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NEW LOW-REYNOLDS-NUMBER K-E TURBULENT MODEL FOR NATURAL AND MIXED CONVECTION IN ENCLOSED CAVITY

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ABSTRACT

In the paper (Zhang et al. 2007) several RANS turbulent models were evaluated by classic natural convection experiment. It can be seen that V2f turbulent model show the best overall performance compared to the other models in terms of accuracy of temperature, velocity and turbulent kinetic energy. However, all compared turbulent models cannot simulate the pseudo laminar phenomenon in the centre zone of the classic natural convection experiment. Moreover, V2f model needs to solve four equations, and the transport equation for the wall normal stress ($\overline{v^2}$) and the elliptic equation for the relaxation function (f) make the model numerically unstable. Based on the idea of V2f model, an anisotropic model (BV2fAM) was developed. In the BV2fAM model, the $\overline{v^2}$ as the turbulent kinetic intensity normal to streamlines, was still be adopted and computed by local turbulent kinetic energy and local flow characteristic variable instead of the $\overline{v^2}$ transport equation. The developed BV2fAM model was testified by two benchmark cases (classic natural convection in a tall cavity, and mixed convection in a square cavity). It can be seen that the developed BV2fAM model can give much better simulated results of temperature, velocity and turbulent kinetic energy than other compared models. Moreover it can simulate the pseudo laminar phenomenon in the centre zone of the classic natural convection experiment.

INTRODUCTION

Among the 90% people's life time spent in enclosed environment in developed countries, 87% is in building and 3% in transport vehicles [1]. So airflow quality in the enclosed environment is very important for the modern human beings. However, airflow in enclosed spaces can be complicated due to complex flow features such as flow transition and lack of stability. As reviewed by Chen and Jiang [2], the standard k-ε turbulence model [3] has been widely used to study the field distributions of air velocity, temperature, turbulence intensity, relative humidity, contaminant concentration and the air quality within ventilated rooms. However, this model should be restricted to regions of fully turbulent flow, while the near-wall layers of viscous-affected flow are bridged by the use of logarithmic wall-functions. In many situations of room ventilation, velocities are small and wall-bounded flow layers are neither fully turbulent and well developed, nor completely laminar. Turbulence models that account for low-Reynolds-number and near-wall turbulence decay effects should therefore be considered. Murakami et al. [4] proposed a low-Reynolds-number k-ε model including damping effect due to buoyancy in a stratified flow field. However, the temperature profile from the computations was higher than the results given from the experiment. In the research of Costa et al. [5] , a problem of confined, mixed convection air flow generated by two non-isothermal plane wall jets was investigated numerically and experimentally. Eight low-Reynolds-number k-ε turbulence models were comparatively tested, together with a simplified version of the two-layer wall-function model of Chieng and Launder [6]. Even the results from the low-Re model of Nagano and Tagawa [7] were much close to those of experiment. However there are more than 7% error between calculated mean temperature in the central region at mid-height of the cavity and measured value. Zhang et al. [8] investigated the representative airflows in enclosed environments, such as forced convection and mixed convection in ventilated spaces, natural convection with medium temperature gradient in a tall cavity, and natural convection with large temperature gradient in a model fire room. In their research, the RANS turbulence models including the indoor zero-equation model, three two-equation models (the RNG k-ε [9], low Reynolds number *k-ε* [10], and SST *k-ω* models [11]), a three-equation model (V2f model), and a Reynolds-stress model [12], were evaluated. The results reveal that among the RANS models studied, the RNG *k-ε* and a modified V2f model by Davidson et al. [13] perform the best overall in four cases studied. However, studies from the literature [14, 15] show that the transport equation for the wall normal stress $\overline{v^2}$ and the elliptic equation for the relaxation function f make the model numerically unstable.

In the paper, based on the idea of V2f model, an anisotropic model (BV2fAM) was developed. In the BV2fAM model, the $\overline{v^2}$ as the turbulent kinetic intensity normal to streamlines, was still be adopted and computed by local turbulent kinetic energy and local flow characteristic variable instead of the $\overline{v^2}$ transport equation. The developed BV2fAM model was testified by two benchmark cases (classic natural convection in a tall cavity, and mixed convection in a square cavity).

