HEFAT2014
10th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics
14 – 16 July 2014
Orlando, Florida

NON-NEWTONIAN RIMMING FLOW: STABILITY ANALYSIS

Fomin S.[1]*, Shankar R. [1], Haine P. [2], and Gartrell A.[3]
[1]Department of Mathematics and Statistics, California State University, Chico, CA 95929, USA
[2]Department of Mathematics, MIT, Cambridge, MA 02139, USA
[3]Department of Mathematics, University of Maryland, College Park, MD 20740, USA
*Author for correspondence
Email: sfomin@csuchico.edu

ABSTRACT
The rimming flow of a thin polymeric film inside a rotating horizontal cylinder is studied theoretically. The non-Newtonian fluid viscosity is described by the Generalized Newtonian Fluid (GNF) constitutive model. With linear stability analysis, it is found that, analogously to Newtonian fluids, rimming flow of viscous non-Newtonian fluids is neutrally stable.

INTRODUCTION
The problem of rimming flow has been investigated for many years because of its many applications in industry. In most applications, a uniform, smooth film coating is desired [1]. However, experimentally, it was shown that rimming flow is characterized by wide variety of uneven, bulging steady-state film distributions and instabilities [2-7] . Depending on various physical parameters including cylinder rotation rate and cylinder filling fraction, this desirable smooth flow regime may or may not exist.

Previous theoretical studies of rimming flow, however, have largely considered Newtonian fluids. Moffatt [8] first derived the value of the maximum amount of fluid a rotating cylinder can sustain. For masses above this critical value, gravitational forces overcome the cylinder's rotational drag and cause a fluid puddle to accumulate on the rising wall of the cylinder. In the lubrication approximation, O'Brien [9] showed that the position of the puddle on the cylinder wall can be represented by shock solutions. O'Brien [10], Villegas [11-12], Johnson [13], Hinch and Kelmanson [14] and Badali et al [15] later showed these shock solutions are stable. However, these "pooling" solutions exhibit uneven bulges, and can be undesirable for coating applications.

Rimming flow does exhibit smooth free surfaces for subcritical loads. The subcritical regime is characterized by small cylinder filling fraction and fast rotation rate. While rather smooth and uniform, the subcritical film has shown instability in experimental investigations. O'Brien [16] first considered the stability of the subcritical regime for Newtonian fluids. In a linear stability analysis, he showed that

NOMENCLATURE

B	Bond number
g	gravitational acceleration
G	inverse function to $\mu(\dot{\gamma})\dot{\gamma}$
h	thickness of the liquid layer
h_s	steady-state film thickness
η_0	characteristic thickness of the liquid layer
$\boldsymbol{n}$	normal to the free surface
p	pressure
r	radial coordinate
r_0	radius of the cylinder
R	modified radial coordinate
Re	Reynolds number
t	time
$\boldsymbol{v}$	fluid velocity vector
v_R, v_θ	radial and angular components of velocity
$\boldsymbol{e}_r, \boldsymbol{e}_\theta$	radial and angular basis vectors
Greek Symbols	
δ	$\delta = \eta_0/r_0$
$\dot{\gamma}$	shear rate
$\boldsymbol{\gamma}$	rate of deformation tensor
$\gamma_{R\theta}, \gamma_{\theta\theta}, \gamma_{RR}$	components of the deformation rate tensor
κ	mean curvature of the free surface
λ	polymer time constant
μ	dynamic viscosity
μ_0	characteristic viscosity
θ	angular coordinate
Φ	mass flux through the liquid layer
ρ	liquid density
σ	surface tension
$\boldsymbol{\tau}$	stress tensor deviator
$\tau_{R\theta}, \tau_{\theta\theta}, \tau_{RR}$	components of the stress tensor deviator
Ω	characteristic angular velocity of cylinder
$\boldsymbol{\nabla}$	Del operator
Superscripts	
*	dimensional quantity
Subscripts	
0	characteristic quantity
θ	angular component
R	radial component

for this class of fluids, the uniform subcritical solutions are neutrally stable. To find an instability mechanism, Hosoi and Mahadevan [17] and Benilov et al [18-22] extended this linear stability analysis by including higher order effects normally ignored in the lubrication approximation. Pressure differences at the top of the cylinder proved destabilizing, but adding surface tension stabilized the solution [23]. Inertia, however, demonstrated a significant destabilizing effect [21,24].

