{bs}}\sqrt{RT_p}\,\Gamma(\gamma).$$

The formula for the parameter θ is:

$$\theta = \frac{2A}{S_{bs}}\frac{\sqrt{RT_p}\,\Gamma(\gamma)}{R_k}\frac{kD_0^p}{\alpha_0^{0.9}} \tag{5}$$

The necessary conditions for the complete combustion of metal in the propellant grain are $\Phi<1$ and $\theta<1$. Following by equations (2), (4) the case $\Phi>1$ ($\Phi<1$) corresponds to $z > z_{lim}$ ($z < z_{lim}$).

Analysis of equations (4) – (5) shows that the increase of the mass fraction of metal leads to the increase in the parameter Φ and the decrease in the parameter θ, and the increase of the mole fraction of oxidizer α_0 decreases both Φ and θ. To evaluate parameters Φ and θ let us take $A/S_{bs} = 1.12\cdot10^{-2}$, T_p= 3500°K, z = 0.2, α_0= 0.4, γ = 1.2, M_g = 24 kg/Kmole, R = 285 J/(kg·K), we have Φ=0.8333, θ = 0.366 for D_0= 50 μm and θ = 1.038 for D_0 =100 μm.

The residence time of combustion products in the chamber of SRM defined by equation (3) and used in equation (5) is an average characteristic, so even if $\theta<1$ it does not mean that all the metal from the burning surface of channel is completely burned. Indeed, the metal particle leaving the area located closer to the front end of grain requires more time for the motion before the exiting the channel than the particles leaving the area located near to the exit of channel. If $\Phi<1$ there exists a coordinate X_{lim} of the point where the particle left the burning propellant surface is completely burned at the exit of channel. Therefore, when $X < X_{lim}$ particles are completely burned before the exit, and when $X > X_{lim}$ they are unburned.

To estimate the coordinate X_{lim} and to define the residence time of metal particle t_d one can use the approximation of the equilibrium two-phase flow. The rotational flow field in a channel of constant cross-section with a lateral burning surface is given as follows [6]:

$$U_g = \pi\frac{X}{R_k}w\cdot\cos\left[\frac{\pi}{2}\left(\frac{Y}{R_k}\right)^2\right]; \; V_g = -\frac{R_k}{Y}w\cdot\sin\left[\frac{\pi}{2}\left(\frac{Y}{R_k}\right)^2\right]$$

and the irrotational one is given as follows [7]:

$$U_g = 2\frac{X}{R_k}w; \qquad V_g = -\frac{Y}{R_k}w.$$

In the case of one-dimensional approach, the continuity equation gives the velocity of product combustion in the form $U = 2Xw/R_k$, which coincides with the velocity of the irrotational flow field for two-dimensional case.

One can see from the above correlations the axial velocity in the case of rotational flow field depends on both X and Y coordinates, and in the other cases it depends only on X coordinate. Therefore to find a dependence $X(t)$ for the metal particle in the case of the rotational flow field it is need to solve the system of two equations $dX/dt=U_g$; $dY/dt=V_g$; $X(0)=X_0$; $Y(0)=R_k$ and one equation $dX/dt = U_g$; $X(0)= X_0$ otherwise. Solving the equations gives for the rotational flow-field:

$$X = X_0\cosh\left(\frac{\pi w}{R_k}t\right) \tag{6}$$

and for the irrotational flow-field:

$$X = X_0\exp\left(\frac{2w}{R_k}t\right) \tag{7}$$

From the formulas (6) and (7) it follows that if the particle burning time equals to the average residence time of the particle in the channel, $\theta = 1$, $t_b = R_k/(2w)$, the value of X_{lim}/L equals to 0.398 for the rotational flow and 0.367 for irrotational flow. It means that particles from more than half of the propellant grain surface do not burn out.

In the limiting case when $\Phi \to 0$ and p=1.5 the metal combustion efficiency is yielded by:

for one-dimensional flow

$$E(\theta) = \frac{2}{\theta}\left(1 + \frac{\exp(-\theta) - 1}{\theta}\right) \tag{8}$$

for two-dimensional flow

$$E(\theta) = 2\int_0^1 \frac{1-\zeta}{\cosh(\pi\theta\zeta/2)} d\zeta \tag{9}$$

So, for the uniform distribution of particles along the channel the value of the metal combustion efficiency is less unity, $E(1)$=0.686 and $E(1)$=0.605 for two-dimensional and one-dimensional cases, respectively.