NOMENCLATURE

C_P	[J/kg·K]	Air specific heat at constant pressure
D	[m]	Cavity depth
g	[m/s^2]	Gravitational acceleration
H	[m]	Cavity height
k	[m^2/s^2]	Turbulent kinetic energy
L	[m]	Turbulent space scale
T	[s]	Turbulent time scale
T	[K]	Temperature
u_i	[m/s]	Velocity component
V	[m/s]	Vertical velocity
W	[m]	Cavity width
x	[m]	Cartesian axis direction
y	[m]	Cartesian axis direction
z	[m]	Cartesian axis direction

Special characters

β	[1/K]	Thermal expansion coefficient
ε	[m^2/s^3]	Turbulent kinetic energy dissipation rate
ρ	[kg/m^3]	Density of air
μ	[Pa·s]	Molecular viscosity
μ_t	[Pa·s]	Turbulent viscosity
ν_t	[m^2/s]	Turbulent kinematic viscosity
θ	[-]	Dimensionless temperature

Superscripts

−	Time-averaged quantities
'	Fluctuating quantities

Subscripts

air	Air
cell	Calculation cell
cold	Cold wall
fl	Floor
Hot	Hot wall
in	Inlet
max	Maximum
min	Minimum
ref	Reference condititions
rsm	Root mean square
w	Wall

MODEL DEVELOPMENT

Reynolds (Ensemble) Averaging

Airflow in a enclosed environment is governed by the Navier-Stokes (N-S) equations. For solving N-S equations, there are three methods: direct numerical simulation (DNS), large eddy simulation (LES), and Reynolds-Averaged Navier-Stokes equation (RANS) model. DNS requires a very fine grid resolution to capture the smallest eddies in the turbulent flow. According to the turbulence theory [16], the number of grid points, N, required to describe turbulent motions should be at least N ~ $Re^{9/4}$. In addition, the DNS method requires very small time steps, which makes the simulation extremely long. Neither existing nor near-future personal computers can meet these needs, so the application of DNS for enclosed environment flows is not feasible now or in the near future. LES always requires much more computing time (at least two orders of magnitude longer) than Reynolds-Averaged Navier-Stokes equation (RANS) model for a steady state flow. A more realistic and widely used approach for enclosed environment airflow simulations is by RANS modeling.

The instantaneous turbulent flow field in an enclosed cavity is governed by the conservation laws of mass, momentum and energy. To simplify the buoyancy effects of cavity airflows, we introduce the Boussinesq assumption, which assumes the density of air is constant and considers the buoyancy effects only in the momentum equation. The Reynolds- average method assumes that an instantaneous quantity is decomposed into the mean (ensemble-averaged or time-averaged) and fluctuating components. For the velocity components:

$$u_i = \overline{u}_i + u_i' \tag{1}$$

where $\overline{u}_i$ and u_i' are the mean and fluctuating velocity components (i = 1, 2, 3).

Substituting expressions of this form for the flow variables into the instantaneous continuity, momentum and energy equations and taking a time (or ensemble) average yields the ensemble-averaged continuity, momentum and energy equations. They can be written in Cartesian tensor form as:

$$\frac{\partial}{\partial x_i}(\rho \overline{u}_i) = 0 \tag{2}$$

$$\frac{\partial}{\partial t}(\rho \overline{u}_i) + \frac{\partial}{\partial x_j}(\rho \overline{u}_i \overline{u}_j) = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}(\mu \frac{\partial \overline{u}_i}{\partial x_j} - \overline{\rho u_i' u_j'}) + \rho g_i \beta (\mathrm{T}_{ref} - \overline{\mathrm{T}}) \tag{3}$$

$$\frac{\partial}{\partial t}(\rho C_P \overline{\mathrm{T}}) + \frac{\partial}{\partial x_j}(\rho C_P \overline{\mathrm{T}}) = \frac{\partial}{\partial x_j}(\lambda \frac{\partial \overline{\mathrm{T}}}{\partial x_j} - \overline{\rho u_i \mathrm{T}'}) + S_{\mathrm{T}} \tag{4}$$