Although previous studies take many complicated effects into account, they do not consider non-Newtonian rheological effects when studying steady-state stability. Coating industries use polymers that exhibit complex rheology that greatly deviates from a Newtonian behavior [25]. Polymers exhibit Newtonian rheology for small strains, but transition to shear-thinning for larger shear rates. To completely describe this important manufacturing process, the effects of these non-Newtonian properties need to be characterized.

However, the non-Newtonian rimming flow, in general, and its stability, in particular, has not been extensively studied. Fomin et al [26-27] proved that shear-thinning fluids described by the power-law, Ellis, and Carreau models lowered the maximal supportable load of the cylinder. Because shear-thinning inhibits the shear-force of the viscous drag of the cylinder, higher rotation rates are required to offset this gravitational-viscous imbalance.

In our study, the linear stability results obtained for Newtonian fluids are extended to non-Newtonian fluids. The effects of non-Newtonian shear thinning on the stability of subcritical steady-state rimming flow are studied. To solve the evolution equation governing time-dependent film thickness, the film's free surface is expanded as a normal mode perturbation of the steady-state and the resulting eigenvalue problem is solved. It is shown that, within the lubrication approximation, shear-thinning rimming flow modeled by the GNF is neutrally stable.

SYSTEM MODEL AND SCALE ANALYSIS

Figure 1 contains a schematic of rimming flow. A horizontal cylinder of radius r_0 is rotating in a counterclockwise direction θ with constant angular velocity Ω. A thin liquid film of thickness $h^*(\theta,t)$ moves along the inner cylinder wall due to the gravity and the cylinder's rotational drag force. A cylindrical system of coordinates (r, θ, z) is used such that the z-axis coincides with the axis of the cylinder. The rest of the cylinder is modeled as being filled with rarefied gas of uniform pressure and negligible viscous traction at the liquid-gas interface. It is assumed the cylinder is sufficiently long such that the flow is two-dimensional.

It is further assumed that inertial effects are of negligibly small Reynolds number ($Re \to 0$) and that the film of density ρ is incompressible. Given these assumptions, the governing equations can be presented in the following vector form:

$$\boldsymbol{\nabla}^* \cdot \boldsymbol{v}^* = 0 \qquad (1)$$

$$\rho \boldsymbol{g} - \boldsymbol{\nabla}^* p^* + \boldsymbol{\nabla}^* \cdot \boldsymbol{\tau}^* = \boldsymbol{0} \qquad (2)$$

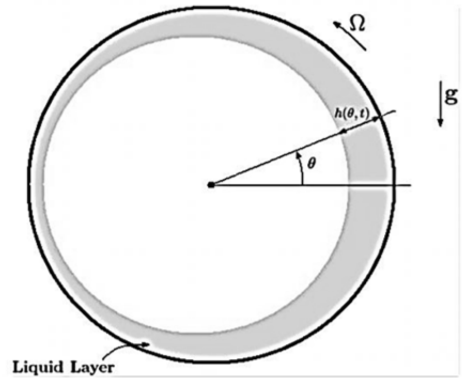

Fig. 1 A simple sketch of the rimming flow system.

where vectors and tensors are denoted in boldface and dimensional variables with asterisks, $\boldsymbol{\nabla}^*$ is the Del operator, $\boldsymbol{v}^*$ is the fluid velocity vector with radial and angular components v_r^* and v_θ^*, p^* is the pressure, $\boldsymbol{g}$ is the gravity acceleration vector, and $\boldsymbol{\tau}^*$ is the stress tensor deviator. The equations (1)-(2) in polar coordinates can be presented as follows:

$$\partial_{r^*}(r^* v_r^*) + \partial_\theta v_\theta^* = 0 \qquad (3)$$

$$-\rho g \sin\theta - \partial_{r^*} p^* + \partial_{r^*} \tau_{rr}^* + \frac{\tau_{rr}^* - \tau_{\theta\theta}^*}{r^*} + \frac{\partial_\theta \tau_{r\theta}^*}{r^*} = 0 \qquad (4)$$

$$-\rho g \cos\theta - \frac{\partial_\theta p^*}{r^*} + \frac{\partial_\theta \tau_{\theta\theta}^*}{r^*} + \partial_{r^*} \tau_{r\theta}^* + \tau_{r\theta}^*/r^* = 0 \qquad (5)$$