OPTIMUM DISTRIBUTION ALONG THE GRAIN

To provide the equality of the metal combustion efficiency to unity it is necessary that the residence time of the particle from the point with X_0–coordinate should be greater than its burning time. One can see from the expression (5) for θ, this may be achieved by variation of D_0 or w at constant values of R_k and α_0. Let us define the dependence $\theta(X)$ giving the value of E equal to unity.

As far as the gas is assumed to be incompressible and flow field is equilibrium, the mole fraction of oxidizer along the particle trajectory can be written as [5]:

$$\alpha = \alpha_0 + \frac{\rho_m}{\rho_g}\omega\left[(D/D_0)^3 - 1\right]$$

which is valid for both one-dimensional and two-dimensional approaches. The burn rate law for the particle can be represented as:

$$\frac{dD}{dt} = -\frac{1}{kp}D^{1-p}\alpha^{0.9} \tag{10}$$

Passing to the dimensionless variables $\delta = D/D_0$, $\tau = t\,2w/R_k$ gives:

$$\frac{d\delta}{d\tau} = -\frac{1}{p\theta}\delta^{1-p}\left[1 + \Phi\left(\delta^3 - 1\right)\right]^{0.9} \tag{11}$$

Denoting $f(\delta,\Phi) = \delta^{p-1}\left[1+\Phi\left(\delta^3-1\right)\right]^{-0.9}$ and integrating (11) by the time from 0 to the burning time of particle τ_b we obtain

$$\tau_b = p\theta\int_0^1 f(\delta,\Phi)d\delta$$

Using previously obtained relations (6), (7) and introducing the variable $\xi = X_0/L$, we have for two-dimensional flow:

$$\theta(\xi) = \frac{2}{\pi p}\operatorname{arcosh}(1/\xi)\Big/\int_0^1 f(\delta,\Phi)d\delta \tag{12}$$

and for one-dimensional flow:

$$\theta(\xi) = \frac{1}{p}\ln(1/\xi)\Big/\int_0^1 f(\delta,\Phi)d\delta\,. \tag{13}$$

Thus, it follows from equations (12), (13), which can be considered as the optimal distribution of the metal particles, that E = 1 can be reached by a variation of θ along ξ at Φ =*const* and by a variation of Φ along ξ at θ = *const.* Assuming Φ = 0 we have:

$$\theta(\xi) = \frac{2}{\pi}\operatorname{arcosh}(1/\xi) \qquad \text{– for two-dimensional flow;}$$

$$\theta(\xi) = \frac{1}{p}\ln(1/\xi) \qquad \text{– for one-dimensional flow.}$$

At $\Phi \to 1$ in both cases we have $\theta(\xi) \to \delta(\xi)$ – Dirac delta function. Meaning of the latter is that when Φ=1 the value of parameter θ (and hence the particle size) should be zero along the entire channel, except for the point ξ=0.

Figure 1 shows characteristic profiles of $\theta(\xi)$ at different values of the parameter Φ. The dashed curves correspond to the one-dimensional flow, and the solid ones – to the two-dimensional flow. Qualitative difference can be clearly seen in the behavior of functions $\theta(\xi)$ when $\xi \to 1$, and for both cases $\theta \to \infty$ when $\xi \to 0$.

Behavior of Φ along the length of the channel at different values of the parameter θ is depicted in Figure 2. It is seen, to achieve the equality E=1 the metal particles should be absent in the propellant to the right of the intersection point between the abscissa axis and the curve $\Phi(\xi)$. The increase (decrease) of the value of θ leads to the shift of the intersection point to the left (right), and for one-dimensional flow (dashed curves) the region free of particles being greater than for two-dimensional flow (solid curves).