where ρ is fluid density; p is pressure; μ is fluid molecular viscosity; t is time; and g_i are gravitational acceleration components, T is air temperature, and S_{T} is the heat source term. The Reynolds stresses, $-\rho\overline{u_i' u_j'}$ and turbulent heat flux $\rho\overline{u_i \mathrm{T}'}$, must be modeled in order to close Equation. A common method employs the Boussinesq hypothesis [17] to relate the Reynolds stresses to the mean velocity gradients:

$$-\rho\overline{u_i' u_j'} = \mu_t (\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}) - \frac{2}{3}\overline{\rho} k \tag{5}$$

$$-\rho\overline{u_i \mathrm{T}'} = \frac{\mu_t}{\mathrm{Pr}_t}\frac{\partial \overline{\mathrm{T}}}{\partial x_i} \tag{6}$$

μ_t is the turbulent viscosity and computed as a function of k and ε or k and ω, and Pr_t is the turbulent Prandtl number. The disadvantage of the Boussinesq hypothesis as presented is that it assumes μ_t is an isotropic scalar quantity, which is not strictly true.

k-ε turbulent model and V2f turbulent model

In the *k-ε* turbulent model, the turbulence kinetic energy, *k*, and its rate of dissipation ε, are obtained from the following transport equations [9,17]:

$$\frac{\partial}{\partial t}(\bar{\rho}k)+\frac{\partial}{\partial x_i}(\rho k\bar{u}_i)=\frac{\partial}{\partial x_j}[(\mu+\frac{\mu_t}{\sigma_k})\frac{\partial k}{\partial x_j}]+G_k+G_b-\rho\varepsilon \quad (7)$$

$$\frac{\partial}{\partial t}(\bar{\rho}\varepsilon)+\frac{\partial}{\partial x_i}(\bar{\rho}\varepsilon\bar{u}_i)=\frac{\partial}{\partial x_j}[(\mu+\frac{\mu_t}{\sigma_\varepsilon})\frac{\partial \varepsilon}{\partial x_j}] + C_{1\varepsilon}\frac{\varepsilon}{k}(G_k+C_{3\varepsilon}G_b)-C_{2\varepsilon}\bar{\rho}\frac{\varepsilon^2}{k} \quad (8)$$

In these equations, G_k represents the generation of turbulence kinetic energy due to the mean velocity gradients. G_b is the generation of turbulence kinetic energy due to buoyancy. $C_{1\varepsilon}$, $C_{2\varepsilon}$, and $C_{3\varepsilon}$ are constants. σ_k and σ_ε are the turbulent Prandtl numbers for *k* and ε, respectively. The turbulent viscosity, μ_t, is computed by combining *k* and ε as follows:

$$\mu_t=\bar{\rho}C_\mu\frac{k^2}{\varepsilon} \quad (9)$$

where C_μ is a constant.

The V2f model is similar to the standard *k*-ε model, but incorporates near-wall turbulence anisotropy and non-local pressure-strain effects. Besides k and ε equations, the $\overline{v^2}$ and relaxation function *f* equations are as follows [13,14]:

$$\frac{\partial}{\partial x_j}(\bar{u}_j\overline{v^2})=\frac{\partial}{\partial x_j}[(\nu+\nu_t)\frac{\partial\overline{v^2}}{\partial x_j}]+kf-6\frac{\overline{v^2}}{k}\varepsilon \quad (10)$$

$$L^2\frac{\partial^2 f}{\partial x_i\partial x_j}-f-\frac{1}{T}[(C_1-6)\frac{\overline{v^2}}{k}-\frac{2}{3}(C_1-1)]+C_2\frac{P_k}{k}=0 \quad (11)$$

where

$$P_k=\nu_t(\frac{\partial\bar{u}_i}{\partial x_j}+\frac{\partial\bar{u}_j}{\partial x_i})\frac{\partial\bar{u}_i}{\partial x_j},\quad T=\max\{\frac{k}{\varepsilon},6(\frac{\nu}{\varepsilon})^{1/2}\},$$

$$L=C_L\max\{\frac{k^{3/2}}{\varepsilon},C_\eta(\frac{\nu^3}{\varepsilon})^{1/4}\}$$