where ∂_{r^*} and ∂_θ denote the partial derivatives with respect to r^* and θ, and g is the gravitational acceleration. At the film's free surface $r^* = r_0 - h^*(\theta, t^*)$, the normal force balance, the tangential force balance, and the kinematic condition are presented below:

$$-p^* + \boldsymbol{n}^* \cdot \boldsymbol{\tau}^* \cdot \boldsymbol{n}^* = 2\kappa^* \sigma,$$

$$\boldsymbol{n}^* \cdot \boldsymbol{\tau}^* \cdot \boldsymbol{t}^* = 0,$$

$$\partial_t^* h^* + v_r^* + v_\theta^* \partial_\theta h^*/r^* = 0 \qquad (6)$$

where $\boldsymbol{n}^*$ is the unit normal vector external to the liquid layer, $\boldsymbol{t}^*$ is the unit tangent vector, σ is the surface tension, and κ^* is the mean curvature of the free surface. The curvature is calculated from $2\kappa^* = \boldsymbol{\nabla}^* \cdot \boldsymbol{n}^*$. The vectors tangent and the normal to the free surface are given by the following equations:

$$\boldsymbol{t}^* = (-\partial_\theta h^* \boldsymbol{e}_r + r^* \boldsymbol{e}_\theta)/\left((\partial_\theta h^*)^2 + r^{*2}\right)^{1/2},$$

$$\boldsymbol{n}^* = -(\boldsymbol{e}_r + \partial_\theta h^*/r^*\, \boldsymbol{e}_\theta)/(1 + (\partial_\theta h^*/r^*)^2)^{1/2}$$

where $\boldsymbol{e}_r$ and $\boldsymbol{e}_\theta$ are the radial and angular basis vectors. On the wall of the cylinder $r^* = r_0$, the no-slip conditions are applied such that $\boldsymbol{v}^* = (v_r^*, v_\theta^*) = (0, \Omega r_0)$.

GENERALIZED NEWTONIAN FLUIDS

Generalized Newtonian Fluids (GNFs) admit a straightforward constitutive model that well describes polymer shear-thinning (see Chapter 4 in [25] for a thorough treatment of this rheological model). This simple constitutive law is given by $\boldsymbol{\tau}^* = 2\mu^* \boldsymbol{\gamma}^*$, where $\boldsymbol{\tau}^*$ is the stress tensor deviator, $\boldsymbol{\gamma}^*$ is the rate of deformation tensor (see the Appendix for tensor components), and $\mu^* = \mu^*(\dot{\gamma}^*)$ is the dynamic viscosity dependent on the shear rate $\dot{\gamma}^* = \sqrt{2\mathrm{tr}(\boldsymbol{\gamma}^* \cdot \boldsymbol{\gamma}^*)}$. The power-law fluid, for example, is governed by this viscosity: $\mu^* =$

$k(\dot{\gamma}^*)^{m-1}$, where m is a flow index and k is a material constant. This formulation also encompasses all other models within the category of Generalized Newtonian fluids, e.g. such as the Ellis and Carreau constitutive equations.

1) *Scale Analysis*

It is convenient for further analysis to convert variables to a nondimensional form. Rimming flow scaling laws for velocity, pressure, time, etc. are well-documented [8-10]. Taking Ω^{-1} as the fast time scale as proposed in [21] and denoting $\delta = h_0/r_0$ as a small parameter that represents the ratio of the unknown characteristic film thickness h_0 to the cylinder radius, the nondimensional variables can be defined as follows:

$v_\theta^* = \Omega r_0 v_\theta, \quad v_r^* = \Omega r_0 \delta v_R, \quad r^* = r_0(1-\delta R), \quad p^* = \rho g r_0 p, \quad t^* = \Omega^{-1} t$ (7)

where R is a modified radial coordinate. Using the velocity scaling laws given in (7) and the dimensional forms of the deformation tensor components given in the Appendix A, the following rate of deformation tensor and GNF stress scaling is presented for each tensor component (see [27]):

$\gamma_{rr}^* = \Omega\gamma_{RR}, \;\; \gamma_{r\theta}^* = \Omega/\delta\gamma_{R\theta}, \;\; \gamma_{\theta\theta}^* = \Omega\gamma_{\theta\theta}$ (8)

$\tau_{rr}^* = \mu_0\Omega\tau_{RR}, \;\; \tau_{r\theta}^* = \mu_0\Omega/\delta\tau_{R\theta}, \;\; \tau_{\theta\theta}^* = \mu_0\Omega\tau_{\theta\theta}$ (9)

where μ_0 is a characteristic viscosity. Assuming the film is very thin, such that terms of $O(\delta)$ are negligibly small and can be omitted, the nondimensional stress-strain correlations will take the following forms:

$\tau_{RR} = 2\mu\,\gamma_{RR}, \;\; \tau_{R\theta} = 2\mu\,\gamma_{R\theta}, \;\; \tau_{\theta\theta} = 2\mu\,\gamma_{\theta\theta}$ (10)

$\gamma_{RR} = -\partial_R v_R, \;\; \gamma_{R\theta} = -\partial_R v_\theta/2, \; \gamma_{\theta\theta} = \partial_\theta v_\theta$ (11)

with $\mu = \mu(|\dot{\gamma}|)$ and $\dot{\gamma} = \partial_R v_\theta$. Scale analysis of the boundary conditions on the free surface equations (6) gives a non-dimensional capillary parameter $B = \delta^{-2} r_0\mu_0\Omega/\sigma$, where the Bond number B reflects the importance of viscous forces over surface tension. Since $B \gg 1$ for rimming flow, surface tension effects can be neglected. Neglecting terms of $O(\delta)$, the nondimensional free surface force balances and the kinematic condition at the free surface given in (6) yield:

$R = h\!: \;\; p = 0, \;\; \tau_{R\theta} = 0, \;\; \partial_t h + v_R + v_\theta \partial_\theta h = 0$ (12)

The no-slip conditions reduce to $\boldsymbol{v} = (v_R, v_\theta) = (0,1)$ at $R = 0$. By balancing the viscous and gravitational forces in the momentum equations (4) and (5), the characteristic thickness of the liquid layer $h_0 = \delta r_0$ is defined in terms of $\delta = \sqrt{\mu_0\Omega/\rho g r_0}$. The nondimensional mass conservation (3) and momentum equations (4), (5) to $O(\delta)$ take the following form:

$\partial_R v_R - \partial_\theta v_\theta = 0$ (13)

$\partial_R p = 0$ (14)

$\cos\theta + \partial_\theta p + \partial_R \tau_{R\theta} = 0$ (15)

2) *Solution*

The solutions of equations (13)-(15) subject to free surface boundary conditions (12) are given below:

$p = 0, \qquad \tau_{R\theta} = (h - R)\cos\theta$ (16)

The next step is to equate the relation for $\tau_{R\theta}$ from the formulae (10), (11) with solution (16) and to solve for v_θ. At this stage, Fomin et al [26-27] introduce a monotonically increasing analytic function G given as the inverse of function $F(\dot{\gamma}) = \dot{\gamma}\mu(|\dot{\gamma}|)$ such that $G\big(x\mu(|x|)\big) = x$. This ansatz allows expression of the shear rate in explicit form:

$\dot{\gamma} = -\mathrm{sgn}(\tau_{R\theta})G(|\tau_{R\theta}|)$ (17)

where the shear rate is given by $\dot{\gamma} = \partial_R v_\theta$. Integrating equation (17) and accounting for the no-slip conditions yields:

$v_\theta = 1 - \mathrm{sgn}(\tau_{R\theta})\int_0^R G(|\tau_{R\theta}|)dR$ (18)

Using the mass conservation equation (13) and the kinematic equation at the free surface given in equation (12), the evolution equation governing the film's free surface thickness variation is obtained:

$\partial_t h(\theta,t) + \partial_\theta \Phi(\theta,t) = 0$ (19)

For GNFs, the non-dimensional mass flux $\Phi = \int_0^h v_\theta dR$ through the liquid layer takes the following form [27]:

$\Phi = h - \mathrm{sgn}(\cos\theta)\int_0^h RG(R|\cos\theta|)dR$ (20)

3) *Linear Stability Analysis*

We are interested in the stability of the evolution equation (19), where Φ is defined by the expression (20). In several studies of non-Newtonian effects on thin films of other geometries [28], shear-thinning was shown to have marked effects on the systems' instabilities. A stability analysis of the Generalized Newtonian Fluid can show whether or a similar phenomenon exists.