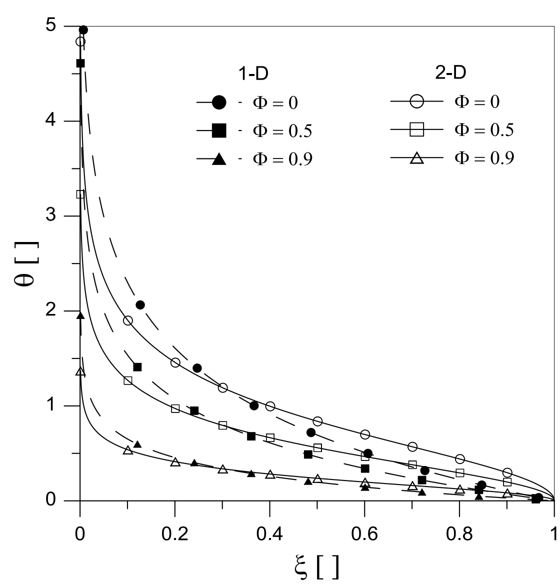

Figure 1 Parameter θ distribution along the channel

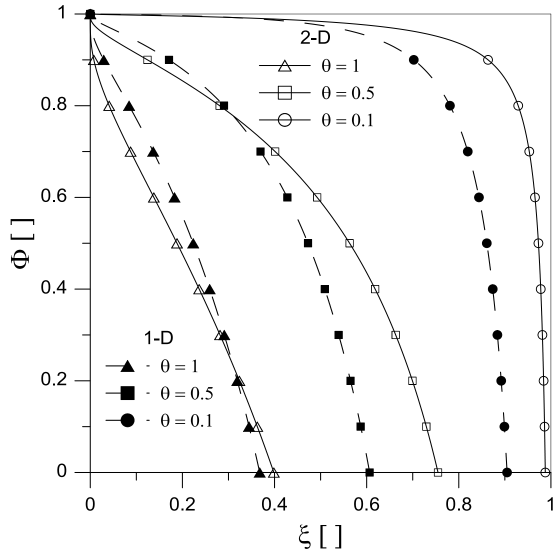

Figure 2 Parameter Φ distribution along the channel

OPTIMUM DISTRIBUTION ACROSS THE GRAIN

During the propellant combustion process the channel radius decreases, that leads to the change of the value of θ for a given value of ξ. Assuming that the increase of the combustion chamber volume does not affect the temperature in the combustion chamber, the value of θ is inversely proportional to the channel radius squared in the case of the SRM with the geometry of $A/S_{bs} \sim 1/R_k$ as follows from equation (5). The inverse proportionality to the channel radius takes place for SRM with the geometry of $A/S_{bs} = const$. Therefore when the value of R_k increases, the value of θ decreases, and dependences $\Phi(\xi)$ and $\theta(\xi)$ will changes. From equations (12) and (13) one can obtain next correlation

$$p\theta\int_0^1 f(\delta,\Phi)d\delta = g(\eta)h(\xi) \qquad (14)$$

where

$$h(\xi)=\begin{cases}\frac{2}{\pi}\operatorname{arcosh}(1/\xi) & -\ 2\text{-}D \text{ flow}\\ \ln(1/\xi) & -\ 1\text{-}D \text{ flow}\end{cases};$$

$$g(\eta)=\begin{cases}\eta & -\ A/S_{bs}=const\\ \eta^2 & -\ A/S_{bs}\sim 1/R_k\end{cases}$$

η is the ratio of the current radius of the channel to the initial one. The correlation (14) should be treated either as a relation between the parameters Φ and θ at constant value of its RHS, which provides E=1, or as an equation for determining the function $\theta(\xi,\eta)$ at given Φ (position of the equal size particles inside the propellant which can completely burn at the channel outlet), or as an equation for determining the function $\Phi(\xi,\eta)$ at given θ (position of propellant portions with the equal mass fraction of metal providing the complete combustion of particles).

Profiles of $\eta(\xi)$ for 2-D flow (solid curves) and for 1-D flow (dashed curves) obtained at Φ=0.99, $g(\eta)=\eta^2$ and different values of θ are depicted in Figure 3. Curves corresponding to different kinds of flow are close to each other near the front end of the grain and are quite different near the rear end of the grain. The distribution of equal size particles (θ=1) in accordance with their mass content inside the propellant grain is shown in Figure 4.