The turbulent kinematic viscosity is computed from

$$\nu_t=C_\mu\overline{v^2}T \quad (12)$$

The distinguishing feature of the V2f model is its use of the velocity scale, $\overline{v^2}$, instead of the turbulent kinetic energy, *k*, for evaluating the eddy viscosity $\overline{v^2}$, which can be thought of as the velocity fluctuation normal to the streamlines, has shown to provide the right scaling in representing the damping of turbulent transport close to the wall, a feature that *k* does not provide. However, this model exhibits the numerical stiffness problem in a segregate solution procedure, such as the SIMPLE algorithm, which requires remedy [18].

New low-Reynolds-number k-ε turbulent model

Based on the idea of V2f model, an anisotropic model (BV2fAM) was developed. In the BV2fAM model, the $\overline{v^2}$ as the turbulent kinetic intensity normal to streamlines, was still be adopted and computed by local turbulent kinetic energy and local flow characteristic variable instead of the $\overline{v^2}$ transport equation. According to the Equation 4, the normal component of Reynolds stress can be written as shown in Equation 13.

$$\overline{u_1'^2}=\frac{2}{3}k-2\nu_t\frac{\partial\bar{u}_1}{\partial x_1},\quad \overline{u_2'^2}=\frac{2}{3}k-2\nu_t\frac{\partial\bar{u}_2}{\partial x_2},\quad \overline{u_3'^2}=\frac{2}{3}k-2\nu_t\frac{\partial\bar{u}_3}{\partial x_3} \quad (13)$$

So the streamline Reynolds stress component $\overline{s^2}$ can be written as follows:

$$\overline{s^2}=(\overline{u_1'^2}\vec{e}_1+\overline{u_2'^2}\vec{e}_2+\overline{u_3'^2}\vec{e}_3)\bullet\vec{V}/|\vec{V}| \quad (14)$$

where $\vec{V}$ is local airflow velocity.

$\overline{v^2}$ as the turbulent kinetic intensity normal to streamlines can be written as follows:

$$\overline{v^2}=2k-\overline{s^2} \quad (15)$$

The turbulent viscosity is computed from

$$\nu_t=\min\{C_\mu^*\frac{k^2}{\varepsilon},\ 0.22\overline{v^2}T\} \quad (16)$$

where

$$T=\min\{0.2\frac{L}{\sqrt{k}},\ \max[\frac{k}{\varepsilon},\ 6(\frac{\nu}{\varepsilon})^{1/2}]\},$$

$$L=C_L\min\{\frac{k^{3/2}}{\varepsilon},\ V_{Cell}^{1/3}\},$$

$$C_\mu^*=C_\mu*(1-\exp(-\frac{y^*}{14}))*(1.0-0.0286^{Frt}),$$

$$Frt=\frac{\varepsilon}{k\sqrt{g\beta\, dT/dx_g}},\ C_\mu=0.09,\ C_L=0.3.$$

Here, *g* is gravitational acceleration, β is thermal expansion coefficient, dT/dx_g is temperature gradient in the gravitational direction, and $V_{Cell}^{1/3}$ is the volume of the calculation cell.

TURBULENT MODEL TEST AND ANALYSIS

The developed BV2fAM model was testified by two benchmark cases, one is classic natural convection in a tall cavity, and another is mixed convection in a square cavity.

Natural convection in a tall cavity

Betts and Bokhari [19] conducted an experimental investigation of natural convection in a tall cavity, as shown in Figure 1. The dimensions of the cavity were 2.18m high × 0.076m wide × 0.52m deep. The cold and hot walls had uniform temperatures of 15.1°C and 34.7°C, respectively. The Rayleigh number based on the cavity width was 0.86×10^6. The numerical results presented were based on a 25×150×60 nonuniform three-dimensional grid for three RANS turbulent models (RNG k-ε, V2f, and BV2fAM).