We assume $h(\theta,t)$ can be given as the steady-state solution $h_s(\theta)$ of equations (19), (20) perturbed by a small, angularly periodic, time-dependent disturbance of $O(\epsilon)$:

$h(\theta,t) = h_s(\theta) + \epsilon\xi(\theta,t)$ (21)

Substituting (21) into (20), the mass flux Φ becomes:

$\Phi = h_s + \epsilon\xi - \mathrm{sgn}(\cos\theta)\int_0^{h_s+\epsilon\xi} RG(R|\cos\theta|)dR$ (22)

The integral term in (22) can be rewritten as such:

$\int_0^{h_s+\epsilon\xi} RG(R|\cos\theta|)dR = \int_0^{h_s} RG(R|\cos\theta|)dR + \int_{h_s}^{h_s+\epsilon\xi} RG(R|\cos\theta|)dR$ (23)

Applying the mean-value theorem to the second integral on the right-hand side of (23) yields:

$\int_{h_s}^{h_s+\epsilon\xi} RG(R|\cos\theta|)dR = \epsilon\xi(h_s + \chi\epsilon\xi)G[(h_s + \chi\epsilon\xi)|\cos\theta|]$ (24)

where $0 < \chi < 1$. Since G is a continuous function [27],

$G[(h_s + \chi\epsilon\xi)|\cos\theta|] = G(h_s|\cos\theta|) + p_\epsilon(\theta,t)$ (25)

where $p_\epsilon(\theta,t) \to 0$ as $\epsilon \to 0$. With equations (23)-(25), and neglecting terms of $o(\epsilon)$ (i.e. linearizing in ϵ), the linearized mass flux Φ of (22) is given by:

$\Phi = h_s - \mathrm{sgn}(\cos\theta)\int_0^{h_s} RG(R|\cos\theta|)dR + \epsilon\xi\big(1 - h_s\mathrm{sgn}(\cos\theta)G(h_s|\cos\theta|)\big)$ (26)

Notice that the leading order terms in (26) cancel since $h_s(\theta)$ satisfies the steady-state form of equations (19), (20), as defined. Combining equations (19) and (26) and dropping ϵ's, the linear evolution equation for the traveling disturbance $\xi(\theta,t)$ is obtained:

$\partial_t\xi(\theta,t) + \partial_\theta[\alpha(\theta)\xi(\theta,t)] = 0$ (27)

Here, $\alpha(\theta) = 1 - h_s(\theta)\mathrm{sgn}(\cos\theta)G(h_s(\theta)|\cos\theta|)$ represents the speed of the disturbance's propagation about the free surface. In [27] it was shown that the steady-state solution

$h_s(\theta)$ exists only if $\alpha(\theta) > 0$ for all θ. For a normal-mode analysis (see [29]), we assume that the disturbance ξ is harmonic in time, such that $\xi(\theta, t) = \mathcal{R}e[f(\theta)e^{st}]$. Here, $\mathcal{R}e[]$ denotes the real part, $f(\theta)$ is an unknown complex periodic function, and s is an unknown complex growth factor. Depending on whether the growth rate $\mathcal{R}e[s]$ is greater than, equal to, or less than zero, the steady-state $h_s(\theta)$ can be either unstable, neutrally stable, or asymptotically stable with respect to disturbance ξ. Dropping e^{st}'s, the evolution equation (27) reduces to the following eigenvalue problem for $f(\theta)$ and s:

$$-sf(\theta) = \partial_\theta[\alpha(\theta)f(\theta)] \qquad (28)$$

Since $\alpha(\theta) \neq 0$ according to [27], equation (28) has nonzero solution (θ) . Solution of this equation is straightforward:

$$f(\theta) = A\alpha(\theta)^{-1}\exp(-s\int_0^\theta \alpha(\phi)^{-1}d\phi) \qquad (29)$$

where A is an integration constant. This is analogous to what O'Brien obtains when specifically considering Newtonian fluids [16]. Applying to $f(\theta)$ the periodic boundary condition $f(\theta) = f(\theta + 2\pi)$, and recognizing that $\alpha(\theta)$ is 2π-periodic, it becomes clear that $1 = \exp(-s\int_0^{2\pi} \alpha(\phi)^{-1}d\phi)$. Since $\alpha(\theta) > 0$ for all θ, the integral term is nonzero, such that we must have the following condition for s:

$$s = 2\pi in/\int_0^{2\pi} \alpha(\phi)^{-1}d\phi \qquad (30)$$

Since α is real, s must be purely imaginary ($\mathcal{R}e[s] = 0$) for all integers (wavenumbers) n, which is related to the disturbance's spatial frequency. Therefore, since $\mathcal{R}e[s] = 0$, it can be said that steady-state solutions for Generalized Newtonian Fluid rimming flows are neutrally stable to small perturbations. This result holds for many constitutive models, including the simple power-law model as well as the more sophisticated and realistic Ellis and Carreau models.