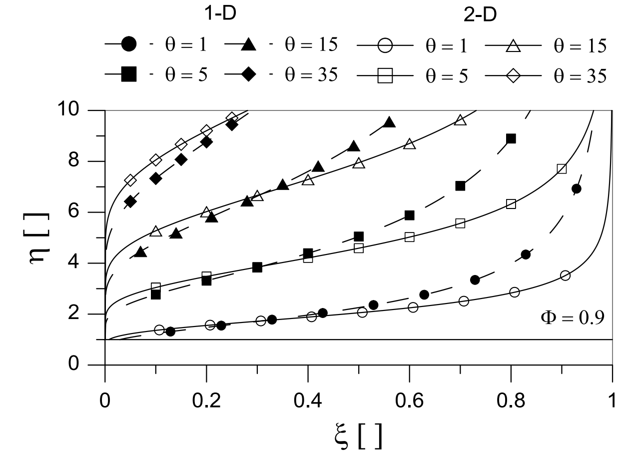

Figure 3 Optimum profiles of metal content at Φ=0.9*,* $g(\eta)=\eta^2$

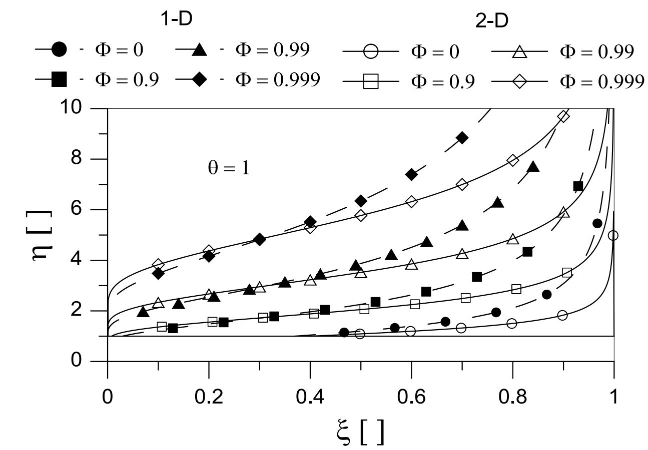

Figure 4 Optimum profiles of metal content at θ=1, $g(\eta)=\eta^2$

The increase of θ (Figure 5) leads to enhancing the region where Φ changes from 0 to 0.9 and increasing a zone free of particles (part of the grain below the curve Φ=0). The same takes place for $g(\eta)=\eta$ and θ=1 (Figure 6). The fact that the particle residence time in the case of the 1-D flow is less than that in the case of the 2-D flow leads to a displacement of profiles θ=*const* (Figure 4) and Φ=*const* (Figure 6) downwards at the front end of the grain and upwards at the rear end of the grain with respect to the similar profiles for 2-D flow.

Figure 7 shows the dependence of the parameter θ to complex $g(\eta)h(\xi)$ ratio on the parameter Φ at $g(\eta)h(\xi)$=*const*. It is seen that at Φ = 0 (the flux of metal is negligible in compare with the flux of oxidizing gas) the parameter θ can be arbitrary due to the arbitrariness of values of $g(\eta)h(\xi)$, and at Φ = 1 (stoichiometric ratio of mole fluxes) the value of θ should be equal to zero.

EFFECT OF THE VELOCITY LAG

The moving inertial particle has a velocity less than the velocity of the gas whereby the residence time of the particles in the channel increases, which increases the metal combustion efficiency.