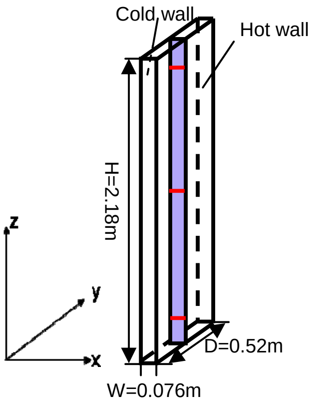
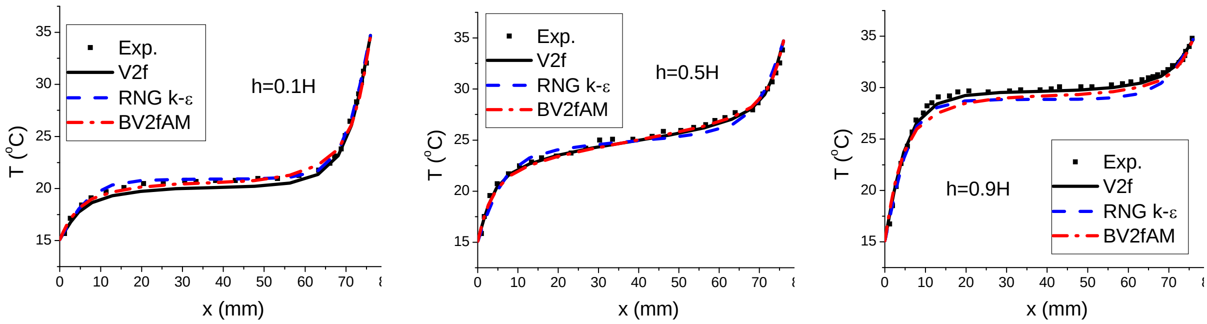
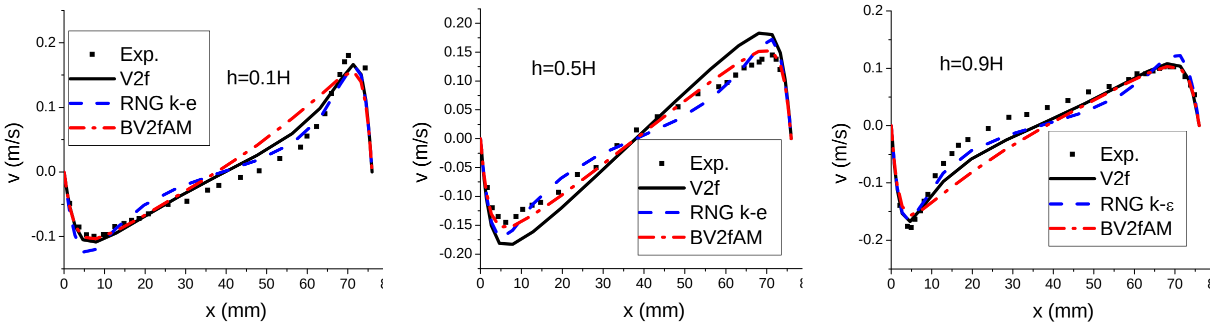


Figure 1 Natural convection in a tall cavity (Ref. [19])