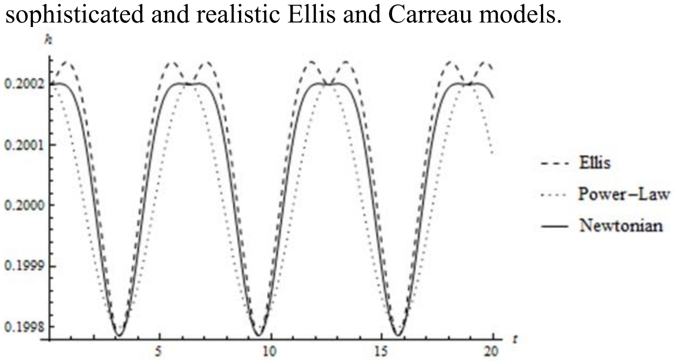

Fig. 2 Numerical solution of equation (19) and (20) for Newtonian, power-law, and Ellis fluids viewed at $\theta = 0$ for $h_0 = 0.2$, $m = 0.15$, $Wi = 4$, and $\epsilon = 0.001$.

To illustrate these neutral stability results numerically, the power-law and Ellis models are considered. For the power-law model, we have $\mu(\dot{\gamma}) = |\dot{\gamma}|^{m-1}$ and $G(x) = |x|^{1/m}$, where m is a flow index [27]. For $m = 1$, the model is Newtonian, and for $0 < m < 1$, the model describes shear-thinning that occurs for polymers at large shear rates [25]. The Ellis model includes a more realistic and gradual transition from Newtonian to shear-thinning behavior as the shear-rate is increased [25]. For the Ellis model, we have $G(x) = x\left(1 + (Wi\,|x|)^{\frac{1}{m}-1}\right)$, where $0 < m < 1/3$ is a flow index and Wi is shear-thinning number [27]. For $Wi = 0$, the model is Newtonian. Equations (19) and (20) are solved with the Newtonian, power-law and Ellis models. The initial condition is given for each model as the corresponding steady-state distribution perturbed by a small periodic disturbance, specifically $\mathrm{h}(\theta, 0) = \mathrm{h_s}(\theta) + \epsilon \mathrm{h_0} \cos\theta$, where ϵ is a small parameter, and $\mathrm{h_0} = \mathrm{h_s}(0)$. Each solution in Fig. 2 appears periodic and has bounded, unchanging amplitude given by the initial perturbation. These computations illustrate the analytically proven neutral stability of the viscous non-Newtonian steady-state rimming flow.

CONCLUSION

To summarize, non-Newtonian effects on rimming flow stability have been investigated theoretically. Shear-thinning has been simulated with the Generalized Newtonian Fluid model. Analytical conclusions have been drawn using a perturbative approach in limiting two-dimensional flow regimes, namely the lubrication limit of $\delta \ll 1$ and $Re \ll 1$. For the GNF model of shear-thinning, all purely viscous steady-states were shown to be neutrally stable, as in the Newtonian case.

Acknowledgments
This research was funded by the NSF award DMS-1156612.

REFERENCES

[1] O'Brien, S. B. G. (1998). A model for the coating of cylindrical light bulbs. *Progress in Industrial Mathematics at ECMI*, 98, 45-54.
[2] Throne, J. L., & Gianchandani, J. (1980). Reactive rotational molding. *Polymer Engineering & Science*, 20(13), 899-919.
[3] Melo, F. (1993). Localized states in a film-dragging experiment. *Physical Review E*, 48(4), 2704.
[4] Debler, Walter R. and Yih, Chia, Shun. (1962). Formation of Rings in a Liquid Film Attached to the Inside of a Rotating Cylinder. *Journal of the Aerospace Sciences*, 29(3), p. 364.
[5] Eres, M. H., Schwartz, L. W., & Roy, R. V. (2000). Fingering phenomena for driven coating films. *Physics of Fluids*, 12, 1278.
[6] Thoroddsen, S. T., & Mahadevan, L. (1997). Experimental study of coating flows in a partially-filled horizontally rotating cylinder. *Experiments in fluids*, 23(1), 1-13.
[7] Tirumkudulu, M., & Acrivos, A. (2001). Coating flows within a rotating horizontal cylinder: Lubrication analysis, numerical computations, and experimental measurements. *Physics of Fluids*, 13, 14.
[8] Moffatt, H. K. (1977). Behaviour of a viscous film on the outer surface of a rotating cylinder. *Journal de mécanique*, 16(5), 651-673.
[9] O'Brien, S.B.G., Gath, E.G. (1990). Location of a shock in rimming flow. *Physics of Fluids*, 10, 1040–1042.