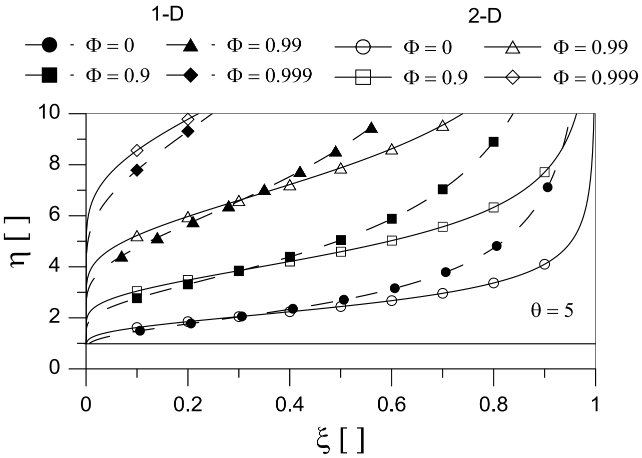

Figure 5 Optimum profiles of metal content at θ=5, $g(\eta)=\eta^2$

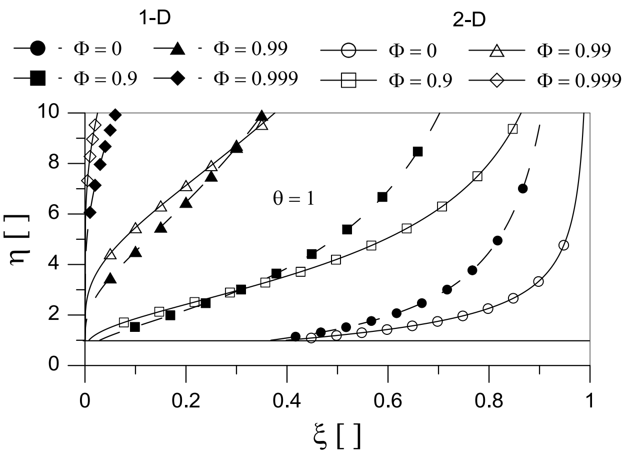

Figure 6 Optimum profiles of metal content at θ=1, $g(\eta)=\eta$

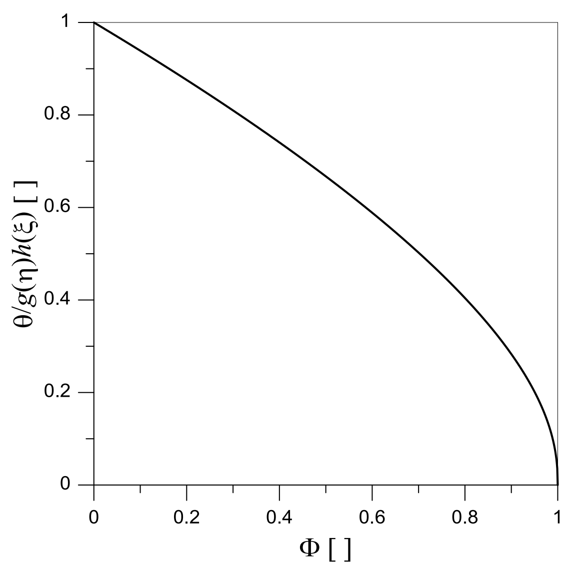

Figure 7 Relationship between parameters of Φ and θ

Consider the two-dimensional approximation. Following [5], we can write the system of equations describing the motion of a particle in the channel flow-field taking into account the fact that the particle velocity lag is negligible at the calculation of the oxidizer concentration along the particle trajectory (so called King's model):

$$\frac{d\delta}{d\tau}=-\frac{1}{p\theta}\delta^{1-p}\left[1+\Phi\left(\delta^3-1\right)\right]^{0.9}$$
$$\frac{du_m}{d\tau}=\frac{9\lambda}{\psi\delta^2}\left(u_g-u_m\right);\ \frac{dv_m}{d\tau}=\frac{9\lambda}{\psi\delta^2}\left(v_g-v_m\right) \tag{16}$$
$$\frac{dx}{d\tau}=u_m;\ \frac{dy}{d\tau}=v_m$$

where

$$\lambda=1+0.15\,\mathrm{Re}_1^{0.687};\ \mathrm{Re}_1=2\delta\,\mathrm{Re}\sqrt{(u_g-u_m)^2+(v_g-v_m)^2}$$
$$\mathrm{Re}=\frac{D_0\rho_g w}{(1-\beta_m)\mu_g};\ \psi=\frac{\rho_m D_0^2 w}{\beta_m\mu_g R_k}.$$

The system of equations (16) is dimensionless, where the length scale is chosen to be the radius of the channel, and the velocity scale is chosen to be $2w$.