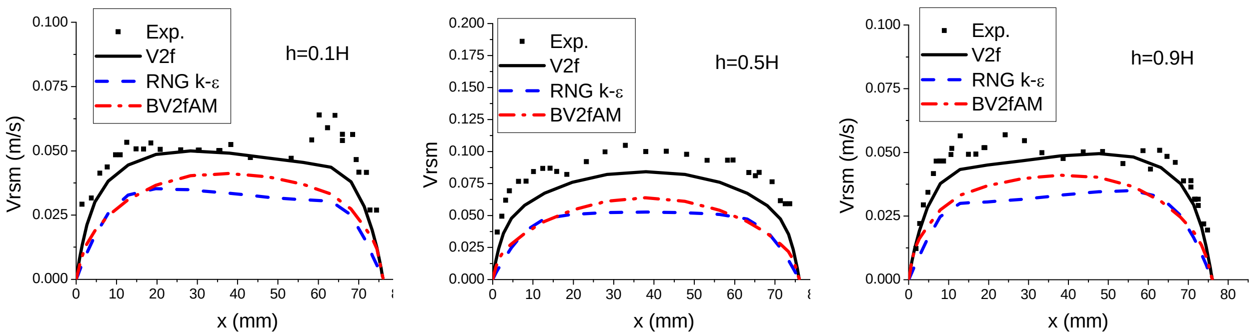
Figure 2 compares the simulated results with the measured data at mid-depth of the cavity. From Figure 2(a), it can be seen that above three RANS turbulent models can simulate the temperature field well. For vertical velocity field shown in Figure 2(b), above three RANS turbulent models, each has its own merits and demerits near the top and bottom wall. However, the BV2fAM model achieved a better accuracy than RNG k-ε and V2f models in the center zone of the cavity. In center zone of the cavity, the flow becomes pseudo-laminar in which the turbulence intensity is large shown in Fig. 2(c) and velocity is close to zero. So the BV2fAM model can reappear the pseudo laminar phenomenon.



(a) Temperature



(b) Vertical velocity



(c) rms vertical velocity fluctuation

Figure 2 Comparison of experimental data and numerical simulation for natural convection in the tall cavity on red lines in Fig.1

Mixed convection in a square cavity

Mixed convection is the most common airflow form in an air-conditioned environment. Blay et al. [20] studied a mixed-convection flow in a square cavity, as shown in Fig. 3. Experiments were performed on a laboratory model composed of a 1040×1040×700 mm cavity equipped with one inlet slot (18 mm wide) and one outlet slot (24 mm wide). The measured inlet conditions were u_{in}= 0.57 m/s; $v_{in} = w_{in} = 0$; T_{in} = 15°C; $\varepsilon_{in} = 0$; and $k_{in} = 1.25 \times 10^{-3}$ m^2/s^2. The wall temperature, T_w , was 15°C and the floor temperature, T_{fl} , was 35.5°C. The Reynolds number based on the inlet condition was 684. The air properties at the reference temperature T_{mean} = 298 K are respectively ρ_{air}= 1.2 kgm^{-3} for density, ν_{air} =1.5×10-5 m^{2}s^{-1} for the kinematic viscosity. The numerical results presented were based on a 60×60×30 nonuniform three-dimensional grid for three RANS turbulent models (RNG *k*-ε, V2f, and BV2fAM).