[10] O'Brien, S. B. G. (2002). A mechanism for linear instability in two-dimensional rimming flow. *Quarterly of Applied Mathematics*, 60(2), 283-299.
[11] Villegas-Díaz, M., Power, H., & Riley, D. S. (2005). Analytical and numerical studies of the stability of thin-film rimming flow subject to surface shear. *Journal of Fluid Mechanics*, 541(1), 317-344.
[12] Villegas-Díaz, M., Power, H., & Riley, D. S. (2003). On the stability of rimming flows to two-dimensional disturbances. *Fluid dynamics research*, 33(1), 141-172.
[13] Johnson, R.E. (1990). Coating Flow Stability in Rotational Molding. *Journal of Engineering Science*, 435-449.
[14] Hinch, E. J., Kelmanson, M. A., & Metcalfe, P. D. (2004). Shock-like free-surface perturbations in low-surface-tension, viscous, thin-film flow exterior to a rotating cylinder. *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, 460(2050), 2975-2991.
[15] Badali, D., Chugunova, M., Pelinovsky, D. E., & Pollack, S. (2011). Regularized shock solutions in coating flows with small surface tension. *Physics of Fluids*, 23, 093103.
[16] O'Brien, S. B. G. (2002). Linear stability of rimming flow. *The Quarterly Journal of Applied Mathematics*, 60(2), 201-211.
[17] Hosoi, A. E., & Mahadevan, L. (1999). Axial instability of a free-surface front in a partially filled horizontal rotating cylinder. *Physics of Fluids*, 11, 97.
[18] Benilov, E. S. (2004). Explosive instability in a linear system with neutrally stable eigenmodes. Part 2. Multi-dimensional disturbances. *Journal of Fluid Mechanics*, 501(1), 105-124.
[19] Benilov, E. S. (2006). Does surface tension stabilize liquid films inside a rotating horizontal cylinder? Part 2: Multidimensional disturbances. *Studies in Applied Mathematics*, 116(1), 1-20.
[20] Benilov, E. S., Kopteva, N., & O'Brien, S. B. G. (2005). Does surface tension stabilize liquid films inside a rotating horizontal cylinder? *The Quarterly Journal of Mechanics and Applied Mathematics*, 58(2), 185-200.
[21] Benilov, E. S., & O’Brien, S. B. G. (2005). Inertial instability of a liquid film inside a rotating horizontal cylinder. *Physics of Fluids*, 17, 052106.
[22] Benilov, E. S., O'Brien, S. B. G., & Sazonov, I. A. (2003). A new type of instability: explosive disturbances in a liquid film inside a rotating horizontal cylinder. *Journal of Fluid Mechanics*, 497, 201-224.
[23] Hinch, E. J., & Kelmanson, M. A. (2003). On the decay and drift of free-surface perturbations in viscous thin-film flow exterior to a rotating cylinder. *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, 459(2033), 1193-1213.
[24] Kelmanson, M. A. (2009). On inertial effects in the Moffatt-Pukhnachov coating-flow problem. *Journal of Fluid Mechanics*, 633, 327.
[25] Bird, R. B., Armstrong, R. C., & Hassager, O. (1987). Dynamics of polymeric liquids. Vol. 1: Fluid mechanics.
[26] Fomin, S., Watterson, J., Raghunathan, S., & Harkin-Jones, E. (2002). Steady-State Rimming Flow of the Generalized Newtonian Fluid, *Physics of Fluids,* 14(9)**,** 3350-3353.
[27] Fomin, S., Hashida, T., & Watterson, J. (2003). Fundamentals of steady-state non-Newtonian rimming flow. *Journal of non-newtonian fluid mechanics*, 111(1), 19-40.
[28] Lin, J., and Hwang, C. "Finite amplitude long-wave instability of power-law liquid films." *International journal of non-linear mechanics* 35(5) (2000): 769-777.
[29] Chandrasekhar, S. (1961). Hydrodynamics and Hydromagnetic Stability. Oxford University.