For rotational flow

$$u_g=\frac{\pi}{2}x\cdot\cos\left(\frac{\pi}{2}y^2\right);\ v_g=-\frac{1}{2y}\sin\left(\frac{\pi}{2}y^2\right),$$

and for irrotational flow $u_g=x;\ v_g=-y/2$.

At the initial moment of time the following conditions are set: $\delta(0)=1;\ x(0)=x_0;\ y(0)=1;\ v_m(0)=-0.5$.

For irrotational flow $u_g(0)=x_0$; for rotational flow $u_g(0)=0$. The condition that the particle resides in the channel is defined by the inequality: $x_0<x<l$, here l is dimensionless length of channel.

To calculate the combustion efficiency of burning particles considering the velocity lag we use the formula [5]:

$$E=1-\frac{1}{l}\int_0^1\delta^3(x_0,l)dx_0 \tag{17}$$

Here $\delta(x_0,l)$ is a diameter of particle at the exit area ($x = l$) which left the burning propellant surface at the point with coordinate x_0. To provide $E = 1$ it is necessary to resolve the system of equation (16) and to define a value of the x_0 coordinate which gives a value of $\delta(x_0,l)$ equal to zero.

From the above system of equation (16 – 17) one can see the solution depends on, besides Φ and θ, also three dimensionless parameters l, Re, and ψ, where ψ is the relaxation time to average residence time ratio. When $\psi\to 0$ the flow tends to the equilibrium one.

As the propellant burns in the case of a cylindrical channel grain, the outflow velocity of the combustion products from the burning propellant surface changes. It is known [8], the outflow velocity of the combustion products in this case varies inversely proportional to the channel radius, and mass outflow rate varies proportional to the channel radius raised to the power of $n / (n-1)$. Where n is an exponent in the burning propellant rate law. Thus, dimensionless parameters l, θ, ψ, Re are also variable. If we denote these parameters at the initial moment of time as l_0, θ_0, ψ_0, Re_0, then at another moment of time (in the case of $A/S_{bs} \sim 1/R_k$) they are as follows:

$$\psi = \psi_0 / \eta^2 \; ; \; \theta = \theta_0 / \eta^2 \; ; \mathrm{Re} = \mathrm{Re}_0 \, \eta^{n/(n-1)} \; ; \; l = l_0 / \eta \qquad (18)$$

Solving the system of equation (16) – (17), the dependence $\eta(\xi)$ which represents the optimum disposition of metal particles in the propellant can be obtained.

The influence of parameters l_0, ψ_0, Re_0 on the position of the curve $\eta(\xi)$ for rotational flow is shown in Figure 8 to Figure 10, respectively. The decrease of values of l_0, Re_0 and the increase of value of ψ_0 lead to the increase of the deviation of the curve position in the direction of the section $\xi = 1$ from that corresponding to the equilibrium flow. As the shift of the curve $\eta(\xi)$ in that direction means the increase of the combustion efficiency, then the increase of the value of ψ_0 and the decrease of the values of l_0, Re_0 is preferable with the energetic point of view in the case of the non-equilibrium flow. It is clear, the closer the curve $\eta(\xi)$ passes near to the curve corresponding to the equilibrium flow, the better the dependence (14) responds to the solution of the system of equation (16). Therefore, one can say, the dependence (14) describes the solution of the system (16) at large values of l_0 and Re_0 and small ψ_0.