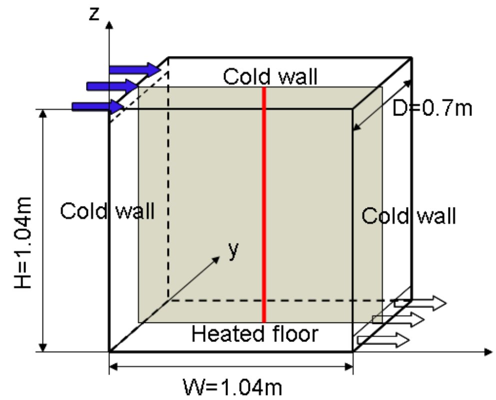
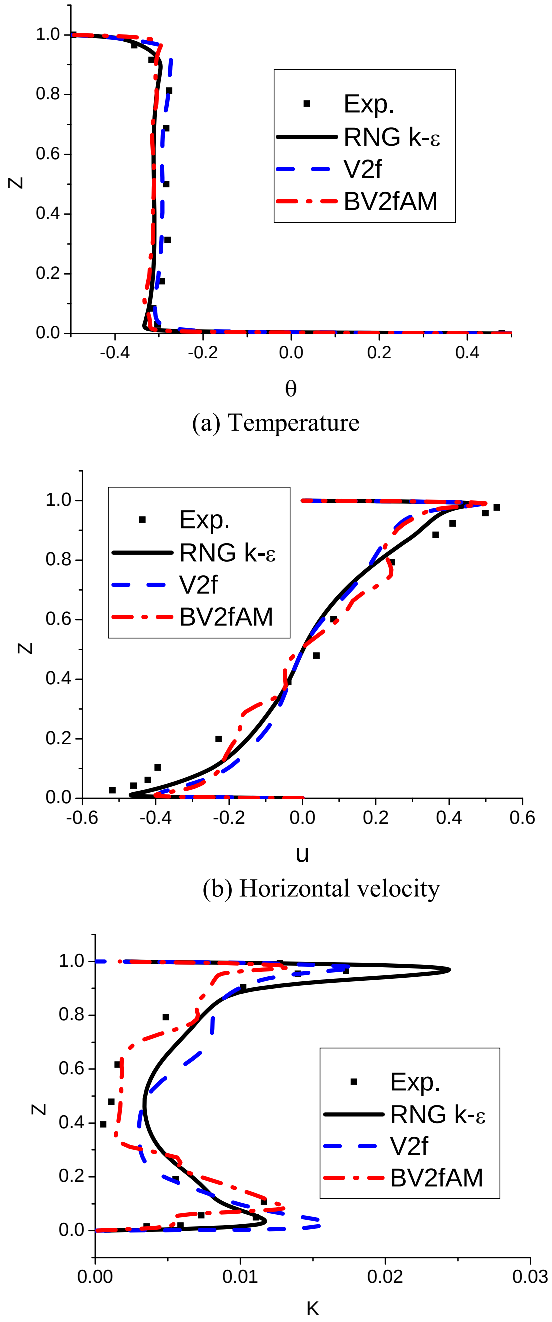


Figure 3 Mixed convection in a square cavity (Ref. [20])

Figure 4 compares the simulated results with the measured data on the centre line at mid-width profile of the cavity shown in Figure 3. In the Figure 3, the ordinate is dimensionless by using the cavity height, H, as the reference length. The dimensionless temperature θ is defined as $\theta = (T-T_{mean})/(T_{Hot}-T_{Cold})$, the horizontal velocity and turbulent kinetic energy are made dimensionless by the mean supply air velocity, u_{in}, as reference velocity. From Figure 4(a), it can be seen that above three RANS turbulent models are able to reproduce the fact that the core of the cavity remains almost isothermal, and the agreement with the vertical experimental temperature profile is excellent. For horizontal velocity field shown in Figure 4(b), above three RANS turbulent models each has its own merits and demerits near the top and bottom wall. However, the BV2fAM model achieved a better accuracy than RNG k-ε and V2f models in the centre zone of the cavity. For the turbulent quantities on Figure 4(c), it can be seen that the turbulence is quite low in the central part of the cavity but significant near the walls and corresponds to a main flow encircling the isothermal and quiet core region. The BV2fAM model's results reproduce the evolution of the experimental profiles, but generally under-predict the turbulence level, while RNG k-ε and V2f model's profiles are not in a good agreement with the experimental ones.



(a) Temperature

(b) Horizontal velocity

(c) Turbulent kinetic energy

Figure 4 Comparison of experimental data and numerical simulation for mixed convection in square cavity on red lines in Figure 3

CONCLUSIONS

Based on the idea of V2f model, an anisotropic turbulent model (BV2fAM) was developed. In the BV2fAM model, the $\overline{v^2}$ as the turbulent kinetic intensity normal to streamlines, was still be adopted and computed by local turbulent kinetic energy and local flow characteristic variable instead of the $\overline{v^2}$ transport equation. The developed BV2fAM model was testified by two benchmark cases (classic natural convection in a tall cavity, and mixed convection in a square cavity). It can be seen that the developed BV2fAM model can give much better simulated results of temperature, velocity and turbulent kinetic energy than other compared models. Moreover it can simulate the pseudo laminar phenomenon in the center zone of the classic natural convection experiment. However, further investigations are necessary to improve the simulation accuracy of turbulent kinetic energy and velocity near wall zone. The stability of the BV2fAM model remains to be further improved.

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