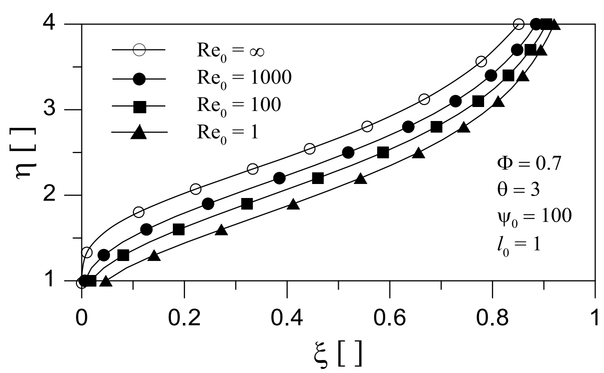

Figure 8 Effect of Re on optimum profile of metal content

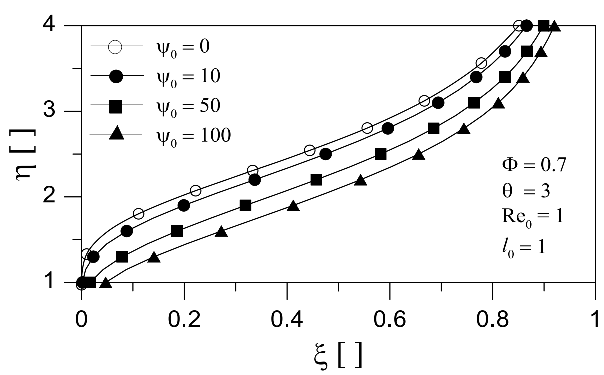

Figure 9 Effect of ψ_0 on optimum profile of metal content

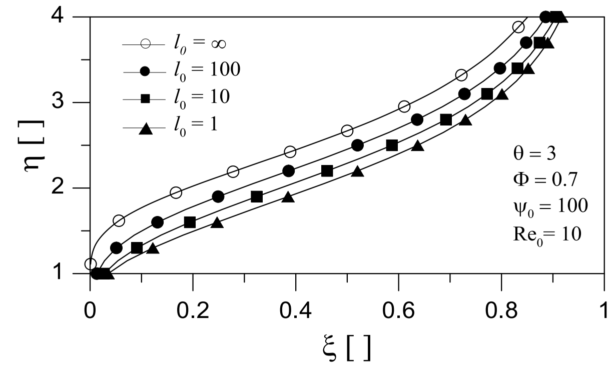

Figure 10 Effect of l_0 on optimum profile of metal content.

CONCLUSION

The analytical correlation for the optimum disposition of metal particles in the case-bounded propellant grain of tube cross-sectional type under the assumption of equilibrium two-phase flow is deduced.

Analysis of the dimensionless system of equations describing a motion of burning particle in the channel of circular cross section with taking into account the velocity lag shows the optimum disposition of metal particles in the propellant grain depends on five parameters: the relative length of the channel; the particle relaxation time; the molar metal to oxidizer flux ratio at the propellant grain surface; particle combustion to residence time ratio and Reynolds number of particle.

The increase of the particle relaxation time and the decrease of the particle Reynolds number and the relative channel length are preferable from the combustion efficiency point of view in the case of non-equilibrium flows.

REFERENCES

[1] Daidzo F., Tadahiko N., Tekamega K., and Sedzo S., Composition of the Propellant, *Journal of Abstracts* 1978, No.14 (III), 14П244П (in Russian)

[2] Vilyunov, V.N., Vorozhtsov, A.B., and Feshchenko, Yu.V., Modeling of two-phase flow of a gas mixture with burning metal particles in a semienclosed channel, *Combustion, Explosion and Shock Waves*, 1989, Vol. 25, 3, pp296-300.

[3] Terzi E.D. Analytical approach to the problem of incomplete combustion of metal additives in the reactors, *Mechanics of fast processes*, Tomsk, 1989, pp. 50-57. (in Russian)

[4] Belyaev A. F., Frolov Yu. V., and Korotkov A. I., Combustion and ignition of particles of finely dispersed aluminum, *Combustion, Explosion and Shock Waves*, 1968, Vol.4, 3, – pp.182-185.

[5] Larson R.S., Prediction of Aluminum Combustion Efficiency in Solid Propellant Rocket Motors, *AIAA Journal*, 1987, Vol.25, 1, pp.82-91.

[6] Jane S.W., and Finkelstein A.B., Laminar Pipe Flow with Injection and Suction through a Porous Wall, *Transactions of ASME*, 1956, Vol.78, 4, pp.719-724.

[7] Loitsyanskii L. G., Mechanics of Liquids and Gases, Pergamon Press, 1966, 804 pp.

[8] Erokhin B.T. Theory of intrachamber processes and design of SRM, Moscow, Mashinostroenie, 1991, 560 pp. (in Russian)